General Guidelines on Construction of SFD and BMD

Before we go on to solving problems, several standard procedures (or practices) in relation with construction of shear force and bending moment diagrams need to be noted.

1) The load, shear and bending moment diagrams should be constructed one below the other, in that order, all with the same horizontal scale.
2) The dimension on the beam need not be scaled but should be relative and proportionate (a 3 m span length should not look more than 5 m length!).
3) Ordinates (i.e., BM and SF values) need not be plotted to scale but should be relative. Curvature may need to be exaggerated for clarity.
4) Principal ordinates (BM and SF values at salient points) should be labeled on both SFD and BMD.
5) A clear distinction must be made on all straight lines as to whether the line is horizontal or has a positive or negative slope.
6) The entire diagram may be shaded or hatched for clarity, if desired.

Common mistakes when calculating shear force and bending moments

Forgetting to add the distance from the reference point when calculating bendings moments.

Not including the correct sign i.e positive or negative

Using incorrect units

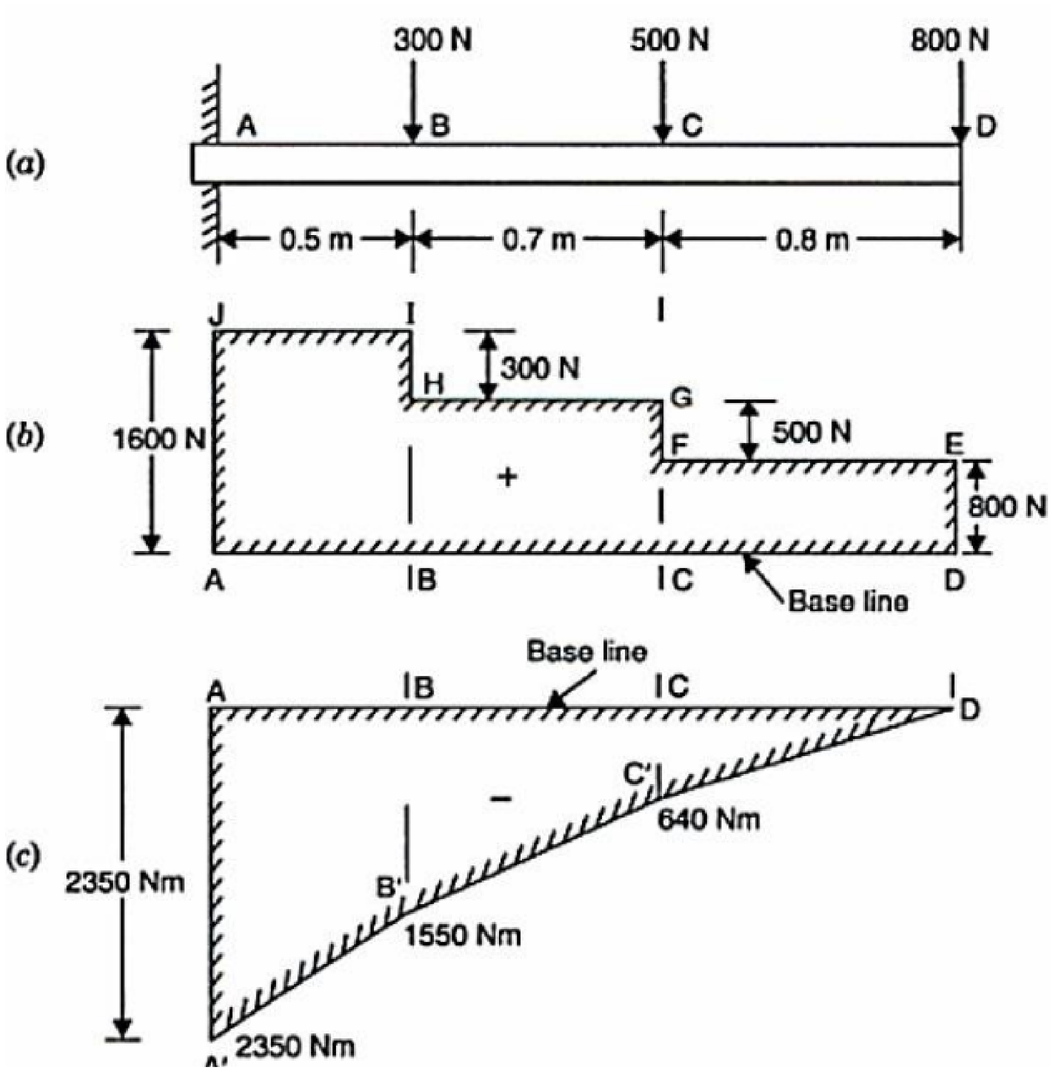
Using the wrong type of curves or lines when drawing the shear force and bending moments diagrams.

Variation of shear force and bending moment diagrams

S.N	**Point Load**	**UDL**	**UVL**
Shear Force	Constant	Linear	Parabolic
Bending Moment	Linear	parabolic	Cubic

WORKED EXAMPLES

1) A cantilever beam of length 2 m carries the point loads as shown in Fig. Draw the shear force and B.M. diagrams for the cantilever beam.



Shear Force Diagram

S.F. at D, $F_D = +800$ N

S.F. at C, $F_c = +800 + 500 = +1300$ N

S.F. at B, $F_B = +800 + 500 + 300 = 1600$ N

S.F. at A, $F_A = +1600$ N.

The shear force, diagram is shown in Fig.

Bending Moment Diagram

The bending moment at D is zero

B.M. at C, $M_C = -800 \times 0.8 = -640$ Nm.

B.M. at B, M_B $= -800 \times 1.5 - 500\ (1.5 - 0.8)$

$= 1200 - 350 = -1550$ Nm.

The B.M. at A, M_A $= -800 \times 2 - 500\ (2 - 0.8) - 300\ (2 - 1.5)$

$= -800 \times 2 - 500 \times 1.2 - 300 \times 0.5$

$= -1600 - 600 - 150 = -2350$ Nm.

Summary

$M_D = 0$

$M_c = -640$ Nm

$M_B = -1550$ Nm

$M_A = -2350$ Nm.

3) Draw the S.F. and BM. diagrams for the overhanging beam carrying uniformly distributed load of 2 kN/m over the entire length and a point load of 2 kN as shown in Fig. Locate the point of contra-flexure.

First calculate the reactions R_A and R_B.

Upward forces = Downward forces

$R_A + R_B = 2 X 6 + 2 = 14 kN$

Taking moments of all forces about *A*, we get

R_B x 4 = 2 x 6 x 3 + 2 x 6 = 48 kNm

R_B = 12 kN ; R_A = 2 kN

Shear force diagram

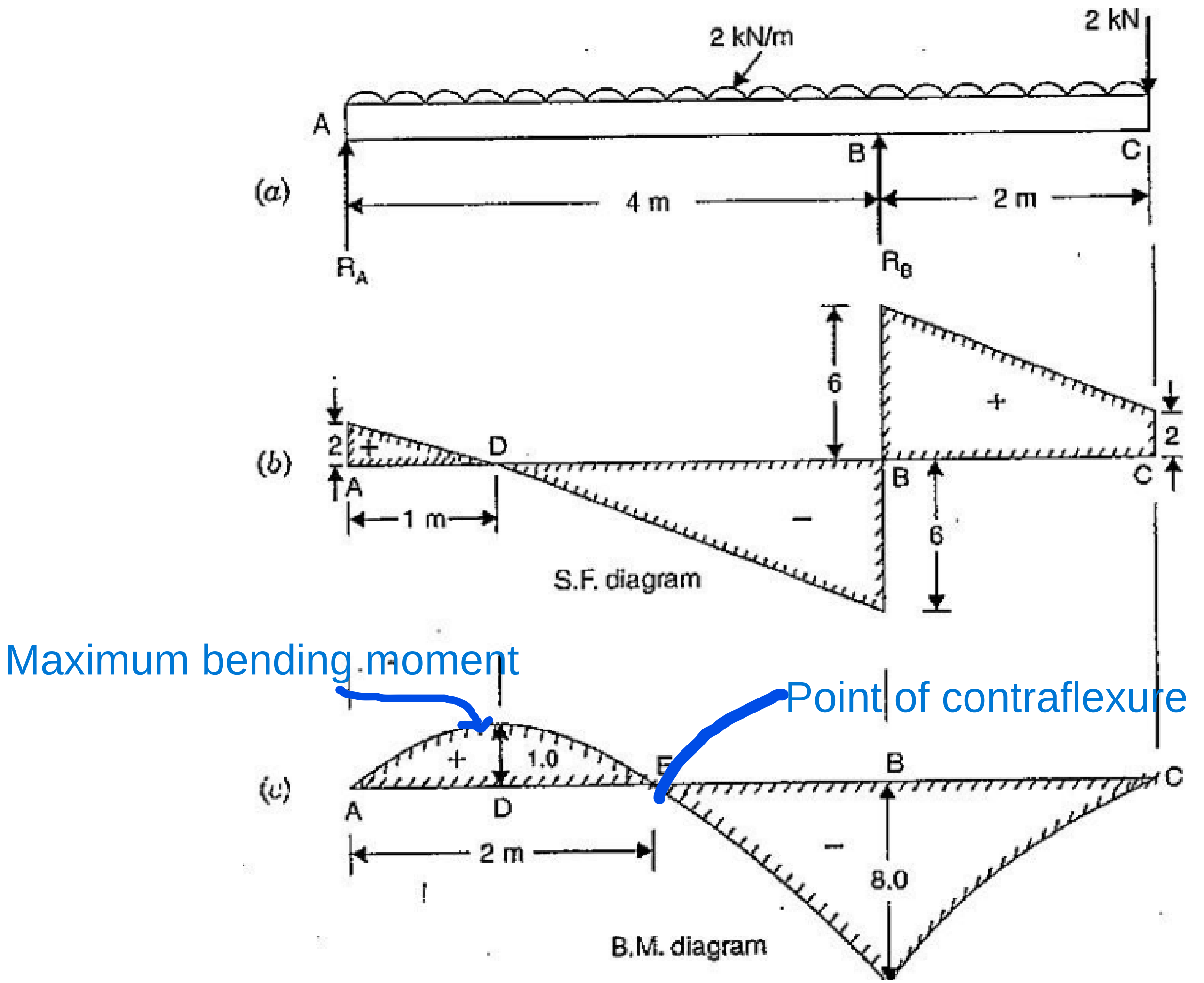
Shear force / At point	Shear force towards	
	Left of the section	Right of the section
A	--	2x6 + 2 – 12 = 2kN
B	2 – 2 x 4 = -6 kN	2 x 2 +2 = 6 kN
C	2 + 12 – 2 x 6 = 2 kN	--

Bending Moment Diagram:

$M_A = 0$

$M_B = -(2 \times 2) \times 1 - 2 \times 2 = -8 kN\text{-}m$

$M_C = 0$



To find maximum bending moment:

WKT, bending moment is maximum where shear force is zero.

Therefore, FD = $0 = R_A - 2 \times x$; $x = 1m$

$M_D = 2 \times 1 - 2 \times 1 \times 0.5 = 1$ kN-m

Point of Contra-flexure

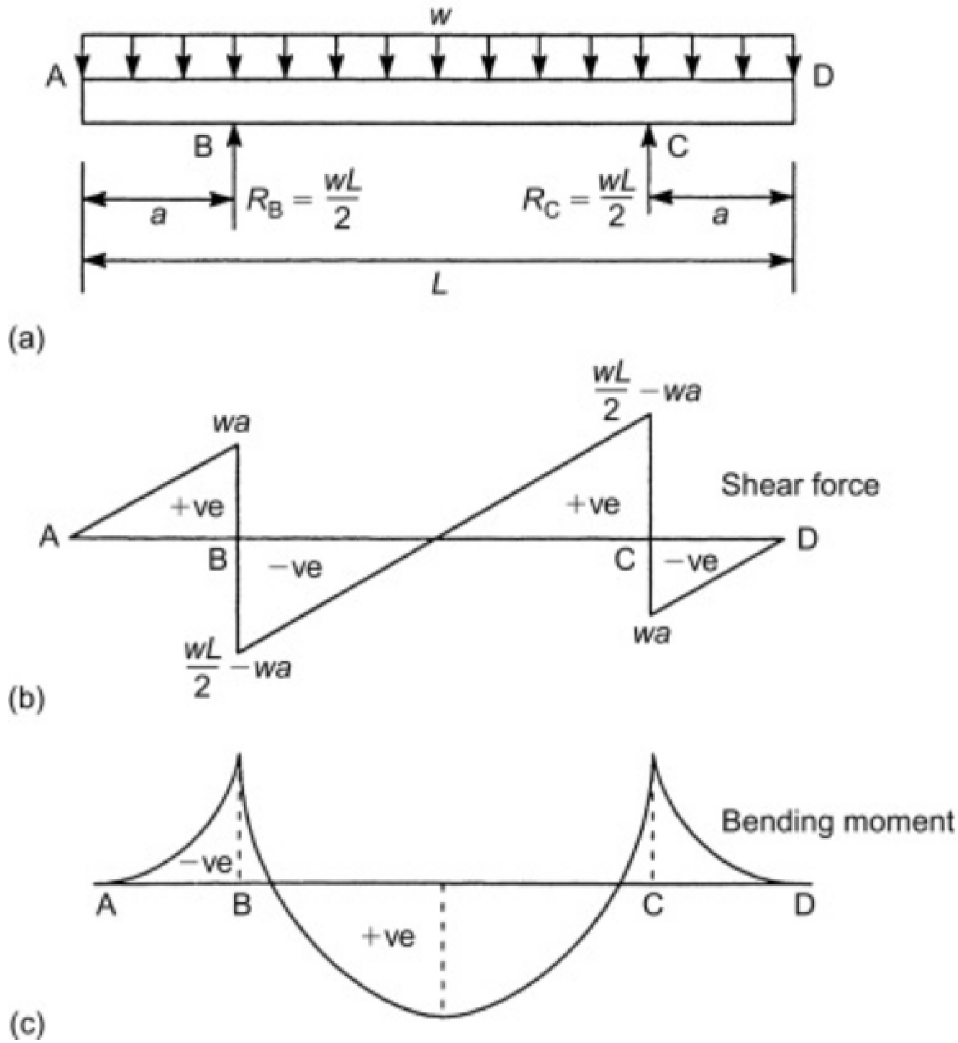
This point is at E between A and B, where B.M. is zero after changing its sign. The distance of E from A is obtained by putting $M_x = 0$,in the following equation

$$M_x = R_A \times x - 2 \times x \times \frac{x}{2} = 2x - x^2$$

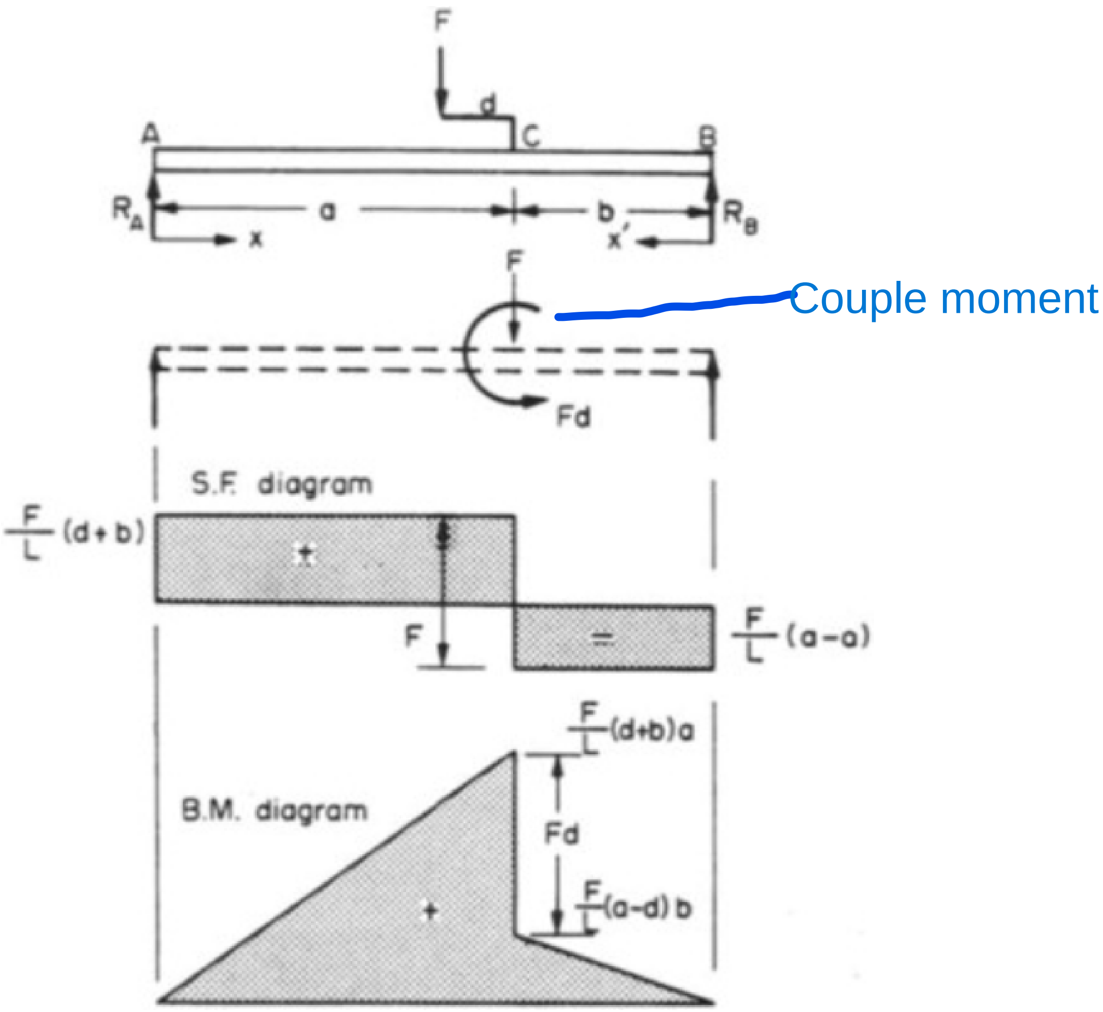
$$0 = 2x - x^2 = x(2 - x)$$

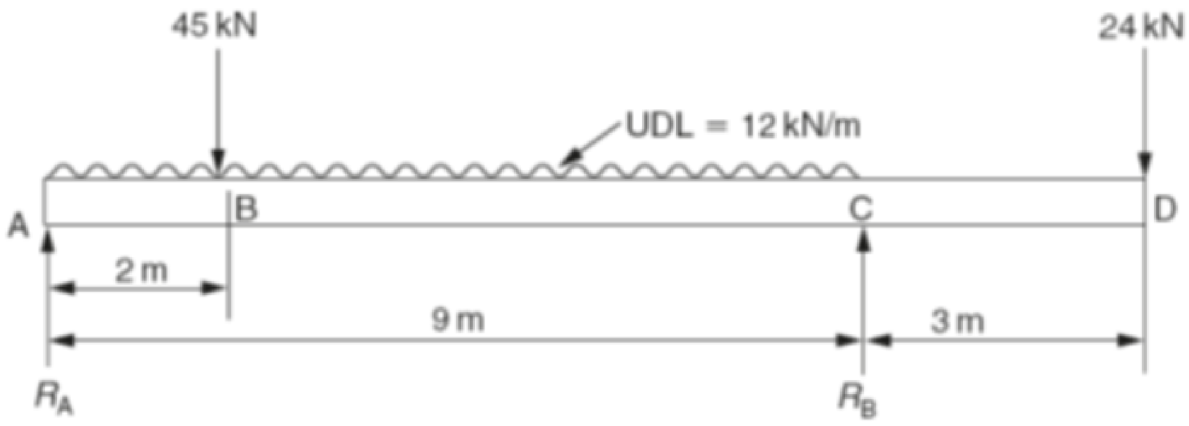
$$2 - x = 0$$

$$x = 2 \text{ m.} \quad \textbf{Ans.}$$

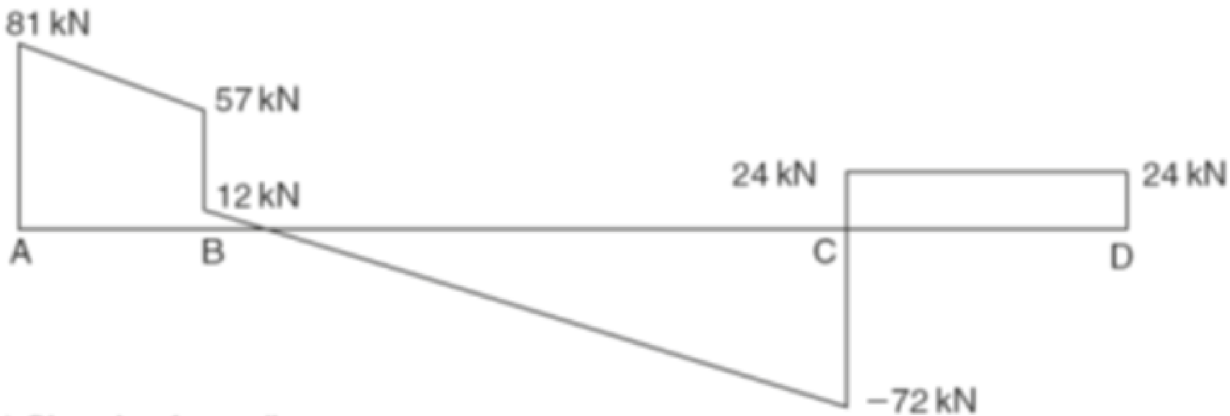


w
A
D
B
C
$R_B = \frac{wL}{2}$
$R_C = \frac{wL}{2}$
a
a
L
(a)
$\frac{wL}{2} - wa$
wa
Shear force
+ve
+ve
A
D
B
−ve
C
−ve
wa
$\frac{wL}{2} - wa$
(b)
Bending moment
−ve
A
B
C
D
+ve
(c)

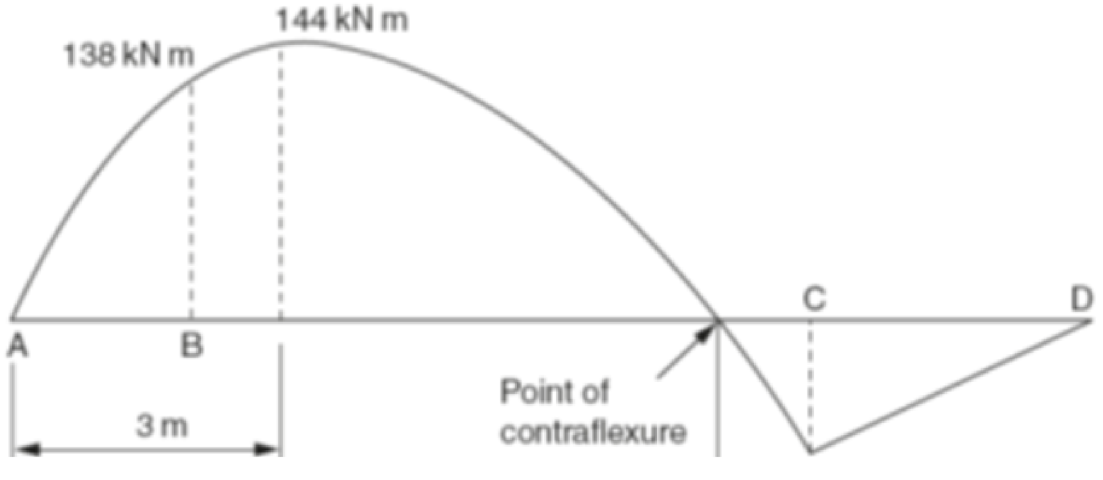

F
d
A
C
B
R_A
a
b
R_B
x
x'
F
Couple moment
Fd
S.F. diagram
$\frac{F}{L}(d+b)$
F
$\frac{F}{L}(a-a)$
B.M. diagram
$\frac{F}{L}(d+b)a$
Fd
$\frac{F}{L}(a-d)\,b$



(a) Loading diagram



(b) Shearing force diagram



In theory of Simple Bending Questions, always use the Ixx

3. A beam of an I-section shown in [redacted] is simply supported over a span of 4 *m*. Find the uniformly distributed load the beam can carry if the bending stress is not to exceed 100 N/mm^2.

Solution:

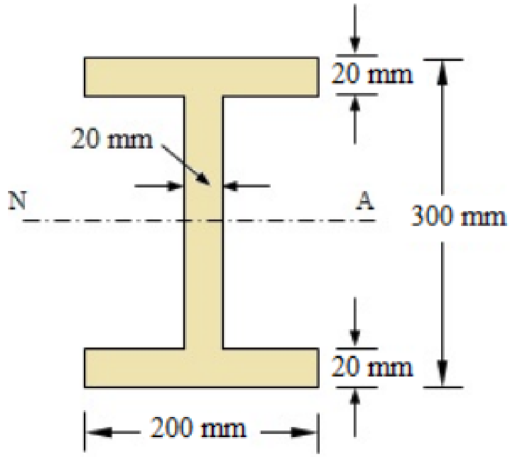
Moment of inertia, $I = \frac{1}{12}\left(BD^3 - bd^3\right) = \frac{1}{12}\left(200 \times 300^3 - 180 \times 260^3\right)$

$$= 180.36 \times 10^6 \ mm^4$$

Maximum bending stress, $\sigma_{\max} = 100 \ N/mrn^2$

Span of beam, $l = 4 \ m$

Extreme fibre distance, $y_{\max} = 150 \ mm$



Section modulus, $$Z = \frac{I}{y_{\max}} = \frac{180.36 \times 10^6}{150} = 1242400\, mm^3$$

Maximum bending moment, $$M = \sigma_{\max} Z = 100 \times 1242400$$

$$= 124240000\, N.mm$$

$$= 124.24\, kN.m$$

But $$M = \frac{wl^2}{8}$$

$$124.24 = \frac{w \times (4)^2}{8}$$

$$w = \frac{124.24 \times 8}{16} = 64.12\, kN/m$$

The maximum uniformly distributed load the beam can carry = 64.12 *kN/m*.

4. A timber beam of rectangular section carries a load of 2 kN at mid-span. The beam is simply supported over a span of 3.6 m. If the depth of section is to be twice the breadth, and the bending stress is not to exceed 9 N/mrn^2, determine the cross-sectional dimensions.

Solution:

Span of the beam, $l = 3.6\ m$

Uniformly distributed load, $w = 2\ kN$

Allowable bending stress, $\sigma_{\text{allow}} = 9\ N/mm^2$

Maximum bending moment at centre of beam, $M = \dfrac{WL}{4} = \dfrac{2 \times 3.6}{4} = 1.8\,kN.m$

$$= 1.8 \times 10^6\ N.mm$$

From the flexural relationship, we have $\quad Z = \dfrac{M}{\sigma_{allow}}$

$$\frac{1}{6} bd^2 = \frac{1.8 \times 10^6}{9}$$

$$bd^2 = \frac{1.8 \times 10^6}{9} \times 6 = 1.2 \times 10^6$$

Depth of section is to be twice the breadth, i.e., $d = 2b$

So, we have $\quad b(2b)^2 = 1.2 \times 10^6$

$$b^3 = \frac{1.2\times10^6}{4} = 0.3\times10^6$$

$$b = 64.94\,mm$$

$$d = 2\times64.943 = 129.886\,mm$$

Therefore, width of beam = 65 *mm*, and depth of beam = 130 *mm*

5. A rectangular beam of width 200 *mm* and depth 300 *mm* is simply supported over a span of 5 *m*. Find the safe uniformly distributed load that the beam can carry per metre length if the allowable bending stress in the beam is 100 N/mm^2.

Solution:

Span of beam, $l = 5\ m$

Width Breadth of the beam, $b = 100\ mm$

Depth of beam, $d = 200\ mm$

Allowable bending stress, $\sigma_{\text{allow}} = 100\ N/mm^2$

Section modulus, $$Z = \frac{1}{6}bd^2 = \frac{1}{2}\times 200\times 300^2 = 3\times 10^6\ mm^3$$

Moment of resistance of the beam, $M = \sigma_{allow} Z = 100\times 3\times 10^6$

$$= 300\times 10^6\ N.mm = 300\,kN.m$$

Maximum bending moment at the centre of the beam,

$$M = \frac{wl^2}{8}$$

$$300 = \frac{w\times (5)^2}{8}$$

$$\therefore \quad w = \frac{300\times 8}{25} = 96\,kN.m$$