The Actuarial Control Cycle – 28/2/20

What do actuaries do?

- Actuaries analyse risks and are interested in the financial consequences of such risks.
- They manage OPM (other people's money)
- They work in Banks, Insurance Companies, Superannuation funds, Investment banks, etc
- Traditionally, they have worked in
    - Life and health insurance
    - General insurance
    - Superannuation
    - Banking
- However the field is rapidly growing into a wide range of problems

Actuary Problems vs Math Problems

- Math problems usually have an exact answer (either right or wrong)
- Actuarial studies might have a range of different solutions to any problems → might have to choose between various alternatives

Risk Management: EXAMPLE

Flood Risk.

- **Avoid** the risk = do not build anymore homes in areas where flooding is likely to occur
- **Manage** the risk =
    - Build dams to control water levels
    - Build levees to support homes
    - Build stronger homes to withstand floods
- **Insure** the risk = encourage people to buy insurance
- **Accept** the risk = Accept that there is a real risk and that costs will be incurred from time to time

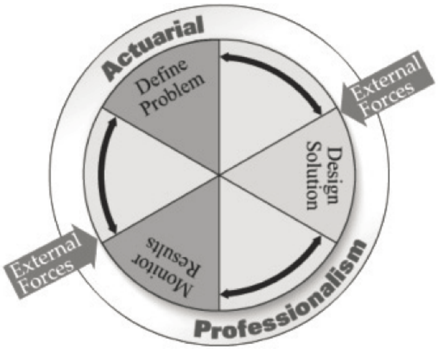
Building Models

- Models use mathematical formulae and algorithms to represent essential features of a real-world situation
- Long term models – since the world is changing in unpredictable ways, it is hard to build a model that looks very far into the future (e.g. Superannuation – 80years)
    - The longer term your model, the more likely it is that unpredictable events will occur that invalidate your model: "The Expanding Funnel of Doubt"
- Types of models – mathematical, statistical, economic and financial

The Problem-Solving Cycle

- Define the problem
- Consider possible solutions
- Build a model to test different scenarios
- Choose the best approach (pros and cons)
- Monitor the results overtime
- Use these results to build a better model and find a better solution

The Actuarial Control Cycle



Inaccurate Models

- It is easy to make models that are mathematically perfect by unrealistic
- All models must be critically examined

<u>Savings and Retirement Models – 6/3/20</u>

<u>EXAMPLE:</u>

Defining the problem: Where will people get the money they will need to survive when they are 75, 85 or 90 years old?

Initial Steps in solving this problem

- Understanding the problem - Old people have differing levels of financial stability
- Looking into the future – What can we do to improve the financial security of older Australians in the future?

List at least 5 sources of income for people over 65?

1. Superannuation
2. Government (Old age pension)
3. Voluntary savings & Investment
4. Keep working
5. Children (supporting their parents)
6. Parents (Inheritance)

Option 1: Voluntary savings?

- Historically, most Australians have been very good at voluntarily saving enough money to provide for their own retirement.
    - Pensioner vs "Self-funded retiree"
- Proportion of the population receiving the Age pension vs self-funded retirees
    - Insert graph from slides

Option 2: Parents?

- Family wealth passed down from one generation to the next

    - e.g. inheritance, family farm, family business
- Will this work? Reasons it won't:
    - Reason 1: Change in social attitudes - SKI-ers = spending the kids inheritance.
    - Reason 2: Old people are living much longer – Need the money for their own financial needs, high cost of aged care
- Potential Problem: Elder abuse
    - Many older Australians have lots of money and valuable assets → exploited by children into giving their kids their money/assets (taking and inheritance)

Option 3: Children?

- In many countries, it is traditional that children have responsibility to look after their parents financially.
- Old people expect to be able to live with their grown-up children, and expect their families to provide care and support.
- Fertility rate has dropped recently

Option 4: Government?

- The Australian government provides an old-age pension to people over 67 BUT ONLY IF THEY NEED MOEY.
- The old age pension is NOT paid to every old person – it is only paid to people who pass a "means test"
    - Means test = prove that you have a low income AND low assets
- The higher your income & assets -> the lower the pension payments
- Reliance on Old Age pension
    - At 2012: More than 80% of people over age 65 were pensioners
    - Approx. 50% full pensioners
    - Approx. 30% part pensioners
- **Reason 1**: Spending Effect
    - People slowly spend their savings as wealth declines. Self-funded become part pensioners.
- **Reason 2**: Generational effect?
    - Compulsory superannuation started in 1992
- Means Test – Consists of both the **assets test** and **income test**

Pension Benefits

- Full old age pension = $23,254 p.a. (2018)
- This is the rate for a single person who owns their own home, but is poor enough to pass the means test and become a full pensioner
- Paid on a PAYG (pay as you go) system -> every pay check is taxed individually
    - Government does not save money for pension

Ageing Australia

- Ratios of working age to Old people:
    - 1970 -> 7 people of working age to every 1 person over the age of 65
    - 2015 -> 4.8:1
    - 2033 -> 3.3:1 (projected)
    - 2060 -> 2.7:1

- What do we do in 2060 when the ratio is 2.7:1?
    - More kids
    - More compulsory super
    - Drawdown restrictions
    - Increase retirement age
    - Tax Incentives
    - Government save money
    - Reduce age pension
- **Solution 1**: High Tax Rates?
- **Solution 2**: Lower old age pension benefits?
    - This might make the government unpopular
    - Sneaky way of doing the same thing: Old age pension increases in line with inflation each year. 2014 Budget proposed a lower rate increase each year.
- **Solution 3**: Stricter Means Test
    - Only give money to people who are really poor.
    - The government made changes to the means test in the 2014 budget
        - Higher benefits for some "part pensioners"
        - Lower benefits to many "part pensioners"
        - About 30,000 people will no longer be eligible at all
- **Solution 4**: Increase GDP?
    - Over the long term – How can we increase the size of people in the workforce in 2050, which represents the Gross Domestic Product of our working population.
        - Encourage people to have more kids
        - Increase Participation rates – few stay at home mums, etc
        - Better productivity – more education, better technology
    - This would allow the government to meet the cost of the additional old age pensions payable to our ageing population.

Option 5: Keep Working

- How can we make people keep working even when they are old?
    - **A. Pension Eligibility Age** -> Old age pension age was increased from 65 to 67
    - **B. Preservation Age** -> (Currently 55 years old) Once you put money into a superannuation fund, it is "locked in" until:
        - you die
        - you become disabled
        - you retire after the preservation age.
    - The preservation age will most likely increase in the future as this encourages people to keep working.
    - **C. Employment Market Changes** -> Encourage employers to hire older workers by:
        - PR Campaigns (older workers are reliable, experienced)
        - Financial incentives
        - Retain Older workers
        - Age discrimination laws (cant force people to retire at age 65)

Option 6: Superannuation

- If people save up money for retirement, by putting money into a superannuation fund, then perhaps they will be less reliant on the government old age pension.
- Encourage more super savings through:
    - Educating people about it -> superannuation calculators at Moneysmart
    - Tax Incentives -> to encourage superannuation savings
    - Compulsory Superannuation savings -> 1992 amendment
- Compulsory Super: every employer must pay 9.5% of a salary into a superannuation fund for each employee (Started as 3% in 1992)
    - Exceptions: self-employed people, part-time workers under age 18, people earning less than $450 per month.
- Labour party in 2007 planned to gradually increase superannuation from 9.5% to 12%.
- Liberal Party won election in 2013 and decided to freeze this increase.
- Problem: Is 9.5% enough? Can it provide a comfortable retirement? Can employers afford to pay higher rates? Would employees rather have cash instead?
- Superannuation is a highly controversial issue and many people have different opinions about it (employees, employers, financial services industry, government).

Building a Model

- We can build a superannuation model which can be used to:
    - Assess the adequacy of superannuation savings
    - Test out different policy proposals
    - Assess the risks for savers

<u>Accumulated Values – 13/3/20</u>

Calculating Superannuation Balances

- Aim: To calculate the amount of money in a retirement savings account.
- We start with a simple situation: *If I put $C into an account at the end of each year, and the account earns compound interest at rate i p.a, how much will I have at the end of n years?*
- Real-life complication also occur within these calculations, including:
    - variable contributions
    - taxes on contributions
    - administration fees
    - insurance costs
    - variable investment returns
    - taxes on investment income
    - salary increases, inflation
    - missing payments (when out of the workforce)

Sum of a Geometric Progression

- If numbers are in a geometric progression then each number in the sequence is equal to the previous number, multiplied by a constant factor $\boldsymbol{r}$.
- If:
    - the first term is denoted $\boldsymbol{a}$**,**

- and the constant ratio is $\boldsymbol{r}$,
- then the first n terms in the sequence are

$$a \ ar \ \ ar^2 \ ar^3 \ldots \ ar^{n-1}$$

- We can prove that (if r is not equal to 1), the sum of the first n terms is $S_n = a\left[\frac{1-r^n}{1-r}\right]$ or $S_n = a\left[\frac{r^n-1}{r-1}\right]$

Accumulated Value of a Single payment

- If we make a payment of C_0 at time t = 0, and invest it to earn compound interest at the rate i p.a. payable annually, the accumulated value at the end of n years is

$$B_n = C_0 \ * \ (1+i)^n$$

- Similarly, if we have a single payment of C_t made at time t, $(t \ = \ 0,1,2, \ldots n)$ then the accumulated value at time n is

$$B_n = C_t \ * \ (1+i)^{n-t}$$

  - That is, the payment earns compound interest for $n - t$ years

Accumulated Value of Multiple Payments

- Suppose we have a series of payments, possibly of different amounts, paid into the account at different times.
- Let C_t denote the payment at time t.
- The accumulated value of all of these payments combined is the sum of the accumulated value of each individual payment.
- That is:

$$B_n = \sum_t C_t (1+i)^{n-t}$$

  - where the sum is over all values of t where a payment has been made.

Accumulated Value of Equal Annual Payments

- A special case occurs when all the payments are equal to a constant C, and payments occur at annually at the end of each year (payments occur at t = 1,2,3, … n)

$$B_n = \sum_{t=1}^{n} C \ (1+i)^{n-t}$$

- We can see that the terms of the right-hand side form a geometric progression, with
  - Initial term $C \ (1+i)^{n-1}$
  - Constant ratio $(1+i)^{-1}$
  - Number of terms n
- By applying the formula for the sum of a GP, we can show that the accumulated value at time n, of payments of C paid annually in **arrears** for n years (at $t \ = \ 1,2,3, \ldots n$), is

$$B_n = C\left[\frac{(1+i)^n - 1}{i}\right]$$

- Similarly, the accumulated value at time n, of payments of C paid annually **in advance** for n years (at $t \ = \ 0,1,2,3, \ldots n$-1), is

$$B_n = C * (1+i)\left[\frac{(1+i)^n - 1}{i}\right]$$

Actuarial Notation: Accumulated Values

- The symbol for the accumulated value of *n* payments of $1 per year, made at the end of each year for *n* years (in **arrears**), at rate of interest *i* per annum compound:

  $s_{\overline{n|}}$

- The symbol for the accumulated value of *n* payments of $1 per year, made at the start of each year (in **advance**) for *n* years, at rate of interest *i* per annum compound:

  $\ddot{s}_{\overline{n|}}$

NOTE: Ensure the conduct reasonableness checks on all calculations. Ask *"Does this answer look reasonable?"*.

Excel Functions for Accumulation

- Accumulated Value
  = FV(interest rate, no. of time periods, amount)

<u>Building a Superannuation Model</u>

- Let's suppose that your employer is paying 9% of your salary into a superannuation fund earning 8% p.a.
- Your salary is $50000 per annum, so this means the annual contribution is $4500
- Assume payments are made at the end of each year.
- Using the formula for the accumulated value of regular payments, after 40 years you will have

  4,500 $s_{\overline{40|}}$ **at 8% = $1,165,754.33**
- However, this is inaccurate as there are further complications.
- We must allow for:
  - Taxes on contributions
  - Administration Fees & Insurance costs
  - Taxes on investment income
  - Investment management fees (asset based fees)
  - Salary increases
  - Inflation of the cost of living

Tax on Contribution

- The government charges a tax of 15% on each contribution made by the employer into a super fund.
- Contributions Tax = 15% * 4,500 = 675
- **Net** Contribution after deducting tax
- = 4,500 - 675 = 3,825
- Accumulated value of **<u>net</u>** contributions:

  3,825 $s_{\overline{40|}}$ **at 8% = $ 990,891.18**

Administration Fees & Insurance

- The superannuation fund will also charge you an **annual administration fee.**
- Note that some funds charge a flat fee say $X each year but others charge a percentage of the amount you contribute (say 1% of 4500 = $45), and some charge both a flat fee AND a percentage fee.
- Also, most superannuation funds buy **life insurance policies** for all their members so if you die, your orphaned children will have enough money to survive and prosper.
- Some also buy **disability insurance** for their members so that you receive a payout if you are injured or become ill and unable to work.
- Most funds let each fund member choose
    - whether or not he wants insurance, and
    - the type of insurance, and
    - the level of cover (i.e. the size of benefit payable if you die or become disabled).
- **If you choose to buy insurance, the cost of the insurance is deducted from your account each year.**
- The cost of insurance will vary from fund to fund.
- Let's say that you pay $150 per annum for insurance. So, the net contribution, after taxes, insurance, and admin fees, is:

Gross contribution	4500
Less tax	675
Less admin fee	50
Less insurance	150
Net Contribution	3,625

- So the accumulation is down to

3625 $s_{\overline{40|}}$ **at 8% = 939,079.88**

Tax on Investment Income

- Investment income is also affected by taxes and fees.
- Most superannuation investment income is taxed at 15%.
    - If the gross interest rate is i
    - And the government takes tax of $t * i$
    - Then the net interest rate is $i - i * t = i * (1 - t)$
- **Consider the interest rate of 8%**

  Net interest after tax = 8.00 – 1.20 = 6.80

- The **net interest rate after tax** is 6.8%
- Our accumulated value is now down to

3625 $s_{\overline{40|}}$ **at 6.8% = $ 687,402.88**

- Note the **large drop** in the account balance, as a result of the tax on investment income.

Investment Management Fees

- Superannuation funds also charge you an **investment management fee**.
    - This money is used to pay the investment managers who provide their expert skills in managing your investments, and also covers investment-related transaction costs such as brokerage.
- This fee is an **asset-based fee**, i.e. it is usually a percentage of the balance of your account.
- The percentage usually varies according to the type of the investment –
    - if you invest in something simple like government bonds then the IMF might be just 0.30% of your assets,
    - but if you invest in something which requires a lot of research and expertise, like private equity or hedge funds, it might be 2% or more.
- Let's assume that you will pay investment management fees of 1% per annum, and the fee is applied by reducing the net interest rate credited to your account.
- (In practice there are different methods of calculation, but this is just a simple approximation).
- To work out the **net-of-tax-and-fees interest rate**, let's assume that

  Gross interest rate = 8%

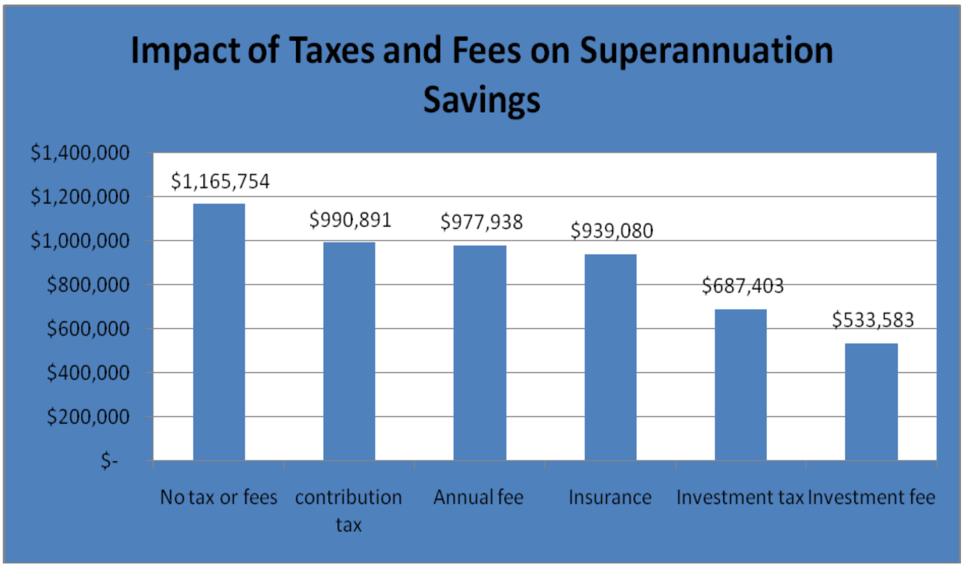
  Net of tax interest rate = 6.8%

  And if investment management fees are 1%

  Net of tax and fees interest rate = 5.8%
- Our accumulated value is now down to

3625 $s_{\overline{40|}}$ at 5.8% = $ 533,582

Overall Impact of Fees and Taxes



- Note that small differences in fees and taxes make a large different to the outcome.

Adjustments to Final Payment

- Some funds also charge an **exit fee**, which is theoretically designed to cover the cost of processing the paperwork for payments. It is usually a relatively small amount.
- If you are **under the age of 60**, you will have to pay a **benefits tax** to take the benefit out of the fund.

Summary: Allowing for Fees, Insurance and Tax

- We started with the formula for the accumulated value of C per annum in arrears for n years at rate i p.a

$$= C * \left[\frac{(1+i)^n - 1}{i}\right]$$

- To allow for fees, insurance, and tax, we can use the same formula, but this time we use NetC instead of C and use NetI instead of i where:
    - NET C = the amount of net contributions which goes into the fund after deducting admin fees, insurance costs, and tax on contributions
    - NET i = the net interest rate after allowing for tax on investment income and investment management fees
- In our simple example
    - NetC = $C * (1 - t_c) - P - E$
        - Where t_c is the tax rate on contributions
        - And P is the insurance premium
        - And E is the administration expense
    - NetI = $i * (1 - t_i) - F$
        - Where t_i is the tax rate on investment income
        - And F is the investment management fee

Accumulated Value $= \boldsymbol{NetC} * \left[\frac{(\boldsymbol{1+NetI})^{\boldsymbol{n}} \boldsymbol{- 1}}{\boldsymbol{NetI}}\right]$

<u>Superannuation Model – 13/3/20</u>

Contributions as a Percentage of Salary

- Under Australian law, the employer must contribute at least 9.5% of an employee's salary to superannuation.
- If salaries increase, then contributions will increase.
- **But what makes salaries increase?**
    - Economic conditions
        - Boom or recession
        - Unemployment Rates
        - Inflation
    - Industrial Relations System
        - Awards (industry) and Enterprise Agreements (employer)

Measures of Inflation

- **CPI** (Consumer Price Index): Increases to keep up with increases in cost of living
- **AWOTE** (Average weekly ordinary time earnings): Increases to improve the standard of living
- Over the long term, salaries tend to increase at a rate which exceeds the CPI
  - i.e. average AWOTE increase > average CPI increase
- The Reserve Bank (RBA) has the task of keeping inflation under control

Salary Increases due to Promotions

- AWOTE measures general increases in earnings for all employees
- Salaries also increase as a result of **promotions** of individuals.

Superannuation Model (Salary Increases)

- Superannuation models should also allow for promotional salary increases as well as AWOTE increases.
  - We assume that salaries increase by a constant percentage each year (for simplicity)
- Suppose Salary during year 1 is denoted S_1
- Salary increases at rate *f* per annum
- Assume all increases occur at end of each year
- Salary paid during year t is

  $$S_t = S_1 * (1+f)^{t-1}$$
- When contributions are variable:

  $$B(n) = \sum_{t=1}^{n} C_t * (1+i)^{n-t}$$

  $$B(n) = \sum_{t=1}^{n} 0.095 S_t * (1+i)^{n-t}$$

  $$B(n) = \sum_{t=1}^{n} 0.095 S_1 (1+f)^{t-1} * (1+i)^{n-t}$$
- **Summing the terms**:

  $$\begin{aligned} B(n) = 0.095S(1+i)^{n-1} \\ +0.095S(1+f)^{1}(1+i)^{n-2} \\ +0.095S(1+f)^{2}(1+i)^{n-3} \\ +\ldots \\ +0.095S(1+f)^{n-2}(1+i)^{1} \\ +0.095S(1+f)^{n-1} \end{aligned}$$

- Note that this forms a geometric progression with:
  - First term **a** = 0.095 S_1 * (1+i) $^{n-1}$
  - Constant ratio **r** = (1+f)/(1+i)
  - Number of terms **n**

- Therefore, the total accumulation is:

$$0.095 * S(1+i)^{n-1}\left[\frac{\left(\frac{(1+f)}{(1+i)}\right)^n - 1}{\frac{(1+f)}{(1+i)} - 1}\right]$$

  - where S denotes the salary during year 1
- This can be further simplified to:
  Total Accumulation =

$$0.095 * S * \left[\frac{(1+i)^n - (1+f)^n}{i - f}\right]$$

Inflation of Prices

- As salaries increase, the prices of goods and services have also been increasing.

EXAMPLE: **Measuring Wealth**

- Joe only eats pizza. He has $1000 at present and pizzas cost $10 each
- His wealth measured in pizza is: 100 pizzas.
- Joe invests money for one year at 10% p.a. interest and his nominal wealth increases to $1100
- The price of Pizza increases by 10% to $11
- His real wealth is still 100 pizzas, i.e. he has NOT increased his **real** wealth.
  - We could say his **real rate of return** is 0%.
- Using the same example, but the price of pizza increases from $10 to $10.50 during the year.
- Therefore, Joe's nominal wealth = $1100 and his wealth measured in pizza = 1100/10.50 = 104.76.
  - His **real rate of return** = 4.76%

Real Rates of Return

- Real rate of return = (1+i) / (1+f) – 1
- If investment returns **>** inflation rate
  - Real rate of return is **positive**
  - Your account is growing faster than price increases
- If investment returns **=** inflation rate
  - Real rate of return is **zero**
  - Your account is growing at the same rate as prices
- If investment returns **<** inflation rate
  - Real rate of return is **negative**
  - Your real wealth is going backwards

Superannuation Risks – 3/4/20

Assessing our Model

- Whenever we build a model we make assumptions. We should consider the following:
- Are they reasonable?
  - Historical Data
  - Understanding of factors which affect assumptions
  - Factors which might affect our assumptions in the future
- How would the answer (accumulated value) be affected if we change our assumptions?
  - Referred to as the **robustness** of the model
- There are also sensitivity tests and scenario tests
  - Sensitivity test – change one assumption
  - Scenario test – e.g. what if there is a recession? These may affect several assumptions at once

Model for Risk Management

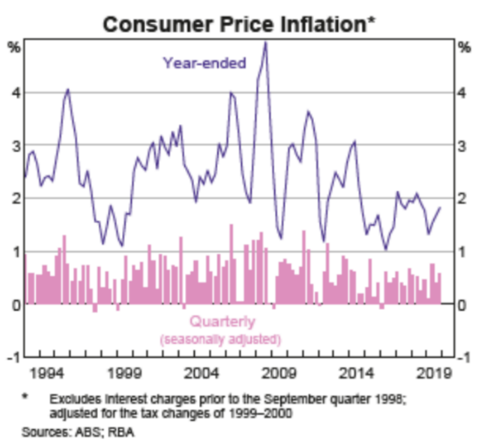
1. Identify the risks
2. Assess the risks
   a. How likely is it that it will happen?
   b. How severe are the consequences if it does?
3. Identify alternative strategies
   a. Accept the risk
   b. Avoid the risk
   c. Manage the risk
   d. Transfer the risk (e.g. insurance, derivatives, etc)

Risks for Superannuation Savings

1. Financial
2. Regulatory
3. Personal
4. Operational Risk
5. Leakages

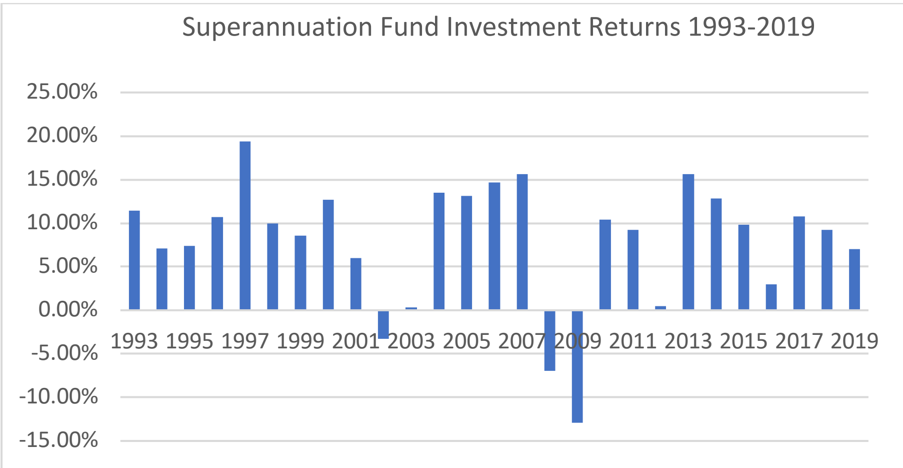
1. Financial Risks

- **Price Inflation** is higher than expected

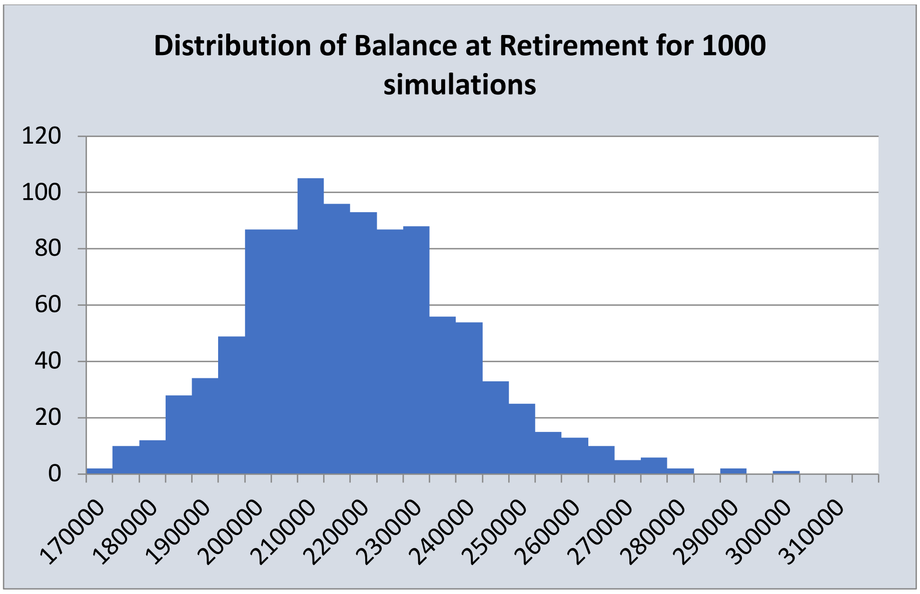


- **Salary Growth** rates are lower than expected

- **Investment returns** are lower than expected



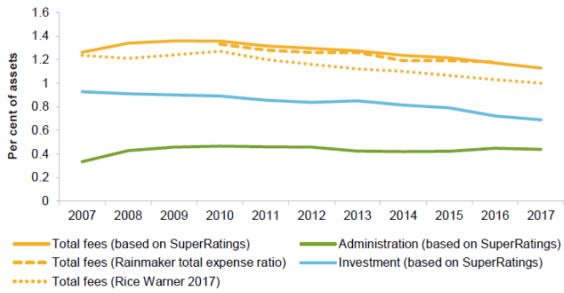
  - Variation in Investment Returns: Stochastic Modelling. These models allow for random variation and are based on statistical models of random variables



  - The graph above shows a distribution of the probability of different outcomes.
  - This is used to measure risk for each investment strategy (e.g. What is the probability that I will have less than $X?

- **<u>Fees</u>** are higher than expected
    - Fees have been slightly declining in recent years.

Figure 3.3 **Total fees have fallen slightly**[a]
Advertised fees as percentage of assets, APRA-regulated funds, 2007–2017



Sources	Rainmaker (2017a, 2017b, table 2), Rice Warner (2017g, 2018c), PC analysis of SuperRatings data.		
Coverage	362 products covering 78% of total assets and 76% of member accounts in APRA-regulated funds in 2017.		
Survivor Bias	Yes.	**Selection Bias**	Yes.

[a] Average product fees are calculated as the asset-weighted average of the fees of individual products.

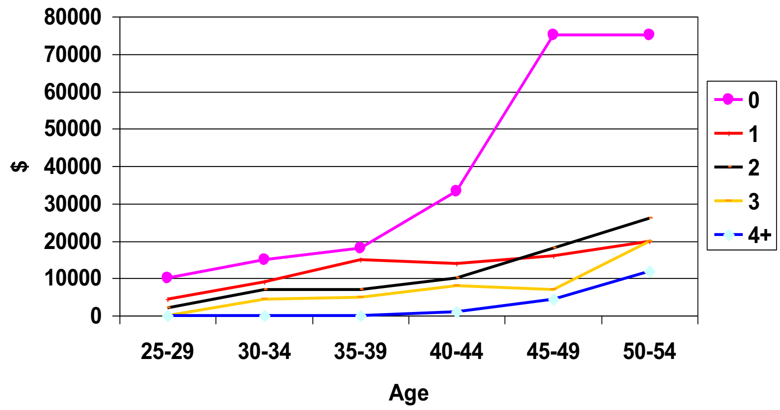
2. Regulatory Risk

- The Australian superannuation industry is plagued by high levels of regulatory risk
- The government frequently changes superannuation rules.
- Superannuation tax changes are made in nearly every budget (every year)
- **Past changes** (changes that have occurred in recent years):
    - Limits on amount you can contribute
    - Limits on total amount you can have in your fund in retirement phase
    - Higher taxes on superannuation for some people (high incomes)
    - Tighter means test so fewer people are eligible for the old age pension
- Possible **Future changes**:
    - High pension eligibility age (67 → 70)
    - Higher preservation age (60 → 67)
    - Higher taxes on contributions
    - Impose controls on the use of superannuation by retired people
- Changes make it difficult to plan ahead
- People are reluctant to put extra money into super because of the regulatory risk.

3. Personal Risks

- **<u>Death/Disability</u>** (permanent or temporary, full or partial)
    - These risks can be managed by buying insurance.
    - Insurance premiums are deducted from super accounts
- **<u>Unemployment</u>** or a lower-paid job
    - Hard to manage this risk (not many insurance possibilities against the risk of becoming unemployed)
- **<u>Children</u>** and other carer responsibilities.

- Variation in Median Value of Superannuation for Women by Age and No. of Children



- Managing the impact of children:
  - **Avoid**
  - **Accept**: hope that your children support you in old age
  - **Manage**: minimise time out of workforce, share responsibilities with husband, super contributions while on maternity leave, etc.

4. Operational Risks

- **Definition**: The risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. (i.e. the system does not work the way it is supposed to)
- Examples:
  - People who aren't entitled to SG contributions
  - Employer does not pay his compulsory contributions into the fund (and may even steal your voluntary contributions as well)
  - Incompetent or corrupt trustees
  - Fraud
- **<u>People who aren't entitled</u>** to SG Contributions:
  - By law, superannuation is compulsory, and employers must pay 9.5% of an employee's salary into a superannuation fund
  - **Self-employed contractors** (e.g. plumber) own their own business and people who hire them do not have to pay super for them.
  - **Gig Economy Workers**: These are not employees either and therefore the people they work for do not pay super for them.
  - Sometimes it is difficult to decide whether a worker is an employee or a self-employed contractor as it is a legal grey area. We must consider a range of factors regarding whether the worker:
    - Works under control of an employer
    - Works set hours…
  - Sham Contracting: Employers may not want to pay super so they tell employees to become self-employed contractors. This is illegal
- **<u>Employer non-compliance</u>**:
  - Employers must pay contributions, however, not all employers obey the law.
  - This is investigated by the ATO (Australian Tax Office)

- Some reasons that this may occur are: **ignorance** (employer doesn't know the rules), **exploitation** and **desperation** (likely when employer is in financial difficulties).
- The Australian National Audit Office (ANAO) reports that between 11% and 20% of employers aren't paying super contributions.
- This problem may not necessarily be solved by going to the ATO as: the employer might be bankrupt, or the legal fees may be too expensive.

- **<u>Incompetent Trustees:</u>**
  - Super funds are managed by trustees who are required to invest money prudently and with care
  - In the past there have been problems with including:
    - Trustees being poor investors
    - Trustees investing into companies owned by people they know
    - Trustees hiring a company owned by someone they know to provide admin service, even if that company charges high fees
    - Trustees buying insurance from related-company insurer, even if that company charges high insurance premiums.
  - **Stealing via fees**: Sometime funds keep charging service fees after they've stopped providing service. For example, the financial advisor is retired, or the customer is dead.
- **<u>Fraud by third parties</u>**
  - Manny trustees hire expert investment managers
  - They give the superannuation fund's money to the investment managers, who then invest the money on behalf of the super fund
  - Often, financial advisors help the trustees choose – they recommend the best fund managers.
  - An example of large scale fraud is the Trio Capital case where over $180 million of super money was stolen
  - In some cases, the government will provide compensation for superannuation fund members who suffer losses due to fraud or dishonest conduct.
    - Ministerial discretion
    - Only for APRA-regulated superannuation funds
  - SMSFs vs APRA-regulated funds
    - APRA-regulated funds (most funds): APRA checks on trustees, supervises and members may receive compensation
    - Self Managed Super Funds (SMSFs): Small funds with less than 5 people, all fund members are trustees, NOT supervised by APRA, NOT eligible for compensation.

5. Leakages

- Some economists are concerned that compulsory superannuation is not really helping people save for retirement
  - **Substitution effects** – if people are forced to save more in super, they simply save less in other investments
  - **Increasing Debt levels** – people are running up credit card and home loan debt, then at retirement using their super to pay off their debts.

Scenario Testing

- We can build a model to assess the impact of various events (both good and bad)
  - Stock market crash
  - Period of unemployment/childcare
  - Increase in contribution rate
  - Reduction in fees
  - Tax rate changes
  - Change in investment strategy

Income Needs After Retirement

- Factors that affect the amount of money needed each year after retirement:
  - Desired standard of living
  - Inflation of prices after retirement
  - Likely expenses
  - Other sources of income such as old age pension from government
  - Government subsidiaries for some services
  - Other assets/resources
- Some expenses will go down once you retire
  - House paid off
  - Kids are self-supporting
  - No work-related expenses
- But some expenses will go up:
  - Health care costs
  - Home help for the elderly (cleaning, home maintenance, etc)
- The ASFA and Westpac estimates that for a:
  - Modest standard of living, you will need approx. $28,165 p.a. (single)
  - Comfortable standard of living, you will need approx.. $44,146 p.a. (single)

EXAMPLE:

- Suppose we decide to retire at 65 and we want to have an income of $30,000 p.a. paid annually in arrears for the next 20 years. How much money will we need to have saved up by age 65?
- Working backwards: Start with a lump sum in your account, take out money each year and see how long the money lasts

Present Values

PV of a single payment

- Suppose we want to have $C to spend at time t.
- How much do we have to invest at time t=0?
- This is called the present value of C, denoted P. The present value of an amount C, payable at time t, given interest rates of i per annum, is:
  $P * (1+i)t = C$
  $P = C/(1+i)^t$

$$\boldsymbol{P = C * (1+i)^{-t}}$$

PV of Multiple Payments

- The PV of multiple payments is calculated by finding the PV of each separate payment and then adding them up.
- PV of multiple payments, where payment due at time t is denoted Ct, and interest rate is i p.a. payable annually

$$\boldsymbol{P = \sum_t C_t * (1+i)^{-t}}$$

Using Excel to find PV

- Excel has some built-in functions to calculate present values of a set of payments
- **PV(i,n,C)** gives the present value of n payments of C p.a. payable annually in arrears at rate I p.a.
- **NPV(I, list of payments)** is a more flexible function which finds the present value of a set of variable annual payments (not all equal to C)
- **XNPV(i, list of payments, list of dates)** is even more flexible because it allows for the payments to be variable amounts and the payments may occur at fractional periods

PV of regular annual payments

- Suppose that we want $C per annum to be paid at t = 1,2,3,...20

$$P = \sum_t C_t * (1+i)^{-t}$$

$PV_1 = C\ (1+i)^{-1}$
$PV_2 = C\ (1+i)^{-2}$
$PV_3 = C\ (1+i)^{-3}$
....
$PV_{20} = C\ (1+i)^{-20}$

- Total PV is sum of all separate PVs

$$\boldsymbol{PV = C\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

Standard Actuarial Notation

$a_{\overline{n}|}^{i}$ Is the Standard notation to represent the present value of payments of $1 p.a. payable **in arrears** for n years at rate i.

$\ddot{a}_{\overline{n}|}^{i}$ Is the standard actuarial notation to represent the present value of payments of $1 pa. payable **in advance** for n years at rate i.

Allowing for Inflation

- Suppose that the payment is $C in the first year and increases at f% each year. Payments are for n years in arrears.
- The retirement savings account has investment returns of i% p.a.
- The PV of this group of payments is given by:

  The PV of the first payment is $C * (1+i)^{-1}$
  The PV of the second payment is $C * (1+f) * (1+i)^{-2}$
  The PV of the third payment is $C * (1+f)^2 * (1+i)^{-3}$

and so on
The PV of the final payment is $C * (1.+f)^{n-1} \ * (1+i)^{-n}$

- Using the sum of a GP, we get the present value:

$$\boldsymbol{C} \ * \ \left[\frac{\boldsymbol{1}-\left(\frac{\boldsymbol{1}+\boldsymbol{f}}{\boldsymbol{1}+\boldsymbol{i}}\right)^{\boldsymbol{n}}}{\boldsymbol{i}-\boldsymbol{f}}\right]$$

Account Based Products

- Suppose we want an income of $40,000 pa. Increasing with inflation at 2% p.a. payable at the end of each year for 20 years.
- We can work out the PV of these payments, save up this sum, and put the money in a superannuation account.
- We can then withdraw the required sum each year.
- However, there are risks:
  - **1. Investment Risk**: What if you earn less than 5% per annum on your account?
  - **2. Inflation Risk**: What is prices increase by more than 2%.
  - **3. Longevity Risk**: What if you live longer than 20 years and run out of money?
- There are produces that protect against these risks:
  - Lifetime Annuity
  - Lifetime Pension

Lifetime Annuities/Pensions

- Lifetime Annuity
  - Customer pays a large lump sum at retirement to a life insurer
  - The life insurer promises to pay a specified amount of money each year as long as the customer is still alive
- Lifetime Pension
  - Same as above, but the provider is a superannuation fund instead of a life insurer
- Advantages of each:
- A customer who buys an indexed lifetime annuity has:
  - No investment risk
  - No inflation risk
  - No longevity risk – insurer keeps paying after average life expectancy

Calculating Annuity Prices

- In order to figure out what price an insurance company should charge for an annuity, we use the valuation of **contingent payments.**
- A **<u>contingent payment</u>** is a payment which depends on a random event (such as life or death)
- The payment is a random variable
- For annuities, a payment will be made at time *t* if the customer is still alive at that time

Life Tables – 10/4/20

Life Tables

- Life tables are used to calculate the probabilities of survival and death for people in a hypothetical population.
- There are different life tables for different populations. (by location, lifestyle, and over different time periods).
- The Australian Life Tables are published by the Australian Government Actuary, based on census data of the Australian population. There are separate tables for males and females.

The Life Table

- Let l_0 be the number of people born (aged 0) in our hypothetical population.
- l_0 assumed to be 100,000 in ALT2015-17
- l_x = number of people still alive at exact age x out of l_0 newborn.

Probability of survival: One Year

$$p_x = \frac{l_{x+1}}{l_x} = \frac{number\ alive\ at\ age\ x+1}{number\ alive\ at\ age\ x}$$

EXAMPLE:

- Males $l_{20} = 99288$ and $l_{21} = 99233$
  so $p_{20} = \frac{99233}{99288} = 0.99945$
- Females $l_{20} = 99467$ and $l_{21} = 99445$
  so $p_{20} = \frac{99445}{99467} = 0.99978$

Number of Deaths

- The number of deaths between exact age x and exact age $x + 1$ is denoted d_x.
- d_x = number alive at age x exact – number alive at age $x + 1$ exact
  i.e. $\boldsymbol{d_x = l_x - l_{x+1}}$

One-year Probability of Death

- The probability that someone aged exactly x will die before age $x + 1$ is $\boldsymbol{q_x}$

$$q_x = \frac{d_x}{l_x}\ or\ q_x = \frac{l_x - l_{x+1}}{l_x}$$

$$or\ q_x = 1 - \frac{l_{x+1}}{l_x}\ or\ q_x = 1 - p_x$$

Probability of Surviving *t* years

- The probability of surviving t years.

$$_t p_x = \frac{l_{x+t}}{l_x} = \frac{number\ alive\ at\ age\ x+t}{number\ alive\ at\ age\ x}$$

Probability of dying in a specified age range

- The probability that a person aged *x* now will die between age *x + t* and *x + s*:

$$\frac{\text{number who die between age } x+t \text{ and } x+s}{\text{number alive at age } x}$$

- i.e.

$$\frac{l_{x+t} - l_{x+s}}{l_x}$$

Life Expectancy

- Let T_x be a random variable which represents the time until death for a person no aged *x* exactly.
- The expected value of T_x is called the future life expectancy at age *x* and is denoted as: $\overset{o}{e}_x$
- i.e. $E(T_x) = \overset{o}{e}_x$

Expected Value of a Random Variable

- If Y is a discrete random variable then the expected value of Y is:

$$E[Y] = \sum_y y * \Pr(Y = y)$$

- Which is sometimes written as:

$$E[Y] = \sum_y y * p(y)$$

- Where the summation is over all possible values of the random variable *Y*.

Calculation of Life Expectancy

- Let's assume people die in the middle of the year (at age *x + t + ½*)
- For a person aged *x,* possible values of the random variable T_x are:

If he dies between x and $x+1$, $Tx = 0.5$
If he dies between $x+1$ and $x+2$, $Tx = 1.5$
If he dies between $x+2$ and $x+3$, $Tx = 2.5$
...
If he dies between $x+t$ and $x+t+1$, $T_x = t + 0.5$

Possible Value of T_x are $t + 1/2$	Probability that a person aged x dies between $x+t$ and $x+t+1$	Product of Value * probability of Value
0.5	$\frac{d_x}{l_x}$	$0.5 * \frac{d_x}{l_x}$
1.5	$\frac{d_{x+1}}{l_x}$	$1.5 * \frac{d_{x+1}}{l_x}$
2.5	$\frac{d_{x+2}}{l_x}$	$2.5 * \frac{d_{x+2}}{l_x}$
…..	…	…
$t + 0.5$	$\frac{d_{x+t}}{l_x}$	$(t + 0.5) * \frac{d_{x+t}}{l_x}$

Expected Value of T_x

- Sum the last column of the table to fine ***E(T_x)***
- Assume ω is the oldest possible age where $l_\omega > 0$ (someone is alive)

$$E[T_x] = \sum_{t=0}^{w-x} (t + 0.5) * \frac{d_{x+t}}{l_x}$$

EXAMPLE:

- The life table for a mouse is given below:

Age x	lx	dx
0	100	20
1	80	40
2	40	30
3	10	10
4	0	

- If deaths occur at mid-year, what is $E(T_0)$?
- $E(T_0)$ = 0.5 * 20/100
  + 1.5 * 40/100
  + 2.5 * 30/100
  + 3.5 * 10/100
  …
  = 1.8 years

Shortcut for Life Expectancy

$$E(T_x) = 0.5\frac{l_x - l_{x+1}}{l_x} + 1.5\frac{l_{x+1} - l_{x+2}}{l_x} + 2.5\frac{l_{x+2} - l_{x+3}}{l_x} + \ldots$$

$$E(T_x) = \frac{1}{l_x}[0.5*(l_x - l_{x+1}) + 1.5*(l_{x+1} - l_{x+2}) + 2.5*(l_{x+2} - l_{x+3}) + \ldots]$$

$$E(T_x) = \frac{1}{l_x}[0.5*l_x + l_{x+1} + l_{x+2} + l_{x+3} + \ldots]$$

$$E(T_x) = 0.5 + \frac{1}{l_x}[l_{x+1} + l_{x+2} + l_{x+3} + \ldots]$$

Iterative formula for E(T_x)

- **Q**: Suppose the mouse has a life expectancy of 1.8 years when new-born. A year later, will its life expectancy be 0.8 years?
- No, it will be higher than 0.8.

$$E(T_1) = 0.5 + [40 + 10]/80$$
$$= 1.125$$

- The iterative formula for life expectancy is:

$$\overset{o}{e}_x = 0.5 + p_x(0.5 + \overset{o}{e}_{x+1})$$

Multiple Lives

- We may want to find probabilities involving multiple lives:
- For example:

- What is that probability that two people are still alive in *t* years?
- Is there are three people A, B and C alive now, what is the probability that exactly one person is alive in *t* years?
- If there are two people alive now (A and B), what is the probability that A dies first?

Probability Rules

- If A and B independent events:

  $P(A\ and\ B) = P(A) * P(B)$

  $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
  $= P(A) + P(B) - P(A) * P(B)$

  $\mathrm{P(Exactly\ one\ of\ A\ and\ B)}$
  $= \mathrm{P(A\ and\ Not\ B) + P(B\ and\ Not\ A)}$
  $= P(A) * [1 - P(B)] + P(B) * [1 - P(A)]$
  $= P(A) + P(B) - 2\,P(A)\,P(B)$

EXAMPLE:

- Joe is age 21 and Bill is age 23 when they graduate uni. What is the probability that both are alive for their 40th reunion?
- The probability that Joe is alive = P(J)

  P(J) = $l61/l21$ = 0.91890
- The probability that Bill is alive = P(B)

  P(B) = $l63/l23$ = 0.90651
- Assuming independence,

  P(Both alive) = P(J) * P(B)
  = 0.83299

EXAMPLE: Multiple Lives

- A superannuation fund is required to pay pensions to retired employees and their spouses.
  - If both husband and wife are alive at year end, the payment is $10,000
  - If only one is alive, payment is $7000.
- Joe is 65 and his wife Mary is 60. Assume that the lives are independent.
- Q: What is the expected value of the payment due at the end of the year?
  - The probability that Joe is alive = P(J)

    P(J) = p_{65} males = 0.99033
  - The probability that Mary is alive = P(M)

    P(J) = p_{65} females = 0.99626
  - P(Both alive) = P(J) * P(M) = 0.98663
  - P(J alive and M dead) = P(J) * [1 – P(M)] = 0.00370
  - P(M alive and J dead) = P(M) * [1 – P(J)] = 0.00963
- Expected Value of payments

  = 10000 * 0.98663 + 7000 * (0.00370 + 0.00963)
  = 9960 (nearest dollar)

Binomial Distribution

- If you have a group of *n* people who all have the same probabilities of survival or death, then you can use the binomial distribution to find the probability that there will be:
    - Exactly *s* survivors
    - Exactly *d* deaths
- Excel has an in-built function for calculating probabilities for Binomial Random Variables

$= BINOM.DIST(r, n, p, 0)$

Gives $\Pr(S = r)$ where S is Bi(n,p)

$= BINOM.DIST(r, n, p, 1)$

Gives $\Pr(S \leq r)$ where S is Bi(n,p)

<u>Contingent Payments – 10/4/20</u>

Retirement Products

- <u>Account based product</u>
    - Lump sim goes into an account which is invested
    - Retiree withdraws money until he runs out of money or dies
    - Any money left over at death goes to heirs
    - ***Retiree*** *bears investment, inflation, and longevity risk*
- <u>Fixed Term Annuity</u>
    - Lump sum is paid to an annuity provider
    - Provider guarantees fixed payments of C p.a. for n years
    - ***Provider*** *bears the investment risk (or reward) and* ***retiree*** *bears inflation and longevity risks*
- <u>Indexed fixed term annuity</u>
    - Lump sum is paid to an annuity provider
    - Provider guarantees payments of C p.a. increasing in line with CPI, for n years.
    - ***Provider*** *bears investment and inflation risk and* ***retiree*** *bears longevity risk.*
- <u>Lifetime Annuity</u>
    - Customer pays a large lump sum at retirement to a life insurer
    - The life insurer promises to pay $C each year as long as the customer is still alive
    - ***Provider*** *bears investment and longevity risk and* ***retiree*** *bears inflation risk.*
- <u>Indexed Lifetime Annuity</u>
    - Same as above, but the payments increase with CPI
    - ***Provider*** *bears investment, inflation and longevity risk and* ***retiree*** *bears no risk* (unless insurer becomes insolvent)

Prices of Lifetime Annuities

- The life insurer must figure out how much to charge for a lifetime annuity
- The number of payments depends on the lifespan of the customer which is a **random variable**

EPV of a Contingent Payment

- A contingent payment is a payment which depends on a random event (such as life or death)
- The payment is now a **random variable**
- When pricing an insurance product which provides contingent payments, we start by working out the present value of the expected value of future payments.
- This is known as the **Expected Present Value** (EPV)
- If payments C_t are sure to be paid, the present value is:

$$PV = \sum_t C_t * (1+i)^{-t}$$

- If payments C_t are **contingent payments**, we find the PV of the expected value of the payments:

$$\boldsymbol{EPV = \sum_t E[C_t] * (1+i)^{-t}}$$

PV of a Lifetime Annuity

- Simplest contingent payment: Pay $C at time *t* if a person aged *x* is alive at that time.
- The payment C_t is now a random variable:
  - $C_t = C$ if person is alive at time *t*
  - $C_t = 0$ is person is dead at time *t*
- The expected value of the payment is:
- $E[C_t] = C * \frac{l_{x+t}}{l_x} + 0 * \left(1 - \frac{l_{x+t}}{l_x}\right)$

$$\boldsymbol{E[C_t] = C * \frac{l_{x+t}}{l_x}}$$

Pricing Lifetime Annuities

- EPV of payments of $C annually **<u>in arrears</u>** payable if a person aged *x* now is still alive at the annual payment date:

$$\boldsymbol{EPV = \sum_{t=1}^{w-x} C * \frac{l_{x+t}}{l_x} * (1+i)^{-t}}$$

  - Where *w* is the oldest age of survival in the Life Table
- EPV of payments of $C annually **<u>in advance</u>** payable for a person aged *x* now is:

$$\boldsymbol{EPV = \sum_{t=0}^{w-x} C * \frac{l_{x+t}}{l_x} * (1+i)^{-t}}$$

  - Same as above except summation starts are *t* = 0 instead of *t* = 1

Setting out the EPV calculation

- Setting out the calculation in a table:
  - Column 1 – t, all possible times of payment
  - Column 2 – Amount C_t, payment which might be made at time *t*
  - Column 3 – Probability that payment will be made at *t*
  - Column 4 – Discount factor $(1 + i)^{-t}$
  - Column 5 – Amount*Probability*Discount for each *t* (APD)
- The EPV is the sum of column 5

EXAMPLE:

- A dog has a vet check at the end of each year, costing $100. What is the expected present value of the vet costs for a dog aged 0 now? Interest rate 5% p.a.

Age x	l(x)
0	1000
1	980
2	670
3	430
4	210
5	0

Time	Amount	Probability	Discount Factor	A*P*D
1	100	0.980	1.05^-1	$ 93.33
2	100	0.670	1.05^-2	$ 60.77
3	100	0.430	1.05^-3	$ 37.15
4	100	0.210	1.05^-4	$ 17.28
5	100	0		$ -
Total				$ 208.53

- EPV = $208.53

Actuarial Notation

a_x is the PV of $1 per annum payable in arrears for life to a person now aged *x*

$\ddot{a}_x$ is the PV of $1 per annum in advance for life to a person now aged *x*

$\ddot{a}_{x:\overline{n}|}$ is the PV of $1 per annum payable in advance to a person now aged *x* until either the person dies OR *n* payments have been made

- Therefore:

$$a_x = \sum_{t=1}^{w-x} \frac{l_{x+t}}{l_x} * (1+i)^{-t}$$

- And:

$$\ddot{a}_x = \sum_{t=0}^{w-x} \frac{l_{x+t}}{l_x} * (1+i)^{-t}$$

- It can be shown that the relationship between $\boldsymbol{a_x}$ ***and*** $\boldsymbol{\ddot{a}_x}$ is:
$\ddot{a}_x = 1 + a_x$

Iterative Formula for a_x

- When making tables of a_x manually, it is useful to use the following iterative formula:

$$a_x = \frac{p_x}{1+i}(1 + a_{x+1})$$

- When constructing annuity tables in excel, use the iterative formula, starting at the bottom of the table (with $a_{110} = 0$) and working your way to the top.

Variations on Annuities

- The formula above assumes a constant payment of C paid annually, which is payable if the person is alive and ceases when they die.
- However, the same method can be used to find the present value of payments where these assumptions are varied.
- Example: **Indexed Annuity**
    - Annual amount increases with inflation at f% p.a.
- Example: **Probability of payment depends on the survival of two or more people**
    - Joint life is paid while BOTH husband and wife are alive
    - Last survivor is paid while either one of the two people are alive
- Example: **Number of payments has a minimum or maximum**
    - Annuities with a maximum term of *n* years
    - Annuities with a guaranteed periof of *n* years.
- A general formula for the present value of contingent payments can be written in this format:

$$PV = \sum_{t} C(t) * \Pr(t) * (1+i)^{-t}$$

    - C(t) is amount which might be paid at time t
    - Pr(t) is the probability that a payment will be made
- Now we have a standard approach for the valuation of all sorts of annuities.
- This same approach can be used for any other contingent payment.
- **Problem**: Model Assumptions
    - Choosing the best estimate of probabilities
    - Choosing the best estimate of investment returns
    - (if indexed) choosing the best estimate of inflation rate

<u>Lifetime Annuities – 17/4/20</u>

Popularity of Lifetime Annuities

- At retirement: receive a lump sum
- Option 1: Put money into an account, make withdrawals
    - Longevity risk
    - Investment risk
    - Inflation risk
- Option 2: Buy an indexed lifetime annuity from a life insurance company
    - No longevity risk
    - No investment risk
    - No inflation risk
- Fact: 94% of retirees choose option 1
- Thus, some people run out of money and must rely on old age pension

Disadvantages of annuities

- **Poor Health**
    - Annuity payments continue as long as you live but stop when you die.

    - Therefore, they are not a good deal if you are in poor health
    - Possible solution: guarantee periods (insurer promises to pay at least *n* payments.
- **Bequests**
    - Many people want to leave money to family members when they die. But with lifetime annuities, this is not an option.
    - On the other hand, with a lifetime annuity, your children won't need to support you in your old age
    - But if you choose an account-based product and live a long time, you might run out of money, you might need financial support from your family.
- **No Liquidity**
    - The amount paid in each year is fixed
    - If extra money is needed for any reason, you cannot take it out
    - This is called surrendering – life insurers usually do not allow surrendering as people would always do this as soon as they got sick.
    - This would mean that they could never make profits or have extra funds for people who outlive their life expectancy.
- **People prefer to spend their retirement savings and then become eligible for the old age pension (pass the means test)**
    - This means you get more money from the government
    - The government worries about this, but evidence shows that most people don't do this
- **Trusting the insurance company**
    - You are handing your life savings to the life insurance company
    - Since the *Life Insurance Act 1945*, these companies have become a lot more trustworthy and less likely to become insolvent.
    - APRA is responsible for solvency of Banks and Insurers, and makes sure that life insurers have enough assets so that there is a very probability that they will be able to pay the benefits
    - <u>Capital Requirement</u>: to prevent life insurers from going broke, they are required to hold
        - Enough assets to provide EPV of future benefits + Capital (extra money to cover risk of losses)
- **Cost**
    - Annuities are expensive because the life insurer must hold high levels of capital to reduce the risk of insolvency.
    - The shareholders who provide this capital are paid in dividends which are covered by the price of annuities.
- For the above reasons, annuities are not very popular in Australia
- The government thinks this is a problem because they end up having to pay more in terms of the old age pension

MyRetirement (CIPRs)

- Comprehensive income products for retirement – renamed 'MyRetirement'
- It is a product that provides protection against longevity risk but without the disadvantages of lifetime annuities.
- (still in progress)

Pensions from Super Funds

- So far, we have considered annuities (sold by life insurance companies)
- Pensions are the same things, except they are offered by superannuation funds
    - The retiree hands over a lump sum payment are receives regular payments until they die.
- Two types of superannuation funds might offer to pay pensions:
    - **Public Sector Fund** – a fund run by the government for its own employees (separates funds for each state and for the Commonwealth)
    - **Private Sector Funds**

Public Sector Funds

- The government has a weakness – they like to promise generous superannuation benefits to their employees, but they don't like to pay the contributions for these benefits
- This is called "unfunded liabilities"
    - Occurs when: assets in the fund <<< EPV of promised benefits
    - Unfunded Liability = EPV of Promised benefits – Assets
- Unfunded liabilities are really high, and are still increasing
- Future taxpayers will have to pay additional taxes to cover this

Private Sector Super Funds

- Only a few private sector superannuation funds pay pensions (most pay lump sums)
- Sometimes these funds can get into difficulties when the vested benefits (amount payable if everyone quit now) falls below the assets.

<u>Annuitant Mortality – 17/4/20</u>

Risks for the customer

- If a customer buys an indexed lifetime annuity, the customer has:
    - No longevity, investment, or inflation risk

Risks for the insurer

- The insurer will charge a price which is the EPV of the payments plus a profit margin.
- However, the insurer now bears the risks:
    - **Longevity risk**: if the customers live longer than expected, the insurer will have reduced profits
    - **Investment risk**: if the investment returns are lower than expected, the insurer will suffer reduced profits
    - **Inflation risk**: if inflation is higher than expected, they suffer reduced profits

Managing the Longevity Risks

- The first step in managing the longevity risk is to use the right life table when doing your calculations
- The Australian Life Tables represent the mortality rates for the whole Australian population and therefore are not suitable for calculating annuity rates for the following reasons:
- **Self-Selection:** unhealthy people usually don't buy lifetime annuities, so annuitants are "self-selected" to be healthier than average.

- **The Wealth Effect**: People who buy annuities tend to be people who are wealthier than average, and these people tend to have lower mortality rates
- **Mortality Improvements:** The ALT is based on recent mortality rates, but historically, life expectancy tends to increase as time goes on. If people live longer than expected, a life insurer that sells lifetime annuities will suffer a loss. Thus, they should allow for future improvements in life expectancy when setting prices.

Annuitant Tables

- Instead of using the ALTs, actuaries use special life tables which are suitable for annuitants.
- They are usually based on the past experience of their own customers, but a lot of data is required to make accurate estimates.
- It is a common problem for companies to not have enough data to create reliable estimates, but they can source data from different life insurance companies.

Impaired Life Annuities

- Some customers have long life expectancies, and some have short.
- If we charge the same price to all customers, short life expectancy people pay too much and long life expectancy people get a bargain
- In Australia, there are no discounts for unhealthy people (no health questions when you buy a lifetime annuity)
- In the UK unhealthy people can get cheaper annuities called "enhanced annuities"
- If an Australian life insurer decides to sell enhanced annuities, they would have to develop skills in assessing how life expectancy is affected by certain health conditions
    - This process is called **underwriting**

Life Tables for Insured Lives

- Insured lives are not a random sample from the Australian population
- Group life product (sold via superannuation fund)
    - Different life tables for different funds
    - Depends on occupation of fund members
- Individual Product
    - Different life tables for males, females, smokers, etc.
    - Insurer applies an underwriting process
    - People with high risk may be **rejected**, **loaded** or might have **exclusions**
- As a result of selection by the insurer, insured lives have a LOWER mortality rate than the average population

Changes in Mortality Rates – 17/4/20

- It is important to analyze this to:
  - Advise insurers: insurance companies which sell annuities face longevity risk
  - Advising government: planning for the future, health services, nursing home car, old age pension

History: Life Expectancy in Australia

- Over the last century, life expectancy has improved substantially

Year of birth	Male Life Expectancy	Females Life Expectancy
1900	51	54
1960	68	74
2016	81	85

Mortality Rates for Smokers

- Life insurance policies pay a benefit if the customer dies
- The premium rate depends on the risk of death. Thus, the charge different premiums to smokers and non-smokers.
- So they need to know the ratio:

$$\frac{smoker\ mortality\ rates}{non\ smoker\ mortality\ rates}$$

- For males:
  - Smoker mortality rates are roughly double the non-smoker rates and more than double at older ages
- For females:
  - Smoker mortality ratios are roughly 1.3 at younger ages but increase sharply at older ages
- **Public Policy**: education/warnings, gruesome health warnings on packets, plain packaging, bans on smoking in certain places, bans on advertising, taxes on cigarettes, quit-lines to help people quit

Car Accidents

- Reasons for reductions in motor vehicle accidents:
  - Compulsory seatbelts (1970)
  - RBT
  - Better cars
  - Better roads
  - Tougher rules for getting a license

Food Quality

- Life expectancy can be improved by
  - Better nutrition
  - Preventing food from containing poison
  - Additives which prevent disease/illness

Increasing Threats

- Although we have managed to reduce the number of deaths due to smoking, car accidents, diseases, poor food quality, etc. some causes of death are actually increasing.
- An example of this is **drug overdosing** in the USA which is primarily caused by Opioids
- Drug overdoses are now the number 1 cause of death in America for people under 50.
- A few reasons for this are that they are very addictive, companies that sell these make a lot of money, unethical marketing, etc.

Life Expectancy in the Future

- We have seen the historical trend of life expectancy increasing in Australia for more than 100 years, but will this trend continue?
- USA – life expectancy has been reducing for certain sub-groups of the population

Introduction to Investment – 17/4/20

Superannuation Fund Choices

- Most funds have many investment options available
- Fund members can choose which investment options they prefer
- Usually you can switch your choices up to 2 times per year for free (after that there is a switching fee)
- Many funds will allow members to specify how much should be invested in each type of asset:
    - Australian shares, Australian sustainable shares, cash, Australian fixed interest, diversified fixed interest, international fixed interest, international shares (hedged), property.
- Each investment contains a combination of many securities.

Pre-mixed Strategies

- For a pre-mixed strategy, the trustees of the fund will decide how to money will be allocated across the different asset classes
- Pre-mixed strategies include:
    - Capital guaranteed – lowest risk
    - Capital stable
    - Balanced
    - Growth
    - High Growth – highest risk
- Each superannuation has a default option which is usually the balance fund.
- The default option is selected if the fund member doesn't make a choice
- Some funds set the default based on age (younger members are put into balanced, while older people are put into more conservative funds)
- A surprisingly high number of people end up in the default option.

Information about Investment Options

- By law, superannuation fund trustees are supposed to give fund members information about each investment option so that fund members can make an informed decision. Including:
    - **Asset allocation** – what proportion is invested in each asset class
    - **Limits on the asset allocation** – how much flexibility does the fund manager have?
    - **Any liquidity restrictions**
    - **Historical Performance** – rates of return over the past 10 years
    - **Risk measure** – defined as the number of years that the fund is likely to have a negative return over 20 years
    - **Fees charged** for the management of assets (including switching fees)
    - **Investment objectives** – there are often measured relative to some benchmark

Measuring Risk

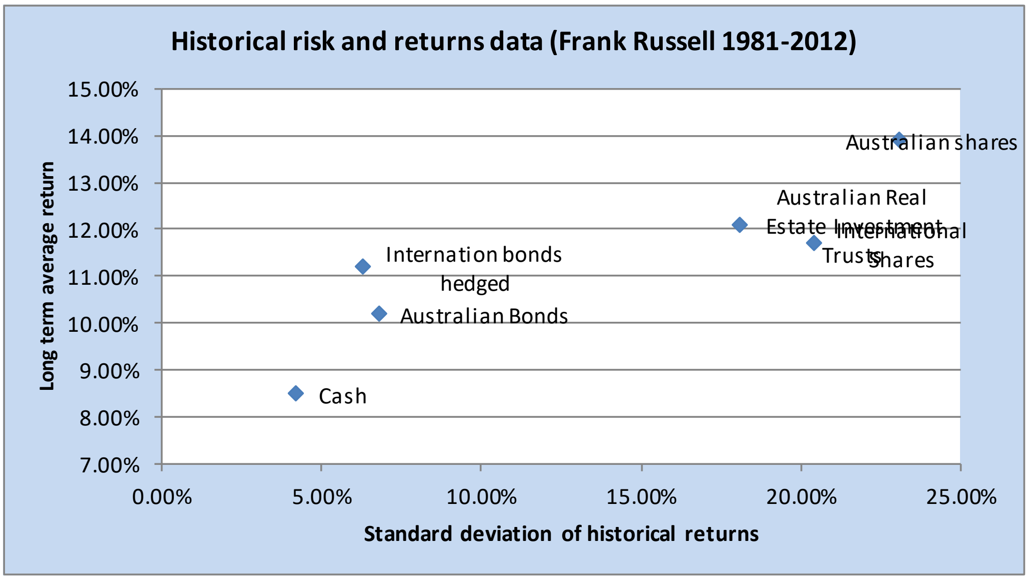
- There are different ways to measure and describe investment risk
- Measure #1 – **Negative Returns**
    - Riskiness is described using the "standard risk measure": the expected number of years of negative returns in 20 years (e.g. Australian Shares – negative returns in 4-6 years out of 20 years)
    - This is not a very effective way to describe risk because it does not indicate the magnitude of the losses. However, it is easy to understand for the general public.
- Measure #2 – **Range of Outcomes**
    - Giving the range of outcomes helps to illustrate the magnitude of gains or losses (e.g. Australian Shares range from -40% to +60%)
    - Still not a very good indicator of risk because it doesn't show the probability of gain or loss
- Measure #3 – **Standard Deviation**
    - Standard deviation is often used as a measure of risk (e.g. Australian Shares have 22% standard deviation)
- Measure #4 – **Asset-liability Matching**
    - All of the above measures only look at the variability of assets
    - Actuaries prefer to look at the variability of assets relative to the liabilities
    - Risk is reduced if assets and liabilities can be matched (i.e. assets and liabilities tend to move up and down together)

ALM and Superannuation Objectives

- A superannuation fund member will have needs such as food, rent, electricity, clothes, phone, etc.
- Ideally, we want the long term investment to exceed the increases in cost of living
- Superannuation Fund Objectives may be expressed as the **real** rate of return (relative to inflation)
- In this case, investment risk would be defined as either:
    - The probability that the investments do not meet this target, or
    - The standard deviation of $(i - f)$ calculated over 5 years.

- From now on, we will use standard deviation as a measure of risk
- Investment alternatives are often shown on graphs
    - X-axis: standard deviation

    - Y-axis: expected rate of return
- Example:



Risk-Return Trade-offs

- We usually assume that people are risk averse (prefer lower levels of risk)
    - Investments with low risk offer low expected returns
    - Investments with high risk offer higher expected returns

Evanescent Investment Opportunities

- If you are offered an investment which offered low risk and high returns, it is most likely a scam (Ponzi Scheme)
    - In an efficient market, investors would quickly become aware of any such investment opportunity and will rush to buy this asset
    - As a result, prices of the asset will increase, and the rate of return will decrease.
    - It will quickly become a low risk, low expected return investment
- If there is an investment with high risk and low returns, it will be very unattractive to investors
    - Hence, no one will want to buy it and the price will fall
    - The people selling this investment will have to offer high returns to entice investors into buying this asset
    - This, it becomes a high risk, high expected return investment.

The Correct Time Frame for Risk Measures

- Actuaries say that risk measures should allow for the investor's time horizon
    - A 20 year old investing in a super fund has a 50 year time frame whereas a 90 year old will have around a 5 year time frame.
- This will affect the investment decision
- **Short termism**: Making long term investment decisions based on short term measures. This leads to poor decision making.

<u>**Investment Portfolios – 17/4/20**</u>

Expected Value of a Portfolio

- We can use the following rules to calculate the expected value of a portfolio of two or more assets:

$$E(aX + bY) = a\,E(X) + b\,E(Y)$$

<u>EXAMPLE:</u>

- You are going to invest \$120: \$50 in an asset A where the annual return is a random variable denoted I_A, where the expected rate of return is 5% p.a. and standard deviation is 4%. \$70 in an asset B where the annual return is a random variable denoted I_B where the expected rate of return is 8% p.a. and standard deviation 9%.
- What is the expected value of your wealth at the end of the year, denoted W? (Assume that the assets' returns are independent)
    - Let W be the random variable which denotes your wealth at the end of the year. Start by writing W as a function of I_A and I_B:
        - ⇨ $W = 50 * (1 + I_A) + 70 * (1 + I_B)$
        - ⇨ $E(W) = E[50 * (1 + I_A) + 70 * (1 + I_B)]$
        - ⇨ $= 50 * E[(1 + I_A)] + 70 * E[(1 + I_B)]$
        - ⇨ $= 50 * [1 + E(I_A)] + 70 * [1 + E(I_B)]$
        - ⇨ $= 50 * 1.05 + 70 * 1.08$
        - ⇨ $= 128.10$

Variance of a Portfolio

- To calculate the variance and standard deviation of the wealth at the end of the year we use the following rule:

$$VAR(aX + bY) = a^2\,Var(X) + b^2\,Var(Y)$$

    - This rule applies only if X and Y are independent.

<u>EXAMPLE:</u>

- You are going to invest \$120 in total: \$50 in an asset where the return has a standard deviation of 4% and \$70 in an asset where the return has a standard deviation of 9%. The two assets are independent.
- What is the standard deviation of your wealth at the end of the year, denoted W?
    - Let W be the random variable which denotes your wealth at the end of the year. Start by writing W as a function I_A and I_B

$$W = 50 * (1 + I_A) + 70 * (1 + I_B)$$

    - Find the variance using the rule:
        - ⇨ $VAR(W) = VAR\{50 * (1 + I_A) + 70 * (1 + I_B)\}$
        - ⇨ $= 50^2 * VAR(1 + I_A) + 70^2\,VAR(1 + I_B)$
        - ⇨ $= 50^2 * VAR(I_A) + 70^2\,VAR(I_B)$
        - ⇨ $= 50^2 * 0.04^2 + 70^2 * 0.09^2$
        - ⇨ $= 43.69$
    - The standard deviation is the square root of the Variance, i.e. 6.61
- So our expected wealth at the end of the year is \$128.10 and the standard deviation is \$6.61

Expected Value and Standard Deviation of The Rate of Return

- In order to find the expected value and the standard deviation of the return on the portfolio, we must first define the investment return:

  Investment return = $\frac{Final\ Value\ of\ Portfolio}{Initial\ Value\ of\ the\ Portfolio} - 1$

- Mathematically, we would say that:

  $I_p = \frac{W_p}{w} - 1$

  Where w is the wealth at the start of the year (not a random variable)

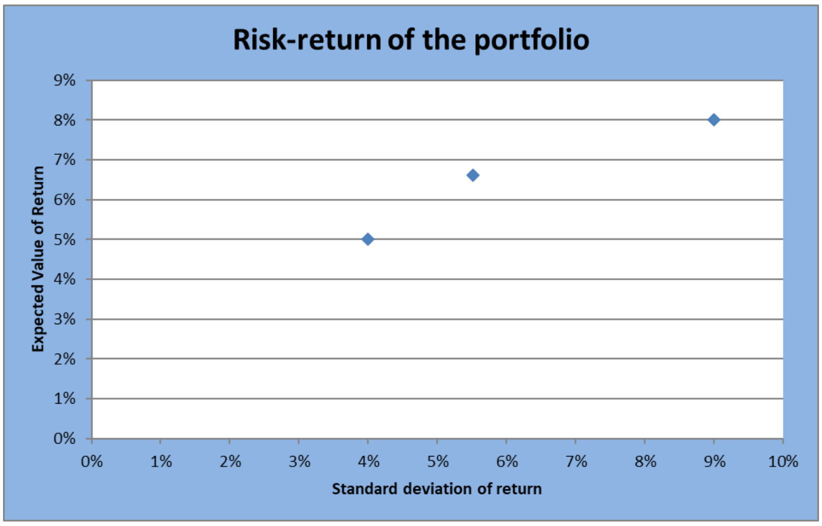
  W_p is wealth of the portfolio at the end of the year (a random variable)

  I_p is the rate of return on the portfolio

- To find the expected value and variance of the investment return on the portfolio,
  - ⇨ $E[I_p] = E\left[\frac{W_p}{w} - 1\right]$
  - ⇨ $= \frac{E[W_p]}{w} - 1.$
  - ⇨ $Var[I_p] = Var\left[\frac{W_p}{w} - 1\right]$
  - ⇨ $= \left(\frac{1}{w}\right)^2 Var[W_p]$

EXAMPLE:

- Extending on the previous example:
  - ⇨ $E(I_p) = 128.10/120 - 1$
  - ⇨ $= 6.75\%$
- And:
  - ⇨ $Var(I_p) = \frac{43.69}{120^2}$
  - ⇨ $= 0.003034$
- And the standard deviation of the investment return is 5.508%
- So, the annual return of the portfolio has an expected value of 6.75% p.a. with standard deviation 5.51%. We can plot the portfolio on a risk-return diagram:



General Formulae for Expected Value and Variance of Two Assets

- A general formula for calculating the expected value of wealth and variance of the return on a portfolio consisting of $\$x_1$ invested in Asset 1 and $\$x_2$ invested in Asset 2, where
  - The return for Asset 1 has expected value μ_1 and standard deviation σ_1
  - The return for Asset 2 has expected value μ_2 and standard deviation σ_2

⇨ $E(I_p) = w_1 * \mu_1 + w_2 * \mu_1$

⇨ $Var(I_p) = w_1^2 * \sigma_1^2 + w_2{}^2 * \ \sigma_2^2$

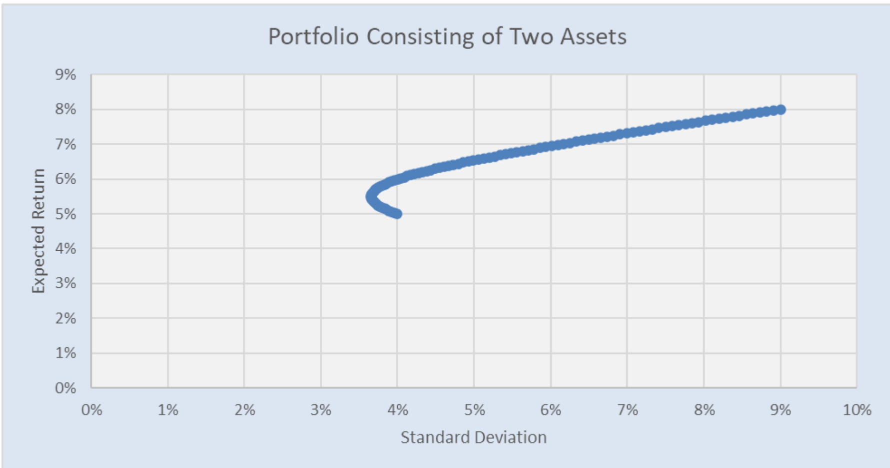
- This can be extended to cover a portfolio consisting of n independent assets:
- Let:
  - w_i be the proportion of the fund which is invested in Asset i
  - μ_i be the expected value of returns on Asset i, and
  - σ_i be the standard deviation of the return on Asset i.

⇨ $E(I_p) = \sum_{i=1}^{n} w_i \ \mu_i$

⇨ $Var(I_p) = \sum_{i=1}^{n} w_i{}^2 * \ \sigma_i^2$

Finding all Possible Portfolios Consisting of Two Independent Assets

- Let's suppose that we have $100 to invest, and we can only invest in Asset 1 and Asset 2, and we are trying to decide how much to invest in each asset.
- Let $x be invested in Asset 1 and $(100-x) be invested in Asset 2, assuming that x is an integer between 0 and 100.
- Then, using excel, find the expected values and standard deviation of all the possible portfolios which can be created from these two assets.
- This is the graph of expected return and standard deviation for all possible portfolios:



Lowest Risk Portfolio

- Putting all your money into the lowest-risk asset is not always the investment strategy with the lowest overall risk.
- A diversified portfolio of risky assets allows the investor to reduce risk.
- The following example illustrates the benefits of **diversification**.

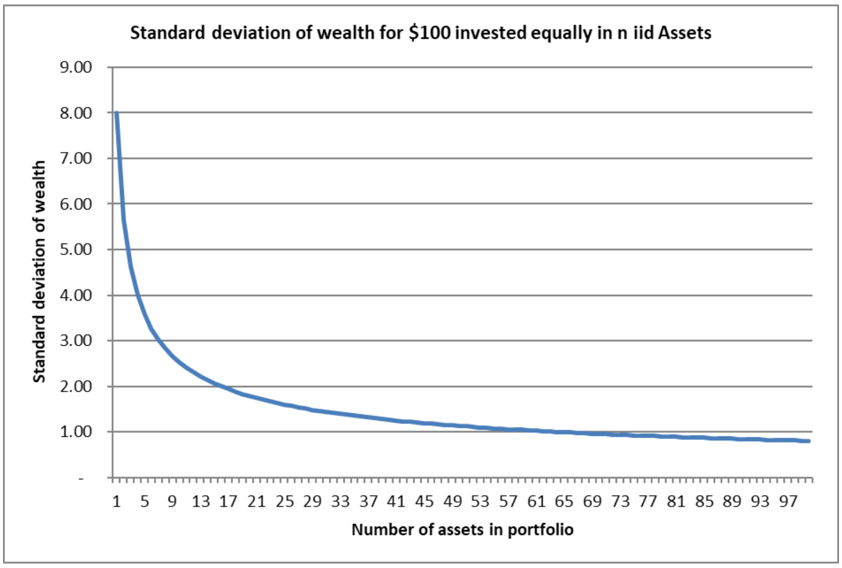
EXAMPLE:

- How can we find the portfolio with the lowest risk? (based off Asset 1 and 2 from earlier)
  - Assume that we can put *\$x* into Asset 1 and *\$(100 – x)* into Asset 2.
  - **Using Excel Solver**: set the Cell that contains the total variance to be minimized by varying the Cell that contains the value of *x*.
  - **Using Calculus**:
    - ⇨ $W = x * (1 + I_1) + (100 - x) * (1 + I_2)$
    - ⇨ $VAR(W) = x^2\, Var(I_1) + (100 - x)^2 * Var(I_2)$
    - ⇨ $= x^2\, 0.04^2 + (100 - x)^2 * 0.09^2$
  - To find the value of *x* that minimizes the variance of *W*, find the first derivative, set it equal to 0 and then solve for *x*.
    - ⇨ $0 = 2x * 0.0016 - 2\,(100 - x) * 0.0081$
    - ⇨ $0 = 0.0032x - 1.62 + 0.0162x$
    - ⇨ $x = 1.62 / 0.0194$
    - ⇨ $x = 83.51\ (rounded\ to\ nearest\ cent)$
- Therefore, we should invest \$83.51 into Asset 1 and \$16.49 into Asset 2 to minimize risk.
- This will produce a portfolio with expected return 5.49% and standard deviation of return 3.66%.
-
- Note the standard deviation of this portfolio is LOWER AND the expected return is HIGHER than if we had simply invested all our money into the lowest risk asset. (illustrating the benefits of diversification)
- Of course, in real life, there are more than two available assets – there might be hundreds or thousands of investments available on the market.
- An investment fund manager has to choose the portfolio which has the best risk-return profile for his customer.

Diversification Across *n* IID Assets

- Suppose we have \$100 to invest and there are an infinite number of assets which are independent and identically distributed
  - They all have the same expected return μ and the same standard deviation of returns σ.
- We have decided to divide out money equally among *n* assets (100/*n* in each asset)
- We can then observe how the risk reduces as the number of assets increases.
- Using the formulas for expected value and SD with $w_i = 1/n$ for each *i*.
- The expected value of the wealth at the end of the year is:
  - ⇨ $E(I_p) = \sum_{i=1}^{n} w_i * \mu_i$
  - ⇨ $E(I_p) = \sum_{i=1}^{n} \frac{1}{n} * \mu$
  - ⇨ $E(I_p) = \mu$
- This is as expected, because investing in multiple identical assets will result in the same expected return.

- The variance of the wealth is:
  - ⇨ $Var(I_p) = \sum_{i=1}^{n} w_i{}^2 * \sigma_i^2$
  - ⇨ $Var(I_p) = \sum_{i=1}^{n} \left(\frac{1}{n}\right)^2 * \sigma^2$
  - ⇨ $Var(I_p) = n * \left(\frac{1}{n}\right)^2 * \sigma^2$
  - ⇨ $Var(I_p) = \left(\frac{1}{n}\right) * \sigma^2$
- Therefore, the standard deviation is: $100 \times \frac{\sigma}{\sqrt{n}}$
- This shows that if you split your money between n assets which have independent and identically distributed assets, **the standard deviation falls as n increases**.



- Once again, this illustrates the benefits of diversification.
- This type of analysis might be useful in deciding how many different assets you would need to own in order to reduce your risk to an acceptable level.
- You might think that it is desirable to hold hundreds of different assets, so that you could reduce your risk to a very low level.
- But in practice this is not a good idea – it costs money to conduct research into each asset, and there are transaction costs involved in buying and selling small amounts of a lot of different assets.

<u>EPV of Life Insurance Products</u>

- We can use the method of contingent payments to calculate the expected present value of **life insurance benefits**.
  - These are policies which pay benefits when someone dies
- For now, we assume that the customer pays a large single premium at the start of the policy
- Two types of policies (for now):
  - Term insurance
  - Whole of Life Insurance

Term Insurance

- A term insurance pays a benefit only if a customer dies within *n* years (*n* is the term of the policy).
- If the customer dies after the term, no benefit is paid.
- The benefit amount is called the **sum insured**.
- This is a reasonable policy for someone who has dependent children. The term can cease once the children are grown up and self-supporting.
- In order to calculate the benefit payments, for each time of payment *t*, we must
    - Consider the amount payable at time *t*
    - The probability that a payment will be made at time *t*
    - The discount factor.
    - And then acquire the sum of the APD
- Assuming that people only die at the end of the year, we have the following notation for a term insurance:

$$A^{1}_{x:\overline{n|}} = \sum_{t=0}^{n-1} \frac{d_{x+t}}{l_x} * (1+i)^{-(t+1)}$$

- This is the present value of benefits of a term insurance policy with a sum insured of \$1, payable to a customer who is age *x* at commencement, with a term of *n* years, with benefits payable at the end of the year of death.

Whole of Life Insurance

- A whole of life Insurance policy has an unlimited term. The insurer will pay the benefit whenever the customer dies, no matter what age.
- If the life table shows that everyone is dead by age $w + 1$, then the highest non-zero value of d_{x+t} occurs when $x + t = w$, i.e. when $t = w - x$.
- The formula for the present value of a Whole of Life Policy with sum insured of \$1 is:

$$A_x = \sum_{t=0}^{w-x} \frac{d_{x+t}}{l_x} * (1+i)^{-(t+1)}$$

<u>Further Investment Risk – 17/4/20</u>

The Shape of the Distribution

- Suppose we are investing \$100 in a portfolio of assets.
- In order to calculate the probability of getting a wealth below or above a certain number, we need to know the distribution of the portfolio returns.

Case 1: Sum of Normal Random Variables

- If returns on all the individual assets in the portfolio are independent normally distributed random variables, then the portfolio will also be **normally distributed**.
- This is because the sum of independent normal random variables is also a normal random variable.

EXAMPLE:

- The annual return on Asset A is normally distributed with expected value 10% and standard deviation 12%.
- The annual return on Asset B is normally distributed with expected value 8% and standard deviation 9%.
- If an investor invests \$80 into Asset A and \$50 into Asset B, what is the probability that he will have more than \$150 at the end of the year.
    - His wealth at the end of the year will be:
        - ⇨ $W = 80 * (1 + I_A) + 50 * (1 + I_B)$
    - Taking the expected value of both sides:
        - ⇨ $E[W] = 80 * 1.10 + 50 * 1.08 = \mathbf{142}$
    - Taking the Variance on both sides:
        - ⇨ $Var[W] = 80\text{^}2 * 0.12\text{^}2 + 50\text{^}2 * 0.09\text{^}2 = \mathbf{112.41}$
    - Standard Deviation of W = **10.60**
    - His wealth at the end of the year has an expected value of \$142 and a standard deviation of \$10.60.
    - Since we have been told that the returns on the underlying assets are normally distributed, the wealth at the end of the year is also normally distributed:
        - ⇨ $\Pr(W > 150) = 1 - \Pr(W < 150)$
        - ⇨ $= 1 - Norm.Dist(150, 142, 10.60236, 1)$
        - ⇨ $= \mathbf{22.53}\%$

Case 2: When the CLT applies

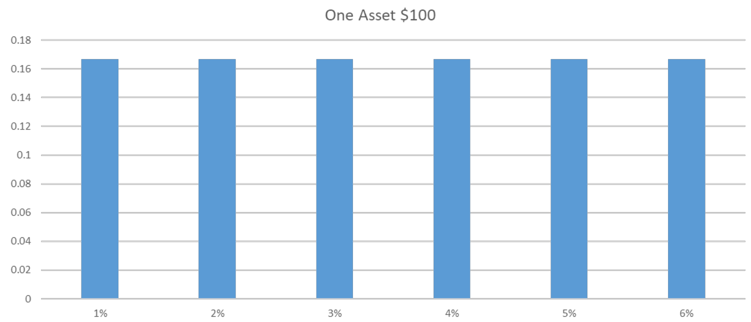
- If:
    - We invest in a large number of assets (*n*)
    - Returns (I_j) on all the assets are independent and identically distributed
    - And we invest equal amounts into each asset
- then the portfolio return I_p will approach the normal distribution.
- This is a result of the **Central Limit Theorem**

EXAMPLE:

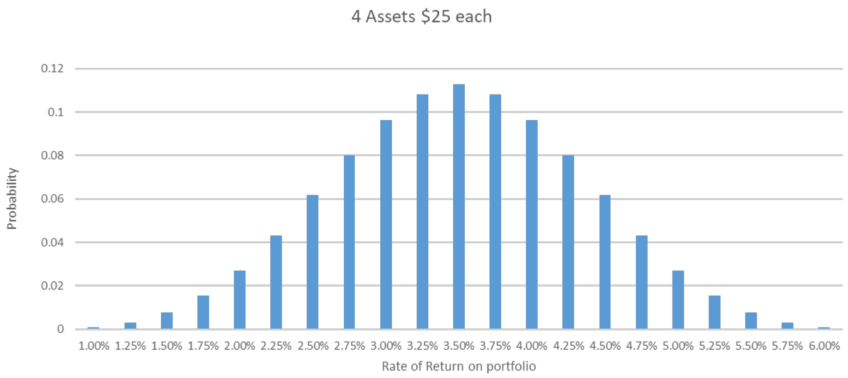
- Suppose that there are a lot of IID assets available for investment (Assets 1 to n)
- All their returns are independent:

I_j	$Prob(I_j = i)$
1%	1/6
2%	1/6
3%	1/6
4%	1/6
5%	1/6
6%	1/6

- Joe has $100 to invest:
  - If he invests his money into ONE of these assets, his distribution will be:



  - If he invests his money into FOUR of these assets equally, the distribution will be:



- As the number of assets increases, the probability distribution becomes more prevalent around the expected value.
- i.e. **it approaches a normal distribution**
- This is true for any underlying distribution with finite variance.
- As a result of the CLT:
  - The wealth arising from each asset's returns are random variables that have identical distributions and are independent
  - The wealth for the entire portfolio is the sum of these n iid random variables.
- According to the CLT, as n increases, the distribution of these n iid random variables will **approach the normal distribution**.

Limitations

- The proof of the CLT assumes that the variance of the n individual components must be finite
- The distribution is only **approximately** Normal, not exactly Normal.
  - The larger the value of n, the closer it comes to a Normal distribution
- This approximation works best if the distribution of the returns on the n individual components is not too skewed.
- If you use the Normal approximation to calculate probabilities, they will be less accurate near the tails of the distribution.

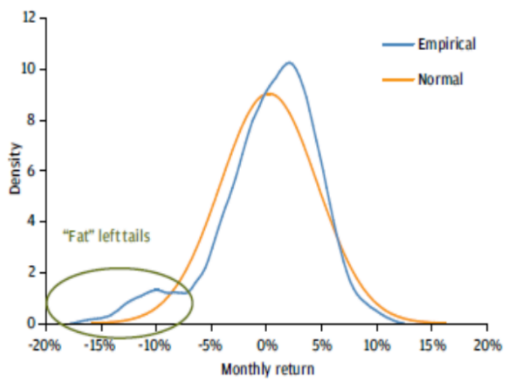
Distributions in Other Situations

- The two cases we just looked at were:
    - **Case 1**: Returns on all assets are independent and Normal
    - **Case 2**: Portfolio is equally weighted investment in a large number of assets which have iid distributions
- In other cases, in order to determine the distribution of the portfolio return, we need to know the distributions of all the assets in the portfolio (not just expected values and variances)
- Otherwise, we can use simulations to estimate the distributions.

Distribution of Portfolios in Real Life

- Investors use investment models to assess their risks, and these models often assume that the returns on a portfolio of shares will follow a Normal Distribution.
- As we have seen, the Normality assumption is reasonable in the cases we looked at above. However, in real life, portfolios may not meet these conditions.
    - Assets returns are not independent
    - Amounts invested in each asset are not equal
    - Returns on underlying assets may not be normally distributed
- So, in real life, the annual returns on a diversified portfolio of shares are NOT normally distributed
- For example, the actual monthly returns on the stock markets have "fat tails" at the left-hand end of the distribution (i.e. too many negative returns)

EXHIBIT 1: INTERNATIONAL EQUITIES–"FAT" LEFT TAILS IN HISTORICAL RETURNS



Source: J.P. Morgan Asset Management. For illustrative purposes only.

Model Assumptions

- Despite these limitations, many investment models still assume that returns are normally distributed.
- This is a simplifying assumption, which is used because:
    - It makes calculation easier
    - The distribution of returns if often approximately normal, and therefore can give answers that are good enough for certain purposes.

Correlated Returns

- If assets are not independent, then we say that the returns are correlated.
- In this topic, we will look at a simple sort of correlation, i.e. the case where correlation arises because both assets are affected by the same external factors.
    - E.g. shares in two different iron ore mining companies will be correlated because the probability of both companies is affected by the price of iron ore
- The degree of correlation between two shares might be positive or negative, strong or weak, and may vary over time
- Returns could be **perfectly positively** correlated:
    - When Asset 1 goes up, Asset 2 goes up and vice versa
    - If assets are perfectly correlated and you know the return on one asset, then you can accurately predict the return on the second asset.
- Returns could be **partially positively** correlated
    - In practice, it is not always possible to understand exactly what is causing the correlation between different companies – there are too many factors which affect the probabilities
    - Instead, investors examine past data on prices, and use this to estimate the correlations between different pairs of companies
    - Excel has a function *CORREL()* which is useful for estimating correlations between two sets of numbers.
- Returns could also be **negatively** correlated.
    - E.g. an umbrella manufacturer and sunscreen manufacturer
    - Theoretically, negative correlation might occur when two companies are competing against each other to attract customers, in a market of fixed size. If one company gains customers, the other one loses them.
- Most shares in the Australian share market tend to be partially positively correlated, because they are all affected by similar economic factors
    - Most shares go up during a boom, and down during a crash
- Many investors invest in a well-diversified combination of Australian Shares, however, this diversification does not completely eliminated risk, because the shares are correlated.
    - According to historical data, a diversified portfolio of Australian Shares has a SD of around 22% over the period from 1981 to 2012
- Even though a diversified portfolio of n IID assets could achieve lower levels of risk, whenever assets are partially positively correlated, diversification is less effective in managing risk.

The Impact of Correlation on Portfolio Risk

- If you know the expected value of returns, variance of returns and the correlations between assets, you can calculate the expected returns and variances of a portfolio of those assets.
- These relationships can be complicated and vary over time (which means past data is not so useful in predicted future data)
- In this unit we only consider the simple case that correlation is caused by one external factor which affects both companies.

EXAMPLE:

- Suppose two companies have the following return distributions:

	Return on A	Return on B
Expected Return	10%	10%
Standard Deviation of return	14%	12%

- If the two companies have independent returns and you invest $50 into each, what is the expected value and standard deviation of your wealth at the end of the year?
  - E(W) = $110
  - Var(W) = $85
  - SD = $9.22
- Now suppose that the two companies are both in the gold mining industry. For both companies, the profitability depends on the price of Gold in the international markets at the end of the year. Let *G* denote the rate of return on an investment in gold, where E(G) = 10% and standard deviation of *G* is 20%
  - Suppose that the return on Company A is given by R_A = 0.70G + 0.03
    - ⇨ $E(R_A)$ = E[0.70 * G + 0.03]
    - ⇨ = 0.70 E[G] + 0.03
    - ⇨ = 0.10
    - ⇨ $Var(R_A)$ = Var[0.70*G+0.03]
    - ⇨ = 0.70^2 * Var(G) = 0.70^2 * 0.20^2
    - ⇨ = 0.0196
    - ⇨ $StDev(R_A)$ = 0.14
  - Similarly, if the Return on Company B is R_B where R_B= 0.60G + 0.04
    - ⇨ $E(R_B)$ = 0.10
    - ⇨ $StDev(R_B)$ = 0.12
- What is the expected return and variance of your wealth at the end of the year if you invest $50 in each company?
  - ⇨ W = 50 * (1+RA) + 50 * (1+RB) = 50 * (1 + 0.70G + 0.03) + 50 * (1+0.60G+0.04)
  - ⇨ = 100 * (1+0.65G + 0.035)
  - ⇨ E(W) = E{100 * (1+0.65G + 0.035)} = 100 * {1+0.65E(G) + 0.035}
  - ⇨ = 110
  - ⇨ Var(W) = Var{100 * (1+0.65G + 0.035)} = Var{100 * 65G+3.5} = 65^2 * Var(G)
  - ⇨ = 65^2 * 0.20^2
  - ⇨ = 169
- Note that when we allow for the correlation between the two assets, the standard deviation of the portfolio is actually 13 compared to the standard deviation before which was just 9.22.
- That is, if we ignored correlation, we would have seriously underestimated the true risk.

- If you have a lot of different assets, then there would be different correlations between each pair of assets, so you would need to know all these correlations to calculate the risk.
- In the past, investors have often seriously underestimated the correlations between assets, and this has caused a lot of trouble in the past.
  - An investment bank creates a portfolio which contains a large number of high risk assets.
  - They tell people that the assets have low correlation.

    - People BELIEVE that they are investing in a well-diversified portfolio which has low risks but if the assets are really highly correlated, then the risks of the portfolio may be much higher than expected.
    - This might lead investors into inadvertently taking on too much risk (e.g. by making too many risky loans)

CASE STUDY: Subprime Debt Markets

- During the period 2001 to 2007, American banks made a lot of home loans to subprime borrowers
    - Subprime borrower: person who had a bad credit rating and hence a high risk of defaulting on their loan repayments
- The banks charged a high interest rate to these subprime borrowers and made millions of these loans, all across the country.
- They knew that each individual loan was risky, but they assumed that diversification would reduce the overall risk of the portfolio.
- The following is a simplified description of how this works:
    - An investment bank would create a Special Purpose Vehicle (SPV), which is basically an investment fund.
    - The SPV would raise money by issuing bonds to investors (often these investors were superannuation funds). These were called Residential Mortgage Backed Securities (RMBS).
    - This money would be used to make loans to thousands of sub-prime borrowers across the country.
    - The sub-prime borrowers would make repayments on their loans and the money would go back to the SPVs.
    - The SPVs would then pay interest to the bond-holders (e.g. the superannuation funds).
- The subprime borrowers would pay high interest rates on their loans, so the investors who purchased RMBS's could earn high returns.
- Of course, if many of the home loan borrowers defaulted, then the SPV might not have enough money to pay the bond-holders the amounts they expected to receive.
- The bondholders knew that some customers would default, but they assumed that:
    - The high interest rates charged on the subprime loans would be enough profit to make up for the occasional default.
    - It was considered unlikely that there would be a large number of defaults at the same time.
- Ratings agencies used statistical methods to estimate the probability distribution of losses on these portfolios.
    - They assumed that the correlation between the loans would be low, hence diversification would be reasonably effective in reducing risk.
    - Therefore, portfolios of these loans were given high credit ratings
    - The investors relied on the advice provided by the credit rating agencies, believed that the risk of loss was low, and therefore invested in the SPVs.
- The correlation assumptions used by the Rating Agency models was based on historical experience for prime loans (i.e. loans to good customers):
    - Loans would only default if someone died or became ill or lost their job
    - These were random events which were uncorrelated

- It turned out that the ratings agencies were wrong, and the correlation between the subprime loans was much higher than expected.
- The historical data used to calculate the correlations had significantly different conditions in which default rates were low and not highly correlated.
- However, when interest rates increased, default rates increased, and a lot of borrowers all defaulted at the same time.
  - Subprime loans are loans made people who are on low incomes and/or have a past history of having difficulty paying loans. These borrowers are much more vulnerable to economic risks.
- **Result**: Total losses were much larger than expected, and these problems led to the GFC.

Leverage

- Some people are willing to take on more risk in order to produce higher expected returns
- Leveraging is the process of borrowing money to invest (aka **gearing up**)
- Leveraging magnifies potential profits, but also magnifies potential losses (i.e. it increase risk)
- The basic formula for leverage is:
  - Initial wealth at time 0 is w_0 (known)
  - Borrow *$L* at interest rate *j* for one year
  - Invest $(w_0 + L)$ in a risky asset with return I_R
  - Repay loan with interest at end of year

$$W = (w_0 + L) * (1 + I_R) - L * (1 + j)$$

- One of the crucial factors in assessing leverage statistics is the interest rate on borrowed funds
- If the interest is **less than** that expected return on the portfolio, expected returns on the leverage portfolio are higher, but the risk is also much higher.
- If the interest is **more than** that expected return on the portfolio, expected returns on the leverage portfolio are lower, and the risk is also much higher.

EXAMPLE:

- Suppose you have $100 and you can invest it in an asset that will return either:
  - 0% with probability 0.5
  - 10% with probability 0.5
- Possible outcome $100 and $110.
- You can decide to take on more risk by borrowing $900 at 4% p.a. and then investing all $1000 into the same asset.
  - $W = 1000(1 + I_1) - 900 * (1.04)$
- If the investment return is 0%, you make a loss of 36%
- If the investment return is 10%, you make a profit of 64%

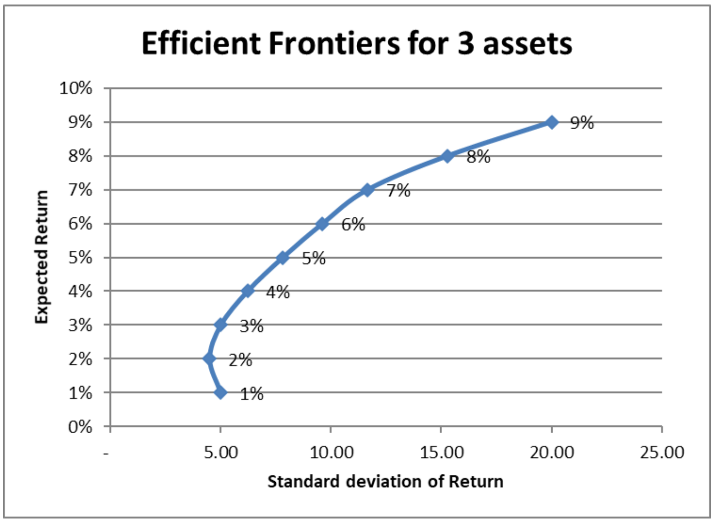
- Some financial advisors tell investors to invest in highly leveraged portfolios without explaining the possible losses.
- Reasons they do this are:
  - **Incompetency**: the financial advisors don't fully understand the risks themselves
  - **Unethical**: they might make a higher commission off this advice, even though they know it is unsuitable for the customer.

Creating a set of Efficient Portfolios

- Suppose that we have three assets available (more than two), with expected returns and variances as shown below (assume independent):

	Asset A	Asset B	Asset C
Expected Return	1%	5%	9%
Standard deviation	5%	12%	20%

- An investor wishes to aim for a portfolio with an expected return of 5% p.a., but with the lowest possible standard deviation. How much should be invested in each asset?
- Let the amount invested in Asset 1 be *x*, the amount in Asset 2 by *y*, and the amount invested in Asset 3 be $(1 - x - y)$.
- Our conditions require that:
  - $x + y + z = 1$
  - Expected return *E(W) = 5%*
  - Standard deviation of Return is minimized
- You can solve this algebraically (using 3 simultaneous equations) or use Excel *solver*.
- **Mean-Variance Efficient Portfolio**: Portfolio which has the highest expected return for a given level of risk, where risk is measured by standard deviation.
  - The MV efficient portfolio is defined for a specified set of risky assets with known expected returns, variance and correlations.
- **Efficient Frontier**: the set of all the mean-variance efficient portfolios (usually displayed as a line on a risk-return diagram)



- The efficient frontier is the part of the line which is above the turning point. E.g. the point labelled 1% is not efficient, because there is a portfolio with same SD but a higher expected return.
- A larger number of assets that are not independent makes it more difficult to determine the efficient frontier.

<u>Variable Investment Returns – 17/4/20</u>

Variable Investment Returns

- So far, we have calculated Accumulated values in which the investment rates each year are known.
- In reality, investment returns are likely to vary from year to year so we will now consider accumulated values in which investment returns each year are random variables

Annual Portfolio Rates of Return

- Suppose I_P is the rate of return in just one year for a portfolio consisting of a combination of assets.
- We can take these one year rates of return and apply them to work out the accumulated value on long term investments.
- Assume that the investment return each year is a random variable:
    - Let I_t denote the investment return in year t (t = 1,2...,n)
- Then the accumulated value of our investment at time n is **also** a random variable.
    - Let ***S* denote the accumulated value of the investment** (S is a random variable)
- In order to assess the riskiness of our investment, we need to know the distribution of the random variable S.
    - i.e. we need to know all the possible values of S and the probability of each outcome $P(S = s)$

The distribution of *S*

- Suppose that we know the distribution of I_t.
- There are three approaches to finding the distribution of S.
    - Find the exact distribution of S by listing each possibility
    - Estimate the distribution of S using computer simulation
    - Find the exact or approximate distribution of S using advanced computational and statistical theory

Method 1: Exact Distribution of *S*

- We can find the exact distribution of S by listing all possible outcomes.
- This is feasible if I_t is a discrete random variable.

<u>EXAMPLE:</u>

- One cash flow of $1000 at time 0. The money is invested for 5 years.
- The investment return in any year is a binary random variable (only 2 possible values).
- Denote the investment returns as I_t (for t = 1,2,3,4,5)
- The investment returns in each year are independent and identically distributed with the discrete distribution:

Value of I_t	Probability
4%	0.40
6%	0.60

    - Note: in this case, the order of interest rates does not matter, so we only need to know how many years had either 4% or 6% interest.

- There are 6 possible outcomes for the accumulated value:

	Possible Outcomes	Accum.
4% every year	1000 * 1.04^5	1216.65
4% in four years, 6% in one year	1000 * 1.04^4 * 1.06	1240.05
4% in three years, 6% in two years	1000 * 1.04^3 * 1.06^2	1263.90
4% in two years, 6% in three years	1000 * 1.04^2 * 1.06^3	1288.20
4% in one year, 6% in four years	1000 * 1.04^1 * 1.06^4	1312.98
6% every year	1000 * 1.06^5	1338.23

- In order to calculate the probability of each possible outcome, we can use the binomial distribution.
    - Let n be the number of years (n = 5)
    - Let p be the probability that the interest rate is 4% in any year (p = 0.4)
    - Let X be the number of years where the interest rate is 4%, i.e. $X \sim B(5,0.4)$
- Hence, the distribution of S is:

Years 4%	Years 6%	Accum	Prob
5	0	$ 1,216.65	0.010240
4	1	$ 1,240.05	0.076800
3	2	$ 1,263.90	0.230400
2	3	$ 1,288.20	0.345600
1	4	$ 1,312.98	0.259200
0	5	$ 1,338.23	0.077760
		sum	1

- What is the probability that I will have less than $1300 at the end of 5 years?
    - P($S < 1300$) = 66.3%
- What is the expected value of wealth at the end of 5 years?

Years 4%	Years 6%	s	Pr (S=s)	s * Pr(S=s)
5	0	$ 1,216.65	0.010240	$ 12.46
4	1	$ 1,240.05	0.076800	$ 95.24
3	2	$ 1,263.90	0.230400	$ 291.20
2	3	$ 1,288.20	0.345600	$ 445.20
1	4	$ 1,312.98	0.259200	$ 340.32
0	5	$ 1,338.23	0.077760	$ 104.06
		sum	1	$ 1,288.48

Theorem about Expected Value

- If we want to calculate E(*S*), we don't need to calculate the whole distribution
- There is a shortcut formula that can be used whenever the investment returns in each year are IID random variables.
  - Let *i* be the expected value of the investment return in each year
  - Let *S* be the random variable which represents the accumulated value at time *n*.
- **Theorem**: E(*S*) is equal to the accumulated value of the cash flows using a fixed investment return of *i*, where $E(I_t) = i$.

Shortcut for P(*S* > *x*)

- Suppose that we have a single cash flow at time 0, and the annual interest rates are IID and binary.
- In order to find the probability that the accumulated value at time *n* exceed some value *x* (i.e. P(*S* > *x*)), we can do the following:

EXAMPLE:

- Interest rate distribution:

Value of I_t	Probability
4%	0.40
6%	0.60

- Let the cash flow at time 0 be \$1000, *n* = 20 years and *x* = \$2500.
- What is P(*S* > 2500)?
  - Let *Y* be the number of years where the return is 6%
    - ⇨ $S = 1000 * 1.06^Y * 1.04^{20-Y}$
  - Work out the minimum value of *Y* which would make the accumulated value *S* exceed \$2500.
    - ⇨ $1000 * 1.06^y * 1.04^{20-y} > 2500$
    - ⇨ $\left(\frac{1.06}{1.04}\right)^y * 1.04^{20} > 2.5$
    - ⇨ $y > \frac{ln\,(1.14097)}{\ln\left(\frac{1.06}{1.04}\right)}$
    - ⇨ $y > 6.9233\ years$
  - Now we know that *S* > 2500 when *Y* ≥ 7, and we know that *Y* is a binomial random variable with parameters *n* = 20 and *p* = 0.6.
  - We can find the required probability using:
    - ⇨ 1-BINOM.DIST(6,20,0.60,1)
    - ⇨ = 0.993534

More Complex Problems

- Situations can become more complex when:
  - There are multiple cash flows
  - There are many possible values for the investment returns
  - The investment returns are no longer IID
  - There is a longer time frame

EXAMPLE:

- Consider the case where there are contributions of $1000 at time t = 0 and $1000 at time t = 1, and we want to know the accumulated value at the end of 2 years.
- Investment returns are equally likely to be either 4%, 6% or 8%.
- Since we have multiple cash flows, the order of investments is now important, so there are 9 equally likely possibilities.

Year 1	Year 2	Accum	Prob
4%	4%	$ 2,121.60	0.111111
4%	6%	$ 2,162.40	0.111111
4%	8%	$ 2,203.20	0.111111
6%	4%	$ 2,142.40	0.111111
6%	6%	$ 2,183.60	0.111111
6%	8%	$ 2,224.80	0.111111
8%	4%	$ 2,163.20	0.111111
8%	6%	$ 2,204.80	0.111111
8%	8%	$ 2,246.40	0.111111
	Exp Value	$ 2,183.60	

- What is the expected value of the accumulation S?
  - Firstly, $E(I_t)$ = 6%
  - And then assuming the interest rate is 6% every year, the accumulated value would be:
    - ⇨ 1000*1.06^2 + 1000*1.06 = 2,183.60

- For more complex cases, the number of possible outcomes become exponentially large. It is better to do these complex cases using a computer.
- However, cases can become so complex that not even a computer can process them, so in this case we move onto Method 2.

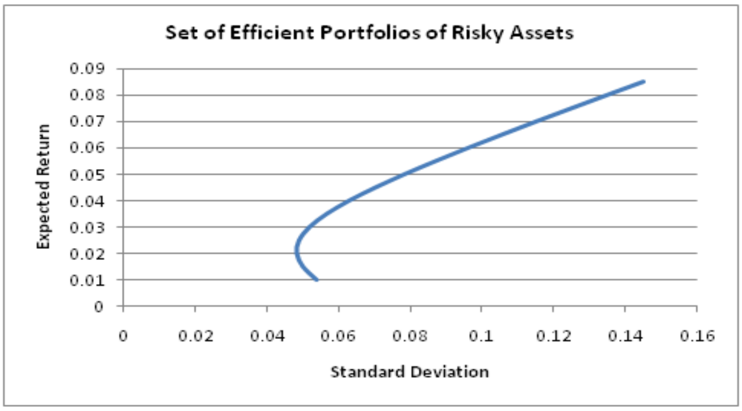
Method 2: Simulation

- When we have more complex cases, or when the interest rate is a continuous random variable, it is often easier to use simulation to estimate the distribution of outcomes and answer questions about risk.
- This requires a computer program such as **Excel** or **R**.
- **Advantages** of Simulation:
  - Can deal with more complex situations
  - Can handle discrete or continuous interest rate distributions
  - Can incorporate economic data into the model
  - Relatively quick and easy to build simulation models
- **Disadvantages** of Simulation:
  - Simulation produces only an estimate of the answer
  - May need to do a lot of simulations to get a reliable estimate (based off confidence intervals)

Method 3: Statistical Models

- In some cases, we can use statistical models to find the distribution of *S*
  - Sometimes we can find the exact distribution
  - Sometimes we can make an estimate of the distribution

Utility Theory – 3/5/20



- This diagram shows the efficient frontier of a combination of risky assets
- We can choose any portfolio on the efficient frontier
  - We could reduce risk by investing some proportion of money into a risk-free asset
  - We could increase risk by using leverage (gearing up)
- This involves assessment of risk-return trade off

Risk Return Trade Offs

- People choose to take on more risk because it generally results in greater average long-term returns.
- Asset classes with higher standard deviation tend to have higher long-term average returns
- Each individual must determine their own risk-return preferences
- Under the code of ethics for Financial Planners, financial advisors are required to consider a client's risk preferences before making recommendations about investments.
  - i.e. advisor should not recommend high risk investments to a risk averse customer
- Financial advisors will often attempt to measure a client's attitude to risk (risk preferences) by giving them a questionnaire.
  - However, it is questionable whether these questionnaires are reliable or accurate.
- Financial advisors usually conduct a **Financial Needs Analysis** (FNA) before making recommendations.
  - The FNA is designed to collect information about a client's financial affairs, e.g. assets, income, debts, typical levels of expenditure, social security benefits, insurance, and so on.
- For example:
  - **Insurance needs**: a person who has dependants might need life insurances, whereas a single person might not.

- **Liquidity Needs**: A person who is retired may need to invest in assets which produce a steady income, whereas a person who is working may be more willing to invest in assets which provide a higher return
  - **Tax Effects**: A person who is a high-income earner (i.e. has a high marginal tax) may be more interested in investments which are taxed at a lower rate or have tax benefits.
  - **Risk Return Preference**: Some people are willing to take more risk for higher returns than others. Retired people are more risk averse because there is no easy way for them to recover from financial losses, whereas younger people have the opportunity to make up for losses in the future.
- Superannuation funds must specify a default investment option – this applies top fund members who do not make deliberate investment choices.
  - In some funds, the default options depends on the age of the customer (called **age-phasing**)

Financial Planners and Risk Preferences

- In the past, there have been several scandals involving poor financial advice, where advisors have recommended high risk investments to risk averse customers in the hopes of a greater commission.
- Many of these clients suffered large losses during the GFC.
- As a result of these scandals, there have been several changes to the legislation to improve the quality of financial advice, and to provide better protection for customers. These include:
  - A ban on conflicted remuneration (e.g. higher commissions to recommend certain products)
  - Stronger educational standards for financial advisors, including ethics training.
  - Stricter supervision of financial advisors by the regulators.

Theoretical Model for Measuring Risk-Return Preferences

- Suppose you have \$100. You can either:
  - Keep the money
  - Pay \$100 to play a gambling game. In this game you toss a fair coin and if it comes up as heads you \$200, if tails, you get \$0.
- The gambling game is called a "fair game" because the expected payoff is \$0.
- Different people will make different decisions about this game, depending on their risk preferences.
  - **Risk averse** people will choose to keep the money
  - **Risk-seeking** people will prefer to play the game.
- If we have a group of risk averse people, we can determine how risk averse they are by changing the expected pay off slightly.
- As we offer a greater pay off, we can tempt more risk averse people into playing the game (i.e. offering a reward for taking the risk)
- By asking people to make such decisions, we can develop a mathematical model of their risk preferences

- The most common model uses **Utility Theory**.

Utility Theory

- Basic idea: If you have a certain level of wealth *w*, this provides a certain level of utility, *U(w)*.
- We can work out the correct shape of the utility function for the investor and use this to work out which risky investment they would prefer.
- In order to determine the most reasonable utility function, we have to make some assumptions:
- **Assumption 1**: (Monotonic Increasing Utility) The more money you have, the greater your utility.
  - People prefer to have more money
  - This means the graph of U(w) against w is upward sloping (first derivative is positive)
- **Assumption 2**: (Decreasing Marginal Utility) The more money you have, the less extra Utility you will receive per extra dollar of wealth
  - This says that an extra dollar is valuable to poor people, but does not provide much extra utility for a wealthy person
  - Mathematically, the curve of U(w) is concave down, so the second derivative is negative.
- **Theorem**: When faced with a choice of risky investments, people should try to maximise their expected utility

EXAMPLE 1: Using Utility theory to make an Investment Decision

- Assume that an investor has $100 in his pocket.
- Let's suppose that the investor has an exponential utility function:

$$U(w) = 200 * (1 - e^{-0.005w})$$

- The investor has a choice about playing a fair game. He can have:
  - $100 for sure
  - $200 or $0 depending on a coin toss
- If he is aiming to maximise his expected utility, which will he choose?
  - The Utility of $100 is 200(1-exp(-0.005*100)) = 78.6939
  - The Utility of $0 is 200(1-exp(-0.005*0)) = 0
  - The Utility of $200 is 200(1-exp(-0.005*200)) = 126.4241
- If we keep $100 and refuse to play the game, our Expected Utility is **78.6939**
  - If we play the game, our Expected Utility
  - = 0.5 * U(0) + 0.5 * U(200) = 0.5 (0) + 0.5 * (126.4241)
  - **= 63.2121**
- We have a higher expected utility if we don't play the same, so we choose not to play.

EXAMPLE 2:

- Assume that an investor has $100 in his pocket.
- Let's suppose that the investor has an exponential utility function:

$$U(w) = 200 * (1 - e^{-0.005w})$$

- The investor has a choice about playing a fair game. He can have:
  - $100 for sure

  - $x or $0 depending on a coin toss
- How much would *x* have to be to entice him to play?
- We already know that the Expected Utility of Option 1 (NOT playing) is 78.6939 and the Expected Utility of Option 2 (Playing the game) **= 0.5 * (0) + 0.5 * 200(1-exp(-0.005x))**
- In order to entice him to play, we would need the Expected Utility of Option 2 (Playing the Game) to be higher than the Expected Utility of Option 1 (Not playing)
  - ⇨ 0.5 * 200(1-exp(-0.005x)) > 78.6939
  - ⇨ x > 309.24
- x must be greater that $309 to maximise the utility function

EXAMPLE 3: Certainty Equivalent

- Let's suppose that the investor has an exponential utility function
$$U(w) = 200 * (1 - e^{-0.005w})$$
- This investor can pay $X to play a game where he wins $200 or $0 depending on a coin toss (fair coin).
- What is the value of $X which would give him the same Expected Utility as playing the game? (This is known as the **Certainty Equivalent** of the game)
  - Note: this solution assumes the investor has exactly $X in his pocket initially
  - The Utility of $X is 200(1-exp(-0.005*X))
    - ⇨ If we play the game, the Expected Utility = 0.5 * U(0) + 0.5 * U(200)
    - ⇨ = 63.2121
  - Setting these to be equal gives
    - ⇨ 200(1-exp(-0.005*X)) = 63.2121
    - ⇨ and solving for x gives
    - ⇨ **X = 75.98** (rounded to nearest cent)
- This implies that the investor is quite risk averse because he would rather have $76 for sure instead of having a 50-50 chance of winning $200 or $0

- Improvement: The solution above is flawed because the investor might initially have more than $100 in his pocket. Suppose he has $100.
- If he refuses to play the game, he will save $100. U(100) = 78.6939
- If he plays the game and pays $X to play (where X < 100), then he will have (100 – X) + 0 if he loses or 200 if he wins.
  - Expected Utility = 0.50 U(100-X) + 0.50 U(200+100-X)
  - = 0.5 * 200[1-exp(-0.005*(100-X)] + 0.5 *200[1-exp(-0.005*(300-X)]
- Setting these to be equal gives, X = 75.98
- In this case, the answer has not changed due to the nature of this particular utility function.
- But for some utility functions, the decisions made will depend on the amount of money the investor has initially.

Insurance Implications of Utility Theory

- Utility theory can be used with any type of risky cash flow (including buying insurance)

EXAMPLE:

- Sherman owns a house worth \$10,000. He is worried that his house might be destroyed by bushfire, flood, or earthquake (and then it will worth \$0). The probability of such an event is 0.20.
- He can buy insurance for \$500. (He will borrow the money to pay the premiums with a 0% loan).
- If his utility function is $U(w) = w - 0.000005w^2$, should he buy insurance?
    - Consider the two possibilities:
    - No insurance: He will have a house worth \$10000 with probability 0.80, but he will have \$0 with probability 0.20
        - ⇨ Expected Utility = 0.80 U(10000) + 0.20 U(0) = 0.80 (10000 - 0.000005 * 10000^2) + 0.20 (0)
        - ⇨ = 7600
    - With insurance: If there is no catastrophe, he will have a house worth \$10,000 and a debt of \$500. If his house burns down or is otherwise destroyed, the insurance company will pay him \$10,000 to rebuild the house and he will still have a debt of \$500. Either way, he will have net assets worth \$9500 overall, with certainty.
        - ⇨ Expected utility = U(9500) = (9500-0.000005 * 9500^2)
        - ⇨ = 9048.75
    - The expected utility is higher if he buys insurance. So, he **should buy the insurance**.

- In general, if customers are risk averse, they will pay premiums which are more than the expected value of the claims cost in order to avoid risk.
- This explains how insurance companies make profits:
    - On average, for a large group of policies, premium income will exceed expected claims payment, hence producing a profit.
- In marketing, if an insurance company can emphasise the severity of the risk, this may help them sell more insurance.

What is the Correct Utility Function?

- So far, all we know is that the utility function U(w) should be slope upwards and be concave down.
- This is not very restrictive (could be log, exponential, power, etc)
- The exact function is still argued, but in practice investors don't usually use this sort of calculation for making investment decisions.
- The utility theory is more often used by economists as it provides a logical framework for decision-making. It has been applied to develop some mathematical models for valuing assets and managing investment portfolios.

Managing Risk and Return – 9/5/20

Management of a Life Insurer

- The first step is to design a product which meets customer needs (annuities, term life, etc)
- The, you need to determine the price to charge for the product (**premium rate**)

Premium Rate

- Calculate the EPV of future payments to determine the premium rate using:

$$EPV = \sum_{t} B_t \, * \, P_t * (1+i)^{-t}$$

- AKA the **Risk Premium** (EPV of Benefit Payments)

Expense and Safety Loadings

- Next step: add the present value of expected future expenses (expense loading). These include
  - Initial costs
  - Annual administration costs
  - Benefit payment costs
  - General overheads
- Each policy holder should pay their fair share of expenses (i.e. costs are included in premium rate)
- The company also faces many risks:
- If there is an adverse experience (worse than expected), then the risk premium may not be enough to pay all promised benefits, rendering the company insolvent.
- To reduce risk of insolvency, companies introduce **safety loadings** in the premiums.
  - Premiums are higher so that they are very likely to be enough to pay claims

Capital

- Another way to reduce insolvency risk is to hold capital (extra money)
- Capital can come from the company's shareholders:
  - Company issues shares to investors
  - Money is raised from investors buying these shares
- Capital can come from retained profits:
  - Company makes profits in previous years
  - Part of these profits pay dividends
  - Part of the profits are retained to build up company strength

Profit Loading

- In order to pay dividends to shareholders, the company may introduce **profit loading** into premium prices (i.e. raising premium rates)
- However, profit loading and safety loading do overlap.
- If the experience is better than expected:
  - Safety loadings are not needed to pay claims, so they contribute to profits
- If experience is worse than expected:
  - Safety loadings were used up in paying higher-than-expected claims; hence profits were lower.

Office Premium

- Office Premium: Premium rate actually charged by the insurer
  - *Office Premium = Risk Premium + Expenses Loading + Safety/Profit Loading*
- In order to set office premiums, we must first understand company objectives.

Insurance Company Objectives

- <u>Maintain Solvency</u>
  - APRA requires very high level of solvency (99.5% chance of remaining solvent)
  - If company does not meet this standard, APRA can prevent them from selling products
- <u>Satisfy the Shareholders</u>
  - Shareholders expect a return on their investments. What they expect depends on the Risk-Return trade off.

Building a Model

- Managing risks involves building a model of the company's income and outgo (including random variables such as mortality, investments, expenses)
- This is an example using a very simplified scenario:

- A company sells one-year-term group life insurance policies.
- There are n members of the fund, all with sum insured S. All the fund members are aged x with mortality rate q.
- We charge an office premium of P per dollar of sum insured.
  - *Office Premium = P*S per person.*
  - Note: ignore income and expenses
- The company has initial capital of C.
- **Assumptions**: $n = 100$, $q = 0.04$
- <u>Profit formula</u>
  - *Profits = Income – Outgo*
  - *Profits = (Premiums + Investment Income) – (Claims + Expenses)*
- <u>Solvency Formula</u>
  - Solvent if Capital at end of year > 0
  - *Capital at end of year = Capital at start + Profits*
- <u>Return on Equity</u>
  - *ROE = Profits/Capital at Start of Year*
- We only consider **Mortality Risk** in this example, and we model it using the Binomial distribution:
  - D = number of deaths
  - $D \sim B(n,q)$
- Total claims cost = $S * D$

Calculating Profits

- *Profit = Premium Income – Death Benefits =* $(n * P * S) - (D * S)$
  - Note: this is simplified

- So, profit is a random variable which depends on number of deaths *D*
- Therefore:
    - *E[Profit] = E[n* P * S – S * D]*
      *= n * P * S – S * E[D]* *(but E[D] = nq)*
      *= 100 * 5,000 – 100,000 * 4*
      = 100,000
- What is the probability that profits will exceed $150,000?
    - *P(Profits > 150,000)*
      *= Pr(100*5000 – 100,000 D > 150,000) = Pr(D < (150,000 – 500,000)/-100,000)*
      = Pr(D < 3.5) = Pr (D <= 3)
      = BINOM.DIST(3, 100, 0.04, 1)
      = 0.42948

Impact of Premium Increases

- Of course, shareholders would like to have higher expected profits, so why not increase premium rates?
- It is simply because this could cause a loss in customers which will end up lowering overall profits.
- In order to work out the premium rate which will produce the highest expected profits, we need to know the **demand function** (relationship between premium rate and number of policies).

- In this example, suppose that the demand function is
$$n = 350 - 5000\,P$$
- This says that as the premium goes up, number of policies sold goes down.
- In order to maximise expected profits, incorporate the demand function:
    - Expected Profit = n * P * S – n *q * S
      = n * (P-q) * S
      replace **n** with n = 350-5000P
      Expected Profit = (350-5000P) (P-q) S
- Then take the first derivative with respect to P and solve for max P.
- This gives a maximum EP of $112,500

Practical Difficulties with Maximising Profits

- It is quite hard to determine in advance how demand will change according to premium rates.
- The demand if also impacted by competitor premium rates
- The demand doesn't depend solely on price, by also marketing (sales team won't be happy with premium increases)

- So far, our model has not included expenses and investment income. Profits may also arise if:
    - *Actual expenses < expense loading*
    - *Investment income > investment assumptions*

Solvency Objectives

- Other than profit objectives, we also want to look at solvency objectives:
  - i.e. What is the risk that the insurer will become insolvent due to too many deaths?
- Insolvency occurs when the capital at the end of the year is less than 0.
- This can be interpreted as when:
  - *Claims > Capital + Premiums + Investment Income - Expenses*
- **Probability of Ruin** = Pr (Capital at end of year < 0)
- **Probability of Sufficiency** = 1 – Pr(Ruin)

EXAMPLE:

- If the company has:
  - Initial Capital C = $220,000 and Premium = $4,500 for a sum insured of $100,000,
  - n = 100 customers
  - q = 0.04
  - Expected profits = 50,000
- *What is the probability of ruin?*
  - Probability of ruin =

    = Pr(Death Benefits > Amount Available to Pay)

    = Pr(100,000 D > 220,000 + 100 * 4500) = Pr(D > 6.7) = 1- Pr(D<=6)

    = 1 – BINOM.DIST(6,100,0.04,1)

    = 10.639%
- Probability of sufficiency = 89.361%

Probability of Sufficiency

- Insurance regulator APRA will not allow insurers to sell policies unless they have a **very high** probability of being able to pay claims (*99.5%*)
- So how can we increase POS?

D~Binomial (100,0.04)		
d	P(D=d)	P(D<=d)
0	0.01687	0.01687
1	0.070293	0.087163
2	0.144979	0.232143
3	0.197333	0.429476
4	0.199388	0.628864
5	0.159511	0.788375
6	0.105233	0.893608
7	0.05888	0.952488
8	0.02852	0.981008
9	0.012147	0.993156
10	0.004606	0.997761

- Looking at the cumulative probability column, find the first number that exceeds 99%.
- We see that: Pr(D ≤ 9) = 99.316%
- So if we have enough money to pay 9 claims, we will meet the requirement.

Strategies for Increasing POS

Strategy 1: Increasing Capital

- We need enough to pay 9 claims of \$100,000 each (i.e. \$900,000)
- We will have premium income of \$450,000 so the shareholders will have to provide a total capital of \$450,000. This can be done by **issuing more shares**.
- Otherwise, capital can be increased through **retained profits** from prior years (keep some profits to build financial strength)
- Advantages of increasing capital
  - Improves POS. Lower risk may attract more customers.
  - It will also ensure compliance with APRA
- Disadvantages:
  - Shareholders get a lower return on their investments which will make them unhappy (calculated using *ROE = Profits/Capital*)

Strategy 2: Increase Premiums

- In order to determine the premium rate required to have a POS of 99%, we require:
  - Premium income + capital ≥ 900,000
  - Premium income + 220,000 ≥ 900,000
  - Premium income ≥ 680,000
- Or \$6,800 per policy (this is a large increase from \$4500)
- Disadvantages:
  - Likely to lose customers, resulting in lower profits

Strategy 3: Change Policy Design

- We can use a "**With Profit**" policy.
  - Charge customers higher premiums, but promise them that if the company makes a profit, they will share the profits with the customer
  - Alternative: insurer gives a discount next year rather than sharing profits.
- In the example, the insurer charges a high premium but promises to pay 25% of the profits to customers at the end of the year (only if profits > 0)
- Same as before:
  - S = 100,000, n = 100, q = 0.04
  - Insurer increases capital to \$360,000
  - Charges a premium of \$5,500 per policy. But gives a profit share of 25% of profits
- *What is the probability of ruin, expected profit and expected return on capital?*
  - P(ruin) = Pr(Claims > Capital + premiums)
    = Pr(100,000D > 360,000+550,000)
    = Pr(D > 9.1) = 1 – Pr (D ≤ 9)
    = 1 – Binom.dist(9,100,0.04,1) = 1- 0.993
    = **0.7%**
  - Easiest way to calculate EP is to calculate EP ignoring the profit share, and then deduct the expected profit share.
  - Expected Profit ignoring profit share
    = *premium income – expected claims*
    = 5500 * 100 – 100,000 * 100*0.04
    = 150,000

- Expected value of Profit Shares:

Outcome Deaths	Profits	Profit Share	Probability
0	$ 550,000	$ 137,500	0.016870319
1	$ 450,000	$ 112,500	0.070292997
2	$ 350,000	$ 87,500	0.144979307
3	$ 250,000	$ 62,500	0.197332946
4	$ 150,000	$ 37,500	0.199388497
5	$ 50,000	$ 12,500	0.159510798
6 or more	negative	0	0.211625136
	expected value	$ 44,717.58	

- Expected net profits for insurer
  = 150,000 – 44,717.58
  = **105,282**
- Expected Return on capital = 105,282/360,000 = **29.2%**

- Advantages of "With Profit":
  - Reduces probability of market ruin
  - Possible marketing advantages
- Customer bears some of the mortality risk. Some super funds like this, and some don't (depends on the mortality risk itself)

Strategy 4: Changing Sales

- If POR is too high, we can potentially change the number and size of policies.
- What would happen if we doubled the number of policies sold, while cutting the sum insured in half?
  - A company which sells a large number of small policies will have lower risk than a company which sells a small number of large policies (found by calculating SD)
  - This illustrates the benefits of diversification
- However, it is not reasonable or easy to increases sales like that in a competitive market where competitors are trying to do the same.
- It can be done by setting extremely low premium rates, spending a lot on marketing or relaxing underwriting standards. But this is completely UNREASONABLE as it will not improve profitability or solvency!

- Regarding the strategy of reducing the sum insured, it is typically not practical to do so.
- Usually an insurer will not want to reduce the sum insured even though historically they have observed that it is risky to sell policies with high sums insured.
  - This can be solved with *reinsurance* (helps maintain a low POR)
- By increasing the number of policies sold, we can improve expected profits, improve diversification, and spread fixed costs over a larger number of policies.
- However, a company that sells a large number of policies will need to hold additional capital to reduce the POR.
- Problems may arise if a fast-growing company does not increase its capital to match the additional risk (these companies have higher POR)
- One flaw in this model arises from the assumption that all the policies are independent (meaning that diversification is beneficial)
  - But if risks are not independent, then selling more policies may increase our risk more than expected.

Strategy 5: Reinsurance

- A company can transfer some of its risk to a reinsurance company (i.e. the insurance company buys insurance)
- There are many types of reinsurance including:
    - Quota share (co-insurance)
    - Stop loss
- **A. Quota Share**:
    - Under a quote share arrangement, the reinsurer takes *x*% of the premium income and pays *x*% of each claim.

EXAMPLE:

- Consider an insurer with:
    - C = 220,000, n = 100, q = 0.04, Premiums = 4500, With quota share reinsurance arrangement where x = 40%.
- What is the POS?

  Premium income = 100*4500
  LESS payment to reinsurer = 40% * 450,000
  NET premium income = 270,000

  Claims cost = 100,000D
  LESS claims paid by reinsurer = 40,000 * D
  NET claims cost = 60,000 D

    - Ruined if *net claims* are more than *initial capital + net premium income*
    - Probability of Ruin = P(60000D > 220,000+270,000)
      = P(D >8.167) [Where D~Bi(100,0.04)]
      = 1- Pr(D<=8) = 1- BINOMDIST(8,100,0.04,1) = 1-0.981
      = 1.9%
- Note: POR without reinsurance was 10.64% so this is a big improvement.
- However, reinsurance has the drawback of reducing your expected profits.
    - *Expected [Net Profit]* = Net Premium Income – Expected Net claims
      = 270,000 – 60,000 E(D) = 270,000 – 60,000 * 100 * 0.04
      = 30,000
    - E[Profits] was $50,000 without reinsurance

- **B. Stop Loss Reinsurance**
    - Insurer is willing to pay the first *x* claims, and then the reinsurer agrees to pay all subsequent claims.

EXAMPLE:

- Consider an insurer with:
    - C = 220,000, n = 100, q = 0.04, Premiums = 4500.
- The insurer buys a reinsurance policy whereby the reinsure pays all claims after the 6th claim. The reinsurer charges a premium of $40,000.
- What is the new POS and Expected profit?
    - Net premium income = 100*4500 – 40,000 = $410,000

- Expected net claims cost = $381,711

d	Insurer Pays	Probability
0	_	0.016870319
1	100,000	0.070292997
2	200,000	0.144979307
3	300,000	0.197332946
4	400,000	0.199388497
5	500,000	0.159510798
6 or more	600,000	0.211625136
Exp. Value	381710.924	

  - E[Net Profit] = 410,000 – 381,711 = 28,289
- Finding the POS:
  - Initial Capital + Premium Income – Reinsurance Premium
    = 220,000 + 450,000 – 40,000
    = $710,000
- Net amount available to pay claims is $710,000. Since at worst we only have to pay 6 claims, our POS is not 100%.

- Stop loss passes a lot of risk to the reinsurer.
- The expected profit for the reinsurer is given by:
  - Reinsurance premium income = 40,000
  - Expected claims cost for insurer = *total expected claims cost – insurers expected claims cost* = 400,000 – 381,711 = 18,289
  - Reinsurer expected profit = **$21,711**
- The reinsurer is collecting just $40,000, but if they have to pay even one claim of $100,000 they will make a loss.
- But the reinsurer diversifies the risk by selling reinsurance policies to many different insurers.

Reinsurance Risk

- Reinsurance can allow us to get a high POS, but a couple of things can go wrong.
  - Reinsurer goes broke.
  - Reinsurer disputes the claim
- In the second scenario, insurance companies can protect themselves by:
  - Always buying reinsurance from reputable companies with good credit rating
  - Having a legal team check the contract to make sure it is unambiguous
  - Making sure they provide accurate information to the reinsurer

Strategy 6: Reduce Death Claims

- Death claims can be reduced through:
    - Stricter underwriting standards. (i.e. try to avoid insuring unhealthy people/high risk people at normal premium rates)
    - Stricter claims management. (sensible strategy if the insurer is paying claims which should not be paid.
        - This strategy could be too strict (unhappy customers)

Business Decisions

- In order to meet objectives, the company must make business decisions about:
    - Premium rates to charge
    - The level of capital needed
    - Number of dividends to pay to shareholders
    - Number of policies to sell (sales targets)
    - Policy design (with-profits? Exclusions?)
    - Underwriting standards
    - Claims management standards (disability)
    - Amount and type of reinsurance to buy

<u>The Role of the Actuary</u>

Historical Perspective

- Friendly societies:
    - A form of life insurance common in the early days.
    - Society collected money from the members and used it to pay sickness and death benefits to them (generally small amounts)
    - Friendly societies were *mutuals* (i.e. non-profit) and they did not have shareholders. All profits belonged to the members.
    - Some of these societies were poorly managed in terms of premium rates, and so the government passed a law in 1819 saying that friendly societies had to have premium rates which were approved by an actuary.
    - The first "official" role for actuaries. However at the time, there were no legal qualifications to become an actuary.
- Life Insurers:
    - Life insurers provided much larger sums insured than Friendly Societies.
    - The first successful life insurer in the UK was the Equitable (est. 1762). They appointed one of their employees to be the actuary, and this actuary was responsible for:
        - <u>Experience Analysis</u>: Checking whether life tables accurately corresponded with actual numbers of deaths.
        - <u>Valuation</u>: Checking to see whether the assets of the insurer would be enough to pay future expected death benefits.

    - Surplus: Since the Equitable was charging premiums which were too high, and it had quite good investment returns. The excess of the Assets over Liabilities was called the Surplus. The actuary had to make a recommendation about what to do with the Surplus (usually recommended that it be retained).
    - Profit Share (bonuses): Some of the surplus was also used to distribute to the policy holders. The actuary was given the responsibility of working out how to evenly distribute this money to the policyholders.
    - Surrender Values: If a policyholder decides to cancel their long-term policy, the amount payable on the policy at the time of cancellation is called the surrender value. Actuaries were required to calculate this value.
- At first, actuaries did not have legal responsibilities to the public, and were just employees to life insurers.
- The British system was based on *"freedom with publicity*" – insurers could run their business any way they liked, as long as they provided financial statements.
- In 1870 the UK government passed a law which required life insurers to provide financial statements which included the Actuarial Valuation of the Liabilities and the value of the insurer's Assets. The policyholders could then make their own assessment of the insurer's solvency, before deciding whether or not to buy a policy from a particular insurer.
- This made the reputation of the actuary very important as trustworthy actuaries made their employing life insurers more attractive.
- Eventually the "*freedom with publicity"* system didn't work, so in 1945 the Australian government passed the Life Insurance Act 1945, which set minimum solvency standards for life insurers.
- The Life Insurance Act was updated in 1955, requiring actuaries to help maintain a financially strong and healthy life insurance industry. They now have responsibilities to the public as well.

Life Insurance Act 1955

- The Life Insurance Act 1995 sets out the responsibilities or the Actuary.
- The Act allows the government regulator, the Australian Prudential Regulation Authority (APRA) to make rules relating to the management of insurance companies.
- Each life insurance company must have an **Appointed Actuary.** The Appointed Actuary must meet certain standards and qualifications.

Role of the Appointed Actuary

1. New Policies: A life insurance company must not issue any new policies unless the AA has given a written report on the terms and conditions of the policy.
2. Modification of Policies: A life insurance company must not modify the terms and conditions of any policies unless the AA has given a written report on the proposed changes.
3. Reinsurance: A life insurance company must not buy a reinsurance policy, change a reinsurance policy, or cancel a reinsurance policy unless they get a written report from the AA explaining the consequences.

4. Valuation of Policy Liabilities: The AA must value the policy liabilities of the insurance company.
5. Capital: The legislation requires each life insurer to maintain adequate amounts of capital. The actuary must set the amount of capital after considering all the different types of risks the insurance company faces. APRA has a set of long and complicated rules for working out the amount of capital required.
6. Payment of Profits to Shareholders/Policyholders: The insurance company must not pay profits to the shareholders or policyholders unless they have received advice from the Appointed Actuary about the likely consequences of such a payment.
7. Allocation of Expenses: The company must obtain the appointed actuary's advice about the allocation of expenses.
8. Financial Condition Report: Every year, the Actuary must make a report on the financial condition of the insurance company. A copy is given to the Board of Directors and to the government regulator.
9. Transfers and Amalgamations: If one insurance company wants to merge or takeover another insurance company, the regulator will usually request an actuarial report on the proposed deal.

Powers and Responsibilities of the AA

- Access to information: The AA must be given access to all the information and data needed to do his job properly. The Actuary can talk to any employee, in order to collect the necessary information.
- Access to Board of Directors: The AA should have the right to talk to the Board of Directors in relation to any of his actuarial responsibilities.
- Whistle-blower Responsibilities: Under certain circumstances, the actuary must report to the government regulator, APRA. (e.g. if they are concerned about the life insurer doing something unethical)

Financial Condition Reports (FCRs)

- FCRs are the key tool for the actuarial management of life insurance companies. FCRs basically cover every aspect of the company's business, including:
  - Data
  - Experience Analysis
  - Assets
  - Policy Liabilities
  - Solvency and Capital Adequacy
  - Premium Rates and Charges
  - Reinsurance
  - Risk Management
  - Material Risks

Professional Standards

- The Actuaries Institute is the professional body for Actuaries in Australia. The Actuaries Institute publishes professional standards which set out the requirements for certain tasks.

Issues in the Life Insurance Industry – 22/5/20

- Average return on net assets was about 12% p.a. over the past 10 years.
- But it has been trending downwards to 0nly 3.5% in 2019. *So what is causing this problem*?
- First we will look at products sold to individuals:
    - Individual Lump Sum (Conventional)
    - Individual Lump Sum (YRT)
    - Individual Disability Income
    - Low Value Policies

Product 1: Individual Lump Sum (Conventional)

- These conventional products include: Whole of Life Policies, Term Insurance and Endowment Assurance policies.
- These are long-term products and are causing some issues:
- Problem 1: Legacy Issues
    - Dozens of different products and several policies sold over decades, all of which require administration (IT systems, actuaries, staff, tax, legislation)
- Problem 2: Low Interest Rates
    - Life insurers make assumptions about future interest rates. If the interest rate is lower than expected they will suffer a loss.
    - Interest rates are particularly important for long term policies with level premiums.
    - Due to events such as the GFC and COVID-19, the RBA has set low interest rates in order to boost the economy (insurers are suffering)

Product 2: Individual Lump Sum (YRT)

- Conventional products are not commonly sold anymore. Instead Yearly Renewable Term (YRT) are more commonly sold to individuals.
    - Person pays for a one year policy which may cover death or death + TPD (total permanent disability)
    - It can be automatically renewed at year end.
    - Premiums go up each year in line with age (due to increased risk of death)
- This product has been profitable in recent years, but not as much as insurers would like.
- The most serious risk for these products is **high lapse rates** (customers cancelling policies)
- Lapse rates have been experiencing an upward trend in recent years with rates of up to 15%.
- Reasons why Lapses cause trouble:
    - **Selective Withdrawal** – Healthy people tend to lapse as they don't need insurance or can find better policies elsewhere. Unhealthy customers do need insurance and can struggle to find it elsewhere, which means that as time goes on the remaining customers are less and less healthy. This results in higher mortality rates than expected and a worse claims experience.
    - **New Business Strain** – Policies typically result in a loss for the insurer in the first few years. The insurer hopes that the product will be profitable in future years, and the future profits will outweigh the first year losses. However this means that the product will only be profitable if the customer renews his/her policy for several

years. At present the product will usually reach profitability after about 8 years, which means that high lapse rates can cause many policies to never be profitable.

- What causes Lapses?
    - **Stepped Premiums** – in YRT, the premium increases as you get older, which means that customers tend to doubt whether they need the insurance after every year.
    - **Bad Economic Conditions** – people can't afford it. Lapse rates may go up if there is high unemployment, high debt or low wage rises.
    - **Impact of Compulsory Super** – life insurance is for the purpose of providing for dependents. However, superannuation savings can sometimes be enough to do the same thing.
    - **Direct Marketing/Poor Sales Methods** – about 10% of policies are sold via direct marketing (TV ads, internet, etc). Policies sold by direct marketing have very high lapse rates possibly due to the fact that people are persuaded to buy products they don't need.
    - **Churning** – A lot of individual YRT is sold via financial advisors, and the advisor gets a large commission every time they sell a new policy. As such, they have incentive to persuade customers to switch policies, which leads to lapsation.
    - **Competitive Market** – Competition in the market means that new, better products are constantly being developed. With YRT, people can easily cancel one policy and switch to another.
- Customers find out about new deals through the internet and financial advisors.
    - **Product Cannibalism** - To win market share, the insurer creates a new better product with slightly lower premium rates or better terms and conditions. If customers find out they lapse the old product and switch. But this new product is less profitable than the old one which harms the insurer's profitability.
    - **Inertia Pricing** – Data analytics can be used to identify which customers are most likely to switch. Retention teams contact people who are likely to lapse and offer them better deals.

Product 4: Low Value Policies

- Low Value Policies tend to have high lapse rates, a high proportion of rejected claims and a low payout ratio.
- An example is life insurance sold via car dealers, who provide you with a loan when you buy a new car.

<u>Issues with Group Products</u>

- Most Australians don't buy individual life insurance policies.
- Instead they belong to a superannuation fund which buys insurance for all its members under a group policy.
- The superannuation fund trustees negotiate terms
- Fund members can choose whether they want each type of insurance and how much:
    - Life insurance – pays a lump sum on death
    - TPD – pays a lump sum on total and permanent disability
    - Disability Income – pays a regular income while the fund member is unable to work, until they can return to work.

Default Cover

- When a person starts a new job, they usually join a superannuation fund, and when they join this new fund they will usually be given a certain amount of insurance cover by default.
    - If they do not want the insurance, they must specifically say so (opt out)
- Typically, the default cover is:
    - Life insurance + TPD
    - And maybe Disability insurance Income (DII)
- The cost is usually a few dollars per week and this premium is deducted from the members superannuation account.
- The benefits and premiums also depend on the member's age:
    - Usually benefits decrease as you get older
    - Don't need as much insurance because you already have money in super and your kids are no longer dependent

Automatic Acceptance

- Every super fund member can have insurance at standard premium rates up to the AA limit with NO underwriting.
    - No questions bout medical health, no medical examinations
- This saves time and money for the insurer but increasing the possibility of insuring high risk people.
- Life insurers realise that some of the fund members may not be healthy, but it saves them a lot of time and trouble and on average the overall group will have fairly predictable death and disability rates.
    - Further reasoning: if you have just started a new job you are probably not seriously ill.

Adverse Selection

- However, healthy people might not need insurance and are more likely to opt out or choose the lower amount.
- This increases adverse selection and the average mortality rates go up.
- Sensible precautions for insurers:
    - Don't give too much flexibility (AA limits should not be too high)
    - Monitor experience (check if a lot of people are opting out)

Voluntary Insurance

- A fund member might as for a higher sum insured than the AA limit, or ask for an increase in the sum insured after joining the fund.
- In these circumstances, the life insurance company will usually ask underwriting question.
- If the underwriting reveals extra risk factors, the insurer might then:
    - Charge a higher premium
    - Refuse to insure
    - Impose exclusions
- The insurer charges different premium rates for additional voluntary insurance (above AA limits)
- Each super fund will have different rules for:
    - Amount of default insurance offered
    - Premium rates

    - Terms and conditions of insurance (e.g. DII waiting periods, DII benefit periods, etc)
    - AA Limits

Problems in Group Life Insurance

- Why is the group life business so unprofitable?
- Reason 1: **Economic**
    - Having a disability makes it hard to work in a usual job.
    - This leads to high unemployment, and the customer is more likely to make a TPD claim
- Reason 2: **Mental Health Claims**
    - In recent years there has been a strong upward trend in mental health disability claims
- Reason 3: **More Lawyers**
    - Increased lawyer involvement in the claims process leads to increased awareness of customer entitlements.
- Reason 4: **More Late Notified Claims**
    - There is an increased number of large, late notified claims, many of which are associated with increased lawyer involvement in the claims process.
    - This means that insurers thought they made profit in prior years, but in reality they didn't. Hence, they didn't increase premiums when they should have.
- Reason 5: **Competition**
    - A desire to win the business makes insurers offer a better product at a cheaper price.
    - As such, premium rates are cut, terms and condition become more generous, AA limits are increased, and underwriting is weaker.
    - Super funds have large numbers of members which means the trustees are in an excellent bargaining position.
    - If there is some uncertainty about the correct premium to charge for insurance, this could lead to the Winner's Curse
- **Winner's Curse**: If an insurer is optimistic and sets premium rates too low, they will "win" the business for that year. However, they will actually make a large loss because they are under-charging and as a result they end up "losing" the business the next year.

The Underwriting Cycle

- Many insurance markets have underwriting cycles. They go as follows:
    - Business is profitable
    - Every insurer wants to gain market share
    - Competition leads to price cutting and weaker underwriting
    - Insurers end up making losses
    - Eventually, insurers realise that they are making losses, so they have large premium increases and underwriting become stricter.
    - The industry returns to profitability and the cycle starts again.

Possible Solutions

- APRA suggests the following possible solutions:
- Better Monitoring:

  - APRA suggests that insurers should be monitoring their experience more closely (noticing trends such as mental health claims and lawyer involvement)
  - The sooner insurers notice the problem, the sooner they can fix it.
- Better Data:
  - Insurers typically collect data from the super fund about past experience, and the actuary uses this data to estimate the correct premium.
  - This is usually not very reliable.
  - Thus, APRA encourages super fund trustees to improve their record-keeping.
  - Furthermore, trustees can't be sued for providing inaccurate data because the have a data quality waiver.
- More Influential Actuaries:
  - Theoretically, the appointed actuary should be checking whether the premium is too low. However, management is under a lot of pressure to have the best premium rates and end up discarding the actuary's advice.
  - As such, the role of actuaries should be strengthened.
- Reinsurance:
  - If group life insurers think that there is a risk of high claims, they could buy reinsurance.
  - However, APRA says that some reinsurers are refusing to reinsure this industry because losses had been too high.
  - This leads to problems because insurers might start buying reinsurance from weaker reinsurers who are not as reliable (Some of these are international so APRA cannot even check the reliability)
- Profit Share:
  - Insurers could sell insurance with profit sharing deals (charge a higher premium but promise to give some back if they make a profit)
- Tighter Terms and Conditions:
  - Lower default cover
  - Lower Automatic Acceptance Limits
  - Stricter underwriting
  - Tighter definitions of TPD
- Better Claims Management:
  - Increasing focus on early intervention and rehabilitation;
  - Increasing focus on mental health issues;
  - Increasing staff training;
  - Increasing resourcing in the claims department;
  - Utilising additional specialist medical officers
  - However, there is a risk that insurers might try to unfairly reject legitimate claims.
- Increase Premiums

- Due to regulatory risk, it could get harder for the financial services industry.
- The government noticed that many young people had been "defaulted in" and a lot of them didn't even know they had insurance.
- As a result, a change in legislation occurred which stated that people under 25 will not get insurance by default when they join a super fund.
- They can "opt in" if they want insurance.

- The government also noticed that a lot of people have multiple small super accounts due to different jobs from the past.
- Many of these accounts are no longer getting new contributions and this poses a problem of duplicate insurance for the customers.
- So there are now rules stating that super funds must cancel insurance for small inactive accounts (less than $6000, no new contributions for 16 months)
- The fund member must be informed before cancellation and they can opt in if they still want insurance.

Insurable Interest

- A person who is likely to suffer a pecuniary or economic loss as a result of the death of some other person has an insurable interest in the life of that person.
- The Gambling Act 1774 states that you can't buy insurance on the life of another person unless you have an insurable interest.
- It was decided that a man has an insurable interest in his wife and vice versa.
- This Act was introduced to prevent undesirable situations in which a person had something to gain from the death of another person.

Moral Hazard

- The principle of indemnity states that on the happening of a loss, the insured shall be placed back into the same financial position as they used to occupy before the loss.
- i.e. The insured should get neither more nor less than the actual amount of the loss sustained.
- Moral hazard is created when the existence of an insurance policy gives the policyholder an incentive to behave in a way which makes the insured event more likely to occur.
    - It occurs when the principle of indemnity is broken AND the policyholder has some control over the probability that the insured event occurs.
- Certain types of insurance might be banned if it is not in the public interest.

<u>General Insurance Pricing – 29/5/20</u>

Life Insurance vs General Insurance

- Life insurance can be relatively simpler as it usually just one claim per policy and the amount of the benefit is known in advance.
- General insurance is more difficult because:
    - There might be multiple claims in a year and
    - The size of the claim is not known in advance.

General Insurance Modelling

- Let N be a random variable which represents the number of claims in one policy in one year.
- Let X be a random variable which represents the size of one claim.
- If we know the distributions of N and X and we assume that they are all independent random variables, we can combine them to calculate the distribution of S.

- *S* is a random variable for the total cost of all claims payable for one policy for one year of insurance.

Claim Frequency Distribution

- For some types of general insurance policies, the Poisson distribution is a good fit for the claim frequency distribution.

$$\Pr(N = n) = \frac{e^{-\lambda}\,\lambda^n}{n!}$$

- Note that:
    - $E[N]$ =λ
    - $Var[N]$ =λ
- Poisson probabilities in Excel:
    - $\Pr(N = n) = Poisson.dist(n,\ \lambda, 0)$
    - $\Pr(N \leq n) = Poisson.dist(n,\ \lambda, 1)$

Claim Size Distribution

- Denote the claim size for one claim as the random variable *X*.
- In general insurance, *X* will depend on the amount of financial loss by the policyholder
    - If the amount of benefit paid exactly equals the amount of the loss, the policyholder is fully indemnified against loss.
    - Sometimes the benefit paid is less than the amount of the loss (deductibles, caps on benefits)
- In reality, claim size distributions are usually continuous distributions, but in this topic we use a discrete distribution.
- For example:

x	Pr(X=x)
100	0.5
200	0.3
300	0.2

    - Expected Value of Claim Size = 100*0.50+200*0.30+300*0.20 = $170
- **Assumptions**:
    - Suppose that the policyholder has made multiple claims throughout the year.
    - We usually assume that these random variables are independent and identically distributed

Expected Value of Claims Cost

- The total cost of claims for one policy in one year is denoted by the random variable *S*.
- The distribution of *S* is the combination of *N* and *X*.
- The expected value (called the risk premium) is given by:

$$\boldsymbol{E[S] = E[N] * E[X]}$$

Deductibles

- In some cases, the insurance policy requires the policyholder to pay the first $D of any loss.
- This is called a deductible or excess.
- Advantages of a Deductible:
    - Eliminates a lot of small claims (people don't submit claims if the loss is less than the deductible)

    - Reduces moral hazard (people might be more careful)
    - Reduces the premium rate (since the insurer does not have to pay for all those small claims, the premium can be lower)

Upper Limits on Benefits Paid

- Insurers try to protect themselves from very large claims by setting an upper limit of the amount payable
- The insurer can calculate the risk premium to allow for caps on claims.
- Alternatively, the insurer might decide to buy excess-of-loss reinsurance.
    - In that case, the cost of the reinsurance policy would be included in the risk premium.

Excess of Loss Reinsurance

- General Insurance companies can buy reinsurance in order to manage their risks (reduce the probability of ruin).
- There are many different ways to arrange reinsurance.
- We will look at just one type of reinsurance: excess of loss.
- Insurers might use the term "excess of loss" in different ways, e.g.
    - the reinsurer pays the excess when one claim exceeds $R
    - the reinsurer pays the excess when claims on one policy exceeds $R
    - the reinsurer pays the excess when total claims for all policies, arising from on one event (e.g. a storm) exceeds $R
    - the reinsurer pays the excess when total claims for one portfolio for the year exceeds $R (e.g. motor vehicle claims)

Gross Premiums

- The insurance company must charge premiums which are sufficient to pay:
    - The expected claims cost (risk premium)
    - Commission (often a percentage of the gross premium)
    - Claims management expenses
    - Overhead expenses
    - Reinsurance costs
    - A profit loading or safety loading
- The premium charged to the customer is called the gross premium.

Gross Premium *= Risk Premium + Expected Claims Processing Expenses + Commission + Other expenses + Profit Loading*