Programming Assignment - I

Introduction to
Time series Analysis

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Importing the necessary libraries for the assignment

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

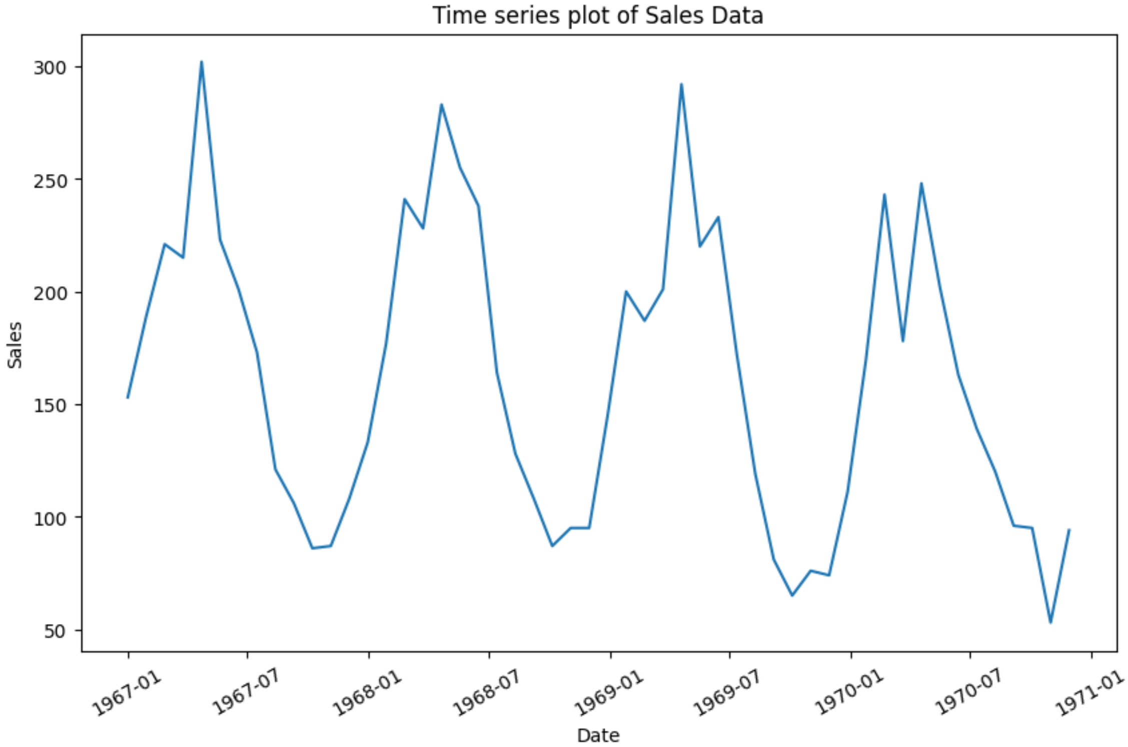
Question 1: The following data shows the sales of company X in successive 4-week periods over 1967-1970.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13
1967	153	189	221	215	302	223	201	173	121	106	86	87	108
1968	133	177	241	228	283	255	238	164	128	108	87	95	95
1969	145	200	187	201	292	220	233	172	119	81	65	76	74
1970	111	170	243	178	248	202	163	139	120	96	95	53	94

1.a) Plot the data

```
data = [153, 189, 221, 215, 302, 223, 201, 173, 121, 106, 86, 87, 108,
        133, 177, 241, 228, 283, 255, 238, 164, 128, 108, 87, 95, 95,
        145, 200, 187, 201, 292, 220, 233, 172, 119, 81, 65, 76, 74,
        111, 170, 243, 178, 248, 202, 163, 139, 120, 96, 95, 53, 94]
date_range = pd.date_range(start='1967-01-01', periods=52, freq='4W')
df = pd.DataFrame({'date': date_range, 'sales': data})
```

```
plt.figure(figsize=(10, 6))
plt.plot(df['date'], df['sales'])
plt.xticks(rotation=30)
plt.xlabel('Date')
plt.ylabel('Sales')
plt.title('Time series plot of Sales Data')
plt.show()
```



1.b) Assess the trend and seasonality in the data

Seasonality: The data is seasonal as we can see that the sales are increasing and decreasing in a cyclic manner. The sales are high around in the middle of the year and low at the end of the year.

Trend: There is a decrease in trend

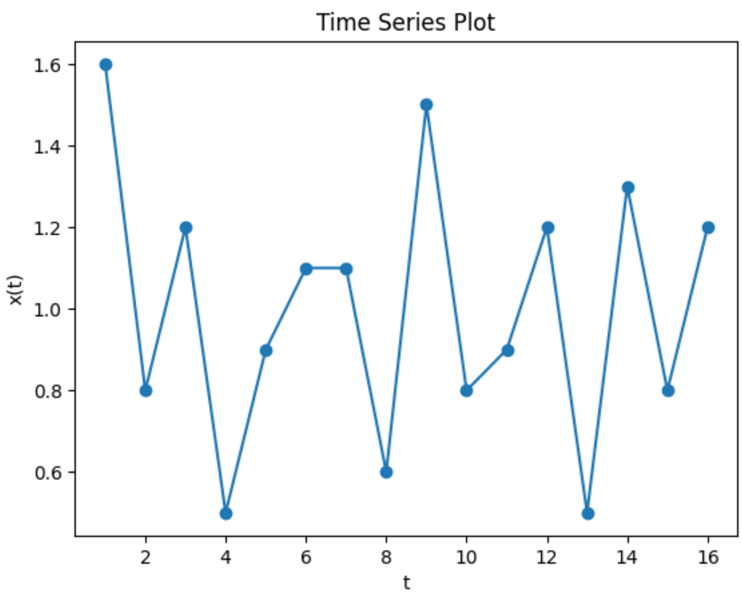
Question 2: Sixteen successive observations on a stationary time series are as follows:

1.6, 0.8, 1.2, 0.5, 0.9, 1.1, 1.1, 0.6, 1.5, 0.8, 0.9, 1.2, 0.5, 1.3, 0.8, 1.2

```
data = [1.6, 0.8, 1.2, 0.5, 0.9, 1.1, 1.1, 0.6, 1.5, 0.8, 0.9, 1.2, 0.5, 1.3, 0.8, 1.2]
df = pd.DataFrame({'data': data})
```

2.a) Plot the Observations

```
fig1 = plt.figure()
plt.plot(range(1, len(data) + 1), df['data'], marker='o')
plt.title('Time Series Plot')
plt.xlabel('t')
plt.ylabel('x(t)')
plt.show()
```

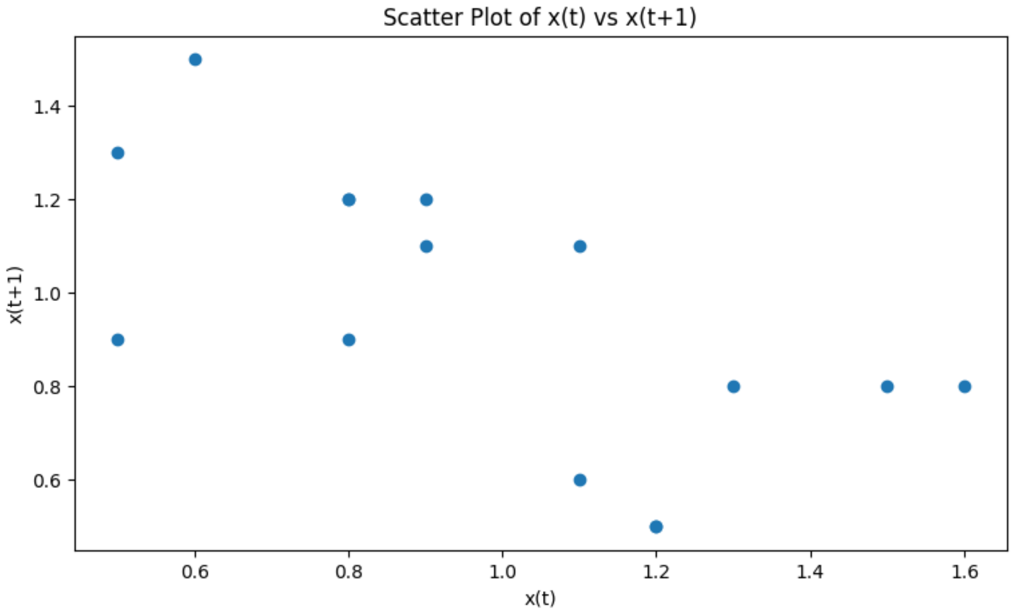


2.b) Looking at the graph, guess an approximate value for the autocorrelation coefficient at lag 1.

From the graph above, it's not possible to make any guesses for autocorrelation coefficient at lag 1

2.c) Plot x_t vs x_{t+1} and try to guess the value of r_1.

```
fig2 = plt.figure(figsize=(9,5))
plt.scatter(df['data'][:15], df['data'][1:16], marker='o')
plt.xlabel('x(t)')
plt.ylabel('x(t+1)')
plt.title('Scatter Plot of x(t) vs x(t+1)')
plt.show()
```



From the above plot we can say that it definitely has negative autocorrelation. Therefore, r_1 value lies in between 0 & -1

2.d) Calculate r_1.

Formula of ACF: $r_h = \frac{Cov(X_t, X_{t+h})}{Var(X_t)} = \frac{\sum_{t=1}^{n-h}(x_t - \bar{x})(x_{t+h} - \bar{x})}{\sum_{t=1}^{n}(x_t - \bar{x})^2}$

```
def acf(dt, tc, lag):
    """
    Function to calculate auto-correlation coefficient of given lag
    :param dt: pd.DataFrame - dataset
    :param tc: string - target column
    :param lag: int - lag
    :return: int - auto-correlation coefficient
    """
    n = len(dt)
    d1 = dt[tc]
    mean_data = np.mean(dt[tc])
    acf_value = sum(
        (d1[:n - lag] - mean_data).reset_index(drop=True) * (d1[lag:] - mean_data).reset_index(drop=True)) / sum(
        (d1 - mean_data).reset_index(drop=True) ** 2)
    return acf_value

lag = 1
r1 = acf(df, 'data', lag)
print("Auto-correlation coefficient r1:", r1)
```

```
Auto-correlation coefficient r1: -0.548780487804878
```

$\therefore r_1 = -0.54878$

Question 3. For the airline passengers data already available in R, plot the autocorrelation function (ACF) for a range of lag values. Interpret the results.

#Load data

```
data("AirPassengers")
data <- AirPassengers
```

#acf_value function for lag `lag`

```
acf_value <- function(data, lag){
  data_size <- length(data)
  mean_data <- mean(data)
  acf_v <- sum((data[1:(data_size - lag)] - mean_data) * (data[(lag + 1):data_size] - mean_data)) / sum((data - mean_data)^2)
  return (acf_v)
}
```

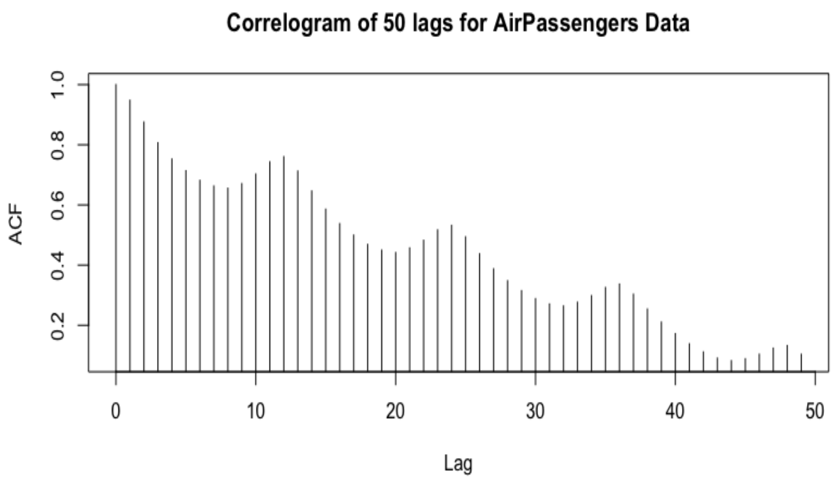
#acf_value_t function from lag 0 to `total_lags-1`

```
acf_value_t <- function(data, total_lags){
  acf_values <- numeric(total_lags-1)
  for (lag in 0:total_lags-1){
    acf_values[lag+1] <- acf_value(data,lag)
    }
  return (acf_values)
  }
```

#acf plot

```
total_lags<-50
acf_values<-acf_value_t(data, total_lags)
plot((1:total_lags)-1,acf_values, type = "h", xlab = "Lag", ylab = "ACF",
  main = "Autocorrelation Function")
```

```
#to display image in jupyter notebook
from IPython.display import Image
Image(filename='img.png',width=800,height=400)
```



Question 4. Consider a time series with both trend and seasonal effects present. Model the time series as $X_t = a_0 + a_1 t + b_1 cos(\lambda t) + c_1 sin(\lambda t) + \epsilon_t, t = 1, \ldots, 25$. Assume that $\lambda = \pi$. Estimate the above coefficients for the following time series data:

(2.7, 7.8, 6.2, 10.7, 9.6, 14.0, 13.2, 16.1, 17.9, 22.2, 23.7, 24.6, 24.6, 28.7, 28.6, 34.5, 34.1, 39.0, 38.7, 43.2, 42.3, 46.2, 46.3, 48.5, 49.8)

```
data = [2.7, 7.8, 6.2, 10.7, 9.6, 14.0, 13.2, 16.1, 17.9, 22.2, 23.7, 24.6, 24.6, 28.7, 28.6, 34.5, 34.1, 39.0,
        38.7, 43.2, 42.3, 46.2, 46.3, 48.5, 49.8]
```

Y is the target vector & $\mathbf{X}$ is the design matrix

```
Y = np.matrix(data).reshape(len(data), 1)
a_0 = np.matrix(np.ones((len(data), 1)))
a_1 = np.matrix(range(1, len(data) + 1)).reshape(len(data), 1)
b_1 = np.matrix([np.cos(np.pi * t) for t in range(1, len(data) + 1)]).reshape(len(data), 1)
c_1 = np.matrix([np.sin(np.pi * t) for t in range(1, len(data) + 1)]).reshape(len(data), 1)
X = np.hstack([a_0, a_1, b_1, c_1])
```

For time series model $X_t = a_0 + a_1 t + b_1 cos(\lambda t) + c_1 sin(\lambda t) + \epsilon_t, t = 1, \ldots, 25$

The coefficient vector $\beta = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T Y$

```
beta = (X.T * X).I * X.T * Y
```

```
print("The estimated coefficients are:")
print(f"a_0: {beta[0, 0]}")
print(f"a_1: {beta[1, 0]}")
print(f"b_1: {beta[2, 0]}")
print(f"c_1: {beta[3, 0]}")
```

```
The estimated coefficients are:
a_0: 1.1217961766203466
a_1: 1.9892275567016904
b_1: 1.0807941181036353
c_1: 47714090888061.21
```

$$\beta = \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1.121796176620368 \\ 1.989227556701689 \\ 1.0807941181036353 \\ 47714090888061.12 \end{bmatrix}$$