

# Lazy Evaluation



# Substitution Revisited

(define (f x) e1)

(f e2)



Evaluate e1 after  
substituting e2 for  
all occurrences of x

# Substitution Example

(define (inc x) (+ x 1))

(inc 5)



(+ 5 1)

# Two Strategies

(define (inc *x*) (+ *x* 1))

(inc (+ 2 3))

Simplify, then substitute  
(and simplify some more...)

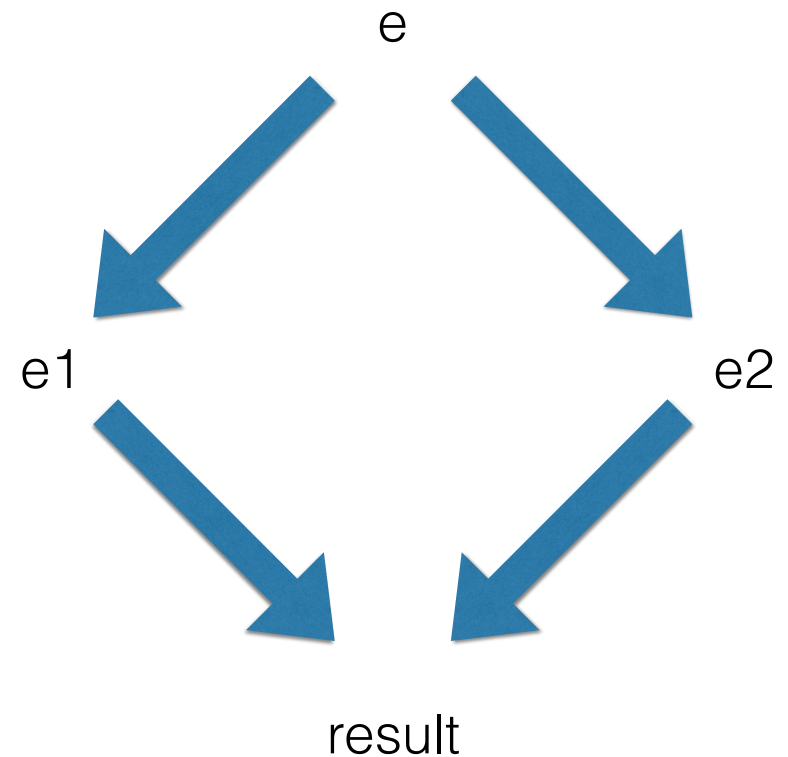
(inc 5)  
(+ 5 1)  
6

Substitute, then simplify

(+ (+ 2 3) 1)  
(+ 5 1)  
6

# Does the Order Matter?

- Church-Rosser Theorem:
  - In the absence of side effects,
  - If both methods terminate,
  - They will terminate with the same answer



# Applicative vs. Normal Order

## Applicative Order (“Eager”)

- Pass the evaluated actual parameter
- Evaluate the body of the function using that

## Normal Order (“Lazy”)

- Pass the unevaluated actual parameter
- Evaluate the body of the function using that
- If you ever use the formal parameter, then evaluate the actual parameter

# Two Strategies (Revisited)

```
(define (inc x) (+ x 1))
```

```
(inc (+ 2 3))
```

Applicative Order  
(Eager)

```
(inc 5)  
(+ 5 1)  
6
```

Normal Order  
(Lazy)

```
(+ (+ 2 3) 1)  
(+ 5 1)  
6
```

# “In The Absence of Side Effects...”

```
y = 0;
```

```
int f(int x)
{
    return x * 2;
}
```

```
f(y + 1);
```

```
y = 0;
```

```
int f(int x)
{
    y = 3;
    return x * 2;
}
```

```
f(y + 1);
```



# “If They Both Terminate...”

```
int a = ... ;
```

```
int f(int x, int y)
{
    return x > 0? y : 0;
}
```

```
f(a, 1 / 0);
```

# Strategic Laziness

```
int a = ... ;
```

```
int f(int x, int y)
{
    return x > 0? y : 0;
}
```

```
f(a, function_that_takes_two_hours());
```

# But...

```
int a = ... ;
```

```
int f(int x, int y)
{
    return x > 0? (y * y * y) : 0;
}
```

```
f(a, function_that_takes_two_hours());
```

# Memoization

```
int a = ... ;
```

```
int f(int x, int y)
```

```
{
```

```
    return x > 0? (y * y * y) : 0;
```

```
}
```

```
f(a, function_that_takes_two_hours());
```

Evaluate on first use

Re-use after that



# Call by Name vs. Call by Need

- Lazy evaluation uses call-by-name parameter passing
  - In the absence of side-effects, and if it terminates, results are the same as call-by-value parameter passing
  - Can terminate even when eager evaluation would fail
  - Can avoid unnecessary computation but may do redundant computation
- Or more modern lazy languages use call-by-need
  - Memoization
  - Benefits of call-by-name but just as fast as call-by-value

# Special Forms

## **Applicative Order (“Eager”)**

- Default behavior is eager
- Lazy special forms

## **Normal Order (“Lazy”)**

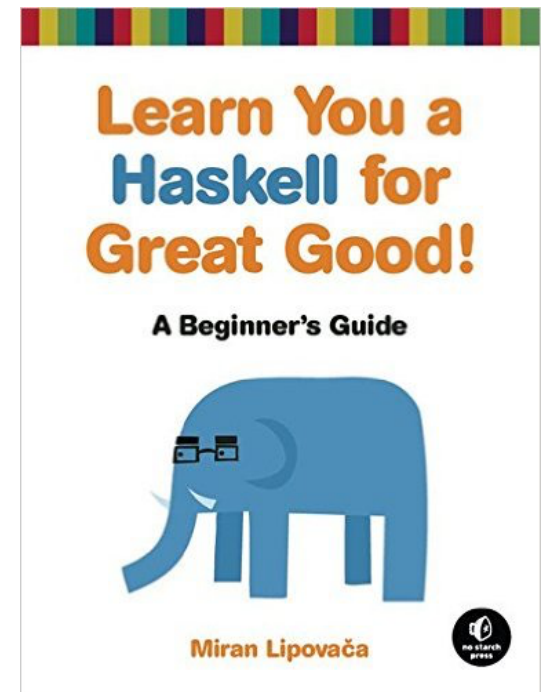
- Default behavior is lazy
- Eager special forms  
(i.e., “do it!”)

# Haskell

- Lazy evaluation (call-by-need parameter passing)
- Haskell is a pure functional language
  - No side effects (more on that later...)

# Getting Started

- We will use the Glasgow Haskell Compiler
  - GHC = compiler
  - GHCi = interactive REPL
- Online tutorials at
  - <http://book.realworldhaskell.org/read/>
  - <http://learnyouahaskell.com>





# Basics

- REPL interpreter
- Infix mathematical operations
- Pattern matching function definitions / calls  
(a bit like what we saw in Prolog)
- All the things you'd expect from a good functional language:
  - List handling
  - First-class and higher-order functions
  - Etc.

# Let (local bindings)

let  $x = 3$  in  $x + 1$

let  $y = 4 + 9$  in  $y * x$

# Modules

- Function definitions go in modules (files that are loaded)
- Load modules with “:load”
- Global definitions (  $x = \dots$  ) allowed only in modules, not in the REPL

In file “Fac.hs”:

```
fac 0 = 1
fac n = n * fac (n-1)
```

In REPL:

```
:load Fac.hs
fac 10
```

# Lists

- Can be defined by just listing elements
- Can be defined by a range
- Can be defined by simple repeating interval over a range
- Can be defined using list comprehension
- Operations:
  - “head” - returns the first element
  - “tail” - returns a list of the rest
  - “take” - returns the first n things in a list
  - “takeWhile” - returns elements of the list as long as a supplied predicate is true
  - “:” - like “cons” in Lisp / Scheme / Racket
  - “++” - concatenation operator
  - “zip” - pairs elements of two lists and returns a list of two-element sublists

```
let x = [4, 8, 15, 16, 23, 42] in  
  tail x
```

```
[1..10]
```

```
[1,3..11]
```

```
1 : [2..10]
```

```
let x = [1,3..11] in  
  let y = [2,4..12] in  
    zip x y
```

# List Comprehension

- Can define lists using list comprehension (like we saw in Erlang)
  - Filter based on criteria
  - Map to produce resulting list

```
myData = [1,2,3,4,5,6,7]
```

```
twiceData = [2 * x | x <- myData]  
-- [2,4,6,8,10,12,14]
```

```
twiceEvenData =  
  [2 * x | x <- myData,  
           x `mod` 2 == 0]
```

# Functions

- Can define functions based on pattern matching
- Can use multiple boolean cases

```
fac 0 = 1
fac n = n * fac (n-1)
```

```
dupfirst (x:xs) = x:x:xs
```

```
scoreToLetter n
| n > 90 = 'A'
| n > 80 = 'B'
| n > 70 = 'C'
| n > 60 = 'D'
| otherwise = 'F'
```

# First-Class and Higher-Order Functions

- Anonymous functions (lambda)

$\lambda x \rightarrow x + 1$

- Usual array of higher-order functions:

- map
- filter
- foldl / foldr
- etc

let x = [1..10] in  
map ( $\lambda x \rightarrow x + 1$ ) x

# Types

- In Haskell, everything has a type
  - Don't have to declare unless necessary to avoid ambiguity
  - Can declare if you want
  - Can use type variables

`x :: Int`

`ones :: [Int]`

`scoreToLetter :: Int -> Char`

`filter :: (a -> Bool) -> [a] -> [a]`



# Let's Get Lazy

- All parameter passing is lazy  
(the formal parameter is bound to the unevaluated actual parameter)
- “let” is also lazy



# Infinite Lists

- With lazy evaluation you can make conceptually infinite lists and other data structures
- Evaluated only as needed
- Use “head” or “take” to force evaluation and get elements
- Higher-order functions on infinite lists can produce other infinite lists

`x = [1.. ]`

`ones = 1 : ones`

# What's This?

```
let x = 1 : 1 : zipWith (+) x (tail x) in  
take 5 x
```

Hint: `zipWith (+) [1,3,4] [2,5,8]`  
returns `[1+2, 3+5, 4+8]`