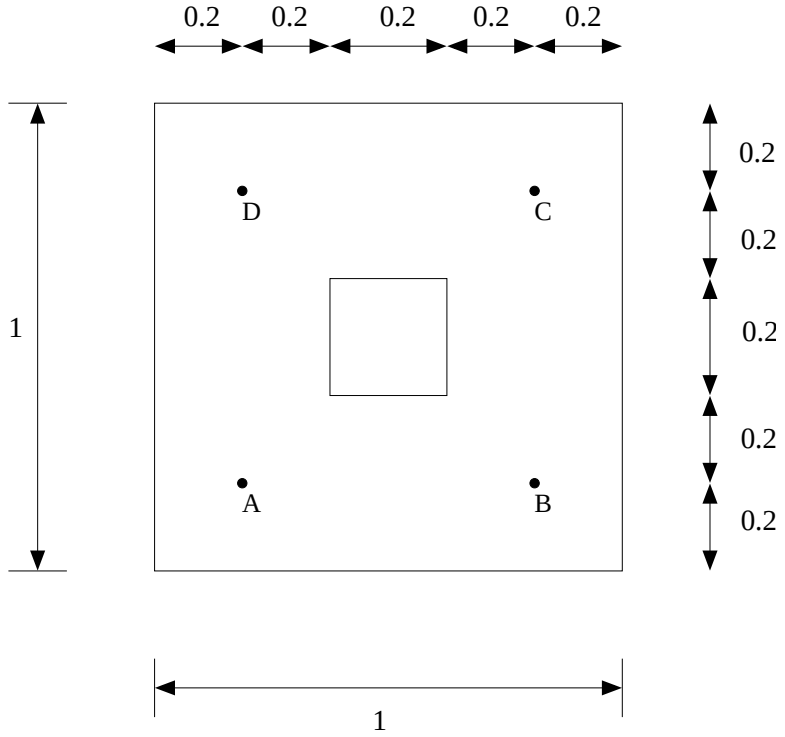
SOLUTION OF AN ELLIPTIC PDE

The non-dimensional transverse displacement of an elastic membrane, u, under a transverse load distribution, f, is governed by the Poisson equation:

$$\frac{\partial u}{\partial x^2} + \frac{\partial u}{\partial y^2} = f$$

Consider a unit square domain with a centered square hole as shown below. On the outer boundaries transverse displacement is zero. On the inner boundaries an upward transverse displacement of magnitude 0.01 is applied. There are 4 downward point forces of magnitude 4 applied at points A, B, C and D.



Solve the Poisson equation for the displacement by using iterative solution methods. Initially take $\Delta x = \Delta y = 0.01$.

- Employ point Jacobi, Gauss-Seidel and SOR methods; plot the convergence histories and compare the convergence rates.
- Compare the u distributions in contour plots and in $x - y$ plots along the mid-sections.
- Experiment with various point force distributions.
- Experiment with different Δx and Δy values.
- For a bonus, solve the system of equations by using the ADI or the direct solution methods.