CBSE Board

Class X Mathematics

Sample Paper 5 (Standard) – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. Correct option: (d)

Explanation:

$(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$

$\text{LCM} = 2^4 \times 3 \times 5 \times 7 = 1680$

2. Correct option: (d)

Explanation:

Range is not a measure of central tendency.

3.

Correct option: (c)

Explanation :

The largest number that divides each one of

1152 and 1664 exactly will be the HCF of the numbers.

Using Euclid's Division Algorithm,

$1664=1152\times1+512$

$1152=512\times2+128$

$512=128\times4+0$

So, HCF(1152,1664)=128

Hence, the largest number is 128.

4.

Correct option: (a)

Explanation :

$$\frac{2x}{3} - \frac{y}{2} + \frac{1}{6} = 0$$

Multiply by the LCM, 6.

$\Rightarrow 4x - 3y + 1 = 0$

$\Rightarrow 4x - 3y = -1 \quad ....(i)$

$\frac{x}{2}+\frac{2y}{3}=3$

Multiply by the LCM, 6.

$\Rightarrow 3x+4y=18$ ....(ii)

Multiply equation (i) and (ii) by 4 and 3 respectively.

$16x-12y=-4$ ....(iii)

$9x+12y=54$ ....(iv)

Adding equations (iii) and (iv), we get

$25x=50$

$\Rightarrow x=2$

Substituting $x=2$ in (ii), we get $y=3$.

5.

Correct option: (b)

Explanation:

$\tan 5°\tan 25°\tan 30°\tan 65°\tan 85°$

$=\tan 5°\times\tan 25°\times\frac{1}{\sqrt{3}}\times\tan(90°-25°)\times\tan(90°-5°)$

$=\tan 5°\times\tan 25°\times\frac{1}{\sqrt{3}}\times\cot 25°\times\cot 5°$

$=\tan 5°\times\cot 5°\times\tan 25°\times\cot 25°\times\frac{1}{\sqrt{3}}$

$=1\times1\times\frac{1}{\sqrt{3}}$

$=\frac{1}{\sqrt{3}}$

6.

Correct option: (c)

Explanation:

Since $\cos 90°=0$

$\cos 1°\cos 2°\cos 3°...\cos 90°...\cos 180°=0$

7.

Correct option: (d)

Explanation:

$\frac{2\sin^2 63^\circ+1+2\sin^2 27^\circ}{3\cos^2 17^\circ-2+3\cos^2 73^\circ}$

$=\frac{2\sin^2 63^\circ+2\sin^2 27^\circ+1}{3\cos^2 17^\circ+3\cos^2 73^\circ-2}$

$$= \frac{2\sin^2 63^\circ + 2\cos^2 63^\circ + 1}{3\cos^2 17^\circ + 3\sin^2 17^\circ - 2}$$
$$= \frac{2(\sin^2 63^\circ + \cos^2 63^\circ) + 1}{3(\cos^2 17^\circ + \sin^2 17^\circ) - 2}$$
$$= \frac{2 \times 1 + 1}{3 \times 1 - 2}$$
$$= \frac{2+1}{3-2}$$
$$= 3$$

8.

Correct option: (c)

Explanation:

The distance of the point $P(-3,4)$ from the x-axis

$=$ y-coordinate of the point

$= 4$ units

9.

Correct option: (b)

Explanation:

Since the point lies on the x-axis, let the point be P

and its coordinates be $(x,0)$.

Given that the point is equidistant from the points A and B.

$\Rightarrow PA = PB$

$\Rightarrow \sqrt{(x+1)^2} = \sqrt{(x-5)^2}$

$\Rightarrow (x+1)^2 = (x-5)^2$

$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$

$\Rightarrow 2x + 1 = -10x + 25$

$\Rightarrow 12x = 24$

$\Rightarrow x = 2$

Hence, the point is $(2,0)$.

10.

Correct option: (b)

Explanation:

Given that R is the mid-point of the line segment AB.

The y-coordinate of $R = \dfrac{5+y}{2}$

$$\Rightarrow 6 = \frac{5+y}{2}$$

$$\Rightarrow 12 = 5 + y$$

$$\Rightarrow y = 7$$

11. The area of a square field is 6050 m^2. The length of its diagonal is <u>110 m</u>

Explanation:-

We know that all the sides of a square are equal.

Let each side of the square $= x$ m

Area of the square $= (\text{side})^2$

$$\Rightarrow 6050 = x^2$$

$$\Rightarrow x = 77.78$$

$\Rightarrow$ Each side of the square $= 77.8$ m

We know that,

Length of the diagonal $= \sqrt{2}\,x$

$$= 1.414 \times 77.8$$

$$= 110 \text{ m}$$

12. If one zero of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -4, then the value of k is <u>5/4</u>

Explanation:-

Since -4 is a zero of $f(x) = (k-1)x^2 + kx + 1$, we have

$f(-4) = 0$

$$\Rightarrow (k-1)(-4)^2 + k(-4) + 1 = 0$$

$$\Rightarrow (k-1)16 - 4k + 1 = 0$$

$$\Rightarrow 16k - 16 - 4k + 1 = 0$$

$$\Rightarrow 12k - 15 = 0$$

$$\Rightarrow 12k = 15$$

$$\Rightarrow k = \frac{15}{12} = \frac{5}{4}$$

OR

If one zero of $3x^2 + 8x + k$ be the reciprocal of the other then k = <u>3</u>

Explanation:-

Let α and $\frac{1}{\alpha}$ be the zeros of $3x^2 + 8x + k$.

Then, we have

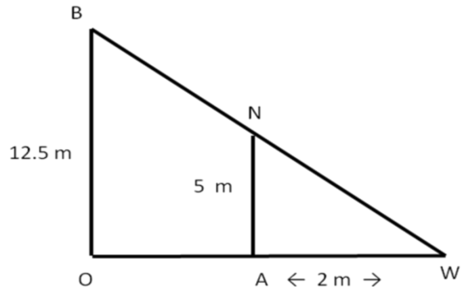
$$\alpha \times \frac{1}{\alpha} = \frac{k}{3}$$

$$\Rightarrow 1 = \frac{k}{3}$$

$$\Rightarrow k = 3$$

13. The shadow of a 5 m long stick is 2 m long. At the same time the length of the shadow of a 12.5 m high tree (in m) is 5m

Explanation:-



Let AN be the long stick and AW be its shadow.

Let OB be the tree and OW be its shadow.

$AW = 2\text{ m}$

$AN = 5\text{ m}$

$OB = 12.5\text{ m}$

Ratio of actual lengths = Ratio of their shadows

$$\Rightarrow \frac{OB}{AN} = \frac{OW}{AW}$$

$$\Rightarrow \frac{12.5}{5} = \frac{OW}{2}$$

$$\Rightarrow OW = \frac{12.5 \times 2}{5}$$

$$\Rightarrow OW = 5.0\text{ m}$$

So, the length of the shadow is 5.0 m

14. The sum of first n terms of an AP is $(3n^2 + 6n)$. The common difference of the AP is 6

Explanation:-

The sum of first n terms of an AP is $(3n^2 + 6n)$.

$$S_n = 3n^2 + 6n$$

$$\Rightarrow S_{n-1} = 3(n-1)^2 + 6(n-1)$$

$$= 3(n^2 - 2n + 1) + 6(n-1)$$

$$= 3n^2 - 6n + 3 + 6n - 6$$

$$= 3n^2 - 3$$

$$a_n = S_n - S_{n-1}$$

$$= 3n^2 + 6n - 3n^2 + 3$$

$$= 6n + 3$$

Let d be the common difference of the AP.

$$\begin{aligned} d &= a_n - a_{n-1} \\ &= (6n+3) - [6(n-1)+3] \\ &= (6n+3) - 6(n-1) - 3 \\ &= 6 \end{aligned}$$

15. If the probability of occurrence of an event is p then the probability of non-happening of this event is 1-p

Explanation:-

Let E be the event.

So, the probability of the event happening will be P(E).

Thus, the probability of the event not happening will be P(E').

Given that, $P(E) = p$

We know that, $P(E) + P(E') = 1$

$\Rightarrow p + P(E') = 1$

$\Rightarrow P(E') = 1 - p$

16.

Let the numbers be a and 81.

HCF×LCM=product of the two numbers

$\Rightarrow 27 \times 162 = 81a$

$\Rightarrow a = 54$

So, the other number is 54.

17.

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \dfrac{AB}{DE} = \dfrac{BC}{EF}$

$\Rightarrow \dfrac{1}{2} = \dfrac{6}{EF}$

$\Rightarrow EF = 12$ cm

18.

$PT = 24$ cm

$OT = 7$ cm

Since PT is a tangent to the circle at T.

$\angle PTO = 90°$ ....(tangent is perpendicular to the radius of a circle)

In ΔPTO,

By Pythagoras theorem,

$OP^2 = PT^2 + OT^2$

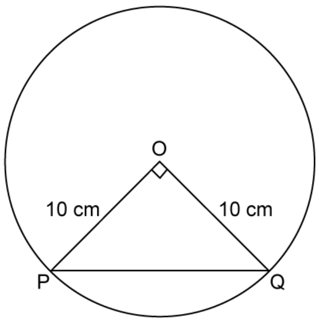
$\Rightarrow OP^2 = 24^2 + 7^2$

$\Rightarrow OP^2 = 576 + 49$

$\Rightarrow OP^2 = 625$

$\Rightarrow OP = 25 \text{ cm}$

OR



In ΔPOQ,

By Pythagoras theorem,

$PQ^2 = PO^2 + OQ^2$

$\Rightarrow PQ^2 = 10^2 + 10^2$

$\Rightarrow PQ^2 = 100 + 100$

$\Rightarrow PQ^2 = 200$

$\Rightarrow PQ = 10\sqrt{2} \text{ cm}$

So, the length of the chord is $10\sqrt{2}$ cm.

19.

The given AP is $21, 18, 15, \ldots$

$a = 21$ and $d = 18 - 21 = -3$

$a_n = a + (n-1)d$

$\Rightarrow -81 = 21 + (n-1)(-3)$

$\Rightarrow -81 = 21 + (n-1)(-3)$

$\Rightarrow -81 = 21 - 3n + 3$

$\Rightarrow 3n = 105$

$\Rightarrow n = 35$

So, -81 is the 35th term.

20.

$x^2+12x+35=0$

$\Rightarrow x^2+7x+5x+35=0$

$\Rightarrow x(x+7)+5(x+7)=0$

$\Rightarrow (x+7)(x+5)=0$

$\Rightarrow x+7=0$ or $x+5=0$

$\Rightarrow x=-7$ or $x=-5$

Section B

21.

To find the HCF of 12, 15, 18, 27

we will find the prime factorization of each number.

$12=2^2\times3$

$15=3\times5$

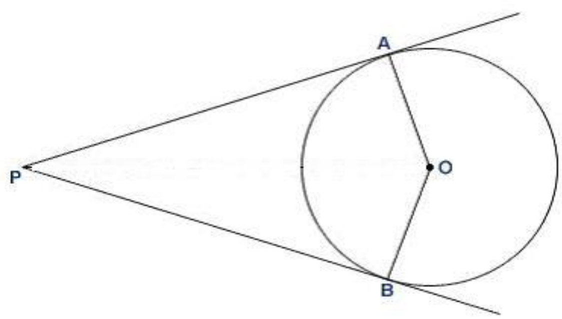
$18=2\times3^2$

$27=3^3$

So, the HCF=3

$LCM=2^2\times3^3\times5=540$

22.



Given: PA and PB are the tangents drawn from a point P to a circle with centre O.
Also, the line segments OA and OB are drawn.

To prove: $\angle APB+\angle AOB=180°$

Proof:

We know that the tangent is perpendicular to the radius through the point of contact.

$\therefore PA \perp OA \Rightarrow \angle OAP = 90°$

$\therefore PB \perp OB \Rightarrow \angle OBP = 90°$

$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$ ....(i)

But, we know that the sum of all the angles of a quadrilateral is 360°.

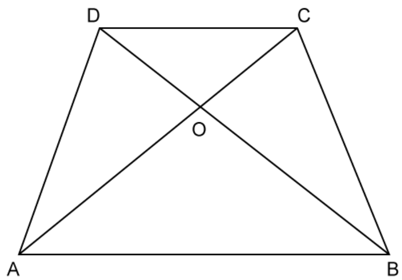
$\therefore \angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$ ....(ii)

From (i) and (ii), we get

$\angle APB + \angle AOB = 180°$

Hence proved.

23.



The diagonals of a trapezium divide each other proportionally.

$\angle CDO = \angle OBA$ ....(alternate angles)

$\angle COD = \angle AOB$ ....(vertically opposite angles)

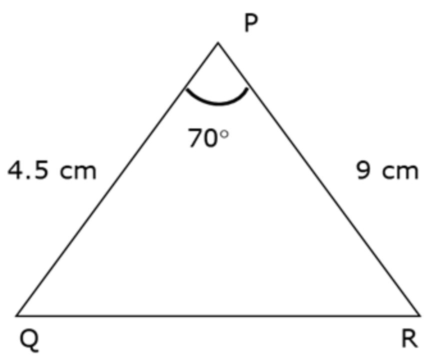
$\Rightarrow \Delta COD \sim \Delta AOB$ ...(AA criterion for similarity)

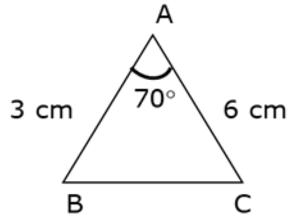
$\Rightarrow \dfrac{ar(\Delta COD)}{ar(\Delta AOB)} = \dfrac{CD^2}{AB^2}$

$\Rightarrow \dfrac{ar(\Delta COD)}{84} = \dfrac{1^2}{2^2}$

$\Rightarrow ar(\Delta COD) = 21 \text{ cm}^2$

OR





In ΔABC and ΔPQR,

$\angle A = \angle P = 70^\circ$ ....(Given)

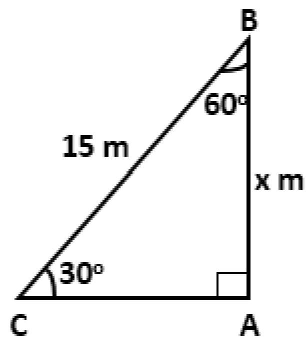
$\frac{AB}{PQ} = \frac{3}{4.5} = \frac{2}{3}$

$\frac{AC}{PR} = \frac{6}{9} = \frac{2}{3}$

$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$

So, $\Delta ABC \sim \Delta PQR$ ....(SAS criterion for similarity)

24.



Let BC be the ladder and AB be the wall.

Then, $BC = 15$ m

$\angle ABC = 60^\circ$

$\Rightarrow \angle ACB = 90^\circ - 60^\circ = 30^\circ$

Let the height of the wall $AB = x$ m

Now, $\sin 30^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{1}{2} = \frac{x}{15}$

$\Rightarrow x = \frac{15}{2}$ m

25.

Let E be an event of winning the game, then E' will be losing it.

$P(E) + P(E') = 1$

$\Rightarrow 0.7 + P(E') = 1$

$\Rightarrow P(E') = 0.3$

Hence, the probability of losing the game is 0.3.

OR

There are 35 students in a class of whom 20 are boys and 15 are girls.

(i) $P(\text{choosing a boy}) = \frac{20}{35} = \frac{4}{7}$

(ii) $P(\text{choosing a girl}) = \frac{15}{35} = \frac{3}{7}$

26.

Let the number of solid spheres be n.

Given Diameter of sphere $= 6$ cm $\Rightarrow$ radius $= 3$ cm

Diameter of cylinder $= 4$ cm

$\Rightarrow$ radius $= 2$ cm and height of the cylinder $= 45$ cm

Now,

Volume of the cylinder $=$ Volume of the sphere $\times$ n

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r^3 \times n$

$\Rightarrow \pi \times 2 \times 2 \times 45 = \frac{4}{3} \times \pi \times 3 \times 3 \times 3 \times n$

$\Rightarrow 45 = 9 \times n$

$\Rightarrow n = \frac{45}{9}$

$\Rightarrow n = 5$

Hence, number of solid spheres is 5.

Section C

27.

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers.

Let $x = 3.\overline{1416}$

$\Rightarrow x = 3.141614161416.....$ ....(i)

Since there are four repeating digits, we multiply by 1000.

$\Rightarrow 1000x = 31416.14161416.....$ ....(ii)

Subtracting (i) from (ii), we get

$999x = 31416$

$\Rightarrow x = \frac{31416}{999}$ which is of the form $\frac{p}{q}$.

So, $3.\overline{1416}$ is a rational number.

OR

We have $\frac{2}{\sqrt{7}}=\frac{2}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{2\sqrt{7}}{7}$ ...(i)

Let $\frac{2}{\sqrt{7}}$ be rational.

Then, from (i), $\frac{2}{7}\sqrt{7}$ is rational.

Now, $\frac{7}{2}$ is rational, $\frac{2}{7}\sqrt{7}$ is rational.

$\Rightarrow\left(\frac{7}{2}\times\frac{2}{7}\sqrt{7}\right)$ is rational.

$\Rightarrow\sqrt{7}$ is rational.

Thus, from (i), it follows that $\sqrt{7}$ is rational.

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction arises by assuming that $\frac{2\sqrt{7}}{7}$ is rational.

Hence, $\frac{2\sqrt{7}}{7}$ is irrational.

28.

The man arranges to pay off a debt of Rs. 36000 by 40 monthly installments.

So, n=40 and S_{40}=36000

Let the first installment be Rs. a, and let d be the common difference.

one-third debt is unpaid, that means two-third is paid.

$\frac{2}{3}(36000)$=Rs.24000

$S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow S_{30}=\frac{30}{2}[2a+29d]$

$\Rightarrow 24000=15[2a+29d]$

$\Rightarrow 1600=2a+29d$

$\Rightarrow 2a+29d=1600$ .....(i)

$S_{40}=\frac{40}{2}[2a+39d]$

$\Rightarrow 36000=20[2a+39d]$

$\Rightarrow 1800=2a+39d$

$\Rightarrow 2a+39d=1800$ .....(ii)

Subtracting (i) from (ii), we get

$10d=200$

$\Rightarrow d=20$

Substituting in (i) ,we get

$2a+29(20)=1600$

$\Rightarrow 2a=1020$

$\Rightarrow a=510$

Hence, the first installment he paid was Rs. 510.

29.

$23x+29y=98$ .....(i) and

$29x+23y=110$ .....(ii)

Adding (i) and (ii), we get

$52x+52y=208$

$\Rightarrow x+y=4$ .....(iii)

Subtract (i) from (ii), we get

$6x-6y=12$

$\Rightarrow x-y=2$ .....(iv)

Adding (iii) and (iv), we get

$2x=6$

$\Rightarrow x=3$

Substituting $x=3$ in (iii), we get $y=1$.

Hence, $x=3$ and $y=1$.

OR

$6x+3y=7xy$ and $3x+9y=11xy$

Dividing throughout by xy, we get

$\frac{6}{y}+\frac{3}{x}=7$ and $\frac{3}{y}+\frac{9}{x}=11$

$\frac{3}{x}+\frac{6}{y}=7$ and $\frac{9}{x}+\frac{3}{y}=11$

Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$

So, we get

$3u+6v=7$ and $9u+3v=11$

.....(i) .....(ii)

Multiply (i) by 3 and subtract (ii) from the resultant.

$\Rightarrow 9u+18v=21$ and $9u+3v=11$

$\Rightarrow 15v=10$

$\Rightarrow v=\frac{2}{3}$

Substituting $v=\frac{2}{3}$ in (i), we get $u=1$.

$\Rightarrow \frac{1}{x}=1$ and $\frac{1}{y}=\frac{2}{3}$

$\Rightarrow x=1$ and $y=\frac{3}{2}$

30.

The given polynomial is $p(x)=2x^4-3x^3-3x^2+6x-2$

Since $\sqrt{2}$ and $-\sqrt{2}$ are the zeros of p(x), it follows that

each one $\left(x-\sqrt{2}\right)$ and $\left(x+\sqrt{2}\right)$ is a factor of p(x).

Consequently, $\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)=\left(x^2-2\right)$ is a factor of p(x).

On dividing p(x) by (x^2-2), we get

$$
\begin{array}{r|l}
 & 2x^2-3x+1 \\
\hline
x^2-2 & 2x^4-3x^3-3x^2+6x-2 \\
 & (-)\,2x^4 \qquad -4x^2 \\
 & \quad - \qquad\quad + \\
\hline
 & -3x^3+x^2+6x-2 \\
 & (-)\,-3x^3 \qquad +6x \\
 & \quad + \qquad\quad - \\
\hline
 & x^2-2 \\
 & (-)\,x^2-2 \\
 & \quad - \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore p(x)=0$

$\Rightarrow (x^2-2)(2x^2-3x+1)=0$

$\Rightarrow \left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)\left(2x^2-2x-x+1\right)=0$

$\Rightarrow \left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)[2x(x-1)-1(x-1)]=0$

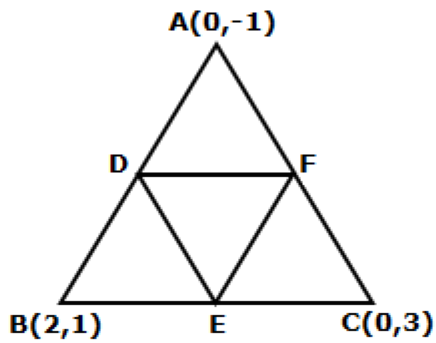
$\Rightarrow \left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)(x-1)(2x-1)=0$

$\Rightarrow \left(x-\sqrt{2}\right)=0$ or $\left(x+\sqrt{2}\right)=0$ or $(x-1)=0$ or $(2x-1)=0$

$\Rightarrow x=\sqrt{2}$ or $x=-\sqrt{2}$ or $x=1$ or $x=\frac{1}{2}$

Thus, the other two zeros are 1 and $\frac{1}{2}$.

31.



$$\text{Area of } \Delta ABC = \frac{1}{2}\left|0(1-3)+2(3+1)+0(-1-1)\right|$$
$$= \frac{1}{2}\left|0+8+0\right|$$
$$= \frac{1}{2}\times 8$$
$$= 4 \text{ sq. units}$$

Let D, E, F be the mid-points of AB, BC and CA respectively.

Then, coordinates of D, E and F are given as

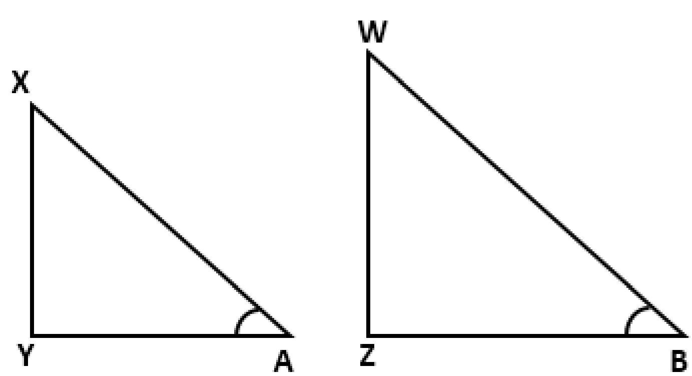
$$D\left(\frac{0+2}{2}, \frac{-1+1}{2}\right), E\left(\frac{2+0}{2}, \frac{1+3}{2}\right) \text{ and } F\left(\frac{0+0}{2}, \frac{3-1}{2}\right)$$

i.e. D(1,0), E(1,2) and F(0, 1)

$$\therefore \text{ Area of } \Delta DEF = \frac{1}{2}\left|1(2-1)+1(1-0)+0(0-2)\right|$$
$$= \frac{1}{2}\left|1+1+0\right|$$
$$= \frac{1}{2}\times 2$$
$$= 1 \text{ sq. unit}$$

Thus, Area of ΔABC : Area of $\Delta DEF = 4:1$

32. Consider two right triangles XAY and WBZ such that tan A = tan B



We have,

$$\tan A = \frac{XY}{AY} \text{ and } \tan B = \frac{WZ}{BZ}$$

Since $\tan A = \tan B$

$$\Rightarrow \frac{XY}{AY} = \frac{WZ}{BZ}$$

$$\Rightarrow \frac{XY}{WZ} = \frac{AY}{BZ} = k(\text{say}) \quad ....(i)$$

$$\Rightarrow XY = k \times WZ \text{ and } AY = k \times BZ \quad ....(ii)$$

Using Pythagoras theorem in triangles XAY and WBZ, we have

$$XA^2 = XY^2 + AY^2 \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow XA^2 = k^2 WZ^2 + k^2 BZ^2 \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow XA^2 = k^2 (WZ^2 + BZ^2) \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow \frac{XA^2}{WB^2} = \frac{k^2 (WZ^2 + BZ^2)}{(WZ^2 + BZ^2)} = k^2$$

$$\Rightarrow \frac{XA}{WB} = k \quad ....(iii)$$

From (i), (ii) and (iii), we get

$$\frac{XY}{WZ} = \frac{AY}{BZ} = \frac{XA}{WB}$$

$$\Rightarrow \Delta AYX \sim \Delta BZW$$

$$\Rightarrow \angle A = \angle B$$

OR

$$\text{L.H.S.} = \frac{\tan A}{(1 - \cot A)} + \frac{\cot A}{(1 - \tan A)}$$

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1 - \tan A}$$

$$= \frac{\tan A}{\frac{\tan A - 1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A(\tan A - 1)}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)} \quad ....[a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= \frac{\tan^2 A + \tan A + 1}{\tan A}$$

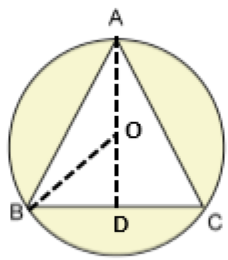
$$= \frac{\tan^2 A}{\tan A} + \frac{\tan A}{\tan A} + \frac{1}{\tan A}$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \tan A + \cot A$$

$$= \text{R.H.S.}$$

33.



Let O be the centre of the circumcircle.

Join OB and draw $AD \perp BC$.

Then, $OB = 42\text{cm}$ and $\angle OBD = 30°$

In ΔOBD,

$$\sin\ 30° = \frac{OD}{OB}$$

$$\Rightarrow \frac{1}{2} = \frac{OD}{42}$$

$$\Rightarrow OD = 21\text{ cm}$$

Now, $BD^2 = OB^2 - OD^2 = 42^2 - 21^2 = (42+21)(42-21) = 63 \times 21$

$$\Rightarrow BD = \sqrt{63 \times 21} = \sqrt{3 \times 21 \times 21} = 21\sqrt{3}\text{ cm}$$

$$\Rightarrow BC = 2 \times 21\sqrt{3} = 42\sqrt{3}\text{cm}$$

Now, area of the shaded region

= Area of the circle – Area of an equilateral ΔABC

$$= \frac{22}{7} \times 42 \times 42 - \frac{\sqrt{3}}{4} \times 42\sqrt{3} \times 42\sqrt{3}$$

$$= (5544 - 2291.5)\text{ cm}^2$$

$$= 3252.5\text{ cm}^2$$

34.

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
$0-10$	16	5	80
$10-20$	p	15	15p
$20-30$	30	25	750
$30-40$	32	35	1120
$40-50$	14	45	630
	$\sum f_i = 92+p$		$\sum f_i x_i = 2580+15p$

Now, Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 25 = \dfrac{2580+15p}{92+p}$

$\Rightarrow 25(92+p) = 2580+15p$

$\Rightarrow 2300+25p = 2580+15p$

$\Rightarrow 10p = 280$

$\Rightarrow p = 28$

Section D

35.

Steps of construction:

1) Draw a line segment $BC = 5$ cm

2) With B as centre and radius 6 cm, draw an arc.

3) With C as centre and radius 7 cm, draw another arc intersecting previous arc at A.

4) Join AB and AC to obtain ΔABC.

5) Below BC, make an acute $\angle CBX$.

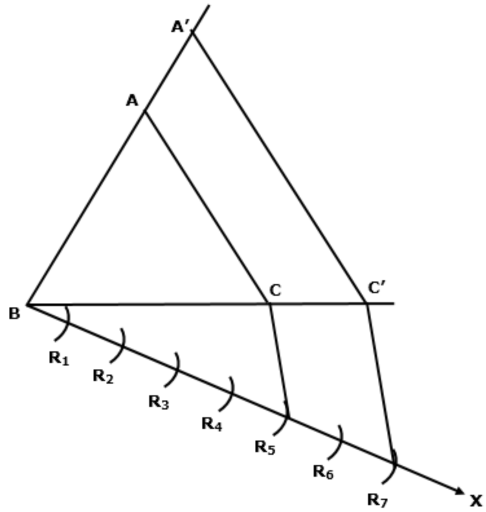
6) Along BX, mark off 7 points $R_1, R_2, R_3, R_4, R_5, R_6, R_7$ such that $BR_1 = R_1R_2 = R_2R_3 = R_3R_4 = \ldots\ldots = R_6R_7$

7) Join R_5C

8) From R_7, draw $R_7C' \parallel R_5C$, meeting BC produced at C'.

9) From C', draw $C'A' \parallel CA$, meeting BA produced at A'.

Thus, $\Delta A'BC'$ is the required triangle.

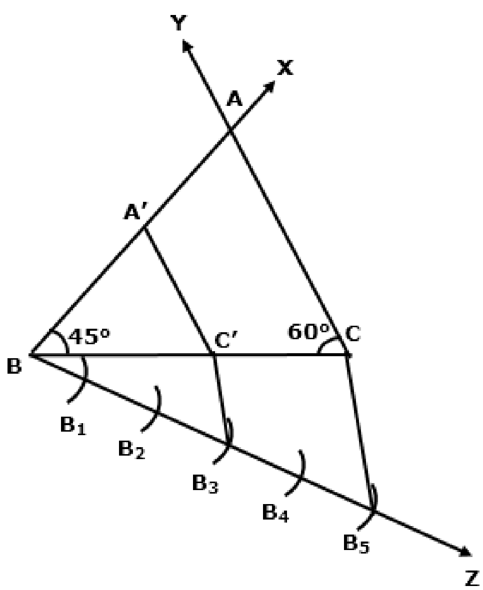


OR

Steps of construction:

1) Draw a line segment $BC = 8$ cm
2) At B, construct $\angle XBC = 45°$ and at C, construct $\angle YCB = 60°$
   Suppose BX and CY intersect at A.
   ΔABC so obtained is the given triangle.
3) Below BC, make an acute $\angle CBZ$.
6) Along BZ, mark off 5 points B_1, B_2, B_3, B_4, B_5
   such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$
7) Join B_5C
8) From B_3, draw $B_3C' \parallel B_5C$, meeting BC at C'.
9) From C', draw $C'A' \parallel CA$, meeting AB at A'.

Thus, $\Delta A'BC'$ is the required triangle.



36.

In ΔPAC and ΔQBC,

$\angle PAC = \angle QBC = 90°$

$\angle APC = \angle BQC$ .....(corresponding angles)

$\Delta PAC \sim \Delta QBC$ ....(AA criterion for similarity)

$\Rightarrow \frac{AP}{BQ} = \frac{AC}{BC}$

$\Rightarrow \frac{x}{z} = \frac{a+b}{b}$

$\Rightarrow a+b = \frac{bx}{z}$ .....(i)

In ΔRCA and ΔQBA,

$\angle RCA = \angle QBA = 90°$

$\angle CRA = \angle BQA$ .....(corresponding angles)

$\Delta RCA \sim \Delta QBA$ ....(AA criterion for similarity)

$\Rightarrow \frac{RC}{QB} = \frac{AC}{AB}$

$\Rightarrow \frac{y}{z} = \frac{a+b}{a}$

$\Rightarrow a+b = \frac{ay}{z}$ .....(ii)

From (i) and (ii),

$\frac{ay}{z} = \frac{bx}{z}$

$\Rightarrow \frac{a}{b} = \frac{x}{y}$ .....(iii)

From (i), we have

$\frac{x}{z} = \frac{a+b}{b}$

$\Rightarrow \frac{x}{z} = \frac{a}{b} + 1$

$\Rightarrow \frac{x}{z} = \frac{x}{y} + 1$ ....(from (iii))

$\Rightarrow \frac{1}{z} = \frac{1}{y} + \frac{1}{x}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

Hence proved.

37.

Suppose the faster pipe takes x minutes to fill the tank.

Then, the slower pipe will take $(x+5)$ minutes to fill the tank.

$\therefore$ Portion of the tank filled by the faster pipe in one minute $=\frac{1}{x}$

$\Rightarrow$ Portion of the tank filled by the faster pipe in $\frac{100}{9}$ minutes

$$=\frac{1}{x}\times\frac{100}{9}$$
$$=\frac{100}{9x}$$

Similarly, portion of the tank filled by the slower pipe in $\frac{100}{9}$ minutes

$$=\frac{1}{x+5}\times\frac{100}{9}$$
$$=\frac{100}{9(x+5)}$$

It is given that the tank is filled in $\frac{100}{9}$ minutes.

$$\therefore \frac{100}{9x}+\frac{100}{9(x+5)}=1$$
$$\Rightarrow \frac{100}{x}+\frac{100}{x+5}=9$$
$$\Rightarrow \frac{100x+500+100x}{x^2+5x}=9$$
$$\Rightarrow 200x+500=9x^2+45x$$
$$\Rightarrow 9x^2-155x-500=0$$
$$\Rightarrow 9x^2-180x+25x-500=0$$
$$\Rightarrow 9x(x-20)+25(x-20)=0$$
$$\Rightarrow (x-20)(9x+25)=0$$
$$\Rightarrow x-20=0 \text{ or } 9x+25=0$$
$$\Rightarrow x=20 \text{ or } x=-\frac{25}{9}$$

Since time cannot be negative, $x\neq-\frac{25}{9}$.

$$\Rightarrow x=20$$
$$\Rightarrow x+5=20+5=25$$

Hence, the faster pipe fills the tank in 20 minutes and the slower pipe fills the tank in 25 minutes.

OR

Suppose B alone takes x days to finish the work.

Then, A alone can finish it in $(x-10)$ days.

Now, $(\text{A's one day's work}) + (\text{B's one day work}) = \dfrac{1}{x-10} + \dfrac{1}{x}$

And, $(A+B)\text{'s one day's work} = \dfrac{1}{12}$

$$\therefore \frac{1}{x-10} + \frac{1}{x} = \frac{1}{12}$$

$$\Rightarrow \frac{x+x-10}{x(x-10)} = \frac{1}{12}$$

$$\Rightarrow 12(2x-10) = x(x-10)$$

$$\Rightarrow 24x - 120 = x^2 - 10x$$

$$\Rightarrow x^2 - 34x + 120 = 0$$

$$\Rightarrow x^2 - 30x - 4x + 120 = 0$$

$$\Rightarrow x(x-30) - 4(x-30) = 0$$

$$\Rightarrow (x-30)(x-4) = 0$$

$$\Rightarrow x-30=0 \text{ or } x-4=0$$

$$\Rightarrow x=30 \text{ or } x=4$$

Since x cannot be less than 10, $x \neq 4$.

$$\Rightarrow x = 30$$

Hence, B alone can finish the work in 30 days.

38.

Let x hours be the time taken by the pipe to fill the tank.

$\because$ The water is flowing at the rate of 4 km/hr,

$\therefore$ Length of the water column in x hours is $4x \text{ km} = 4000x$ m.

$\therefore$ The length of the pipe is 4000x m

The diameter of the pipe $= 20$ cm

$\Rightarrow$ radius $= 10$ cm

$$= \frac{10}{100} \text{ m}$$

$$= 0.1 \text{ m}$$

$\therefore$ Volume of the water flowing through the pipe in x hours $= V_1$

$$= \pi r^2 h$$

$$= \pi \times (0.1)^2 \times 4000x \quad \text{...(i)}$$

Given Diameter of the cylindrical tank $= 10$ m

$\Rightarrow$ radius $= 5$ cm and

Volume of the water that falls into the tank in x hours $= V_1$

$$= \pi r^2 h$$

$$= \pi \times (5)^2 \times 2 \quad \text{...(ii)}$$

$\therefore$ Volume of the water flowing through the pipe in x hours

$=$ Volume of the water that falls into the tank in x hours

$\Rightarrow \pi \times (0.1)^2 \times 4000x = \pi \times (5)^2 \times 2$

$\Rightarrow 40x = 50$

$\Rightarrow x = \dfrac{50}{40}$ hour

$\Rightarrow x = \dfrac{50}{40} \times 60$ minutes

$\Rightarrow x = 75$ minutes $= 1$ hour 15 mins

Thus, the water in the tank will be filled in 1 hour 15 minutes.

OR

The total height $= 40$ cm which includes the height of the base.

So, the height of the frustum of the cone $= 40 - 6 = 34$ cm

$\therefore$ Slant height of frustum $(l) = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{34^2 + \left(\frac{45}{2} - \frac{25}{2}\right)^2}$$

$$= \sqrt{34^2 + (22.5 - 12.5)^2}$$

$$= \sqrt{34^2 + (10)^2}$$

$$= \sqrt{1256}$$

$$= 35.44 \text{ cm}$$

Area of the metallic sheet used

$=$ Curved surface area of frustum of cone

$+$Area of circular base

$+$Curved surface area of cylinder

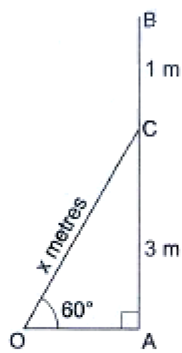
$$= \pi \times 35.44 \times (22.5 + 12.5) + \pi \times (12.5)^2 + 2\pi \times 12.5 \times 6$$
$$= \frac{22}{7} \times 35.44 \times 35 + \frac{22}{7} \times 156.25 + 2 \times \frac{22}{7} \times 12.5 \times 6$$
$$= \frac{27288.8}{7} + \frac{3437.5}{7} + \frac{3300}{7}$$
$$= \frac{27288.8 + 3437.5 + 3300}{7}$$
$$= \frac{34026.3}{7}$$
$$= 4860.9 \text{ cm}^2$$

Now,

Volume of the water that the bucket can hold $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \frac{1}{3} \times \frac{22}{7} \times 34\left((22.5)^2 + (12.5)^2 + 22.5 \times 12.5\right)$$
$$= \frac{748}{21} \times 943.75$$
$$= 33615.12$$
$$= 33.62 \text{ litres (approx.)} \quad \text{.....(Since 1 litres} = 1000 \text{ cm}^3\text{)}$$

39.



Let AB be the electric pole such that AB = 4 m.

Let C be a point 1 m below B.

$\Rightarrow$ AC = 4 m – 1 m = 3 m

Let OC be the ladder = x metres.

Then, $\angle AOC = 60°$.

In right ΔOAC,

$$\text{cosec}\,60° = \frac{OC}{AC}$$

$$\Rightarrow \frac{2}{\sqrt{3}} = \frac{x}{3}$$

$$\Rightarrow x = \frac{6}{\sqrt{3}}$$

On rationalising we get,

$$x = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{6\sqrt{3}}{3}$$

$$\Rightarrow x = 2\sqrt{3}$$

$$\Rightarrow x = 2 \times 1.73 = 3.46 \text{ m}$$

Hence, the length of the ladder should be 3.46 m.

40.

We make the classes exclusive.

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$	Cumulative frequency
10.5 – 15.5	2	13	26	2
15.5 – 20.5	3	18	54	5
20.5 – 25.5	6	23	138	11
25.5 – 30.5	7	28	196	18
30.5 – 35.5	14	33	462	32
35.5 – 40.5	12	38	456	44
40.5 – 45.5	4	43	172	48
45.5 – 50.5	2	48	96	50
	$\sum f_i = 50$		$\sum f_i x_i = 1600$	

Mean:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1600}{50} = 32$$

Median:

$$N = 50 \Rightarrow \frac{N}{2} = 25$$

The cumulative frequency just greater than 25 is 32.

Hence, median class is $30.5 - 35.5$.

$\therefore\ l = 30.5,\ h = 5,\ f = 14,\ cf = cf$ of preceding class $= 18$

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\} = 30.5 + \left\{ 5 \times \frac{25 - 18}{14} \right\} = 30.5 + 2.5 = 33$$

Mode:

Maximum frequency $= 14$

Hence, modal class is $30.5 - 35.5$

$$\text{Now, Mode} = x_k + h\left\{ \frac{\left(f_k - f_{k-1}\right)}{\left(2f_k - f_{k-1} - f_{k+1}\right)} \right\} = 30.5 + 5\left\{ \frac{14 - 7}{2(14) - 7 - 12} \right\} = 30.5 + 3.8 = 34.4$$