Homework Report

AE305 - Numerical Methods

Aerospace Engineering Department

Homework #4
Team #8

Authors:
Arda Özuzun (Id:2172765)
Muhammed Kürşat Yurt (Id:2172971)
Yusuf Uzuntaş (Id:1882364)

Date: February 6, 2021

1 Introduction

In this homework, unsteady heat conduction and linear convection partial differential equations are given to develop solutions with proper boundary and initial conditions.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

While the boundary conditions at the each ends are $T(0,t) = 20$ and $T(10,t) = 100$, initial temperature distribution of the body is $T(x;0) = 10$

$$\frac{\partial w}{\partial t} = V \frac{\partial w}{\partial x} \tag{2}$$

For the linear convection equation the boundary conditions at the each ends are $w(-0,t) = 0$ and $\frac{\partial w}{\partial x}|_{(20,t)} = 0$. Also, the initial conditions are defined in this problem as follows:
for $0 \leq x \leq \pi$

$$w(x,0) = 10cos(x)$$

for$x < 0 and x > \pi$

$$w(x,0) = 0$$

The heat conduction equation is used to find out a temperature distribution of a one dimensional body over the time. In this case one dimensional body has a initial temperature distribution and it is exposed to boundary conditions from its ends. The linear convection equation which has also boundary and initial conditions is used to for examination of wave.

These partial differential equations are solved by finite difference method. The FORTRAN code is completed for explicit and implicit solutions. Also, for both equation different types of finite difference expressions (forward-central-backward difference) are used. In addition, solutions of different diffusion and courant number are compared.

For the bonus part initial condition given by:
for $x < -3$ or $x > 3$

$$w(x) = 0$$

for $x \leq -3$ or $x \geq 3$

$$w(x) = |sin(x)|$$

where $-10 \leq x \leq 30$

From nature of the equation for explicit solution FTCS scheme is used for get a convergent solution. Also for implicit solution BTCS scheme is used.

2 Method

Finite Difference Method is a method to solve differential equations by discrediting them. It widely used for partial differential equations (PDE's). In this method differential equations written as finite difference expressions of discrete points in solution domain and then solved numerically.

2.1 Finite difference expressions of heat conduction/diffusion equation

2.1.1 Explicit Forward Time-Backward/Central/Forward Space FDEs

Forward Time Backward Space Formula can be written as follows in terms of finite difference expression:

$$T_i^{n+1} = \frac{\alpha \Delta t}{\Delta x^2}\left(T_i^n - 2T_{i-1}^n + T_{i-2}^n\right) + T_i^n \tag{3}$$

Forward Time Forward Space Formula can be written as follows in terms of finite difference expression:

$$T_i^{n+1} = \frac{\alpha \Delta t}{\Delta x^2}\left(T_i^n - 2T_{i+1}^n + T_{i+2}^n\right) + T_i^n \tag{4}$$

Forward Time Central Space Formula also can be written as follows in terms of finite difference expression:

$$T_i^{n+1} = \frac{\alpha \Delta t}{\Delta x^2}\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + T_i^n \tag{5}$$

2.1.2 Implicit Backward Time-Central Space FDE Constant Temperature Case

The Implicit Backward Time Centered Space Formula can be written as follows in terms of finite difference expression:

$$-\alpha \frac{\Delta t}{\Delta x^2} T_{i-1}^{n+1} + (1 + \alpha \frac{2\Delta t}{\Delta x^2}) T_i^{n+1} - \alpha \frac{\Delta t}{\Delta x^2} T_{i+1}^{n+1} = T_i^n \tag{6}$$

where:

$$d = \alpha \frac{\Delta t}{\Delta x^2} \tag{7}$$

Therefore:

$$-dT_{i-1}^{n+1} + (1 + 2d) T_i^{n+1} - dT_{i+1}^{n+1} = T_i^n \tag{8}$$

Matrix form of this solution is:

$$\underbrace{\begin{bmatrix} (1+2d) & -d & 0 & \dots & 0 & 0 \\ -d & (1+2d) & -d & \dots & 0 & 0 \\ 0 & -d & (1+2d) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (1+2d) & -d \\ 0 & 0 & 0 & \dots & -d & (1+2d) \end{bmatrix}}_{\textbf{A}} \underbrace{\begin{bmatrix} T_2^{n+1} \\ T_3^{n+1} \\ T_4^{n+1} \\ \vdots \\ T_{imax-2}^{n+1} \\ T_{imax-1}^{n+1} \end{bmatrix}}_{\textbf{T}} = \underbrace{\begin{bmatrix} (T_2^n + dT_1^{n+1}) \\ T_3^n \\ T_4^n \\ \vdots \\ T_{imax-2}^n \\ (T_{imax-1}^n + dT_{imax}^{n+1}) \end{bmatrix}}_{\textbf{RHS}}$$

2.1.3 Explicit Backward Time-Central Space FDE Constant Isolated Boundary Case

The Explicit Backward Time Centered Space Formula can be written as follows in terms of finite difference expression:

$$T_i^{n+1} = \frac{\alpha \Delta t}{\Delta x^2} \left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + T_i^n \tag{9}$$

where :

$$d = \alpha \frac{\Delta t}{\Delta x^2} \tag{10}$$

And because the boundary condition at x = 0 as an insulated wall, this equation is used:

$$T_1^{n+1} = T_2^{n+1} \tag{11}$$

Therefore:

$$T_i^{n+1} = d\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + T_i^n \tag{12}$$

2.1.4 Implicit Backward Time-Central Space FDE Constant Isolated Boundary Case

The Implicit Backward Time Centered Space Formula can be written as follows in terms of finite difference expression:

$$-\alpha \frac{\Delta t}{\Delta x^2} T_{i-1}^{n+1} + (1 + \alpha \frac{2\Delta t}{\Delta x^2}) T_i^{n+1} - \alpha \frac{\Delta t}{\Delta x^2} T_{i+1}^{n+1} = T_i^n \tag{13}$$

where :

$$d = \alpha \frac{\Delta t}{\Delta x^2} \tag{14}$$

Therefore:

$$-dT_{i-1}^{n+1} + (1+2d)T_i^{n+1} - dT_{i+1}^{n+1} = T_i^n \tag{15}$$

And also there is isolated surface when x equals to zero so T_1^{n+1} equals to T_2^{n+1}

Matrix form of this solution is:

$$\underbrace{\begin{bmatrix} (1+d) & -d & 0 & \dots & 0 & 0 \\ -d & (1+2d) & -d & \dots & 0 & 0 \\ 0 & -d & (1+2d) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (1+2d) & -d \\ 0 & 0 & 0 & \dots & -d & (1+2d) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} T_2^{n+1} \\ T_3^{n+1} \\ T_4^{n+1} \\ \vdots \\ T_{imax-2}^{n+1} \\ T_{imax-1}^{n+1} \end{bmatrix}}_{\mathbf{T}} = \underbrace{\begin{bmatrix} (T_2^n) \\ T_3^n \\ T_4^n \\ \vdots \\ T_{imax-2}^n \\ (T_{imax-1}^n + dT_{imax}^{n+1}) \end{bmatrix}}_{\mathbf{RHS}}$$

2.2 Finite difference expressions of linear convection equation

2.2.1 Explicit Backward Time-Backward/Central Space FDEs

Forward Time Backward Space Formula can be written as follows in terms of finite difference expression:

$$w_i^{n+1} = w_i^n - \nu \frac{\Delta t}{\Delta x} \left(w_i^n - w_{i-1}^n \right) \tag{16}$$

Forward Time Central Space Formula can be written as follows in terms of finite difference expression:

$$w_i^{n+1} = w_i^n - \nu \frac{\Delta t}{2\Delta x} \left(w_{i+1}^n - w_{i-1}^n \right) \tag{17}$$

Also, because $\frac{\partial w}{\partial x} \mid_{(20,t)}$ equal to $zero$, not only explicit forward time backward space but also explicit forward time central space utilize following equation:

$$w_{imax}^{n+1} = w_{imax-1}^{n+1} \tag{18}$$

2.2.2 Implicit Backward Time-Central Space FDE

Implicit Backward Time Central Space Formula can be written as follows in terms of finite difference expression:

$$-\nu \frac{\Delta t}{2\Delta x} w_{i-1}^{n+1} + w_i^{n+1} + \nu \frac{\Delta t}{2\Delta x} w_{i+1}^{n+1} = w_i^n \tag{19}$$

And :

$$\sigma = \nu \frac{\Delta t}{\Delta x} \tag{20}$$

Therefore:

$$-\frac{\sigma}{2}w_{i-1}^{n+1}+w_i^{n+1}+\frac{\sigma}{2}w_{i+1}^{n+1}=w_i^n \tag{21}$$

And also:

$$\frac{\partial w}{\partial x}\mid_{(20,t)=20} \tag{22}$$

Thus, w_{imax}^{n+1} equals to w_{imax-1}^{n+1} Matrix form of this solution is:

$$\underbrace{\begin{bmatrix} 1 & \frac{\sigma}{2} & 0 & \dots & 0 & 0 \\ -\frac{\sigma}{2} & 1 & \frac{\sigma}{2} & \dots & 0 & 0 \\ 0 & -\frac{\sigma}{2} & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & \frac{\sigma}{2} \\ 0 & 0 & 0 & \dots & -\frac{\sigma}{2} & 1+\frac{\sigma}{2} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} w_2^{n+1} \\ w_3^{n+1} \\ w_4^{n+1} \\ \vdots \\ w_{imax-2}^{n+1} \\ w_{imax-1}^{n+1} \end{bmatrix}}_{\mathbf{w}} = \underbrace{\begin{bmatrix} (w_2^n+\frac{\sigma}{2}w_1^{n+1}) \\ w_3^n \\ w_4^n \\ \vdots \\ w_{imax-2}^n \\ w_{imax-1}^n \end{bmatrix}}_{\mathbf{RHS}}$$

2.3 Finite difference expressions of 1-D convection-diffision equation

1-D convection-diffusion equation is as following:

$$\frac{\partial w}{\partial t}+\nu\frac{\partial w}{\partial x}=\alpha\frac{\partial^2 w}{\partial x^2} \tag{23}$$

This equation is solved by Explicit Forward Time Central Space FDE and Implicit Backward Time Central Space FDE.

2.3.1 Explicit Forward Time-Central Space FDE

Explicit Forward Time Central Space Formula can be written as follows in terms of finite difference expression:

$$w_i^{n+1}=(-\nu\frac{\Delta t}{2\Delta x}+\alpha\frac{\Delta t}{\Delta x^2})w_{i+1}^n+(1+\alpha\frac{2\Delta t}{\Delta x^2})w_i^n+(\nu\frac{\Delta t}{2\Delta x}+\alpha\frac{\Delta t}{\Delta x^2})w_{i-1}^n \tag{24}$$

where:

$$d = \alpha \frac{\Delta t}{\Delta x^2} \tag{25}$$

$$\sigma = \nu \frac{\Delta t}{\Delta x} \tag{26}$$

Thus:

$$w_i^{n+1} = (-\frac{\sigma}{2} + d)w_{i+1}^n + (1 + 2d)w_i^n + (\frac{\sigma}{2} + d)w_{i-1}^n \tag{27}$$

2.3.2 Implicit Backward Time-Central Space FDE

Implicit Backward Time Central Space Formula can be written as follows in terms of finite difference expression:

$$(-\nu\frac{\Delta t}{2\Delta x} - \alpha\frac{\Delta t}{\Delta x^2})w_{i-1}^{n+1} + (1 + \alpha\frac{2\Delta t}{\Delta x^2})w_i^{n+1} + (\nu\frac{\Delta t}{2\Delta x} - \alpha\frac{\Delta t}{\Delta x^2})w_{i+1}^{n+1} = w_i^n \tag{28}$$

And:

$$d = \alpha \frac{\Delta t}{\Delta x^2} \tag{29}$$

$$\sigma = \nu \frac{\Delta t}{\Delta x} \tag{30}$$

Thus:

$$(-\frac{\sigma}{2} - d)w_{i-1}^{n+1} + (1 + 2d)w_i^{n+1} + (\frac{\sigma}{2} - d)w_{i+1}^{n+1} = w_i^n \tag{31}$$

Matrix form of this solution is:

$$\underbrace{\begin{bmatrix} (1+2d) & (\frac{\sigma}{2}-d) & 0 & \dots & 0 & 0 \\ (-\frac{\sigma}{2}-d) & (1+2d) & (\frac{\sigma}{2}-d) & \dots & 0 & 0 \\ 0 & (-\frac{\sigma}{2}-d) & (1+2d) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (1+2d) & (\frac{\sigma}{2}-d) \\ 0 & 0 & 0 & \dots & (-\frac{\sigma}{2}-d) & (1+2d) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} w_2^{n+1} \\ w_3^{n+1} \\ w_4^{n+1} \\ \vdots \\ w_{imax-2}^{n+1} \\ w_{imax-1}^{n+1} \end{bmatrix}}_{\mathbf{w}} = \underbrace{\begin{bmatrix} (w_2^n + (\frac{\sigma}{2}+d)w_1^{n+1}) \\ w_3^n \\ w_4^n \\ \vdots \\ w_{imax-2}^n \\ (w_{imax-1}^n + (-\frac{\sigma}{2}+d)w_{imax-1}^{n+1}) \end{bmatrix}}_{\mathbf{RHS}}$$

2.4 Crank-Nicolson Method for Heat Diffusion Equation

Crank-Nicolson Method is a $2^n d$ order accurate Implicit Method. Crank-Nicolson Method Formula can be written as follows in terms of finite difference expression:

$$-\frac{\alpha\Delta t}{2\Delta x^2}T_{i-1}^{n+1} + (1 + \frac{\alpha\Delta t}{\Delta x^2})T_i^{n+1} - \frac{\alpha\Delta t}{2\Delta x^2}T_{i+1}^{n+1} = \frac{\alpha\Delta t}{2\Delta x^2}T_{i-1}^n + (1 - \frac{\alpha\Delta t}{\Delta x^2})T_i^n + \frac{\alpha\Delta t}{2\Delta x^2}T_{i+1}^n \tag{32}$$

where:

$$d = \alpha \frac{\Delta t}{\Delta x^2} \tag{33}$$

Thus:

$$-\frac{d}{2}T_{i-1}^{n+1} + (1+d)T_i^{n+1} - \frac{d}{2}T_{i+1}^{n+1} = \frac{d}{2}T_{i-1}^{n} + (1-d)T_i^{n} + \frac{d}{2}T_{i+1}^{n} \tag{34}$$

where:

$$RHS(i) = \frac{d}{2}T_{i-1}^{n} + (1-d)T_i^{n} + \frac{d}{2}T_{i+1}^{n} \tag{35}$$

So the final version of this equation is as follows:

$$-\frac{d}{2}T_{i-1}^{n+1} + (1+d)T_i^{n+1} - \frac{d}{2}T_{i+1}^{n+1} = RHS(i) \tag{36}$$

Matrix form of this solution is:

$$\underbrace{\begin{bmatrix} (1+d) & -\frac{d}{2} & 0 & \dots & 0 & 0 \\ -\frac{d}{2} & (1+d) & -\frac{d}{2} & \dots & 0 & 0 \\ 0 & -\frac{d}{2} & (1+d) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (1+d) & -\frac{d}{2} \\ 0 & 0 & 0 & \dots & -\frac{d}{2} & (1+d \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} T_2^{n+1} \\ T_3^{n+1} \\ T_4^{n+1} \\ \vdots \\ T_{imax-2}^{n+1} \\ T_{imax-1}^{n+1} \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} (RHS(2) + \frac{d}{2}T_1^{n+1}) \\ RHS(3) \\ RHS(4) \\ \vdots \\ RHS(imax-2) \\ RHS(imax-1) + \frac{d}{2}T_{imax}^{n+1} \end{bmatrix}}_{\mathbf{RHS}}$$

3 Results and Discussion

3.1 Conduction Diffusion Equation

3.1.1 Explicit Solution

Explicit Forward Time Backward Space

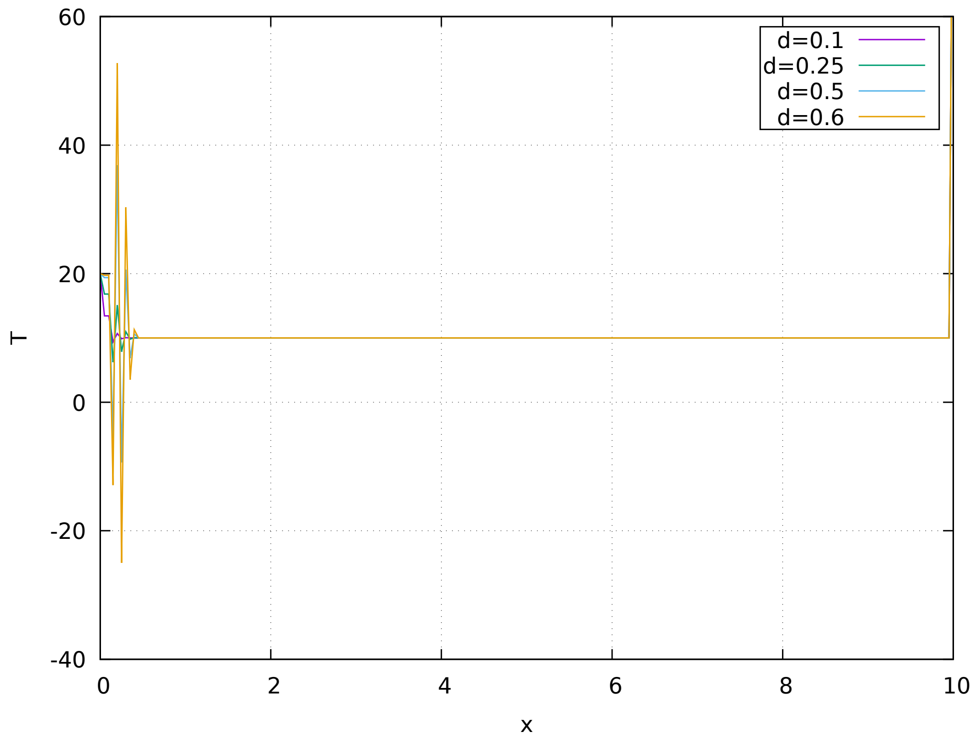

Figure 1: x versus Temperature for different diffusion number at the same steptime

It can be observed from figure 1, for all diffusion number the solution fluctuates. However, as the diffusion number increase fluctuation become more apparent for this stepsize. This solution method is not appropriate in this problem.

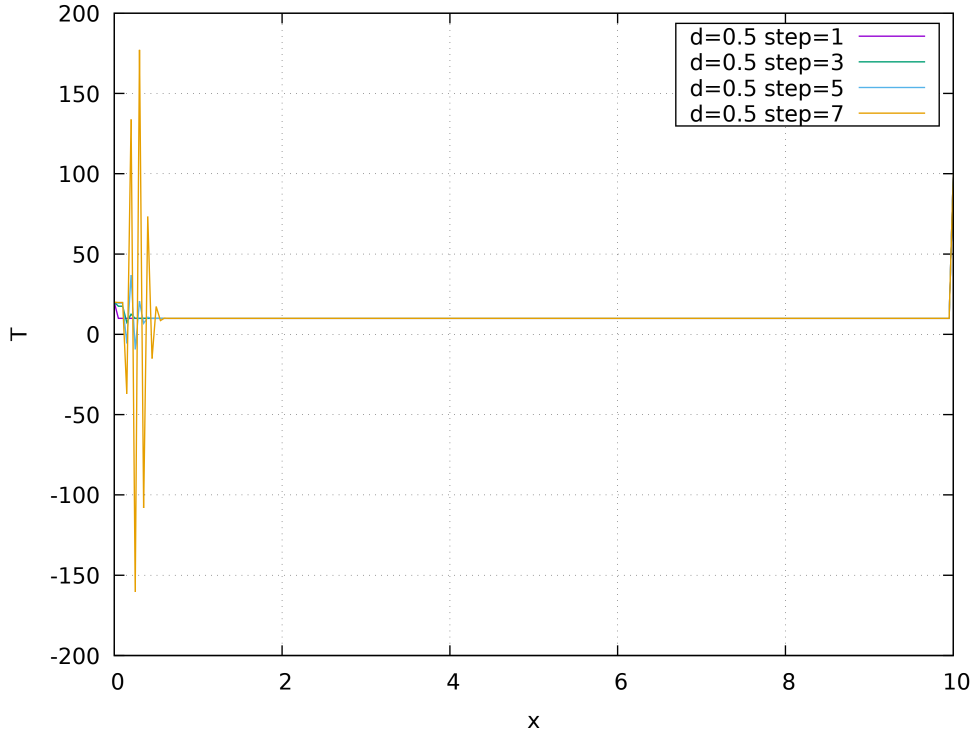

Figure 2: x versus Temperature at different timesteps for a single diffusion number

It can be seen from figure 2 that temperature fluctuates in all step and also it can been said that there are more fluctuation in the high step. This solution method is not appropriate in this problem.

Explicit Forward Time Central Space

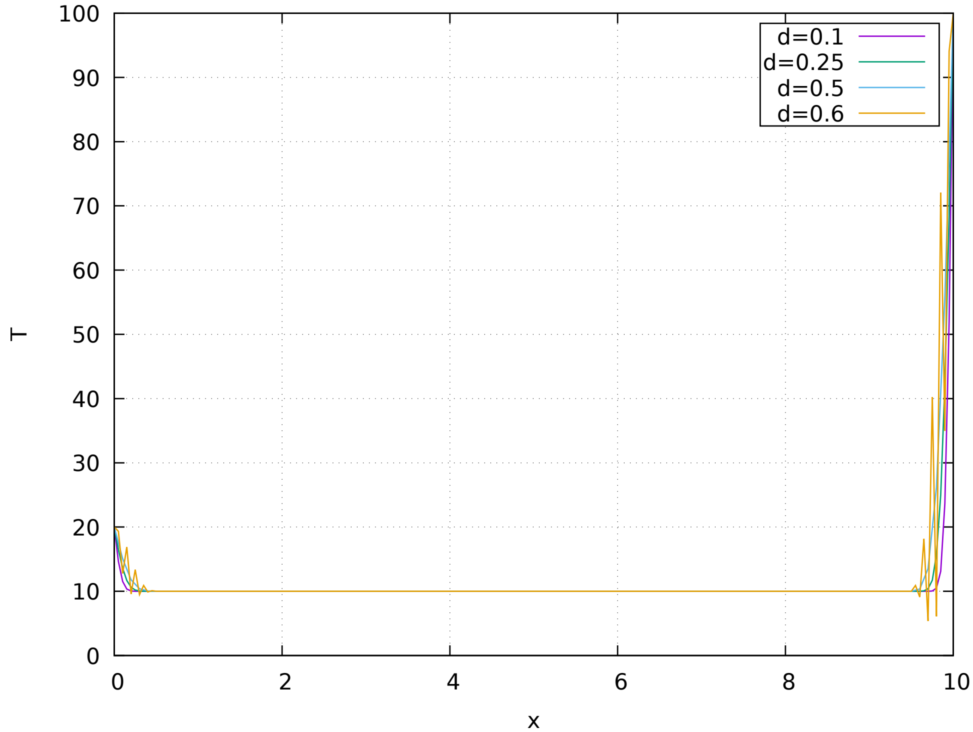

Figure 3: x versus Temperature for different diffusion number at the same timestep

In the Figure 3, the best diffusion number is 0.5 because the solution converges slower when diffusion number is less than 0.5. Also the solution diverges when diffusion number is more than 0.5. Hence, to optimize calculation time it should be chosen diffusion number of 0.5.

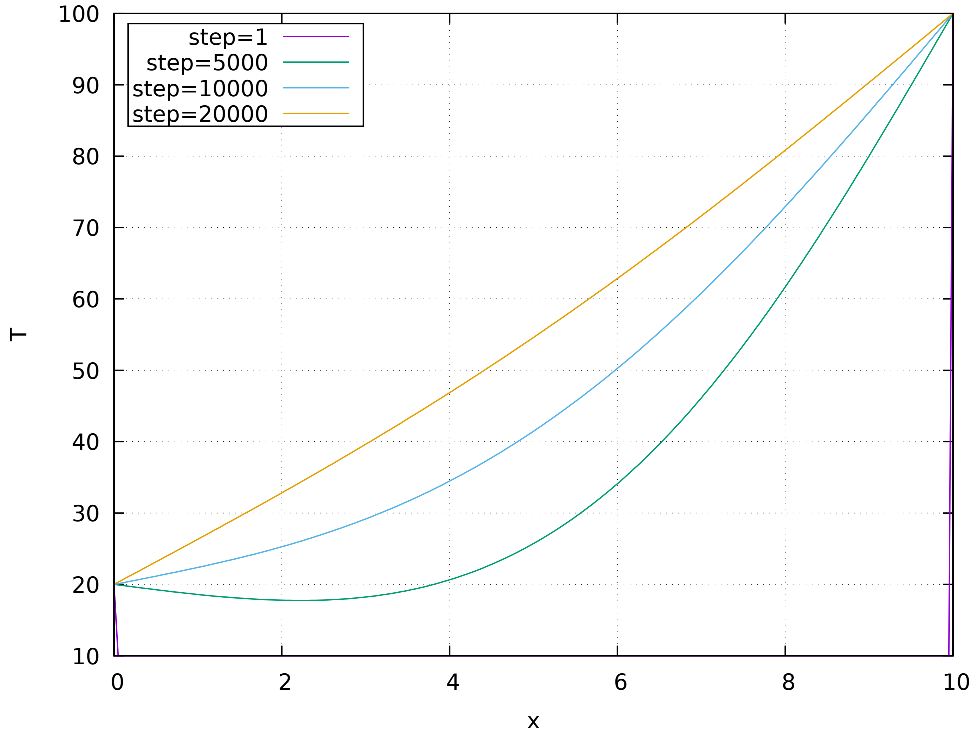

Figure 4: x versus Temperature at different timesteps for a single diffusion number

It can be seen from figure 4 that for as the timestep increases it converges to steady state solution as expected. After it is converged it has a linear temperature distribution.

Explicit Forward Time Forward Space

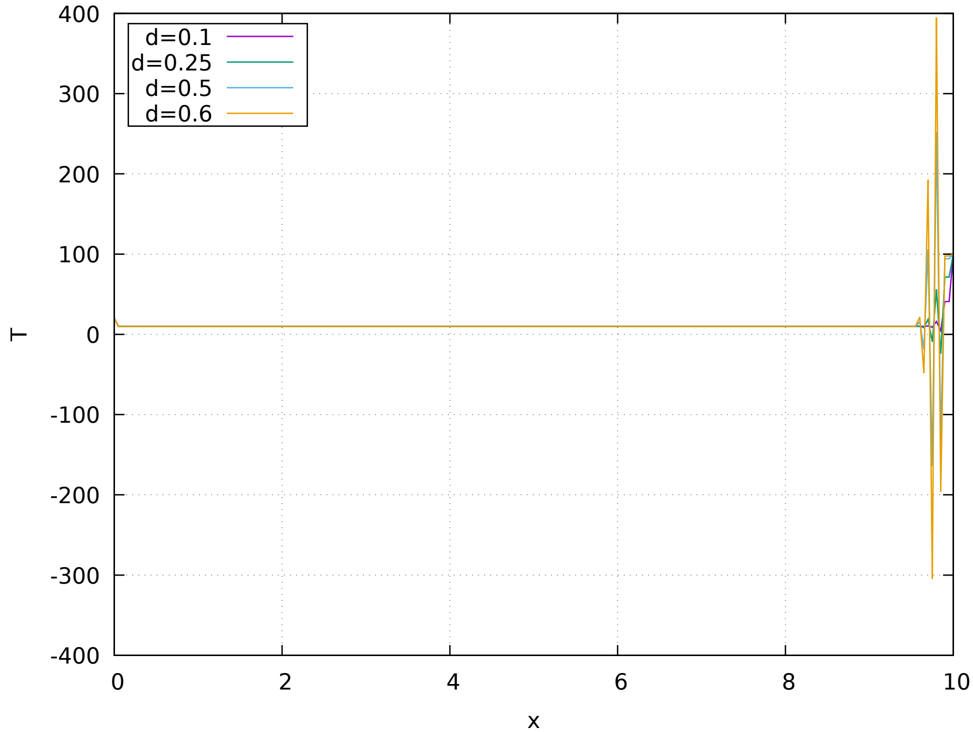

Figure 5: x versus Temperature for different diffusion number at the same timestep

It is clear that from figure 5 for all diffusion number the solution starts to fluctuate from right boundary. However,as the diffusion number increase fluctuation become more apparent. This solution method is not appropriate in this problem.

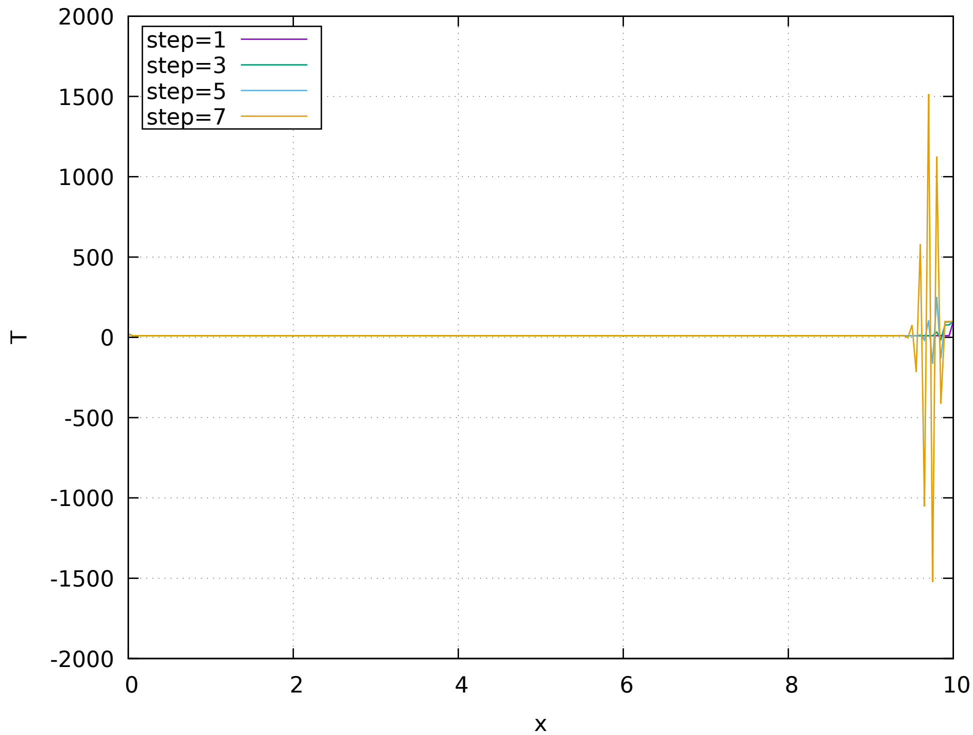

Figure 6: x versus Temperature at different timesteps for a single diffusion number

It is observable that in figure 6 the temperature fluctuates for all steps and also it can been said that there are more fluctuation as timestep increase. This solution method is not appropriate in this problem.

Discussing Observations

From that part of the homework it is can be learned that for every different problem requires different solution scheme. Also, convergent solution schemes have some limitations to converge.

3.1.2 Implicit Backward Time Central Space

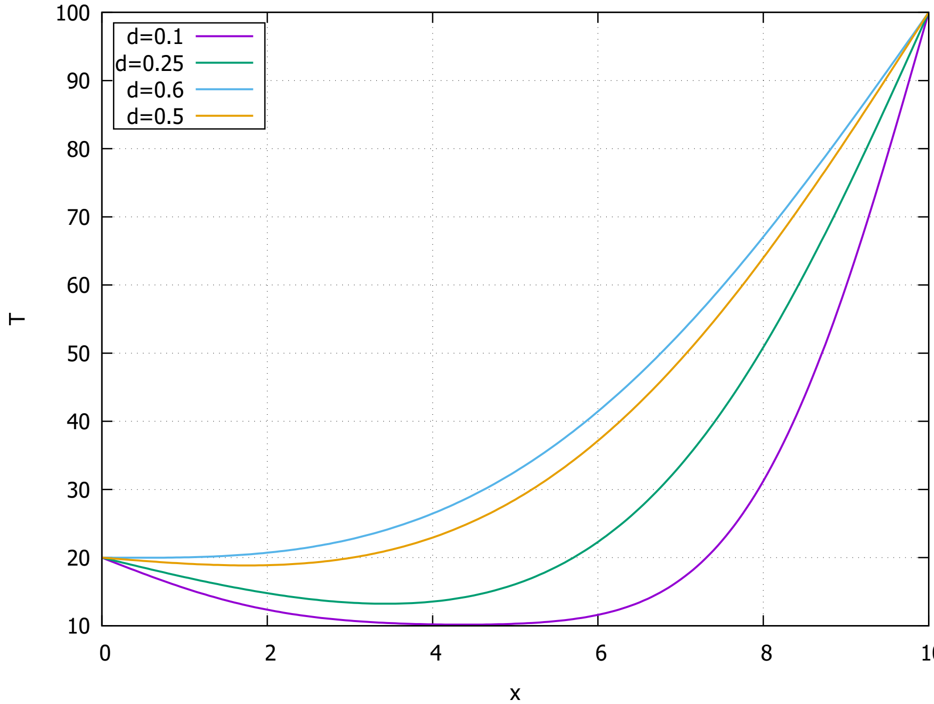

Figure 7: x versus temperature at different diffusion number at the same timestep

In figure 7, it can be seen that when diffusion number increases, solution converges faster in the implicit backward time central space scheme.

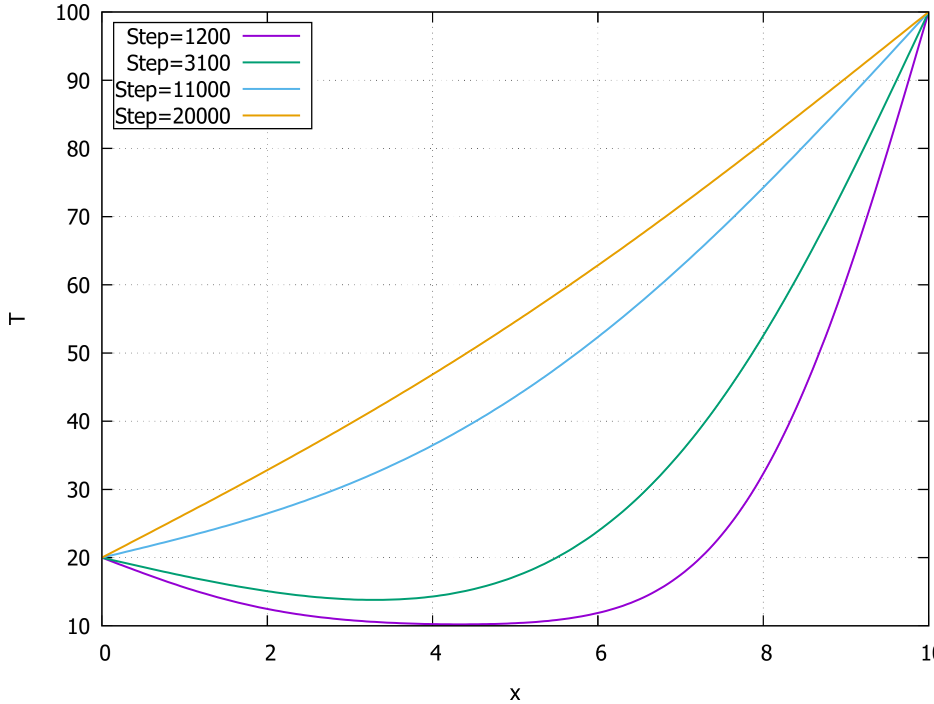

Figure 8: x versus temperature at different timesteps for a single diffusion number

In figure 8, it is obvious that as the timestep increases, solution is getting closer to converge. After solution is converged, linear temperature distribution is clearly seen.

3.1.3 Insulated wall

Explicit Forward Time Central Space

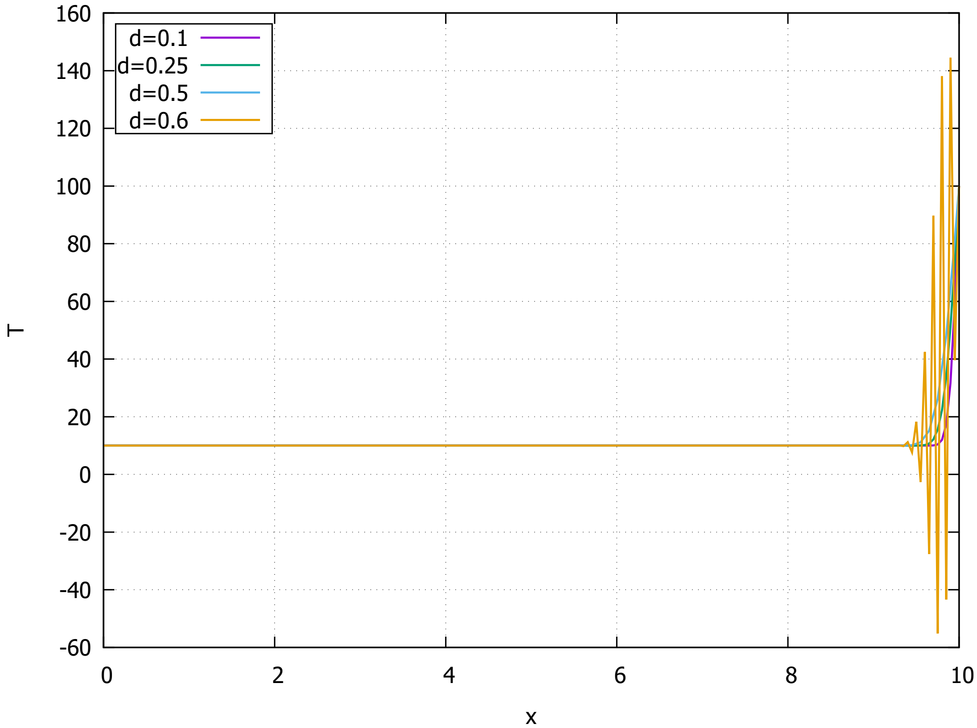

Figure 9: x versus Temperature for a different diffusion number at same timestep

It can be observed from figure 9, as the diffusion number increase until 0.5, the solution converge faster but after diffusion number passes to 0.5, such as 0.6 in the figure 9, the solution diverges when explicit forward time central space method is used.

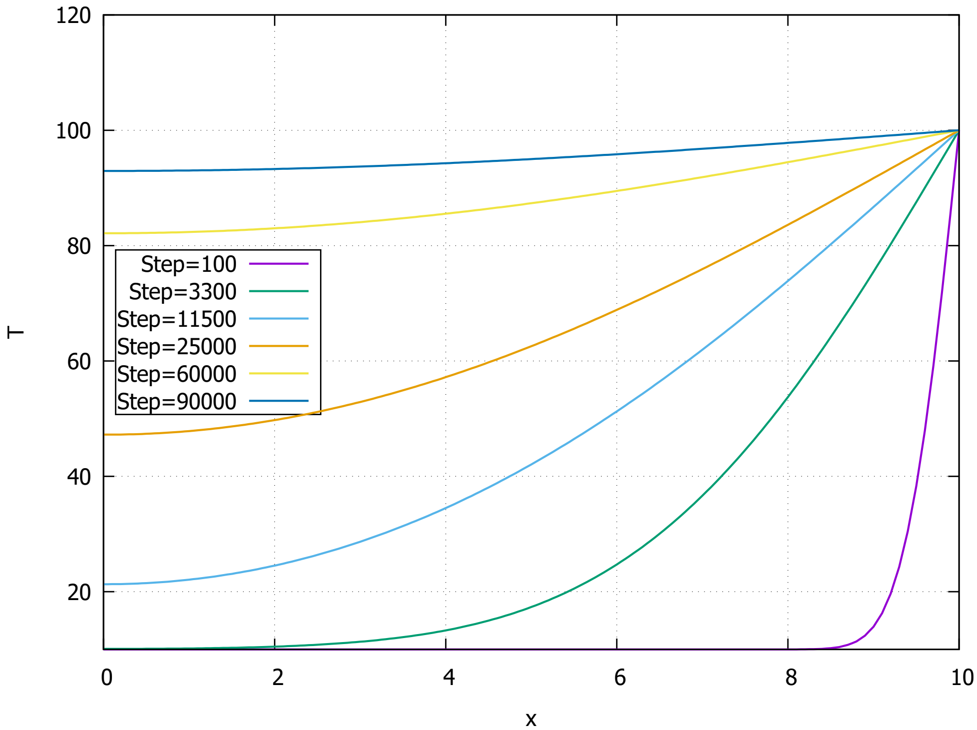

Figure 10: x versus T for same diffusion number at different timestep

In figure 10, it is obvious that as the timestep increases temperature value of the right boundary is constant and equals to 100 ° C whereas value of left boundary which is insulated increase with time. At the beginning heat diffuses from right to the left as expected. As well as higher timestep it closes to the steady state solution.

Implicit Backward Time Central Space

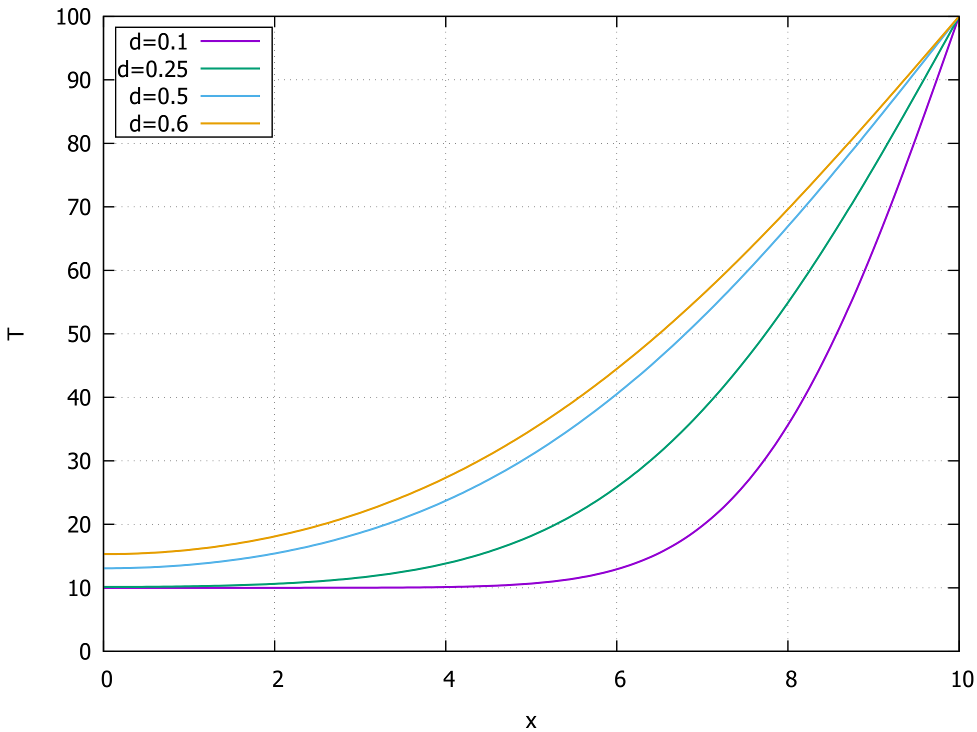

Figure 11: x versus T for different diffusion number at same timestep

The Figure 11 shows that for high diffusion number solution converges faster. Also, right boundary value is fixed for all diffusion numbers because it is the boundary condition and independent of the diffusion number.

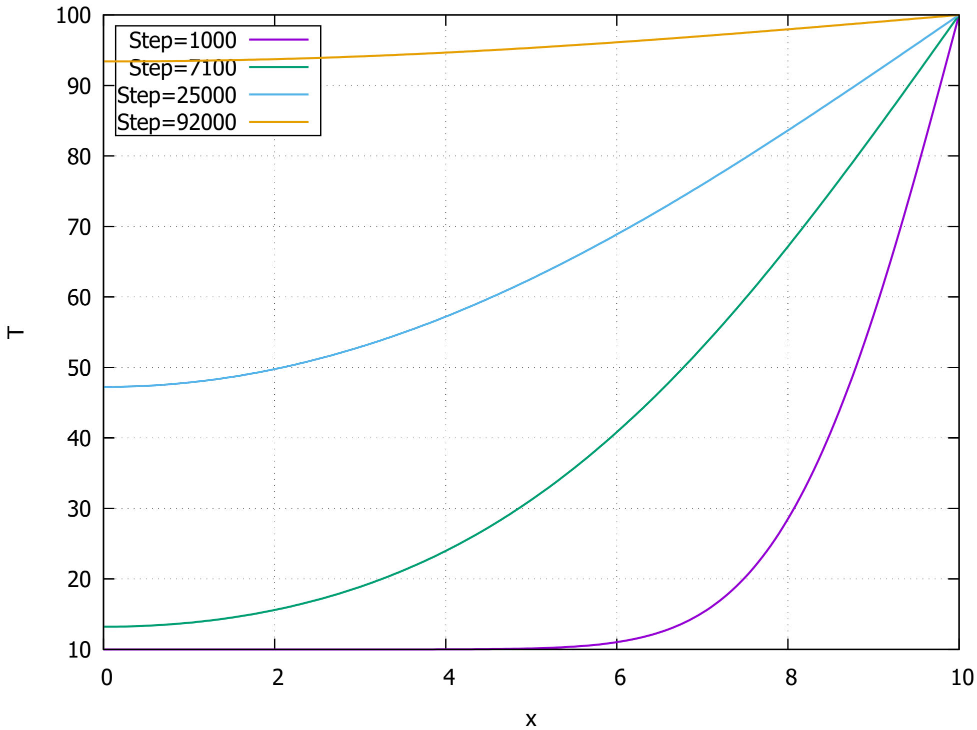

Figure 12: x versus T at different timestep for same diffusion number

In figure 12, it is obvious that as the timestep increases temperature value of the right boundary does not change and equals to 100°C whereas value of left boundary which is insulated increase with time.

3.2 Linear Convection Equation

3.2.1 Explicit Solutions

Explicit Forward Time Central Space

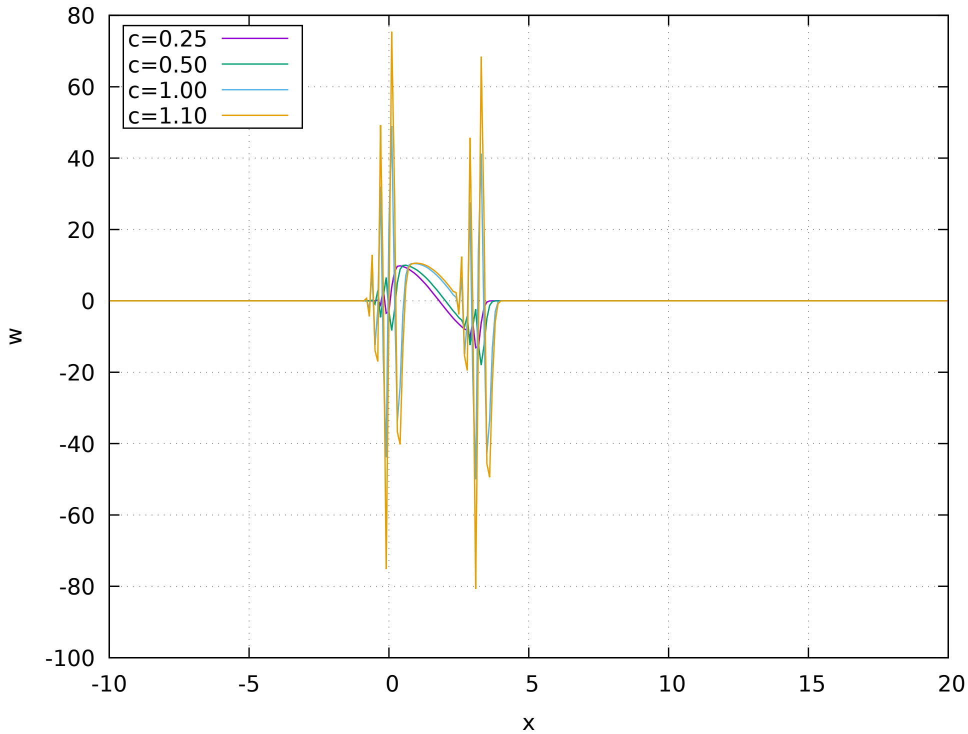

Figure 13: x versus w for different courant numbers at the same time

In Figure 13 it is clear that for all courant numbers fluctuation can be seen. Also as the courant number increase fluctuation is more visible. This method is not appropriate for solution.

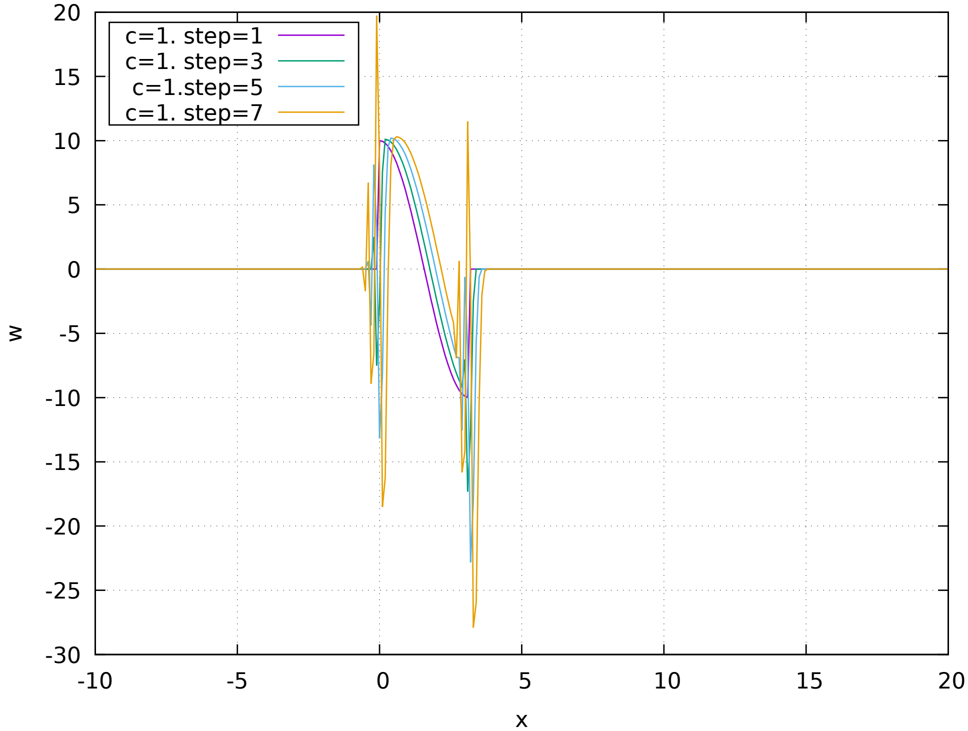

Figure 14: x versus w for same courant number at different timesteps

From figure 14 it can be observable that for courant number 1 the fluctuation of the solution gives higher values as well as timestep increase. Hence, this solution method is not applicable for the given problem.

Explicit Forward Time Backward Space

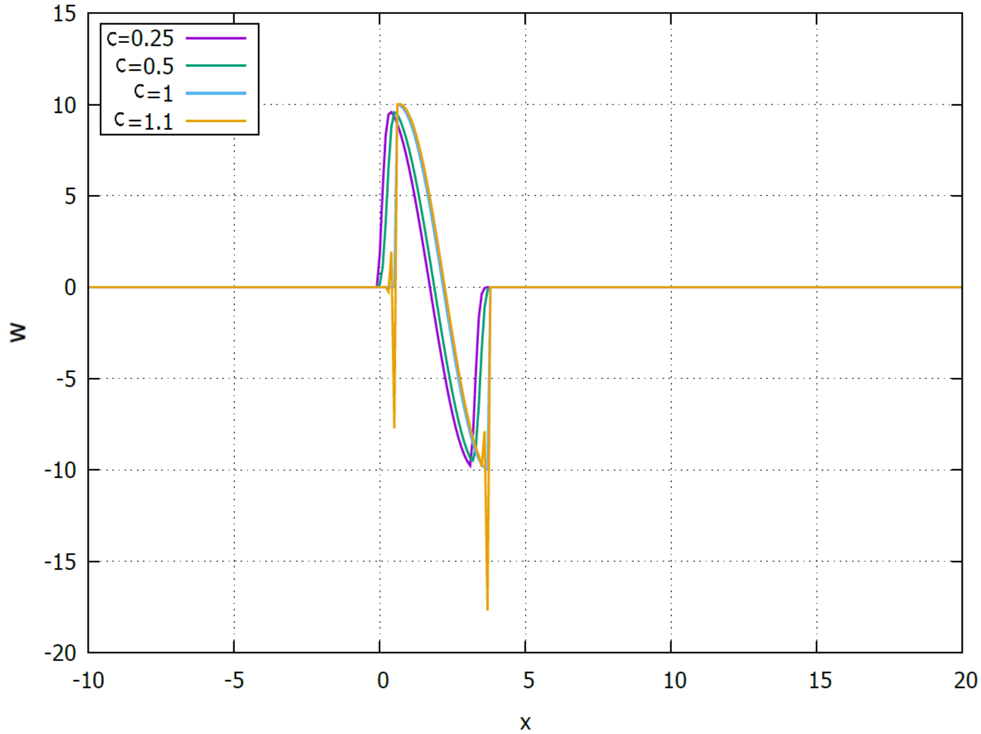

Figure 15: x versus w for different courant numbers at the same time

In figure 15, as courant number are increase until 1, there are no significant changes happen just some numerical diffusion occurs. However, fluctuation rises while courant number exceeds to 1.

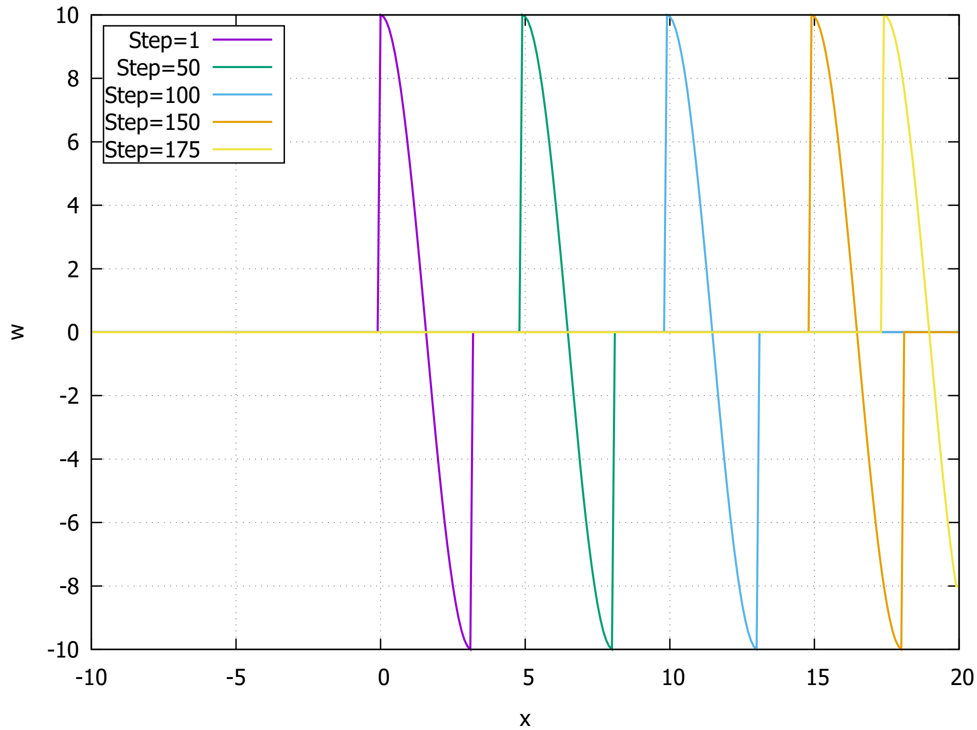

Figure 16: x versus w for same courant number at different timesteps

From figure 16 it can be said that heat convects without any promlem.

3.2.2 Implicit Backward Time Central Space

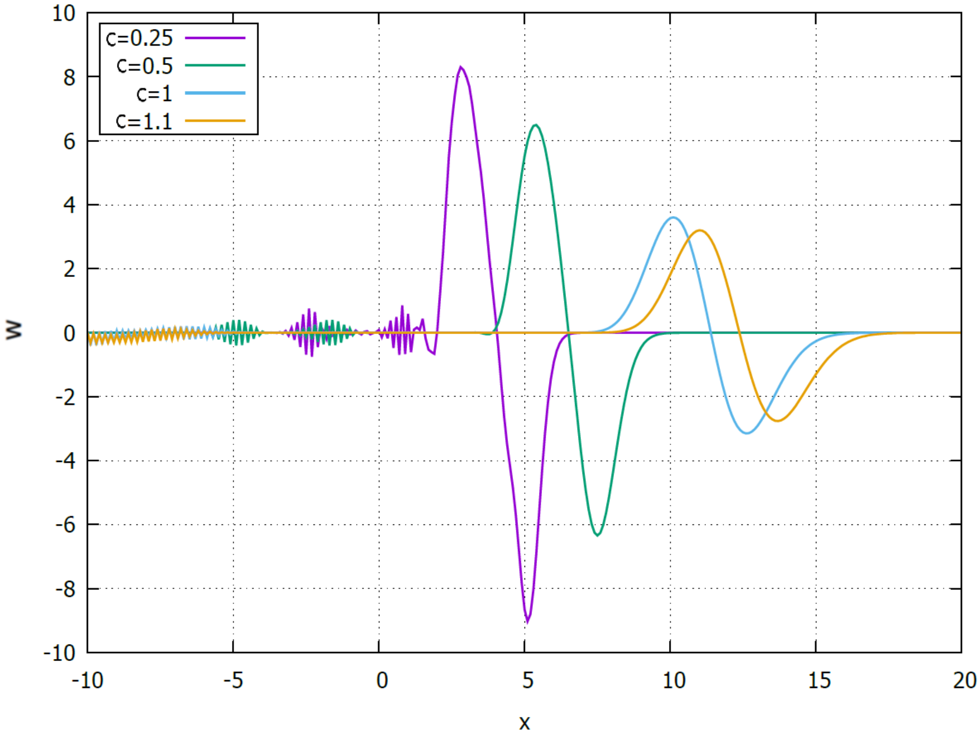

Figure 17: x versus w for different Courant numbers at the same time

From figure 17, it is observed that even though we use implicit solution there some unintentional fluctuations observed behind the initial condition.

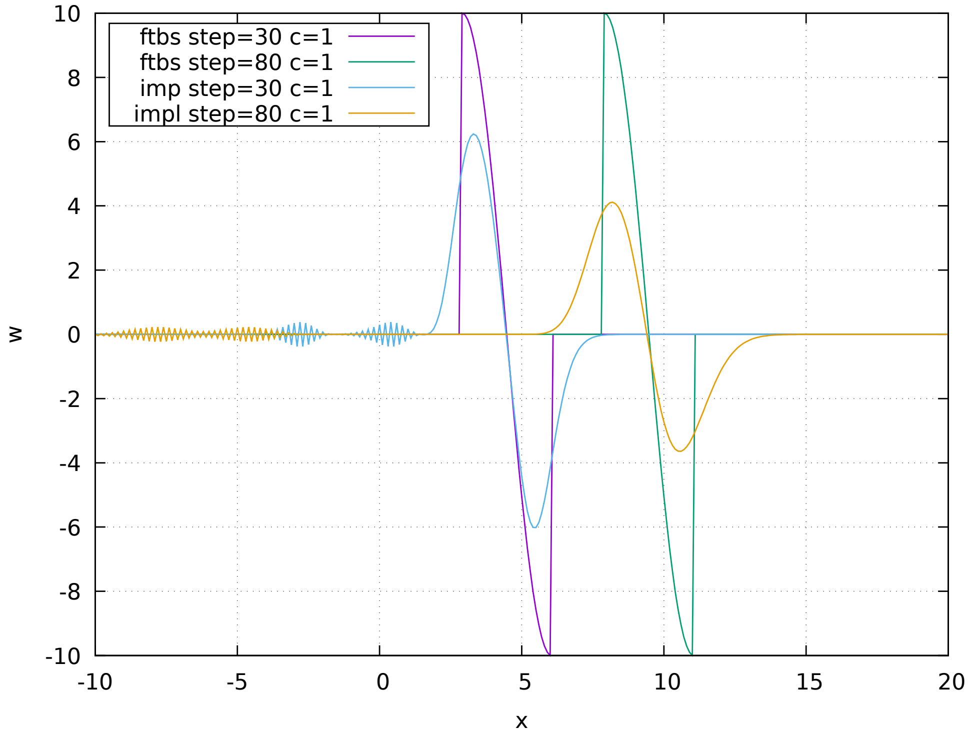

Figure 18: x versus w for same Courant number at different timesteps

3.3 Convection Diffusion Equation

Explicit Solution using Forward Time Central Difference Scheme

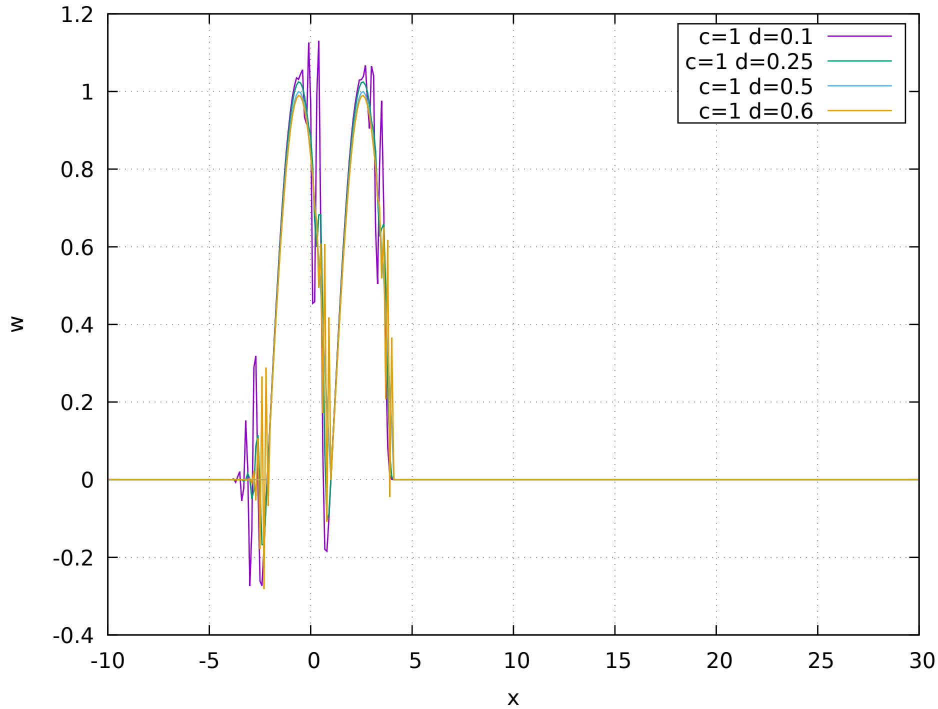

Figure 19: At same timestep with courant number of 1 different diffusion numbers

It is obvious at figure 19 all solutions expect c=1 and d=0.5 fluctuates, which means does not give the solution for given problem expect mentioned case.

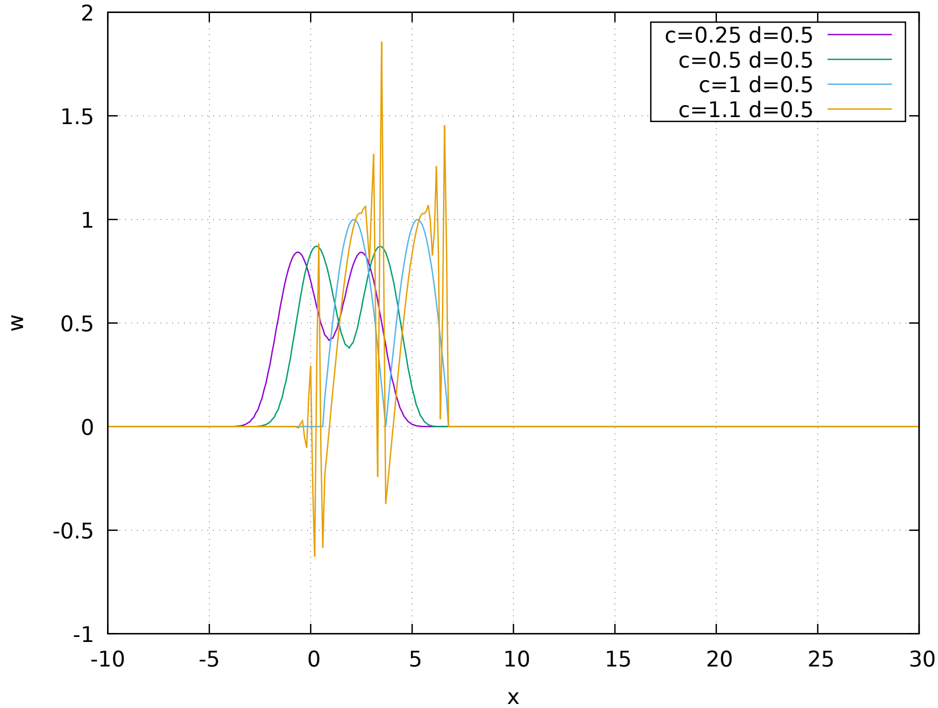

Figure 20: At same timestep with diffusion number of 0.5 different courant numbers

From figure 20, if courant number is less than one, it can be observed that solution is numerically diffuses. Whenever, it is clear that courant number is 1 and diffusion number is equal to 0.5 solution can be obtained without any problem.It is also observable that, whenever courant number is higher than 1 the solution diverges.

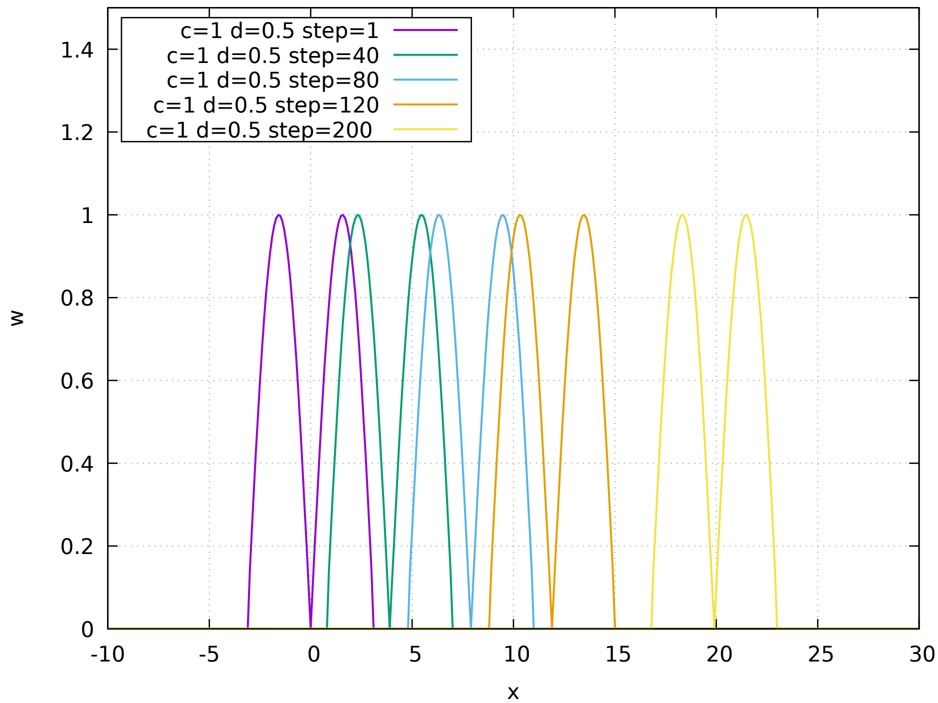

Figure 21: Different timesteps with diffusion number of 0.5 and courant number of 1

As mentioned above courant number is 1 and diffusion number is 0.5 the solution can be obtained. Hence, courant number and diffusion number for convergent case is only 1 and 0.5 respectively.

Implicit Solution using Backward Time Central Space Scheme

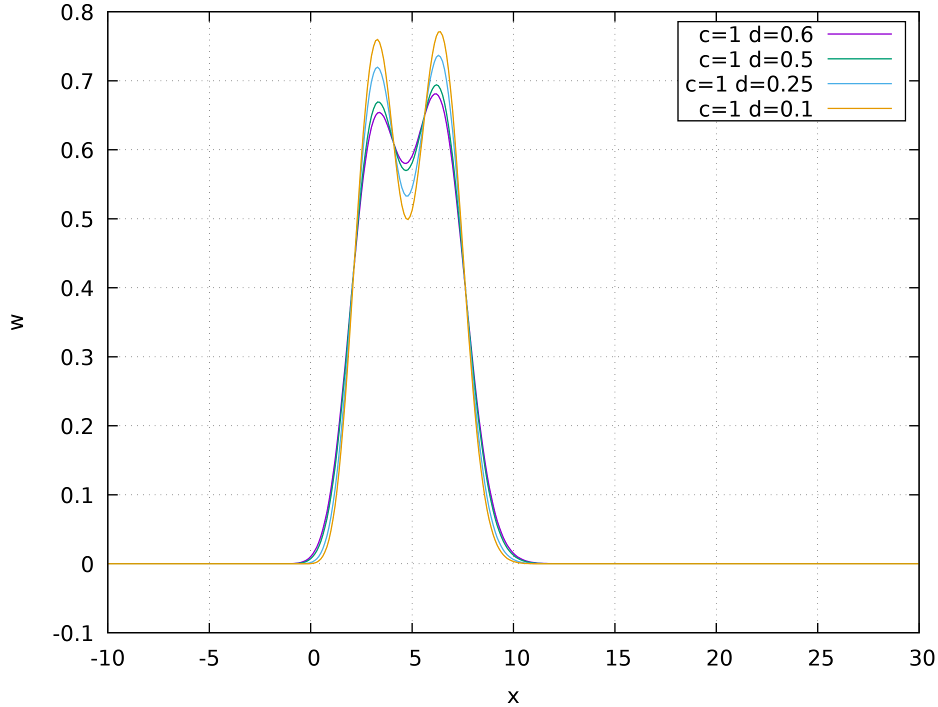

Figure 22: Implicit method courant number of 1 at timestep

From figure 22, it can be observable that as well as the diffusion number is get higher the solution diffuses more rapidly.

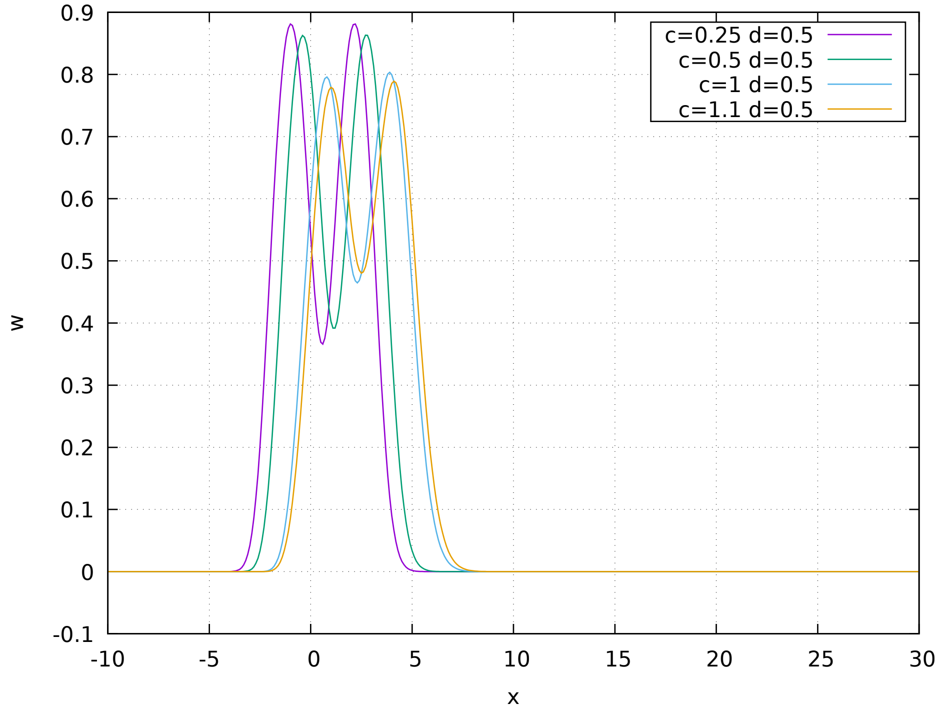

Figure 23: Implicit method diffusion number of 1 at timestep

From figure 23, it can be observable that as well as the courant number is increase the solution diffuses more rapidly.

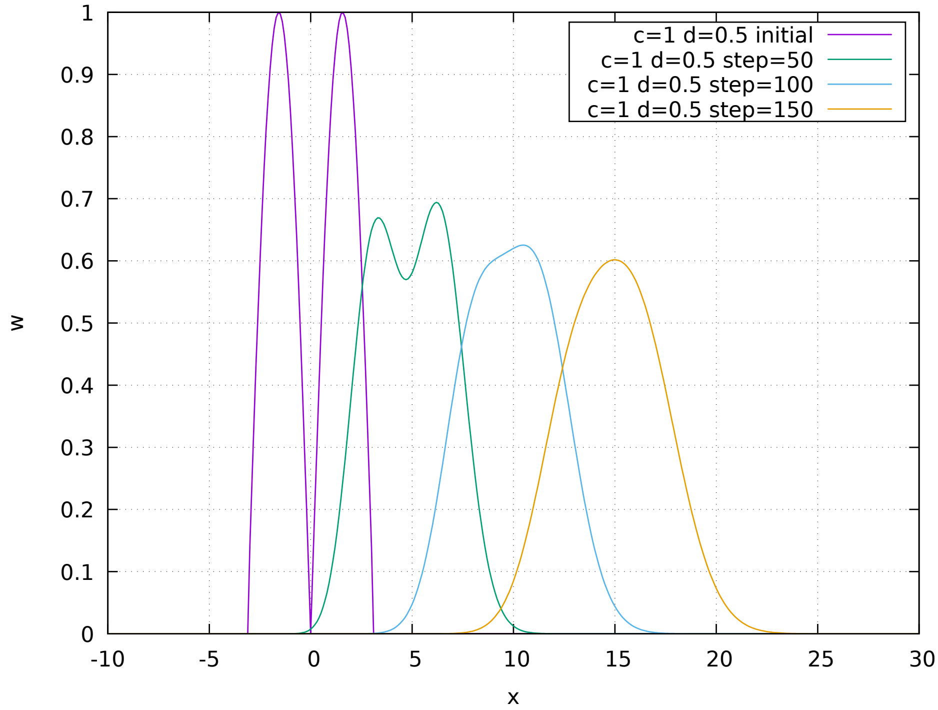

Figure 24: Implicit solution using courant number of 1 and diffusion number of 0.5 at different timesteps

From figure 24, it is observable that as well as the solution moves forward it diffuses same time it convects. From our heat classes knowledge this is expected solution.

4 Conclusion

In conclusion, temperature distribution of a one dimensional body is found by utilization partial differential heat conduction/diffusion equation. Linear convection equation is used in order to observe to how heat convects in a body.Heat conduction and linear convection equation are solved by using either explicit or implicit method and also it is observed that different explicit and implicit methods affect solution. When heat diffusion conduction equation are solved by using explicit forward time backward space or explicit forward time forward space, the solution diverges. Thus, this methods are not appropriate in heat conduction equation. On the contrary, when heat diffusion equation is solved by using explicit forward time central space, the solution converges but the solution also diverges while diffusion number is greater than 0.5. It can been said that diffusion number restrict the explicit methods. Furthermore, diffusion number approaches to 0.5 in order to converge faster. Implicit method is more useful than explicit method because the solution never diverges when diffusion number is greater than 0.5. Even if they requires more coding work they give more accurate results than explicit solutions. The linear convection equation are solved by explicit forward time backward space, forward time central space and implicit methods. Implicit method is gives more accurate result than explicit convergent one. The forward time forward backward space is converges whenever required conditions satisfied. However, forward time central space scheme is not appreciate for this equation it will always diverges. For convection diffusion equation both implicit and explicit convergent methods are valid. However, explicit solution has more restricted conditions than implicit method as expected from previous cases.