Part 1.2: Electrostatic Potential in 2D region

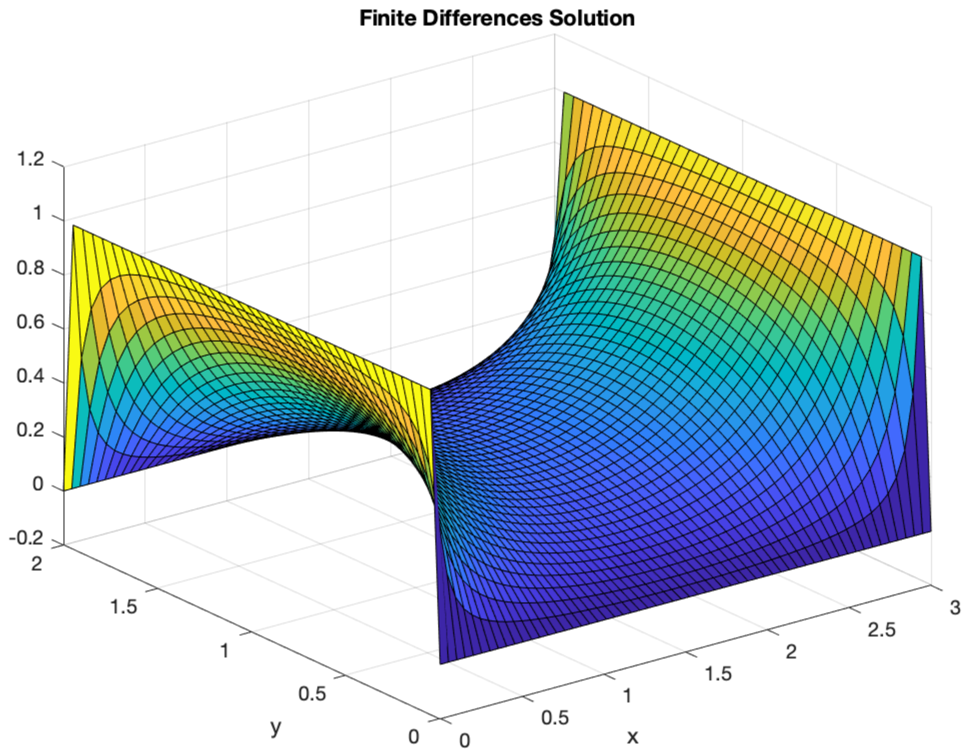
The second stage involved modifying the boundary contitions of the region. The boundary conditions were set such that $V = V_0$ at $x = 0, x = L$ and at $y = 0, andy = W$, $V = 0$. The plot is generated and compared to the analytical solution.

```
close all
clear

L = 3;
W = 2;
V_0 = 1;

dx = 0.05;
dy = 0.05;

nx = L/dx;
ny = W/dy;

c1 = 1/(dx^2);
c2 = 1/(dy^2);
c3 = -2*(1/dx^2 + 1/dy^2);

G = zeros(nx*ny,nx*ny);

for i = 2:nx-1
    for j = 2:ny-1

        n = i + (j - 1)*nx;
        nym = i + (j - 2)*nx;
        nyp = i + j*nx;
        nxm = (i - 1) + (j - 1)*nx;
        nxp = (i + 1) + (j - 1)*nx;

        G(n,n) = c3;
        G(n,nxm) = c1;
        G(n,nxp) = c1;
        G(n,nym) = c2;
        G(n,nyp) = c2;
    end
end

F = zeros(nx*ny,1);
```

Given the changes to the boundary conditions, the matrices are filled using the iterative approach.

```
for j = 1:ny
    n = 1 + (j - 1)*nx;

    G(n, n) = 1;
    F(n) = V_0;

    n1 = nx + (j-1)*nx;
```

```
    G(n1, n1) = 1;
    F(n1) = 1;
end

for i = 1:nx

    n = i;
    G(n,n) = 1;

    n1 = i + (ny - 1)*nx;
    G(n1, n1) = 1;


end

F(1) = 0;
F(1 + (ny - 1)*nx) = 0;
F(nx) = 0;
F(nx + (ny - 1)*nx) = 0;

%Finding solution and reshaping the transpose
V = G\F;
solution = reshape(V,[nx, ny])';

x = linspace(0, L, nx);
y = linspace(0, W, ny);

figure(1)
surf(x, y, solution)
xlabel('x')
ylabel('y')
title('Finite Differences Solution')
```

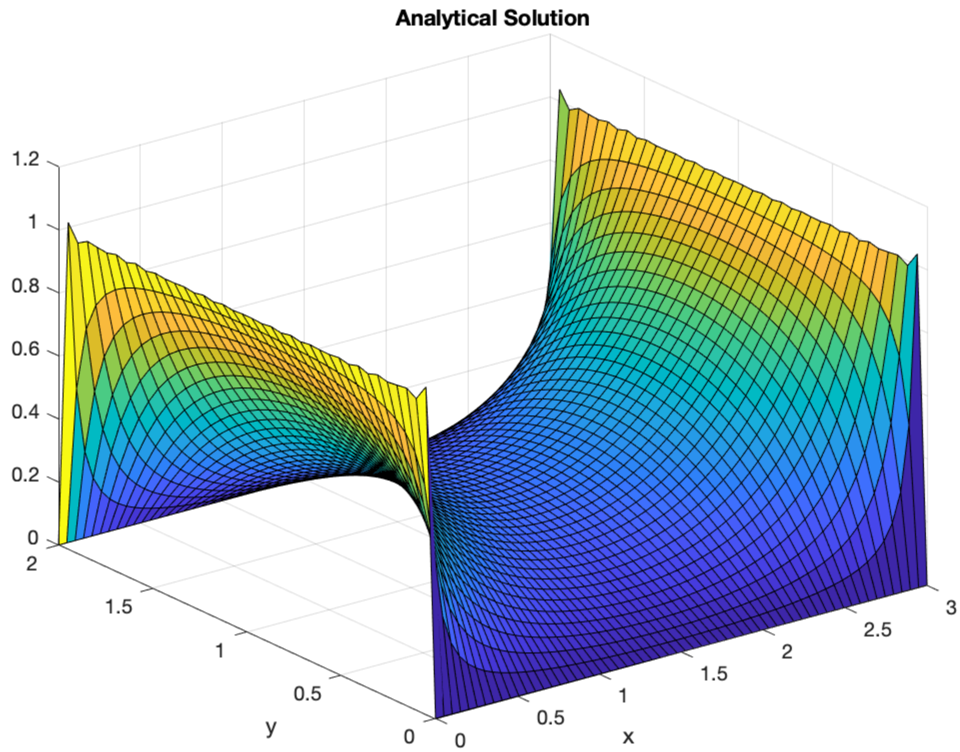


The analytical solution was then determined and compared to the solution generated for the Finite Difference Solution. The analyitcal solution was calculated by using an infinite series provided in the assignment.

```
Asol = zeros(ny, nx);
x_anyl = repmat(linspace(-L/2,L/2,nx),ny,1);
y_anyl = repmat(linspace(0,W,ny),nx,1)';
count = 100;

for i=1:count
    count = 2*i - 1;
    Asol = Asol + 1./count.*cosh(count.*pi.*x_anyl./W)./
cosh(count.*pi.*(L./2)./W).*sin(count.*pi.*y_anyl./W);
end

Asol = Asol.*4.*V_0./pi;
```

The analytical solution can be seen in the figure below. It was noted that these solutions are near identical. The different between the two stems from the time taken to reach a reasonable solution.

```
figure(2);
surf(linspace(0,L,nx),linspace(0,W,ny),Asol);
xlabel('x');
ylabel('y');
title('Analytical Solution');
```



Generating a movie to compare the two solutions, analytical and using Finite Difference, displays the difference between the two. The variable dependance of the analytical solution results in the plot taking longer to take shape. The analytical solution can be stopped when it is within a small error margin of the Finite Difference solution. For more complex solutions, numerical method is advantageous as it is easier to apply compared to the analytical approach. In simple systems such as the rectangular region in this assignment, the analytical solution is valid.

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