CBSE Board

Class X Mathematics

Sample Paper 1 – Solution

Section A

1. Correct option : C

Explanation:

$\sqrt{3}$ is an irrational number. Since, it cannot be written in the form of $\frac{p}{q}$ form.

2. Correct option : B

Explanation:

mode = x(median) – y(mean) then x = 3, y = 2

Since, mode = 3median – 2mean

3. Correct option : B

Explanation:

OA is perpendicular to TA by tangent radius theorem.

OP is perpendicular to TP by tangent radius theorem.

$\Rightarrow \angle ATP + \angle OAT + \angle OPT + \angle POA + \angle ATP = 360°$

$\Rightarrow 90° + 90° + 130° + \angle ATP = 360°$

$\Rightarrow \angle ATP = 50°$

4. Correct option : B

Explanation:

The product of a non-zero rational and an irrational number is always irrational.

5. Correct option : B

Explanation:

The sample space S = {1, 2, 3, 4, 5, 6} ⇒ n(S) = 6

Let A be the event that getting a number less than 3.

A = {1, 2} ⇒ n(A) = 2

$\Rightarrow P(A) = \frac{2}{6} = \frac{1}{3}$

6. Correct option : B

Explanation:

Given quadratic polynomial is $x^2 + 5x + 8 = 0$ where a = 1, b = 5, c = 8

Sum of the zeroes i. e $\alpha + \beta = -b/a = -5$

7. Correct option : A

Explanation:

$144 = 2^4 \times 3^2$

Hence, the exponent of 2 in the prime factorization of 144 is 4.

8. Correct option : D

Explanation:

$5x^3$ consists one term. Hence, it is a monomial.

9. Correct option : D

Explanation:

A(-1, 0), B(5, -2) and C(8, 2) are the vertices of a triangle ABC, then its centroid is $\left(\frac{-1+5+8}{3}, \frac{0-2+2}{3}\right) = (4,0)$

10. Correct option : B

Explanation:

The point (-3, 5) lies in the II quadrant.

11. If P $\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(-6, 5) and B(-2, 3) then a = <u>-12</u>.

P $\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(-6, 5) and B(-2, 3).

$\Rightarrow$ The midpoint of the line segment joining A(-6, 5)and B(-2, 3) is

$\left(\frac{-6-2}{2}, \frac{5+3}{2}\right) = (-4,4)$

According to the question,

$\Rightarrow \left(\frac{a}{3}, 4\right) = (-4,4) \Rightarrow \frac{a}{3} = -4 \Rightarrow a = -12$

12. If one zero of $3x^2 + 8x + k$ be the reciprocal of the other, then k = <u>3</u>.

Let α and $\frac{1}{\alpha}$ be the roots of the polynomial.

Product of the zeros = k/3

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{3} \Rightarrow k = 3$

OR

The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is $\frac{1}{2}ab$.

The x-intercept and y-intercept of the line $\frac{x}{a} + \frac{y}{b} = 1$ are a and b respectively.

Area of triangle = $\frac{1}{2} \times bh = \frac{1}{2}ab$

13. The value of $\sin 45° \sin 30° + \cos 45° \cos 30°$ is $\frac{1+\sqrt{3}}{2\sqrt{2}}$.

$\sin 45° \sin 30° + \cos 45° \cos 30°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

14. Without using trigonometric tables, sin 29° – cos 61° = 0

sin 29° – cos 61°

= sin 29° – cos(90° – 29°) $\quad\quad$ ∵ cos (90° – θ) = sin θ

= sin 29° – sin 29°

= 0

15. ΔABC ~ ΔDEF such that ar(ΔABC) = 36 cm^2 and ar(ΔABC) = 49 cm^2. Then the ratio of their corresponding sides is 6:7.

Since, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

16. (sec A + tan A)(1 – sin A)

$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\frac{1+\sin A}{\cos A}\right)(1 - \sin A)$

$= \frac{1-\sin^2 A}{\cos A} \quad\quad \because\ 1 - \sin^2 A = \cos^2 A$

$= \frac{\cos^2 A}{\cos A}$

= cos A

OR

The value of cot θ × tan θ = 1

17. A circle of radius 28 cm and central angle 45°.

θ = 45°, r = 28 cm

Area of sector = $\frac{\theta}{360} \times \pi r^2 = \frac{45}{360} \times \frac{22}{7} \times 28 \times 28 = 308\ cm^2$

18. P(B) = $\frac{3}{13}$ and n(S) = 52

P(B) = $\frac{3}{13}$

$\Rightarrow \frac{n(B)}{n(S)} = \frac{3}{13}$

$\Rightarrow \frac{n(B)}{52} = \frac{3}{13}$

$\Rightarrow n(B) = \frac{3}{13} \times 52 = 12$

19. -26, -24, -22,….to 36 terms

a = -26, d = 2 and n = 36

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow S_{36} = \frac{36}{2}\left[2 \times (-26) + 35 \times 2\right] = 324$

20. The measures of three angles of a triangle are in the ratio 1:2:3.

Let the angles of a triangle x, 2x and 3x.

The sum of the angles of a triangle is 180°.

⇒ x + 2x + 3x = 180°

⇒ 6x = 180°

⇒ x = 30°

⇒ The angles of a triangle are 30°, 60° and 90°.

21. S is the sample space.

n(S) = 52

i. Let A be the event of getting an ace card

There are 4 ace cards in a pack of well-shuffled 52 playing cards.

n(A) = 4

Hence, P(A) = $\frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

ii. Let B be the event of getting is a spade card.

There are 13 ace cards in a pack of well-shuffled 52 playing cards.

n(A) = 13

Hence, P(A) = $\frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

22. According to the question,

$n(S) = 200$

There are 135 students like Kabaddi.

There are 200 – 135 = 65 students who do not like Kabaddi.

Let a be the event that the student selected do not like Kabaddi.

$n(A) = 65$

$$P(A) = \frac{n(A)}{n(S)} = \frac{65}{200} = \frac{13}{40}$$

OR

Let S be the sample space.

S = {HH, HT, TH, TT}

$n(S) = 4$

i. Let A be the event of getting at least one head.

A = {HH, HT, TH}

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

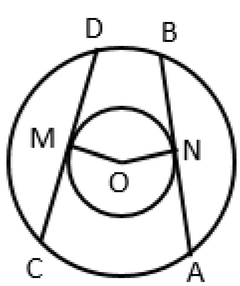
ii. Let B be the event of getting no head.

B = {TT}

$n(B) = 1$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$$

23.



Let AB and CD be two chords of the circle which touch the inner circle at N and M resepctively.

We have to prove that AB = CD.

Since AB and CD are tangents to the smaller circle.

OM = ON = radius of the smaller circle

Then, AB and CD are two chords of the larger circle such that they are equidstant from the centre.

Hence, AB = CD.

24. $\dfrac{\sin^2\theta}{\cos\theta}+\cos\theta$

$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}$ $\qquad\because \sin^2\theta+\cos^2\theta=1$

$=\sec\theta$

$\Rightarrow \dfrac{\sin^2\theta}{\cos\theta}+\cos\theta=\sec\theta$

OR

$\cos^2\theta\,(1+\tan^2\theta)$

$=\cos^2\theta\times\sec^2\theta$

$=\cos^2\theta\times\dfrac{1}{\cos^2\theta}$ $\qquad\because \sec\theta=\dfrac{1}{\cos\theta}$

$=1$

25. Area of a circle = circumference of a circle

$\Rightarrow \pi r^2 = 2\pi r$

$\Rightarrow r^2 = 2r$

$\Rightarrow r^2 - 2r = 0$

$\Rightarrow r(r-2) = 0$

Since, $r \neq 0$ hence $r = 2$ cm.

Hence, the diameter is $2r = 2(2) = 4$ cm.

26.

1. The cubic polynomials are $x^3 + 1$, $x^3 + x$, $x^3 - x^2$.
   Hence, 3 students wrote cubic polynomial.
2.

$$\begin{array}{r|l} & x+6 \\ \hline x-1 & x^2+5x+3 \\ & \underline{x^2-x} \\ & 6x+3 \\ & \underline{6x-6} \\ & 9 \end{array}$$

Section C

27. $f(x) = x^2 - 5x + k$

$\Rightarrow a = 1, b = -5$ and $c = k$

Sum of zeros = $\alpha+\beta=\frac{-b}{a}=5$ ...(i)

Product of zero = $\alpha\beta=\frac{c}{a}=k$ ...(ii)

Also, $\alpha-\beta=1$ ...(iii)

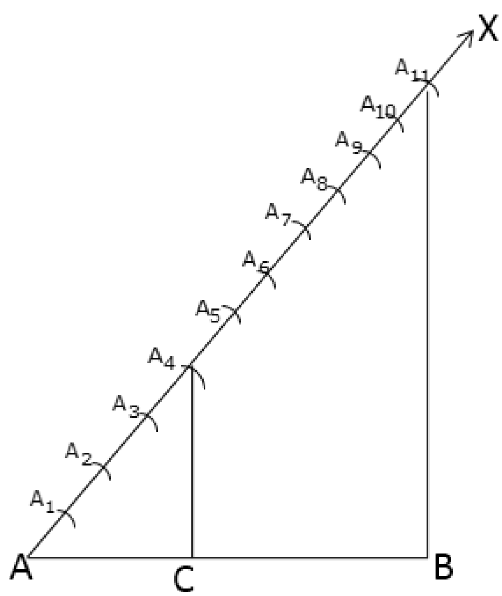
We know that

$(\alpha+\beta)^2-(\alpha-\beta)^2=4\alpha\beta$

$\Rightarrow 5^2-1^2=4k$

$\Rightarrow 24=4k\Rightarrow k=6$

28.



Steps of construction:

Step 1: Draw a line segment AB = 6.5 cm

Step 2: Draw a ray AX making an acute angle $\angle BAX$

Step 3: Along AX, mark (4 + 7) = 11 points

$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}$ such that

$AA_1 = A_1A_2 = ...$

Step 4: Join $A_{11}B$

Step 5: Through A_4, draw a line parallel to $A_{11}B$ meeting AB at C

$\therefore$ C is the point on AB, which divides AB in the ratio 4:7

On measuring, AC = 2.4 cm

CB = 4.1 cm

29. According to the question,

Surface area of sphere = surface area cube

$\Rightarrow 4\pi r^2 = 6a^2$ where r be the radius of a sphere and a be the length of a cube.

$$\Rightarrow \frac{r^2}{a^2} = \frac{6}{4\pi}$$

$$\Rightarrow a^2 = \frac{4}{6}\pi r^2 \text{ ...(i)}$$

$$\Rightarrow a = 2r\sqrt{\frac{\pi}{6}} \text{ ...(ii)}$$

$$\frac{\text{volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3} \times \frac{\pi r^3}{a^2 \times a} = \frac{4}{3} \times \frac{\pi r^3}{\frac{4}{6}\pi r^2 \times 2r\sqrt{\frac{\pi}{6}}} = \frac{1}{\sqrt{\frac{\pi}{6}}}$$

OR

The radii of the circular top and bottom are 20 cm and 15 cm respectively.

$r_1 = 20$ cm and $r_2 = 15$ cm and $h = 21$ cm

Capacity of the tub = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 21\left(20^2 + 15^2 + 20 \times 15\right)$

$= 22 \times 925$

$= 20350 \text{ cm}^3$

$= 20.35$ litres $\quad\quad \because 1 \text{ litre} = 1000 \text{ cm}^3$

The capacity of the tub is 20.35 litres.

30. $\sin\theta = \frac{11}{61}$

We know that $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$= \sqrt{1 - \left(\frac{11}{61}\right)^2}$$

$$= \sqrt{1 - \frac{121}{3721}}$$

$$= \sqrt{\frac{3600}{3721}}$$

$$\cos\theta = \frac{60}{61}$$

OR

$$\tan\theta + \frac{1}{\tan\theta} = 2$$

$$\Rightarrow \left(\tan\theta + \frac{1}{\tan\theta}\right)^2 = 4$$

$$\Rightarrow \tan^2\theta + 2\tan\theta \times \frac{1}{\tan\theta} + \frac{1}{\tan^2\theta} = 4$$

$$\Rightarrow \tan^2\theta + 2 + \frac{1}{\tan^2\theta} = 4$$

$$\Rightarrow \tan^2\theta + \frac{1}{\tan^2\theta} = 2$$

31. According to the question,

The largest number which divides 546 and 764 leaving remainders 6 and 8 respectively.

Hence, the numbers are 546 – 6 = 540 and 764 – 8 = 756

Required largest number = HCF (540, 756)

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$

$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 2^2 \times 3^2 \times 7$

HCF = $2^2 \times 3^3 = 108$

Hence, the largest number is 108 which divides 546 and 764 leaving remainders 6 and 8 respectively.

OR

$4620 = 2 \times 2310$

$= 2 \times 2 \times 1155$

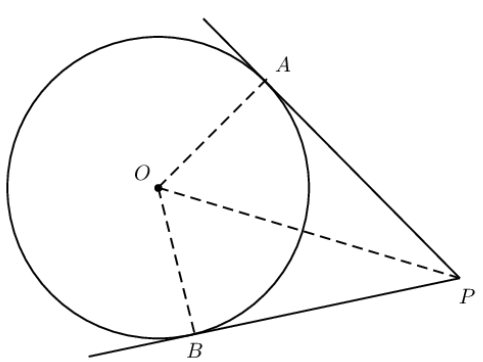
$= 2 \times 2 \times 5 \times 231$

$= 2 \times 2 \times 5 \times 3 \times 77$

$= 2 \times 2 \times 5 \times 3 \times 7 \times 11$

$= 2^2 \times 5 \times 3 \times 7 \times 11$

32.



Here,

PA and PB are tangents to the circle with centre O,

and AO and OB are the radii of the Circle.

$\therefore \left.\begin{matrix} PA \perp AO \\ PB \perp BO \end{matrix}\right\}$ .....tangent $\perp$ to radius

In ΔOPA and ΔOPB

$\angle OAP = \angle OBP$ .....each 90° (radius and tangent are $\perp$ at their poit of contact)

$OA = OB$ .....(radii of the same circle)

$OP = OP$ .....(common)

$\Delta OPA \cong \Delta OPB$........(by RHS Theorem)

$\therefore PA = PB$......(CPCT)

Hence Proved

33. 6x + 3y = 7...(i)

3x + 9y = 11...(ii)

Multiplying by 3 to (i)

18x + 9y = 21 ...(iii)

Subtracting (ii) from (iii) we get 15x = 10

x = 2/3

x = 2/3 Put it in (i) we get y = 1

Hence, x = 2/3 and y = 1

34. From the given figure,

The coordinates of station C, Town A and Town B are (-3, 2), (3, 5) and (5, 0) respectively.

1. To find who will travel more distance, Jay or Ajay to reach to their hometown, we need to find the distance between them.

   $$CA = \sqrt{(3+3)^2 + (5-2)^2} = \sqrt{36+9} = \sqrt{45}$$

   $$CB = \sqrt{(5+3)^2 + (0-2)^2} = \sqrt{64+4} = \sqrt{68}$$

   $\sqrt{68} > \sqrt{45}$ hence, Ajay travel more distance to reach hometown.

2. To find the coordinates of the point represented by the point D we need to find midpoint of points A and B.

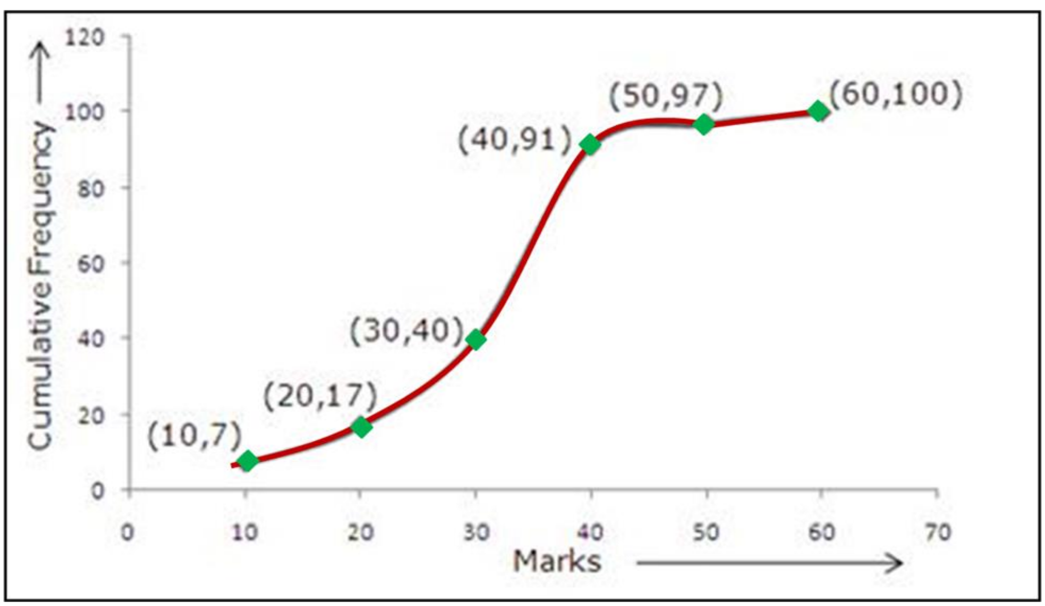
   $$D = \left(\frac{3+5}{2}, \frac{0+2}{2}\right) = (4,1)$$

Section D

35. We first prepare the cumulative frequency distribution table as given below:

Marks	No. of students	Marks less than	Cumulative frequency
0-10	7	10	7
10-20	10	20	17
20-30	23	30	40
30-40	51	40	91
40-50	6	50	97
50-60	3	60	100

Now, we mark the upper class limits along x-axis by taking a suitable scale and the cumulative frequencies along the y-axis by taking a suitable scale.
Thus, we plot the points (10, 7), (20, 17), (30, 40), (40, 91), (50, 97) and (60, 100).
Join the plotted points by a free hand to obtain the required ogive.



OR

x	f	fx
3	6	18
5	8	40
7	15	105
9	p	9p
11	8	88
13	4	52
	$\sum f = 41 + p$	$\sum fx = 303 + 9p$

$\sum f = 41 + p,\ \sum fx = 303 + 9p$

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\Rightarrow 7.5 = \frac{303 + 9p}{41 + p}$$

$\Rightarrow\ 7.5\,(41 + p) = 303 + 9p$

$\Rightarrow 307.5 + 7.5p = 303 + 9p$

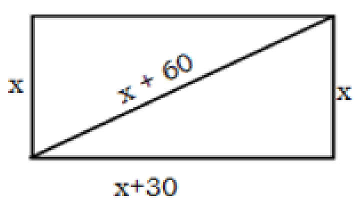
$\Rightarrow 4.5 = 1.5p$

$\Rightarrow\ p = 3$

36. Let the shorter side be x metres.

$\Rightarrow$ Diagonal = (x + 60) metres

$\Rightarrow$ Longer side = (x + 30) metres



By applying Pythagoras theorem,

$(x + 30)^2 + x^2 = (x + 60)^2$

$\Rightarrow x^2 + 60x + 900 + x^2 = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

$\Rightarrow x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow x(x - 90) + 30(x - 90) = 0$

$\Rightarrow (x - 90)(x + 30) = 0$

$\Rightarrow x = 90$ or $x = -30$

But, side cannot be negative.

So, x = 90 = shorter side

$\Rightarrow$ Longer side = x + 30 = 90 + 30 = 120 m

Thus, shorter side = 90 m, longer side = 120 m

37. Let a be the first term and d be the common difference of the given AP.

Let the AP be $a_1, a_2, a_3 \ldots a_n, \ldots$

According to the question,

$a_7 = -1$ and $a_{16} = 17$

a + (7 − 1)d = −1 and a + (16 − 1)d = 17

a + 6d = −1 and a + 15d = 17

Solving these equations simultaneously, we get

d = 2 and a = −13

Hence, the general term = a_n = a + (n − 1)d = −13 + (n − 1)2 = 2n − 15

OR

In an AP, the first term is a and the common difference is d.

$S_1 = \frac{n}{2}\left[2a+(n-1)d\right]$ ... (i)

$S_2 = \frac{2n}{2}\left[2a+(2n-1)d\right]$ ... (ii)

$S_3 = \frac{3n}{2}\left[2a+(3n-1)d\right]$ ... (iii)

$S_2 - S_1 = \frac{2n}{2}\left[2a+(n-1)d\right]-\frac{n}{2}\left[2a+(n-1)d\right]$

$S_2 - S_1 = \frac{n}{2}\left[2a+(3n-1)d\right]$

$3(S_2 - S_1) = \frac{3n}{2}\left[2a+(3n-1)d\right] = S_3$

$S_3 = 3(S_2 - S_1)$

38. In ΔACB,

CA = CB

$\angle CAB = \angle CBA$

In ΔACP and ΔBCQ,

$\Rightarrow 180^\circ - \angle CAB = 180^\circ - \angle CBA$

$\Rightarrow \angle CAP = \angle CBQ$

$\text{Now}, AP \times BQ = AC^2$

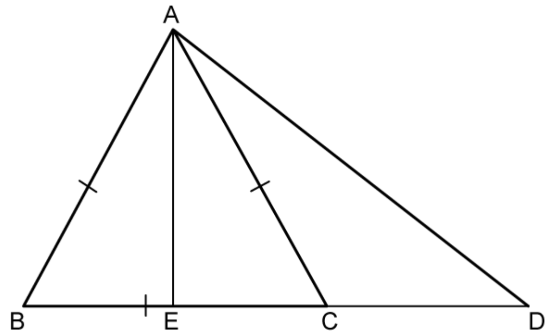
$\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

$\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ ....$(\text{Since } CA = CB)$

Thus, $\angle CAP = \angle CBQ$ and $\frac{AP}{AC} = \frac{BC}{BQ}$

$\therefore \Delta ACP \sim \Delta BCQ$…………..(SAS test)

OR



Construction: Draw a perpendicular AE from A.

Thus, $AE \perp BC$.

Proof:

In ΔABC, AB = AC

And AE is a bisector of BC

Then BE = EC.

In right-angled triangles AED and ACE,

$$AD^2 = AE^2 + DE^2 ---(1)$$

$$AC^2 = AE^2 + CE^2 ---(2)$$

Subtracting (2) from (1),

$$\Rightarrow \left(AD^2 - AC^2\right) = DE^2 - CE^2$$

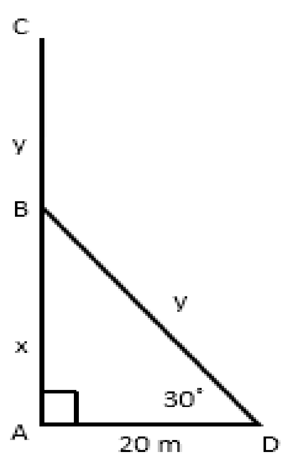
$$\Rightarrow AD^2 - AC^2 = (DE + CE)(DE - CE)$$

$$= (DE + BE)(DE - CE)[\because CE = BE]$$

$$\Rightarrow AD^2 - AC^2 = BD \times CD$$

Hence proved.

39.



Let AC be the height of the tree before it was broken.
BC is the broken part.
The distance of a point from the bottom of the tree is 20 m.
AC = x + y
BC = BD = y
AB = x
AD = 20 m
Angle of elevation = 30°
In ΔABD,

$$\tan 30^\circ = \frac{x}{20}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{20}$$

$$x = \frac{20}{\sqrt{3}}$$

$$\sin 30^\circ = \frac{x}{y}$$

$$\frac{1}{2} = \frac{20}{\sqrt{3}y}$$

$$y = \frac{40}{\sqrt{3}}$$

$$x + y = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

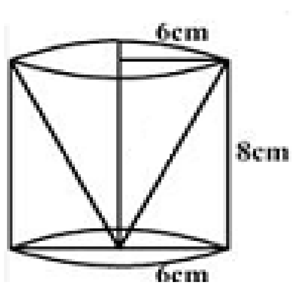
$$x + y = \frac{60\sqrt{3}}{3} = 20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$$

Height of the tree before it was broken = 34.64 m

40. For the conical cavity, radius r = 6 cm and height = 8 cm.
Volume of the remaining solid = Volume of the cylinder – Volume of the cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= 3.14 \times 6^2 \times 8\left(1 - \frac{1}{3}\right) = 602.88 \text{ cm}^3$$



The volume of the remaining solid is 602.88 cm^3.