Homework Report

AE305 - Numerical Methods

Aerospace Engineering Department

Homework #2
Team #8

Authors:
Arda Özuzun (Id:2172765)
Muhammed Kürşat Yurt (Id:2179271)
Yusuf Uzuntaş (Id:1882364)

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1 Introduction

In this homework, boundary layer of an incompressible flow is solved by using adaptive Runge-Kutta 4 method which will be explained in next section. Since governing partial differential equation and transformation to ordinary differential equation is given in homework this part is skipped to prevent repetition. Additionally in this homework automated shooting method implemented and formula which is using for approximate root is given in following section. Allowable error is chosen as 0.01 in this problem as an engineering decision. Since it is expected to converge 1, the allowable error of 0.01 is means %1 error which is suitable for most engineering problems in real world. Also allowable error can be changed just by changing one line of code. From fluid properties table it is founded that kinematic viscosity of air at 15°C is equal to $1.48 \cdot 10^{-5}\ m^2/s$ and assumed and used in following solutions. Lastly, starting guess of shooting method is chosen as $f'' = 0.5$.

1.1 Runge-Kutta 4 Method

Runge-Kutta methods are a series of numerical integration methods. They has some order of accuracy of Taylor series expansion of order n without calculation of higher order derivatives. Runge-Kutta 4 method have an order of accuracy of 4. The general formula of RK4 is given by:

$$y_{n+1} = y_n + \Delta x * \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1}$$

where k_1, k_2, k_3 and k_4 are:

$$k_1 = f(x, y) \tag{2}$$
$$k_2 = f(x + \tfrac{\Delta x}{2}, y + \tfrac{\Delta x}{2} * k_1) \tag{3}$$
$$k_3 = f(x + \tfrac{\Delta x}{2}, y + \tfrac{\Delta x}{2} * k_2) \tag{4}$$
$$k_4 = f(x + \Delta x, y + \Delta x * k_3) \tag{5}$$
$$\tag{6}$$

1.2 Shooting Method

Shooting method is used for reducing a boundary value problem (BVP) to an initial value problem (IVP). We "shoot" different points to achieve the given boundary condition. If the given ODE is linear, it is possible to use superimpose. By addition or subtraction of two results, it is able to obtain the given boundary condition. However, if the ODE is nonlinear in that case, superimposing is not able. By using trial and error, get closer to the given boundary value. While doing that, it can be assumable that ODE's behavior is linear in small intervals. In this homework this assumption is used while searching an acceptable f'' value. To obtain new f'' value the following formula is used in this homework.

$$f''_{n+2}(0) = (1 - f''_n(0)) * \frac{f''_{n+1}(0) - f''_n(0)}{f'_{n+1}(0) - f'_n(0)} \tag{7}$$

Since our problem is nonlinear there some fluctuations may observed while shooting steps however it converges the true $f''(0)$ value within an acceptable range.

1.3 Adaptive RK4 Method

Adaptive methods are used for reducing computation time and cost. Instead of using same step size for every interval in adaptive methods step size changes for different interval. In this homework, it is used that the same interval integrated twice, firstly a step size and half of the step size. If the error is lower than user defined threshold program uses that point which is founded by lower step size and calculate next step with formula given below. If error is greater than threshold program calculates new step size and do same step until reach the solution such that error is less than threshold.The new Δx is obtained by using following formulation:

$$\Delta x_{n+1} = \Delta x_n * \left| \frac{E_{allowable}}{E_{calculated}} \right|^{0.20} \tag{8}$$

2 Results and Discussion

2.1 The dimensionless velocity U and V versus η

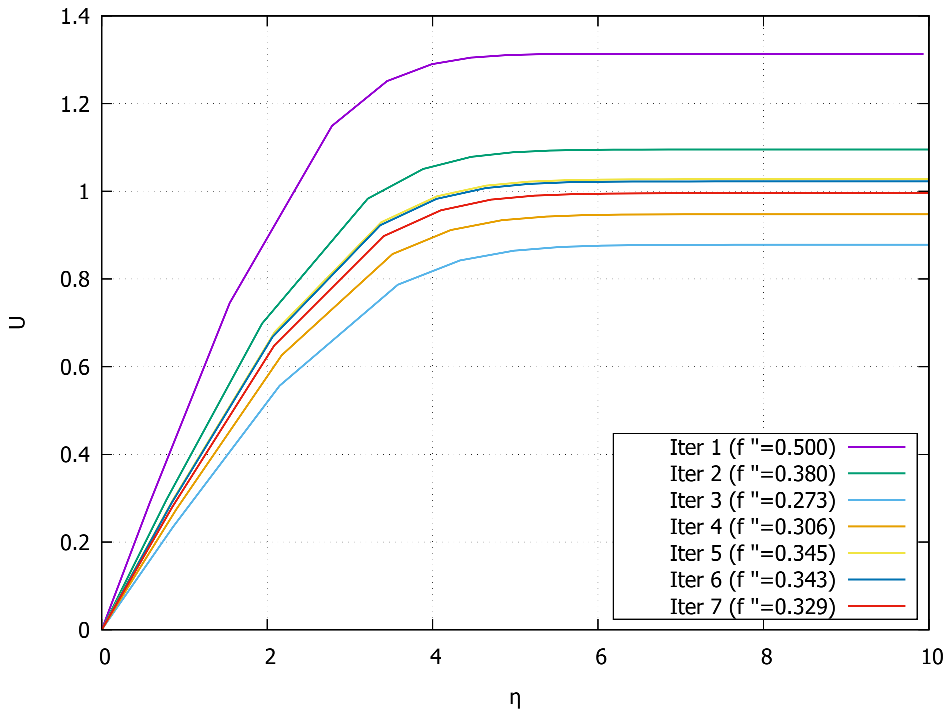

Figure 1: Dimensionless velocity U versus η

In figure 1 it is excepted that dimensionless velocity to converge 1 as η goes infinity since it is a specified boundary condition. U is a function of η. The definition of η is given by $y\sqrt{\frac{U_\infty}{\nu x}}$ which indicates that for different x values y might a take different value for each x such that η is same. As a result, whenever U and η graph is plotted it can be observed that eventually graphs does not change due to reasons mentioned above. To prevent repetition, figure 1 given to represent all x values $x = 0.1$, $x = 0.5$ and $x = 1.0$ because they all same. Also it can be observed that since our equation is nonlinear and shooting method uses linear approximation it is fluctuates between over and under shooting as expected and explained in methods section then find an acceptable guess for f'.

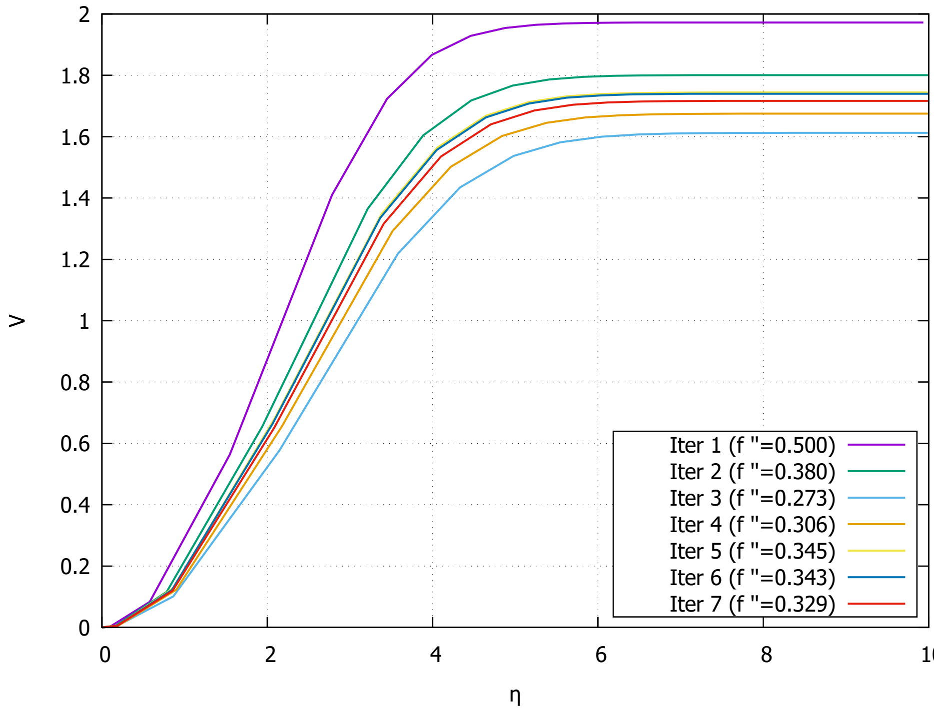

Figure 2: Dimensionless velocity V versus η

The definition of V is given by $\eta f' - f$ again as mentioned above all term are functions of η, for all values of x the graph have to be same. Figure 2 represents V versus η for all x values.

2.2 u and v versus y and η

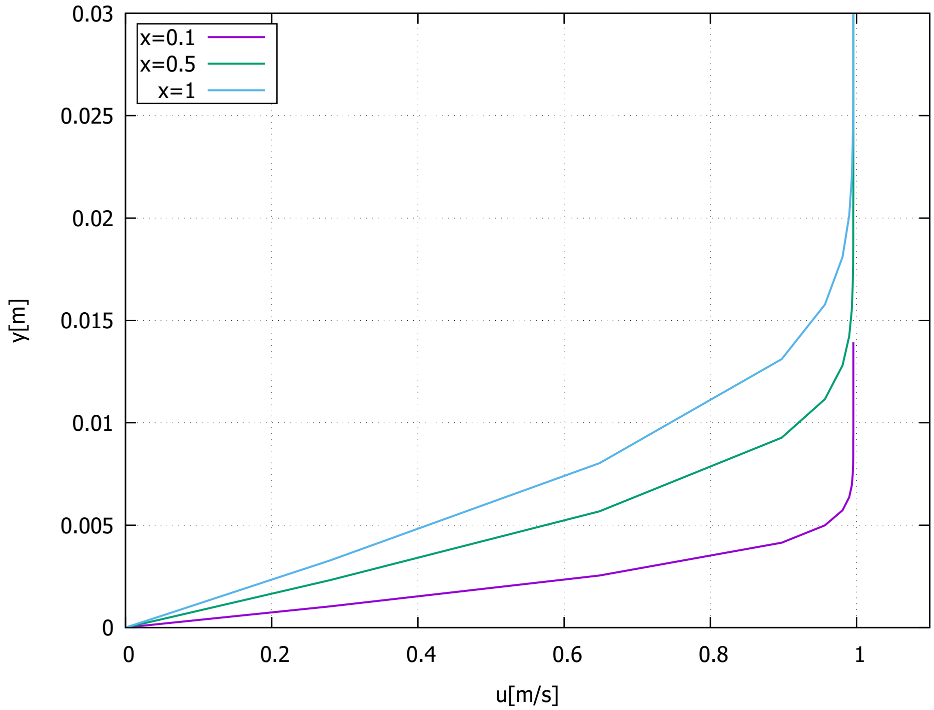

Figure 3: Velocity u versus y

It can be seen from figure 3, as the value of x gets smaller, u converge to 1 at lower y value. Although value of x is change, u always converge to 1. The reason of unchanging u values, $u(x = \infty)$ is constant and specified by boundary condition and U_∞.

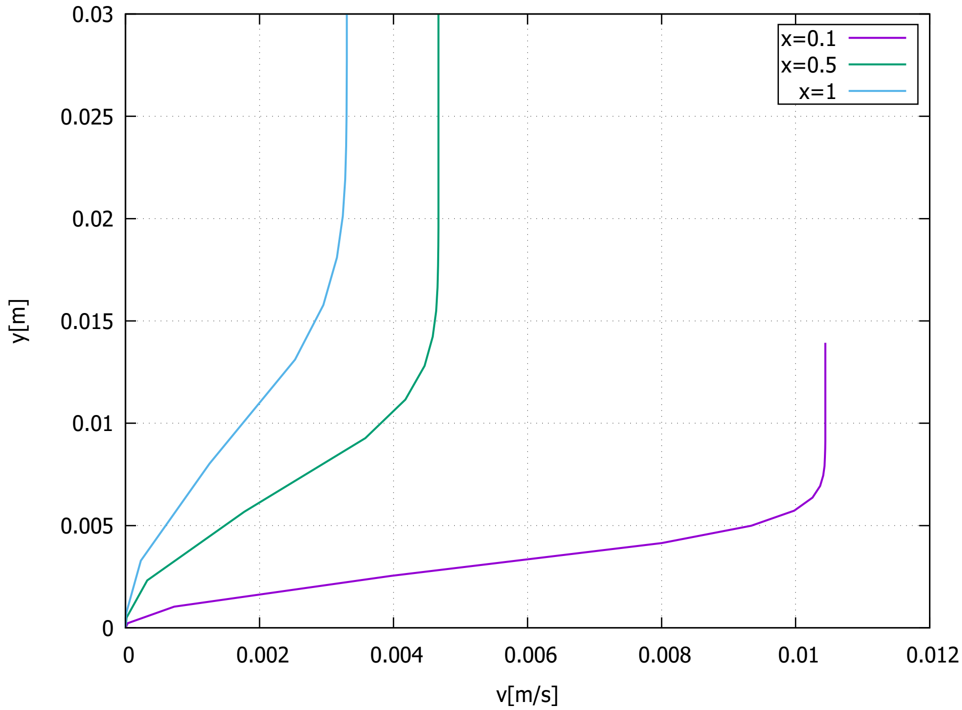

Figure 4: Velocity v versus y

From figure 4, when value of x increase , v value converge to lower than previous value and also values of v rise at same y values. It can also be observed that for lower values of x, v converges its final value at lower height value y and velocity v is higher at the same y value

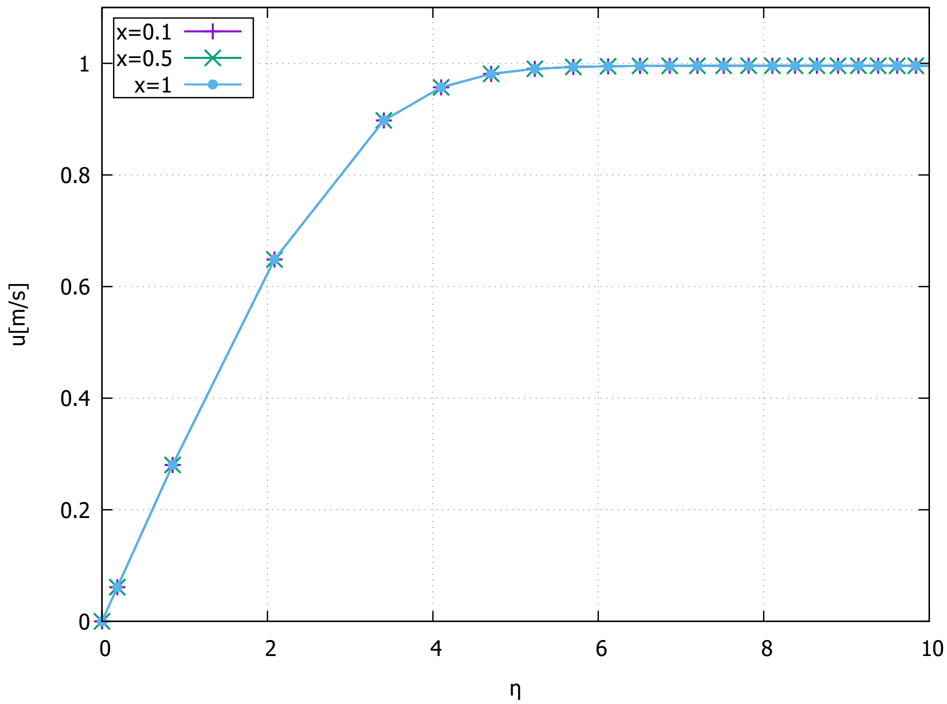

Figure 5: Velocity u versus η

Figure 5 shows that values of u and η do not depend on x values so the plot does not alter when value of x alters.Therefore, the plot overlap with different x values. All the values of u for $x = 0.1, x = 0.5$ and $x = 1.0$ cases converges to 1.

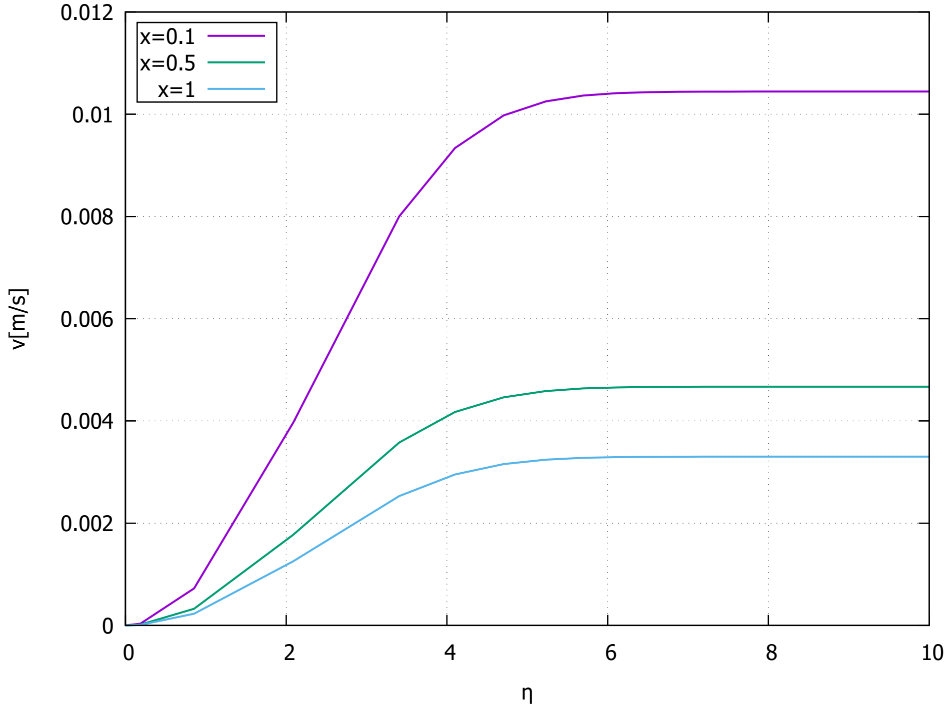

Figure 6: Velocity v versus η

Figure 6 shows 3 different cases. When value of x changes, value of v changes at same η and also convergence value of v changes. The main reason of this, v dependent on the value of x.

2.3 Adaptive Runge-Kutta versus Non-adaptive Runge Kutta

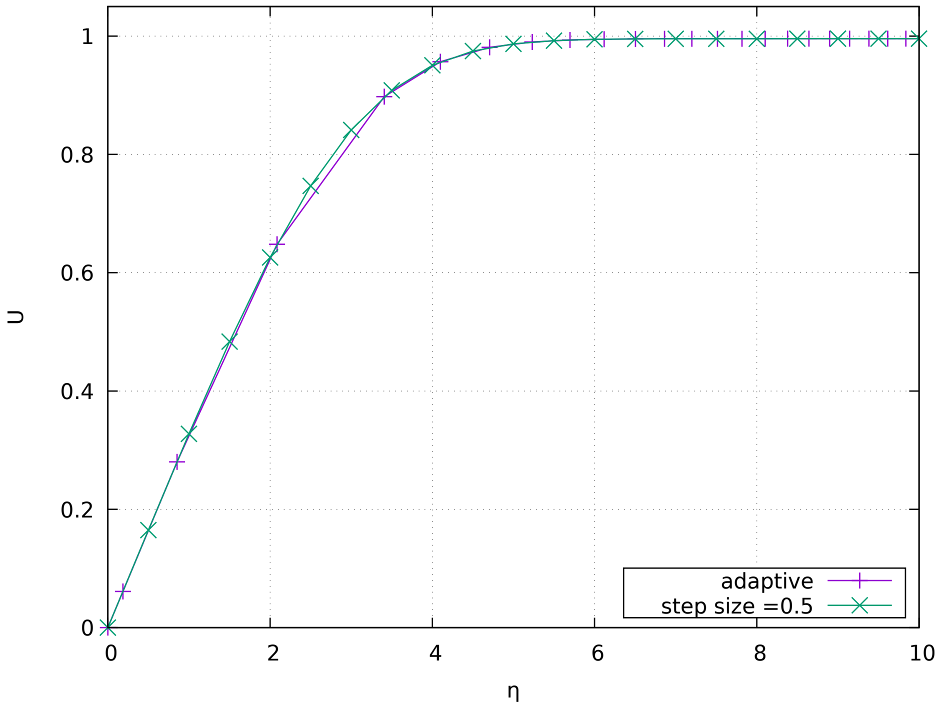

Figure 7: Dimensionless velocity U versus η

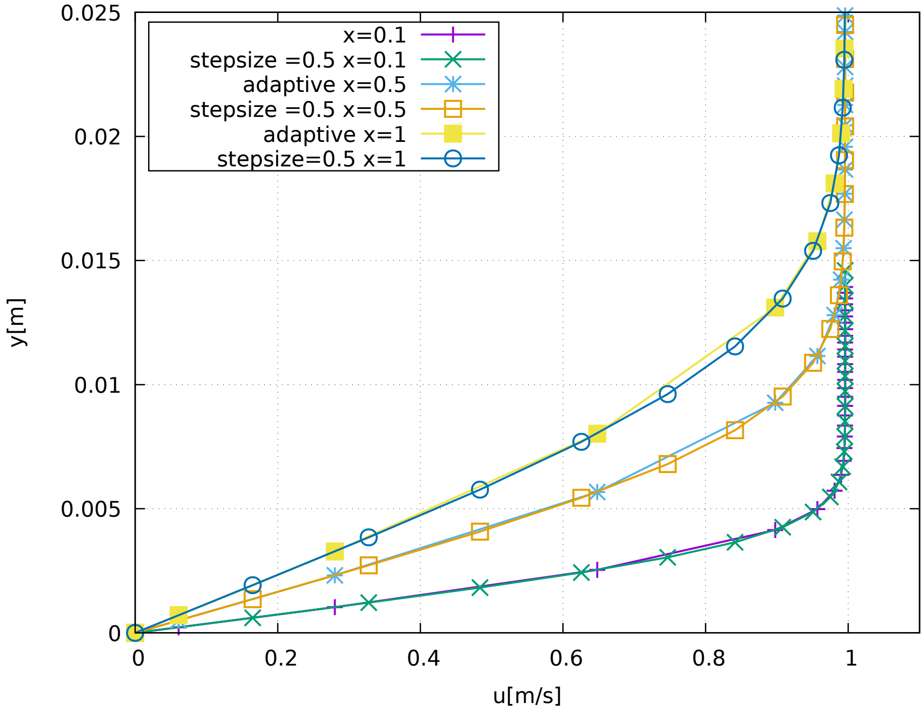

Figure 8: y versus velocity u

It can be observed from figures 7 and 8 adaptive method converges with less iteration when compared the same initial step size 0.5. 8 iteration and 12 iteration requires to converge solution for adaptive and non-adaptive RK4 respectively. To gain %33 percent less iteration, it is given up that getting a smoother graph. It is a trade off between having more data points and smoother graphs and doing less iteration i.e. reducing computing time and cost. That %33 percent saving is might not seem, in that calculation, important, however, in larger computations that much saving saves lots of money and time.

3 Conclusion

In this homework, shooting and adaptive Runge-Kutta 4 methods are used in order to solve given problem. Step size always goes to appropriate value which satisfy error criteria because adaptive RK4 is used. Therefore results converges to final value within less iterations whenever adaptive method is used. The reason why shooting method is used is to modify a boundary value problem (BVP) to an initial value problem (IVP). In conclusion, differences between adaptive and non-adaptive RK4 methods are compared in this homework. Both methods advantages and disadvantages over each other. Choosing the method is completely depend on one's requirement and priority. As a result of this homework, adaptive solution is recommended for tasks in which computation time and cost is important. Also using shooting method manually or and automated code part as done above is totally depend on needs, for more accurate shooting it is highly recommended that using and automated code part since it is required more iterations.