ФИТ-212
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Вариант 31

Работа над ошибками.

① Выполнить действия.

$$\frac{(1-i)^{16}}{(-\sqrt{3}+i)^{12}}(-1-\sqrt{3})\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$$

$(1-i)^{16} \to r=\sqrt{1+1}=\sqrt{2}$ $\quad \varphi=\operatorname{arctg}\left(-\frac{1}{1}\right)=-\frac{\pi}{4}$

$(-\sqrt{3}+i)^{12} \to r=\sqrt{3+1}=2$ $\quad \varphi=\pi+\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)=\frac{5\pi}{6}$

$(-1-i\sqrt{3}) \to r=\sqrt{1+3}=2$ $\quad \varphi=-\pi+\operatorname{arctg}(\sqrt{3})=-\frac{2\pi}{3}$

$$\frac{2^8(\cos(-4\pi)+i\sin(-4\pi))}{2^{12}(\cos 10\pi+i\sin 10\pi)}\left(2\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)\right)\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)=$$

$$=\frac{1}{8}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=-\frac{1}{8}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=-\frac{1}{8}\left(\frac{1}{2}+\frac{3}{4}\right)=-\frac{1}{8}$$

Ответ: $-\frac{1}{8}$

② Извлечь корень и дать геометрическое истолкование

$$\sqrt[4]{\frac{\sqrt{3}-i}{-\sqrt{75}+5i}}=\sqrt[4]{\frac{(\sqrt{3}-i)(-\sqrt{75}-5i)}{(-\sqrt{75}+5i)(-\sqrt{75}-5i)}}=\sqrt[4]{\frac{-20}{100}}=\sqrt[4]{-\frac{1}{5}}$$

$r=\sqrt{\frac{1}{25}}=\frac{1}{5}$

$\varphi=\pi+\operatorname{arctg}0=\pi$

$z=\frac{1}{5}(\cos\pi+i\sin\pi)$

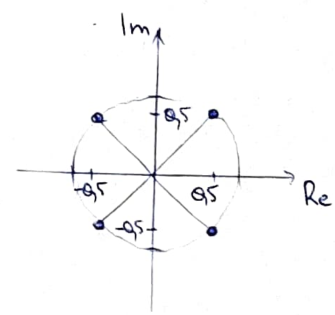
$z^{\frac{1}{4}}=\sqrt[4]{\frac{1}{5}}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\Rightarrow\sqrt[4]{\frac{1}{5}}\left(\cos\frac{\pi+2\pi k}{4}+i\sin\frac{\pi+2\pi k}{4}\right)$, $k=0,1,2,3$

$k=0$ $\quad w_1=\sqrt[4]{\frac{1}{5}}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=\sqrt[4]{\frac{1}{5}}\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)$

$k=1$ $\quad w_2=\sqrt[4]{\frac{1}{5}}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=\sqrt[4]{\frac{1}{5}}\left(-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)$

$k=2$ $\quad w_3=\sqrt[4]{\frac{1}{5}}\left(\cos\frac{5\pi}{4}+i\sin\frac{3\pi}{4}\right)=\sqrt[4]{\frac{1}{5}}\left(-\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)$

$k=3$ $\quad w_4=\sqrt[4]{\frac{1}{5}}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)=\sqrt[4]{\frac{1}{5}}\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)$



③ Найти геометрическое место точек, изображающих комплексные числа z, для которых одновременно: $\arg z<\frac{\pi}{3}$ и $\arg z\leq\frac{\pi}{6}$

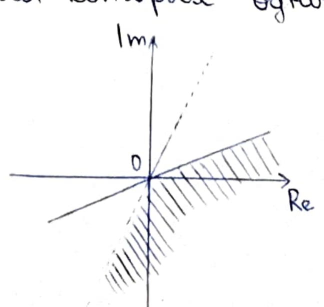
$\arg z<\frac{\pi}{3}$ $\quad \arg z\leq\frac{\pi}{6}$

$\arg\frac{y}{x}<\frac{\pi}{3}$ $\quad \arg\frac{y}{x}\leq\frac{\pi}{6}$

$\frac{y}{x}<\operatorname{tg}\frac{\pi}{3}$ $\quad \frac{y}{x}\leq\operatorname{tg}\frac{\pi}{6}$

$\frac{y}{x}<\sqrt{3}$ $\quad \frac{y}{x}\leq\frac{1}{\sqrt{3}}$

$y<\sqrt{3}x$ $\quad y\leq\frac{x}{\sqrt{3}}$



4) 1) $z^3 - \bar{z} + z = 0$

$z = x + iy$

$(x+iy)^3 - (x-iy) + (x+iy) = 0$

$(x^3 + 3x^2iy + 3x(iy)^2 + (iy)^3) + 2iy = 0$

$(x^3 - 3xy) + i(3x^2y - iy^3 + 2iy) = 0$

$\begin{cases} x^3 - 3xy = 0 \to y = \frac{x^2}{3} \\ 3x^2y - iy^3 + 2iy = 0 \end{cases}$

$3x^2\left(\frac{x^2}{3}\right) - \left(\frac{x^2}{3}\right)^3 + 2\left(\frac{x^2}{3}\right) = 0$

$x^4 - \frac{x^6}{3} + \frac{2}{3}x^2 = 0$; Пусть $t = x^2$, тогда:

$t^2 - \frac{t^3}{3} + \frac{2}{3}t$

$t(t^2 - 3t - 2) = 0$

$t_1 = 0; t_2 = 1; t_3 = 2$

$x_{12} = \pm 1; x_{34} = \pm\sqrt{2}; x_5 = 0$

$y_{12} = \frac{1}{3}; y_{34} = \frac{2}{3}; y_5 = 0$

2) $2z^2 + 3|z|^2 = 0$

$2(x+iy)^2 + 3(\sqrt{x^2+y^2})^2 = 0$

$2(x^2 + 2xiy - y^2) + 3x^2 + 3y^2 = 0$

$2x^2 + 4xiy - 2y^2 + 3x^2 + 3y^2 = 0$

$5x^2 + 4xiy + y^2 = 0$

$(5x^2 + y^2) + 4xiy = 0$

$\begin{cases} 5x^2 + y^2 = 0 \to x = \sqrt{\frac{-y^2}{5}} \\ 4x^2iy = 0 \end{cases}$

Ответ: 1) $(1; \frac{1}{3}), (-1; \frac{1}{3}), (\sqrt{2}; \frac{2}{3}), (-\sqrt{2}; \frac{2}{3}), (0;0)$

2) $(0;0)$