A solid column of diameter 50 mm is required to be replaced by hollow column whose external diameter is 1.25 times internal diameter. The column is long enough to fail by buckling only. Compute percent saving in material.

Solution

Given,

$$D_s = 50 \text{ mm (solid column)}$$

$$D_o = 1.25\, D_i \text{ (hollow column)}$$

Here, we know,

$$\text{Critical load for solid column} = \text{Critical load for hollow column}$$

$$P_s = P_h$$

$$\frac{\pi^2 E I_s}{L^2} = \frac{\pi^2 E I_h}{L^2}$$

$$I_s = I_h$$

$$\frac{\pi D_s^4}{64} = \frac{\pi (D_o^4 - D_i^4)}{64}$$

$$50^4 = (1.25 \times D_i)^4 - D_i^4$$

$$D_i^4 = 4336043.5$$

$$\therefore\ D_i = 45.6 \text{ mm}$$

Thus,

$$D_o = 1.25 \times D_i = 57.04 \text{ mm}$$

Further,

$$\text{Percentage saving} = \frac{A_{\text{solid}} - A_{\text{hollow}}}{A_{\text{solid}}} \times 100$$

$$= \frac{(\pi/4)\, D_s^2 - (\pi/4)\,(D_o^2 - D_i^2)}{\pi/4 \times D_s^2} \times 100$$

$$= \frac{50^2 - (57.04^2 - 45.6^2)}{50^2} \times 100$$

$$= 53.03\%$$

NB/ use the correct values for different end fixicity of the columns

1. A solid round bar 3 m long and 5cm in diameter is used as a strut with both the ends hinged. Determine the crippling load. Take $E=2x10^5 N/mm^2$.

Step 1: Data

Length of the column = 3000mm

Diameter of the column = 50mm

condition = Both ends hinged

Crippling load = ??

$E=2x10^5 N/mm^2$

Step 2: Calculation of moment of inertia

$I = \pi d^4 / 64$

$I= \pi (50)^4 / 64$

$I = 0.306X10^6 mm^4$

Step 3: Calculation of crippling load

Condition = Both ends hinged

$P = \Pi^2 E I / L^2$

$P = \Pi^{2}(2x10^5) (0.306X10^6) / (3000)^2$

P = 67.11 KN

NB/ Always use the the least second moment of inertia when working with the the Euler's and Rankine's Formula.

4. A column of timber section 15cmx 20cm is 6m long both ends being fixed .E for timber is 17.5KN/mm^2, Determine
a) Crippling load
b) Safe load for the column if factor of safety=3

Step 1: Data
Length of the column = 6000mm
Width of the column = 150mm
Depth of the column = 200mm
Condition = Both ends being fixed
Crippling load =??
FOS=3
E=17.5 KN/mm^2

Step 2: Calculation of moment of inertia
$I_{xx} = bd^3 / 12$
$I_{xx} = 150\ (200)^3 / 12$
$\mathbf{I_{xx} = 100X10^6 mm^4}$

$I_{yy} = db^3 / 12$
$I_{yy} = 200\ (150)^3 / 12$
$\mathbf{I_{yy} = 56.25X10^6 mm^4}$

Choose whichever is least
Therefore, $\mathbf{I = 56.25X10^6 mm^4}$

Step 3: Calculation of crippling load

Condition = Both ends being fixed

$P = 4\Pi^2 E\, I / L^2$

$P = 4\Pi^2 (17.5 \times 1000)\, (56.25 \times 10^6) / (6000)^2$

P = 1079.48 KN

Step 4: Calculation of safe load

Safe load = crippling load /FOS

Safe load = 1079.48/3

Safe load = 359.82KN

7. A solid round bar 4m long and 5cm diameter was found to extend 4.6mm under the tensile load of 50KN.This bar is used as a strut with both ends hinged. Determine the buckling load for the bar and also safe load taking factor of safety as 4.

Step 1: Data

Length of the column = 4000mm

Diameter = 50mm

Extension = 4.6mm

Tensile load = 50KN

FOS = 4

Condition = Both ends hinged

Buckling load=??

Step 2: Calculation of strain

$e = dl / l$

$e = 4.6 / 4000$

$e = 1.15 \times 10^{-3}$

Step 3: Calculation of Area of cross section

$A = \pi d^2/4$

$= \pi (50)^2/4$

$A = 1963 mm^2$

Step 4: Calculation of stress

$\sigma = P/A$

$\sigma = 50 \times 10^3 / 1963$

$\sigma = 25.47 N/mm^2$

Step 4: Calculation of stress

$\sigma = P/A$

$\sigma = 50X10^3 /1963$

$\sigma = 25.47\ N/mm^2$

Step 5: Calculation of Young's modulus

$E = \sigma / e$

$E = (25.47) / (1.15X10^{-3})$

$E = 0.221X10^5\ N/mm^2$

Step 6: Calculation of Moment of inertia

$I= \pi d^4/64$

$= \pi (50)^4/64$

$I = 3.068X10^5 mm^4$

Step 7: Calculation of buckling load

Condition = both ends hinged

$P = \Pi^2 E I / L^2$

$P = \Pi^2 (0.221X10^5) (3.068X10^5) / (4000)^2$

P = 4.182 KN

Step 8: Calculation of Safe load

Safe load = crippling load /FOS

Safe load = 4.182X1000/4

Safe load = 1.0455 KN

A built up column consisting of rolled steel beam ISWB 300 with two plates 200 mm x 10 mm connected at the top and bottom flanges. Calculate the safe load the column carry, if the length is 3m and both ends are fixed. Take factor of safety 3 f_c = 320 N/mm^2 and α ? = 1 / 7500 Take properties of joist: A = 6133 mm^2 I_{XX} = 9821.6 x 10^4 mm^4 ; I_{yy} = 990.1 x 10^4 mm^4

Solution:

Given:

Length of the built up column, l = 3m = 3000 mm
Factor of safety = 3

f_c =320 N/mm^2

$$\alpha = \frac{1}{7500}$$

Sectional area of the built up column,

$$A = 6133 + 2(200 \times 10) = 10133 mm^2$$

Moment of inertia of the built up column section abut xx axis,

$$I_{XX} = 9821.6\times10^{4} + 2\left[\frac{200\times10^{3}}{12} + (200\times10)(155)^{2}\right]$$

$= 1.94 \times 10^{8}\ \text{mm}^{4}$

Moment of inertia of the built up column section abut YY axis,

$$I_{YY} = 990.1\times10^{4} + 2\left(\frac{10\times200^{3}}{12}\right)$$

$= 0.23 \times 10^{8}\ \text{mm}^{4}$

Since I_{yy} is less than I_{xx} , The column will tend to buckle about Y-Y axis.

Least moment of inertia of the column section,

$$I = I_{\min} = I_{YY} = 0.23\times10^{8}\,mm^{4}$$

The column is fixed at both ends.

$\therefore$ Effective length,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500mm$$

$\therefore$ Least radius of gyration o the column section,

$$K = \sqrt{\frac{J}{A}} = \sqrt{\frac{0.23\times10^{8}}{10133}} = 47.64mm$$

. 44. 4. A beam of channel section 120mm x 60mm has a uniform thickness of 15mm.Draw the shear stress diagram if it carries a shear force of 50kN.Find the ratio of maximum and mean shear stresses. (Ans:Shear stress values at significant fibres from bottom: 0, 6.67, 26.67, 35.24, 26.67, 6.67, 0 MPa. Ratio = 2.22) 5. The c/s of a beam is an unsymmetric I - section of overall depth 350mm, topflange 250mmx50mm,bottomflange 150mmx50mm, and web thickness 50mm. Draw the shear stress distribution diagram if it carries a shear force of 80 kN. (Ans:Shear stress values at significant fibres from bottom: 0, 1.378, 4.134, 5.89, 5.06, 1.012, 0 MPa.) 42

Crippling load as given by Rankine's formula,

$$p_{cr} = \frac{f_c \times A}{1+\alpha\left(\frac{L}{K}\right)^2} = \frac{320 \times 10133}{1+\frac{1}{7500}\left(\frac{1500}{47.64}\right)^2}$$

$$= 2864023.3 \text{ N}$$

Safe load = $\frac{\text{Crippling load}}{\text{Factor of safety}}$ $= \frac{2864023.3}{3} = 954674.43N$

Result:

i. Crippling load = 2864023.3 N
ii. Safe load = 954674.43 N