Design and Implementation of Type-II Compensator in DC-DC Switch-Mode Step-up Power Supply

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***Abstract*—DC-DC power supplies have taken place a prime role in various fields of engineering applications. Such applications demand steady, regulated source of voltage irrespective any changes in line and load. The compensators have taken place a key role for maintaining the accurate performance of converters dynamics in DC-DC power supplies. Some DC-DC power-supplies *like* boost (step-up), buck-boost *etc* have a *right-half-plane zero i.e.* there is a problem of *non-minimum phase*, so it is difficult for the PID controller to exhibit good performance. For this reason a special kind of classical controller *i.e. Type-II compensator* is introduced for obtaining better performance. This paper describes the designing of a Type-II compensator for a DC-DC step-up (boost) power supply.**

Keywords—switch-mode-power-supply; step-up converter; non-minimum phase system; Type-II compensator designing;

I. INTRODUCTION

The DC-DC power supplies have been framed an important role almost every domain of engineering applications. The introduction of these kinds of power supplies has increased the technical and economical qualities with extensive area of applications from a few tens of watts to several hundreds of megawatts *like as* battery chargers, laptop/personal computers, microcontroller/DSP kit, power systems, telecommunication system, utility grid, spacecraft, high voltage dc transmission, adjustable motor drives and many others [1][2].

In the early days, the potential dividers (PD's) used to take a key role for control and transfer the dc power from one level to another level to the load. The operations of the PD's are simpler but its applications have been reduced due to several limitations *like as* sufficient energy loss, difficulties for stepping up the load voltage, problems in regulations *etc*. That's why the linear regulators were introduced to make the system energy efficient. The operating principle of linear regulators are almost similar to the PD's, is embedded with load regulation features and the series resistance is replaced by solid state device. In linear regulators the solid state device operates in active zone incurring a significant amount of power losses across it. So, heat sink is required to dissipate the generated heat. Due to the presence of heat sinks, and line-frequency transformer the size and the weight of the power supply is bulky in nature and it is not suitable and economical for large power applications in terms of energy efficiency. But in Switch-Mode Power Supplies (SMPS) the solid state device works *like* a switch *i.e.* either completely *on* or *off*. When the switch *on*, large current flows through it with taking almost zero voltage across it. Similarly for *off* condition, the voltage across the switch is high with almost zero current through it. In both the cases the total power losses across the switch is almost zero, so there are no conduction losses in switch-mode power supply. So, energy efficiency is very high (extended up to 95%) in SMPS.

Over the last decades the technical developments are taken place by the introduction of different types of controller [8-16] for achieving fast dynamic responses as well as better reliable DC-DC switch-mode power supply. Actually the dynamic performance of the power supply is maintained by the control scheme *i.e.* nature of the controllers. There are several classical controllers *like* PI, PID controllers; have been developed over the years to ensure desired performance of the converter under specific conditions. Some converters *like* boost (step-up), buck-boost, and fly-back have a *right-half-plane zero* (*non minimum phase* system) [3], so it is difficult for the normal PID controller to exhibit good performance with load, line variations and parametric uncertainty. For this reason *Type-controllers* [4] are best suited. The *state-space averaging* [5-7] methods have been used for mathematical modelling of converter. The main objective of this paper is to design *Type-II controller* for a step-up *i.e.* boost type power supply and the closed loop stability analysis of the power supply in frequency domain.

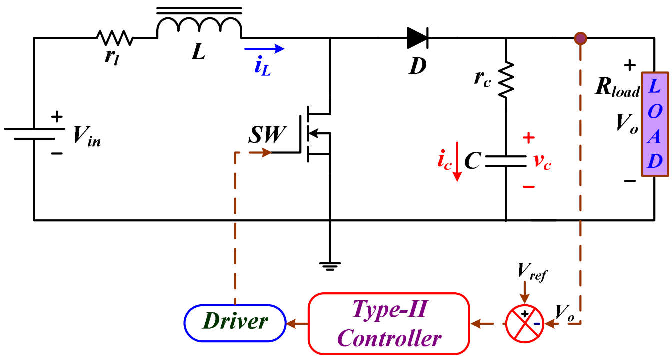

Figure 1: Circuit diagram of switch-mode step-up converter associated with Type-II compensator.

II. CONVERTER SMALL SIGNAL MODEL

A. Transfer Function of Step-up Converter

The *small signal modeling* [5-7] of DC-DC switch-mode step-up converter *i.e.* boost converter is written in equ (1).

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$$T_p(s)=\frac{\tilde{v}_0(s)}{\tilde{d}(s)}\simeq G_{do}\frac{\left(1+\dfrac{s}{\omega_{z-ESR}}\right)\left(1-\dfrac{s}{\omega_{z-RHP}}\right)}{\left\{1+\dfrac{s}{\omega_o Q}+\dfrac{s^2}{\omega_o{}^2}\right\}} \qquad (1)$$

where, Gain $(\mathrm{G}_{do})\simeq V_{in}/(1-D)^2$;

Zero due to ESR $(\omega_{z-ESR})=1/(r_c C)$ rad/sec;

RHP Zero $(\omega_{z-RHP})\simeq(1-D)^2(R_{load}-r_l)/L$ rad/sec;

Natural Freq $(\omega_o)\simeq\sqrt{\left\{r_l+(1-D)^2 R_{load}\right\}/LCR_{load}}$ rad/sec;

Quality Factor $(Q)=\omega_o/\left[(r_l/L)+\left\{1/C(R_{load}+r_c)\right\}\right]$

From the equ (1), it can be observed that a *right-half plane zero* (RHP) present in the converter transfer function. So, there will be an effect of *non-minimum phase* problem in converter dynamics. That's why the initial under-shoot is coming in time domain waveform (Figure 2) and bode plot shows the sufficient phase-lag at gain crossover frequency (Figure 3).

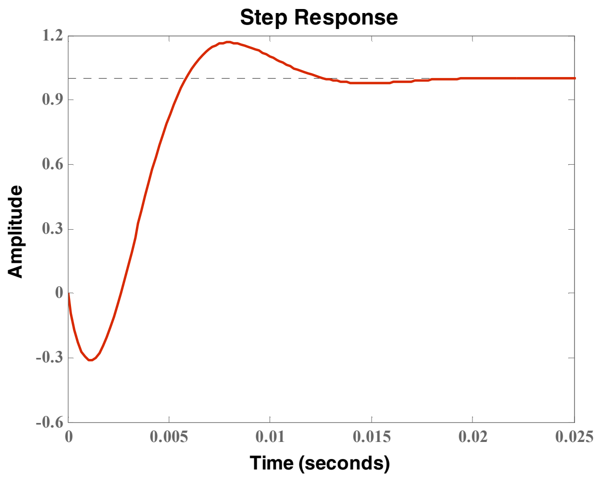

Figure 2: Time response of step-up converter.

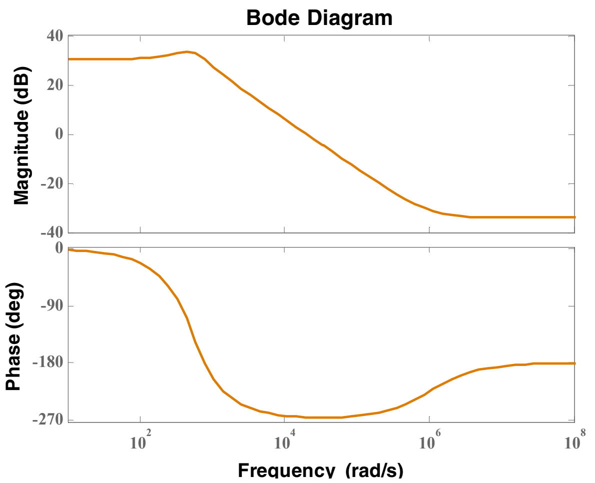

Figure 3: Frequency response of boost-converter.

TABLE I: PARAMETERS OF BOOST CONVERTER

Circuit Components	Values
Input Voltage V_{in}	5 Volt
Output Voltage V_o	12 Volt
Inductance L	250 µH
Output Capacitance C	1056 µF
Inductor Resistance r_l	10 mΩ
ESR of Capacitor r_c	30 mΩ
Load Resistance R_{load}	25 Ω
Switching Period T	50 μs

III. TYPE-II COMPENSATOR DESIGN

The compensator design is an important part for assuring good performance and regulation of a power supply. The compensator, the poles-zeros combinations, is used to provide the classical loop shaping by modifying the gain and phase characteristics of open-loop frequency response as well as ensures the converter robustness. The *"Type-II"* compensator is a lead compensator with a pole at origin. So, this compensator provides 0° to 90° *phase boost* with *zero steady state error*. Even though boost converter having *non-minimum phase* system, it exhibits a better closed loop performance utilizing a cascaded Type-II controller. With proper tuning of this compensator the converter performs faster response, with *minimal overshoots* and *zero steady-state error*.

A. Mathematical Approach

Type-II compensator is a pair of pole-zero combination with a pole at origin.

The expression of compensator transfer function:

$$T_c(s)=\frac{(1+s/\omega_z)}{(s/\omega_{po})(1+s/\omega_p)} \qquad (2)$$

where the locations of pole and zero have been considered at ω_p and ω_z.

The magnitude of compensator transfer function is found

$$|T_c(j\omega)|=\frac{|1+j\,\omega/\omega_z|}{|j\,\omega/\omega_{po}||1+j\,\omega/\omega_p|}=\frac{\sqrt{1+(\omega/\omega_z)^2}}{(\omega/\omega_{po})\sqrt{1+(\omega/\omega_p)^2}} \qquad (3)$$

The argument is written as

$$\arg T_c(j\omega)=\tan^{-1}(\omega/\omega_z)-\tan^{-1}(\omega/\omega_p)-\pi/2 \qquad (4)$$

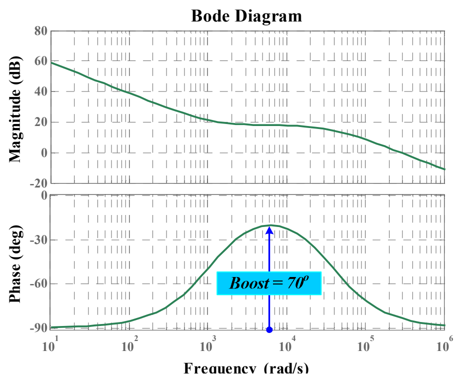

Figure 4: The Bode diagram of Type-II compensator

The frequency domain response of Type-II compensator is plotted in Figure 4. Here the combined action of the pole-zero creates a localized *phase boost* (70°). If the pole and the zero coincident, they perfectly neutralize each other *i.e.* a flat 0-dB magnitude with 0° phase contribution have found over the frequency. The phase boost starts again if both of the pole and zero have been spited together. That's way the phase lead is to be varied from 0° to 90° by changing the pole-zero position.

The *frequency* where the *maximum phase boost* will be occurred can easily obtain by derivate equation (4)

$$\frac{d}{df}\left(\arg T_c\left(j\omega\right)\right)=\frac{d}{df}\left(\tan^{-1}\left(f/f_z\right)-\tan^{-1}\left(f/f_p\right)-\pi/2\right)$$

$$or, \quad =\frac{1}{f_z\left(\frac{f^2}{f_z^2}+1\right)}-\frac{1}{f_p\left(\frac{f^2}{f_p^2}+1\right)}=0 \quad (5)$$

By solving the equation (5), the *maximum phase boost* is obtained at the *geometric means* of the pole-zero frequencies:

$$f_{\max}=\sqrt{f_p f_z} \quad (6)$$

Ultimately it can be concluded that the maximum phase boost is occurred at *geometric mean* of pole- zero frequencies. Generally this geometric mean frequency is considered as crossover frequency of the compensator.

B. Derivation of k in Type-II Compensator

The *'k'* is defined as the ratio of the pole frequency to the zero frequency in Type-II compensator. This pole-zero combinations provide an adjustable phase boost from 0° to 90° at the cross over frequency. So equitation should need to find the relation between *k* and the *phase boost* of the compensator.

Initially both terms are unknown, so there must be two equations:

$$phase\ boost=\tan^{-1}\left(f_c/f_z\right)-\tan^{-1}\left(f_c/f_p\right) \quad (7)$$

$$\text{and } f_c=f_{\max}=\sqrt{f_p f_z} \quad (8)$$

$$\text{So, } f_z=f_c^2/f_p \quad (9)$$

Now f_z is substituted in (7)

$$phase\ boost=\tan^{-1}\left(f_p/f_c\right)-\tan^{-1}\left(f_c/f_p\right) \quad (10)$$

To solve this equation, *k* is introduced where $k=f_p/f_z$.

$$phase\ boost=\tan^{-1}\left(k\right)-\tan^{-1}\left(\frac{1}{k}\right) \quad (11)$$

From the trigonometric formula

$$\tan^{-1}\left(k\right)+\tan^{-1}\left(\frac{1}{k}\right)=\frac{\pi}{2} \quad (12)$$

By solving *k* equ. (11) and (12)

$$2\tan^{-1}\left(k\right)=phase\ boost+\frac{\pi}{2}$$

$$or,\ k=\tan\left(\frac{phase\ boost}{2}+\frac{\pi}{4}\right) \quad (13)$$

The expression has been known as "*k factor*" approach [4].

So the pole location will be

$$f_p=k.f_c=\tan\left(\frac{phase\ boost}{2}+\frac{\pi}{4}\right)f_c \quad (14)$$

and the zero location will be

$$f_z=\frac{f_c}{k}=\frac{f_c}{\tan\left(\frac{phase\ boost}{2}+\frac{\pi}{4}\right)} \quad (15)$$

If the *crossover frequency* and the necessary *phase boost* are known, the compensator pole-zero locations can be easily found from equ. (14) and (15).

C. Mid-Band Gain Adjustment for the Compensator

Type-II compensator is combination of *single pole-zero pair* with *an origin pole*. Compensator has been described by equ. (2). Now rewrite the numerator by factoring s/ω_z.

$$T_c\left(s\right)=\frac{s}{\omega_z}\frac{\left(\omega_z/s+1\right)}{\left(s/\omega_{po}\right)\left(1+s/\omega_p\right)}=\frac{\omega_{po}}{\omega_z}\frac{1+\omega_z/s}{1+s/\omega_p}$$

$$or, \quad =G_o\frac{1+\omega_z/s}{1+s/\omega_p} \quad (16)$$

In this expression, ω_{po} is 0-dB crossover pole and the term G_o is called the mid-band gain and G_o is equal to ω_{po}/ω_z.

IV. Design Example with a Type-II

Let's assume a power supply that has a gain deficit of - 18 dB at a 1 kHz selected crossover frequency. The necessary phase boost is 68°. From (14) and (15), a pole can be placed at

$$f_p=\tan\left(68^o/2+45^o\right)\times1000=5.14\text{ kHz} \quad (17)$$

and the zero is placed at

$$f_z=\frac{1000}{\tan\left(68^o/2+45^o\right)}=194.38\text{ Hz} \quad (18)$$

So the transfer function of the designed Type-II compensator is given by $\frac{1000\left(s+1221.3\right)}{s\left(s+32324\right)}$.

V. Stability Analysis

A. Closed-loop performances: Adjustment of Compensator Crossover Frequency (f_c)

The closed-loop performances of a step-up (boost) converter cascaded with Type-II compensator has been studied with different cross over frequencies (f_c) for a constant gain (G_o). Since crossover frequency directly influences the pole-zero location of compensator, it has to be judiciously selected to get optimum performance. From Figure 5 & 6 it is clear that for f_c=1 kHz, both the time and frequency responses are better than f_c=300 Hz and f_c=2 kHz. It is understood that if the f_c is increased beyond certain value than the performance of the closed loop converter deteriorates. Again for a lower value of f_c=300 Hz, the time response plot is not impressive and it exhibits more undershoots before reaching final steady-state point. The respective comparative performance details are also reported in TABLE II.

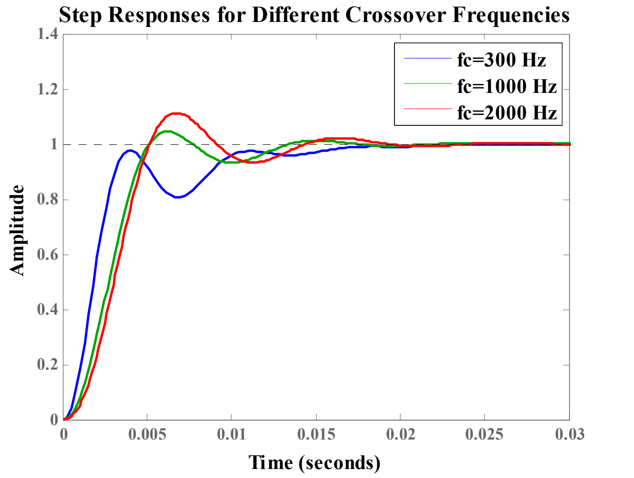

Figure 5: The comparative study of closed-loop boost converter for different cross over frequencies with constant compensator gain.

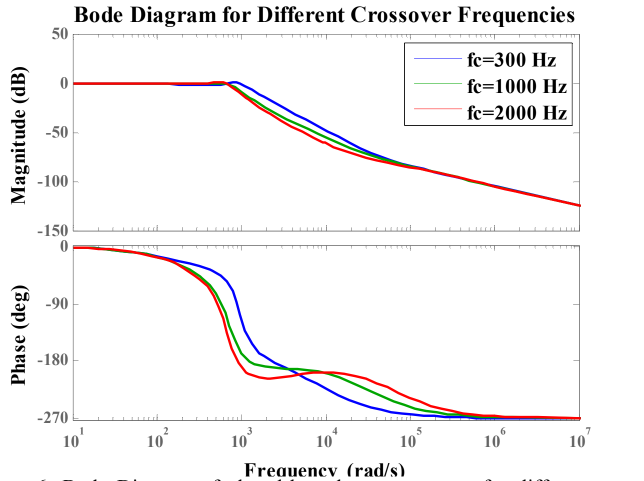

Figure 6: Bode Diagram of closed-loop boost converter for different cross over frequencies.

TABLE II: COMPARITIVE STUDY OF CLOSED-LOOP PERFORMANCES

Crossover Frequency (f_c)	300 Hz	1000 Hz	2000 Hz
Maximum Overshoot (M_p)	0 %	4.52 %	11.2 %
Rise time (t_r)	0.0173 sec	0.00324 sec	0.0050 sec
Phase margin (PM)	63.6°	67.4°	60.7°
Gain crossover frequency (GCF)	699 rad/sec	391 rad/sec	376 rad/sec
Gain margin (GM)	19.6 dB	15.8 dB	10.9 dB
Phase Crossover Frequency (PCF)	2250 rad/sec	1210 rad/sec	896 rad/sec
Compensator Gain (G_o)	1000	1000	1000
Compensator Transfer Function (T_c)	$\frac{1000(s+366.4)}{s(s+9697.3)}$	$\frac{1000(s+1221.3)}{s(s+32324)}$	$\frac{1000(s+2442.7)}{s(s+64648)}$
Closed-Loop Stability	Stable	Stable	Stable

B. Closed-loop performance: Adjustment of Compensator Gain

The closed-loop performances of a step-up power-supply have been shown in Figure 7 & 8 with different values of compensator gain for a fixed crossover frequency of 1000 Hz. Increasing the gain means the damping ratio ξ will be decreased and consequently the rise time as well as the peak overshoot may be increased. The comparative analysis of performance details are also reported in TABLE III. It can be observed (from TABLE III) that the best response is found at gain (G_o) 1000 for cross-over frequency (f_c) of 1 kHz.

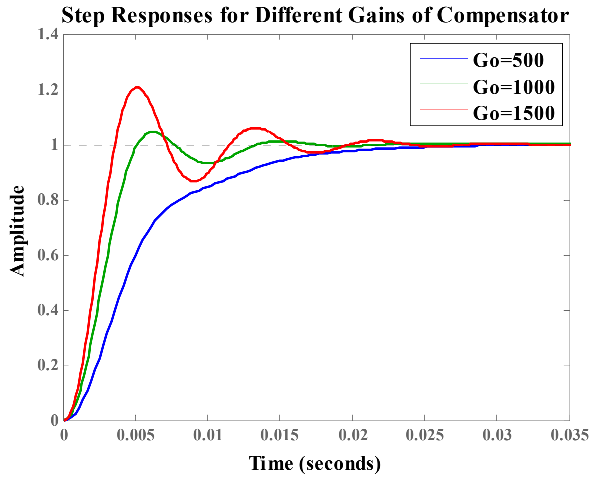

Figure 7: The comparative study of closed-loop boost converter for different gains of compensator with constant crossover frequency.

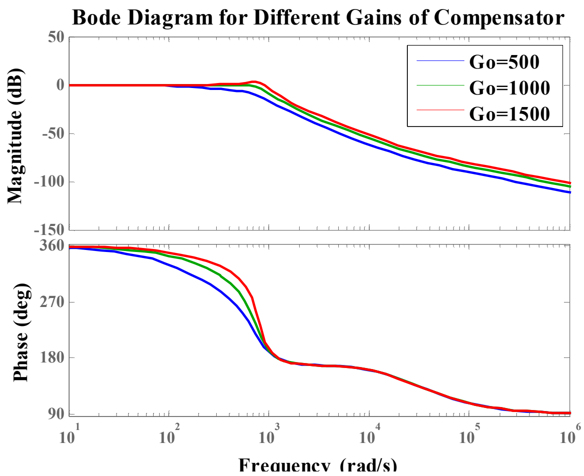

Figure 8: Bode Diagram of closed-loop boost converter for different gains of compensator.

TABLE-III : COMPARITIVE STUDY OF CLOSED-LOOP PERFORMANCES

Compensator Gain (G_o)	500	1000	1500
Crossover Frequency (f_c)	1 kHz	1 kHz	1 kHz
Maximum Overshoot (M_p)	0 %	4.52 %	20.6 %
Rise time (t_r)	0.01100 sec	0.00324 sec	0.00232 sec
Phase margin(*PM*)	81.9°	67.4°	45.7°
Gain crossover frequency (*GCF*)	179 rad/sec	391 rad/sec	597 rad/sec
Gain margin (*GM*)	21.8 dB	15.8 dB	12.3 dB
Phase Crossover Frequency (*PCF*)	1210 rad/sec	1210 rad/sec	1210 rad/sec
Compensator Transfer Function (T_c)	$\frac{500(s+1221.3)}{s(s+32324)}$	$\frac{1000(s+1221.3)}{s(s+32324)}$	$\frac{1500(s+1221.3)}{s(s+32324)}$
Closed-Loop Stability	Stable	Stable	Stable

VI. CONCLUSION

This work describes design of a Type-II compensator for a DC-DC switch-mode step-up power supply. The cascade compensated closed loop control system has been simulated in MATLAB. The effect of crossover frequency (f_c) and gain (G_o) on the *closed loop stability* & *performance* of converter has been investigated. These two parameters finally decide the compensator transfer function. The closed loop behavior of power supply with Type-II compensation is satisfactory. But for obtaining better results the parameters of compensator have to be optimized by soft-computing techniques and this may be a part of future work.

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