CBSE Board
Class X Mathematics
Sample Paper 9 (Standard) – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. Correct option: B
Explanation:
$117 = 1\times 65 + 52$
$65 = 1 \times 52 + 13$
$52 = 4 \times 13 + 0$
$\therefore$ HCF of 65 and 117 is 13.
$65m - 117 = 13$
$65m = 117 + 13 = 130$
$\therefore m = 130/65 = 2$

2. Correct option: B
Explanation:
The cumulative frequency table is useful in determining the median.

3. Correct option: A
Explanation:
Product of two numbers = LCM $\times$ HCF
$\therefore 1600 = \text{LCM} \times 5$
$\therefore \text{LCM} = 320$

4. Correct option : B
Explanation:
$6x - 2y + 9 = 0$ and $3x - y + 12 = 0$
$a_1 = 6, b_1 = -2, a_2 = 3, b_2 = -1, c_1 = 9, c_2 = 12$

$$\Rightarrow \frac{a_1}{a_2} = \frac{6}{3} = 2, \frac{b_1}{b_2} = \frac{-2}{-1} = 2, \frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given system has no solution.
Hence, the lines are parallel.

5. Correct option : D
Explanation:
$\sin 40° - \cos 50° = \sin (90° - 50°) - \cos 50° = \cos 50° - \cos 50° = 0$

6. Correct option : D

Explanation:

$$\frac{\tan 30^\circ}{\cot 60^\circ}=\frac{\tan\left(90^\circ-60^\circ\right)}{\cot 60^\circ}=\frac{\cot 60^\circ}{\cot 60^\circ}=1$$

7. Correct option : C

Explanation:

sin A = cos B

$\Rightarrow$ cos (90° – A) = cos B

$\Rightarrow$ 90° – A = B

$\Rightarrow$ A + B = 90°

8. Correct option: B

Explanation:

The point P(3, 4) lies at a distance of 4 units from x-axis.

9. Correct option: A

Explanation:

Mid-point of AB is P $\left(\frac{-3+1}{2},\frac{b+b+4}{2}\right)=\left(-1,b+2\right)$

$\left(-1,b+2\right)=\left(-1,1\right)$

b + 2 = 1 i.e. b = -1

10. Correct option : C

Explanation:

A(2, 3), B(5, k) and C(6, 7)

$x_1 = 2$, $y_1 = 3$, $x_2 = 5$, $y_2 = k$, $x_3 = 6$ and $y_3 = 7$

Since the points are collinear,

$\Rightarrow x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)=0$

$\Rightarrow 2\cdot\left(k-7\right)+5\left(7-3\right)+6\left(3-k\right)=0$

$\Rightarrow$ k = 6

11. Each side of an equilateral triangle measures 8 cm. Its area is $\underline{16\sqrt{3}}$.

Each side of an equilateral triangle measures 8 cm.

Its area is $\frac{\sqrt{3}}{4}a^2=\frac{\sqrt{3}}{4}\times 8^2=16\sqrt{3}$

12. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively. The quadratic polynomial is $\underline{x^2 - 3x - 10 = 0}$.

OR

If two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, then the third zero is $\underline{\frac{-b}{a}}$.

13. $\Delta ABC \sim \Delta PQR$. If $ar(\Delta ABC) = 4\ ar(\Delta PQR)$ and BC = 12 cm, then QR = <u>6 cm</u>.

Explanation:-

Given:

$ar(\Delta ABC) = 4\ ar(\Delta PQR)$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1}$$

$$\Rightarrow \frac{BC^2}{QR^2} = \frac{4}{1}$$

$$\Rightarrow \frac{12^2}{QR^2} = \frac{4}{1}$$

$$\Rightarrow QR^2 = 36 \Rightarrow QR = 6 \text{ cm}$$

14. The sum of first 16 terms of the AP 5, 8, 11, 14,…. is <u>440</u>.

Explanation:-

Given AP is 5, 8, 11, 14,….

a = 5, d = 3 and n = 16

$$\Rightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{16} = \frac{16}{2}\left[2\times5 + (16-1)\times3\right]$$

$$\Rightarrow S_{16} = 440$$

15. The probability of an impossible event is <u>0</u>.

16. $\frac{17}{30} = \frac{17}{2\times3\times5}$

Since, the denominator has 3 as its factor it is a non-terminating decimal.

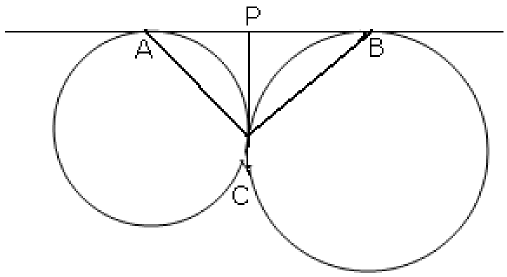
17. $\angle A = \angle R = 80^\circ$

$\angle B = \angle Q = 60^\circ$

Therefore, using the angle sum property, we have:

$\angle P = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$

18.



Lengths of tangents drawn from an external point to a circle are equal.

PA = PC (from P)

Therefore, $\angle$PAC= $\angle$PCA = x (say)

Also, PC = PB (from P)

$\angle$PBC= $\angle$PCB=y

In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180^o$

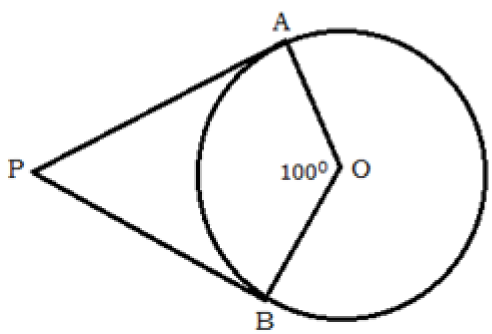
$y + (x + y) + x = 180^o$

$2(x + y) = 180^o$

$x + y = 90^o$

$\angle ACB = 90^o$

OR



We know that radius is perpendicular to the point of contact of tangents.

$\angle PAO = 90^o$ and $\angle PBO = 90^o$

Using angle sum property of quadrilaterals, we get,

$\angle P + \angle O = 180°$

$\angle P + 100° = 180°$

$\therefore \angle P = 80°$

19. Given that the first and last terms of an AP are 1 and 11, i.e., a=1 and l =11.

Let the sum of its n terms is 36, then,

$S_n = \frac{n}{2}(a + l)$

$36 = \frac{n}{2}(1+11)$

$n = \frac{36}{6} = 6$

Thus, the number of terms in the AP is 6.

20. The given quadratic equation is $kx^2 - 5x + k = 0$.

For repeated roots, we have

$b^2 - 4ac=0$

$\Rightarrow (-5)^2 - 4 \times k \times k = 0$

$\Rightarrow 25 - 4k^2 = 0$

$\Rightarrow 4k^2 = 25$

$\Rightarrow k= \pm \frac{5}{2}$

21. $m\angle ABC = 90°$

Since AB being diameter is perpendicular to tangent BC at the point of contact.

So $m\angle ABP + m\angle PBC = 90°$ (i)

Also $m\angle APB = 90°$ (angle in the semi-circle)

So $m\angle BAP + m\angle ABP = 90°$ (ii) (using angle sum property of triangles)

From (i) and (ii), $\angle PBC = \angle BAP$

22. A rational number will have a terminating decimal representation only if the denominator can be expressed in terms of prime numbers 2 and 5.

We see that,

$343 = 7 \times 7 \times 7$

So according to the condition given above, the denominator of $\frac{29}{343}$ cannot be expressed fully in terms of 2 and 5.

Hence, the number cannot have a terminating decimal representation.

23. Let n be the required number of spheres.

Since, the spheres are melted to form a cylinder. So, the volume of all the n spheres will be equal to the volume of the cylinder.

$n \times \frac{4}{3} \times \pi \times 3 \times 3 \times 3 = \pi \times 2 \times 2 \times 45$

$\therefore n = 5$

Thus, the required number of spheres which are melted to form the cylinder is 5.

OR

Let x cm be the edge of the new cube.

Volume of the new cube = Sum of the volumes of three cubes

$x^3 = 3^3 + 4^3 + 5^3$

$x^3 = 27 + 64 + 125$

$x^3 = 216$

$x = 6$ cm.

Edge of the new cube is 6 cm long.

24.

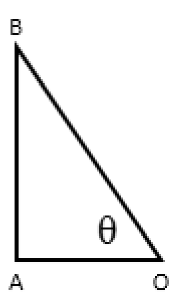
Classes	Class Mark (x_i)	Frequency (f_i)	f_ix_i
0-10	5	7	35
10-20	15	3	45
20-30	25	15	375
30-40	35	5	175
		$\Sigma f_i = 30$	$\Sigma f_ix_i = 630$

Mean = $\dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{630}{30} = 21$

OR

A die is thrown at once then S = {1, 2, 3, 4, 5, 6}
$\therefore$ n(S) = 6
Let E be an event of getting a prime number.
$\therefore$ E = {2, 3, 5}
$\therefore$ n(E) = 3
$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

25.



Let AB be the tower and O be the point of observation.

Then, AB = $100\sqrt{3}$ m and OA = 100 m.

$$\tan\theta = \frac{AB}{OA} = \frac{100\sqrt{3}}{100} = \sqrt{3}$$

$$\theta = 60^\circ$$

26. Using Thale's theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{3x+4} = \frac{x+3}{3x+19}$$

$$\Rightarrow x(3x+19) = (x+3)(3x+4)$$

$$\Rightarrow 3x^2 + 19x = 3x^2 + 4x + 9x + 12$$

$$\Rightarrow 6x = 12 \Rightarrow x = 2$$

27. Let 6 + $\sqrt{2}$ be rational and equal to $\frac{a}{b}$.

Then, $\frac{6+\sqrt{2}}{1} = \frac{a}{b}$, where a and b are co-primes, $b \neq 0$

$$\therefore \sqrt{2} = \frac{a}{b} - 6 = \frac{a-6b}{b}$$

Here a and b are integers.

So, $\frac{a-6b}{b}$ is rational.

Therefore, $\sqrt{2}$ is rational. This is a contradiction as $\sqrt{2}$ is irrational.

Hence, our assumption is wrong.

Thus, 6 + $\sqrt{2}$ is an irrational number.

28. Let a − d, a and a + d be three terms in an A.P.

According to the question,

a − d + a + a + d = 3

3a = 3 or a = 1

(a − d) (a) (a + d) = −8

$a(a^2 - d^2) = -8$

Putting the value of a = 1, we get,

$1 - d^2 = -8$

$d^2 = 9$ or $d = \pm 3$

Thus, the required three terms are −2, 1, 4 or 4, 1, −2.

29. Assume the fixed charge = Rs. x
And, the subsequent charge = Rs. y
According to the question, we have
$x + 4y = 27$ ... (i)
$x + 2y = 21$ ... (ii)
Subtracting (ii) from (i), we have
$2y = 6$ or $y = 3$
$\Rightarrow x = 27 - 12 = 15$ [from (i)]
Thus, the fixed charge is Rs. 15 and the charge for each extra day is Rs. 3.

OR

Let the fraction be $\frac{x}{y}$.
According to the question,
$x + y = 8$ ....(1)
$$\frac{x+3}{y+3} = \frac{3}{4}$$
$\Rightarrow 4x + 12 = 3y + 9$
$\Rightarrow 4x - 3y = -3$ ....(2)
Multiplying (1) by 3, we get
$3x + 3y = 24$ ....(3)
Adding (2) and (3), we get
$7x = 21$
$\Rightarrow x = 3$
$\Rightarrow y = 8 - x = 8 - 3 = 5$
Thus, the fraction is $\frac{3}{5}$.

30. $$\text{L.H.S.} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$$
$$= \frac{\left(\sqrt{\sec\theta - 1}\right)^2 + \left(\sqrt{\sec\theta + 1}\right)^2}{\left(\sqrt{\sec\theta + 1}\right)\left(\sqrt{\sec\theta - 1}\right)}$$
$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}}$$
$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}}$$

$$= \frac{2\sec\theta}{\tan\theta}$$
$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$
$$= 2\operatorname{cosec}\theta$$
$$= \text{R.H.S.}$$

OR

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$$
$$= \frac{\sin(90^\circ\text{-}70^\circ)}{\sin 20^\circ} + \frac{\sin(90^\circ\text{-}59^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$$

$= \dfrac{\sin 20^\circ}{\sin 20^\circ} + \dfrac{\sin 31^\circ}{\sin 31^\circ} - 8 \times \dfrac{1}{4} = 1 + 1 - 2 = 0$ .... Since, $\cos\theta = \sin(90^\circ - \theta)$

31. Let P(x, y), Q(a + b, b – a) and R(a – b, a + b) be the given points.

It is given that $PQ = PR \Rightarrow PQ^2 = PR^2$

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2 + y^2 - 2y(b - a) + (b - a)^2$

$= x^2 + (a - b)^2 - 2x(a - b) + y^2 - 2y(a + b) + (a + b)^2$

$\Rightarrow -2x(a + b) - 2y(b - a) = -2x(a - b) - 2y(a + b)$

$\Rightarrow -ax - bx - by + ay = -ax + bx - ay - by$

$\Rightarrow 2bx = 2ay$

$\Rightarrow bx = ay$

32. Out of 52 cards, one card can be drawn in 52 ways.

So, the total number of outcomes = 52

i. There are 26 red cards, including two red kings, in a pack of 52 playing cards.

   Also, there are 4 kings, two red and two black.

   Therefore, card drawn will be either a red card or a king if it is any one of 28 cards (26 red cards and 2 black kings).

   So, favourable number of elementary events = 28

   Hence, the required probability = $\dfrac{28}{52} = \dfrac{7}{13}$

ii. There are 6 red face cards, 3 each from diamonds and hearts.

Out of these 6 red face cards, one card can be chosen in 6 ways.

So, favorable number of elementary events = 6

Hence, the required probability = $\frac{6}{52} = \frac{3}{26}$

iii. There are two suits of black cards, viz., spades and clubs.

Each suit contains one card bearing number 10.

So, favorable number of elementary events = 2

Hence, the required probability = $\frac{2}{52} = \frac{1}{26}$

OR

Class Interval	Frequency f_i	Mid-value x_i	$f_i x_i$
0-10	10	5	50
10-20	6	15	90
20-30	8	25	200
30-40	12	35	420
40-50	5	45	225
	$\sum f_i = 41$		$\sum f_i x_i = 985$

From the table,

$\sum f_i = 41$ and $\sum f_i x_i = 985$

Mean $(\bar{x})$ = $\frac{\sum f_i x_i}{\sum f_i} = \frac{985}{41} = 24.02$

33. Diameter of the cylinder = 12 cm

Radius of cylinder = 6 cm

Height of the cylinder = 15 cm

Volume of ice-cream in the cylinder = $\pi r^2 h = \pi \times 36 \times 15 = 540\,\pi$

Diameter of cone = 6 cm

Radius of cone = 3 cm

Height of cone = 12 cm

Volume of one ice cream = volume of ice cream cone + volume of hemispherical top of ice cream = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$$=\frac{1}{3}\pi(9)(12)+\frac{2}{3}\pi(27)$$
$$=36\pi+18\pi$$
$$=54\pi$$

So, the number of ice cream cones = $\frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}}$ = $\frac{540\pi}{54\pi}=10$

Hence, the number of ice cream cones is 10.

34. Since a – b, a and a + b are the zeros of f(x) = $x^3 - 3x^2 + x + 1$.

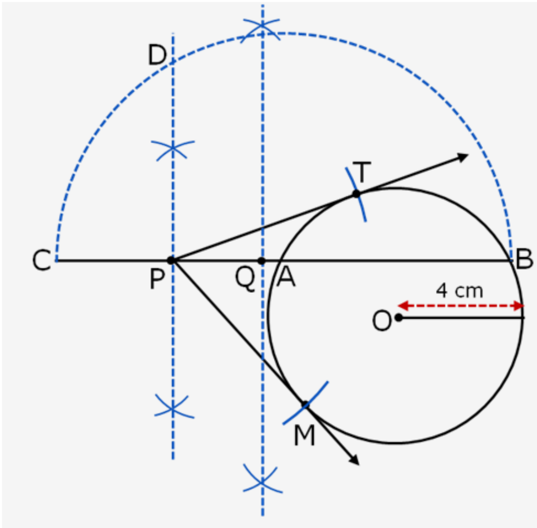
$$\therefore\ (a-b)+a+(a+b)=-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$
$$\Rightarrow 3a=-\frac{-3}{1}$$
$$\Rightarrow 3a=3$$
$$\Rightarrow a=1$$

And, $(a-b)\times a\times(a+b)=-\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

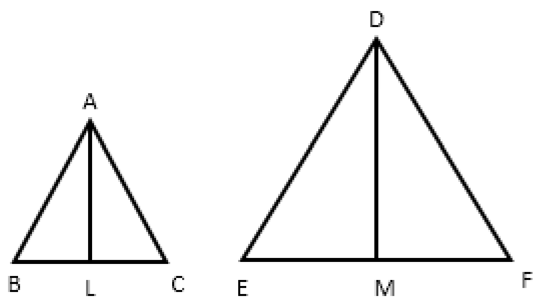
$$\Rightarrow a(a^2-b^2)=-\frac{1}{1}$$
$$\Rightarrow 1(1-b^2)=-1$$
$$\Rightarrow b^2=2$$
$$\Rightarrow b=\pm\sqrt{2}$$

35. Steps of construction:

- Draw a circle of radius 4 cm.
- Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.
- Produce AP to C such that AP = CP.
- Draw a semi-circle with CB as diameter.
- Draw PD perpendicular to CB outside the circle, intersecting the semi-circle at D.
- With P as centre and PD as radius, draw arcs to intersect the given circle at points T and M.
- Join PT and PM.
- Then PT and PM are the required tangents.



36.



Given : Two triangles ABC and DEF in which ∠A = ∠D, ∠B = ∠E, ∠C = ∠F and AL is perpendicular to BC and DM is perpendicular to EF

To Prove : $\frac{BC}{EF} = \frac{AL}{DM}$

Proof :

Since equiangular triangles are similar.

$\Delta ABC \sim \Delta DEF$

$\frac{AB}{DE} = \frac{BC}{EF}$ ....(i)

In ΔALB and ΔDME,

$\angle ALB = \angle DME = 90^\circ$

$\angle B = \angle E$

$\Delta ALB \sim \Delta DME$ A-A criterion

$\frac{AB}{DE} = \frac{AL}{DM}$ ....(ii)

$\frac{BC}{EF} = \frac{AL}{DM}$ from (i) and (ii)

OR

We have, $\angle A + \angle D = 90°$

In ΔAPD, by angle sum property,

$\angle A + \angle D + \angle P = 180°$

$\Rightarrow 90° + \angle P = 180°$

$\Rightarrow \angle P = 180° - 90° = 90°$

In ΔAPC, by Pythagoras theorem,

$AC^2 = AP^2 + PC^2$ ....(1)

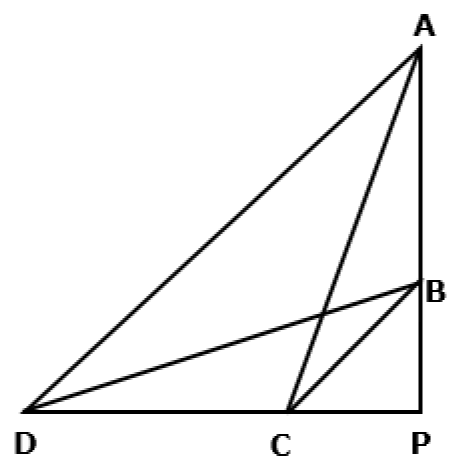
In ΔBPD, by Pythagoras theorem,

$BD^2 = BP^2 + DP^2$ ....(2)

Adding equations (1) and (2),

$AC^2 + BD^2 = AP^2 + PC^2 + BP^2 + DP^2$

$\Rightarrow AC^2 + BD^2 = (AP^2 + DP^2) + (PC^2 + BP^2) = AD^2 + BC^2$



37. Let list price of the book = Rs. x

So, the number of books purchased = $\frac{1200}{x}$

And increased price of the book = Rs. (x + 10)

So, the number of books purchased = $\frac{1200}{x+10}$

According to the condition, if the list price of a book is increased by Rs. 10, then a person can buy 10 less books.

$\therefore \frac{1200}{x} - \frac{1200}{x+10} = 10$

$\therefore (1200)\left[\frac{1}{x} - \frac{1}{x+10}\right] = 10$

$\therefore (1200)\left[\frac{x+10-x}{x(x+10)}\right] = 10$

$\therefore 1200 = x(x + 10)$

$\therefore\ x^2 + 10x - 1200 = 0$

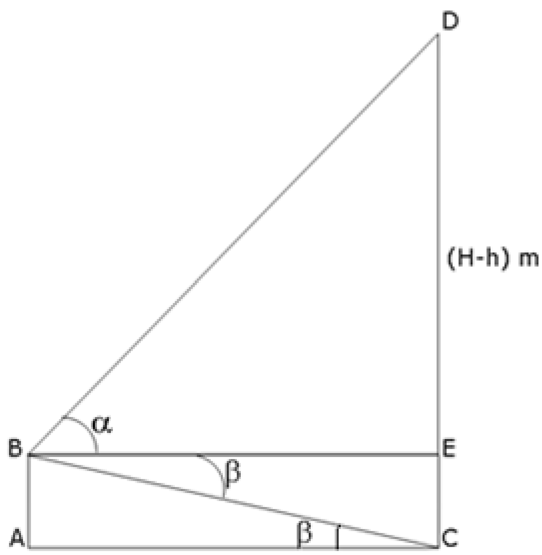
$\therefore (x + 40)(x - 30) = 0$

$\therefore x = -40$ or $x = 30$

But x is the list price of the book and hence can't be negative.

Therefore, the original list price of the book is Rs. 30.

38. Let B be the window of a house AB and let CD be the other house.



Then, AB = EC = h metres.

Let CD = H metres.

Then, ED = (H − h) m

In ΔBED,

$\cot \alpha = \frac{BE}{ED}$

$BE = (H - h) \cot \alpha$ ... (a)

In ΔACB,

$\frac{AC}{AB} = \cot \beta$

$AC = h.\cot \beta$ .... (b)

But BE = AC

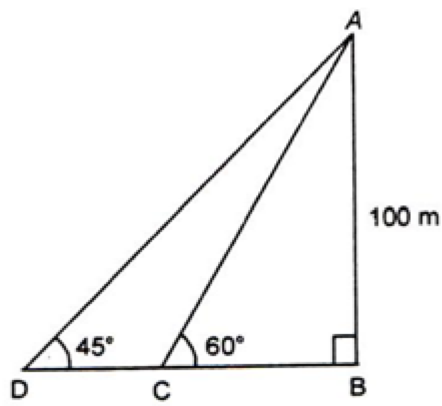
$\therefore (H - h) \cot\alpha = h\cot\beta$ ....[From (a) and (b)]

$$H = h\,\frac{(\cot\alpha + \cot\beta)}{\cot\alpha}$$

$H = h(1 + \tan\alpha \cot\beta)$

Thus, the height of the opposite house is $h(1 + \tan\alpha.\cot\beta)$ metres.

OR



Here, the man has covered the distance CD in 2 minutes.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

Now, in ΔABC,

$$\frac{100}{BC} = \tan 60^{o} = \sqrt{3}$$

$$\Rightarrow BC = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

In ΔABD,

$$\frac{100}{BD} = \tan 45^{o} = 1$$

$$\Rightarrow \quad BD = 100$$

$$\therefore \quad CD = BD - BC$$

$$= \left(100 - \frac{100\sqrt{3}}{3}\right) = 100\left(\frac{3-\sqrt{3}}{3}\right)$$

$$\text{Thus, Speed} = \frac{100\left(\frac{3-\sqrt{3}}{3}\right)}{2}$$

$$= 50\left(\frac{3-\sqrt{3}}{3}\right) \text{m / min}$$

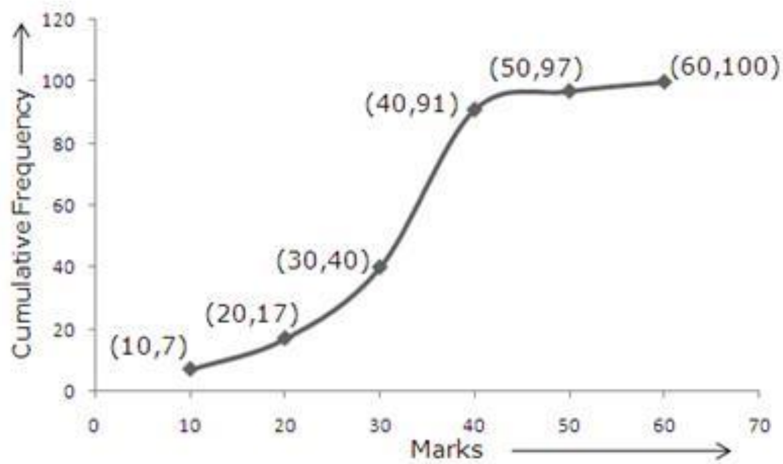
39. We first prepare the cumulative frequency distribution table as given below:

Marks	No. of students	Marks less than	Cumulative frequency
0-10	7	10	7
10-20	10	20	17
20-30	23	30	40
30-40	51	40	91
40-50	6	50	97
50-60	3	60	100

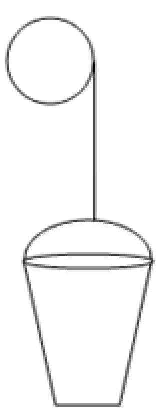
Now, we mark the upper class limits along x-axis by taking a suitable scale and the cumulative frequencies along the y-axis by taking a suitable scale.

Thus, we plot the points (10, 7), (20, 17), (30, 40), (40, 91), (50, 97) and (60, 100).

Join the plotted points by a free hand to obtain the required ogive.



40.



Time taken by bucket to ascend = 1 min 28 secs = 88 secs
Speed = 1.1 m/ sec
Length of the rope = distance covered by bucket to ascend

$= (1.1 \times 88)$ m $= (1.1 \times 88 \times 100)$ cm $= 9680$ cm

Radius of the wheel = 38.5 cm = $\frac{77}{2}$ cm

Circumference of the wheel = $2\pi r$ = $\left(2\times\frac{22}{7}\times\frac{77}{2}\right)$ cm = 242 cm

$\therefore$ Number of revolutions = $\frac{\text{Length of the rope}}{\text{Circumference of the wheel}} = \left(\frac{9680}{242}\right) = 40$

Hence, the wheel makes 40 revolutions to raise the bucket.

OR

Surface area of sphere = $4\pi R^2$

$4\pi R^2 = 616$

$4\times\frac{22}{7}\times R^2 = 616 \Rightarrow R^2 = \frac{616\times 7}{4\times 22}$

$\Rightarrow R^2 = 49 \Rightarrow R = 7$ cm

Diameter of the smaller sphere = 3.5 cm

$\therefore$ Radius of the smaller sphere = 1.75 cm $\Rightarrow$ r = 1.75 cm

Let the number of smaller spheres be = x.

Volume of x smaller spheres = Volume of larger metal sphere

$x\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$x\times 1.75\times 1.75\times 1.75 = 7\times 7\times 7$

$x = \frac{7\times 7\times 7}{1.75\times 1.75\times 1.75} = 64$

$\therefore$ No. of smaller spheres = 64