Colour, colour spaces and the human visual system

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Abstract

Colour is the high level result of biological processing within the eye and brain and as such presents problems to standard computer vision theory. To be able to make sensible use of colour it is important to understand some of the early processing stages within the eye and how these can help to explain the perception we have of colour. This paper discusses our perception of colour and outlines these early biological processing stages. A discussion then follows explaining the various methods, standards and colour spaces, with particular reference to the *Commission Internationale de l'Eclairage* standard, that can be used to make use of colour information within a computer system.

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1 What is colour?

Colour has been a major concern of western philosophy since the Presocratics in the seventh century B.C. [9]. Although there were many incorrect ideas about colour vision such as those held by Empedocles in the fifth century B.C. in which the eye acted as a lantern emanating particles that bounced back off surfaces, an idea that remained prevalent up until the philosopher Abu Ali Mohammed Ibn Al Hazen in 1000 A.D. [5], there were some notable ones which are still very much part of colour vision theory. For instance Democritus of Abdera said that "colour and the other senses exist only by convention; in reality there exist only atoms in the void ¹. Such sensations are due to human beings being what they are and having the kind of senses that they have and it is only because they have these senses that they have that they have come to attribute colour etc. to objects" [9]. Democritus meant by this that colour and the other senses only exist because human beings happen to interpret the world in a particular way.

As discussed by Chamberlin [6] Sir Isaac Newton wrote in his book Opticks "The rays [Of light] are, to speak properly, not coloured. In them is nothing else than a power to stir up a sensation of this or that colour". It is within this sentence that is distilled the most important aspect of colour vision:

Colour vision is a perception. Without the mind there is no such thing as colour.

Chamberlin also points out that "There is no such thing as colour in the absence of an observer; 'colour' is a subjective sensation, experienced through the light-sensitive receptor mechanism of the eye" [6]. Zeki wrote in his study of cortical cell responses to surface colours: "The results described here suggest that the nervous system, rather than analyze colours, takes what information there is in the external environment, namely, the reflectance of different wavelengths of light and transforms that information to construct colours." [19], in other words he is arguing that colour is a property of the brain, not the world outside [17].

1.1 Newtons pioneering work on colour

Newton first split white light into its spectral parts over 300 years ago by creating a beam of sunlight with the use of a small circular aperture and passing this beam through a prism. What he saw was a band of seven colours, including red, orange, yellow, green, blue, indigo and violet, which we now call the visible spectrum. Newton viewed these results as suggesting that sunlight was not pure but instead consisted of

¹Democritus is not referring to *atoms* in the sense we know them today although they do have some remarkable similarities. To him and the other Presocrastics they were the elements that constitute what-is and filled the void which is what-is-not. They were invisible and moved in the void, collided with each other and formed compounds by becoming hooked with one another [9].

seven separate parts. To verify his theory that the seven parts were pure he attempted to split one of the seven parts into further colours with the use of a second prism, in this he failed. We now call these separate parts hues.

Newton went on to do many other experiments on the nature of colour. In particular he experimented with the mixing of different hues. He found that certain hues could be formed by mixing other hues and that these could then be split into their component hues again. Most excitingly he found that white light could be formed by a mixture of the seven principle hues and also by various pairs of hues. He called these pairs of hues that when combined form white *complementary*.

1.1.1 Metameric Colours

It is interesting to note the apparent paradox that Newton's original "seven parts" could not be broken down into further hues but the same hue formed by a combination of other hues could. This paradox is further shown by the fact that white light can either be formed by a combination of all the "seven parts" or any set of complementary pairs. We call these different sets of hues that when combined yield the same hue metameric. Metameric colours or metamers are important because they "imply that the visual system is "blind" to certain aspects of the physical world" [16]. We don't know if we are seeing an orange formed from red and yellow or green and red.

1.1.2 Newtons colour circle

Newton's colour circle is one of the first attempts to classify colours and understand their mixing rules. It is based on arranging the seven spectral hues in a circle as shown below in figure 1. To find the colour produced by mixing two hues a line is drawn connecting them and if they are mixed in equal parts the hue produced will lie at the center point of this line. If there is more of one hue present than the other the final hue will shift towards the most prolific hue. As white can be formed by a mixture of all seven spectral hues it is found at the centre of the circle.

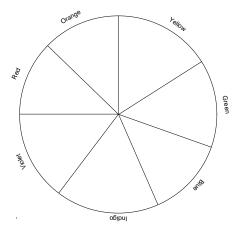


Figure 1: Newton's colour circle

The colour circle provides an insight into one of the results noticed by Newton when performing colour mixing experiments. If he mixed two spectral hues to produce a different spectral hue it was less vivid than when that hue was produced by a prism. A clue to this can be see in figure 2 where it can be seen that the position in the circle of the "new" hue is not on the circumference. As white is at the centre it is plausible to assume that as colours approach the centre they become more and more diluted with white; this property of colour is called *saturation*.

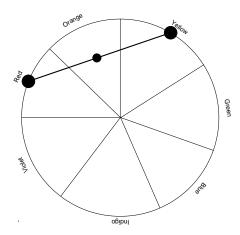


Figure 2: Newton's colour circle showing the effect of mixing two colours

1.1.3 Non spectral hues

An interesting omission from Newton's colour circle are the *nonspectral* colours. Newton noticed these colours when he mixed certain spectral hues and produced a colour unlike any of the seven spectral hues. For instance when red and blue are mixed the result is purple. This is now recognized as a hue but as a non spectral hue. The existence of these non spectral hues reinforces the original premise that colour is a subjective perception depending on the interpretation of the mixture of wavelengths striking the eye.

1.1.4 A revised version of Newton's colour circle

To include the non spectral colours Newton's colour circle has been extended as can be seen in figure 3, from Sekuler/Blake [16]. This circle has been arranged with the complimentary colours opposite each other and the non spectral colours added between the red and blue end of the spectrum. The approximate wavelength of the relative spectral lights have also been included.

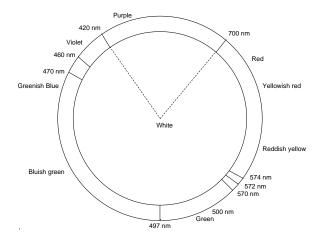


Figure 3: Extended version of Newton's colour circle

1.2 The the Young-Helmholtz theory of the trichromacy of colour

Following Newton's work one of the most remarkable minds to be attracted to the study of colour was Sir Thomas Young, a British Physician, who published "The Trichromatic Theory of Colour Vision" in 1801. A German mathematician, physicist and biologist Hermann von Helmholtz greatly developed this work formulating the Young-Helmholtz theory of Trichromacy [4] which states that:

Any colour can be formed by combining three properly chosen primary colours. Any primaries can be used as long as none can be formed from a mixture of the other two.

It is this trichromatic theory that allows any colour to be formed by mixing three different coloured lights, allows colour televisions to operate with three different coloured "guns" and the human eye to make sense of the world with three receptors each responsive to different "colours".

1.3 Phenomenal colour spaces

Newton's colour circle is based on hue and saturation and although neglecting a colour's brightness is the first example of a phenomenal colour space.

A phenomenal colour space is a colour space which uses as classifying descriptors hue, saturation and brightness.

More importantly, phenomenal colour spaces attempt to classify colours *purely in* relation to how we perceive them and as such represent the minds perception of colour.

Both hue and saturation have briefly been explained before but the concept of brightness has not. As all these terms are very important to the understanding of phenomenal colour spaces they are laid out fully in the following list.

Hue is that quality of colour that separates between red, yellow, green, blue, purple etc.

Saturation characterises a colour as being pale or vivid. It is also a measure of the amount of white that has been added to a colour. A highly saturated colour has had no white added, the extreme being the spectral hues. A highly desaturated colour has had a large amount of white added and in its extreme becomes the colour white.

Brightness is a measure of the intensity of light. Two colours differ only in brightness if they have the same relative spectral makeup but these components have a higher intensity. It is this quality of a colour that allows us to describe something as bright or dim.

Two examples of a phenomenal colour space are Munsell's and Ostwalds's colour spaces. These are both three dimensional representations of hue, saturation and brightness and are based on a colour circle similar to Newton's in that they have the spectral hues around the circumference. Brightness varies along a line perpendicular to the colour circle plane, the principle brightness axis is known as the achromatic axis along which colours vary from white through greys to black. Saturation is represented as the distance from the achromatic axis. There are many other examples of phenomenal colour spaces see Chamberlin [6] for a comprehensive list.

1.3.1 Munsell's colour space

Munsell's colour space was published in 1905 [4] and consists of 1500 systematically ordered colour samples. It is a three dimensional representation of a phenomenal colour space with a hue circle divided into five principle hues red, yellow, green, blue and purple. These are further split into five intermediate hues red–yellow, yellow–green, green–blue, blue–purple and red–purple. These are again further divided each into ten divisions giving 100 separate hues.

The saturation, as explained above, is represented as the distance from the achromatic axis with brightness represented as the achromatic axis. One important property about Munsell's colour space is that the steps of saturation and brightness are arranged to be at equal perceptual intervals.

In three dimensions Munsell's colour space becomes a solid as shown below in figure 4. Its strange shape is due to the saturation and brightness varying by equal perceptual intervals.

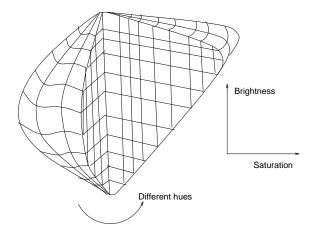


Figure 4: Munsell's colour space

1.3.2 Ostwald's colour space

The Ostwald system is similar to the Munsell system except that "each hue is combined, in a number of fixed proportions, with each of eight equally spaced neutral colours varying from white to black" [14]. An example of Ostwald's colour space is shown below in figure 5. It is interesting to compare its regular shape with that of the Munsell system which is due to the Ostwald's space lack of dependence on equal perceptual differences.

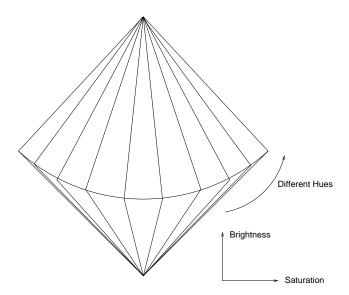


Figure 5: Ostwald's colour space

2 The human visual system

The human visual system is highly complex and has evolved as part of the solution to understanding and surviving in the environment around us [8]. It is important to realise that the visual system does not consist solely, or even mainly, of the eye but that the eye is just the first stage in a complex and highly effective system which culminates in the perceptions we have of the world. By understanding the eye we can gain clues about this visual system but they are only clues. To attempt to understand how human beings perceive the world the clues put forward from many disciplines including neuroscience, physiology, biology, physics, biochemistry and psychology also have to be collected and studied.

2.1 Structure of the eye

A sagittal section of the human eye is shown below in figure 6, from Gregory [8]. When light enters the eye it first passes through the *conjunctiva* a delicate mucous membrane covering the front of the eye. It then passes through the cornea where the light is bent most. The cornea is an interesting organ because it is virtually isolated from the rest of the body, it has no blood supply but instead relies on the aqueous humour which fills the cornea for nutrient. Sitting behind the cornea is the crystaline lens which is used for accommodation (focusing), accommodation is achieved by the ciliary muscle adjusting the shape of the lens by means of the zonula. The iris sits between the cornea and crystaline lens and varies the amount of light entering the eve through a small hole called the pupil. It is the iris that gives each eye its distinctive colour. Although the iris does cut down on the amount of light entering it is not it's sole purpose, "firstly the iris varies the size of the pupil in the ratio 16:1 whilst the eye has a brightness range of approximately 100,000:1. It seems that the pupil contracts to limit the rays of light to the central region, it also closes for near vision which increases the depth of field for near objects" [8]. The main body of the eye is filled with vitreous humour a transparent jelly like substance [11].

2.1.1 The retina

At the rear of the eye is the *retina* where the eyes light receptors reside. These receptors are made up of two different types of cells called rods and cones. Both rods and cones contain a pigment called rhodopsin to convert the incoming light to nerve impulses. Each molecule of rhodopsin can be broken down or "bleached" by a photon of light. It is this bleaching of rhodopsin that gives rise to nerve impulses [11]. When all the rhodopsin within a light receptor has been bleached the receptor then fails to operate.

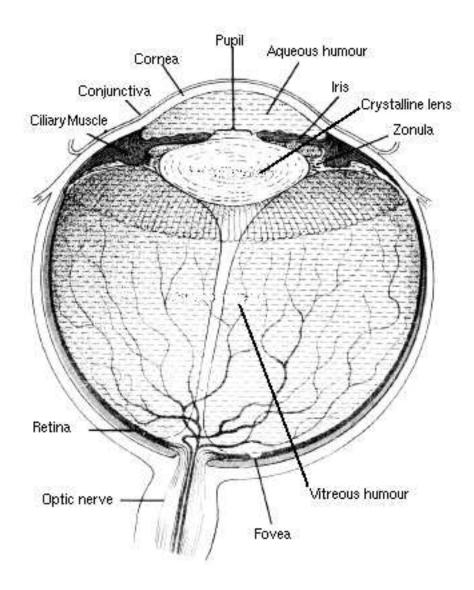


Figure 6: Sagittal section of the eye

2.1.2 The rods

The human eye contains around 125 million rods [11] which are primarily used in dim light and provide black and white vision. This form of vision is called *scotopic* vision. The world appears black and white at night and at other times of low illumination because at these times vision is almost solely provided by the rods.

2.1.3 The cones

There are 6–7 million [11] cones in the human eye; they function best in bright light and are essential for acute vision. There are three kinds of cones which are each responsive to different parts of the visual spectrum. It is these cones that allow us to perceive colours which is known as *photopic vision*. The three cones are often referred to as Red, Green and Blue or R, G and B which approximately reflects the area of the visible spectrum to which they respond or L, M, and S referring to Long, Median and Short wavelengths. For the purposes of this report L, M and S will be used. Figure 7 shows the response curve of the three cones along with the overall photopic sensitivity curve.

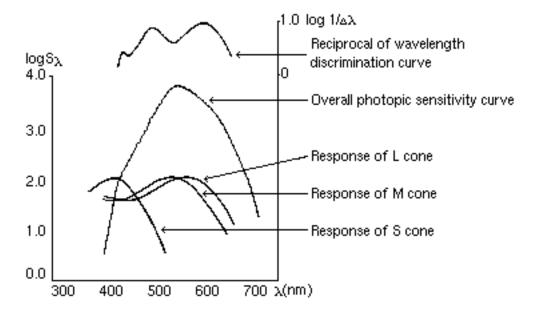


Figure 7: Response curve of cones

2.1.4 Geography of the rods and cones

The distribution of the rods and cones is not uniform. The cones almost exclusively exist in the *fovea* which is a small depression in the retina incident with the central area of vision [11] while the rods populate the non-foveal region. When the retina is examined it appears to be made inside out with the incident light rays having to

traverse a jumble of connectors and cells before striking the photoreceptors. Within the foveal region the collector cells are "peeled" off into the non-foveal region reducing the retinas thickness. This allows the fovea to achieve the necessary acuity with cones which are far less sensitive to light than the rods which fill the non-foveal region.

2.2 Psychophysical and Neurological theories of human vision

Due to the huge complexity of neurological processing within the brain when psychophysics and neuroscience are applied to human vision they have normally been applied to the early processing of visual information, typically those within the eye itself. Although this may appear limiting a very large amount of useful information can still be obtained. It is important to realise that many layers of further processing can occur before the concept of "perception" takes place and so these ideas should not and indeed cannot be expected to explain human vision. As Thompson puts it "Visual science is still far from able to provide the full story of how the activity in multiple neuronal areas becomes integrated to form our experience of colour" [17].

This is not to say that research into higher level processing has has not taken place a notable example being that of Semir Zeki [15] who approached the problem of colour vision at the physiological level. Zeki inserted microelectrodes in the visual cortex of anaesthetised monkeys and measured the neuronial potentials generated when they were given coloured stimuli. In 1970 he was able to pinpoint a small area of cells on each side of the brain which seemed to be responding to colour in the prestriate cortex of monkeys known as V4, Zeki referred to these as "colour-coding cells". Further evidence for the specialisation of V4 was given by the generation of coloured rings and halos when this area of the brain is stimulated magnetically.

2.2.1 The importance of phenomenal colour space

As discussed in section 1.3 a phenomenal colour space represents how we perceive colours, and as such one of the goals of understanding human vision is to provide a mapping from the frequencies of light incident on the eye to a phenominal colour space. Indeed as phenominal colour spaces are the only measure we have of a human's perception of colour this mapping is one of the most important results for psychophysics and neuroscience in the field of colour vision. The phenominal colour space of a human can be thought of as a representation of how colour is interpreted at a high level within the brain. Again, as Thompson said when writing about the mapping of visual information to a phenominal colour space "We do not intend to suggest that they [neurological and psychophysical theories] provide the full-fledged "linking propositions" needed to identify chromatic perceptual states and states of the visual substrate." [17] they do however provide very useful clues.

2.2.2 The response of the three retinal cones and the Young-Helmholtz theory of trichromacy

When incident light of a particular colour falls on the eye a set of responses in the three retinal cones is exhibited. For instance if light in the red end of the spectrum falls on the retina the response from the L cones will be greater than that of the M and S cones. It is this initial response that is the input into the neuronal system leading to colour perception. The theory that any colour can be represented in the retina as three different responses in the L, M and S cones is of course the "Young–Helmholtz theory of trichromacy" as discussed in section 1.2 [16]. Young had in fact postulated the existence of three different receptors within the eye in 1801:

As it is almost impossible to conceive each sensitive point of the retina to contain an infinite number of particles, each capable of vibrating in perfect unison with every possible undulation, it becomes necessary to suppose the number limited; for instance to the three principle colours, red, yellow and blue, and that each of the particles is capable of being put in motion more or less forcibly by undulations differing less or more from perfect unison. Each sensitive filament of the nerve may consist of three portions, one for each principle colour [5].

The correspondence of the trichromacy theory of colour and the existence of three separate cone types is not an accident the former leading to the later.

2.2.3 The wavelength discrimination curve and opponent processes

An interesting result obtained by psychologists is the wavelength discrimination curve [17]. This curve shows the smallest change in wavelength of light required to register a change in perceived colour and is sometimes referred to as the hue discrimination curve [8]. A reciprocal of this curve is shown in figure 7 in which the areas of highest discrimination register as peaks. As can be seen the human visual system has two distinct peaks where wavelength discrimination is highest, these are around the redgreen region of the spectrum and the blue–yellow region. If the human eye has three receptors where do these two distinct peaks originate from?

A clue to this can be found from one set of colour names associated with maximum hue discrimination ie red and green. It seems as if the L and M cones, which absorb red and green wavelength light respectively, work together and a measurement of the difference in output of these cones is made. This is indeed the theory put forward by Ewald Hering (1834–1918) [6] known as the opponent colour theory. This theory argues that the human visual system responds to variations between the opposite primaries red and green, blue and yellow and a third pair black and white or brightness. Postreceptor cells, known as ganglion cells, have been found which could be responsible for this behaviour [17]. If the surface of the retina is studied in detail it can be seen that the inputs to these ganglion cells come from two concentric rings

of cones. These rings of cones are either L and M which correspond to red and green or a mixture of M and L and S with S corresponding to blue and the mixture of L and M corresponding to yellow. These concentric rings are shown schematically in figure 8.

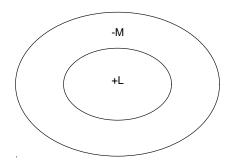


Figure 8: Schematic diagram of ganglion cells

If these cells are investigated further it is found that the cones that make up the outer portion of the two concentric rings act to inhibit the output signal of the ganglion cell while the cones that make up the inner portion amplify the output signal. The result of this is that the prescence of one of the opponent colours will reduce the output whilst the prescence of the other increases it and that during darkness the output is non zero. This output is shown in figure 9. It seems likely that it is the prescence of these ganglion cells taking as input the output of the cones and feeding the optic nerve which gives rise to the peaks in the wavelength discrimination curve. This hypothesis is given further strength by the work of Wiesel and Hubel from [3] "who examined the receptive fields of single neurons in the lateral geniculate nucleus of the monkey brain extending the work to cover the effects of wavelength. Cells in this nucleus have receptive fields that are virtually identical to those of the retinal ganglion cells that drive them, so that the recording might as well have been made from ganglion cells."

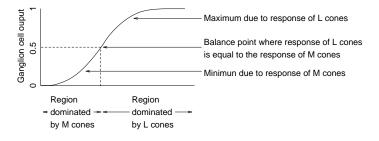


Figure 9: Output of ganglion cells

2.2.4 Overlapping cone responses lead to metamers

As can be seen from figure 7 there is considerable overlap of the cone response functions, this is necessary to allow the visual system to distinguish light of different wavelengths as discussed in section 2.2.3. If, for example wavelengths in the range 540–570nm, the area of the spectrum we refer to as green-yellow, only exhibited a response in one of the cones the visual system would not be able to differentiate between intensity and wavelength differences in this range. In practice, as explained by the opponent process theory, both the L and M cones exhibit a response and it is the ratio of the two responses that distinguishes between the wavelengths thus allowing us to perceive yellow and green.

As stated by Robertson [13] "A fundamental consequence of the existence of only three [overlapping] types of photoreceptors is that many different spectral radiance distributions can produce the same perceived colour. For example, approximately 6 watts of 540nm radiation mixed with approximately 25 watts of 650nm radiation will have the same effect on the three cone types as approximately 10 watts of 580nm radiation. Thus no matter what subsequent processing the visual system applies to the signals from the three cone types, it cannot distinguish between the two stimuli". This explains the existence of metamers discussed in section 1.1.1 where mixtures of different hues exhibited the same perception of colour.

2.2.5 The phenominal colour space of other animals

To further emphasis the importance of the phenomenal colour space it is helpful to try and imagine the phenomenal colour spaces belonging to other animals. Most animal species have some members that exhibit trichromatic vision with many exhibiting dichromacy (squirrels, rabbits, shrews, dogs and cats), tetrachomacy (goldfish and turtles) and even pentachromacy (pigeons and ducks) [17]. For an animal exhibiting trichromatic vision it is "possible" to try and imagine what form the phenomenal colour space would take. It is likely to be similar to our own being fully specified with three separate axis but with different areas of maximum hue discrimination; however we cannot imagine how these hues are perceived by the animals, for instance how does the colour orange appear to a horse?

For animals exhibiting a higher dimensionality of colour vision than our own the form of their phenomenal colour space becomes even harder to imagine, a tetrochromat does not simply have greater hue discrimination but has a completely different phenomenal colour space. The colour space will firstly require four dimensions to represent it [17], one for each psychophysical channel, and is as such called a colour hyperspace. The difference between the phenomenal colour spaces of two trichromats is not the same as the difference between a trichomat and a tetrachromat. The two colour spaces are incommensurable, there is no way to form a mapping between the two. The point is that a certain species exhibiting colour vision has a phenomenal colour space which is the high level result of processing within the eye and brain. All

of the processes which occur to map from the incident light to this colour space are not, at present, fully understood what is true is that there appears to be a mapping and that psychophysical and neurological investigations are helping to understand this mapping.

2.3 Changes in the perception of an objects colour

In the discussion so far the perception of a colour has been considered in isolation to other stimuli in the visual field. This is a useful simplification to make in understanding colour perception but does not correspond well to real world problems where the visual field may be cluttered and complicated. Robertson wrote "In practice there is no one-to-one relationship between a colour stimulus and a perceived colour. The perceived colour is determined not only by the colour stimulus to which it is directly associated but also by the state of adaptation of the visual system, by neighbouring stimuli in the field of view and by other physical, physiological and psychological factors."

3 Colour constancy and adaptation

One of the most powerful properties of the human visual system is its ability to operate over a wide range of sensitivities and conditions. The sensitivity range is in the order of approximately 10^{10} allowing the stars to be viewed at night and objects to be discriminated on a bright summers day. On a logarithmic scale this enormous range is roughly equally divided between scotopic and photopic vision [5].

3.1 Dark and light adaptation

If you move from being outside on a bright sunlit day to a dark room your vision is a first poor but slowly becomes more acute. This process is called dark adaptation and gives the power to see with remarkable clarity on a moon lit night. The opposite occurs when we move to a brightly lit area from a dimly lit one, in this case a process called light adaptation prevents us from being dazzled. Although it is possible to imagine a visual system with no form of adaptation it would generate a very large band width of data and would be likely to suffer from a lack of contrast particularly in the extremes of the brightness range. Very bright scenes would suffer from "white out" whilst dimly lit scenes would appear almost totally black. An analogous problem is that of balancing exposure time and aperture in an SLR camera so as to produce a picture with the desirable contrast.

3.1.1 Rhodopsin

As discussed in section 2.1.1 the rods and cones within the retina use rhodopsin to convert light to nerve impulses through a process called bleaching. If bleaching is occurring when a visual scene is viewed there must be a process regenerating the rhodopsin, if this was not true then the scene would slowly fade to darkness. According to Boynton [5] "It is certain that bleaching and regeneration of photopigments is an important factor in adaptation.". In the case of light adaptation more and more rhodopsin molecules become bleached thus reducing the available number to react with photons and lowering the output of the photoreceptor. For dark adaptation the opposite can be imagined with more and more rhodopsin molecules being generated and the photoreceptor output increasing. This process of regeneration and bleaching of rhodopsin molecules has been investigated by Rushton [5] who drew up a series of equations to model the behaviour.

3.1.2 Bleaching of rhodopsin

For bleaching to occur there must be rhodopsin molecules to bleach. This suggests a proportionality between the rate of bleaching and the number of unbleached rhodopsin molecules remaining. Rushton [5] suggests the corresponding differential equation where p is the number of unbleached rhodopsin molecules, I is the intensity of the bleaching light and N is a scaling factor.

$$\frac{dp}{dt} = \frac{-Ip}{N} \tag{1}$$

This then implies that the number of rhodopsin molecules varies with time exponentially as

$$p = e^{\frac{-It}{N}}. (2)$$

3.1.3 Regeneration of rhodopsin

Rushton found that the rate of regeneration depended on the amount of rhodopsin molecules to be regenerated with 0.25% of the bleached rhodopsin being regenerated per second. This also suggests an exponential behaviour with regeneration proportional to 1-p the amount of bleached rhodopsin assuming that p is 1 when the rhodopsin is fully regenerated.

$$\frac{dp}{dt} = \frac{1-p}{400} \tag{3}$$

and

$$p = 1 - e\frac{-t}{400}. (4)$$

3.1.4 The equilibrium amount of rhodopsin

If the rate of rhodopsin bleaching is proportional to p and the rate of regeneration is proportional to 1-p there must be a balance point when the two processes are equal. To find this consider equation 5 showing the total rate of change of rhodopsin, from equations 1 and 3.

$$\frac{dp}{dt} = \frac{1-p}{400} - \frac{Ip}{400I_0} \tag{5}$$

The scaling factor in the bleaching term has been replaced by $400I_0$ where I_0 is the intensity that bleaches 1/400th of the rhodopsin per second, from Boynton [5]. The steady state solution is then easily found by setting dp/dt to zero thus yielding

$$1 - p = \frac{I}{I + I_0} \tag{6}$$

and

$$p = \frac{I_0}{I + I_0}. (7)$$

3.1.5 Rate of change of rhodopsin level with light intensity

From equation 7 it can seen that the rate of change of rhodopsin with light intensity is

$$\frac{dp}{dt} = \frac{-I_0}{\left(I + I_0\right)^2} \tag{8}$$

and by combining this with equation $7 \frac{dp}{P}$ is found to be

$$\frac{dp}{P} = \frac{\frac{-I_0}{(I+I_0)^2}}{\frac{I_0}{I+I_0}} dI \tag{9}$$

which simplifies to

$$\frac{dp}{P} = \frac{-1}{I + I_0} dI. \tag{10}$$

For light intensities where $I \ll I_0$ equation 10 simplifies to

$$\frac{dp}{P} \approx \frac{-1}{I_0} dI \tag{11}$$

and for light intensities where $I \gg I_0$ it simplifies to

$$\frac{dp}{P} \approx \frac{-1}{I} dI. \tag{12}$$

This suggests that in the region of low to medium bleaching the visual system obeys Weber's law which states that $\frac{\Delta p}{p}$ is constant. Above this half way point the visual system begins to saturate. In practice this saturation can largely be ignored because as Rushton discovered during his investigations of bleaching a high light level is required to bleach a large fraction of rhodopsin and that even in bright daylight could bleaching of only 50% could be achieved [5].

3.2 Colour constancy

The light being reflected from a surface depends not only on the properties of the surface but also on the light illuminating the surface. If this illuminating light changes then the reflected light will also change. As the output of the three retinal cones depends on this reflected light their output will also vary with changes in illumination. If the perceived colour depends on these three cones then surely it must also change with illumination? This would mean that the colour of an object should change as we walk from sunlight to an artificially illuminated room. In practice, however, we hardly notice these changes due to a property of the visual system called *colour constancy*. As Sekuler and Blake wrote in their book Perception [16] "To achieve colour constancy, the visual system must disentangle a surface's spectral reflectance from the spectral quality of light that is illuminating that surface".

Colour constancy works by the visual system adjusting to allow for small differences in illumination. For instance if you have become adapted to daylight, which contains a great deal of blue light, the visual system becomes less responsive to the blue end of the spectrum. It is likely that colour constancy evolved to compensate for small changes in the spectral distribution of natural light [16] and as such can break down when objects are illuminated by a restricted wavelength belonging to an artificial light for instance a red or blue bulb.

3.2.1 Colour constancy and bleaching

Boynton suggests that the process of rhodopsin molecule bleaching used in dark and light adaptation is very likely to be the same process that governs colour constancy. In a red rich light the rhodopsin molecules within the L cone will suffer from more bleaching with respect to the M and S cones thus reducing their relative output and desensitising the eye to red.

4 Tristimulus space

Tristimulus space uses the theory of Trichromacy as discussed in section 1.2, to represent a colour by the amount of three fixed primaries needed to exactly match that colour. It is also commonly known as RGB space or LMS space which refers to the

three cones within the eye, see section 2.1.3, and is represented using three orthogonal axis.

As any three primaries can be used to represent a colour, providing that they fulfill the requirements of the trichromacy theory, it is likely that a colour may be represented by different primaries in different colour spaces. Indeed this is frequently the case and as Jan Allebach showed [2] creates problems in communication between devices and computer systems which do not share a common set of primaries.

4.1Colourmetric transformations

Colourmetric transformations allow us to move from one set of primaries to another e.g. LMS to $L^1M^1S^1$ therefore providing a mapping from one trichromatic colour space to another.

As any primary stimulus can be made from a mixture of the other set of primary stimuli this transformation can be written in the form shown below where a_{11} to a_{33} are constants.

$$L^{1} = a_{11}L + a_{21}M + a_{31}S$$

$$M^{1} = a_{12}L + a_{22}M + a_{32}S$$
(13)
(14)

$$M^1 = a_{12}L + a_{22}M + a_{32}S (14)$$

$$S^{1} = a_{13}L + a_{23}M + a_{33}S (15)$$

or $P^1 = PT$ where T is the transformation matrix and P and P^1 are the different colour spaces.

$$\begin{pmatrix}
a_{11} & a_{21} & a_{31} \\
a_{12} & a_{22} & a_{32} \\
a_{13} & a_{23} & a_{33}
\end{pmatrix}$$

4.2 CIE color space

Until 1931, the concept of colour had no scientific basis and colours could only be specified by comparing them with physical samples. In that year the Commission Internationale de l'Eclairage or CIE adopted a system of colour specification [5].

The CIE system specifies colours using a standard set of primaries, it is this standardisation of the primaries that allows the CIE system to be portable between different computer systems and devices. Although it is possible to match any colour with any set of primaries which satisfy the trichromacy theory it is sometimes necessary to use negative amounts of a primary. This is possible in theory but when using an additive mixture of lights is impossible.

To simplify this the CIE chose an "imaginary set of primaries" called the colour matching functions [6] which ensure that the visible colours all lie within the positive octant of the colour space ². These imaginary primaries are a super saturated red a super saturated green and an unattainable blue. Although they are imaginary they are all defined in terms of real lights. As a further simplification the CIE ensured that the M primary matched the photopic sensitivity curve for humans [12] which is of course brightness. This allows the luminance of a colour to be easily calculated and the hue to be altered, by adjusting the L and S primaries, without varying the luminance. The unit vectors controlling the relative brightness of the three primaries were then adjusted so that when L, M and S were all equal to one the colour white was defined [6].

Any realisable colour can now be represented by a point in the three dimensional LMS space. As the three primaries represent unattainable colours the volume of realisable colours will not fill this entire space but is rather a cone shape lying within the positive octant [12].

4.2.1 The chromaticity diagram

Many applications are not concerned with the luminance of a colour but purely its hue. For these it is customary to transform the LMS coordinates to the *chromaticity* coordinates l, m and s³. These coordinates are found by projecting the LMS coordinates onto the surface L + M + S = 1. The transformation equations found from simple geometry are therefore

$$l = \frac{L}{L + M + S},\tag{16}$$

$$m = \frac{M}{L + M + S} \text{ and}$$
 (17)

$$s = \frac{S}{L + M + S}. (18)$$

As the chromaticity coordinates are a projection onto the surface L + M + S = 1 it follows that l + m + s = 1. Since s = 1 - m + l any position on this surface can be uniquely specified by just m and l. A two dimensional plot transformed so that m and l are orthogonal is shown in figure 10. This is known as the *chromaticity diagram* and is a standard way of representing colours. When using such a diagram it is important to realise that it cannot uniquely represent a colour, to do so requires the knowledge of the brightness or M primary. It is also necessary to realise that the chromaticity diagram is a heavily distorted version of the original CIE colour space.

²In the original literature concerning the CIE colour space the primaries were referred to as X, Y and Z where Z corresponds to the S primary, Y to the M primary and Z to the L primary.

³l, m and s are often referred to as z, y and x respectively.

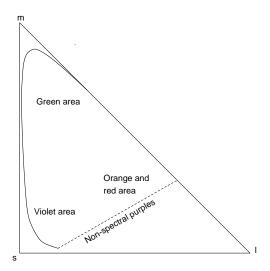


Figure 10: CIE chromaticity diagram

5 Perceptually uniform colour spaces

Although the 1931 CIE colour space is a very useful construct in the measurement and specification of colour it does not provide a measure of the perceived difference between two colours. The determination of a quantity that describes the difference between two colour stimuli depends on the ability of an observer to judge the relative magnitude of such a difference. The observer's judgement depends greatly on the conditions of observation and the kind of stimuli presented [18]. There is, however, a great deal of experimental data available on colour discrimination which has led to a number of empirical formulas designed to predict colour differences. The important similarity between these formulae is that they are firstly approximate and secondly can only be said to operate correctly under certain experimental conditions.

5.1 CIE uniform colour spaces

In 1976 [18] the CIE recommended two approximate uniform colour spaces that were chosen from several of similar merit to promote uniformity of practice. These would be recommended until a colour space giving substantially better correlation with visual judgement was developed.

5.1.1 $L^*u^*v^*$ uniform colour space

The first of these uniform colour spaces is produced by plotting the quantities L^* , u^* and v^* . L^* is a measure of the colour's brightness, u^* is redness–greenness and v^* is

blueness-yellow and they are defined by

$$L^* = 116 \left(\frac{M}{M_n}\right)^{\frac{1}{3}} - 16 \tag{19}$$

$$u^* = 13L^*(u^1 - u_n^1) (20)$$

$$v^* = 13L^*(v^1 - v_n^1) (21)$$

where

$$u^{1} = \frac{4L}{L + 15M + 3S} \tag{22}$$

$$v^1 = \frac{9M}{L + 15M + 3S} \tag{23}$$

$$u_n^1 = \frac{4L_n}{L_n + 15M_n + 3S_n} \tag{24}$$

$$v_n^1 = \frac{9Y_n}{L_n + 15M_n + 3S_n} \tag{25}$$

The tristimulus values L_n , M_n and S_n are those found from a white stimulus which help ensure colour constancy.

The above equations are valid in the region $\frac{M}{M_n} > 0.008856$ [18]. For values of $\frac{M}{M_n}$ equal or less than this the following calculation for L^* is recommended

$$L_m^* = 909.3 \frac{M}{M_p} \text{ for } \frac{M}{M_p} \le 0.008856$$
 (26)

The magnitude of the difference between two colours is then given by the colour difference formula

$$\Delta E_{u^*v^*} = \left[(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2 \right]^{\frac{1}{2}} \tag{27}$$

5.1.2 $L^*a^*b^*$ uniform colour space

The second uniform colour space is based on Munsell's uniform colour space, as discussed in section 1.3.1, and is produced by plotting the quantities L^* , a^* and b^* defined by

$$L^* = 116 \left(\frac{M}{M_n}\right)^{\frac{1}{3}} - 16 \tag{28}$$

$$a^* = 500 \left[\left(\frac{L}{L_n} \right)^{\frac{1}{3}} - \left(\frac{M}{M_n} \right)^{\frac{1}{3}} \right]$$
 (29)

$$b^* = 200 \left[\left(\frac{M}{M_n} \right)^{\frac{1}{3}} - \left(\frac{S}{S_n} \right)^{\frac{1}{3}} \right]$$
 (30)

the tristimulus values L_n , M_n and S_n are again those found from a white stimulus which help ensure colour constancy.

As with the $L^*u^*v^*$ uniform colour space there are constraints specifying when the equations above are valid and they are that $\frac{L}{L_n}$, $\frac{M}{M_n}$ and $\frac{S}{S_n} > 0.008856$ [18]. Values of $\frac{L}{L_n}$, $\frac{M}{M_n}$ and $\frac{S}{S_n}$ outside this range may be included if they are replaced by the following formula.

$$L_m^* = 903.3 \frac{M}{M_n} \text{ for } \frac{M}{M_n} \le 0.008856$$
 (31)

and

$$a_m^* = 500 \left[f\left(\frac{L}{L_n}\right) - f\left(\frac{M}{M_n}\right) \right] \tag{32}$$

$$b_m^* = 200 \left[f\left(\frac{M}{M_n}\right) - f\left(\frac{S}{S_n}\right) \right] \tag{33}$$

where

$$f\left(\frac{L}{L_N}\right) = \left(\frac{L}{L_n}\right)^{\frac{1}{3}} \qquad \text{for } \frac{L}{L_n} > 0.008856 \tag{34}$$

$$f\left(\frac{L}{L_N}\right) = 7.787\left(\frac{L}{L_n}\right) + \frac{16}{116} \quad \text{for } \frac{L}{L_n} \le 0.008856$$
 (35)

$$f\left(\frac{M}{M_n}\right) = \left(\frac{M}{M_n}\right)^{\frac{1}{3}} \qquad \text{for } \frac{M}{M_n} > 0.008856 \tag{36}$$

$$f\left(\frac{M}{M_n}\right) = 7.787\left(\frac{M}{M_n}\right) + \frac{16}{116} \quad \text{for } \frac{M}{M_n} \le 0.008856$$
 (37)

$$f\left(\frac{S}{S_n}\right) = \left(\frac{S}{S_n}\right)^{\frac{1}{3}} \qquad \text{for } \frac{S}{S_n} > 0.008856 \tag{38}$$

$$f\left(\frac{S}{S_n}\right) = 7.787\left(\frac{S}{S_n}\right) + \frac{16}{116} \quad \text{for } \frac{S}{S_n} \le 0.008856$$
 (39)

The magnitude of the difference between two colours in the $L^*a^*b^*$ uniform colour space is given by

$$\Delta E_{a^*b^*} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{\frac{1}{2}}$$
(40)

A Summary of colour spaces

A.1 RGB or LMS trimstimulus additive colour representation.

Colour space relying on the additive nature of colour and the theory of trichromacy discussed in section 4. Within this space colour is represented as a triplet of LMS coordinates and it is this representation that is commonly used by many computer systems and file formats.

A.2 Cyan Magenta Yellow (Black)

This is a colour space designed to represent subtractive colour and as such is often used in the printing and photography industries. Printers often use the fourth component to represent black ink which increases the density range and therefore the available gamut.

A.3 Hue Saturation and Lightness

This represents a set of colour spaces based on the phenominal colour spaces discussed in section 1.3. They include HSI (hue, saturation, intensity), HSV(hue, saturation, value), HCI(hue, chroma, intensity) etc.

A.4 YIQ, YUV, YCbCr, YCC (Luminance - Chrominace)

These are the television transmission colour spaces, also known as transmission primaries. They separate luminance from chrominance (lightness from colour) and are useful in compression and image processing applications. They are device dependent and, unless you are a TV engineer, unintuitive. Kodaks PhotoCD system uses a type of YCC colour space, PhotoYCC, which is a device calibrated colour space.

A.5 CIE XYZ (1931)

LMS colour space based on three imaginary primaries referred to as X,Y and Z. The three primaries are specified by the CIE(Commission Internationale de l'Eclairage) and as such provide a device independent colour space, see section 4.2.

A.6 CIE $L^*u^*v^*$

Uniform colour space where the distance between two colours within the colour space represents the perceptual difference measured by human observers. It is produced by plotting the quantities L^* , u^* and v^* where L^* is a measure of the colour's brightness, u^* is redness–greenness and v^* is blueness–yellow. See section 5.1.1.

A.7 CIE $L^*a^*b^*$

Uniform colour space based on Munsell's uniform colour space. See section 5.1.2

B Image file formats

There are a multitude of file formats available for the storage of images. These different formats have various merits and disadvantages depending on the application for which they are to be used.

B.1 Different image representations

There are two principle methods for image representation, bitmaps and vector notation.

B.1.1 Bitmap representation

Bitmap representation is by far the most common form used in file formats. The image is split into a grid and the light value, either brightness or colour, for each part of the grid is then recorded. With a bitmap notation 'any' image can be represented.

B.1.2 Vector representation

With vector representation the image is described as a series of lines and geometrical shapes. When examined vector notation often looks like a computer program with 'english' type terms being used to describe the image. For instance a circle center (100,100) and radius 10 could be described as CIRCLE(100,100,10). As vector notation depends on the use of geometrical shapes it is therefore limited to images that can be constructed from these. Vector notation does not require every point in the image to be recorded and as such typically produces far smaller file sizes than bitmap representation.

B.1.3 Choice of representation

The choice of representation to use depends on the application the format is intended for. If the image is complex with varying colours and complex shapes such as photographs, paintings etc. the bitmap representation is the only really suitable format. Vector notation works very well for applications which produce line art such as CAD packages with the additional advantage of smaller file sizes.

B.2 Monochrome images

B.2.1 Binary monochrome images and dithering

In their simplest form monochrome images consist of regions of either pure black or pure white represented as either 0 or 1. The files holding these images often make

⁴The word 'any' is used with reservation because although the format is flexible enough to store most scenes information can still be lost through lack of resolution both spatially and chromatically.

no reference to the colours they are supposed to represent leaving this to the output device. These files are relatively compact and suitable for storing images and graphs designed to be displayed by black and white output devices for instance a black and white laser printer.

The effect of different shades of grey can be produced in a binary monochrome image by using a method called dithering or halftoning. Dithering relies on the eye interpreting a sufficiently fine pattern of black and white as grey. Dithering is used for a large amount of printed material including many newspapers as the printing process is far cheaper than one able to produce true shades of grey and/or colours. A disadvantage with dithering is that, once dithered, an image is very hard to edit since most graphics programs are unable to distinguish between dots used for dithering and dots which represent lines. Because of this it is advantageous to postpone dithering until very late in the image production process.

B.2.2 Greyscale images

When an image is recorded as a greyscale image it can contain a certain number of shades of grey. This number defines the resolution of the image and can be found by the number bits⁵ used to represent each point. The human eye can easily discern about 64 shades of grey [10] so therefore to create realistic images 6 bits per pixel is necessary⁶.

B.2.3 Accuracy of greyscale images

When an image is transformed from a physical media to an electronic one perturbations can be introduced due to the response curve of the device used not correlating to that of the human eye. Most scanners, for instance, respond linearly to brightness whereas the human eye responds roughly to the cube root of intensity.

To handle these problems requires a knowledge of the limitations of the various media used throughout the image capture process. A popular method to correct for the linear response of scanners is gamma correction where gamma is the constant relating the output image intensity to input image intensity as in the following equation.

$$OUTPUT = INPUT^{\gamma}$$

Gamma correction although being able to correct for the linear behaviour of most scanners cannot allow for scanners and other forms of image capture with a more complex nonlinear response. In particular the response curve of a photograph is non linear in the extremes of intensity. To handle this, many file formats allow for a

⁵Number of bits refers to the size of the binary number used to represent the greyscale value. If 4 bits were used then 2⁴ or 16 shades of grey are available. In the extreme when the number of bits becomes 1 the greyscale image becomes a binary monochrome image.

 $^{62^6 = 64}$

greylevel response curve mapping the measured image intensity to the actual image intensity.

Other data that may be included within a file is information about the contrast of the original image [10]. This is necessary if the image data has to scaled to fit within a particular representation.

B.3 Colour images

B.3.1 Spot colour

If the file format is designed to apply to an output device with a limited number of colours, for instance a flat bed plotter equipped with various coloured pens, a representation called spot colour can be used. With files of this type the relevant colour pen to be used is simply included with each graphic element in the image. Since spot colour generally applies to lines and shapes it is often used with vector representation [10].

B.3.2 Tristimulus additive colour representation RGB or LMS

As discussed in section 4 colours can be represented as a triplet representing LMS coordinates and it is this representation that is commonly used by colour file formats. As with greyscale images the resolution of the colour image depends on the number of bits per pixel used to represent the colours. The standard method for expressing the resolution of a tristimulus colour representation is to give the total number of bits available to represent all three colours. For instance 24 bit colour allows 8 bits per colour.

B.3.3 Device dependent and independent colour spaces

The tristimulus colour representation relies on specifying the amount of three primaries that when combined produce the desired colour. As discussed in section 4 the colour produced will depend on the set of primaries used by the output device. This dependency on the output colour relying on a particular set of primaries is called device dependent colour. Device dependent colour manifests itself as the same tristimulus colour representation producing different colours on output devices with different sets of primaries.

Device independent colour spaces use a standard set of primaries to represent the LMS coordinates. These are then transformed, see section 4.1, to make use of the particular primaries used by the output device.

B.3.4 Colour gamut

A colour gamut is the area enclosed by a colour space in three dimensions. It is usual to represent the gamut of a colour reproduction system graphically as the range of

colours available in some device independent colour space. Often the gamut will be represented in only two dimensions, for example on a CIE chromaticity diagram (see 4.3).

B.3.5 Accuracy of colour images

Colour images suffer from the same perturbations introduced from non linear imaging devices as those suffered by greyscale images. The usual solution for this is to treat each image plane as a separate greyscale image and use the greyscale correction techniques discussed in section B.2.3. Although this corrects for errors within each image plane problems still arise with device dependent colour spaces because of the different choice of primaries used by imaging devices. If this is not corrected the final image hue will appear shifted towards one primary. To become portable the output of an imaging device must therefore be calibrated to some standard. In the CIE 1931 standard the colour white is represented by equal amounts of the L, M and S primaries and this allows a calibration to take place if the tristimulus output of the imaging device corresponding to white is recorded [10].

Therefore for an accurate colour image to be recorded the image file must contain the white point of the original image along with response curves for each colour plane or the gamma values corresponding to each image plane.

B.4 Choice of file format

There are many file formats available the most popular of which on UNIX systems are TIFF, GIF, JPEG and PPM. It is not the inner working details of these formats that is important here but rather what information is held and whether any information is lost. For a detailed discussion of the inner structure of these any many other file formats see "Graphics File Formats" [10].

The first three of these file formats include compression algorithms to reduce the final size of the file. The important point with these is that the compression algorithm employed in the JPEG format is a lossy algorithm. This is not a problem for most of the uses JPEG is employed, particularly those connected with showing moving images, but renders it unsuitable for image processing.

The TIFF (Tag Image File Format) format is a very flexible format developed by the Aldus Corporation [1] which allows a colour resolution of up to 16 bits per colour triplet. TIFF includes the ability to store colour response curves for all three primaries and the white point from an image input device. GIF (Graphics Interchange Format) [7] was developed by CompuServe Incorporated primarily as a transmission format for a data stream. It also includes colour response curves but has no provision to hold the image white point.

Both TIFF and GIF have a complicated structure which contrasts with PPM which is described as the lowest common denominator colour image file format. PPM

stands for portable pixmap and is an extremely simple format with no provision for recording colour response curves or the image white point.

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