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A Comparison of the CIE 1976 Colour Spaces

The CIE 1976 colour spaces, CIELUV and CIELAB, have been compared by recalculating the results of a number of reported sets of experimental data. These include the results of just-noticeable-difference observations, colour-difference scaling, colour-matching ellipses, and acceptability ellipses. As a means of representing the colour-difference data uniformly, it is shown that neither colour space is significantly better than the other. Attention is drawn to some anomalies in the CIELAB space.

Introduction

The International Commission on Illumination (CIE) has recommended the use of two approximately uniform colour spaces and their associated colour-difference formulae (CIE, 1978). They are designated CIE 1976 (L^* u^* v^*) Colour Space and CIE 1976 (L^* a^* b^*) Colour Space. The abbreviation CIELUV is recommended for the former and CIELAB for the latter. The choice of formula for a particular situation is often dictated by familiarity and common practice in a particular industry; no official recommendation is given by the CIE as to which formula should be used in any particular circumstance.

The background to the formulae is adequately presented by Robertson (1977). The purpose of this article is to make a comprehensive review of the available experimental data on perceived colour differences and use it to compare the two colour spaces.

The Formulae

The first approximately uniform colour space is produced by plotting in rectangular coordinates the quantities L^*, u^*, v^* defined by

$$L^* = 116(Y/Y_n)^{1/3} - 16, \quad Y/Y_n > 0.008856,$$

$$u^* = 13L^*(u' - u_n'),$$

$$v^* = 13L^*(v' - v_n'),$$

with

$$u' = 4X/(X + 15Y + 3Z),$$

$$v' = 9Y/(X + 15Y + 3Z).$$

The chromaticity coordinates u_n', v_n' define the nominally white object-colour stimulus.

The total difference ΔE_{uv}^* between any two colours each given in terms of L^*, u^*, v^* is calculated from

$$\Delta E_{uv}^* = [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{1/2}.$$

The second approximately uniform colour space is produced by plotting in rectangular coordinates the quantities L^* a^* b^* defined by

$$L^* = 116(Y/Y_n)^{1/3} - 16,$$

$$a^* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}],$$

$$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}],$$

$$X/X_n, \quad Y/Y_n, \quad Z/Z_n > 0.008856.$$

The tristimulus values X_n, Y_n, Z_n define the colour of the nominally white object-colour stimulus.

The total difference ΔE_{ab}^* between any two colours each given in terms of L^*, a^*, b^* is calculated from

$$\Delta E_{ab}^* = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2}.$$

It should be noted that the function for CIE 1976 psychometric lightness L^* is the same for both sets of formulae; this function is a substitute for the Munsell Renotation Value Function (see Newhall, Nickerson, and Judd, 1943). Stenius (1978) has shown that at very low values of lightness the L^* function deviates appreciably from the Munsell Renotation Value Function. However, these deviations are of little practical importance for reflecting samples, and the equation for L^* given above is simple to express and is easily used, whereas the Munsell Renotation Value Function requires the solution of a fifth-order polynomial to obtain a measure of psychometric lightness.

The limitation on the ratios X/X_n, Y/Y_n, Z/Z_n is necessary to prevent the values of the functions from going negative. For most object colours the conditions are satisfied,

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TABLE I. Results of Friele analysis.

Study reference	No. of sample centres	Correlation coefficient CIELUV	CIELAB
Morley, Munn, and Billmeyer (1975)	19	0.69	0.70
Robinson (1969)	1	0.76	0.82
Davidson and Friede (1953)	19	0.57	0.55
Metropolitan Section (1971)	10	0.71	0.67
VVVR (Selier, Haeflaak, and Friele, 1978)	10	0.61	0.58

TABLE II. Results of Ikeda et al. (1979) analysis.

Illuminant	Correlation coefficient CIELUV	CIELAB
D_{65} set 1	0.73	0.78
D_{65} set 2	0.82	0.88
D_{65} mean	0.78	0.83
S_A set 1	0.85	0.71
S_A set 2	0.88	0.73
S_A mean	0.87	0.72

but Z/Z_n for deep yellows, oranges, and reds may assume a value below 0.008856. Pauli (1976) suggested a modification to the formulae to overcome this problem. The values of the ratios are extended down to zero by linear interpolation such that, at the changeover point, both the function and its first derivative are continuous.

Thus, in calculating L^* where values of $Y/Y_n \leqslant 0.008856$ the formula to be used is

$$L^* = 903.3\,(Y/Y_n), \quad Y/Y_n \leqslant 0.008856.$$

In calculating a^* and b^* the following formulae can be used:

$$a^* = 500[f(X/X_n) - f(Y/Y_n)],$$
$$b^* = 200[f(Y/Y_n) - f(Z/Z_n)],$$

where

$$\begin{aligned} f(X/X_n) &= (X/X_n)^{1/3}, & X/X_n &> 0.008856,\\ &= 7.787(X/X_n) + 16/116, & X/X_n &\leqslant 0.008856,\\ f(Y/Y_n) &= (Y/Y_n)^{1/3}, & Y/Y_n &> 0.008856,\\ &= 7.787(Y/Y_n) + 16/116, & Y/Y_n &\leqslant 0.008856,\\ f(Z/Z_n) &= (Z/Z_n)^{1/3}, & Z/Z_n &> 0.008856,\\ &= 7.787(Z/Z_n) + 16/116, & Z/Z_n &\leqslant 0.008856. \end{aligned}$$

This modification was added as an Appendix to the original recommendations (CIE, 1978) and has been used in all the calculations in this article.

Ohta (1977) has shown that the relationship between the two colour spaces is complex; no simple conversion factor of the form $\Delta E_{uv}^*/\Delta E_{ab}^*$ can be determined. Thus, we have two colour spaces derived from different roots but both attempting to represent colour-difference data in a uniform manner.

Earlier Comparisons

A comprehensive evaluation of the colour-difference formulae associated with the two colour spaces has been published by Friele (1978). In Table I are summarised the correlation coefficients he obtained when colour differences calculated using the two formulae were compared with the results of five studies of the colour differences seen in practice. It is clear that these results show neither formula to be significantly superior to the other. Lozano (1977) found a higher correlation for the $L^*\ a^*\ b^*$ formula: 0.91 and 0.76 for two groups of colours, compared with 0.85 and 0.73 for the $L^*\ u^*\ v^*$ formula. In the first group of colours, lightness was varied while maintaining almost constant colourfulness and hue; in the second group of colours, colourfulness was varied, with the other two variables remaining approximately constant. Grum, Saunders, and MacAdam (1978) found that the $L^*\ u^*\ v^*$ formula was superior for predicting correlated colour temperatures.

Zeller and Hemmendinger (1979) investigated colour differences in three regions of colour space: a moderate-chroma red, a moderate-chroma yellow, and a high-chroma yellow. The correlation coefficients for least-squares analysis between perceived and calculated colour differences were 0.55, 0.75, and 0.73 for CIELUV and 0.81, 0.42, and −0.58 for CIELAB. This shows that, although one formula may be generally more consistent, another may give higher correlation for some colours and lower for others.

Kuehni and Marcus (1979) investigated colour differences associated with six sample centres: four paint samples and two fabric samples. The average correlation coefficient between visual and calculated colour differences was 0.79 for CIELUV and 0.84 for CIELAB.

Ikeda, Nakayama, and Obara (1979) scaled the relative values of colour differences between the colour chips in two sets of colours that had equal lightness and approximately constant Munsell Chroma. Standard Illuminants A and D_{65} were used and the results compared with calculated values using the CIELUV and CIELAB formulae. The correlation coefficients are shown in Table II. Ikeda, Nakayama, and Obara (1979) also showed that the ratios of perceived colour differences under Illuminant A to those under Illuminant D_{65} are almost constant (the correlation coefficient was 0.96). However, the ratios of measured colour differences under Illuminant A to those under Illuminant D_{65} varied considerably, the degree of variation being higher in CIELUV than in CIELAB.

The overall conclusion from the results of the studies quoted above is that neither of the two colour-difference formulae appears to be of clearly superior merit in predicting the magnitudes of colour differences.

New Analysis of Earlier Data—I

The two sets of data that are traditionally used to analyse new colour spaces are the ellipses derived from colour-matching data by MacAdam (1942) and the renotation data derived from measurements of the original Munsell spacing of colours (Newhall, Nickerson, and Judd, 1943). In this section, these sets of data have been analysed in the new colour spaces, together with the colour-discrimination data of Wright (1941) and the experimental data of MacAdam used in the derivation of the OSA uniform colour scales (MacAdam, 1974).

The data have been analysed by computing the mean and standard deviation of various parameters in both colour spaces. From these numbers the limits for 95% confidence in the mean were calculated using the formula

$$\chi = \text{mean} \pm (\text{standard error} \times t),$$

where χ defines the upper or lower limit of the mean and t is a tabulated number that is a function of the number of degrees of freedom of the data and the required confidence level.

Thus, this formula defines a range in which there is a 95% chance of obtaining the mean value. If ranges of values are defined for the two colour spaces and they overlap, then the difference between the colour spaces in terms of that particular data set are considered insignificant.

The Wright discrimination data consists of just-noticeable colour differences plotted as steps on 35 lines on a chromaticity diagram. These data were transformed to the CIELUV and CIELAB colour spaces and are shown in Fig. 1. (When transformed to the CIELAB space these lines become slightly curved, because of the nonlinear transformation used to obtain CIELAB coordinates. In this analysis, chord length, not arc length, was used, in order to comply with the convention that in the CIELAB space colour differences are regarded as proportional to distances in the space.) The data were derived using a colorimeter with a bipartite field and a dark surround. The illuminant used for transforming the data was Illuminant E; this illuminant was chosen because of its traditional use with stimuli that appear luminous.

In Fig. 1, values of L^* of 70, 50, and 30 have been used. These correspond to luminance factors of 0.41, 0.18, and 0.06 or Munsell Values of (very nearly) 7, 5, and 3, respectively. Also plotted in Fig. 1 are the optimal colour limits calculated for the respective values of L^* for Illuminant E. These graphs show that some of the lines fall outside the optimal limits, which implies that they are not realisable at the particular value of L^* for nonfluorescing reflecting colours. These values have not been included in the analysis.

The data for each value of L^* and colour space have been compared by computing the standard deviation of the line lengths for each colour space. The results are shown in Table III. CIELUV gives smaller standard deviations than CIELAB, but the difference is only significant at a 95% confidence level for the result at $L^* = 70$.

The MacAdam ellipse data (from Wyszecki and Stiles, 1967) were transformed into the CIELUV and CIELAB colour spaces using values of L^* of 70, 50, and 30; the results are plotted in Fig. 2 together with optimal limits for Illuminant C. These data have been computed using Illuminant C because this was the surround illuminant used by MacAdam in his original experiment.

TABLE III. New analysis of earlier data—I.

		CIELUV	CIELAB
Wright discrimination lines			
$L^* = 70$ (152 lines) standard deviation		0.28	0.50
$L^* = 50$ (221 lines) standard deviation		0.29	0.55
$L^* = 30$ (260 lines) standard deviation		0.32	0.54
MacAdam discrimination ellipses			
$L^* = 70$ (16 ellipses) eccentricity		2.16	2.71
$L^* = 50$ (24 ellipses) eccentricity		2.48	3.50
$L^* = 30$ (25 ellipses) eccentricity		2.52	3.74
$L^* = 70$ (16 ellipses) s.d. of mean radius		0.14	0.30
$L^* = 50$ (24 ellipses) s.d. of mean radius		0.20	0.25
$L^* = 30$ (25 ellipses) s.d. of mean radius		0.21	0.24
Munsell Renotation of real samples judged			
$L^* = 70$	s.d. of hue spacing	5.9°	4.3°
$L^* = 50$	s.d. of hue spacing	5.2°	3.7°
$L^* = 30$	s.d. of hue spacing	4.6°	3.3°
$L^* = 70$	mean hue curvature	1.33%	1.12%
$L^* = 50$	mean hue curvature	1.70%	1.48%
$L^* = 30$	mean hue curvature	1.68%	1.50%
$L^* = 70$	mean chroma eccentricity	1.40	1.31
$L^* = 50$	mean chroma eccentricity	1.42	1.33
$L^* = 30$	mean chroma eccentricity	1.43	1.22
OSA real samples judged			
$L^* = 60.4$ (104 comparisons among 43 samples) s.d. of perceived colour difference per distance in colour space		0.24	0.24

When transformed to the CIELAB colour space, the ellipses are no longer exactly elliptical but slightly ovoid in shape because of the nonlinear transformation used to obtain the CIELAB coordinates.

The departures from exact ellipticity were regarded as being too small to warrant any special treatment in this analysis; the departures do not illustrate any deficiency in the CIELAB space, since the original data on which the MacAdam ellipses were based were only approximately represented by ellipses.

The data have been compared by computing the mean eccentricity (the ratio of the length of the semimajor axis to the length of the semiminor axis) and the standard deviation of the mean radius of the ellipses. This number is calculated by finding, for each ellipse, the mean radius, computing the standard deviation for all the ellipses, and dividing this standard deviation by the mean radius for all ellipses. This last step serves to normalise the standard deviation so that the results for the two spaces can be compared. A low value for the mean eccentricity implies that the ellipses are approaching circular shape, i.e., local uniformity; a low value for the standard deviation of the mean radius implies that the circles (or ellipses) are all of approximately similar size, i.e., overall uniformity.

The results, in Table III, show that CIELUV gives smaller values than CIELAB although the smallest mean

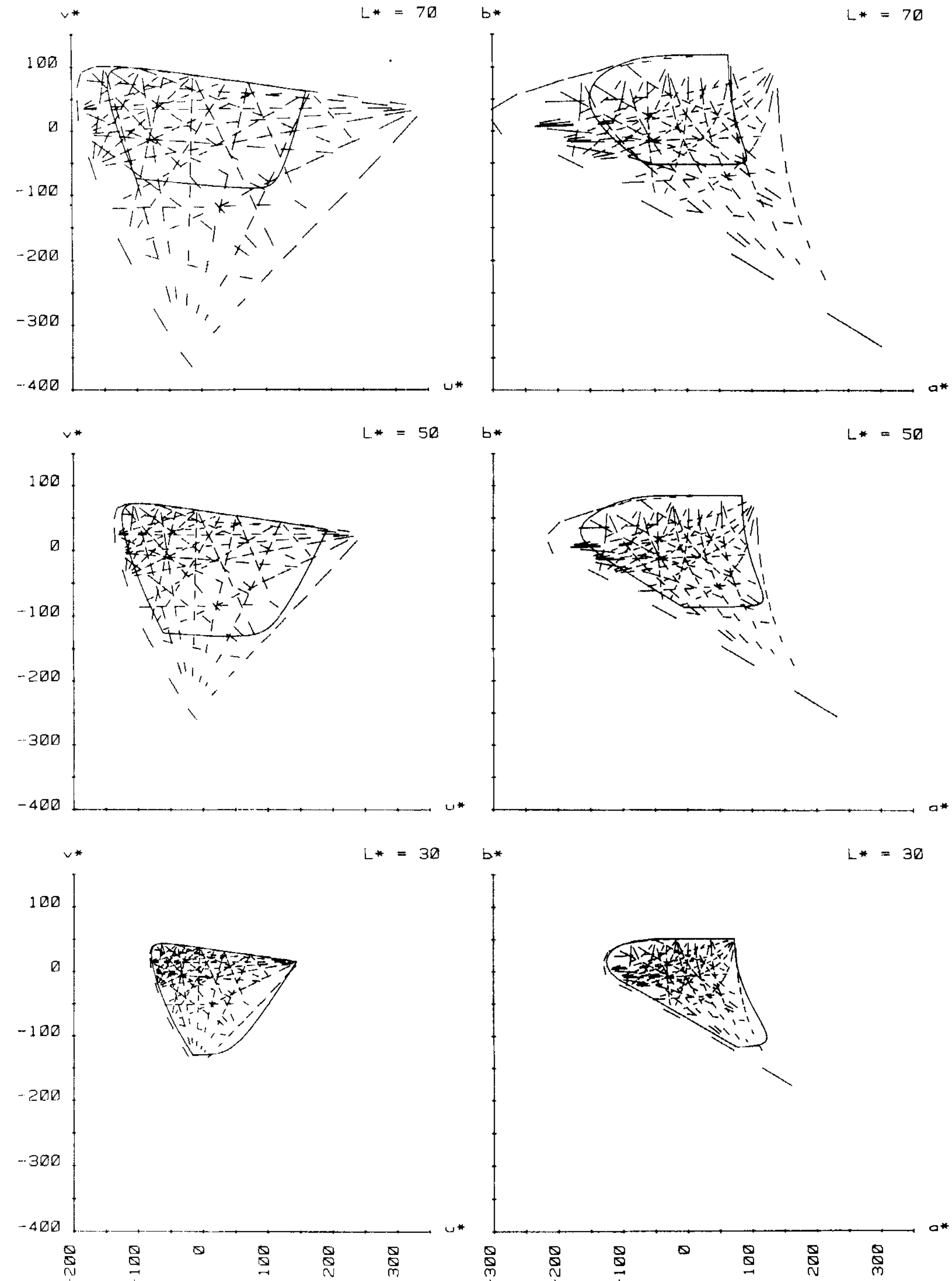

FIG. 1. The Wright (1941) data plotted on CIELUV and CIELAB psychometric chroma diagrams for values of L^* of 70, 50, and 30. The optimal limit for each value is also plotted.

eccentricity is 2.16. However, none of these differences are statistically significant at a 95% confidence level. It should be noted that, for each value of L^*, only those ellipses whose centres are inside the optimal limit have been included in the calculations.

The Munsell Renotation data for Values 7, 5, and 3 were transformed into CIELUV and CIELAB colour spaces; the results are plotted in Fig. 3 using the same scale that was used for Figs. 1 and 2. The optimal limits for Illuminant C are also shown. The Munsell data plotted in these figures are limited to those samples used in the original scaling experiments (see Kelly, Gibson, and Nickerson, 1943). Thus, only Munsell Hues corresponding to 5 and 10 units are plotted and the values of Munsell Chroma considered are much smaller than those given in the complete renotation data. The reason for this limitation is that the data used here are confined to those for the smoothed visual observations, whereas the complete Munsell Renotation data

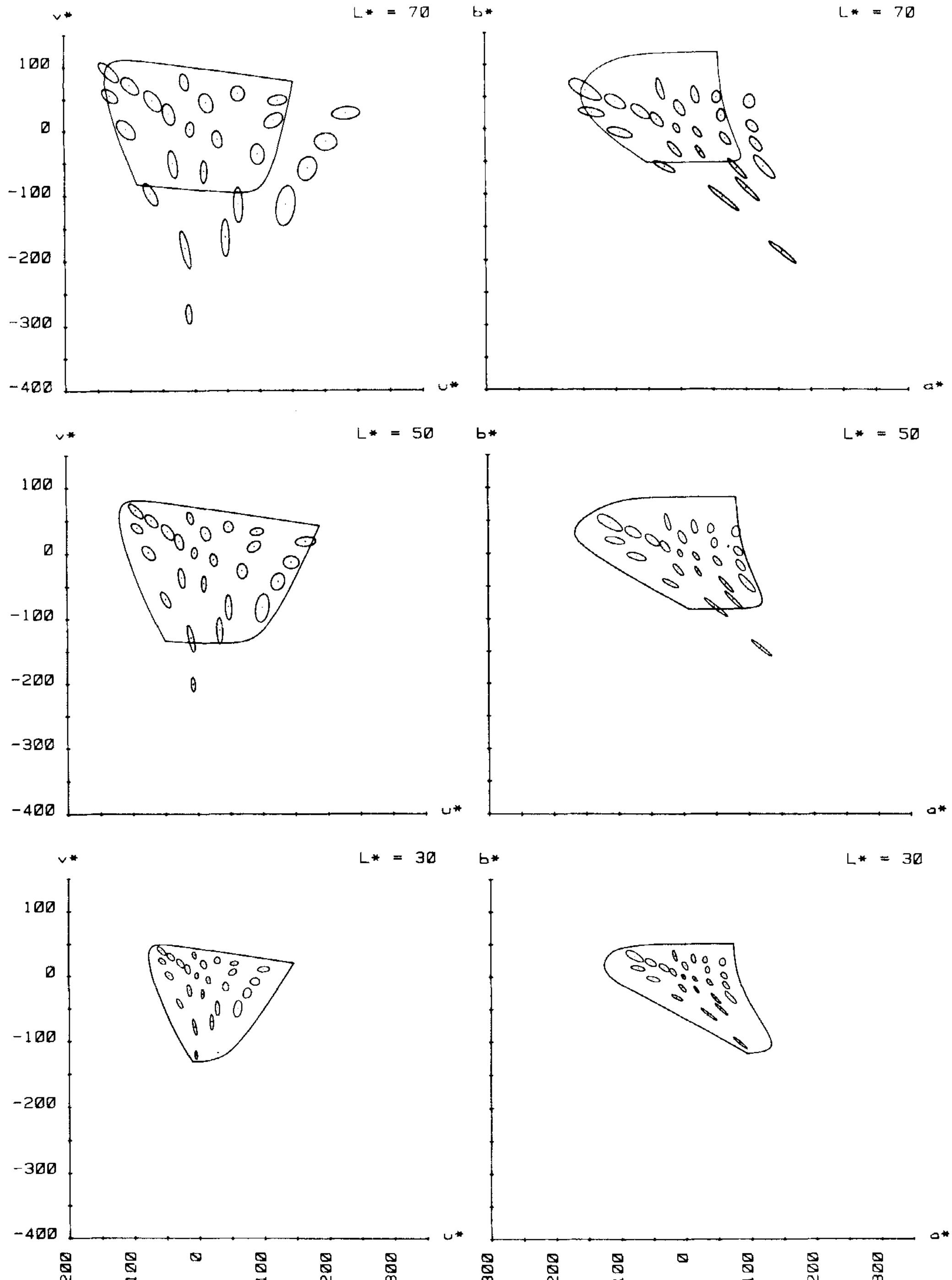

FIG. 2. The MacAdam (1942) data plotted on CIELUV and CIELAB psychometric chroma diagrams for values of L^* of 70, 50, and 30. The optimal limit for each value is also plotted.

involve the extension of this smoothed visual data by extrapolation. The extrapolation was aided by use of the Adams Chromatic Value colour space that is itself an earlier version of the CIELAB space. The extrapolated data are clearly of no value for comparing spaces for visual uniformity. By limiting the data in this way it is noticeable, from Fig. 3, that only a small area of the colour spaces is considered in the comparisons.

Three numbers were computed to compare these data. First, the mean hue angle was computed for each hue line at each value considered. The hue angle is defined by

$$\text{CIELUV} \qquad h_{uv} = \arctan(v^*/u^*),$$

$$\text{CIELAB} \qquad h_{ab} = \arctan(b^*/a^*).$$

The standard deviation of the mean hue spacing was then calculated for each value and colour space. If the spacing were perfect the mean hue spacing would be equal to 18° in both colour spaces. Second, the mean hue curvature was

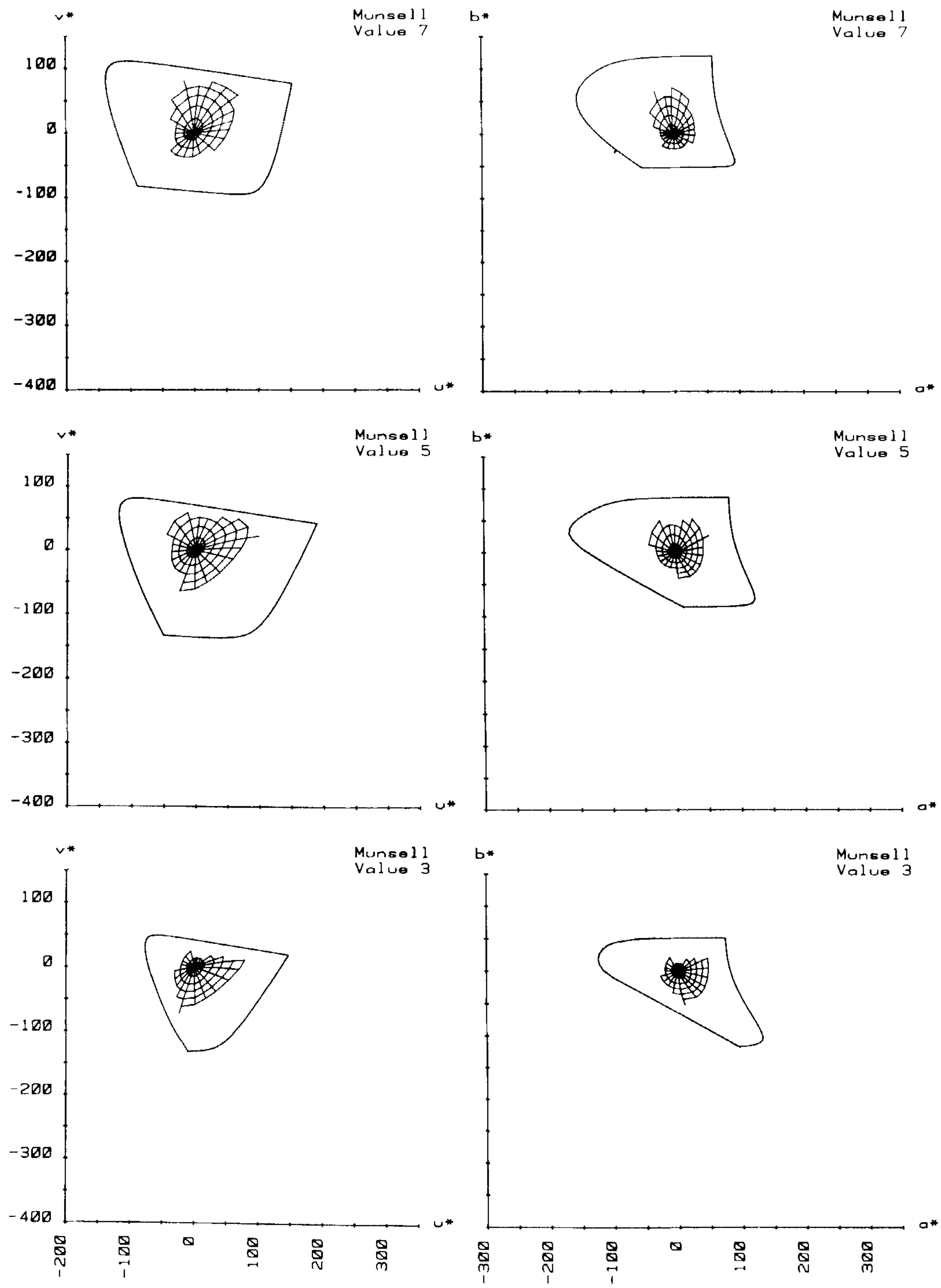

FIG. 3. The Munsell Renotation data (limited to those samples used in the original scaling experiments) plotted on CIELUV and CIELAB psychometric chroma diagrams for values of L^* of 70, 50, and 30. The optimal limit for each value is also plotted.

calculated for each hue line. This was calculated by drawing a straight line from the illuminant point to the maximum chroma on each Munsell Hue line and expressing the greatest distance of the hue line from the straight line as a percentage of the length of the straight line. Third, the eccentricities of the chroma contours were measured as the ratio of the maximum diameter to the minimum diameter, with both diameters passing through the illuminant point. The results are shown in Table III. It is seen that all these measures give lower values for CIELAB colour space, suggesting that this space gives a more uniform spacing of the Munsell data considered in this analysis. However, none of these differences is statistically significant at a 95% confidence level.

The last set of data analysed in this section is that derived by MacAdam in association with the OSA uniform colour scales (MacAdam, 1974). One hundred four pair comparisons were made involving 43 test colours and 76 ob-

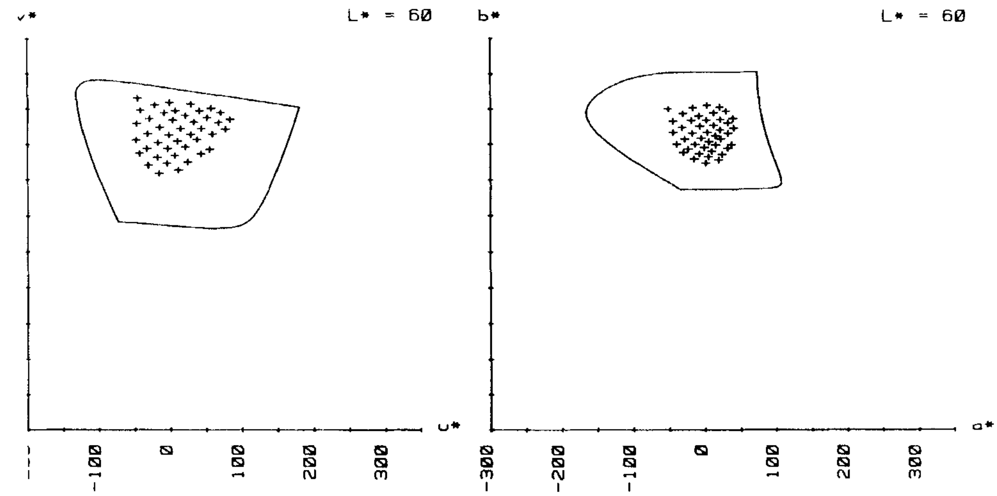

FIG. 4. The experimental OSA data (from MacAdam, 1974) plotted on CIELUV and CIELAB psychometric chroma diagrams for $L^* = 60$. The optimal limit for $L^* = 60$ is also shown.

servers. The test colours were all nominally of 28% reflectance; the mean value of L^* was 60.4. The data were derived by scaling the perceived colour differences of the pairs. The coordinates of the 43 test colours plotted in CIELUV and CIELAB colour spaces are shown in Fig. 4. Again the same scale as in Figs. 1–3 is used. The illuminant used is D_{65}, and the optimal colour limit shown is also for that illuminant.

In order to compare the two colour spaces, the scaled value representing the perceived colour difference was divided by the colour difference calculated using the CIELUV or CIELAB colour-difference formulae. The standard deviation of this ratio was used to compare the spaces. As can be seen from Table III, the standard deviation was the same for both spaces.

Thus, for four sets of data analysed, no consistently significant difference has been found between the CIELUV and CIELAB colour spaces. The largest difference is for the Wright discrimination data where the standard deviation of line length is 76% higher for CIELAB compared with CIELUV, but at the 95% confidence limit this is only significant at one of the three values of L^* studied.

New Analysis of Earlier Data—II

Several other sets of ellipses have been produced as a result of colour-matching or colour-scaling experiments. Brown and MacAdam (1949) produced ellipsoids by a statistical analysis of colour-matching data. Both this study and the earlier study of MacAdam (1942) used a colorimeter with a 2° bipartite field; Brown (1957) used a 10° field and produced average results for 12 observers. Wyszecki and Fielder (1971a) analysed statistically the colour-matching results produced by three observers using a colorimeter with two 3° fields. Robertson (1976) also produced average ellipsoids for two 3° fields and seven observers. Kuehni (1971, 1972) produced 50% acceptability ellipses from the results of assessing the colour differences between three standards and 113 fabric samples; ten observers were used. Kuehni (1977) has analysed the HATRA data (see Jaeckel, 1973) in terms of acceptability ellipsoids. These data were derived from an extensive study involving the acceptability of 400 samples as matches to ten standards.

All these sets of ellipses have been analysed using the technique applied earlier to the MacAdam ellipses, i.e., the mean eccentricity and the standard deviation of the mean radius have been calculated. For the Brown and MacAdam, the Brown, and the Wyszecki and Fielder ellipses, values of L^* of 70, 50, and 30 were used for the calculations; the original data were derived using test samples viewed with surrounds of a lower luminance than that of the actual test colours. Only those ellipses with centres inside the optimum limit for the specified value of L^* were used in the calculations. For the other sets of data, where real surface colours were used, the values of L^* of the samples themselves were used. The results are shown in Table IV.

As before, neither colour space is shown to be superior. For the eccentricity of the Wyszecki and Fielder ellipses one observer's results give lower mean values for CIELAB while the other two observers favour CIELUV. It is sometimes the case that the mean eccentricity is lower for CIELUV but the corresponding standard deviation of the mean radius is lower for CIELAB. This implies good local uniformity in CIELUV but better overall uniformity in CIELAB. However, the lowest value of mean eccentricity quoted in Table IV is 1.80, which represents an ellipse of quite large eccentricity, and the lowest value of standard deviation of mean radius quoted is 0.12, which represents a fluctuation in radius of 12%. Thus, although one colour space may be relatively better than the other for a particular set of data, on an absolute basis it still may not be very uniform.

This is also shown by an analysis of other sets of data that have been derived by scaling colour differences. Sugiyama and Wright (1963, 1964) used six pairs of colours of approximately the same luminance factor (mean $L^* = 60.9$). Eleven observers scaled the colour differences between the 21 possible pair combinations. Macbeth artificial daylight

TABLE IV. New analysis of earlier data—II (ellipses).

	CIELUV	CIELAB
Brown and MacAdam (1949) ellipses		
Observer 1		
L^* = 70 (28 ellipses) eccentricity	2.09	2.29
L^* = 50 (34 ellipses) eccentricity	2.24	2.34
L^* = 30 (37 ellipses) eccentricity	2.42	2.25
L^* = 70 (28 ellipses) s.d. of mean radius	0.28	0.27
L^* = 50 (34 ellipses) s.d. of mean radius	0.27	0.26
L^* = 30 (37 ellipses) s.d. of mean radius	0.27	0.25
Observer 2		
L^* = 70 (28 ellipses) eccentricity	1.97	1.90
L^* = 50 (34 ellipses) eccentricity	2.34	2.10
L^* = 30 (37 ellipses) eccentricity	2.54	2.02
L^* = 70 (28 ellipses) s.d. of mean radius	0.32	0.27
L^* = 50 (34 ellipses) s.d. of mean radius	0.32	0.27
L^* = 30 (37 ellipses) s.d. of mean radius	0.31	0.26
Brown (1957) ellipses		
L^* = 70 (16 ellipses) eccentricity	2.39	2.33
L^* = 50 (21 ellipses) eccentricity	2.53	2.66
L^* = 30 (22 ellipses) eccentricity	2.65	2.56
L^* = 70 (16 ellipses) s.d. of mean radius	0.21	0.35
L^* = 50 (21 ellipses) s.d. of mean radius	0.24	0.31
L^* = 30 (22 ellipses) s.d. of mean radius	0.24	0.27
Wyszecki and Fielder (1971a) ellipses		
Observer 1		
L^* = 70 (27 ellipses) eccentricity	2.22	2.18
L^* = 50 (28 ellipses) eccentricity	2.32	2.18
L^* = 30 (28 ellipses) eccentricity	2.32	2.10
L^* = 70 (27 ellipses) s.d. of mean radius	0.27	0.36
L^* = 50 (28 ellipses) s.d. of mean radius	0.30	0.39
L^* = 30 (28 ellipses) s.d. of mean radius	0.30	0.36
Observer 2		
L^* = 70 (27 ellipses) eccentricity	1.80	2.12
L^* = 50 (28 ellipses) eccentricity	1.86	2.11
L^* = 30 (28 ellipses) eccentricity	1.86	2.06
L^* = 70 (27 ellipses) s.d. of mean radius	0.25	0.30
L^* = 50 (28 ellipses) s.d. of mean radius	0.24	0.30
L^* = 30 (28 ellipses) s.d. of mean radius	0.24	0.28
Observer 3		
L^* = 70 (27 ellipses) eccentricity	2.22	2.48
L^* = 50 (28 ellipses) eccentricity	2.33	2.51
L^* = 30 (28 ellipses) eccentricity	2.33	2.43
L^* = 70 (27 ellipses) s.d. of mean radius	0.34	0.39
L^* = 50 (28 ellipses) s.d. of mean radius	0.35	0.42
L^* = 30 (28 ellipses) s.d. of mean radius	0.35	0.37
Robertson (1976) ellipses		
6 ellipses eccentricity	1.90	2.25
s.d. of mean radius	0.29	0.35
Kuehni (1971) ellipses		
3 ellipses eccentricity	2.89	2.39
s.d. of mean radius	0.23	0.12
Kuehni (HATRA) (1977) ellipses		
10 ellipses eccentricity	2.09	1.99
s.d. of mean radius	0.28	0.26

(u' = 0.2000, v' = 0.4712) was used to illuminate the samples. In a second experiment, 14 observers made ratio comparisons with the same pairs of colours. The data (taken from Wyszecki and Wright, 1965) were analysed in a way similar to that used for the OSA data above. The number (Q) derived for each colour space represents the standard deviation of perceived colour difference per unit distance in colour space. The results of this analysis are shown in Table V. These results show that there is little to choose between the two scaling methods, the differences being statistically insignificant.

A second set of data was taken from Wright (1965). Seven colours of approximately equal luminance factor (mean L^* = 63.7) were used to form 21 pairs. One observer made ratio comparisons of all possible combinations of the 21 pairs. Ten trials were made and an average perceived colour difference found for each pair. These results were compared with colour differences calculated using the CIELUV and CIELAB formulae. The results are given in Table V. The results from individual trials always favoured CIELUV colour space, as did the average result. This result is significant in that the 95% confidence intervals for the results in the two spaces do not overlap.

Another set of data from Wright (1965) used 117 comparisons chosen among 19 samples. One observer made ratio comparisons four times. Results of comparisons with colour differences calculated using the CIELUV and CIELAB formulae are given in Table V and favour CIELUV. Again this is a significant difference at the 95% level.

Wyszecki and Wright (1965), in a trial of the old 1964 (U^* V^* W^*) colour-difference formulae, derived a set of perceived colour-difference data. The 32 pairs of colours had luminance factors in the range of 0.65–0.03 (L^* = 84.6–20.6). The data were derived using both a white surround and a black surround. The CIELUV and CIELAB results are again shown in Table V calculated from the average results of the observers. The results for the white surround calculated for each individual observer show that one observer gives a lower value of Q for CIELUV. The other nine observers follow the results of the average observer and favour CIELAB: a statistically significant result at the 95% level.

Wright (1966) investigated the temporal factor in judging colour differences. One observer scaled each of eight pairs of colours when viewed for seven different time intervals ranging from 33 ms to unlimited time. The results of comparisons with CIELUV and CIELAB colour differences favour CIELAB; the mean values of Q for all seven time intervals are shown in Table V and are significantly different at the 95% level for these two spaces.

Wyszecki and Fielder (1971b) carried out an experiment, using a colorimeter, in which an observer was presented with two slightly different test colours and asked to make a third test colour that differed from each of the two presented test colours by an amount equal to the difference between them.

TABLE V. New analysis of earlier data—II (colour-difference evaluations).

	CIELUV	CIELAB
Sugiyama and Wright (1963, 1964) colour-difference scaling		
L^* = 60.1 (6 pairs of colours)		
pair comparison Q	0.10	0.21
ratio comparison Q	0.11	0.19
H. Wright (1965) colour-difference scaling		
L^* = 63.7 (21 pairs from 7 samples) Q	0.19	0.22
L^* = 62.6 (117 pairs from 19 samples) Q	0.15	0.22
Wyszecki and Wright (1965) colour-difference scaling		
L^* = 84.6–20.6 (32 pairs)		
white surround Q	0.33	0.26
black surround Q	0.33	0.24
H. Wright (1966) colour-difference scaling		
L^* = 84.6–39.1 (8 pairs) mean Q	0.55	0.26
Wyszecki and Fielder (1971b) colour-difference matches		
Observer 1		
L^* = 70 (34 matches) s.d. of difference	0.76	0.91
L^* = 50 (38 matches) between colour difference	0.66	0.81
L^* = 30 (38 matches) of two sample pairs	0.66	0.82
Observer 2		
L^* = 70 (34 matches) s.d. of difference	0.90	0.82
L^* = 50 (38 matches) between colour difference	0.79	0.72
L^* = 30 (38 matches) of two sample pairs	0.79	0.73
Observer 3		
L^* = 70 (34 matches) s.d. of difference	0.83	0.88
L^* = 50 (38 matches) between colour difference	0.70	0.79
L^* = 30 (38 matches) of two sample pairs	0.70	0.80
Pointer (1974) discrimination lines		
D_{65}		
L^* = 70 (83 lines) standard deviation	0.44	0.40
L^* = 50 (105 lines) standard deviation	0.54	0.47
L^* = 30 (110 lines) standard deviation	0.53	0.47
S_A		
L^* = 70 (68 lines) standard deviation	0.47	0.38
L^* = 50 (88 lines) standard deviation	0.44	0.38
L^* = 30 (109 lines) standard deviation	0.46	0.43

Thus, three colours were formed whose coordinates should plot as the points of an equilateral triangle in a uniform colour space. Wyszecki (1972) has shown that colour-difference-matching ellipses derived from these data correlate highly in terms of orientation and shape with colour-matching ellipses. The size of the colour-difference-matching ellipse is always larger than that of the colour-matching ellipse. The uniformity of CIELUV and CIELAB colour-difference spaces was assessed by calculating the colour difference between the colour generated by the observer and each of the two test colours provided on the colorimeter. These two colour differences should be equal, and the standard deviation of their difference, divided by the difference to normalise the two spaces, provides a measure of the uniformity of the spaces. The test colours were presented with a white surround and with twice the luminance of that surround. The comparisons were made for values of L^* of 70, 50, and 30 and for each of the three observers used in the experiment. The results, shown in Table V, show that one observer favours CIELAB while two favour CIELUV, but the differences are not significant at the 95% level.

Pointer (1974) investigated colour discrimination as a function of observer adaptation. Using a white-light adaptation with a colour temperature in the range 2000–6500 K, the observer adjusted one colorimetric field to be just noticeably different from the other. The test colours were derived using a Burnham colorimeter and results were obtained for 18 lines crossing the chromaticity diagram. These data have been used to compare CIELUV and CIELAB; the analysis was similar to that applied to the Wright (1941) discrimination data above. Values of L^* of 70, 50, and 30 have been used and, again, only those data that lie inside the optimal limit for the specified value of L^* have been included in the calculations. The results, shown in Table V, for adaptation to Illuminants D_{65} and A, favour CIELAB, but the differences are not significant at the 95% level.

The overall results of this analysis of colour-difference data do not show either CIELUV or CIELAB to be clearly superior.

Anomalies of the Spaces

The results of a study of the gamut of available samples of painted, printed, or dyed surface colours were plotted in both the CIELUV and CIELAB spaces (Pointer, 1980). It was found that the CIELAB space showed a pronounced bulge in the limit for optimal colours at hue angles h_{ab} of about 320°, especially at low values of L^* (see, e.g., Fig. 1). Although this bulge contains Munsell Chroma loci at approximately equal spacings, these loci were originally extrapolated by uniform increments in the similar Adams space (Newhall, Nickerson, and Judd, 1943), and are not based on any experimental data. The experimental data on chromaticity discrimination by Wright (1941) and by MacAdam (1942) suggest that the bulge in the L^* a^* b^* space in this region is an artifact of the space (see Figs. 1 and 2).

The use, in the CIELAB space, of two different functions of the ratios X/X_n, Y/Y_n, and Z/Z_n, according to whether their values are above 0.008856, leads to some unexpected effects (McLaren, 1980). For a colour of constant relative spectral composition, as its value of Y/Y_n is decreased, its value of hue angle h_{ab} remains constant until one of the ratios falls below 0.008856; it then gradually changes, changing back to its original value when all three ratios are below 0.008856. In the worst case the maximum change in h_{ab} is 36°. These changes in h_{ab} tend to increase with increasing purity and with decreasing value of Y/Y_n; however, for spectral colours of dominant wavelengths in the range 560–700 nm, changes in h_{ab} occur for all values of Y/Y_n. Because these effects all occur as a result of changes in a^* and b^*, the corresponding values of C^* will also change, and hence changes will occur in CIELAB hue difference ΔH_{ab}^*. None of these effects occurs in the CIELUV system.

The extent to which the CIELUV and CIELAB spaces represent the Swedish Natural Colour System (NCS) uniformly has been studied (Tonnquist, 1975) by plotting loci of constant colour content, white content, and black content, in L^*, C_{uv}^* and L^*, C_{ab}^* plots for four hues (red, yellow, green, and blue). Neither space represents the NCS space uniformly, but compression of reds and blues toward the grey axis in the CIELAB space causes it to differ from the NCS system more than in the case of the CIELUV space.

Conclusions

Twenty-five data sets have been used to compare CIELUV and CIELAB colour spaces and their associated colour-difference formulae. These data sets represent experiments carried out under a wide range of experimental conditions. There are real samples and colorimeter fields, white surrounds and black surrounds, 2° and 10° field sizes, high illuminance level and low illuminance level, juxtaposed test colours and separated test colours, perceptible differences and acceptable differences. Some of the data sets split into subsets, and for some sets the experiments or the analyses have been carried out under different conditions. The result is 68 pairs of numbers that represent the colour spaces. If a particular data set has been analysed in more than one way the results have been normalised and averaged so that only one pair of numbers is considered in the final analysis for each data set under each set of experimental conditions. Of the 68 pairs of numbers, only 8 are statistically different in that their associated 95% confidence intervals do not overlap. Of these 8 results, 4 favour CIELUV and 4 CIELAB.

Thus, the evidence presented in this article suggests that the data analysed will not be represented in a significantly more uniform manner in either the CIELUV or CIELAB colour space. However, it should be noted that none of the statistics presented suggests overall absolute uniformity for either space with any particular data set.

A decision as to which colour space to use is not easy to make. The CIELUV space does have the advantage of having an associated chromaticity diagram; on the other hand, the CIELAB space, or earlier versions of it, is already used in industrial applications (see, e.g., McLaren, 1976). However, the CIELAB space has been shown to have difficulties in terms of hue angle for low tristimulus-value ratios, and the reverse transformation from L^* a^* b^* to X, Y, Z is also very complicated when the Pauli modification is included.

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FORTHCOMING COLOR MEETING

Fourth European Conference on Visual Perception

The fourth European Conference on Visual Perception will be held September 8-11, 1981, at Gouvieux (near Paris), France. The theme of this nontopical meeting on visual research is sensory processes in visual perception, whether studied electrophysiologically or psychophysically, in man and other higher vertebrates. Between 100 and 120 papers, in English, are expected.

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