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MTH2600 1- Elementary Number Theory

# QUIZ 1.

## Problem 1. $123x + 360y = 99$

Euclid Algorithm:

$360 = 2 \times 123 + 114.$
$123 = 1 \times 114 + 9$
$114 = 12 \times 9 + 6$
$9 = 1 \times 6 + 3$
$6 = 2 \times 3 + 0.$

$\Rightarrow \gcd(123, 360) = 3 = d$

And $3 | 99 \Rightarrow$ solution exists.

$\Rightarrow$

$3 = 9 - 6 \times 1$
$= 9 - (114 - 9 \times 12) \times 1.$
$= 9 - 114 + 9 \times 12$
$= 9 \times 13 - 114$
$= (123 - 1 \times 114) \times 13 - 114$
$= 13 \times 123 - 13 \times 114 - 114$
$= 13 \times 123 - 14 \times 114.$
$= 13 \times 123 - 14 \times (360 - 2 \times 123)$
$= 13 \times 123 - 14 \times 360 + 28 \times 123$

$\Rightarrow 3 = 41 \times 123 - 14 \times 360$

$\Rightarrow 99 = 1353 \times 123 - 462 \times 360.$

$\Rightarrow \begin{cases} x_0 = 1353 \\ y_0 = -462 \end{cases}$

General solution: $\begin{cases} x = x_0 + \frac{b}{d}t \\ y = y_0 - \frac{a}{d}t \end{cases} \Rightarrow \begin{cases} x = 1353 + \frac{360}{3}t \\ y = -462 - \frac{123}{3}t \end{cases}$

$\Rightarrow \begin{cases} x = 1353 + 120t \\ y = -462 - 41t \end{cases} (t \in \mathbb{Z}).$

## Problem 2.

Since 19 is prime, by Wilson's theorem,

$$(19-1)! \equiv -1 \pmod{19}$$
$$\Rightarrow 18! \equiv -1 \pmod{19}$$
$$\Rightarrow 18 \times 17! \equiv -1 \pmod{19}$$
$$\Rightarrow (-1) \times 17! \equiv -1 \pmod{19} \quad (\because 18 \equiv -1 \bmod 19).$$
$$\Rightarrow -17! \equiv -1 \pmod{19}$$
$$\Rightarrow 17! \equiv 1 \pmod{19}.$$

Thus, the remainer is 1.