Name: Nguyen Minh Duc

ID: 20202026

HOMEWORK ASSIGNMENT 2.

CSE331 - Intro to algorithm

Problem 1. n = Size of A => $n = r - p + 1$.

⊗ The partition procedure:

```
Partition (A, p, r)
1.    x = A[r]
2.    i = p - 1
3.    for j = p to r-1
4.        if A[j] ≤ x
5.            i = i + 1
6.            exchange A[i] with A[j]
7.    exchange A[i+1] with A[r]
8.    return i + 1
```

⊗ The number of execution times:

- line 1: 1 time
- line 2: 1 time
- line 3: $r-1-p+1 = r-p$ times
- line 4: $r-p$ times
- line 5: At most $r-p$ times ($\because$ when $A[j] \leq x \ \forall j$)
  At least 0 time ($\because$ when $A[j] > x \ \forall j$)
- line 6: At most $r-p$ times ($\because$ when $A[j] \leq x \ \forall j$)
  At least 0 time ($\because$ when $A[j] > x \ \forall j$)
- line 7: 1 time
- line 8: 1 time.

⊗ The total number of ~~execution~~ operations:

- At most: $1+1+(r-p)+(r-p)+(r-p)+(r-p)+1+1 = 4(r-p+1) = 4n$
- At least: $1+1+(r-p)+(r-p)+0+0+1+1 = 2(r-p+1)+2 = 2n+2$

⊗ let $T(n)$ be the running time of Partition(A,p,r):

$$\Rightarrow \begin{cases} T(n) \leq 4n \\ T(n) \geq 2n+2 \end{cases} \Rightarrow \begin{cases} T(n) = O(n) \\ T(n) = \Omega(n) \end{cases} \Rightarrow T(n) = \Theta(n).$$

⊗ Thus, the running time of the PARTITION procedure is $\Theta(n)$.

Problem 2. $N = 10^6 \Rightarrow 100N = 100 \times 10^6 = 100$ million.

⊗ ~~By~~ let X be the number of compares used by Quicksort.

⊗ The desired probability is $P[X > 100N]$.

⊗ By the Chebyshev's inequality, we obtain:

$$P[X - \mu > k\sigma] < \frac{1}{k^2} \Rightarrow P[X > \mu + k\sigma] < \frac{1}{k^2}. \ (1)$$

⊗ Setting $\mu + k\sigma = 100N$, we have:

$$2N\ln N + k(0.65N) = 100N \Rightarrow 2\ln N + 0.65k = 100 \Rightarrow k = \frac{100 - 2\ln N}{0.65} = \frac{100 - 2\ln 10^6}{0.65}$$

$$\Rightarrow k = \frac{100 - 12\ln 10}{0.65}$$

$$\Rightarrow k \approx 111.3369$$

⊛ Plug $k = \dfrac{100 - 12 \ln 10}{0.65} \approx 111.3369$ into (1), we obtain

$$P[X > 100N] < \frac{1}{\left(\frac{100 - 12\ln 10}{0.65}\right)^2} \Rightarrow P[X > 100N] < \left(\frac{0.65}{100 - 12 \ln 10}\right)^2$$

$$\Rightarrow P[X > 100N] < 8.0672 \times 10^{-5}$$

$$\Rightarrow P[X > 100N] < 0.0080672\ \%$$

⊛ Thus, the probability that Quicksort uses more than 100 million compares is less than 0.0080672 %.

Problem #3.