CBSE Board
Class X Mathematics
Sample Paper 7 – Solution

Section A

1. **Correct Option:** B

Explanation:

$5.\overline{69}$ is recurring and non – terminating.

Hence, it is a rational number.

2. **Correct Option:** D

Explanation:

x_i	10	5	p	25	35
f_i	3	5	25	7	5
$f_i\ x_i$	30	25	25p	175	175

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{30 + 25 + 25p + 175 + 175}{3 + 10 + 25 + 7 + 5}$$

$$\Rightarrow 20.6 = \frac{405 + 25p}{50}$$

$$\Rightarrow 405 + 25p = 20.6 \times 50$$

$$\Rightarrow 25p = 625$$

$$\Rightarrow p = 25$$

3. **Correct Option:** C

Explanation:

We know that,

$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$

$\Rightarrow \text{LCM}(26, 169) \times \text{HCF}(26, 169) = 26 \times 169$

$$\Rightarrow \text{LCM}(26, 169) = \frac{26 \times 169}{13} = 2 \times 169 = 338$$

4. **Correct Option:** D

Explanation:

To solve a pair of linear equations in two variables algebraically, we have the following methods:

i. Substitution method

ii. Elimination method

iii. Cross – multiplication method

5. **Correct Option:** B

Explanation:

We know that, $\tan(90° - \theta) = \cot\theta$

For $\theta = 35°$,

$\tan 55° = \cot 35°$ ..... (1)

Also $\cot 90° = 0$

$\Rightarrow \frac{\tan 55°}{\cot 35°} + \cot 1° \cot 2° ......\cot 90° = 1 + 0 = 1$ .... from (1) and (2)

6. **Correct Option:** B

Explanation:

$(1 - \cos^2 A)\,\text{cosec}^2 A$

$= \sin^2 A \times \text{cosec}^2 A$

$= \sin^2 A \times \frac{1}{\sin^2 A}$

$= 1$

7. **Correct Option:** D

Explanation:

$\cos(\alpha + \beta) = 0$

$\Rightarrow \cos(\alpha + \beta) = \cos 90°$

$\Rightarrow \alpha + \beta = 90°$

$\Rightarrow \alpha = 90° - \beta$

$\therefore \sin(\alpha - \beta) = \sin(90° - \beta - \beta) = \sin(90° - 2\beta) = \cos 2\beta$

8. **Correct Option:** A

Explanation:

Let R be the mid-point of PQ, then, the coordinates of mid-point of PQ, i.e., R is $\left(\frac{(-2-6)}{2}, \frac{(8-4)}{2}\right) = (-4, 2)$

9. **Correct answer:** B

Explanation:

(x, -1), (2, 1) and (4, 5) lie on a line if the area of a triangle formed by these three points is 0.

Area of a triangle = 0

$$\frac{1}{2}\left|x(1-5)+2(5+1)+4(-1-1)\right|=0$$

$$\frac{1}{2}\left|-4x+12-8\right|=0$$

$$-4x+4=0$$

$$x=1$$

10. **Correct option:** B

Explanation:

Let the coordinates of the point be P(x, 2x). Let Q be the point (4, 3).

$PQ^2 = (4 - x)^2+(3 - 2x)^2 = 10$

$16 + x^2 - 8x + 9 + 4x^2 - 12x = 10$

$\Rightarrow 5x^2 - 20x + 15=0$

$\Rightarrow x^2 - 4x + 3=0$

$\Rightarrow (x - 3)(x - 1)=0$

$\Rightarrow x = 1$ or $x = 3$

So, 2x = 2 or 6

Hence, the coordinates of the required point are (1, 2) or (3, 6).

11. If the length, breadth and height of a cuboid are equal, then it is called a Cube.

OR

The radii of two cylinders are in the ratio 2:3. If their heights are in the ratio 3:5, then the ratio of their curved surface areas is 2 : 5.

Explanation:

Curved surface area of a cylinder = $2\pi rh$

Let r_1 and r_2 be the radii of the cylinders and h_1 and h_2 be the heights of the cylinders.

Ratio of curved surface areas = $\frac{2\pi r_1 h_1}{2\pi r_2 h_2}=\frac{r_1}{r_2}\times\frac{h_1}{h_2}=\frac{2}{3}\times\frac{3}{5}=\frac{2}{5}=2:5$

12. If x – 2 is a factor of the polynomial $x^3 - 6x^2 + ax - 8$, then the value of a is equal to 12.

Explanation:

x – 2 is a factor of the polynomial $x^3 - 6x^2 + ax - 8$; hence, p(2) = 0.

$2^3 - 6 \times 2^2 + 2a - 8 = 0$

$8 - 24 + 2a - 8 = 0$

$2a - 24 = 0$

$a = 12$

13. If ΔABC and ΔDEF are similar such that 6AB = 4DE and BC = 12 cm, then EF = <u>18</u>.

Explanation:

$$\frac{A(\Delta ABC)}{A(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2$$

$$\left(\frac{4}{6}\right)^2 = \left(\frac{12}{EF}\right)^2$$

EF = 18 cm

14. $\dfrac{\boxed{13}}{\boxed{2}}$ is the next term of the AP 4, $\frac{9}{2}$, 5, $\frac{11}{2}$, 6… .

Explanation:

The general form of an AP is

a, a + d, a + 2d,…

$d = \frac{1}{2}$

$6 + \frac{1}{2} = \frac{13}{2}$

15. The probability of getting a non-face card from a deck of cards is $\dfrac{\boxed{10}}{\boxed{13}}$.

Explanation:

The number of face cards is 12, and the total number of cards is 52.

The probability of a face card is $\frac{12}{52} = \frac{3}{13}$

Probability of getting a non-face card = $1 - \frac{3}{13} = \frac{10}{13}$

16. Let the given number when divided by 143 gives q as quotient and 31 as remaninder. Then, the number = 143q + 31 = 13 × 11q + 13 × 2 + 5 = 13(11q + 2) + 5.

So, the same number when divided by 13 gives 5 as remainder.

17. In ΔABC, $DE \parallel BC$.

Then, by Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2}{3} = \frac{AE}{4}$$

$$\Rightarrow AE = \frac{2 \times 4}{3} = \frac{8}{3} = 2.67 \text{ cm}$$

18. Since tangent $\perp$ radius,
Using Pythagoras theorem,

$OP^2 + PQ^2 = OQ^2$
$3^2 + 4^2 = OQ^2$
$OQ^2 = 25$
$OQ = 5$ cm

19. Using the formula:

$a_n = a + (n - 1)d$

If we reverse the AP, then the last term will become the first term.

$\therefore$ 12^{th} term of the AP from the end $= 60 - (12 - 1) \times 3$

$= 60 - 33$

$= 27$

20. $(x^2 + 1)^2 - x^2 = 0$

$\Rightarrow x^4 + 2x^2 + 1 - x^2 = 0$

$\Rightarrow x^4 + x^2 + 1 = 0$

Let $x^2 = y$

$\Rightarrow y^2 + y + 1 = 0$

Here, Discriminant, $D = b^2 - 4ac = 1 - 4 = -3 < 0$

So, the equation does not have real roots.

OR

Consider, $p(x) = 3x^2 + 2x - 1$

$p(-1) = 3(-1)^2 + 2(-1) - 1 = 3 - 2 - 1 = 0$

Hence, -1 is the root of $3x^2 + 2x - 1 = 0$

Section B

21. $870 = 225 \times 3 + 195$

$225 = 195 \times 1 + 30$

$195 = 30 \times 6 + 15$

$30 = 15 \times 2 + 0$

$\therefore$ HCF (870, 225) = 15

22. Since the lengths of tangents from an exterior point to a circle are equal.

Therefore, XP = XQ (tangents from X) ....(i)

AP = AR (tangents from A) ....(ii)

BQ = BR (tangents from B) ....(iii)

Now, XP = XQ

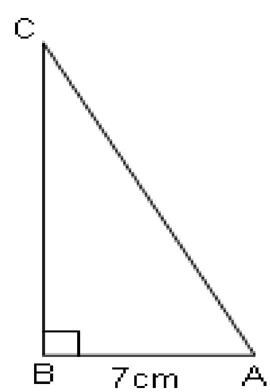
$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR [Using (ii) and (iii)]

23. Each equal side a = 13 cm and base = b = 24 cm

Area of an isosceles triangle = $\dfrac{1}{4}b\sqrt{4a^2 - b^2} = \dfrac{1}{4} \times 24\sqrt{4 \times 169 - 24^2} = 60 \text{ cm}^2$

24. In ΔABC, we have



$AC^2 = BC^2 + AB^2$

$(1 + BC)^2 = BC^2 + AB^2$

$\Rightarrow 1 + BC^2 + 2BC = BC^2 + AB^2$

$\Rightarrow 1 + 2BC = 7^2$

$\Rightarrow 2BC = 48$

$\Rightarrow BC = 24$ cm

$\Rightarrow AC = 1 + BC = 1 + 24 = 25$ cm

Hence, $\sin C = \dfrac{AB}{AC} = \dfrac{7}{25}$ and $\cos C = \dfrac{BC}{AC} = \dfrac{24}{25}$

25. Face cards in a pack of cards are jacks, queens and kings.

$\therefore$ The number of face cards = 4 × 3 = 12

Total number of cards = 52

(i) Probability of getting a face card = $\frac{12}{52} = \frac{3}{13}$

(ii) Number of king cards which are not red = 2

Number of red cards = 26

$\therefore$ Probability of getting a red card or a black king card $= \frac{28}{52} = \frac{7}{13}$

Probability of getting neither a red card nor a king card $= 1 - \frac{7}{13} = \frac{6}{13}$

OR

Total number of outcomes in the experiment of throwing two dice is 36.

Possible outcomes in getting the same number on both dice {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

Required probability = $\frac{6}{36} = \frac{1}{6}$

26. According to the question,

R = 15 cm, r = 10 cm, h = 8 cm

Volume of the frustum = $\frac{\pi \times 8}{3}\left(15^2 + 10^2 + 15 \times 10\right)$

$$= \frac{22}{7} \times \frac{8}{3}\left(15^2 + 10^2 + 15 \times 10\right)$$

$= 3980.95\ cm^3$

OR

D = 10 cm, R = 5 cm, d = 8 cm, r = 4 cm, h = 4 cm

$l = \sqrt{h^2 + (R - r)^2} = \sqrt{4^2 + (5 - 4)^2} = 4.12$ cm

Total surface area of a frustum = $\pi[(R + r)\, l + R^2 + r^2]$

$$= \frac{22}{7}\left[(5 + 4) \times 4.12 + 5^2 + 4^2\right]$$

$= 245.39$

Section C

27. Let us assume, on the contrary that $\sqrt{5}$ is a rational number.

Then, there exist positive integers a and b such that

$\sqrt{5} = \frac{a}{b}$, where a and b are co-prime

$\Rightarrow \left(\sqrt{5}\right)^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow 5 = \frac{a^2}{b^2}$

$\Rightarrow 5b^2 = a^2$

$\Rightarrow 5 | a^2$

$\Rightarrow 5 | a$

$\Rightarrow a = 5c$ for some positive integer c

$\Rightarrow a^2 = 25c^2$

$\Rightarrow 5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow 5 | b^2$

$\Rightarrow 5 | b$

This implies that a and b have at least 5 as a common factor.

This contradicts the fact that a and b are co-prime.

So, our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

OR

Let n be the positive integer such that no prime less than or equal to $\sqrt{n}$ divides n.
Then, we have to prove that n is prime.
Suppose n is not a prime integer.
Then, we may write
$n = ab$ where $1 < a \leq b$
$a \leq \sqrt{n}$ and $b \geq \sqrt{n}$
Let, p be a prime factor of a.
Then, $p \leq a \leq \sqrt{n}$ and p divides a
Hence, p divides ab.
So, p divides n
Hence, a prime less than $\sqrt{n}$ divides n, which is a contradiction.
Therefore, our assumption is wrong.
Hence, n is a prime number.

28. Let A = first term and d = common difference of the given AP.

In an AP consisting of 15 terms, $\left(\frac{15+1}{2}\right)^{th}$ i.e. 8^{th} term is the middle term.

Given: middle term = 56

$\therefore$ a + 7d = 56 ... (i)

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{15} = \frac{15}{2}[2a + (15-1)d]$$
$$= 15(a + 7d)$$
$$= 15 \times 56 \quad .....(i)$$
$$= 840$$

29. Given equations are 2x + 3y = 7; (a – b)x + (a + b)y = 3a + b – 2

The given system of equations will have infinite number of solutions, if

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Consider, $\frac{2}{a-b} = \frac{3}{a+b}$

$\Rightarrow 2a + 2b = 3a - 3b$

$\Rightarrow a = 5b \quad ....(i)$

Consider, $\frac{3}{a+b} = \frac{7}{3a+b-2}$

$\Rightarrow 9a + 3b - 6 = 7a + 7b$

$\Rightarrow 2a - 4b = 6$

$\Rightarrow 2(5b) - 4b = 6 \quad ....[\text{From (i)}]$

$\Rightarrow 6b = 6 \Rightarrow b = 1$

$\Rightarrow a = 5b = 5 \times 1 = 5$

Hence, a = 5 and b = 1

OR

Given system of equations are

$7x - 2y - 3 = 0$

$11x - \frac{3}{2}y - 8 = 0$

By cross multification, we have

$$\therefore \frac{x}{\left[(-2)(-8)-\left(\frac{-3}{2}\right)\times(-3)\right]}=\frac{-y}{\left[7\times(-8)-(-3\times 11)\right]}=\frac{1}{\left[7\times\left(\frac{-3}{2}\right)-11\times(-2)\right]}$$

$$\Rightarrow \frac{x}{16-\frac{9}{2}}=\frac{-y}{-56+33}=\frac{1}{\frac{-21}{2}+22}$$

$$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)}=\frac{y}{23}=\frac{1}{\frac{23}{2}}$$

$$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)}=\frac{1}{\frac{23}{2}} \text{ and } \frac{y}{23}=\frac{1}{\frac{23}{2}}$$

$$\Rightarrow x=1 \text{ and } y=2$$

Hence, the solution of the given system of equations is x = 1 and y = 2.

30. Using long division to divide $x^4 - 4x^3 + x - 1$ by $x + 5$

$$\begin{array}{r l}
 & x^3 - 9x^2 + 45x - 224 \\
x+5 \,\big) & x^4 - 4x^3 + 0x^2 + x - 1 \\
 & x^4 + 5x^3 \\
 & - \quad - \\
\hline
 & -9x^3 \\
 & -9x^3 - 45x^2 \\
 & + \quad + \\
\hline
 & 45x^2 + x \\
 & 45x^2 + 225x \\
 & - \quad - \\
\hline
 & -224x - 1 \\
 & -224x - 1120 \\
 & + \quad + \\
\hline
 & 1119
\end{array}$$

Hence, the remainder is 1119.

31. Since P and Q are trisecting AB and P is nearer to A, so the point P divides AB in the ratio 1 : 2. Let the coordinates of P be (x, y).

Therefore, using section formula, coordinates of P are given by

$$P\left(\frac{1\times5+2\times2}{1+2},\frac{1\times(-8)+2\times 1}{1+2}\right)=P(3,-2)$$

Now P(3, −2) lies on the line $2x - y + k = 0$

$\Rightarrow 2 \times 3 - (-2) + k = 0$

$\Rightarrow 6 + 2 + k = 0$

$\Rightarrow k = -8$

32. The two angles θ and ϕ being the acute angles of a right triangle must be complementary angles.

$\Rightarrow \theta = (90° - \phi)$ and $\phi = (90° - \theta)$ ....(i)

Given $\frac{\sin^2\theta}{\cos^4\phi}+\frac{\sin^4\phi}{\cos^2\theta}=1$

$$\Rightarrow \frac{\sin^2(90°-\phi)}{\cos^4(90°-\theta)}+\frac{\sin^4(90°-\theta)}{\cos^2(90°-\phi)}=1$$

$$\Rightarrow \frac{\cos^2\phi}{\sin^4\theta}+\frac{\cos^4\theta}{\sin^2\phi}=1 \quad [\because \sin(90°-A)=\cos A, \cos(90°-A)=\sin A]$$

$$\Rightarrow \frac{\cos^4\theta}{\sin^2\phi}+\frac{\cos^2\phi}{\sin^4\theta}=1$$

33. Circumference of a circle = 14π

$2\pi r = 14\pi$

$r = 7$ cm

Perimeter of a square = 24

$4x = 24$

$x = 6$ cm

$$\frac{\text{Area of a circle}}{\text{Area of square}}=\frac{\pi\times7^2}{6\times6}$$

$$=\frac{49\times\frac{22}{7}}{36}$$

$$=\frac{154}{36}$$

$$=\frac{77}{18}$$

34.

Age in yrs	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Persons	10	15	25	22	13	10	5

From the data given as above we may observe that maximum class frequency is 25 belonging to class interval 20 – 30.

So, modal class = 20 – 30

Lower class limit (l) of modal class = 20

Frequency (f_1) of modal class = 25

Frequency (f_0) of class preceding the modal class = 15

Frequency (f_2) of class succeeding the modal class = 22

Class size (h) = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 20 + \left(\frac{25 - 15}{2(25) - 15 - 22}\right) \times 10$$

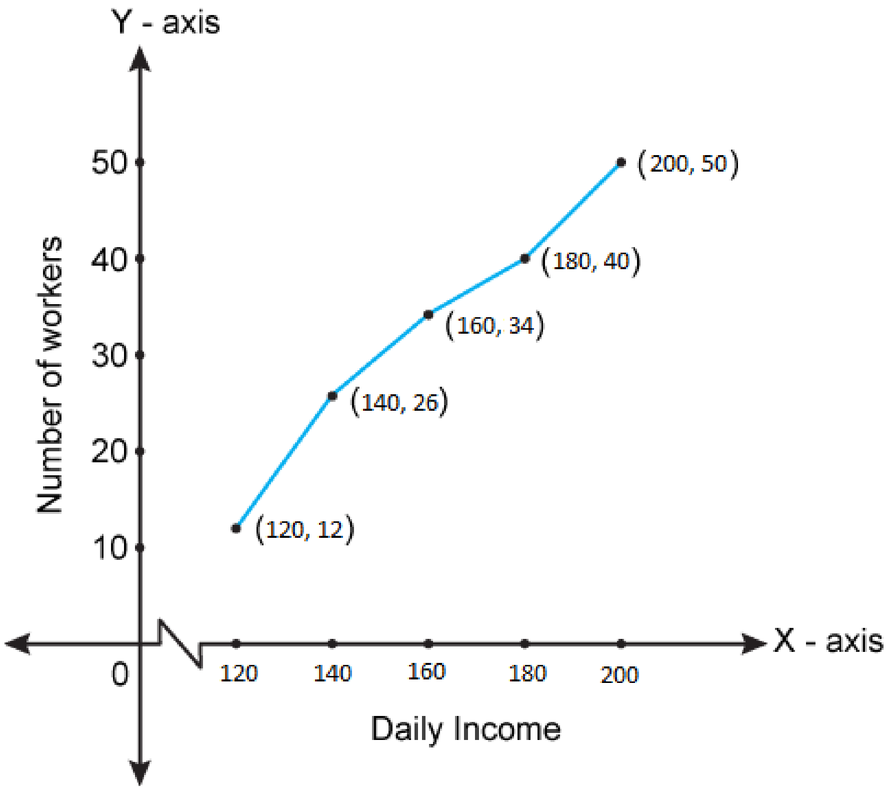
$$= 20 + \frac{10}{13} \times 10$$

$$= 20 + 7.69$$

$$= 27.69 \text{ years (approx.)}$$

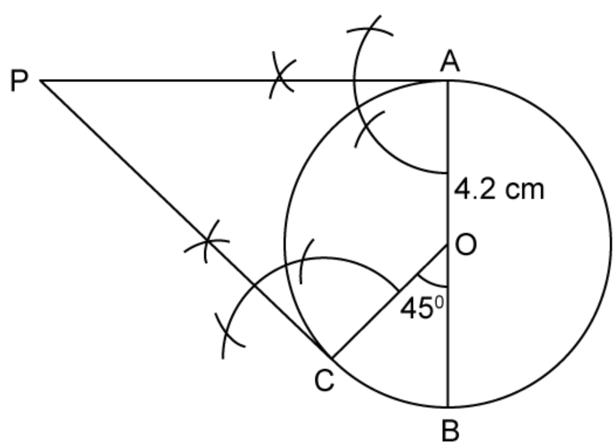
OR

	x	f	cf
100-120	110	12	12
120-140	130	14	26
140-160	150	8	34
160-180	170	6	40
180-200	190	10	50



Section D

35.



Steps of construction:

(i) A circle of radius 4.2 cm at centre O is drawn.

(ii) A diameter AB is drawn.

(iii) With OB as the base, an angle BOC of 45° is drawn.

(iv) At A, a line perpendicular to OA is drawn.

(v) At C, a line perpendicular to OC is drawn.

(vi) These lines intersect each other at P.

PA and PC are the required tangents.

OR

Steps of construction:

Step 1: Draw a line segment BC = 6.5 cm

Step 2: Draw an angle of 60° at B so that $\angle XBC = 60°$

Step 3: With centre B and radius 4.5 cm, draw an arc intersecting XB at A

Step 4: Join AC

ΔABC is the required triangle.

Step 5: Draw a line BY below BC which makes an angle CBY.

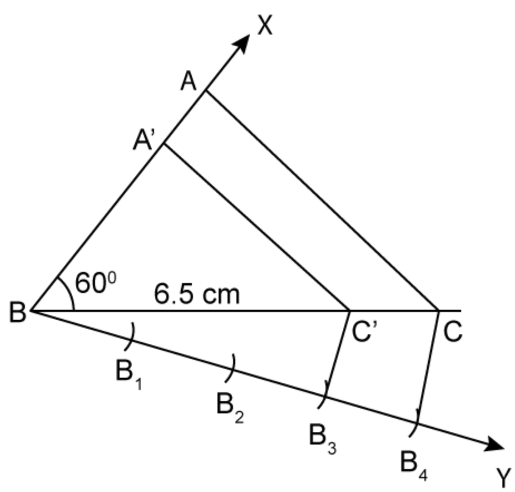
Step 6: Mark B_1, B_2, B_3 and B_4 at equal distances from B such that
$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

Step 7: Join CB_4

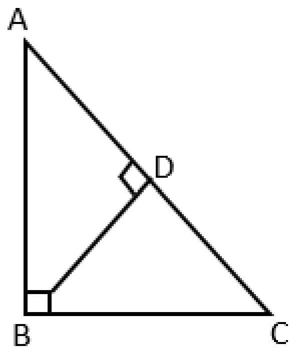
Step 8: Draw B_3C' parallel to CB_4

Step 9: Draw $C'A'$ parallel to CA through C' intersecting BA produced at A'

Δ A'BC' is the required triangle which is similar to ΔABC.



36. Given: In ΔABC, $\angle ABC = 90°$



To prove: $AC^2 = AB^2 + BC^2$

Construction:

Draw seg $BD \perp$ hypotenuse AC and $A - D - C$

Proof:

In ΔABC, seg $BD \perp$ hypotenuse AC ....(construction)

$\therefore \Delta ABC \sim \Delta ADB$ ....(Similarity in right angled triangles)

$\therefore \frac{AB}{AD} = \frac{AC}{AB}$ ....(Corresponding sides of similar triangles)

$\therefore AB^2 = AC \times AD$ ....(i)

Similarly, $\Delta ABC \sim \Delta BDC$ ....(Similarity in right angled triangles)

$\therefore \frac{BC}{DC} = \frac{AC}{BC}$ ....(Corresponding sides of similar triangles)

$\therefore BC^2 = AC \times DC$ ....(ii)

$AB^2 + BC^2 = AC \times AD + AC \times DC$ ....[Adding equations (i) and (ii)]

$= AC(AD + DC)$

$= AC \times AC$

$= AC^2$

37. Let the speed of the boat in still water be x kmph, then

Speed of the boat downstream = (x + 2) kmph

And the speed of the boat upstream = (x – 2) kmph

Time taken to cover 8 km downstream = $\frac{8}{(x+2)}$ hrs

Time taken to cover 8 km upstream = $\frac{8}{(x-2)}$ hrs

Total time taken = $\frac{5}{3}$ hrs

$$\frac{8}{(x+2)}+\frac{8}{(x-2)}=\frac{5}{3}$$

$$\Rightarrow \frac{1}{x+2}+\frac{1}{x-2}=\frac{5}{24}$$

$$\Rightarrow \frac{x-2+x+2}{(x+2)(x-2)}=\frac{5}{24}$$

$$\Rightarrow \frac{2x}{x^2-4}=\frac{5}{24}$$

$$\Rightarrow 5x^2-20-48x=0$$

$$\Rightarrow 5x^2-48x-20=0$$

$$\Rightarrow 5x^2-50x+2x-20=0$$

$$\Rightarrow 5x(x-10)+2(x-10)=0$$

$$\Rightarrow (x-10)(5x+2)=0$$

$$\Rightarrow x=10 \text{ or } x=\frac{-2}{5}$$

$$\Rightarrow x=10 \ (\text{speed cannot be negative})$$

Then the speed of the boat in still water is 10 kmph.

OR

Let the marks obtained by Kamal in Mathematics and English be x and y.

$$\therefore \quad x+y=40 \ldots\ldots(1)$$

$$\text{and } (x+3)(y-4)=360 \ldots\ldots(2)$$

$$\text{From}(1) \ y=40-x$$

$$\text{Putting value of } y \text{ in}(2)$$

$$(x+3)(40-x-4)=360$$

$$\Rightarrow (x+3)(36-x)=360$$

$$\text{or } 36x-x^2+108-3x=360$$

$$\Rightarrow -x^2+33x-252=0 \quad \text{or} \quad x^2-33x+252=0$$

$$\Rightarrow x^2-21x-12x+252=0$$

$$\text{or } x(x-21)-12(x-21)=0$$

$$\Rightarrow (x-21)(x-12)=0$$

$$\therefore \text{when } x-21=0, x=21$$

$$\text{when } x-12=0, x=12$$

$$\text{for } x=21,\ 21+y=40 \quad \therefore y=19$$

$$\text{for } x=12,\ 12+y=40 \quad \therefore y=28$$

The marks obtained by Kamal in Mathematics and English respectively, are (21, 19) and (12, 28).

38. Radius of the hemisphere = r = 4 cm = Radius of cone

Height of cone = h = 4 cm

Volume of toy = Volume of hemisphere + Volume of the cone

$$=\frac{2}{3}\pi r^3+\frac{1}{3}\pi r^2 h$$

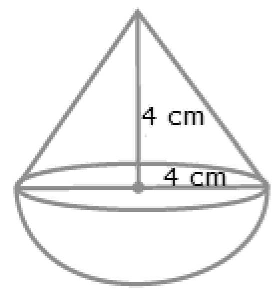
$$=\frac{1}{3}\pi r^2(2r+h)$$

$$=\frac{1}{3}\times\frac{22}{7}\times 4\times 4(2\times 4+4)$$

$$=\frac{1}{3}\times\frac{22}{7}\times 4\times 4\times 12$$

$$=\frac{1408}{7}\text{ cm}^3$$

$$=201.14\text{ cm}^3$$



It is given that a cube circumscribes the given toy.

$\Rightarrow$ Edge of the cube $=8$ cm

$\Rightarrow$ Volume of the cube $=(8)^3=512\text{ cm}^3$

Difference in the volumes of the cube and the toy $=512-201.14=310.86\text{ cm}^3$

Total surface area of the toy $=$ CSA of cone $+$ CSA of hemisphere

$$=\pi rl+2\pi r^2$$

$$=\pi r(l+2r)$$

Now, $l=\sqrt{h^2+r^2}=\sqrt{4^2+4^2}=\sqrt{32}=4\sqrt{2}$ cm

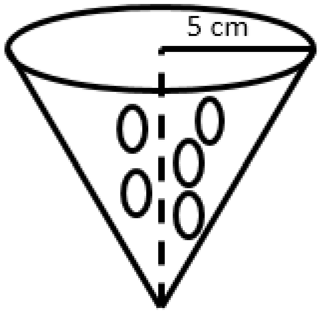
$\therefore$ Total surface area of the toy $=\frac{22}{7}\times 4\left(4\sqrt{2}+2\times 4\right)$

$$=\frac{22}{7}\times 16(1.414+2)$$

$$=\frac{352}{7}\times 3.414$$

$$=171.68\text{ cm}^2$$

OR



Let the number of lead shots dropped be n.
Then total volume of n – lead shots = ¼ of volume of the conical vessel
Lead shots:
r = 0.5 cm

V = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 0.5\times 0.5\times 0.5$

Total volume of n – shots = $n\times\frac{4}{3}\times\frac{22}{7}\times 0.125$

Cone :
r = 5 cm, h = 8 cm

V = $\frac{1}{3}\times\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 5\times 5\times 8 = \frac{1}{3}\times\frac{22}{7}\times 200$

¼ volume = $\frac{1}{4}\times\frac{1}{3}\times\frac{22}{7}\times 200$

$n\times\frac{4}{3}\times\frac{22}{7}\times 0.125 = \frac{1}{4}\times\frac{1}{3}\times\frac{22}{7}\times 200$

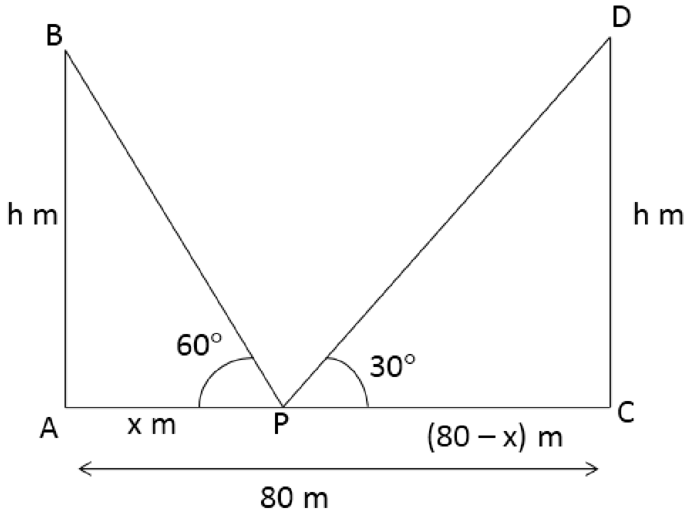
n = $\frac{200\times 1}{4\times 4\times 0.125} = 100$

Number of lead shots = 100.

39. Let AB and CD be the two poles, and let the height of the poles be h metres.

Let AP = x

CP = 80 – x



In ΔPAB,

$$\Rightarrow \tan 60° = \frac{AB}{AP} = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x ----(1)$$

In right ΔPCD

$$\tan 30° = \frac{CD}{PC}$$

$$\Rightarrow \frac{h}{80-x} = \tan 30° \Rightarrow \frac{h}{80-x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{80-x}{\sqrt{3}} ----(2)$$

From (1) and (2), we get

$$\frac{80-x}{\sqrt{3}} = \sqrt{3}x \Rightarrow 80 - x = 3x$$

$$\Rightarrow \quad 80 = 4x$$

$$\Rightarrow \quad x = 20 \text{ m}$$

Putting $x = 20$ in (1), we get

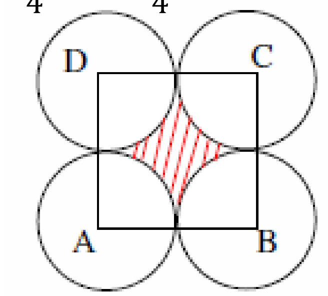
$$h = \sqrt{3} \times 20 = 20\sqrt{3} = 34.64 \text{ m}$$

Hence, the height of the two poles is 34.64 m.

AP = x = 20 m and DC = 80 – x = 60 m

40. Radius of each circle is 1 unit.

Area of each quadrant of a circle of radius 1 cm $= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(1)^2 = \frac{1}{4}\pi$ sq. units



$\therefore$ Area of four quadrants $= 4 \times \frac{1}{4}\pi = \pi$ sq. units

Now, side of the square ABCD = 1 + 1 = 2 units

$\therefore$ Area of square = 2 × 2 = 4 sq. units

Thus, area of shaded region

= Area of square ABCD – Area of four quadrants

= 4 – π

$\therefore$ Required probability = $= \frac{(4-\pi)}{4} = 1 - \frac{\pi}{4}$

OR

Total number of tickets = 100

(i) Even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100.
Total number of even numbers = 50
P(getting an even number) = $\frac{50}{100} = \frac{1}{2}$

(ii) Numbers less than 16 are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
Total number of numbers less than 16 is 14.
P(getting a number less than 16) = $\frac{14}{100} = \frac{7}{50}$

(iii) Numbers which are a perfect square are 4, 9, 16, 25, 36, 49, 64, 81, 100.
Total number of perfect squares = 9
P(getting a perfect square) = $\frac{9}{100}$

(iv) Prime numbers less than 40 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37.
Total number of prime numbers = 12
P(getting a prime number less 40) = $\frac{12}{100} = \frac{3}{25}$