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MTH26001- Elementary Number Theory:
     Student ID: 20202026
    Student Name: Nguyen Minh Duc
                                                      ASSIGNMENT 3.
                                                      823 = 9 ( mod H)
      Sec 4.2. . (1 bemar = 83 = 3888 co
              Problem 3. (+ bow) 1 = 111 sss -
  & Since a = b (mod n), a-b=nk (kEZ)
                                                      => a = b+ nkgd aldisioib ei 111 888 + 188111 c.
  @ Note that ged (b, n) | & b and ged (b, n) | n.
          => b+nk =0 (mod gd(6,n)).
       => a = 0 (mod gcd(b,n)).

=> gcd (b,n) | a. } => gcd(b,n) is adivisor of both a and n.

Moreover, gcd(b,n) | n }

=> gcd(b,n) \ (s,n) 
& Similarly, we have: b = a-nk (-: a-b=nk, k \ 2).
      Note that gcd (a, n) 1 b and gcd (a, n) ( n.
                => a -nk = 0 (mod gcd(a,n)).
       => gcd(a,n) 1 b.7 => gcd(a,n) is adivisor of both bond n.

=> qcd(a,n) 1 b.7 => gcd(a,n) is adivisor of both bond n.

Moreover, gcd(a,n) 1 n )

=> gcd(a,n) Sqcd(b,n). -2.
  = 0 (mod qcd(a,n)).
(3) and (2) => gcd(a,n) = qcd(b,n)
           Problem 5.
B Rove 53103 + 10353 =0 (mod 39).
         Note that: 532 = 2704 = 72 x 39 + 1. =>532 = 1 (mod 39)
                                                                                                =>53102 = 1 (mod 39)
                                                                                                =>53103 =53=14 (mod 39).
          Note that: 1032 = 10609 = 272 +39+1 => 1032 = 1 (mod 39)
                                                                                                    => 10352 = 1 (mod 39)
                                                                                                   =>10353 = 103 = 25 (mad 39).
         Thus, 53103+10353 = 14+25 = 39 =0 (mod 39)
                 => 53103 + 10353 is divisible by 39.
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@ Prove 111333+33311 = 0 (mod 7). Note that 1112=12321 = 1760x7+1 => 1112=1 (mod 7) => 411382 = 1 (mod 7) =7 111333 = 111 = 6 (mod 7).

Note that 333 = 47 x7 + 4 => 333 = 4 (mod 7).

=> 333 = 43 (mod 7) =73333 = 64=1 (mod 7). => 333 111 = 1 (mod 7) (mod 7)

:> bink 30 (wood gul(b)N).

Thus, 111333+33311 = 6+1 =7=0 (mod 7) (mlen) den sois => 111333 + 333111 is divisible by7. + d = 1 (= . N I (N, W) by Dans gold (N, d) loop top yell x.

Problem 11.

@ We have the following table: ((Not) loop hom) O. F. o

20 10 1 2 2 = 4 2 = 8 2 = 16 25 = 32 26 = 64 2 = 128 28 = 256 29 = 512 20 mod 11 0 1 2 4 8 5 10 9 7 3 6

All of the "21 mod 11" are distinct and ranging from 0 to 10. => {0, 1,2,2,2,2, ...,2} forms a complete set of residues modulo 11. of any of the of

8. Considering The following table:

| X | 0 | 12 = 1 | 22 = 4 | 32 = 9 | 4 = 16 | 5 = 25 | 62 = 36 | 72 = 49 | 82 = 64 | 92 = 81 | 102 = 100 |
| X mod 11 0 | 1 | 4 | 9 | 5 | 3 | 3 | 5 | 9 | 4 | 1

There are missing 2, 6, 7 and 8 in "x mod 11" row. => {0,12,22,...,102} does not form a complete set of residues modulo 11.

> 3. Hove 53,00 + 103, 23 0 (mod 39). (1000 that: 53 = 2104 = 72x39+4.3753 = 1 (mod 39) 153 powod 1 5 201 83 (c

=) 53 to \$ = 53 = 14 (mod 39).

(Pr Lemo 1 = 1801 (= 1+68+89+1 =) 103 = 1 (cm 1 89) (1-3 benj) 1 E # ? E of (. . C. & Benn 25 E & U & & 2 80) Co

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Problem S.

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Sec 4.3
    Problem 3.
From the hint + 99 = 9 (mod 10) => 99 = 10K+9, KEZ.
                      +9 = (910) × 99 = (9×99) × 2009 EL989 X 189
                                  =(9x89) x89 (mod 100)
                                  =(801) × 89 (mod 100)
                                   = 1 × 89 (mod 100)
  Thus, the last two digits of 99 are 89.
    Problem 10: 201 = 015+ 040
& let N be such integer. Decimal representation of N:
             N = an10"+ an-10"+ ... + a,10 +a0.
  Note that 10=1 (mod 9) => 10x = 1 (mod 9.) + x & M.
  Hence, N= an+an-1+ ... +00 (mod 9).
       . => N = 15 (mod 9) (-: sum of digits of Nisis).
                6 (mod 9).
&. From the hint, a3 = 0,1 or 8 (mod 9) => Any cube is congruent 100,1008
modulo 9. However, N=6 & (mod 9). Thus, N com't be a cube -
& let a & Z, a = 94+r (q & Z, 0 < r (9). =) a = r (mod 9).
  For r=1, a=1 (mod 9)
 For v = 3, a= 32=9=0 (mod 9)
  For 1 = 4, a2 = 42 = 16 = $7(mod 9)
  For r=5, a2=51=25=7(mod9)
  for r=6, a= 36=36=0 (mod 9)
 For r=7, a =7 = 49=4 [mod 9)
 For Y = 81 a2 = 82 = 64 = 1 (mad 9).
Thus, any square is congruent to 0,1,40r7 modulo 9. However, N=6 (mod9).
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=> N can't be a square -- 2)

· From () and (2), N can't be a square or a cube.

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Problem 11.
0. 407000 273x4945 = 273x105+xx04+49x10+5.
  Note that 10 =10 (mod 495) => 10 = 100 (mod 495)
                            => 104 = 10000 = 20 x 495+100 = 100 (mod 49$
     10 = 10 = 10 = 10 ( mod 495).
BHRNCE, 273×4945 = 273×10 + 100x + 49×100 + 10y+5 (mod 495)
             = 100x+104+7635 (mod495)
           100x +10y +210 (mod 495).
a per have to say solla.
                  = x40 +210 (mod 495).
® Since 273x4945 is divisible by 495, 740 +210 =0 (mod 495)?
                               =) 240 +210 = 495K, KEZ.
@ Note that 29 0 ( 240 +210 & 990 +210 = 1200.
           2) 0 5 495 K $ 1200 + ... + " OI ... D + " OI ... D - M
 => KE 1 1, 27. (=: KEZ).

=> KE 1 1, 27. (=: KEZ).

(71 = 2

(ourstradiction)

For K = 1, 7140 + 210 = 495 => 7140 = 285 => 44 = 8 (constradiction)
           => 0 < k 5 4200 = 2.42 (Plane) 1 = 22 40/11
                                    No solution.
    For K=2, 740 +210 = 495x2=990 => 740 = 780 => fx=7 (soristy).
                   module of However, N = 6 @ (mod 9). Thus, N can't b
Therefore, (x, 4) = (7,8).
                                 k lu a 6 Z, a 2 9 Atr (4 6 Z, 0 Gr
8. For a prime p? 3, phas the form of 6k+1 or 6k+5, k & Z.
   Problem 25.
8. Consider: 105 = 7692×13+4 = 4 (mod 13).
          =7 106 = 40 = 1 ( mod (3)
          => 106K = 1 (mod 13)
          =7/106 Ker = 10 cmod (3)
  (dem) = 1/106K+5 = 4 (mod 13).
8 For P=6K+1, 102410+1=102(6K+1)-106K+1+1=101-10+1 (mod 13)
= 0 (mod 13).
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L> 13 / 1028 + 108+1.

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@ For P = 6 K+5, 102-108+1 = 102(6K+5)-106K+5+1 = 42-4+1 (mod 13)
                              = 13 (mod 13)
   L> 13/102P-10P+1.
& There fore, 13/ 102P-10P+1 to for any prime P>3.
                         S Co (Sebano 1-678 V) (Shemo) 1:
   Problem 2 (b) 12x+254 = 331.
                     => \ \ 12x = -254+331 => \ \ 12x = 331 (mod25) (1) \ 254 = -12x + 331 \ \ \ 254 = 331 (mod 12) (2).
& Consider equation (1), 12x = 331 (mod 25).
                  => 12x = 6 (mod 25)
=> 24x = 12 (mod 25)
                    => -2 = 12 (mod 25) (-, 24=-1 (mod 25)).
     (12 ) = 12 = 13 (mod 25) => x= 13+25E, + EZ.
                   => 4 = 7 (mod 12) (-: 25 = 1 (mod 12))
€ Consider equation (2), 25 y = 331 (mod 12).
                   => Y = 7 +125, SEZ.
& Now, we obtain: 12x+25y=331=> 12(13+25t)+25(7+125)=331.
                           (=) 300+ + 3005 + 331 = 331.
(=) ++5 = 0 (=> 5=-t.
       (6886 pom) 4 x612x St + hall x x 16x + (2-)x 665 x S
(8) Greneral solution: \begin{cases} x = 13 + 25t, t \in \mathbb{Z}. \\ y = 7 - 12t \end{cases}
   Problem 3. 320-74=11 (mod 13).
8. Check the solvability: gcd(3, 7, 13) = 1/11.
  => The equation has a solution.
 Ø. 32-74 = 11 (mod 13) => 32 = 11+74 (mod 13).
                         => 12x = 44+28 y (mod 13)
                         =>->1=5+24 (mod 13) (-:12=-1 (mod 13))
     => - 24 (mod 13)
                          => 2 = 8 + 114 (mod 13).
                          401123456789101112
                          2481614121011191753111210
& We have the solution table:
         (mod 13)
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Problem 4.
     (x=1 (mod3) N=3x5x7=105
     x = 2 (mod 5) . N1= 105 = 35, N2= 105 = 21, N3 = 105 = 45.
     (x=3 (mod7)
                              35x = 1 (mod 3)
  Now, we have 3 congruences: 21x = 1 (mod 5)
                              15x=1(mod7)
                                  821x=1 (mod 5)
 0935 x = 1 (mod 3)
  =) -x = 1 (mod 3) (: 35=-1 (mod 3)) => 2 = 1 (mod 5) (::1=1 mod 5)
                                  @ 15x =1 (mod 7)
  =) x =-1 (mod 3)
                                   =) x = 1 (mod 7) (: 15=1 mod 7)
  27 x = 2 (mod 3)
: &Thus, we obtain (x1, x2, x3) = (2,1,1).
 @ Solution: 400 x = 1 x35x2 + 2x21x1 + 3x15x1 (mod 105)
                => x = 157 (mod 105)
                => X = 52 (mod 105).
     (x=5 (mod 11) N=11x29x31=9889

N=14 (mod 29) N=899, N=2341, N=319.

N=15 (mod 31)

341x=1 (mod 29) 319x=1(v
 (b) (x = 5 (mod 11)
                                           319x = 1 (mod 31)
    899 x =1 (mod 11)
                                           =>9x=1(mod 31)
                        2) 22x =1 (mod 29)
  => 8 x = 1 (mod 11)
                                           2)63x =7 (mod 31)
                        2) -7x =1 (mod 29)
                        =) -28 7 = 4 ( mod 29)
                                            => >c = 7 (mod 24)
   => 32×= 4 (med 11)
   C7 84002
   => -x = 4 (mod 11) =>, x = 4 (mod 24)
       >c =-4 (mod 11)
 @ Thus, we obtain (x1,x1,x3) = (-4,4,7).
 © Solution: x = 5 x 899 x(-4) + 14€x 341 x4+ 15 x319x7 (mod 9889)
         =72= 34611 (mod 9889)
                                                s general solution:
         => 1= 4944 (mod 9889).
 (c) (x=5 (mod 6), N=6x11x17=1122.
                    N1=187, N2=102, Ng=66.
      x =4 (mod/1)
     ( )= 3 (mod 17)
                                       66x = 1 (mod 17)
                                       =1-2x=1 (mod 17)
                     102x =1 (mod 11)
    187x=1 (mod 6)
  27 2 = 1 (mod 6) => 37 = 1 (mod 11)
                    =>21x=7 (mod11) =>418x=-9 (mod 17).
                     27-71 = 7 (mod 11) | 27 71 =-9 (mod 17)
                     => > == 7 (mod 11)
 8 Soluri Cn: 71 = 5 x 187 x 1 + 4 x 102 x (-7) + 3 x 66 x (-9) (mod 1122)
 @ Thus, we obtain (x1, x2, x3) = (1, -7, -9)
        2) 763 -3703 (mod 1122)
         =) x = 785 (mod (122).
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(d) 1 \times 1 \pmod{5} = 1 + 1 = 2 \pmod{5} = 1 - 1 = 2 \pmod{5} = 1 \times 1 = 2 \times 1 = 
 154x=1(mods) | 385x=1(modz) | MOX=1(mod7) | tox=1(mod11)
@ Thus, we obtain (11,1/2, x3, x4) = (-1, 1, 3, 3).
   & Solution: 71= -2×154×(-1)+3(385)×1+2×110×3+-+×70×3.(mod 770)
                                                  2) x= 653 (mod 770)
       Problem 11.
& (=>) let's guppose a solution exists.
            let d = gcd (n,m) => { n=dr (dr,s ∈ Z).
           we have: \ 2 = a (mod n) => { x= a+nt (+, k \in \mathbb{Z}).
                                                                                    => a-b = mk-nt.

=> a-b = mk-nt.

=> a-b = mk-nt.

(m=dr).

=> a-b = d(rk-st).

=> a-b = q(d(n,m))(a-b.—1).

=> d | a-b => q(d(n,m))(a-b.—1).
8((2) (et d= g(d(n,m).a)Suppose d(a-b.=) a-b=dk, KEZ.
              8. Since d=qcd (n,m), 3 x0,40: n26+m40 = d.
                                                                                                                                                 => Nx0k+my0k=dk=d-b.
                                                                                                                                                   => m40k+b= a-nx0k.
             => | x = a (mod n) => There is a simultaneous a solution. - 0.
B. Uniqueness: (et y be other solution, i.e. ( ) = a (mod n).

Then, we have: { x= y=a (mod n)}
         then, we held: \ \x = y = a (modn)
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=> \ \ x-4 = 0 (mod n) => \ \ m/x-4 => Lcm(n,m) 1x-4. Property Parks Jelan => 2-4 =0 (mod (cm(n,m)) => x=y (mod lcm(n,m))

attendical a dimension of intermedia to the thomas a sel

gcd(u, w) |a-b @ Thus, From @ and O, & there exists simultaneous solution iff graterium and it is unique under moduto (cm(n,m). Tistame I forced | 11 boxes 1 f K. Th | whom I for a Till | whom I for a the wind for a till | whom I for a the wind for a till | who will be a till | who w

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