SECOND ORDER RUNGE-KUTTA METHODS

Employ Heun's method to solve a first order ODE of a physical process, which has an analytical/experimental/numerical (non-polynomial) solution. In addition, derive a second order RK method of your own by choosing p_1 arbitrarily between 0 and 1, and solve the same problem again. Modify the Fortran code, $euler.f$ for the numerical solutions.

In your report

- Describe the physical process and the mathematical model.
- Obtain numerical solutions for various step sizes, and compare them to the reference solution and the corresponding Euler solutions. Plot all the solutions in a single graph.
- Compare the relative error distributions for the various step sizes on the same graph.
- For a fixed step size in Heun's solution, decrease the step size of the Euler solution till the error distributions in both solutions are about the same. Plot the solutions in a single graph.
- Determine the maximum step size for a convergent solution. Plot the solutions in a single graph.
- Compare the Heun's , the RK2 and the reference solutions together for the same step sizes, and plot their relative error distributions. (2 graphs)
- Iterate on the corrector step of your RK2 method and compare the relative error distributions for various number subiterations on the same graph.
- Discuss the results obtained and draw your conclusions.
- Include the Fortran code developed, but do not include any printed data.

For a bonus:

- Estimate the truncation error at a certain location for various step sizes and assess the order of accuracy of the numerical solution. Note that the leading truncation error term is in the form of $E = |A|\Delta x^n$. When the logarithm of the error estimation, $\log_{10} E$, is plotted with respect to the step size, Δx;

$$\log_{10} E = \log_{10} |A| + n \log_{10} \Delta x$$

the slope of the curve represents the order of accuracy of the numerial method employed.