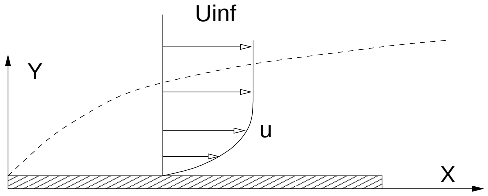
SOLUTION OF A 3^{RD} ORDER ODE



Boundary layer flow on a flat plate

The incompressible boundary layer flow on a flat plate is governed by

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2} \end{aligned}$$

subject to the boundary conditions:

$$\begin{aligned} u(x,0) &= 0 \\ v(x,0) &= 0 \\ u(x,\infty) &= U_\infty \end{aligned}$$

By introducing a new variable $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$ and a stream function $\Psi = \sqrt{\nu x U_\infty} f(\eta)$ such that

$$\begin{aligned} u &= \frac{\partial \Psi}{\partial y} = U_\infty f' \\ v &= -\frac{\partial \Psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f' - f) \end{aligned}$$

the governing equations are reduced to a single 3^{rd} order ODE for the function $f(\eta)$:

$$\boxed{ff'' + 2f''' = 0}$$

subject to the boundary conditions:

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 0 \\ f'(\infty) &= 1 \end{aligned}$$

Integrate the 3^{rd} order ODE given above iteratively by employing the shooting method and the 4^{th} order accurate classical RK method. In your report

- plot the dimensionless velocities U and V versus η at $x = 0.1m,\ 0.5m,\ 1m$ for $U_\infty = 1m/s$ and ν_{air} along the "shooting" steps. (6 plots for U,V and x)

$$\begin{aligned} U &= \frac{u}{U_\infty} \equiv f' \\ V &= \frac{v}{\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}} \equiv \eta f' - f \end{aligned}$$

- plot u and v versus y and η at $x = 0.1m,\ 0.5m,\ 1m$ (12 plots similarly)
- For a bonus, implement an adaptive solution algorithm.