2-3-B

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ex 2.4

$f(x, y) = 0$ … 陰関数 (陰伏関数)

(1) $x^2 + y^2 = 4$ の両辺を x で微分する.

$$(x^2)' + \frac{d}{dy}y^2 \cdot \frac{dy}{dx} = (4)'$$

$$2x + 2yy' = 0$$

$$y' = \frac{-2x}{2y} = -\frac{x}{y} \quad \cdots ①$$

(2) 代入 $x = 1 \to x^2 + y^2 = 4$

$$1^2 + y^2 = 4$$

$$y = \pm\sqrt{3}$$

よって ① より

$$\left.\frac{dy}{dx}\right|_{(x, y) = (1, \pm\sqrt{3})} = \pm\frac{1}{\sqrt{3}}$$

51 (1) 両辺を x で微分する

$$\frac{d}{dy}y^2 \cdot \frac{dy}{dx} = \frac{d}{dy}(4x)$$

$$2y \cdot y' = 4$$

$$y' = \frac{4}{2y} = \frac{2}{y} \quad \cdots ①$$

$x = 4$ を与式に代入すると

$$y^2 = 4 \cdot 4 = 16$$

$$y = \pm 4$$

① より

$$y' = \frac{4}{2(\pm 4)} = \pm\frac{1}{2}$$

(2) 与式の両辺を x で微分する

$$(x^2)' + \frac{d}{dy}y^2 \cdot \frac{dy}{dx} = (6)'$$

$$2x + 2y \cdot y' = 0$$

$$y' = \frac{-2x}{2y} = -\frac{x}{y}$$

$x = -1$ を与式に代入すると

$$(-1)^2 + y^2 = 6$$

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$

① より

$$y' = -\frac{-1}{\pm\sqrt{5}} = \pm\frac{1}{\sqrt{5}}$$

52 (1)

$$y = \frac{4x+1}{(x-2)^{\frac{1}{2}}}$$

$$y' = \frac{(4x+1)'(x-2)^{\frac{1}{2}} - (4x+1)\{(x-2)^{\frac{1}{2}}\}'}{\{(x-2)^{\frac{1}{2}}\}^2}$$

$$= \frac{4(x-2)^{\frac{1}{2}} - (4x+1) \cdot \frac{1}{2}(x-2)^{-\frac{1}{2}} \cdot 1}{x-2}$$

$$= \frac{4(x-2)^{-\frac{1}{2}}(x-2) - (4x+1) \cdot \frac{1}{2}(x-2)^{-\frac{1}{2}} \cdot 1}{x-2}$$

$$= \frac{4(x-2) - (4x+1) \cdot \frac{1}{2}}{(x-2)(x-2)^{\frac{1}{2}}}$$

$$= \frac{8(x-2) - (4x+1)}{2(x-2)\sqrt{x-2}}$$

$$= \frac{4x-17}{2(x-2)\sqrt{x-2}}$$

(2) log の真数 > 0 より $\tan\frac{x}{2} \neq 0$.

$$y' = \frac{1}{\tan\frac{x}{2}} \cdot \left(\tan\frac{x}{2}\right)'$$

↑ log を $\tan\frac{x}{2}$ で微分

↓ $\tan\frac{x}{2}$ を $\frac{x}{2}$ で微分

$$= \frac{1}{\frac{\sin\frac{x}{2}}{\cancel{\cos}\frac{x}{2}}} \cdot \frac{1}{\cos^{\cancel{2}}\frac{x}{2}} \cdot \frac{1}{2} \quad \leftarrow \frac{x}{2} \text{ を } x \text{ で微分}$$

$$= \frac{1}{2\sin\frac{x}{2} \cdot \cos\frac{x}{2}}$$

ここで, $\sin x = \sin\left(\frac{x}{2} + \frac{x}{2}\right)$

$$= \sin\frac{x}{2}\cos\frac{x}{2} + \cos\frac{x}{2}\sin\frac{x}{2}$$

$$= 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$y' = \frac{1}{\sin x}$$

(3)

$$y' = \frac{1}{e^x + 1} \cdot e^x$$

$$= \frac{e^x}{e^x + 1}$$

(4) $y' = (x)'\sin^{-1}\sqrt{x} + x(\sin^{-1}\sqrt{x})'$

ここで $t := \sqrt{x}$, $g := \sin^{-1}\sqrt{x} = \sin^{-1} t$

$\Rightarrow t = \sin g$

両辺を g で微分して

$$\frac{dt}{dg} = \cos g$$

よって

$$\frac{dg}{dt} = \frac{1}{\frac{dt}{dg}} = \frac{1}{\cos g}$$

$$= \frac{1}{\sqrt{1 - \sin^2 g}} \quad \because \sin^2 g + \cos^2 g = 1,\ -\frac{\pi}{2} \le g \le \frac{\pi}{2} \Rightarrow \cos g \ge 0$$

$$= \frac{1}{\sqrt{1 - t^2}}$$

だから,

$$(\sin^{-1}\sqrt{x})' = \frac{dg}{dt}\frac{dt}{dx}$$

$$= \frac{1}{\sqrt{1-t^2}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$y' = \sin^{-1}\sqrt{x} + \overset{\sqrt{x}}{\cancel{x}} \cdot \frac{1}{2\cancel{\sqrt{x}}\sqrt{1-x}}$$

$$= \sin^{-1}\sqrt{x} + \frac{\sqrt{x}}{2\sqrt{1-x}}$$

(5)

$$y = \log(x+2)^{\frac{2}{3}}$$

$$= \frac{2}{3}\log(x+2)$$

$$y' = \frac{2}{3} \cdot \frac{1}{x+2} \cdot 1$$

$$= \frac{2}{3(x+2)}$$

53 (1)

$$f'(x) = \left(\sqrt{2x^2 - x}\right)'$$

$$= \{(2x^2 - x)^{\frac{1}{2}}\}'$$

$$= \frac{1}{2}(2x^2 - x)^{-\frac{1}{2}} \cdot (2 \cdot 2x - 1)$$

$$= \frac{1}{2}\frac{4x-1}{\sqrt{2x^2 - x}}$$

$$= \frac{4x-1}{2\sqrt{2x^2 - x}}$$

$$f'(1) = \frac{4 \cdot 1 - 1}{2\sqrt{2 \cdot 1^2 - 1}}$$

$$= \frac{3}{2}$$

(2)

$$f'(x) = \cos 3x \cdot 3$$

$$= 3\cos 3x$$

$$f'\left(\frac{7}{3}\pi\right) = 3\cos\left(3 \cdot \frac{7}{3}\pi\right)$$

$$= 3\cos 7\pi$$

$$= -3$$

(3)

$$f'(x) = \frac{1}{\cos^2\frac{x}{4}} \cdot \frac{1}{4}$$

$$= \frac{1}{4\cos^2\frac{x}{4}}$$

$$f'(3\pi) = \frac{1}{4\cos^2\frac{3\pi}{4}}$$

$$= \frac{1}{4 \cdot \left(-\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{4 \cdot \frac{1}{2}}$$

$$= \frac{1}{2}$$

(4)

$$f'(x) = \frac{1}{4x-1} \cdot 4$$

$$= \frac{4}{4x-1}$$

$$f'(3) = \frac{4}{4 \cdot 3 - 1}$$

$$= \frac{4}{11}$$

(5)

$$f'(x) = (x)'(e^{x^2}) + (x)(e^{x^2})'$$

$$= 1 \cdot e^{x^2} + x \cdot e^{x^2} \cdot 2x$$

$$= e^{x^2}(1 + 2x^2)$$

$$f'(2) = e^{2^2}(1 + 2 \cdot 2^2)$$

$$= e^4(1 + 8)$$

$$= 9e^4$$

(6)

$$f'(x) = \frac{(2x+1)'(x-3) - (2x+1)(x-3)'}{(x-3)^2}$$

$$= \frac{2(x-3) - (2x+1) \cdot 1}{(x-3)^2}$$

$$= \frac{-7}{(x-3)^2}$$

$$f'(4) = \frac{-7}{(4-3)^2} = -7$$

54 (1)

$$f'(x) = \frac{(2x)'(x^2+1) - (2x)(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$

$$= \frac{-2x^2+2}{(x^2+1)^2} = 0$$

ここで $(x^2+1)^2 \neq 0$

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1$$

$$x \neq \pm i$$

$$\frac{-2x^2+2}{(x^2+1)^2} = 0$$

$$-2x^2 + 2 = 0$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

(2)

$$f'(x) = \frac{(1-x)'(x^2-x+1) - (1-x)(x^2-x+1)'}{(x^2-x+1)^2}$$

$$= \frac{-x^2+x-1 - (1-x)(2x-1)}{(x^2-x+1)^2}$$

$$= \frac{-x^2+x-1 - (2x-1-2x^2+x)}{(x^2-x+1)^2}$$

$$= \frac{x^2-2x}{(x^2-x+1)^2} = 0$$

ここで $x^2 - x + 1 \neq 0$

$$x \neq \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

$$\frac{x^2-2x}{(x^2-x+1)^2} = 0$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, 2$$

55 (1)

$$f(x) = 2\{x(5-x)^{\frac{1}{2}}\}'$$

$$= 2\{(x)'(5-x)^{\frac{1}{2}} + x\{(5-x)^{\frac{1}{2}}\}'\}$$

$$= 2\{(5-x)^{\frac{1}{2}} + x \cdot \frac{1}{2}(5-x)^{-\frac{1}{2}} \cdot (-1)$$

$$= 2(5-x)^{\frac{1}{2}} - x(5-x)^{-\frac{1}{2}}$$

$$= 2(5-x)^{\frac{1}{2}} - \frac{x}{\sqrt{5-x}}$$

$$= \frac{2(5-x) - x}{\sqrt{5-x}}$$

$$= \frac{-3x+10}{\sqrt{5-x}} = 0$$

$\sqrt{5-x} \neq 0$ かつ $5 - x \ge 0 \Rightarrow 5 - x > 0 \Leftrightarrow x < 5$

$$\frac{-3x+10}{\sqrt{5-x}} = 0$$

$$-3x + 10 = 0$$

$$x = \frac{10}{3}$$

(2)

$$f'(x) = \{(x-5)(x^2+3)^{\frac{1}{2}}\}'$$

$$= (x-5)'(x^2+3)^{\frac{1}{2}} + (x-5)\{(x^2+3)^{\frac{1}{2}}\}'$$

$$= 1 \cdot (x^2+3)^{\frac{1}{2}} + (x-5)\frac{1}{2}(x^2+3)^{-\frac{1}{2}} 2x$$

$$= (x^2+3)^{\frac{1}{2}} + x(x-5)(x^2+3)^{-\frac{1}{2}}$$

$$= (x^2+3)^{\frac{1}{2}} + \frac{x(x-5)}{(x^2+3)^{\frac{1}{2}}}$$

$$= \frac{x^2+3+x^2-5x}{\sqrt{x^2+3}}$$

$$= \frac{2x^2-5x+3}{\sqrt{x^2+3}}$$

$$= \frac{(2x-3)(x-1)}{\sqrt{x^2+3}} = 0$$

$\sqrt{x^2+3} \neq 0$ かつ $x^2 + 3 \ge 0 \Rightarrow x^2 + 3 > 0 \Leftrightarrow x^2 > -3$

$(\text{実数})^2 \ge 0$ より $-\infty < x < \infty$

$$\frac{(2x-3)(x-1)}{\sqrt{x^2+3}} = 0$$

$$(2x-3)(x-1) = 0$$

$$x = \frac{3}{2}, 1$$