Always remember to leave your answers using the appropriate units not just numbers unless it's strain or other unitless parameters

Example 1.16 *Calculate the force P_3 and change in length for the following Fig.1.14. Take $E = 200\ GN/m^2$; $P_1 = 120\ kN$; $P_2 = 220\ kN$; and $P_4 = 160\ kN$.*

[MU, Apr. 97]

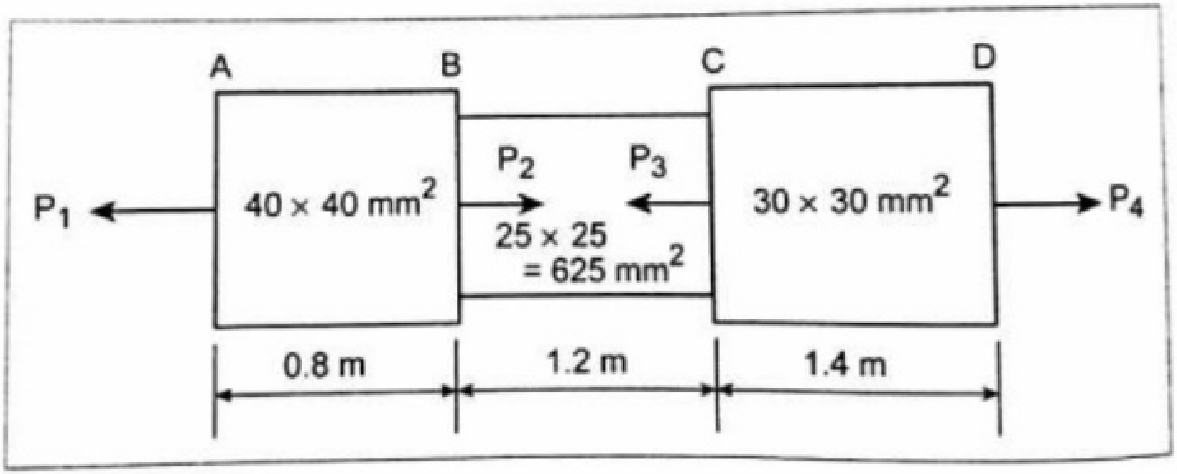

Fig. 1.14.

Given: $E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$

$= 200 \times 10^3\ N/mm^2$

Common mistakes include:

Using measurements of different units together i.e multiplying m and mm

You must convert the units to be the same i.e all m or all mm

$$P_1 = 120 \text{ kN} = 120 \times 10^3 \text{ N};$$
$$P_2 = 220 \text{ kN} = 220 \times 10^3 \text{ N}$$
$$P_4 = 160 \text{ kN} = 160 \times 10^3 \text{ N};$$
$$A_1 = 40 \times 40 = 1600 \text{ mm}^2;$$
$$A_2 = 25 \times 25 = 625 \text{ mm}^2$$
$$A_3 = 30 \times 30 = 900 \text{ mm}^2$$
$$L_1 = 0.8 \text{ m} = 800 \text{ mm};$$
$$L_2 = 1.2 \text{ m} = 1200 \text{ mm};$$
$$L_3 = 1.4 \text{ m} = 1400 \text{ mm}.$$

To find: 1. Force, P_3, 2. Change in length, δL.

☺ ***Solution:*** We know that,

Forces acting towards right = Forces acting towards left

$$\Rightarrow \quad P_2 + P_4 = P_1 + P_3$$

$$\Rightarrow \quad 220 + 160 = 120 + P_3$$

$$\Rightarrow \quad \boxed{P_3 = 260 \text{ kN}}$$

Take, Tensile force is positive.

Compressive force is negative.

Consider part AB:

Left hand side force = $P_1 = 120$ kN (T)

Right hand side force = P_2 (T) – P_3 (C) + P_4 (T)

$$= 220 - 260 + 160$$

$$= 120 \text{ N (T)}$$

So, load on part AB is 120×10^3 N (T).

$$\text{Right hand side force} = -P_3\,(C) + P_4\,(T)$$
$$= -260 + 160$$
$$= -100 \text{ kN (C)}$$

So, Load on part BC is -100×10^3 N (C)

Consider part CD:

$$\text{Left hand side force} = P_1\,(T) - P_2\,(C) + P_3\,(T)$$
$$= 120 - 220 + 260$$
$$= 160 \text{ kN (Tensile)}$$
$$\text{Right hand side force} = P_4\,(T)$$
$$= 160 \text{ kN (T)}$$

So, load on part CD $= 160 \times 10^3$ N (T)

We know that,

$$\text{Total Elongation} = \frac{(\text{Load on part AB}) \times L_1}{A_1 E} + \frac{(\text{Load on part BC}) \times L_2}{A_2 E} + \frac{(\text{Load on part CD}) \times L_3}{A_3 E}$$

$$\Rightarrow \quad \delta L = \frac{120 \times 10^3 \times 800}{1600 \times 200 \times 10^3} - \frac{100 \times 10^3 \times 1200}{625 \times 200 \times 10^3} + \frac{160 \times 10^3 \times 1400}{900 \times 200 \times 10^3}$$

$$\boxed{\delta L = 0.58 \text{ mm}}$$

Result: 1. Force, $P_3 = 260$ kN

2. Change in length, $\delta L = 0.58$ mm

Example 1.30 *A reinforced concrete column 300 × 300 mm is reinforced with 8 steel rods with a total area of 1820 mm². The column carries an axial load of 400 kN. If the modulus of elasticity of steel is 18 times that of concrete, find the stresses in concrete and in steel.*

[MU, Oct. '96 & Oct. '98]

Given:

$$\text{Area of column} = 300 \times 300$$
$$= 90{,}000 \text{ mm}^2$$

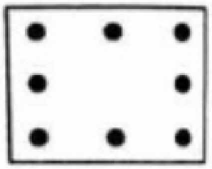

Fig. 1.21.

$$\text{Area of column} = 90{,}000 \text{ mm}^2$$
$$\text{Total area of steel rod, } A_s = 1820 \text{ mm}^2$$
$$\text{Axial load, } P = 400 \text{ kN} = 4{,}00{,}000 \text{ N}$$
$$\left.\begin{matrix}\text{Modulus of Elasticity}\\ \text{of steel}\end{matrix}\right\} = \left\{\begin{matrix}\text{18 times Modulus}\\ \text{Elasticity of concrete}\end{matrix}\right.$$
$$\Rightarrow \quad E_s = 18\, E_c$$

To find: Stresses in concrete and steel.

N.B/ Area of the column is not the same as the area of concrete

$$\text{Area of concrete} = \text{Area of column} - \text{Area of steel}$$

$$= 90{,}000 - 1820$$

$$\boxed{A_c = 88{,}180 \text{ mm}^2}$$

We know that,

$$\text{Total load, P} = \begin{Bmatrix} \text{Load on} \\ \text{steel bar} \end{Bmatrix} + \begin{Bmatrix} \text{Load on} \\ \text{concrete} \end{Bmatrix}$$

$$4{,}00{,}000 = P_s + P_c \qquad \ldots (A)$$

We know that,

Change in length of steel bar = Change in length of concrete

$$\Rightarrow \quad \frac{P_s L_s}{A_s E_s} = \frac{P_c L_c}{A_c E_c}$$

Length of steel bar and concrete bar are equal.

$$\text{So, } L_s = L_c$$

$$\Rightarrow \quad \frac{P_s}{A_s E_s} = \frac{P_c}{A_c E_c}$$

$$\Rightarrow \quad \frac{P_s}{A_s \cdot 18\, E_c} = \frac{P_c}{A_c E_c} \qquad [\text{Since, } E_s = 18\, E_c]$$

$$\Rightarrow \quad P_s = \frac{1820 \times 18\, E_c \times P_c}{88{,}180 \times E_c}$$

$$\boxed{P_s = 0.371\, P_c}$$

$$\boxed{P_s = 0.371\ P_c}$$

Substituting in equation (A),

$$\Rightarrow \quad 4,00,000 = 0.371\ P_c + P_c$$

$$\boxed{\text{Load on concrete,} \quad P_c = 291757.8\ \text{N}}$$

$$\text{Load on steel,} \quad P_s = 0.371\ [291757.8]$$

$$\Rightarrow \boxed{P_s = 108242.1\ \text{N}}$$

$$\text{Stress in concrete, } \sigma_c = \frac{\text{Load}}{\text{Area}} = \frac{P_c}{A_c} = \frac{291757.8}{88,180}$$

$$\boxed{\sigma_c = 3.3\ \text{N/mm}^2}$$

$$\text{Stress in steel, } \sigma_s = \frac{\text{Load}}{\text{Area}} = \frac{P_s}{A_s} = \frac{1,08,242.1}{1820}$$

$$\boxed{\sigma_s = 59.47\ \text{N/mm}^2}$$

Result: Stress in concrete, $\sigma_c = 3.3\ \text{N/mm}^2$

Stress in steel, $\sigma_s = 59.47\ \text{N/mm}^2$

Example 1.39 *A steel tube of 50 mm external diameter and 30 mm internal diameter encloses a copper rod of 25 mm diameter. The tube is closed at each end by rigid plates of negligible thickness. If, at a temperature of 20 °C there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to 220 °C.*

Take $E_s = 200 \text{ GN/m}^2$ $\quad E_c = 100 \text{ GN/m}^2$

$\alpha_s = 12 \times 10^{-6} /°C$ $\quad \alpha_c = 18 \times 10^{-6} /°C$

Given:

External diameter of steel tube, $D_s = 50$ mm

Internal diameter of steel tube, $d_s = 30$ mm

Diameter of copper rod, $D_c = 25$ mm

Rise in temperature, $T = 220° - 20°$

$= 200°C$

Young's modulus of steel, $E_s = 200 \text{ GN/m}^2$

$= 200 \times 10^9 \text{ N/m}^2$

$= 200 \times 10^3 \text{ N/mm}^2$

$= 2 \times 10^5 \text{ N/mm}^2$

Young's modulus of copper, $E_c = 100 \text{ GN/m}^2$

$= 100 \times 10^9 \text{ N/m}^2$

Coefficient of linear expansion of steel $\left.\right\}$ $\alpha_s = 12 \times 10^{-6}$ /°C

Coefficient of linear expansion of copper $\left.\right\}$ $\alpha_c = 18 \times 10^{-6}$ /°C

To find: 1. Stress developed in steel tube, σ_s.

2. Stress developed in copper, σ_c.

☺ ***Solution:*** Area of steel tube, $A_s = \frac{\pi}{4} [D_s^2 - d_s^2]$

$$= \frac{\pi}{4} [50^2 - 30^2]$$

$$\boxed{A_s = 1256.63 \text{ mm}^2}$$

Area of copper rod, $A_c = \frac{\pi}{4} D_c^2$

$$= \frac{\pi}{4} (25)^2$$

$$\boxed{A_c = 490.87 \text{ mm}^2}$$

Coefficient of linear expansion of copper is more than that of steel. So, copper will be subjected to compressive stress whereas steel will be subjected to tensile stress.

We know that,

$$\text{Stress in copper, } \sigma_c = \frac{\text{Load on the copper}}{\text{Area of copper}}$$

$$\sigma_c = \frac{P_c}{A_c}$$

$$\Rightarrow \boxed{P_c = \sigma_c \times A_c} \quad \ldots (1)$$

Similarly,

$$\text{Load on the steel, } P_s = \sigma_s \times A_s \quad \ldots (2)$$

Under equilibrium condition,

Compression in the copper bar is equal to tension in the steel tube.

i.e., Load on the copper = Load on the steel

$$\Rightarrow \sigma_c A_c = \sigma_s A_s$$

$$\Rightarrow \sigma_s = \sigma_c \times \frac{A_c}{A_s}$$

$$= \sigma_c \times \frac{490.87}{1256.63}$$

$$\boxed{\sigma_s = 0.39\ \sigma_c} \quad \ldots (3)$$

$$\left.\begin{array}{r}\text{Actual}\\\text{expansion}\\\text{of steel}\end{array}\right\} dL_s = \left\{\begin{array}{c}\text{Free}\\\text{expansion}\\\text{of steel}\end{array}\right\} + \left\{\begin{array}{c}\text{Expansion}\\\text{due to}\\\text{tensile stress}\\\text{in steel}\end{array}\right\}$$

$$\Rightarrow dL_s = \alpha_s T L_s + \frac{\sigma_s}{E_s} \times L_s \quad \ldots (4)$$

$$\left.\begin{array}{r}\text{Actual}\\\text{expansion}\\\text{of copper}\end{array}\right\} dL_c = \left\{\begin{array}{c}\text{Free}\\\text{expansion}\\\text{of copper}\end{array}\right\} - \left\{\begin{array}{c}\text{Contraction due to}\\\text{compressive stress}\\\text{induced in copper}\end{array}\right\}$$

$$= \alpha_c T L_c - \frac{\sigma_c}{E_c} \times L_c \quad \ldots (5)$$

We know that,

Actual expansion of steel = Actual expansion of copper

$$\Rightarrow \quad \alpha_s T L_s + \frac{\sigma_s}{E_s} \times L_s = \alpha_c T L_c - \frac{\sigma_c}{E_c} \times L_c$$

$$\Rightarrow \quad L_s\left[\alpha_s T + \frac{\sigma_s}{E_s}\right] = L_c\left[\alpha_c T - \frac{\sigma_c}{E_c}\right]$$

$$\Rightarrow \quad \alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c} \qquad [\because L_s = L_c]$$

$$\Rightarrow 12 \times 10^{-6} \times 200 + \frac{0.39\,\sigma_c}{2 \times 10^5} = 18 \times 10^{-6} \times 200 - \frac{\sigma_c}{1 \times 10^5}$$

$$[\because \sigma_s = 0.39\,\sigma_c]$$

$$\Rightarrow \quad 11.95 \times 10^{-6}\,\sigma_c = 1.2 \times 10^{-3}$$

$$\Rightarrow \quad \boxed{\sigma_c = 100.41 \text{ N/mm}^2}$$

We know that, $\sigma_s = 0.39 \times \sigma_c$

$$\Rightarrow \quad \sigma_s = 0.39 \times 100.41$$

$$\boxed{\sigma_s = 39.16 \text{ N/mm}^2}$$

Result:

1. Stress developed in copper rod, $\sigma_c = 100.41 \text{ N/mm}^2$
2. Stress developed in steel tube, $\sigma_s = 39.16 \text{ N/mm}^2$