1

GRAPHS AND NETWORKS

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1

Disclaimer

2

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- Slides in this course courtesy of
  - Dr. Abish Malik (Purdue)
  - Dr. Yun Jang (Sejong Univ.)
  - Dr. Ross Maciejewski (ASU)
  - Dr. Niklas Elmqvist (UMD)
  - Dr. David Ebert (Purdue)

2

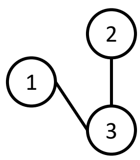
Connections

- Past lectures, we've talked about multivariate data
  - Time series (line plots, cyclical, control chart, prediction)
- In these data, we're seeing connections between events, locations and time
- However, there are other ways to represent these sorts of connections
- We can model a set of connections as a graph

What is a Graph?

- **Vertices** (nodes) connected by **Edges** (links)
- We have several ways to represent a graph
  - Adjacency list

Adjacency List
1:3
2:3
3:1,2



  - Adjacency matrix

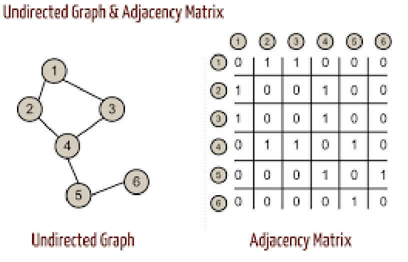
	1	2	3
1	0	0	1
2	0	0	1
3	1	1	0

Graph Representations

5

- Adjacency list representation is usually preferred
  - The most compact way to represent *sparse* graphs

- Adjacency matrix representation may be preferred
  - A highly connected graph



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Graph Terminology

6

- Graphs can have ?? cycle
  - You can follow a path from one node and return back to the same node
- Graph edges can be *directed (uni-, bi-)* or *undirected*
- The ?? degree is the number of ?? edge connected to it
  - For a directed graph, you have an *In-Degree* and an *Out-Degree*
- Graph edges can have values (weights) on them and these values can be measures like we've discussed in our multivariate data (nominal, ordinal or quantitative)

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Some details from this slide borrowed from John Stasko's Graph lecture: http://www.cc.gatech.edu/~stasko/7450/Notes/graph1.pdf

6

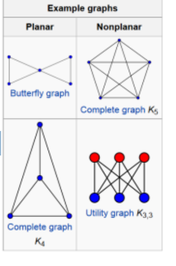
Graph Drawing Research Overview

7

- There is a whole research community that focuses on the creation of graph layout techniques:
- http://www.graphdrawing.org/
- **1970s**
  - Begin to visualize graphs on a computer
  - *Key Ideas*:
    - Aesthetics in the graph layouts
    - Can we mathematically optimize these "aesthetics"???

7

Graph Drawing



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- **1980s**
  - Exciting algorithms and geometry
    - Many fundamental graph layout algorithms were developed
  - Planarity became a key concept
    - planar
    - A ?? is a graph that can be embedded in a plane so that no edges intersect
    - The problem of determining if a graph is planar can be solved in linear time with a simple algorithm
  - Fary's Theorem (1948) – any simple planar graph can be drawn without crossings so that its edges are straight line segments

Fáry, István (1948), "On straight-line representation of planar graphs", *Acta Sci. Math. (Szeged)* **11**: 229–233

8

Graph Drawing

9

- **1990s**
  - Graph drawing matured as a discipline with the advent of the graph drawing conference
  - Information visualization emerges as a discipline
- **2000s**
  - More demand for graph drawing
  - Data volumes become larger – data deluge
  - More usable products, faster computers
  - New customers (social networks, systems biology, security)

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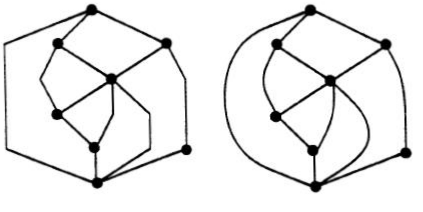
How to Draw a Graph?

10

- Various graphic standards for the representations
  - Vertices: typically circles or boxes
  - Edges: a simple curve
- So, a graph drawing algorithm
  - takes edges and vertices of a graph
  - outputs a pictorial representation based on some algorithmic graphic standard



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How to Draw a Graph?

11

- A graph has infinitely many ways to be laid out
- What we are concerned
  - the usefulness of the drawing
  - the capability of conveying the meaning and relationships
- These are often expressed by means of aesthetics
  - A fundamental and classic aesthetic
    - the minimization of ?? edge crossings
  - Might want to ?? on the screen that the graph takes up minimize the area
  - May want to optimize displays of ?? symmetry

P Eades and R Tamassia, "Algorithms for Drawing Graphs: An Annotated Bibliography," *Technical Report No. CS-89-09*, Brown University, October 1989.

11

Aesthetic Considerations

12

- ?? – minimize towards planar
- ?? – minimize towards proper scale
- ?? – minimize towards efficiency
- ?? – minimize longest edge
- ??– minimize variance
- ??– minimize orthogonal towards straight-line

crossings
total edge length
area
maximum edge length
uniform edge length
total bends

Some details from this slide borrowed from John Stasko's Graph lecture: http://www.cc.gatech.edu/~stasko/7450/Notes/graph1.pdf

12

Experimenting with Aesthetics-Based Graph Layout

13

- Automatic graph layout algorithms are typically judged
  - Speed
  - Extent to which they conform to an aesthetic
  - E.g., minimized crossings, maximized symmetry, etc.
- However, we want to understand if humans can use these
  - Helen Purchase experiments
    - showed that ?? is the most important aesthetic to consider

reducing the number of crossing

Purchase, H.C.; Carrington, D.; Allder, J.-A. Experimenting with aesthetics-based graph layout. In: Anderson, M.; Cheng, P.; Haarslev, V., editors. Vol. LNAI 1889, Theory and Application of Diagrams Conference: Springer Verlag; 2000. p. 498-501

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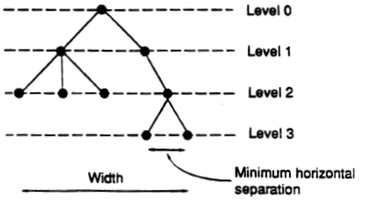
Rooted Trees

14

- A graph might be used to represent some hierarchy, so we often utilize a tree metaphor
- Typically these utilize the following aesthetics
  - Vertices are placed along horizontal lines according to their level (distance from root)
  - There is a minimum separation distance between two consecutive vertices on the same level
  - The width of the drawing is as small as possible



P Eades and R Tamassia, "Algorithms for Drawing Graphs: An Annotated Bibliography," *Technical Report No. CS-89-09*, Brown University, October 1989.

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Free Trees

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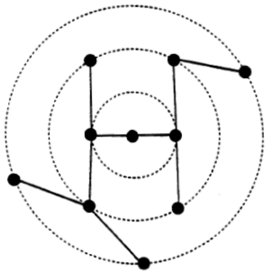
- We can have trees that do not represent hierarchy
- Modify rooted trees to produce radial drawing by arranging the vertices of each level on a concentric circle
- It is NP-complete to construct a planar orthogonal grid drawing of a tree such that the maximum edge length is minimized



P Eades and R Tamassia, "Algorithms for Drawing Graphs: An Annotated Bibliography," *Technical Report No. CS-89-09*, Brown University, October 1989.

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Planarity Testing

16

line crossing

- Clearly, one aesthetic is to remove ??, thus planarity is a desirable graph feature
- This is a well-studied problem, and many methods exist that run in O(n) time where n is the number of edges (or vertices) in a graph
  - *Classic path addition method*[1]
  - *PQ tree vertex addition method*[2]
  - *Edge addition method*[3]

1 - Hopcroft, John; Tarjan, Robert E. (1974), "Efficient planarity testing", *Journal of the Association for Computing Machinery* **21** (4): 549–568

2 - Lempel, A.; Even, S.; Cederbaum, I. (1967), "An algorithm for planarity testing of graphs", in Rosenstiehl, P., *Theory of Graphs*, New York: Gordon and Breach, pp. 215–232.

3 - Boyer & Myrvold (2004), p. 243: "Its implementation in LEDA is slower than LEDA implementations of many other O(*n*)-time planarity algorithms."

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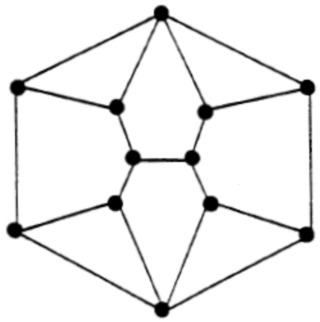
Straight Line Graph Drawings

17

- Tutte's Algorithm
  - The result is a non-crossing, convex drawing



William T. Tutte. How to draw a graph. *Proc. London Math. Society*, 13(52):743–768, 1963.

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Orthogonal Grid Drawings

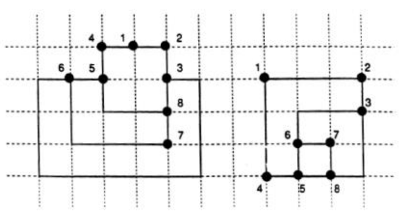
18

- Planar orthogonal grid drawings were first studied in connection with circuit layout
- With this graphic standard, goal is to minimize the ?? (good for readability and for VLSI)

number of bends

degree at most 4

- Any planar graph of ?? admits a planar orthogonal grid drawing with area $O(n^2)$



P Eades and R Tamassia, "Algorithms for Drawing Graphs: An Annotated Bibliography," *Technical Report No. CS-89-09*, Brown University, October 1989.

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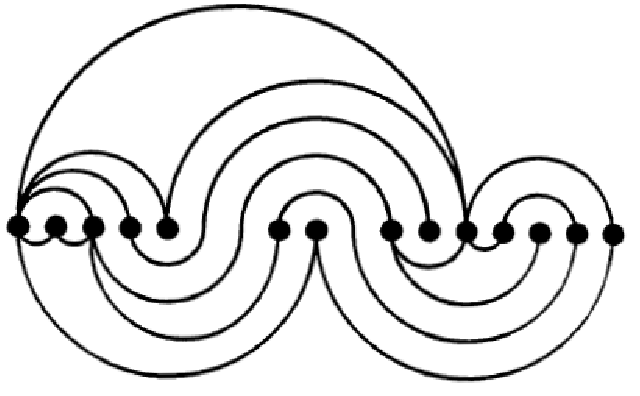
Planar Drawings on a Line

19



P Eades and R Tamassia, "Algorithms for Drawing Graphs: An Annotated Bibliography," *Technical Report No. CS-89-09*, Brown University, October 1989.

19

Force-Directed Methods

20

- Use a physical analogy to draw graphs
- View a graph as a system of objects with forces acting between them
- Assumption is that a balanced system gives a good layout
- Specifically, a system configuration with locally minimum energy
  - The sum of forces on each object is zero
- https://observablehq.com/@d3/force-directed-graph

20

Force-Directed Methods

21

- Consider each node/vertex to be the object of the system
  - Nodes interact with each other based on some forces
  - links do not interact with each other, instead, they add new forces to each node
- Equilibrium configuration is when
  - the sum of forces on each node is zero
- Methods typically have two parts
  - Model – a force system defined by nodes and links
  - Algorithm – a technique for finding equilibrium

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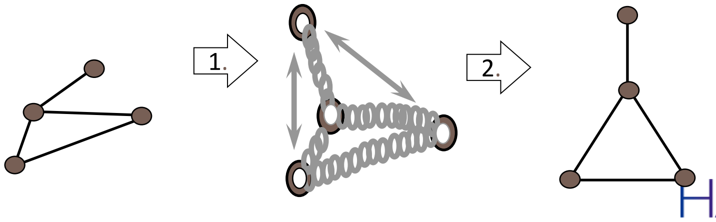
Spring Methods

22

- Node
  - Thought of as an electrically charge particle that repels each other
- Link
  - Thought of as a spring that connects the particles
  - So there is an attraction force when longer than the natural length, and repulsion when shorter



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Springs and Electrical Forces

23

- Use a combination of spring & electrical forces
  - Node: modeled as spring
  - Link: equally charged particles which repel each other
- The force on *v* : *F(v)* = $\Sigma\ f_{u,v} + \Sigma\ g_{u,v}$
- $f_{u,v}$: force on *v* by the spring between *u* and *v*
  : follow Hook's law (proportional to the difference between the distance between *u* and *v* and the zero-energy length of the spring)
- $g_{u,v}$: Electrical repulsion exerted on *v* by vertex *u*
  : follow inverse square law

23

Springs and Electrical Forces

24

- *d(p,q)* : Euclidean distance between points *p* and *q*
- $p_v = (x_v, y_v)$: position of vertex *v*
- *x* component of the force on *v*

$$\sum_{(u,v)\in E} k^1_{uv}(d(p_u,p_v)-l_{uv})\frac{x_v-x_u}{d(p_u,p_v)} + \sum_{(u,v)\in V\times V}\frac{k^2_{uv}}{(d(p_u,p_v))^2}\cdot\frac{x_v-x_u}{d(p_u,p_v)}$$

$l_{u,v}$: natural (zero energy) length of the spring between *u* and *v*

: if the spring has natural length $l_{u,v}$, no force is exerted;

$k^1_{u,v}$: stiffness of the spring between *u* and *v*

: the larger $k^1_{u,v}$, the more tendency for the distance between *u* and *v* to be close to $l_{u,v}$.

$k^2_{u,v}$: the strength of the electrical repulsion between *u* and *v*

24

Springs and Electrical Forces

25

Aim of the force model design:

- Spring force:
  - Ensure the distance between adjacent nodes u and v is approximately equal to $l_{u,v}$.
- Electrical force:
  - Ensure nodes not too close to each other.
- One may choose parameters l_{uv} $k^1_{u,v}$ $k^2_{u,v}$ to customize for specific applications.

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25

Finding Equilibrium

26

- There are many techniques to find an equilibrium configuration (or minimum energy).
- Simple algorithm
  - Initially at random location
  - At each iteration:
    - Force F(v) on each node is computed
    - Each node v is moved in the direction of F(v) by a small amount proportional to the magnitude of F(v)
  - Stops when equilibrium is achieved or some conditions are met.
- Not the fastest, but allow smooth animation.
- Calculating attractive forces only between neighbors: O(|L|)
- Calculating repulsive forces between all pair of vertices: O(|N|^2)
  - Bottleneck of the algorithm in general

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Force directed methods

27

- Many experiments in this area, but the open problem is, “Why do these methods work?”
  - Many experiments, some theorems
- Theorem: If a graph has the right automorphisms, then there is a local minimum of a spring drawing that is symmetric
- Theories
  - Combinatorial rigidity theory
  - Theory of multidimensional scaling

Details from this slide borrowed from Peter Eades’ “The Future of Graph Drawing – and a rhapsody” lecture: http://sydney.edu.au/engineering/it/~peter/downloads.html

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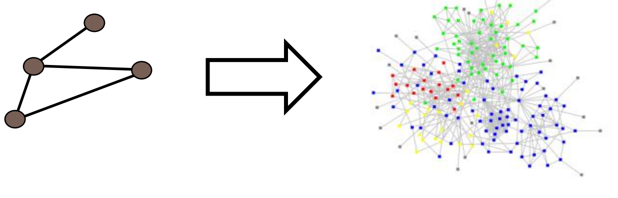
27

Scale Challenge

28

- So, what happens when we get a lot of nodes and edges?
- We run out of space for nodes and links so we get a hairball
- Can slow down the algorithm!



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28

Expanding Graph Drawing

29

- If we want to improve the visual capacity for graph drawing we need to think of extensions
  - 3D
  - Interaction
  - Clustering

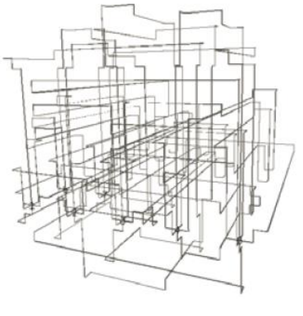
29

3D Graph Drawing

30

- 1990-2005
  - Many theorems on 3D
  - New metaphors introduced
  - New research grants and start-up company
  - However, 3D mostly failed

3D orthogonal drawing of K_7



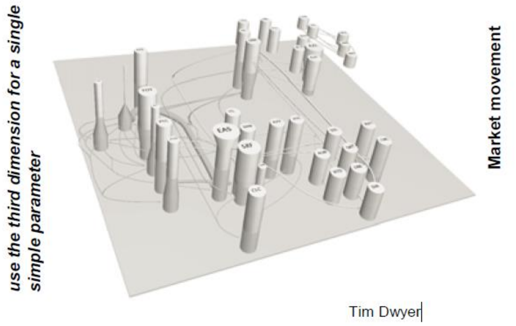
Maurizio Patrignani

30

2.5D Graph Drawing

31

- Colin Ware: "Use 3D with a 2D attitude"
- Tim Dwyer: "use the third dimension for a single simple parameter"



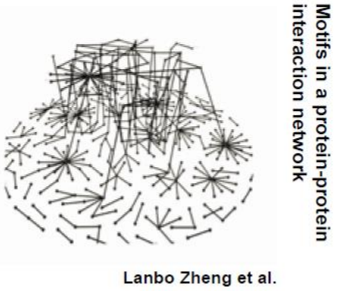
Tim Dwyer and David R. Gallagher. Visualising changes in fund manager holdings in two and a half-dimensions. *Information Visualization*, 3(4):227-244, 2004

31

Multiplane Method

32

- Partition the graph
- Draw each part on a 2D manifold in 3D
- Connect the parts with inter-manifold edges



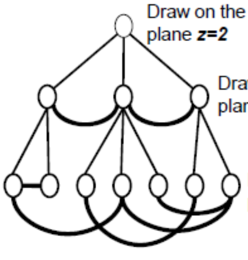
Weidong Huang, Colin Murray, Xiaobin Shen, Le Song, Ying Xin Wu, Lanbo Zheng: Visualisation and Analysis of Network Motifs. IV 2005: 697-702

32

Multilevel Visualization of Clustered Graphs

33

- Draw the clusters on height I of the cluster tree on the plane z=i



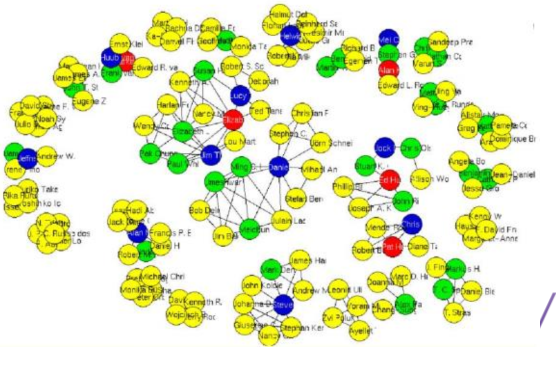
P. Eades and Q. Feng. Multilevel Visualization of Clustered Graphs. In *Proc. 6th Int. Symp. Graph Drawing*, volume 1190 of *LNCS*, pages 101{112. Springer, 1996.

33

Thinking Back on our Visual Variables

34

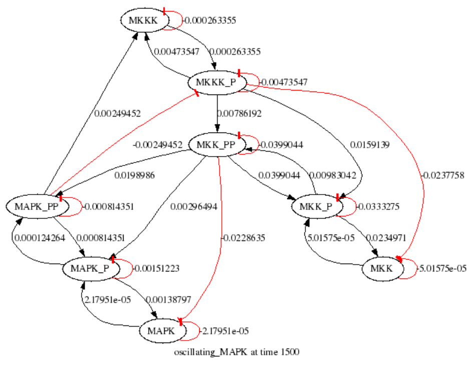
- If we think back onto our early lectures, we need to make decisions on what the appropriate visual variables will be
- For a node, we have
  - Shape
  - Color
  - Size
  - Location
  - Label



34

Thinking Back on our Visual Variables

- For a link, we have
  - Color
  - Size
  - Label
  - Form
    - Polyline
    - Straight
    - Orthogonal grid



Network Visualization Nirvana

- What would be our dream situation for network/graph visualization?
  - ?? Every node is visible
  - ?? For every node we can count its degree
  - ?? For every link...
  - ??

B Shneiderman and A Aris. Network Visualization by Semantic Substrates. *IEEE Transactions on Visualization and Computer Graphics*, 12(5): 733-740, 2006.

Graph Visualization Task Taxonomy

37

- Topology-based tasks
  - ?? Adjacency
    - Find the set of nodes adjacent to a node
  - ?? Accessibility
    - Find the set of nodes accessible to a node
  - ?? Common connection
    - Given nodes, find the set of nodes connected to all
  - ?? Connectivity
    - Find shortest path
    - Identify clusters
    - Identify connected components

B. Lee, C. Plaisant, C. Sims Parr, J.-D. Fekete, N. Henry, "Task Taxonomy for Graph Visualization", Proc. of BELIV '06, April '06, pp. 1-5.
Some details from this slide borrowed from John Stasko's Graph lecture: http://www.cc.gatech.edu/~stasko/7450/Notes/graph1.pdf

37

Graph Visualization Task Taxonomy

38

- Attribute-based tasks
  - ?? On the nodes
    - Find the nodes having a specific attribute value
  - ?? On the edges
    - Given a node, find the nodes connected only by certain kinds of edges
- Browsing tasks
  - ?? Follow path
  - ?? Revisit
- Overview task
  - ?? Compound exploratory task
    - Estimate size of a network
    - Find patterns

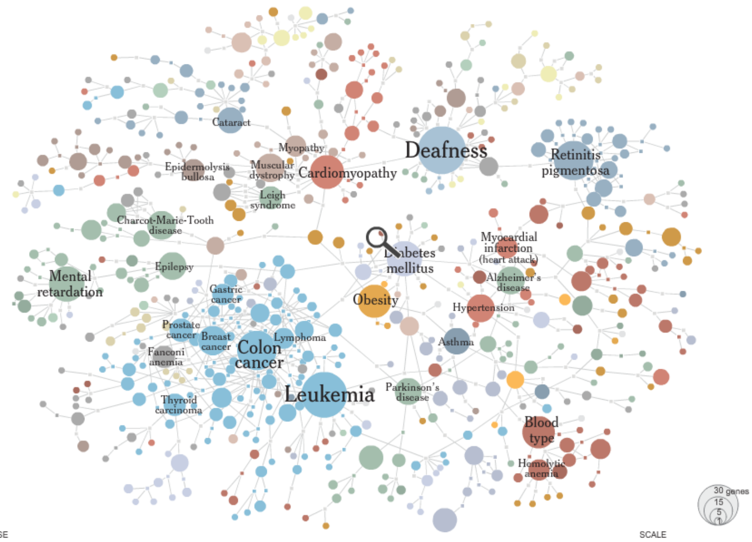
B. Lee, C. Plaisant, C. Sims Parr, J.-D. Fekete, N. Henry, "Task Taxonomy for Graph Visualization", Proc. of BELIV '06, April '06, pp. 1-5.
Some details from this slide borrowed from John Stasko's Graph lecture: http://www.cc.gatech.edu/~stasko/7450/Notes/graph1.pdf

38

39

Mapping the Human 'Diseasome'

Researchers created a map linking different diseases, represented by circles, to the genes they have in common, represented by squares.
Related Article: Redefining Disease, Genes and All



TYPE OF DISEASE

Cancer	Connective tissue	Cardiovascular	Endocrine	Gastrointestinal	Ear, nose, throat	Developmental	Multiple types
Bone	Muscular	Hematological	Immunological	Nutritional	Ophthalmological	Neurological	Unclassified
Skeletal	Dermatological	Renal		Metabolic	Respiratory	Psychiatric	

SCALE

30 genes
15
5
1

Each circle represents a disease or disorder and is scaled in proportion to the number of genes associated with that disease.

http://www.nytimes.com/interactive/2008/05/05/science/20080506_DISEASE.html

39

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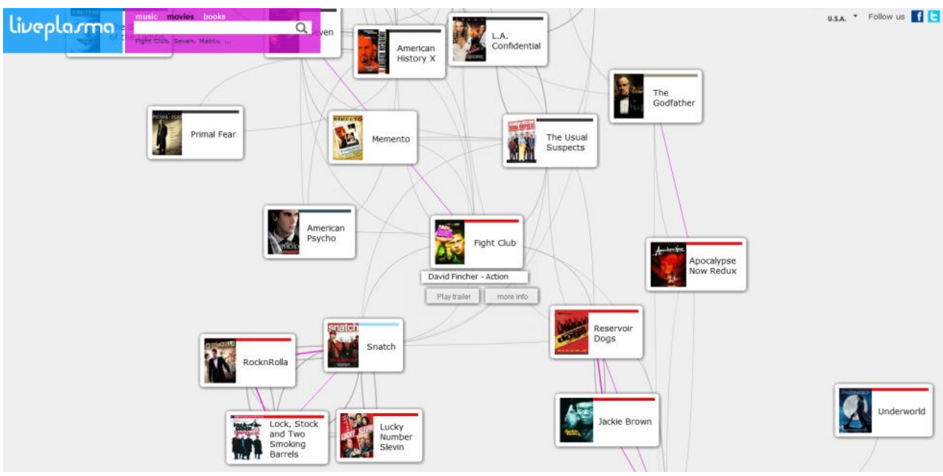

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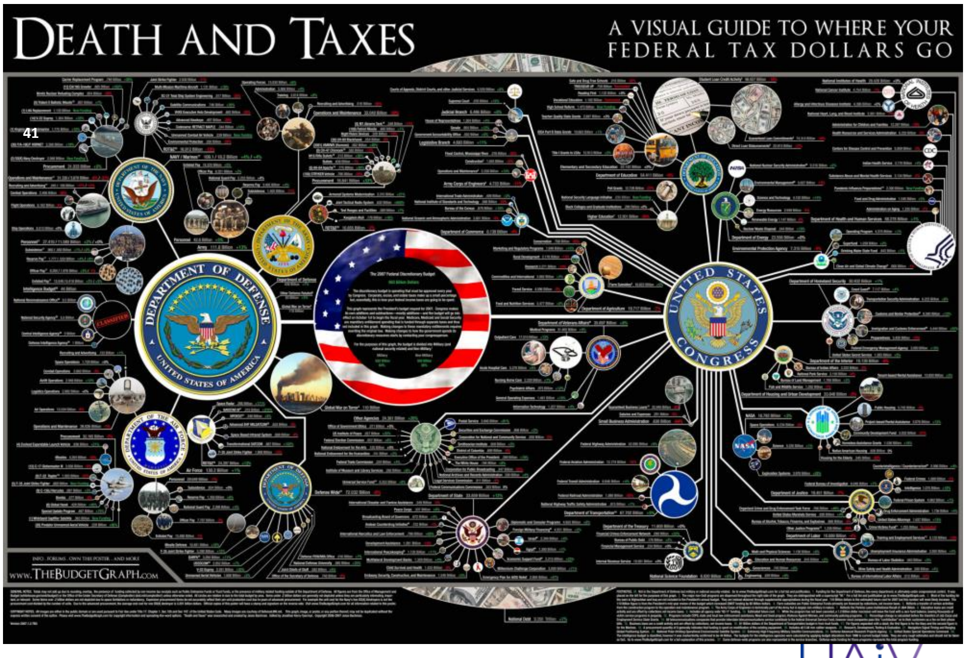
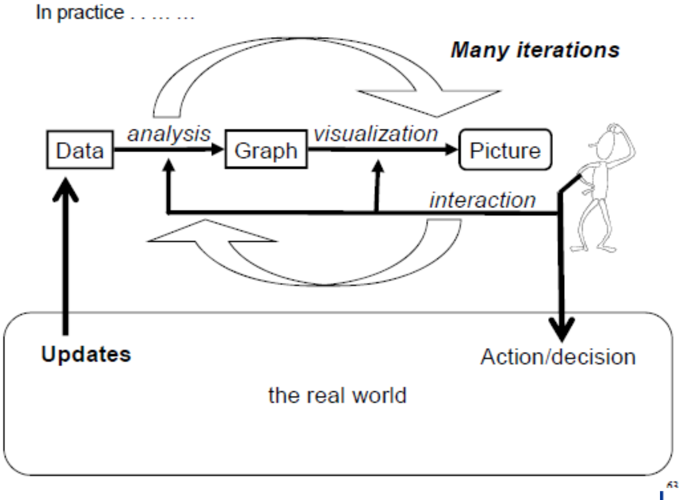


http://www.deathandtaxesposter.com/

41

Interaction Flow in Graphs

42



Details from this slide borrowed from Peter Eades' "The Future of Graph Drawing – and a rhapsody" lecture: http://sydney.edu.au/engineering/it/~peter/downloads.html

42

Using Interaction

43

- By using interaction, we can reduce ??
  - The layout is only computed for the key frame (can be a relatively small graph in comparison)
  - Can use the time that the user is thinking about the visual to compute
- This also reduces the ??
- However, this can increase the ??
  - The user must remember things from key frame to key frame

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43

Implications of Interaction

44

- Drilling down changes the size of a node (zooms in)
- Drilling up is performed by the system, but needs to preserve mental model
- Nodes must move to accommodate these changes in size
- The new picture must also conform to all the aesthetic constraints
- The mental map must be preserved
  - Preserve ordering
  - Preserve proximity
  - Preserve topology

Details from this slide borrowed from Peter Eades' "The Future of Graph Drawing – and a rhapsody" lecture: http://sydney.edu.au/engineering/it/~peter/downloads.html

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44

Readings

45

- Required Reading:
  - Ivan Herman, Guy Melanon, and M. Scott Marshall. 2000. Graph Visualization and Navigation in Information Visualization: A Survey. *IEEE Transactions on Visualization and Computer Graphics* 6, 1 (January 2000), 24-43
  - Adam Perer and Ben Shneiderman: Integrating Statistics and Visualization for Exploratory Power: From Long-Term Case Studies to Design Guidelines, IEEE Computer Graphics & Applications 29, 3 (May/June 2009), 39-51
  - M. Wattenberg, "Visual exploration of multivariate graphs," Proceedings of ACM CHI '06, Montreal, Canada, April 2006, pp. 811-819.

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45