~~К~~Р по теме „Определённые интегралы"

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ФИТ-212
Вариант 4.

7,5 балла

1

2,5

а) $\int_{-1}^{2}\frac{x^4-2}{x^2+1}dx$; $\int\frac{x^4-2}{x^2+1}dx = \int\frac{x^4}{x^2+1}dx - \int\frac{2}{x^2+1}dx$

$\int(x^2-1+\frac{1}{x^2+1})dx = \frac{x^3}{3} - x + \mathrm{arctg}\,x + C$

$\int\frac{2dx}{x^2+1} = 2\,\mathrm{arctg}\,x + C$

$\int\frac{x^4-2}{x^2+1}dx = \frac{x^3}{3} - x - \mathrm{arctg}\,x + C$

+

2,5 + 2 + 1 + 2

$F(2) = \frac{8}{3} - 2 - \mathrm{arctg}\,2 = \frac{2}{3} - \mathrm{arctg}\,2$

$F(-1) = -\frac{1}{3} + 1 + \frac{\pi}{4} = \frac{\pi}{4} + \frac{2}{3}$

$\int_{-1}^{2}\frac{x^4-2}{x+1}dx = \frac{2}{3} - \mathrm{arctg}\,2 - (\frac{\pi}{4} + \frac{2}{3}) = -\frac{\pi}{4} - \mathrm{arctg}\,2$

б) $\int_{1}^{4}\frac{dx}{2+\sqrt{8x-7}}$; $\int\frac{dx}{2+\sqrt{8x-7}} = \left[\begin{matrix} u = 8x-7 \\ dx = -\frac{1}{8}du\end{matrix}\right. = -\frac{1}{8}\int\frac{du}{\sqrt{-u}+2} = \left[\begin{matrix} v = \sqrt{-u}+2 \\ du = -2\sqrt{-u}dv\end{matrix}\right. \Rightarrow$

$\Rightarrow -2\int\frac{v-2}{v}dv = -2\left(\int 1dv - 2\int v^{-1}dv\right) = -2v + 4\ln v = -2\sqrt{-u} - 4 + 4\ln(-\sqrt{u}+2) \Rightarrow$

$\Rightarrow \frac{\sqrt{8x-7}+2}{4} - \frac{\ln(\sqrt{8x-7}+2)}{2} + C$

$F(x) = \frac{\sqrt{8x-7} - 2\ln(\sqrt{8x-7}+2)}{4}$

$F(4) = \frac{\sqrt{32-7} - 2\ln(\sqrt{32-7}+2)}{4}$

$F(1) = \frac{\sqrt{1} - 2\ln(\sqrt{8-7}+2)}{4}$

собрать

$\int_{1}^{4}\frac{dx}{2+\sqrt{8x+7}} = \frac{5-2\ln 7 - 1 + 2\ln 3}{4} = \frac{4-2\ln 7+2\ln 3}{4}$

в) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 4x\,dx$; $\int x\cos 4x\,dx = \frac{x\sin 4x}{4} - \int\frac{\sin 4x}{4}dx \Rightarrow$

$\Rightarrow \int\frac{\sin 4x}{4}dx = \left[\begin{matrix} u = 4x \\ dx = \frac{1}{4}du\end{matrix}\right. = \frac{1}{16}\int\frac{\sin u}{1}du = -\frac{1}{16}\cos u =$

$= -\frac{1}{16}\cos 4x \Rightarrow \frac{x\sin 4x}{4} + \frac{\cos 4x}{16} + C$

$F(x) = \frac{x\sin 4x}{4} + \frac{\cos 4x}{16}$

$F(\frac{\pi}{2}) = \frac{\pi\sin 2\pi}{8} + \frac{1}{16} = \frac{1}{16}$

+

$F(\frac{\pi}{4}) = \frac{\pi\sin\pi}{16} + (-\frac{1}{16}) = -\frac{1}{16}$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 4x\,dx = F(\frac{\pi}{2}) - F(\frac{\pi}{4}) = \frac{1}{16} - (-\frac{1}{16}) = \frac{1}{8}$

2) ②.

Вычислить несобственные интегралы или установить их расходимость:

а) $\int_{-\infty}^{2} \frac{dx}{x^2+2x+10} = \int_{-\infty}^{2} \frac{dx}{(x+1)^2+9}$; $\int_{-\infty}^{3} \frac{1}{u^2+9} du = \int_{-\infty}^{3} \frac{1}{9(\frac{u^2}{9}+1)} du =$

$= \frac{1}{9} \int_{-\infty}^{3} \frac{1}{\frac{u^2}{9}+1} du = \begin{bmatrix} v = \frac{u}{3} \\ dv = \frac{1}{3} du \end{bmatrix} = \frac{1}{3} \int_{-\infty}^{1} \frac{1}{v^2+1} dv \Rightarrow$

$\Rightarrow \lim\limits_{a \to -\infty} \frac{1}{3} \operatorname{tg} v \Big|_a^1$

$\frac{1}{3} \operatorname{tg}^{-1} v \Big|_{-\infty}^{1} = \frac{1}{3} \operatorname{tg}^{-1}(1) - \left(\frac{1}{3} \operatorname{tg}^{-1}(-\infty)\right) = \frac{\pi}{4}$ (+).

б) $\int_{3}^{5} \frac{dx}{\sqrt{-x^2+8x-15}} = \int_{3}^{5} \frac{dx}{1-(x-4)^2} = \begin{bmatrix} u = x-4 \\ du = dx \end{bmatrix} =$

$= \int_{-1}^{1} \frac{1}{\sqrt{1-u^2}} du = 2 \int_{0}^{1} \frac{1}{\sqrt{1-u^2}} du \Rightarrow$

$\Rightarrow \lim\limits_{a \to 1} \arcsin(u) \Big|_0^a$

$2\arcsin(u) \Big|_0^1 = 2\arcsin(1) - 2\arcsin(0) = \pi$ (+).

Ответ: а) $\frac{\pi}{4}$

б) π

3) а) 1 Вычислить площадь фигуры, ограниченной линиями.

Решение:

$y_k = y_0 + y'(x)(x-x_0)$

$y' = 4x^3 \Rightarrow f(1) = 4\cdot(1)^3 = 4$

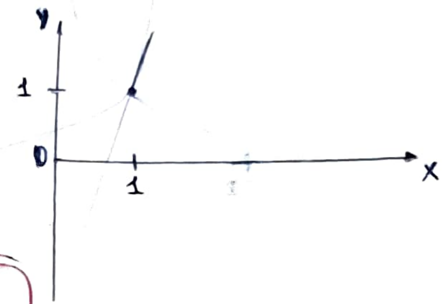
$y_k = y_0 + y'(x_0)(x-x_0)$

$y_k = 1+4(x-1) = 4x-3$

$y_n = y_0 - \frac{1}{y'(x_0)}(x-x_0)$

$y_n = 1 - \frac{1}{4}(x-1)$

$y_n = \frac{5}{4} - \frac{x}{4}$



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б) $r = \cos\varphi$
$r = \sin\varphi$ $\quad (0 \leq \varphi \leq \frac{\pi}{2})$

1) $S_1 = \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin\varphi\, d\varphi = \frac{1}{2}\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}}(1-\cos 2\varphi)\,d\varphi = \frac{1}{4}\left(\varphi - \frac{1}{2}\sin 2\varphi\right)\Big|_0^{\frac{\pi}{4}} =$

$= \frac{1}{4}\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2} - (0-0)\right) = \frac{1}{4}\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{16} - \frac{1}{8}$

+

2) $\left[-\frac{\pi}{4}; \frac{\pi}{2}\right] \qquad \varphi = \frac{\pi}{4} \quad r = \cos\varphi$

$S_2 = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos\varphi^2 = \frac{1}{2}\cdot\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(1+\cos 2\varphi)\,d\varphi = \frac{1}{4}\left(\varphi + \frac{1}{2}\sin\varphi\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$

$= \frac{1}{4}\left(\frac{\pi}{2} + 0 - \left(\frac{\pi}{4} + \frac{1}{2}\cdot 1\right)\right) = \frac{1}{4}\left(\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2}\right) = \cancel{\frac{2\pi}{16}} -$

$S_1 + S_2 = \frac{\pi}{16} - \frac{1}{8} + \frac{\pi}{16} - \frac{1}{8} = \frac{\pi - 2}{8}$ + –

Ответ: а)

б) $S = \frac{\pi - 2}{8}$

4) Вычислить длину дуги кривой. 0.

а) $r=3e^{\frac{3\varphi}{4}}, \quad 0 \le \varphi \le \frac{\pi}{3}$

$$S=\int_0^{\frac{\pi}{3}} \sqrt{1+\left(\frac{d}{dx}\left(3e^{\frac{3\varphi}{4}}\right)\right)^2}\,dx = \int_0^{\frac{\pi}{3}} \sqrt{1+0^2}\,dx = \frac{\pi}{3}$$

Формула неверна

б) $y=4-x^2, \quad x=-2, \quad x=2$

$$S=\int_{-2}^{2} \sqrt{1+\left(\frac{d}{dx}\left(4-x^2\right)\right)^2}\,dx = \int_{-2}^{2} \sqrt{1+(-2x)^2}\,dx =$$

$$= \int_{-2}^{2} \sqrt{1+4x^2}\,dx = \frac{1}{2}\left(4\sqrt{17}+\operatorname{arsh}(4)\right)$$

? —.

Ответ: а) $\frac{\pi}{3}$

б) $\frac{1}{2}\left(4\sqrt{17}+\operatorname{arsh}(4)\right)$

5) а) Вычислить объём тела, ограниченного поверхностями: $x^2+y^2=9$; $z=y$, $z=0$ $(y\geq 0)$

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$$V=\int_{-3}^{3}dx\int_{0}^{\sqrt{9-x^2}}dy\int_{0}^{y}dz=\int_{-3}^{3}dx\int_{0}^{\sqrt{9-x^2}}\frac{y^2}{2}dy=$$

3?

$$=\int_{-2}^{3}\left(\frac{9-x^2}{2}\right)dx=\left(4{,}5x-\frac{x^3}{6}\right)\Bigg|_{-3}^{3}=2\left(4{,}5\cdot 3-\frac{27}{6}\right)=18$$

б) Найти объём тела, образованного вращением вокруг оси Ox фигуры, ограниченной линиями $y=x^2$, $x=2$, $y=1$

2

$$V_{об}=\pi\int_{0}^{2}(x^2)^2dx=\pi\left(\frac{x^5}{5}\right)\Bigg|_{0}^{2}$$

$$V=\pi\int_{1}^{2}x^4dx=\frac{31\pi}{5}$$

$y=1$

$$V=\pi\int_{1}^{2}1dx=\pi$$

$$V=\frac{\pi}{5}+\pi=\frac{6\pi}{5}$$

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$$V_{кон}=\frac{32\pi}{5}-\frac{31\pi}{5}=\frac{\pi}{5}$$

Ответ: а) 18

б) $\frac{6\pi}{5}$