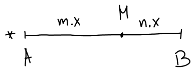
* $(\vec{a} \times \vec{b}) \times \vec{c}$

* 

* уравнение на равнина;
* проектиране: успоредно, ортогонално, централно
* метрично канонично уравнение

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1 зад.

$\vec{a}, \vec{b} : |\vec{a}| = |\vec{b}| = 1, \ \angle(\vec{a}, \vec{b}) = \frac{\pi}{3}$ $\quad \vec{a}^2 = |\vec{a}|^2 = 1, \ \vec{b}^2 = |\vec{b}|^2 = 1, \ (\vec{a}.\vec{b}) = 1.1.\cos\frac{\pi}{3} = \frac{1}{2}$

I $\vec{OA} = (\vec{a} \times \vec{b}) \times \vec{a} \quad \vec{OB} = \vec{a} + \vec{b} \quad \vec{OC} = (\vec{a} \times \vec{b}) \times \vec{b}$

$\vec{OA} = (\underset{1}{\vec{a}} \times \underset{2}{\vec{b}}) \times \underset{3}{\vec{a}} = (\vec{a}.\vec{a}).\vec{b} - (\vec{b}.\vec{a}).\vec{a} = 1.\vec{b} - \frac{1}{2}.\vec{a}$

$\vec{OC} = (\underset{1}{\vec{a}} \times \underset{2}{\vec{b}}) \times \underset{3}{\vec{b}} = (\vec{a}.\vec{b}).\vec{b} - (\vec{b}.\vec{b}).\vec{a} = \frac{1}{2}.\vec{b} - \vec{a}$

$\vec{OA} = \vec{b} - \frac{\vec{a}}{2}$

$\vec{OC} = \frac{\vec{b}}{2} - \vec{a}$

$\vec{OB} = \vec{a} + \vec{b}$

$\in \ell(\vec{a}, \vec{b})$ $\Downarrow$ $\vec{OA}, \vec{OB}$ и $\vec{OC}$ са лин. зависими

Не същ. тетраедър OABC

II $\vec{OA} = (\vec{a} \times \vec{b}) \times a = b - \frac{\vec{a}}{2}$

$\vec{OB} = \vec{a} + \vec{b}$

! $\vec{OC} = \vec{a} \times \vec{b} \Rightarrow \vec{OC} \perp \vec{a}, \ \vec{OC} \perp \vec{b}$

$V_{OABC} = ? \qquad V_{OABC} = \frac{1}{6}.|(\vec{OA}\ \vec{OB}\ \vec{OC})|$

$(\vec{OA} \times \vec{OB}).\vec{OC} = [(\vec{b} - \frac{\vec{a}}{2}) \times (\vec{a} + \vec{b})].\vec{OC} = [\vec{b} \times \vec{a} + \underbrace{\vec{b} \times \vec{b}}_{\vec{0}} - \underbrace{\frac{\vec{a}}{2} \times \vec{a}}_{\vec{0}} - \frac{\vec{a}}{2} \times \vec{b}].\vec{OC} =$

$= [-\vec{a} \times \vec{b} - \frac{\vec{a} \times \vec{b}}{2}].\vec{OC} = -\frac{3}{2}.(\vec{a} \times \vec{b}).(\vec{a} \times \vec{b}) = -\frac{3}{2}.(\vec{a} \times \vec{b})^2 = -\frac{3}{2}.(\vec{a}^2.\vec{b}^2 - (\vec{a}.\vec{b})^2) = -\frac{3}{2}.(1.1 - \frac{1}{4}) = -\frac{3}{2}.\frac{3}{4}$

$(\vec{OA}\ \vec{OB}\ \vec{OC}) = -\frac{9}{8} \Rightarrow V_{OABC} = \frac{1}{6}.\left|-\frac{9}{8}\right| = \frac{3}{16}$ куб. ед.

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2 зад. ОКС $K = Oxy$, търсим метрично канонично уравнение

$k: x^2 + 6xy + y^2 + 18x + 6y + 5 = 0 \qquad K = Oxy = O\vec{e}_1\vec{e}_2$

$a_{11} = 1 \quad a_{12} = 3 \quad a_{22} = 1 \Rightarrow A_1 = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \longrightarrow A_1' = \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix}$

спр. $\vec{e}_1 \vec{e}_2$ $\qquad$ спр. $\vec{b}_1, \vec{b}_2$

I $s_1 = ?, \ s_2 = ?, \ \vec{b}_1$ и $\vec{b}_2$

$|A_1 - s.E| = 0 \Rightarrow \begin{vmatrix} 1-s & 3 \\ 3 & 1-s \end{vmatrix} = 0$ $\qquad (1-s)^2 - 3^2 = 0$

$(1-s-3).(1-s+3) = 0$

$s_1 = 4 \quad s_2 = -2 \Rightarrow k$ е хипербола

$s_1 = 4 \Rightarrow \vec{b}_1(\alpha_1, \beta_1) \quad |\vec{b}_1| = 1 \Leftrightarrow \alpha_1^2 + \beta_1^2 = 1$

$(A_1 - s_1.E).\begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1-4 & 3 \\ 3 & 1-4 \end{pmatrix}.\begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \left| \begin{array}{l} -3\alpha_1 + 3\beta_1 = 0 \\ \alpha_1^2 + \beta_1^2 = 1 \end{array} \right. \Rightarrow \vec{b}_1\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$s_2 = -2 \Rightarrow \vec{b}_2(\alpha_2, \beta_2) \quad |\vec{b}_2| = 1 \quad \begin{pmatrix} 1-(-2) & 3 \\ 3 & 1-(-2) \end{pmatrix}.\begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \left| \begin{array}{l} 3\alpha_1 + 3\beta_1 = 0 \\ \alpha_2^2 + \beta_2^2 = 1 \end{array} \right. \Rightarrow \vec{b}_2\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

II Изв. смяна на ОКС $K = Oxy \xrightarrow{T_1} K' = Ox'y' : \ Ox' \uparrow\uparrow \vec{b}_1, \ Oy' \uparrow\uparrow \vec{b}_2$

$T_1: \begin{cases} x = \frac{\sqrt{2}}{2}.x' + \frac{\sqrt{2}}{2}.y' \\ y = \frac{\sqrt{2}}{2}.x' - \frac{\sqrt{2}}{2}y' \end{cases}$

Спр. K' $\quad A_1' = \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = \begin{array}{cc} x'^2 & x'y' \\ \end{array} \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix}$

$\left(y = \frac{\sqrt{2}}{2} \cdot x' - \frac{\sqrt{2}}{2}\, y' \right.$

Спр. K' $\quad A_1' = \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = \begin{array}{cc} x'^2 & x'y' \\ \begin{pmatrix} 4 & \underline{\underline{0}} \\ \underline{\underline{0}} & -2 \end{pmatrix} \\ & y'^2 \end{array}$

$k: 4x'^2 - 2y'^2 + 18 \cdot \underbrace{\left(\frac{\sqrt{2}}{2} x' + \frac{\sqrt{2}}{2} \cdot y' \right)}_{\text{от } T_1} + 6 \cdot \left(\frac{\sqrt{2}}{2} x' - \frac{\sqrt{2}}{2} y' \right) + 5 = 0$

$k: 4x'^2 - 2 \cdot y'^2 + 12 \cdot \sqrt{2} \cdot x' + 6 \cdot \sqrt{2} \cdot y' + 5 = 0 \quad (*)$

III Изв. смяна на ОКС : $K' = Ox'y' \xrightarrow{T_2} K'' = Cx''y''$: 1) $C(\alpha, \beta)$ е центърът на k

2) $Cx'' \uparrow\uparrow Ox'$

$Cy'' \uparrow\uparrow Oy'$

$T_2 : \begin{cases} x' = x'' + \alpha \\ y' = y'' + \beta \end{cases} \to (*)$

$4(x'' + \alpha)^2 - 2 \cdot (y'' + \beta)^2 + 12\sqrt{2} \cdot (x'' + \alpha) + 6\sqrt{2} \cdot (y'' + \beta) + 5 = 0$

$4x''^2 + 8\alpha x'' + 4\alpha^2 - 2y''^2 - 4\beta \cdot y'' - 2\beta^2 + 12\sqrt{2} \cdot x'' + 12\sqrt{2} \cdot \alpha + 6\sqrt{2} \cdot y'' + 6\sqrt{2}\beta + 5 = 0$

$4x''^2 - 2y''^2 + x'' \cdot \underbrace{(8\alpha + 12\sqrt{2})}_{0} + y'' \cdot \underbrace{(-4\beta + 6\sqrt{2})}_{0} + 4\alpha^2 - 2\beta^2 + 12\sqrt{2}\alpha + 6\sqrt{2}\beta + 5 = 0$

Търсим $\alpha = ?, \beta = ?$: $\left| \begin{array}{l} 8\alpha + 12\sqrt{2} = 0 \Rightarrow \alpha = -\frac{3\sqrt{2}}{2} \\ -4\beta + 6\sqrt{2} = 0 \Rightarrow \beta = \frac{3\sqrt{2}}{2} \end{array} \right. \Rightarrow 4 \cdot \frac{9 \cdot 2}{4} - 2 \cdot \frac{9 \cdot 2}{4} + 12\sqrt{2} \cdot \left(-\frac{3\sqrt{2}}{2} \right) + 6\sqrt{2} \cdot \left(\frac{3\sqrt{2}}{2} \right) + 5 =$

$= 18 - 9 - 36 + 18 + 5 = -4$

Спр. K'', $k: 4x''^2 - 2y''^2 - 4 = 0 \;/ : 4$

$\frac{x''^2}{1^2} - \frac{y''^2}{2} = 1$

$a^2 = 1 \Rightarrow a = 1, \quad b^2 = 2 \Rightarrow b = \sqrt{2}$

$T : \begin{cases} x = \frac{\sqrt{2}}{2} \cdot \left(x'' - \frac{3\sqrt{2}}{2} \right) + \frac{\sqrt{2}}{2} \cdot \left(y'' + \frac{3\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2} x'' + \frac{\sqrt{2}}{2} y'' + 0 \\ y = \frac{\sqrt{2}}{2} \cdot \left(x'' - \frac{3\sqrt{2}}{2} \right) - \frac{\sqrt{2}}{2} \left(y'' + \frac{3\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2} x'' - \frac{\sqrt{2}}{2} y'' - 3 \end{cases}$

МаТ, АГ -> 20

ДГ -> 27+3

КН -> 10+43=53

ИС -> 71+1+50=122

3 зад.

6 зад. Спрямо ОКС $K = Oxy$ са дадени правите:

$h: x - 7y - 6 = 0$, $m: 5x - 13y - 30 = 0$ и точката $B\left(\frac{4}{3}, \frac{2}{3}\right)$.

а) Да се намерят координатите на върховете на триъгълник ABC, ако височината и медианата му през върха C лежат съответно на правите h и m;

б) Да се намерят координатите на центъра и дължината на радиуса на **вписаната** в триъгълника окръжност.

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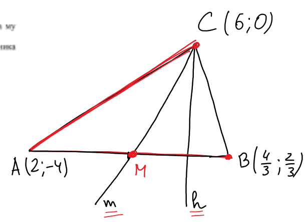

а) 1) т. $C = h \cap m$ $\quad \left| \begin{array}{l} x - 7y - 6 = 0 \quad / \cdot (-5) \\ 5x - 13 \cdot y - 30 = 0 \end{array} \right.$

$(35 - 13)y = 0 \Rightarrow y = 0$

$x = 6$

$C(6; 0)$

2) $AB \begin{cases} \perp h : x - 7y - 6 = 0 \\ \ni B\left(\frac{4}{3}; \frac{2}{3}\right) \end{cases} \Rightarrow AB: 7x + y - 10 = 0$

$7 \cdot \frac{4}{3} + \frac{2}{3} - 10 = 0$

3) ?, т. $M = AB \cap m$ $\quad \left| \begin{array}{l} 7x + y - 10 = 0 \quad / \cdot 13 \\ 5x - 13y - 30 = 0 \end{array} \right. +$ $\quad 96x - 160 = 0$

$x = \frac{160}{96} = \frac{10}{6} = \frac{5}{3}$

$y = 10 - \frac{35}{3} = -\frac{5}{3}$

$M\left(\frac{5}{3}, -\frac{5}{3}\right)$ – среда на

$B\left(\frac{4}{3}, \frac{2}{3}\right)$

$A(x, y)$

$\left(x + \frac{4}{3}\right) \cdot \frac{1}{2} = \frac{5}{3} \Rightarrow \quad x = 2 \cdot \frac{5}{3} - \frac{4}{3} = 2 \qquad A(2; -4)$

$\left(y + \frac{2}{3}\right) \cdot \frac{1}{2} = -\frac{5}{3} \qquad y = 2 \cdot \left(-\frac{5}{3}\right) - \frac{2}{3} = -4$

б) $A(2, -4)$ $\quad$ т. O – начало на к. с.

$B\left(\frac{4}{3}; \frac{2}{3}\right)$ $\quad$ т. I – ц-р на вписана окр.

$C(6, 0)$

$\overrightarrow{OI} = \frac{a \cdot \overrightarrow{OA} + b \cdot \overrightarrow{OB} + c \cdot \overrightarrow{OC}}{a + b + c}$!

$a = |\overrightarrow{BC}| = \frac{10\sqrt{2}}{3}$ $\quad \overrightarrow{BC}\left(\frac{14}{3}, -\frac{2}{3}\right) \Rightarrow |\overrightarrow{BC}|^2 = \frac{196 + 4}{9} = \frac{200}{9} \Rightarrow$

$b = |\overrightarrow{AC}| = 4\sqrt{2}$ $\quad \overrightarrow{AC}(4, 4) \Rightarrow |\overrightarrow{AC}| = 4 \cdot \sqrt{2}$

$c = |\overrightarrow{AB}| = \frac{10\sqrt{2}}{3}$ $\quad \overrightarrow{AB}\left(-\frac{2}{3}, \frac{14}{3}\right)$

$A(2; -4)$

$B\left(\frac{4}{3}, \frac{2}{3}\right)$

$b = |AC| = 4\sqrt{2}$ $\quad \vec{AC}(4, 4) \to$

$A(2; -4)$

$B\left(\frac{4}{3}, \frac{2}{3}\right)$

$C(6; 0)$

$c = |\vec{AB}| = \frac{10\sqrt{2}}{3} \quad \vec{AB}\left(-\frac{2}{3}, \frac{14}{3}\right)$

$a + b + c = \frac{20\sqrt{2}}{3} + 4\sqrt{2} = \frac{32\sqrt{2}}{3}$

$x_I = \left(\frac{10\sqrt{2}}{3} \cdot 2 + 4\sqrt{2} \cdot \frac{4}{3} + \frac{10\sqrt{2}}{3} \cdot 6\right) \cdot \frac{3}{32\sqrt{2}} = \frac{96}{32} = 3$

$y_I = \left(\frac{10\sqrt{2}}{3} \cdot (-4) + 4\sqrt{2} \cdot \frac{2}{3} + \frac{10\sqrt{2}}{3} \cdot 0\right) \cdot \frac{3}{32\sqrt{2}} = \frac{-32}{32} = -1$

$I(3; -1)$

$r = |\delta(I; AB)|$ $\quad AB: 7x + y - 10 = 0$ – общо

$AB: \frac{7x + y - 10}{\sqrt{7^2 + 1^2}} = 0$ – нормално

$AB: \frac{7x + y - 10}{\sqrt{50}} = 0 \qquad I(3; -1)$

$\delta(I; AB) = \frac{7 \cdot 3 + (-1) - 10}{5\sqrt{2}} = \frac{10}{5\sqrt{2}} = \sqrt{2}$

$r = \sqrt{2}$