

FINITE VOLUME METHOD

Compute 2D potential (incompressible, inviscid, irrotational) flow fields by solving Laplace's equation for the potential function Φ . Note that $\vec{\nabla}\Phi = \vec{V} = u\vec{i} + v\vec{j}$.

$$\vec{\nabla} \cdot \vec{\nabla}\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$$

Laplace's equation may be reformulated as an unsteady diffusion equation by adding a pseudo time derivative which goes to zero for steady flows:

$$\frac{\partial\Phi}{\partial t} = \left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} \right)$$

where the diffusion coefficient is assumed to be unity. The differential equation given above is then written in the integral form as follows:

$$\frac{\partial}{\partial t} \int_{\Omega} \Phi d\Omega + \oint_S \vec{F} \cdot d\vec{S} = 0$$

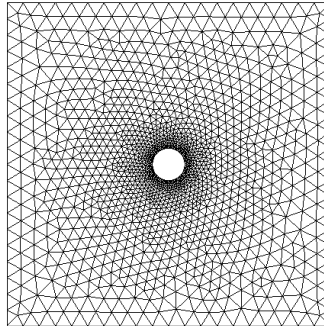
where $\vec{F} = -\vec{\nabla}\Phi$.

The boundary conditions are specified at the farfield boundary and on solid surfaces. At the farfield boundary, the flow velocity is set equal to the free stream velocity:

$$\vec{\nabla}\Phi_{\infty} = V_{\infty}(\cos \alpha \vec{i} + \sin \alpha \vec{j})$$

where α is the angle of attack. The mass flux is set to zero on solid surfaces: $\vec{F} \cdot \vec{S} = 0$

- Use the incomplete Fortran code, *fv.f* and complete the "flux" subroutine.
- Solve the potential flow field over a circle using the grid provided (*grid.dat*)



- Plot the velocity vectors and streamlines of the flow at various α values.
- Replace the circle with a NACA four digit airfoil profile. Use the discrete airfoil data obtained from the web site provided to generate the input file for the *easymesh* program. Employ *easymesh2tec*, *easymesh2fv* programs for the format conversion.
- Obtain solutions (velocity vector, streamlines) at various (small) α values.
- For a bonus, evaluate the surface pressure coefficient by using Bernoulli's equation ($C_p = 1 - V^2$, $V_{\infty} = 1$) and compare with the analytical solution; obtain the lift and drag coefficients for the NACA airfoil; solve a flow field over a multi-element airfoil configuration; ...