

Assignment - 1

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Abstract—This document contains the solution to Exercise 3.39 (b) of Oppenheim.

Problem 1. Suppose the z -transform of $x[n]$ is

$$X(z) = \frac{z^{10}}{\left(z - \frac{1}{2}\right)\left(z - \frac{3}{2}\right)^{10}\left(z + \frac{3}{2}\right)^2\left(z + \frac{5}{2}\right)\left(z + \frac{7}{2}\right)} \quad (1)$$

It is also known that $x[n]$ is a stable sequence.
Determine $x[n]$ at $n = -8$

Solution:

$x[n] = \sum [\text{Residues of } X(z)z^{n-1} \text{ at the pole inside the circle}]$

ROC includes unit circle $|z| = 1$ and ROC is $\frac{1}{2} < |z| < \frac{3}{2}$

$x[-8] = \text{remainder of } \frac{z}{(z - \frac{1}{2})(z - \frac{3}{2})^{10}(z + \frac{3}{2})^2(z + \frac{5}{2})(z + \frac{7}{2})} \text{ inside unit circle}$

But $z = 1/2$ is the only pole inside unit circle

$$x[-8] = \frac{\frac{1}{2}}{\left(\frac{1}{2} - \frac{3}{2}\right)^{10}\left(\frac{1}{2} + \frac{3}{2}\right)^2\left(\frac{1}{2} + \frac{5}{2}\right)\left(\frac{1}{2} + \frac{7}{2}\right)} = \frac{1}{96} \quad (2)$$