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MTH2600 1 - Elementary Number Theory

QUIZ 1.

Problem 1. $123x + 360y = 99$

Euclid Algorithm:

$$360 = 2 \times 123 + 114.$$

$$123 = 1 \times 114 + 9$$

$$114 = 12 \times 9 + 6 \quad \Rightarrow$$

$$9 = 1 \times 6 + 3$$

$$6 = 2 \times 3 + 0.$$

$$\Rightarrow \gcd(123, 360) = 3 = d$$

And $3 \mid 99 \Rightarrow$ solution exists.

$$3 = 9 - 6 \times 1$$

$$= 9 - (114 - 9 \times 12) \times 1.$$

$$= 9 - 114 + 9 \times 12$$

$$= 9 \times 13 - 114$$

$$= (123 - 1 \times 114) \times 13 - 114$$

$$= 13 \times 123 - 13 \times 114 - 114$$

$$= 13 \times 123 - 14 \times 114.$$

$$= 13 \times 123 - 14 \times (360 - 2 \times 123)$$

$$= 13 \times 123 - 14 \times 360 + 28 \times 123$$

$$\Rightarrow 3 = 41 \times 123 - 14 \times 360$$

$$\Rightarrow 99 = 1353 \times 123 - 462 \times 360.$$

$$\Rightarrow \begin{cases} x_0 = 1353 \\ y_0 = -462 \end{cases}$$

$$\text{General solution: } \begin{cases} x = x_0 + \frac{b}{d}t \\ y = y_0 - \frac{a}{d}t \end{cases} \Rightarrow \begin{cases} x = 1353 + \frac{360}{3}t \\ y = -462 - \frac{123}{3}t \end{cases}$$

$$\Rightarrow \begin{cases} x = 1353 + 120t \\ y = -462 - 41t \end{cases} (t \in \mathbb{Z}).$$

Problem 2.

Since 19 is prime, by Wilson's theorem,

$$\begin{aligned} (19-1)! &\equiv -1 \pmod{19} \\ \Rightarrow 18! &\equiv -1 \pmod{19} \\ \Rightarrow 18 \times 17! &\equiv -1 \pmod{19} \\ \Rightarrow (-1) \times 17! &\equiv -1 \pmod{19} \quad (\because 18 \equiv -1 \pmod{19}) \\ \Rightarrow -17! &\equiv -1 \pmod{19} \\ \Rightarrow 17! &\equiv 1 \pmod{19} \end{aligned}$$

Thus, the remainder is $\frac{1}{19}$.

$$\begin{aligned} P11 - 81 \times 8 &= \\ P11 - 81 \times (P11 \times 1 - 851) &= \\ P11 - P11 \times 81 - 851 \times 81 &= \\ P11 \times P11 - 851 \times 81 &= \\ (851 \times 3 - 032) \times P11 - 851 \times 81 &= \\ 851 \times 88 + 032 \times P11 - 851 \times 81 &= \\ 032 \times P11 - 851 \times 81 &= 8 \quad \Leftrightarrow \\ 032 \times 53 - 851 \times 88 &= 88 \quad \Leftrightarrow \\ 8881 &= 0X \quad \Leftrightarrow \\ 53 \times 165 &= 0^p \end{aligned}$$

$$\begin{cases} 1 \frac{032}{8} + 8781 = X \\ 1 \frac{851}{8} - 521 = Y \end{cases} \quad \Leftrightarrow \quad \begin{cases} 1 \frac{p}{8} + 0X = X \\ 1 \frac{p}{8} - 0Y = Y \end{cases} \quad \text{General solution:}$$

$$\begin{cases} 1 \frac{032}{8} + 8781 = X \\ 1 \frac{851}{8} - 521 = Y \end{cases} \quad \Leftrightarrow \quad \begin{cases} 1 \frac{032}{8} + 8781 = X \\ 1 \frac{851}{8} - 521 = Y \end{cases}$$