Assignment 2: Part 1.1 Electrostatic Potential in 1D Region

Kurtis Gassewitz - 100973882

Part 1 of the assignment consisted of determining the electrostatic solution for a region of space defined by dimensions L and W. The regions were to have an L/W ratio of $3/2$. The Finite difference method was used to determine a solution for the problem in the form of $GV = F$, where G, V and F are all matrices. The V matrix will be the potential of each point in the region, while F is used as a 'forcing' matrix.

The solution to the system is found using the Laplace equation in 2D, solved using an iteration approach. The solution involves using the Finite Difference form of the Laplace equation shown below.

$$\frac{V_{x-1,y} - 2V_{x,y} + V_{x+1,y}}{(\Delta x)^2} + \frac{V_{x,y-1} - 2V_{x,y} + V_{x,y+1}}{(\Delta y)^2} = 0$$

The equation above can be split into different iterative stages which each have a different coeffiecient. The number of points as well as coefficients are listed below.

```
close all;
clear

L = 3;
W = 2;
V_0 = 1;

dx = 0.05;
dy = 0.05;

nx = L / dx;
ny = W / dy;

c1 = 1/(dx^2);
c2 = 1/(dy^2);
c3 = -2*(1/dx^2 + 1/dy^2);
```

The G Matrix is generated which will serve to show the interaction of each point on another. The G matrix will be sized as the total number of points in the system on the X-axis as well as on the Y-axis. This allows for a solution of each point on any other point. The matrix is generated such that initial conditions are set on to each points interaction with another.

```
G = zeros(nx*ny,nx*ny);

for i = 2:(nx - 1)
    for j = 2:(ny - 1)

        n = i + (j - 1)*nx;
        nym = i + (j - 2)*nx;
        nyp = i + j*nx;
        nxm = (i - 1) + (j - 1)*nx;
        nxp = (i + 1) + (j - 1)*nx;
```

```
        G(n,n) = c3;
        G(n,nxm) = c1;
        G(n,nxp) = c1;
        G(n,nym) = c2;
        G(n,nyp) = c2;
    end
end
```

The F vector serves to set the boundary conditions for the system. The boundary is set to $V = V_0$ at $x = 0$ and %V = 0$ at $x = L$. It was noted that boundary conditions only existed on the left and right boundaries of the region. The matrices are then setup following the Laplace iterative approach.

```
F = zeros(nx*ny,1);

for j = 1:ny
    n = 1 + (j - 1)*nx;

    G(n, n) = 1;
    F(n) = V_0;

    n1 = nx + (j-1)*nx;
    G(n1,n1) = 1;
end

for i = 2:(nx - 1)

    n = i;
    n1 = i + nx;

    G(n,n) = 1;
    G(n,n1) = -1;

    n = i + (ny - 1)*nx;
    n1 = i + (ny - 2)*nx;
    G(n,n) = 1;
    G(n,n1) = -1;
end
```

The solution to the region is then found using the original formula, $GV = F$.

```
V = G\F;
```

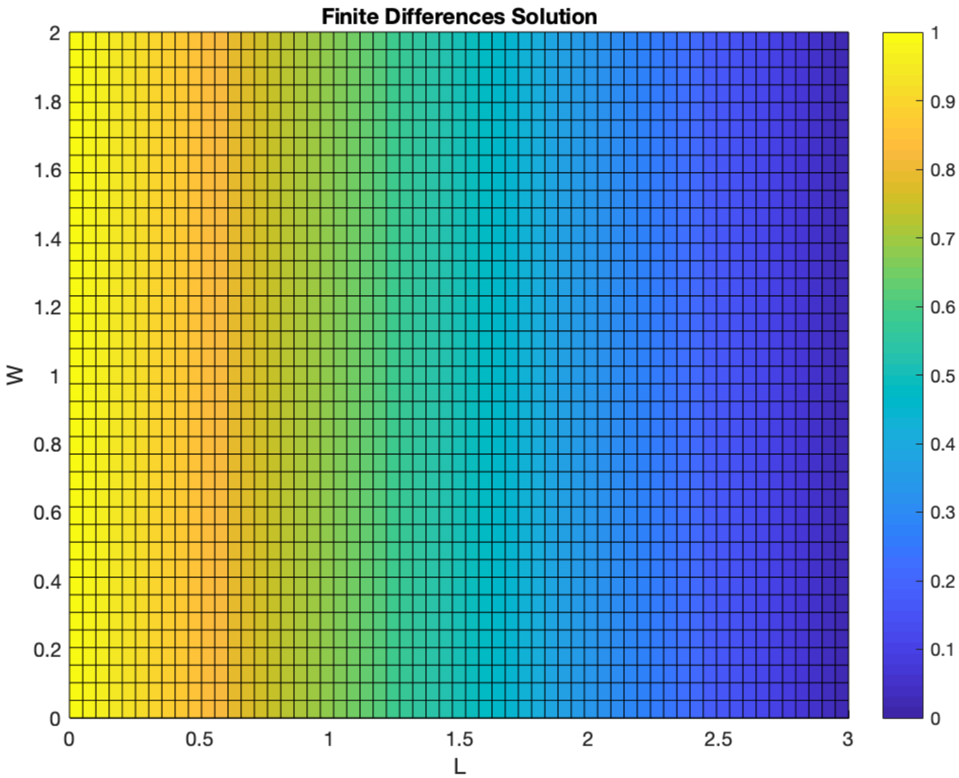
The solution is in vector form and needs to be reshaped such that the the solution takes the form of the region. Given the size of dx and dy chosen, the final matrix should be $40X60$. The solution is the plotted and shown below in figure 1.1.

```
solution = reshape(V,[nx, ny])';

x = linspace(0,L,nx);
y = linspace(0,W,ny);

figure(1)
surf(x, y, solution)
```

```
xlabel('L')
ylabel('W')
title('Finite Differences Solution')
view(0,90);
colorbar;
figLabel = {'Figure 1.1: Finite Difference Solution using Iteration'};
```



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