Homework Assignment 1

CSE33101 Intro to Algorithms (Spring 2022)

Due: 2022-04-05 11:59 pm

Handwrite your answer to the following questions in English, scan it, and submit it to BlackBoard. **Illegible answers will not be graded** (zero points).

Total 10 points

1. Show that if $2^{n+1} = \Theta(2^n)$ or not by using the definition of O and Ω notation (2 points).

2. Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{26}$ of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \ldots, g_{25} = \Omega(g_{26})$. Partition your list into equivalence classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if (iff) $f(n) = \Theta(g(n))$ (2 points).

$n \lg n$	$2^{2^{n+1}}$	$(\sqrt{2})^{\lg n}$	n^2	$n!$	$(\lg n)!$	$(\lg n)^{\lg n}$	e^n	$4^{\lg n}$
$(\frac{3}{2})^n$	n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$	$(n+1)!$	$\sqrt{\lg n}$	$2^{\sqrt{2 \lg n}}$
$\ln \ln n$	2^n	$n \cdot 2^n$	$n^{\lg \lg n}$	$\ln n$	1	$2^{\lg n}$	n	

3. Let's define a sequence S_1, S_2, S_3, ... by the rule that $S_1 = 1$, $S_2 = 1$, $S_3 = 2$ and every further term is the sum of the proceeding two. Thus, the sequence begins 1, 1, 2, 3, 5, 8, 13, ... If $k = (1 + \sqrt{5})/2$, prove if the following is true or not for all positive integers n by using mathematical induction (2 points).

$$S_n \leq k^{n-1}$$

4. Prove that the number of different triples that can be chosen from n items is precisely $n(n-1)(n-2)/6$ by using mathematical induction (2 points).

5. Think of a process moving from the integer x to y via multiple steps based on the following rules.

   A. The length of each step is nonnegative, and the length of the first and last step is one.

   B. The length of the next step is either one less than, equal to, or one greater than the length of the previous step.

   For example, moving from the integer 10 to 15 takes four steps (i.e., 1+2+1+1). Write a pseudocode algorithm that finds the smallest number of steps when moving from the integer x to y (2 points).