Student Information

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Answer 1

a.

Countable infinite. Set D = { -a/b:a,b∈ N and $a < b$ } Since N is countable, and we can map N to D, D is countably infinite.

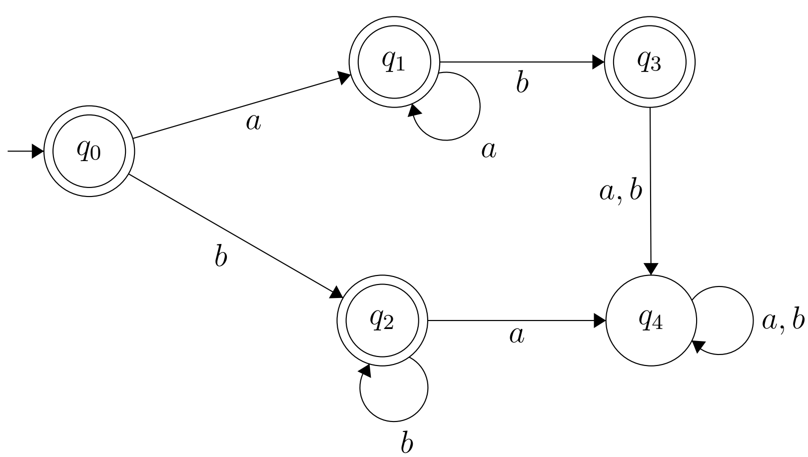
b.

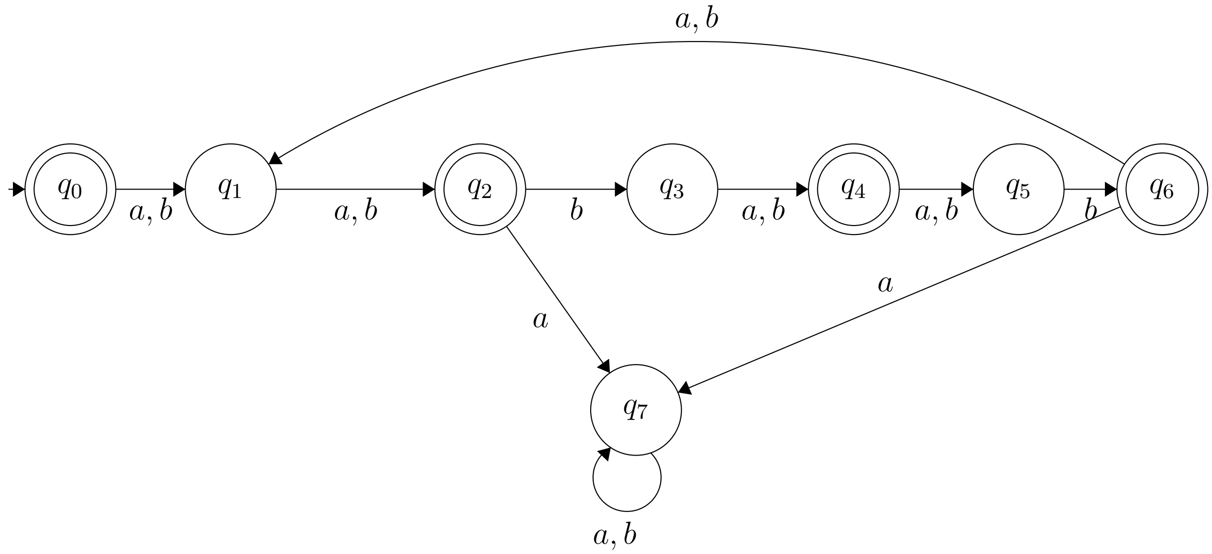
Countable finite. Since L is finite, it is regular and so does L^*. Both L and L^* is regular so L^+ is regular.(Recall that $L^+ = LL^*$). So there are not any non-regular L^+. Hence it is countably finite, because it is an empty set.

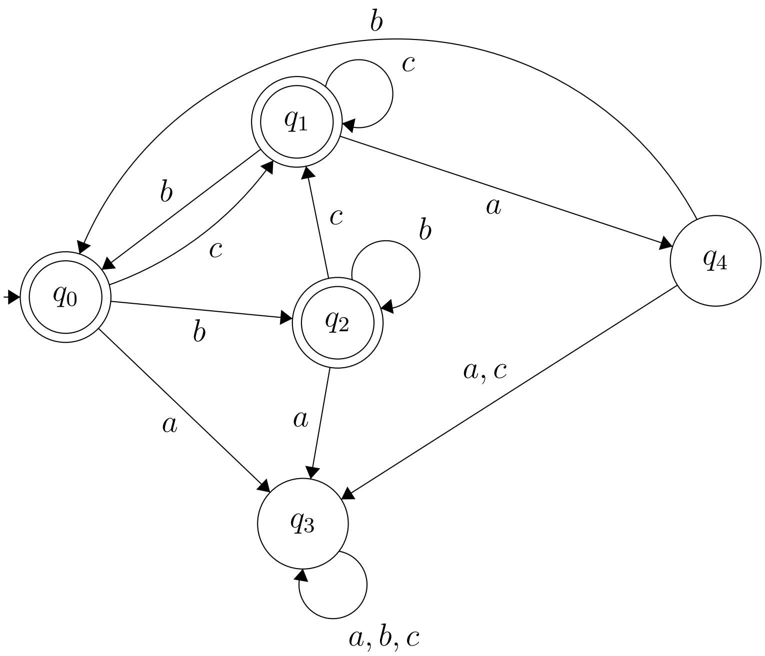
c.

Uncoutably infinite.
There ara uncountably infinite language over the alphabet Σ. Regular ones, have the property being countable due to the Machines which accepts those languages that can be mapped to natural numbers; however, there is no alternative way to map non-regular languages since there does not exist any automota accepts those languages.So it is uncountably infinite.

Answer 2







Answer 3

a.

w_1 = abbb is not in L(N). w_1 ends with "b" and to reach final states(in this case, only q_5), an "a" must be read from input. In this case, input will end in an not-accepting state in all possible computings.Hence, w_1 is not in L(N).

b.

$(q_0, a, q_1), (q_1, e, q_3), (q_3, b, q_3), (q_3, a, q_1), (q_1, e, q_3), (q_3, b, q_3), (q_3, a, q_5)$
Input stop when it is in final state, so it is in L(N)

Answer 4

a.

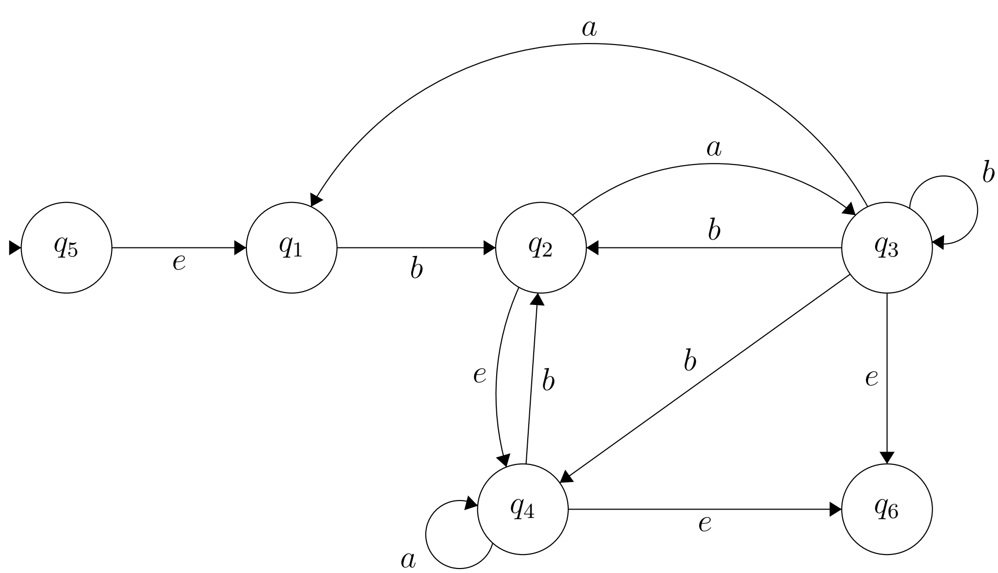
$M_G = (K_G, \Sigma, \Delta, s_G, F_G)$
$K_G = \{q_1, q_2, q_3, q_4, q_5, q_6\}$
$\Sigma = \{a, b\}$
$s_G = q_5$ $F_G = \{q_6\}$
$\Delta_G = \{(q_1, b, q_2),$

$(q_2, a, q_3), (q_2, e, q_4),$

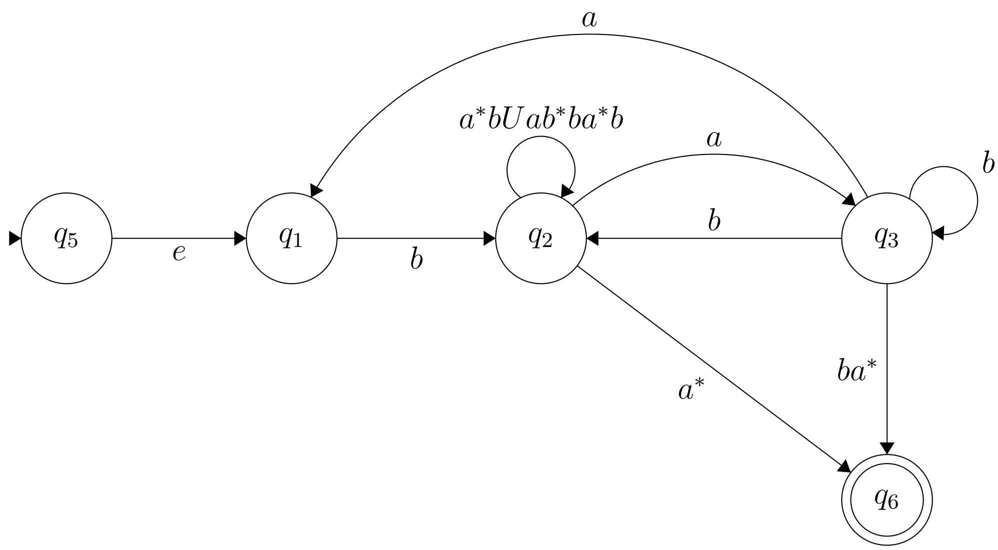
$(q, 3, a, q_1), (q_3, b, q_2), (q_3, b, q_3), (q_3, b, q_4), (q_3, e, q_6),$

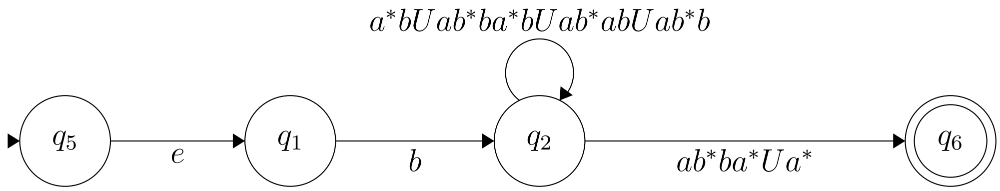
$(q_4, b, q_2), (q_4, a, q_4), (q_4, e, q_6),$

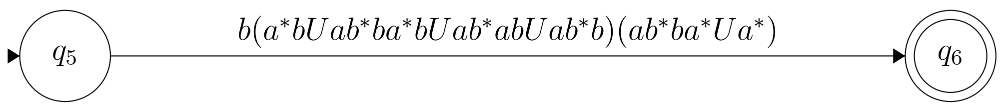
$(q_5, e, q_1)\}$



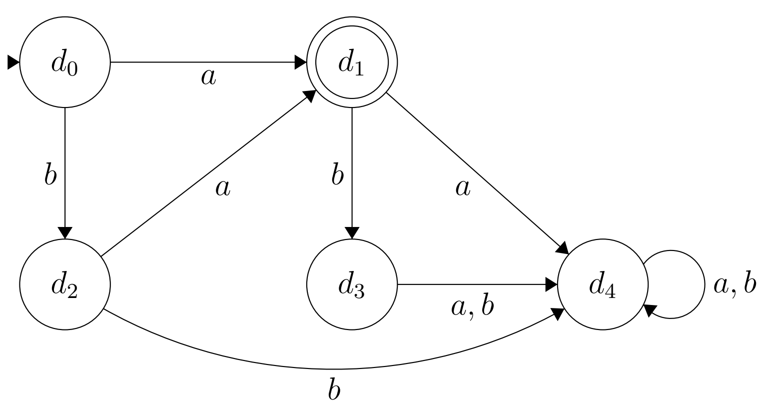
b.







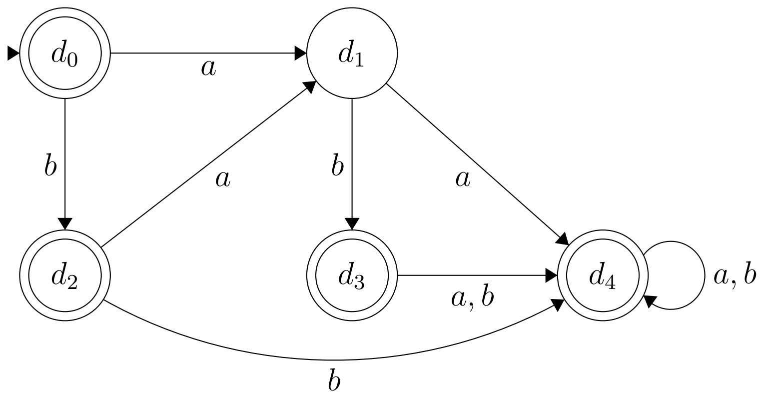
Answer 5



.

where $d_0 = \{q_0, q_1, q_2\}$
$d_1 = \{q_1, q_3\}$
$d_2 = \{q_2\}$

$d_3 = \{q_1\}$
$d_4 = \{\}$



regular expression for above language:
$eUbUabUbabUbaa(aUb)^*Ubab(aUb)(aUb)^*Uab(aUb)(aUb)^*Uaa(aUb)^*$

Answer 6

$M_1 = (K_1, \Sigma_1, \Delta_1, s_1, F_1)$
$M_2 = (K_2, \Sigma_2, \Delta_2, s_2, F_2)$

$M = (K, \Sigma, \Delta, s, F)$

$K = K_1 x K_2$

$\Sigma = \Sigma_1 \bigcup \Sigma_2$
$\Delta = \{((q_1, q_2), w, (q_1', q_2')) | w \in \Sigma, (q_1, w, q_1') \in \Delta_1, (q_2, w, q_2') \in \Delta_2\}$
$s = (s_1, s_2)$
$F = \{(q_1, q_2) | q_1 \in F_1, q_2 \in K_2 - F_2\}$

1)

$L(M) \subseteq L(M_1) - L(M_2)$

Assume $x \in L(M)$, states $\rightarrow (p, q)$

M_1 accepts x, since p is accepting state for M_1
M_2 doesn't accept x, since q is not-accepting state for M_2
so it holds.

$L(M_1) - L(M_2) \subseteq L(M)$

Assume $x \in L(M_1) - L(M_2)$, states $\rightarrow$(p,q)

M_1 accepts x, since p is accepting state for M_1
M_2 doesn't accept x, since q is not-accepting state for M_2
(p,q) is accepted by L(M) by construction
so it holds.

Since both are true $L(M) = L(M_1) - L(M_2)$

Answer 7

p = pumping length

$= a^{p^2}$ $s \in L$

$|s| = p^2 \geq p$
, so the Pumping Lemma will hold. So, S can be split into 3 part as s = xyz satisfying
1) $xy^iz \in L$ for each $i \geq 0$
2) $|y| > 0$
3) $|xy| \leq p$

x = a^k
y = a^l
z = a^{p^2-k-l}, where $k + l \leq p$ and $l \geq 0$
when choose i = 2 to get xy^2z
$a^k a^{2l} a^{p^2-k-l} = a^{p^2+l}$
$l > 0 \rightarrow p^2 + l > p$
$l \leq p \rightarrow p^2 + l \leq p^2 + p < p^2 + 2p + 1 = (p+1)^2$

$p^2 < p^2 + l < (p+1)^2$
So $p^2 + l$ cannot be a square of an integer.