

Chapter 1: Real Numbers

1. Dividend = (divisor \times quotient) + remainder

$$\Rightarrow 1365 = (\text{divisor} \times 31) + 32$$

$$\Rightarrow \frac{1365 - 32}{31} = \text{divisor}$$

$$\Rightarrow \frac{1333}{31} = \text{divisor}$$

$$\therefore \text{Divisor} = 43$$

2. On dividing 2520 by 405, we get

$$\text{Quotient} = 6, \text{remainder} = 90$$

$$\therefore 2520 = (405 \times 6) + 90$$

Dividing 405 by 90, we get

$$\text{Quotient} = 4,$$

$$\text{Remainder} = 45$$

$$\therefore 405 = 90 \times 4 + 45$$

Dividing 90 by 45

$$\text{Quotient} = 2, \text{remainder} = 0$$

$$\therefore 90 = 45 \times 2$$

$$\therefore \text{HCF of } 405 \text{ and } 2520 \text{ is } 45.$$

3. According to the question,

$$\text{HCF}(26, 169) = 13$$

$$\text{HCF} \times \text{LCM} = \text{Product of numbers}$$

$$\text{LCM} = \frac{169 \times 26}{13}$$

$$\text{LCM} = 338$$

4. Let us assume that $5 + 3\sqrt{2}$ is a rational number.

Hence, we can find co-prime p and q ($q \neq 0$) such that

$$5 + 3\sqrt{2} = \frac{p}{q}$$

$$\therefore \frac{p}{q} - 5 = 3\sqrt{2}$$

$$\therefore \frac{p}{3q} - \frac{5}{3} = \sqrt{2}$$

$$\therefore \frac{p-5q}{3q} = \sqrt{2}$$

But $\sqrt{2}$ is irrational.

Thus, our assumption is wrong.

Hence, $5+3\sqrt{2}$ is an irrational.

5.

2	336
2	168
2	84
2	42
3	21
	7

2	240
2	120
2	60
2	30
3	15
	5

2	96
2	48
2	24
2	12
2	6
	3

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$$

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3 \times 5$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$$

$$\text{H.C.F} = 2^4 \times 3 = 16 \times 3 = 48$$

Each stack will contain 48 books.

Number of stacks of the same height

$$= \frac{240}{48} + \frac{336}{48} + \frac{96}{48} = 5 + 7 + 2 = 14$$

Hence, 14 stacks will be there.

6. (i) $\frac{13}{3125}$

$$3125 = 5^5$$

The denominator is of the form 5^m .

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

$$(ii) \quad \frac{17}{8}$$

$$8 = 2^3$$

The denominator is of the form 2^m .

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

7. If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$, then a and b are integers having no common factor other than 1, and $b \neq 0$.

Now,

$$\sqrt{6} = \frac{a}{b}$$

$$\Rightarrow 6 = \frac{a^2}{b^2} \text{ [on squaring both sides]}$$

$$\Rightarrow 6b^2 = a^2 \dots\dots\dots(i)$$

$$\Rightarrow 6 \text{ divides } a^2 \text{ [}\because 6 \text{ divides } 6b^2\text{]}$$

$$\Rightarrow 6 \text{ divides } a$$

Let $a = 6c$ for some integer c.

Putting $a = 6c$ in (i), we get

$$6b^2 = 36c^2 \Rightarrow b^2 = 6c^2$$

$$\Rightarrow 6 \text{ divides } b^2 \text{ [}\because 6 \text{ divides } 6c^2\text{]}$$

$$\Rightarrow 6 \text{ divides } b \text{ [}\because 6 \text{ divides } b^2 = 6 \text{ divides } b\text{]}$$

Thus, 6 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence, $\sqrt{6}$ is irrational.

8. Subtracting 6 from each number

$$378 - 6 = 372, 510 - 6 = 504$$

2	372	2	504
2	186	2	252
3	93	2	126
	31	7	63
		3	9
			3

$$372 = 2 \times 2 \times 3 \times 31 = 2^2 \times 3 \times 31$$

$$504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7$$

$$\text{H.C.F of } 372 \text{ and } 504 = 2^2 \times 3 = 12$$

Therefore the largest number which divides 378 and 510 leaving the remainder in each case is 12.

Chapter 2: Polynomials

1. The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:

$$\begin{array}{r}
 \overline{x-3} \\
 x^2-2 \overline{) x^3-3x^2+5x-3} \\
 \underline{x^3 -2x} \\
 - + \\
 \underline{-3x^2+7x-3} \\
 -3x^2 +6 \\
 \underline{+ -} \\
 \underline{7x-9}
 \end{array}$$

Quotient = $x - 3$

Remainder = $7x - 9$

2. Let $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

It is given that the two zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $p(x)$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$\begin{array}{r}
 \phantom{x^2+0x-\frac{5}{3}} \overline{3x^2+6x+3} \\
 x^2+0x-\frac{5}{3} \overline{) 3x^4+6x^3-2x^2-10x-5} \\
 \underline{3x^4+0x^3-5x^2} \\
 - - + \\
 \underline{6x^3+3x^2-10x-5} \\
 6x^3+0x^2-10x \\
 \underline{- - +} \\
 \underline{3x^2+0x-5} \\
 3x^2+0x-5 \\
 \underline{- - +} \\
 \underline{0}
 \end{array}$$

$$\begin{aligned}
 3x^4 + 6x^3 - 2x^2 - 10x - 5 &= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) \\
 &= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)
 \end{aligned}$$

Now, $x^2 + 2x + 1 = (x+1)^2$

So, the two zeroes of $x^2 + 2x + 1$ are -1 and -1 .

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1 .

3. Given polynomial is $p(x) = ax^2 + 7x + b$

$\frac{2}{3}$ and -3 are the zeroes of p(x)

$$\Rightarrow p\left(\frac{2}{3}\right)=0 \text{ and } p(-3)=0$$

$$\Rightarrow a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0$$

$$\Rightarrow 4a + 9b + 42 = 0 \dots\dots (i)$$

Also, $a(-3)^2 + 7(-3) + b = 0$

$$\Rightarrow 9a + b - 21 = 0 \dots\dots (ii)$$

Solving (i) and (ii) simultaneously, we get

$$a = 3$$

Substituting the value of a in (ii), we get

$$b = -6$$

4. By division algorithm,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\text{Divisor} \times \text{Quotient} = \text{Dividend} - \text{Remainder}$$

$$\text{Divisor} \times \text{Quotient} = x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a$$

$$= x^4 - 6x^3 + 16x^2 - 26x + 10 - a$$

It will be perfectly divisible by $x^2 - 2x + k$.

So, now $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ can be divided by $x^2 - 2x + k$ as follows:

[illegible]

As $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is divisible by $x^2 - 2x + k$, so, the remainder will be 0.

$$\therefore (-10 + 2k)x + (10 - a - 8k + k^2) = 0$$

$$\Rightarrow (-10 + 2k) = 0 \text{ and } (10 - a - 8k + k^2) = 0$$

Now, $(-10 + 2k) = 0 \Rightarrow 2k = 10 \Rightarrow k = 5$

$$10 - a - 8k + k^2 = 0$$

$$\Rightarrow 10 - a - 8 \times 5 + 25 = 0$$

$$\Rightarrow 10 - a - 40 + 25 = 0$$

$$\Rightarrow -5 - a = 0$$

$$\Rightarrow a = -5$$

Thus, $k = 5$ and $a = -5$.

Chapter 3: Pairs of Linear Equations

1. On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and the y-axis, respectively.

Given equations are $2x + 3y = 2$ and $x - 2y = 8$

Graph of $2x + 3y = 2$:

$$y = \frac{2(1-x)}{3}$$

Putting $x = 1$, we get $y = 0$

Putting $x = -2$, we get $y = 2$

Putting $x = 4$, we get $y = -2$

\therefore Table for $2x + 3y = 2$ is

x	1	-2	4
y	0	2	-2

Plot the points A(1, 0), B(-2, 2) and C(4, -2) on the graph paper. Join AB and AC to

get the graph line BC. Extend it both ways.

Thus, line BC is the graph of $2x + 3y = 2$.

Graph of $x - 2y = 8$:

$$y = \frac{x-8}{2}$$

Putting $x = 2$, we get $y = -3$

Putting $x = 4$, we get $y = -2$

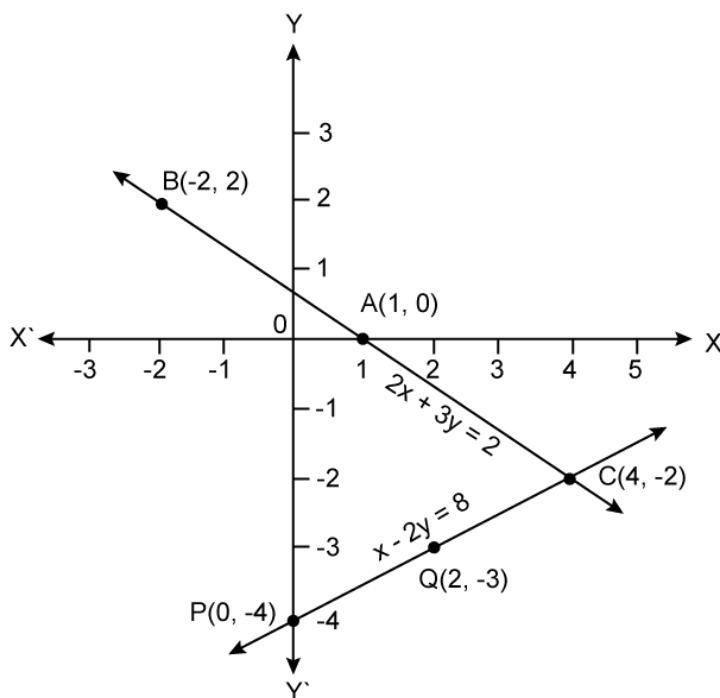
Putting $x = 0$, we get $y = -4$

Table for $x - 2y = 8$ is

x	2	4	0
y	-3	-2	-4

Now, on the same graph paper as above, plot the points P(0, -4) and Q(2, -3). Point C(4, -2) has already been plotted. Join QC and extend it. Thus, the line PC is the graph of $x - 2y = 8$.

Scale: 1 cm = 1 unit on both axes



The two graph lines intersect at C(4, -2)

$\therefore x = 4, y = -2$ is the solution of the given system of equations.

2. Let 1-day work of a man be $\frac{1}{x}$ and that of a boy be $\frac{1}{y}$.

Then, $\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$

$$\Rightarrow 2u + 5v = \frac{1}{4} \text{ --- (1)}$$

$$\text{and } \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$$

$$\Rightarrow 3u + 6v = \frac{1}{3} \text{ --- (2)}$$

Multiplying (1) by 6 and (2) by 5, we get

$$12u + 30v = \frac{6}{4} \text{ --- (3)}$$

$$15u + 30v = \frac{5}{3} \text{ --- (4)}$$

Subtracting (3) from (4), we get

$$3u = \frac{1}{6} \Rightarrow u = \frac{1}{18}$$

Putting $u = \frac{1}{18}$ in (1), we get

$$2 \times \frac{1}{18} + 5v = \frac{1}{4} \Rightarrow \frac{1}{9} + 5v = \frac{1}{4} \Rightarrow 5v = \frac{1}{4} - \frac{1}{9}$$

$$\Rightarrow 5v = \frac{5}{36} \Rightarrow v = \frac{1}{36}$$

$$\text{Now } u = \frac{1}{18} \Rightarrow x = \frac{1}{u} = 18$$

$$\text{and } v = \frac{1}{36} \Rightarrow y = \frac{1}{v} = 36$$

$$\therefore x = 18, y = 36$$

A man will finish the work in 18 days and a boy will finish the work in 36 days when they work alone.

3. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

So,

$$\text{Speed upstream} = (x - y) \text{ km/hr}$$

$$\text{Speed downstream} = (x + y) \text{ km/hr}$$

$$\text{Time taken to cover 12 km upstream} = \frac{12}{x - y} \text{ hrs}$$

$$\text{Time taken to cover 40 km downstream} = \frac{40}{x + y} \text{ hrs}$$

$$\text{Total time taken} = 8 \text{ hrs}$$

$$\therefore \frac{12}{x - y} + \frac{40}{x + y} = 8$$

$$\text{Again, time taken to cover 16 km upstream} = \frac{16}{(x - y)}$$

$$\text{Time taken to cover 32 km downstream} = \frac{32}{(x + y)}$$

$$\text{Total time taken} = 8 \text{ hrs}$$

$$\therefore \frac{16}{(x-y)} + \frac{32}{(x+y)} = 8$$

Putting $\frac{1}{(x-y)} = u$ and $\frac{1}{(x+y)} = v$, we get

$$12u + 40v = 8 \Rightarrow 3u + 10v = 2 \text{ ---(1)}$$

$$16u + 32v = 8 \Rightarrow 2u + 4v = 1 \text{ ---(2)}$$

Multiplying (1) by 4 and (2) by 10, we get

$$12u + 40v = 8 \text{ --- (3)}$$

$$20u + 40v = 10 \text{ --- (4)}$$

Subtracting (3) from (4), we get

$$8u = 2 \Rightarrow u = \frac{1}{4}$$

Putting $u = \frac{1}{4}$ in (3), we get

$$3 \times \frac{1}{4} + 10v = 2 \Rightarrow 10v = \frac{5}{4} \Rightarrow v = \frac{1}{8}$$

$$u = \frac{1}{4} \Rightarrow \frac{1}{x-y} = \frac{1}{4} \Rightarrow x-y = 4 \text{ --- (5)}$$

$$v = \frac{1}{8} \Rightarrow \frac{1}{x+y} = \frac{1}{8} \Rightarrow x+y = 8 \text{ --- (6)}$$

On adding (5) and (6), we get $2x = 12 \Rightarrow x = 6$

Putting $x = 6$ in (6), we get

$$6 + y = 8 \Rightarrow y = 8 - 6 = 2$$

$$\therefore x = 6, y = 2$$

Hence, the speed of the boat in still water = 6 km/hr and the speed of the stream = 2 km/hr.

4. Let the fraction be $\frac{x}{y}$.

When 2 is added to both numerator and denominator, the fraction becomes

$$\frac{x+2}{y+2} = \frac{1}{3} \quad \text{or} \quad 3x+6 = y+2$$

$$\Rightarrow 3x - y = -4 \text{ ---(1)}$$

When 3 is added both to numerator and denominator, the fractions becomes

$$\frac{x+3}{y+3} = \frac{2}{5} \quad \text{or} \quad 5x+15 = 2y+6$$

$$\Rightarrow 5x - 2y = -9 \text{ ----(2)}$$

Multiplying (1) by 22), we get

$$6x - 2y = -8 \text{ ---(3)}$$

$$\text{Eq.(2) is } 5x - 2y = -9 \text{ ---(4)}$$

Subtracting (4) from (3), we get $x = 1$

From (1), $3 - y = -4, y = 7$

$$\therefore \text{ Required fraction is } \frac{1}{7}$$

Chapter 4: Quadratic Equations

1. The given equation is $3x^2 - 2x + 8 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -2, c = 8$$

$$\begin{aligned}\therefore D &= (b^2 - 4ac) \\ &= (-2)^2 - (4 \times 3 \times 8) \\ &= (4 - 96) \\ &= -92\end{aligned}$$

2. Since $x = 1$ is a solution of $x^2 + kx + 3 = 0$, it must satisfy the equation $\therefore (1)^2 + k(1) + 3 = 0 \Rightarrow k = -4$
Hence, the required value of $k = -4$.

3. The given quadratic equation is

$$x^2 - 8x + 18 = 0$$

The above equation can be written as

$$\therefore x^2 - 8x + 16 - 16 + 18 = 0$$

$$\therefore x^2 - 8x + 16 = -2$$

$$\therefore (x - 4)^2 = -2$$

$$\therefore x - 4 = \sqrt{-2}$$

Now square root of a negative number is not possible

So the given quadratic equation has no roots.

4. The given equation is $3x^2 - 32x + 12 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -32, c = 12$$

$$\therefore D = (b^2 - 4ac) = [(-32)^2 - 4 \times 3 \times 12] = (1024 - 144) = 880 > 0$$

So, the given equation has real and distinct roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{32 + \sqrt{880}}{2 \times 3} = \frac{32 + 4\sqrt{55}}{6} = \frac{2(16 + 2\sqrt{55})}{6}$$

$$\Rightarrow \alpha = \frac{16 + 2\sqrt{55}}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{32 - \sqrt{880}}{2 \times 3} = \frac{32 - 4\sqrt{55}}{6} = \frac{2(16 - 2\sqrt{55})}{6}$$

$$\Rightarrow \beta = \frac{16 - 2\sqrt{55}}{3}$$

Hence, $\frac{16 + 2\sqrt{55}}{3}$ and $\frac{16 - 2\sqrt{55}}{3}$ are the roots of the given equation.

5. $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

$$\Rightarrow x^2 - 1.x - \sqrt{2}x + \sqrt{2} = 0$$

$$\Rightarrow x(x - 1) - \sqrt{2}(x - 1) = 0$$

$$\Rightarrow (x - 1)(x - \sqrt{2}) = 0$$

$$(x - 1) = 0 \text{ or } x - \sqrt{2} = 0$$

$$x = 1 \text{ or } x = \sqrt{2}$$

Hence, 1 and $\sqrt{2}$ are the roots of the given equation.

6. Comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = 3$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -7}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{37}}{2}$$

7. Let the speed of the boat in still water be x kmph, then

Speed of the boat downstream = $(x + 2)$ kmph

And the speed of the boat upstream = $(x - 2)$ kmph

Time taken to cover 8 km downstream = $\frac{8}{(x + 2)}$ hrs

Time taken to cover 8 km upstream = $\frac{8}{(x - 2)}$ hrs

Total time taken = $\frac{5}{3}$ hrs

$$\frac{8}{(x+2)} + \frac{8}{(x-2)} = \frac{5}{3}$$

$$\Rightarrow \frac{1}{x+2} + \frac{1}{x-2} = \frac{5}{24}$$

$$\Rightarrow \frac{x-2+x+2}{(x+2)(x-2)} = \frac{5}{24}$$

$$\Rightarrow \frac{2x}{x^2-4} = \frac{5}{24}$$

$$\Rightarrow 5x^2 - 20 - 48x = 0$$

$$\Rightarrow 5x^2 - 48x - 20 = 0$$

$$\Rightarrow 5x^2 - 50x + 2x - 20 = 0$$

$$\Rightarrow 5x(x-10) + 2(x-10) = 0$$

$$\Rightarrow (x-10)(5x+2) = 0$$

$$\Rightarrow x = 10 \text{ or } x = \frac{-2}{5}$$

$$\Rightarrow x = 10 \text{ (speed cannot be negative)}$$

Then the speed of the boat in still water is 10 kmph.

8. $\frac{(x-3)}{(x+3)} - \frac{(x+3)}{(x-3)} = 6\frac{6}{7}$

$$\Rightarrow \frac{(x-3)^2 - (x+3)^2}{(x+3)(x-3)} = \frac{48}{7}$$

$$\Rightarrow \frac{(x^2+9-6x) - (x^2+9+6x)}{(x+3)(x-3)} = \frac{48}{7}$$

$$\Rightarrow \frac{-12x}{x^2-9} = \frac{48}{7}$$

$$\Rightarrow -84x = 48x^2 - 432$$

$$\Rightarrow 48x^2 + 84x - 432 = 0$$

$$\Rightarrow 4x^2 + 7x - 36 = 0$$

$$\Rightarrow 4x^2 + 16x - 9x - 36 = 0$$

$$\Rightarrow 4x(x+4) - 9(x+4) = 0$$

$$\Rightarrow (4x-9)(x+4) = 0$$

$$\Rightarrow 4x-9=0 \text{ or } x+4=0$$

$$\Rightarrow x = \frac{9}{4} \text{ or } x = -4$$

Hence, -4 and $\frac{9}{4}$ are the roots of the given equation.

Chapter 5: Arithmetic Progressions

1. The given AP is 5, 7, 9, ... 201.

The last term $l = 201$, common difference $d = 7 - 5 = 2$

6th term from the end $= l - (n - 1)d$

$$= 201 - (6 - 1) \times 2$$

$$= 201 - 10$$

$$= 191$$

2. Let the three consecutive numbers be $a - d$, a , $a + d$.

According to the question,

$$a - d + a + a + d = 3$$

$$\therefore 3a = 3$$

$$\therefore a = 1$$

$$(a - d)a(a + d) = -35$$

$$(1 - d)(1 + d) = -35$$

$$1 - d^2 = -35$$

$$d^2 = 36 \Rightarrow d = \pm 6$$

3. In the given AP, let the first term = a common difference = d

$$\text{Then, } a_n = a + (n - 1)d$$

$$\Rightarrow a_7 = a + (7 - 1)d, \text{ and } a_{13} = a + (13 - 1)d$$

$$\Rightarrow a_7 = a + 6d, a_{13} = a + 12d$$

$$\Rightarrow a_7 = -4 \Rightarrow a + 6d = -4 \text{ --- (1)}$$

$$\Rightarrow a_{13} = -16 \Rightarrow a + 12d = -16 \text{ --- (2)}$$

Subtracting (1) from (2), we get

$$6d = -12 \Rightarrow d = -2$$

Putting $d = -2$ in (1), we get

$$a + 6 \times (-2) = -4 \Rightarrow a - 12 = -4 \Rightarrow a = 8$$

Thus, $a = 8$, $d = -2$

So, the required AP is 8, 6, 4, 2, 0....

4. Let a be the first term and d be the common difference.

$$p^{\text{th}} \text{ term} = a + (p - 1)d = q \text{ (given) } \text{-----(1)}$$

$$q^{\text{th}} \text{ term} = a + (q - 1)d = p \text{ (given) } \text{-----(2)}$$

Subtracting (2) from (1),

$$(p - q)d = q - p$$

$$(p - q)d = -(p - q)$$

$$\therefore d = -1$$

Putting $d = -1$ in (1),

$$a - (p - 1) = q$$

$$\therefore a = p + q - 1$$

$$\therefore (p + q)^{\text{th}} \text{ term} = a + (p + q - 1)d = (p + q - 1) - (p + q - 1) = 0$$

5. Let the first term of the given AP = a and common difference = d

$$\text{then, } a_n = a + (n - 1)d$$

$$\Rightarrow a_4 = a + (4 - 1)d, a_{25} = a + (25 - 1)d, \text{ and } a_{11} = a + (11 - 1)d$$

$$\Rightarrow a_4 = a + 3d, a_{25} = a + 24d, \text{ and } a_{11} = a + 10d$$

$$\text{Now, } a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$$

$$\therefore a_{25} = a + 24d = (-3d + 24d) = 21d$$

$$\text{And } a_{11} = a + 10d = -3d + 10d = 7d$$

$$\therefore a_{25} = 21d = 3 \times (7d) = 3 \times a_{11}$$

Hence, it is proved that 25th term of the given AP is triple its 11th term.

6. 24, 21, 18, ...

$$a = 24, d = 21 - 24 = -3, S_n = 78$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow \frac{n}{2} [2a + (n - 1)d] = 78$$

$$\Rightarrow \frac{n}{2} [48 - 3(n - 1)] = 78$$

$$\Rightarrow \frac{n}{2} [51 - 3n] = 78$$

$$\Rightarrow 3n^2 - 51n + 156 = 0$$

$$\Rightarrow n^2 - 17n + 52 = 0$$

$$\Rightarrow (n - 4)(n - 13) = 0$$

$$\Rightarrow n = 4 \text{ or } n = 13$$

The number of terms either 4 or 13.

7. Number of logs in the 1st row (from bottom) = 20

$$\text{Number of logs in the 2nd row} = 19$$

$$\text{Number of logs in the 3rd row} = 18$$

Let there be n number of rows

$$\therefore 20 + 19 + 18 + \dots \text{ to } n \text{ terms} = 200$$

$$S_n = \frac{n}{2} [2a + (n-1)d] = 200$$

$$\Rightarrow \frac{n}{2} [2 \times 20 + (n-1) \times (-1)] = 200$$

$$\Rightarrow \frac{n}{2} (40 - n + 1) = 200$$

$$\Rightarrow \frac{n}{2} (41 - n) = 200 \text{ or } 41n - n^2 = 400$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow n(n-16) - 25(n-16) = 0 \text{ or } (n-16)(n-25) = 0$$

$$\therefore n = 16 \text{ or } 25$$

For $n = 16$, the number of logs in the n th row

$$= a + (n-1)d$$

$$= 20 + (16-1) \times (-1)$$

$$= 20 - 15 = 5$$

$n \neq 25$, \therefore number of logs in the 25th row is negative which is not admissible.

Thus, there are 16 rows and the number of logs at the top is 5.

8. Let a_1 and a_2 be the first terms and d_1 and d_2 the common differences of the two given A.P.'s

$$\text{We have } S_n = \frac{n}{2} [2a_1 + (n-1)d_1] \text{ and } S'_n = \frac{n}{2} [2a_2 + (n-1)d_2]$$

$$\Rightarrow \frac{S_n}{S'_n} = \frac{\frac{n}{2} [2a_1 + (n-1)d_1]}{\frac{n}{2} [2a_2 + (n-1)d_2]}$$

$$\text{It is given that } \frac{S_n}{S'_n} = \frac{3n+5}{5n+7}$$

$$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{3n+5}{5n+7}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{3n+5}{5n+7} \dots (i)$$

Now, ratio of their N th terms..... (Considering ' N ' instead of ' n ' to avoid confusion)

$$\Rightarrow \frac{a_1 + (N-1)d_1}{a_2 + (N-1)d_2} \dots (ii)$$

In (i) if we replace

$$\frac{n-1}{2} \text{ by } N-1 \text{ we will get required ratio}$$

$$\Rightarrow \frac{n-1}{2} = N-1$$

$$\Rightarrow n-1=2N-2$$

$$\Rightarrow n=2N-1 \text{ ...substitute in (ii)}$$

$$\Rightarrow \frac{S_n}{S'_n} = \frac{a_1 + (N-1)d_1}{a_2 + (N-1)d_2} = \frac{3(2N-1)+5}{5(2N-1)+27} = \frac{3N+1}{5N+11}$$

Hence, the ratio of their N^{th} terms is $3N + 1 : 5N + 11$

9. Let the three terms of an A.P. be $a - d$, a , $a + d$.

The sum of three terms of an A.P. is 21.

$$\Rightarrow a - d + a + a + d = 21$$

$$\Rightarrow 3a = 21$$

$$\Rightarrow a = 7$$

The product of the first and the third terms exceeds the second term by 6.

$$\Rightarrow (a - d)(a + d) = a + 6$$

$$\Rightarrow a^2 - d^2 = a + 6$$

$$\Rightarrow 49 - d^2 = 13$$

$$\Rightarrow d^2 = 36$$

$$\Rightarrow d = \pm 6$$

When $a = 7$ and $d = 6$ then the three terms are 1, 7, 13.

When $a = 7$ and $d = -6$ then the three terms are 13, 7, 1.

Chapter 6: Triangles

1. The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

$$\therefore \frac{\text{ar}\Delta ABC}{\text{ar}\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{16}{81}$$

2. Given: $\Delta OAB \sim \Delta OCD$,

$AB = 8 \text{ cm}$, $BO = 6.4 \text{ cm}$, $CD = 5 \text{ cm}$, $OC = 3.5 \text{ cm}$

$$\Rightarrow \frac{OA}{OC} = \frac{AB}{CD} = \frac{BO}{DO}$$

$$\Rightarrow \frac{OA}{3.5} = \frac{8}{5} = \frac{6.4}{DO}$$

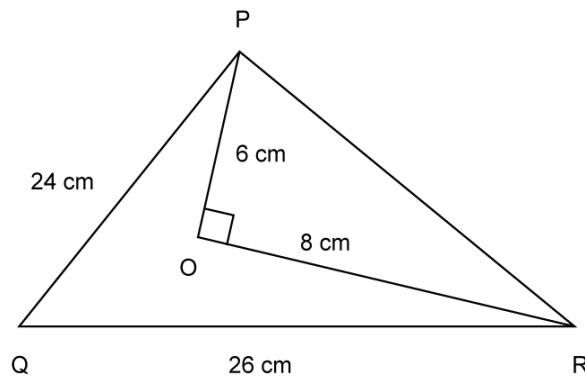
$$\Rightarrow \frac{OA}{3.5} = \frac{8}{5} \text{ and } \frac{6.4}{DO} = \frac{8}{5}$$

$$OA = \frac{3.5 \times 8}{5} = 5.6$$

$$\text{and } DO = \frac{6.4 \times 5}{8} = 4$$

$$\therefore OA = 5.6 \text{ cm and } DO = 4 \text{ cm}$$

3. Given: In $\triangle PQR$, $\angle POR = 90^\circ$, $OP = 6$ cm, $PQ = 24$ cm and $QR = 26$ cm



In $\triangle POR$,

$\angle POR = 90^\circ$ given

\therefore By Pythagoras theorem, we get

$$PR^2 = PO^2 + OR^2$$

$$PR^2 = (6^2 + 8^2)$$

$$= (36 + 64)$$

$$= 100$$

$$\Rightarrow PR = \sqrt{100} = 10 \text{ cm}$$

In $\triangle PQR$,

$$QR^2 = (26)^2 = 676 \text{ cm}^2$$

$$QP^2 + PR^2 = (24^2 + 10^2) = (576 + 100) = 676$$

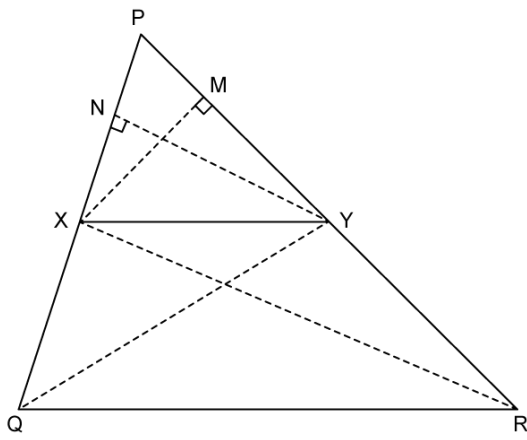
$$\text{Hence, } QR^2 = QP^2 + PR^2$$

(This means that the sum of the square of two sides is equal to the square of the greatest side.)

By the converse of the Pythagoras' theorem,

$\triangle PQR$ is a right-angled triangle which is right angled at P.

4.



Given: ΔPQR in which $XY \parallel QR$, XY intersects PQ and PR at X and Y respectively.

To prove: $\frac{PX}{XQ} = \frac{PY}{YR}$

Construction : Join RX and QY and draw YN perpendicular to PQ and XM perpendicular to PR .

Proof :

$$\text{Since, } \text{ar}(\Delta PXY) = \frac{1}{2} \times PX \times YN \dots\dots(i)$$

$$\text{ar}(\Delta PXY) = \frac{1}{2} \times PY \times XM \dots\dots(ii)$$

$$\text{Similarly, } \text{ar}(\Delta QXY) = \frac{1}{2} \times QX \times NY \dots\dots(iii)$$

$$\text{ar}(\Delta RXY) = \frac{1}{2} \times YR \times XM \dots\dots(iv)$$

Dividing (i) by (iii) we get,

$$\therefore \frac{\text{ar}(\Delta PXY)}{\text{ar}(\Delta QXY)} = \frac{\frac{1}{2} \times PX \times YN}{\frac{1}{2} \times QX \times NY} = \frac{PX}{QX} \dots\dots(v)$$

Again dividing (ii) by (iv)

$$\therefore \frac{\text{ar}(\Delta PXY)}{\text{ar}(\Delta RXY)} = \frac{\frac{1}{2} \times PY \times XM}{\frac{1}{2} \times YR \times XM} = \frac{PY}{YR} \dots\dots(vi)$$

Since the area of triangles with same base and between same parallel lines are equal, so

$$\therefore \text{ar}(\Delta QXY) = \text{ar}(\Delta RXY) \dots\dots(vii)$$

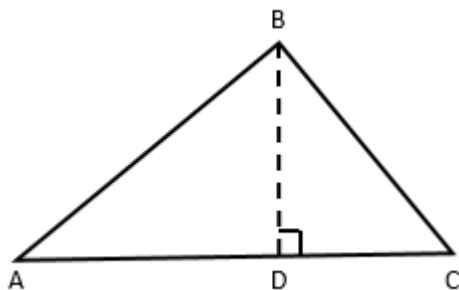
As ΔQXY and ΔRXY are on same base XY and between same parallel lines XY and QR .

Therefore, from (v), (vi) and (vii) we get

$$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$$

Hence proved.

5.



Given: A right angle triangle ABC at B.

To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw BD perpendicular to AC.

Proof :

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

$$\therefore \triangle ADB \sim \triangle ABC$$

$$\therefore \frac{AD}{AB} = \frac{AB}{AC} \quad \because \text{c. p. c. t.}$$

$$\therefore AD \times AC = AB^2 \dots\dots(i)$$

$$\triangle BDC \sim \triangle ABC$$

$$\therefore \frac{CD}{BC} = \frac{BC}{AC} \quad \because \text{c. p. c. t.}$$

$$\therefore CD \times AC = BC^2 \dots\dots(ii)$$

Adding (i) and (ii)

$$\therefore AD \times AC + CD \times AC = AB^2 + BC^2$$

$$\therefore AC(AD + CD) = AB^2 + BC^2$$

$$\therefore AC \times AC = AB^2 + BC^2$$

$$\therefore AB^2 + BC^2 = AC^2$$

Hence proved.

6. i.

In $\triangle AMD$

$$AM^2 + MD^2 = AD^2 \quad (1)$$

In $\triangle AMC$

$$AM^2 + MC^2 = AC^2 \quad (2)$$

$$AM^2 + (MD + DC)^2 = AC^2$$

$$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$$

Using equation (1) we may get

$$AD^2 + DC^2 + 2MD.DC = AC^2$$

$$\text{Now using the result } DC = \frac{BC}{2}$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \cdot \left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

ii.

In $\triangle ABM$ applying Pythagoras theorem

$$AB^2 = AM^2 + MB^2$$

$$= (AD^2 - DM^2) + MB^2 \dots\dots\dots \text{from (1)}$$

$$= (AD^2 - DM^2) + (BD - MD)^2$$

$$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$$

$$= AD^2 + BD^2 - 2BD \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

iii.

In $\triangle AMB$

$$AM^2 + MB^2 = AB^2 \quad (1)$$

In $\triangle AMC$

$$AM^2 + MC^2 = AC^2 \quad (2)$$

Adding equation (1) and (2)

$$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$$

$$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$$

$$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$$

$$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Chapter 7: Coordinate Geometry

1. Let P(a, b + c), Q(b, c + a) and R(c, a + b) be the given points.

$$\text{Then, } (x_1 = a, y_1 = b + c), (x_2 = b, y_2 = c + a), (x_3 = c, y_3 = a + b)$$

$$\begin{aligned} \therefore \text{Area of } \Delta PQR &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)] \\ &= \frac{1}{2} [a(c - b) + b(a - c) + c(b - a)] \\ &= 0 \end{aligned}$$

Hence, the given points are collinear.

2. The line segment joining the points A(3, -4) and B(1, 2) is trisected at the points P and Q.

$$\text{We have } P(a, -2) \text{ and } Q\left(\frac{5}{3}, b\right).$$

The point P is the point of trisection of the line segment AB.

So, P divides AB in the ratio 1:2.

$$(a, -2) = \left(\frac{2 \times 3 + 1 \times 1}{1 + 2}, \frac{2 \times -4 + 1 \times 2}{1 + 2} \right) = \left(\frac{7}{3}, -2 \right)$$

$$\therefore a = \frac{7}{3}$$

Similarly,

The point Q is the point of trisection of the line segment AB.

Hence, Q divides AB in the ratio 2:1.

$$Q\left(\frac{5}{3}, b\right) = \left(\frac{2 \times 1 + 1 \times 3}{1 + 2}, \frac{2 \times 2 + 1 \times -4}{1 + 2} \right) = \left(\frac{5}{3}, 0 \right)$$

$$b = 0$$

3. Let A(6, -1), B(1, 3) and C(k, 8) be the given points.

$$\begin{aligned} AB &= \sqrt{(1-6)^2 + (3+1)^2} = \sqrt{(-5)^2 + (4)^2} = \sqrt{25+16} \\ &= \sqrt{41} \text{ units} \end{aligned}$$

$$BC = \sqrt{(k-1)^2 + (8-3)^2} = \sqrt{k^2 + 1 - 2k + 25} = \sqrt{k^2 - 2k + 26}$$

we know that, AB = BC

$$\sqrt{41} = \sqrt{k^2 - 2k + 26}$$

[on squaring both sides, we get]

$$k^2 - 2k + 26 = 41$$

$$k^2 - 2k + 26 - 41 = 0$$

$$k^2 - 2k - 15 = 0$$

$$k^2 - 5k + 3k - 15 = 0$$

$$k(k-5) + 3(k-5) = 0$$

$$(k-5)(k+3) = 0$$

$$\therefore k = 5 \text{ or } k = -3$$

4. From the diagram given we have Anita, Atharva and Atish are seated at C(2, 4), B(3, 5) and A(5, 7) respectively.

Using distance formula, we have

$$AB = \sqrt{(5-3)^2 + (7-5)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$BC = \sqrt{(3-2)^2 + (5-4)^2} = \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$AC = \sqrt{(5-2)^2 + (7-4)^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$\text{Since, } AB + BC = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2} = AC$$

Hence, A, B and C are collinear.

Thus, Anita, Atharva and Atish are seated in a line.

5. Let A(2, 1), B(5, 2), C(6, 4) and D(3, 3) be the angular points of a parallelogram ABCD. Then

$$AB = \sqrt{(5-2)^2 + (2-1)^2}$$

$$= \sqrt{(3)^2 + (1)^2}$$

$$= \sqrt{10}$$

$$= \sqrt{10} \text{ units}$$

$$BC = \sqrt{(6-5)^2 + (4-2)^2}$$

$$= \sqrt{(1)^2 + (2)^2}$$

$$= \sqrt{1+4}$$

$$= \sqrt{5} \text{ units}$$

$$DC = \sqrt{(6-3)^2 + (4-3)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5} \text{ units}$$

Thus, $AB=DC$ and $AD=BC$

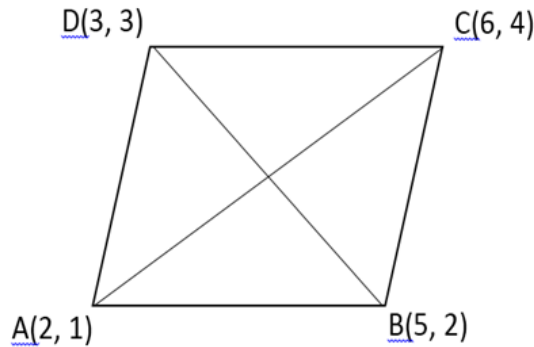
\therefore ABCD is a parallelogram because its opposite sides are equal.

$$\begin{aligned} \text{Diagonal } AC &= \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} \\ &= \sqrt{25} = 5 \text{ units} \end{aligned}$$

$$\text{Diagonal } BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(2)^2 + (1)^2} = \sqrt{5} \text{ unit}$$

Hence, diagonal $AC \neq$ diagonal BD

Thus, ABCD is not a rectangle, because its diagonals are not equal.



6. Let $P(m, 6)$ divides the line segment joining $A(-4, 3)$ and $B(2, 8)$ in the ratio $k:1$.

Hence, the point P is

$$\left(\frac{k \times 2 + 1 \times (-4)}{k+1}, \frac{k \times 8 + 1 \times 3}{k+1} \right) = \left(\frac{2k-4}{k+1}, \frac{8k+3}{k+1} \right)$$

$$\Rightarrow \frac{2k-4}{k+1} = m \quad \text{-----(1)}$$

$$\text{and } \frac{8k+3}{k+1} = 6$$

$$\Rightarrow 8k+3 = 6k+6$$

$$\Rightarrow 2k = 3$$

$$\therefore k = \frac{3}{2}$$

Putting value of k in (1)

$$\frac{2 \times \frac{3}{2} - 4}{\frac{3}{2} + 1} = m$$

$$m = \frac{3-4}{\frac{5}{2}}$$

$$m = -\frac{2}{5}$$

$$\text{Hence, } m = -\frac{2}{5}, k = \frac{3}{2}$$

7. The vertices of the quadrilateral ABCD are A(-4, -2), B(-3, -5), C(3, -2) and D(2, k). Join AC

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

Now, area of ΔABC

$$= \frac{1}{2} [(-4) \times (-5+2) + (-3) \times (-2+2) + (3) \times (-2+5)]$$

$$= \frac{1}{2} [12+0+9] = \frac{21}{2}$$

$$\begin{aligned} \text{Area of } \Delta ACD &= \frac{1}{2} [-4(-2-k) + 3 \times (k+2) + 2 \times (-2+2)] \\ &= \frac{1}{2} [4(k+2) + 3(k+2) + 0] = \frac{1}{2} [7k+14] \end{aligned}$$

Area of quad.ABCD = Area of ΔABC + Area of ΔACD

$$\begin{aligned} &= \frac{21}{2} + \frac{1}{2} (7k+14) \\ &= \frac{1}{2} (21+7k+14) \\ &= \frac{1}{2} (7k+35) \text{ sq.units} \end{aligned}$$

But area of quadrilateral ABCD = 28sq.units

$$\Rightarrow \frac{1}{2} (7k+35) = 28$$

$$\Rightarrow 7k = 56 - 35 = 21$$

$$\therefore k = \frac{21}{7} = 3$$

Chapter 8: Introduction to Trigonometry

1. L.H.S. = $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$

$$= \sin^2 72^\circ - \cos^2 18^\circ$$

$$= \sin^2 72^\circ - \cos^2 (90^\circ - 72^\circ)$$

$$= \sin^2 72^\circ - \sin^2 72^\circ$$

$$= 0$$

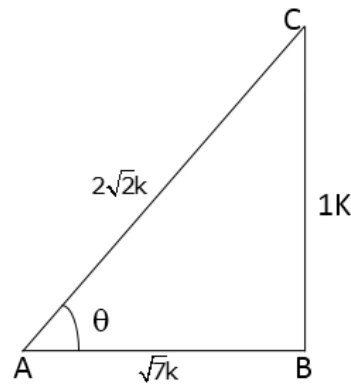
$$= \text{R.H.S.}$$

2. ΔABC is right angled at C.
 Now, $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow A + B + C = 180^\circ$
 $\Rightarrow A + B + 90^\circ = 180^\circ$
 $\Rightarrow A + B = 90^\circ$
 $\therefore \cos(A + B) = \cos 90^\circ = 0$

3. We know that
 $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
 $\Rightarrow (3x)^2 - \left(\frac{3}{x}\right)^2 = 1$
 $\Rightarrow 9x^2 - \frac{9}{x^2} = 1$
 $\Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1$
 $\Rightarrow 3 \times 3\left(x^2 - \frac{1}{x^2}\right) = 1$
 $\Rightarrow 3\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{3}$

4. L.H.S. = $\frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta}$
 $= \frac{1 + (\sec \theta - \tan \theta)}{1 + \sec \theta + \tan \theta}$
 $= \frac{(\sec^2 \theta - \tan^2 \theta) + (\sec \theta - \tan \theta)}{1 + \sec \theta + \tan \theta}$
 $= \frac{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta) + (\sec \theta - \tan \theta)}{1 + \sec \theta + \tan \theta}$
 $= \frac{(\sec \theta - \tan \theta)[\sec \theta + \tan \theta + 1]}{1 + \sec \theta + \tan \theta}$
 $= \sec \theta - \tan \theta$
 $= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}$
 $= \frac{1 - \sin \theta}{\cos \theta}$
 $= \text{R.H.S.}$

5. Let us draw a $\triangle ABC$ in which $\angle B = 90^\circ$ and $\angle A = \theta$.



$$\text{Given: } \tan \theta = \frac{BC}{AB} = \frac{1}{\sqrt{7}}$$

$$\text{Let } BC = 1k \text{ and } AB = \sqrt{7}k$$

where k is positive

By pythagoras theorem, we have

$$AC^2 = (AB^2 + BC^2)$$

$$\Rightarrow AC^2 = \left[(\sqrt{7}k)^2 + (1k)^2 \right]$$

$$= 7k^2 + 1k^2 = 8k^2$$

$$\Rightarrow AC = 2\sqrt{2}k$$

$$\operatorname{cosec} \theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{1k} = 2\sqrt{2}$$

$$\sec \theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$$

$$\begin{aligned} \Rightarrow \frac{(\operatorname{cosec}^2 \theta - \sec^2 \theta)}{(\operatorname{cosec}^2 \theta + \sec^2 \theta)} &= \frac{\left[(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}} \right)^2 \right]}{\left[(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}} \right)^2 \right]} \\ &= \frac{\left(8 - \frac{8}{7} \right)}{\left(8 + \frac{8}{7} \right)} = \frac{\left(\frac{48}{7} \right)}{\left(\frac{64}{7} \right)} = \frac{48}{64} = \frac{3}{4} \end{aligned}$$

$$\text{Hence, } \frac{(\operatorname{cosec}^2 \theta - \sec^2 \theta)}{(\operatorname{cosec}^2 \theta + \sec^2 \theta)} = \frac{3}{4}$$

6. $x = \cot A + \cos A$ and $y = \cot A - \cos A$

Thus, we have

$$x + y = (\cot A + \cos A) + (\cot A - \cos A) = 2 \cot A$$

$$x - y = (\cot A + \cos A) - (\cot A - \cos A) = 2 \cos A$$

$$\begin{aligned} \text{L.H.S.} &= \left(\frac{x-y}{x+y} \right)^2 + \left(\frac{x-y}{2} \right)^2 \\ &= \left(\frac{2 \cos A}{2 \cot A} \right)^2 + \left(\frac{2 \cos A}{2} \right)^2 \end{aligned}$$

Thus, we have

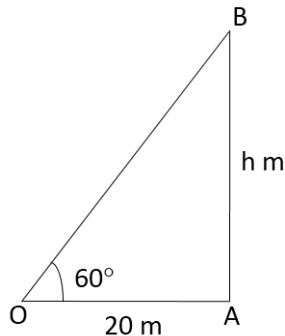
$$x + y = (\cot A + \cos A) + (\cot A - \cos A) = 2 \cot A$$

$$x - y = (\cot A + \cos A) - (\cot A - \cos A) = 2 \cos A$$

$$\begin{aligned} \text{L.H.S.} &= \left(\frac{x-y}{x+y} \right)^2 + \left(\frac{x-y}{2} \right)^2 \\ &= \left(\frac{2 \cos A}{2 \cot A} \right)^2 + \left(\frac{2 \cos A}{2} \right)^2 \\ &= \left(\frac{\cos A}{\cot A} \right)^2 + (\cos A)^2 \\ &= \left(\frac{\cos A}{\cos A / \sin A} \right)^2 + (\cos A)^2 \\ &= (\sin A)^2 + (\cos A)^2 \\ &= \sin^2 A + \cos^2 A \\ &= 1 \\ &= \text{R.H.S.} \end{aligned}$$

Chapter 9: Some Applications of Trigonometry

1. Let AB be the tower standing on a level ground and O be the position of the observer. Then OA = 20 m and $\angle OAB = 90^\circ$ and $\angle AOB = 60^\circ$.



Let AB = h metres.

From the right-angled $\triangle OAB$, we have

$$\frac{AB}{OA} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{h}{20} = \sqrt{3}$$

$$\Rightarrow h = (20 \times \sqrt{3})$$

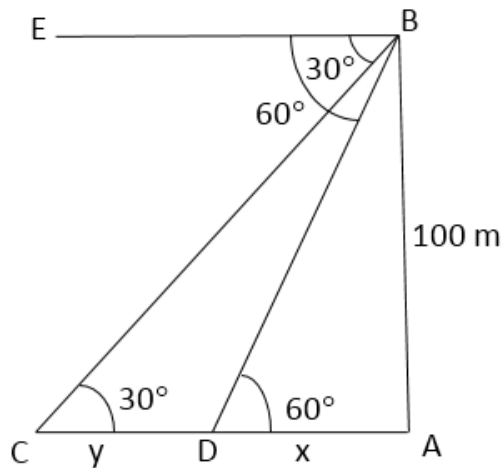
$$\Rightarrow h = 20 \times 1.732$$

$$\Rightarrow h = 34.64 \text{ m}$$

Hence, the height of the tower is $20\sqrt{3} \text{ m} = 34.64 \text{ m}$

2. Let AB be the lighthouse, and let C and D be the positions of the ship.

Let AD = x, CD = y



In $\triangle BDA$,

$$\frac{x}{100} = \cot 60^\circ$$

$$x = \frac{100}{\sqrt{3}} \text{ m}$$

Similarly in $\triangle BCA$, $\frac{x+y}{100} = \cot 30^\circ$

$$\Rightarrow (x+y) = 100\sqrt{3} \text{ m}$$

$$y = (x+y) - x$$

$$= \left(100\sqrt{3} - \frac{100}{\sqrt{3}} \right) \text{ m} = \left(\frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right) \text{ m}$$

$$= 115.46 \text{ m}$$

The distance travelled by the ship during the period of observation = 115.46 m

3. Let CD be a tower of height = 50 m
 AB be another tower of height = 40 m
 The observe is at M and angle of elevation $\angle CMD = \angle AMB = 45^\circ$
 Distance between BD = BM + MD
 In right angle triangle CDM,

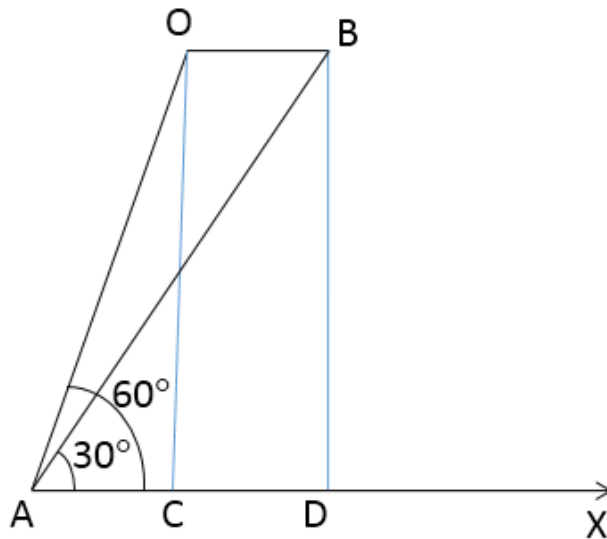
$$\tan \theta = \frac{CD}{DM} \Rightarrow \tan 45^\circ = \frac{50}{DM}$$

$$1 = \frac{50}{DM} \Rightarrow DM = 50 \text{ m}$$
 In right angle triangle AMB,

$$\tan \theta = \frac{AB}{MB} \Rightarrow \tan 45^\circ = \frac{40}{BM}$$

$$1 = \frac{40}{BM} \Rightarrow BM = 40 \text{ m}$$
 Hence, $BD = BM + MD = 40 + 50 = 90 \text{ m}$
 The distance between the windmills on either side of field is 90 m.

4. Let O and B be the two positions of the jet plane, and let A be the point of observation.
 Let AX be the horizontal ground.
 Draw $OC \perp AX$ and $BD \perp AX$.
 Then
 $\angle CAO = 60^\circ$, $\angle DAB = 30^\circ$ and $OC = BD = 1500\sqrt{3} \text{ m}$



From right $\triangle OCA$, we have

$$\frac{AC}{OC} = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{AC}{1500\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow AC = 1500 \text{ m} \text{ --- (1)}$$

From right $\triangle ADB$, we have

$$\frac{AD}{BD} = \cot 30^\circ = \sqrt{3}$$

$$\Rightarrow \frac{AD}{1500\sqrt{3}} = \sqrt{3} \Rightarrow AD = (1500\sqrt{3} \times \sqrt{3}) = 4500 \text{ m}$$

$$\therefore CD = (AD - AC) = (4500 - 1500) \text{ m} = 3000 \text{ m}$$

$$\therefore OB = CD = 3000 \text{ m}$$

Thus, the jet plane covers 3000 m in 15 seconds.

Hence, the speed of the jet plane is

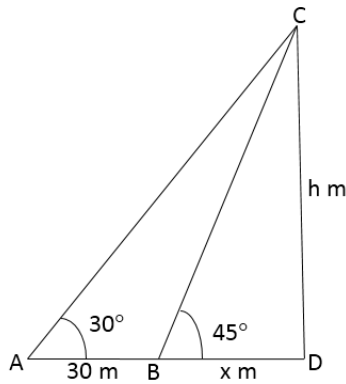
$$= \left(\frac{3000}{15} \times \frac{60 \times 60}{1000} \right) \text{ kmph}$$

$$= 720 \text{ kmph}$$

5. Let CD be the height of the building.

Then $\angle CAB = 30^\circ$, $\angle CBD = 45^\circ$, $\angle ADC = 90^\circ$ and $AB = 30 \text{ m}$

$CD = h$ metres and $BD = x$ metres



From right-angled $\triangle CAD$, we have

$$\frac{CD}{DA} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{h}{30 + x} = \frac{1}{\sqrt{3}} \Rightarrow 30 + x = h\sqrt{3}$$

$$x = (h\sqrt{3} - 30) \quad \dots\dots(1)$$

From right-angled $\triangle BCD$, we have

$$\frac{CD}{BD} = \tan 45^\circ = 1 \Rightarrow \frac{h}{x} = 1 \Rightarrow h = x \quad \dots\dots(2)$$

from (1) & (2), we get

$$h\sqrt{3} - 30 = h \Rightarrow h\sqrt{3} - h = 30$$

$$\Rightarrow h = \frac{30}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{30\sqrt{3}+30}{3-1} = \frac{30(\sqrt{3}+1)}{2}$$

$$\Rightarrow h = 15(1.732+1) = 15 \times 2.732 = 40.98$$

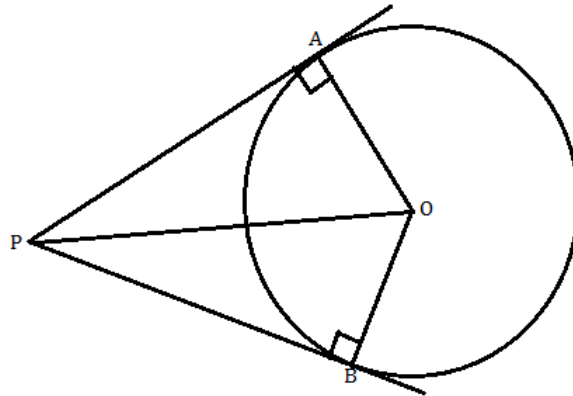
Putting $h = 40.98$ in (2), we get $x = 40.98$ m

Hence, the height of the building = 40.98 m

And the distance of its base from point A = $AD = (30 + x)$ m = $(30 + 40.98)$ m = 70.98 m

Chapter 10: Circles

1. Consider the following diagram.



Let P be an external point and PA and PB be tangents to the circle.

We need to prove that $PA = PB$

Now consider the triangles $\triangle OAP$ and $\triangle OBP$

$m\angle A = m\angle B = 90^\circ$ PA and PB are tangents

$OP = OP$ [common]

$OA = OB$ (radii of the circle)

Thus, by Right Angle-Hypotenuse-Side criterion of congruence we have,

$\triangle OAP \cong \triangle OBP$

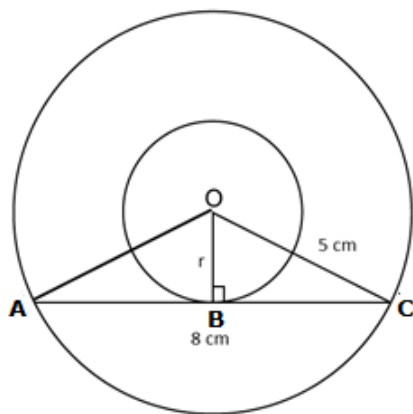
The corresponding parts of the congruent triangles are congruent.

Thus,

$PA = PB$

The lengths of the tangents drawn from an external point to a circle are equal.

2.



Let O be the centre of concentric circles.

Since AC is a tangent to the inner circle.

$\angle OBC = 90^\circ$ (tangent is perpendicular to the radius of a circle)

AC is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

So, $AC = 2BC \Rightarrow BC = 4$ cm

In $\triangle OBC$,

By Pythagoras theorem,

$$OC^2 = OB^2 + BC^2$$

$$\Rightarrow 5^2 = r^2 + 4^2$$

$$\Rightarrow r^2 = 5^2 - 4^2$$

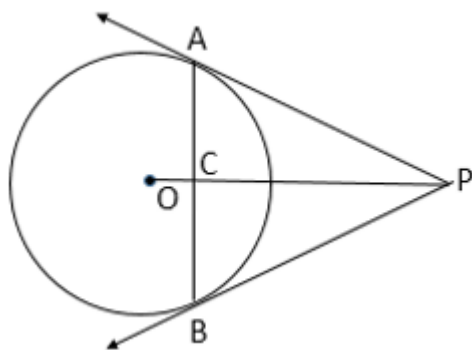
$$\Rightarrow r^2 = 25 - 16$$

$$\Rightarrow r^2 = 9$$
 cm

$$\Rightarrow r = 3$$
 cm

Hence, the radius of the smaller circle is 3 cm.

3.

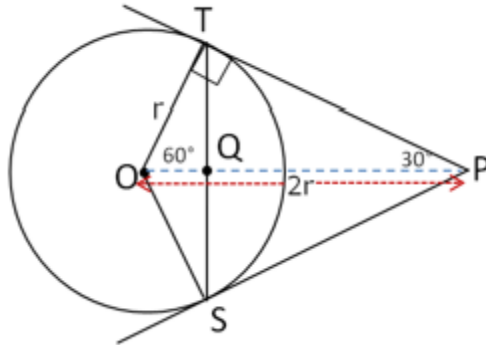


Let AB be a chord of circle with centre O.

Let AP and BP be two tangents at A and B respectively.

Suppose the tangents meet at point P. Join OP.
 Suppose OP meets AB at C.
 Now, in $\triangle PCA$ and $\triangle PCB$,
 $PA = PB$ (tangents from an external point are equal)
 $\angle APC = \angle BPC$ (PC is the angle bisector of $\angle APB$)
 $PC = PC$ (common)
 Hence, $\triangle PAC \cong \triangle PBC$ (by SAS congruence criterion)
 $\Rightarrow \angle PAC = \angle PBC$... C. A. C. T.

4. In the given figure,



$OP = 2r$... (Given)

$\angle OTP = 90^\circ$... (radius drawn at the point of contact is perpendicular to the tangent)

In $\triangle OPT$,

$$\sin \angle OPT = \frac{OT}{OP} = \frac{1}{2} = \sin 30^\circ$$

$$\Rightarrow \angle OPT = 30^\circ$$

$$\therefore \angle TOP = 60^\circ$$

$\therefore \triangle OPT$ is a 30° - 60° - 90° , right triangle.

In $\triangle OTS$,

$OT = OS$... (Radii of the same circle)

$\therefore \triangle OTS$ is an isosceles triangle.

$\therefore \angle OTS = \angle OST$... (Angles opposite to equal sides of an isosceles triangle are equal)

In $\triangle OTQ$ and $\triangle OSQ$

$OS = OT$... (Radii of the same circle)

$OQ = OQ$... (side common to both triangles)

$\angle OTQ = \angle OSQ$... (angles opposite to equal sides of an isosceles triangle are equal)

$\therefore \triangle OTQ \cong \triangle OSQ$... (By S.A.S)

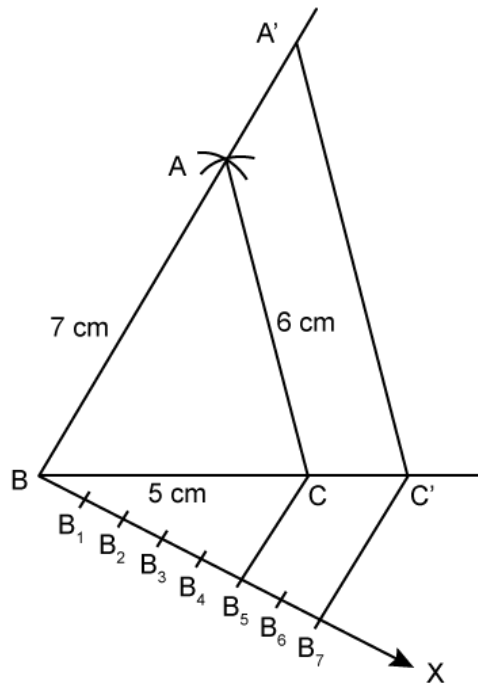
$$\therefore \angle TOQ = \angle SOQ = 60^\circ \text{ ... (C.A.C.T)}$$

$$\therefore \angle TOS = 120^\circ \text{ ... } (\angle TOS = \angle TOQ + \angle SOQ = 60^\circ + 60^\circ = 120^\circ)$$

$$\therefore \angle OTS + \angle OST = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore \angle OTS = \angle OST = 60^\circ \div 2 = 30^\circ \quad \because OT = OS$$

1.



Steps of construction:

Step 1: Draw a line segment $BC = 5\text{ cm}$

Step 2: With B as the centre and radius 7 cm, an arc is drawn

Step 3: With C as the centre and radius 6 cm, another arc is drawn intersecting the previous arc at A

Step 4: Join AB and AC

Step 5: $\triangle ABC$ is the given triangle

Step 6: Draw a line BX below BC

Step 7: Cut off equal distances from DX such that

$$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$$

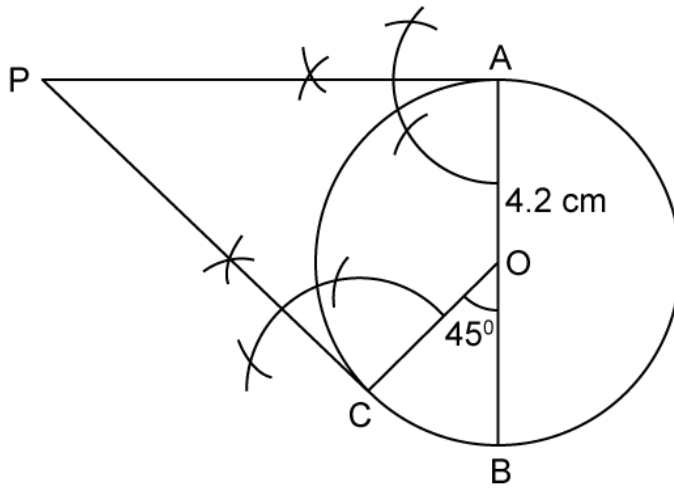
Step 8: Join B_5C

Step 9: Draw a line through B_7 parallel to B_5C cutting BC produced at C'

Step 10: Through C' , draw a line parallel to CA, cutting BA produced at A'

Step 11: $\triangle A'BC'$ is the required triangle.

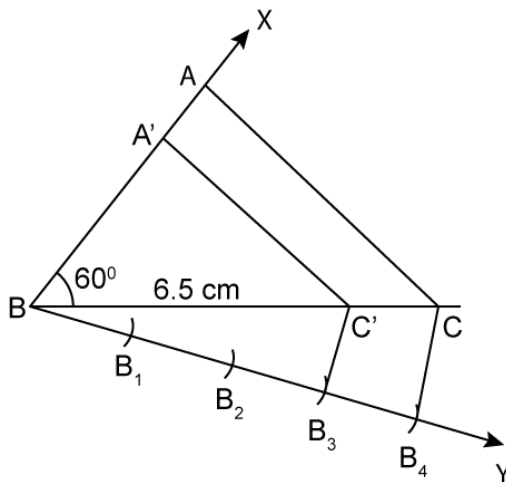
2.



Steps of construction:

- (i) A circle of radius 4.2 cm at centre O is drawn.
 - (ii) A diameter AB is drawn.
 - (iii) With OB as the base, an angle BOC of 45° is drawn.
 - (iv) At A, a line perpendicular to OA is drawn.
 - (v) At C, a line perpendicular to OC is drawn.
 - (vi) These lines intersect each other at P.
- PA and PC are the required tangents.

3.



Steps of construction:

Step 1: Draw a line segment $BC = 6.5 \text{ cm}$

Step 2: Draw an angle of 60° at B so that $\angle XBC = 60^\circ$

Step 3: With centre B and radius 4.5 cm, draw an arc intersecting XB at A

Step 4: Join AC

$\triangle ABC$ is the required triangle.

Step 5: Draw a line BY below BC which makes an angle CBY.

Step 6: Mark B_1, B_2, B_3 and B_4 at equal distances from B.

Such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

Step 7: Join CB_4

Step 8: Draw B_3C' parallel to CB_4

Step 9: Draw $C'A'$ parallel to CA through C' intersecting BA produced at A'

$\triangle A'BC'$ is the required similar triangle.

Chapter 12: Areas Related to Circles

1. Area of the first circle = $\pi r^2 = 962.5 \text{ cm}^2$

$$r^2 = \left(962.5 \times \frac{7}{22} \right) \text{cm}$$

$$r^2 = 306.25$$

$$r = 17.5 \text{ cm}$$

Area of the second circle = $\pi R^2 = 1386 \text{ cm}^2$

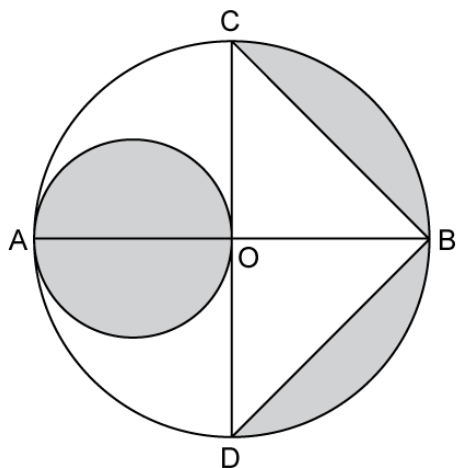
$$R^2 = \left(1386 \times \frac{7}{22} \right) \text{cm}$$

$$R^2 = 441$$

$$\Rightarrow R = 21 \text{ cm}$$

Width of ring $R - r = (21 - 17.5) \text{ cm} = 3.5 \text{ cm}$

2.



Area of the shaded region = (area of circle with OA as the diameter)

+ (area of semi-circle DBC) - (area of $\triangle BCD$)

Area of the circle with OA as the diameter = πr^2

$$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right) \text{cm}^2$$

$$= 38.5 \text{ m}^2$$

OB = 7 cm, CD = AB = 14 cm ... diameter of the same circle

$$\text{Area of semi-circle DBC} = \frac{1}{2} \pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \right) \text{cm}^2$$

$$= 77 \text{ cm}^2$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times DC \times OB$$

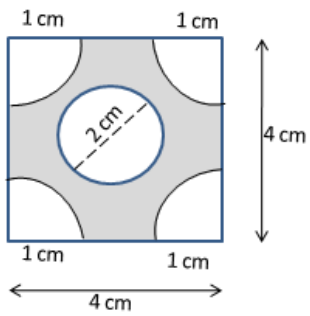
$$= \frac{1}{2} \times 14 \times 7$$

$$= 49 \text{ cm}^2$$

$$\text{Area of shaded region} = (38.5 + 77 - 49)$$

$$= 66.5 \text{ cm}^2$$

3.



$$\begin{aligned}\text{Area of the square} &= (4 \times 4) \text{ cm}^2 \\ &= 16 \text{ cm}^2\end{aligned}$$

Area of four quadrant corners

$$\begin{aligned}&= 4 \left[\frac{1}{4} \pi r^2 \right] \\ &= \pi r^2 \\ &= (\pi \times 1 \times 1) \text{ cm}^2 \\ &= 3.14 \text{ cm}^2\end{aligned}$$

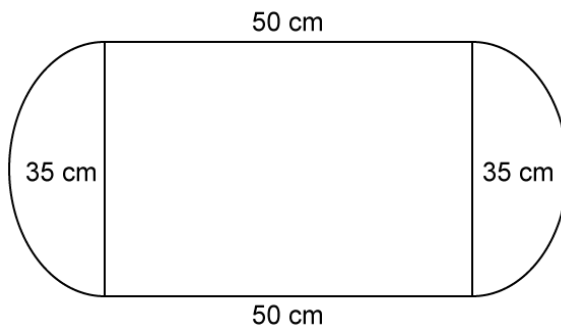
$$\text{Radius of the inner circle} = \frac{2}{2} = 1 \text{ cm}$$

$$\begin{aligned}\text{Area of the circle at the centre} &= \pi r^2 = (3.14 \times 1 \times 1) \text{ cm}^2 \\ &= 3.14 \text{ cm}^2\end{aligned}$$

Area of the shaded region = [area of the square - area of four corner quadrants - area of the circle at the centre] cm^2

$$Q = [16 - 3.14 - 3.14] \text{ cm}^2 = 9.72 \text{ cm}^2$$

4.



Area of the rectangular lawn in the middle

$$= (50 \times 35) \text{ m}^2 = 1750 \text{ m}^2$$

$$\text{Radius of semi-circles} = \frac{35}{2} = 17.5 \text{ m}$$

$$\text{Area of two semicircles} = 2(\text{area of semi circle})$$

$$= \left[2 \left(\frac{1}{2} \pi r^2 \right) \right] \text{m}^2$$

$$= \left(2 \times \frac{1}{2} \times \frac{22}{7} \times 17.5 \times 17.5 \right) \text{m}^2$$

$$= 962.5 \text{ m}^2$$

$$\text{Area of the lawn} = (\text{area of the rectangle} + \text{area of the semi-circle})$$

$$= (1750 + 962.5) \text{ m}^2 = 2712.5 \text{ m}^2$$

Chapter 13: Surface Areas and Volumes

1. $R = 20 \text{ cm}$, $r = 8 \text{ cm}$ and $h = 16 \text{ cm}$

$$\therefore l = \sqrt{h^2 + (R-r)^2} = \sqrt{(16)^2 + (20-8)^2}$$

$$= \sqrt{256 + 144} \text{cm} = 20 \text{ cm}$$

$$\text{Total surface area of the container} = \pi l(R+r) + \pi r^2$$

$$= [3.14 \times 20 \times (20+8) + 3.14 \times 8 \times 8] \text{cm}^2$$

$$= (3.14 \times 20 \times 28 + 3.14 \times 8 \times 8) \text{cm}^2$$

$$= (1758.4 + 200.96) \text{cm}^2$$

$$= 1959.36 \text{ cm}^2$$

$$\text{Cost of the metal sheet used} = \text{Rs} \left(1959.36 \times \frac{15}{100} \right) = \text{Rs.} 293.90$$

2. Radius of the cone = $r = 5 \text{ cm}$ and height $h = 12 \text{ cm}$

$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi 5^2 \times 12$$

$$\text{Diameter of the ball} = 4 \text{ cm, radius} = 2 \text{ cm}$$

$$\text{Volume of the sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi 2^3$$

According to the question,

$$\text{The number of balls} = \frac{\frac{1}{3} \pi 5^2 \times 12}{\frac{4}{3} \pi 2^3} = 9.375 \approx 9$$

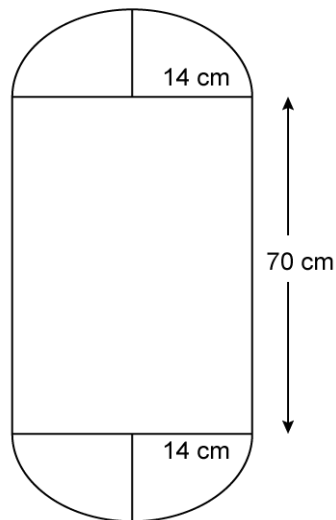
3. Let r be the inner radius, H be the height of the glass and h be the height of conical shaped base.

$$\begin{aligned}
 \text{Capacity of glass with hemispherical raised bottom} &= \pi r^2 H - \frac{2}{3} \pi r^3 \\
 &= \pi r^2 \left(H - \frac{2}{3} r \right) \\
 &= \pi r^2 \left(12 - \frac{2}{3} \times 6 \right) \\
 &= 288\pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Capacity of the glass conical raised bottom} &= \pi r^2 H - \frac{1}{3} \pi r^2 h \\
 &= \pi r^2 \left(H - \frac{1}{3} h \right) \\
 &= \pi r^2 \left(12 - \frac{1}{3} \times 3 \right) \\
 &= 396\pi
 \end{aligned}$$

Hence, Ashok got more quantity of juice.

4.



$$\text{Radius of each hemispherical end} = \frac{28}{2} = 14 \text{ cm}$$

Height of each hemispherical part = Its radius

$$\text{Height of the cylindrical part} = (98 - 2 \times 14) = 70 \text{ cm}$$

Area of surface to be polished

$$= 2(\text{curved surface area of the hemisphere})$$

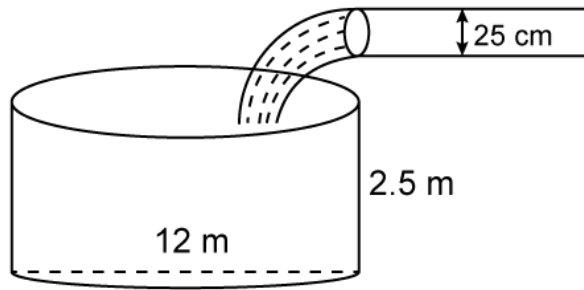
$$+ (\text{curved surface area of the cylinder})$$

$$\begin{aligned}
&= \left[2(2\pi r^2) + 2\pi rh \right] \text{sq.unit} \\
&= 2\pi r(2r + h) \text{cm}^2 \\
&= 2 \times \frac{22}{7} \times 14 \times [2 \times 14 + 70] \text{cm}^2 \\
&= (88 \times 98) = 8624 \text{ cm}^2
\end{aligned}$$

Cost of polishing the surface of the solid
= Rs. (0.15×8624)

= Rs. 1293. 60

5.



Height of the cylindrical tank = 2.5 m

Its diameter = 12 m, radius = 6 m

$$\text{Volume of the tank} = \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 2.5 \text{ m}^3 = \frac{1980}{7} \text{ m}^3$$

Water is flowing at the rate of 3.6 km/ hr = 3600 m/hr

Diameter of the pipe = 25 cm, radius = 0.125 m

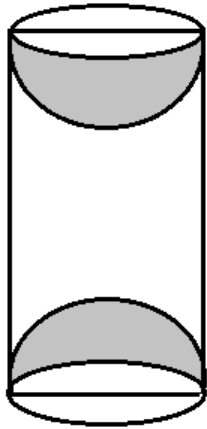
Volume of water flowing per hour

$$\begin{aligned}
&= \frac{22}{7} \times 0.125 \times 0.125 \times 3600 \text{ m}^3 \\
&= \frac{22 \times 3600}{7 \times 8 \times 8} \text{ m}^3 = \frac{2475}{14} \text{ m}^3
\end{aligned}$$

$$\begin{aligned}
\text{Time taken to fill the tank} &= \frac{1980}{7} \div \frac{2475}{14} \text{ hr} \\
&= \frac{1980}{7} \times \frac{14}{2475} \text{ hr} = \frac{792}{495} \text{ hr} \\
&= 1.6 \text{ hr} = 1 \text{ hr } 36 \text{ min } s
\end{aligned}$$

$$\text{Water charges} = \text{Rs. } \frac{1980}{7} \times 0.07 = \text{Rs. } 19.80$$

6.



Radius of a wooden cylinder = 4.2 cm

Height of the wooden cylinder = 12 cm

Lateral surface area

$$= 2\pi rh \text{ sq.cm}$$

$$= 2 \times \pi \times 4.2 \times 12 \text{ cm}^2$$

$$= 100.8\pi \text{ cm}^2$$

Radius of the hemisphere = 4.2 cm

Surface area of two hemispheres

$$= 2 \times 2\pi r^2 \text{ sq.unit}$$

$$= 4\pi \times 4.2 \times 4.2 \text{ cm}^2$$

$$= 70.56\pi \text{ cm}^2$$

Total surface area = $(100.8 + 70.56)\pi \text{ cm}^2$

$$= 171.36\pi \text{ cm}^2$$

$$= 171.36 \times \frac{22}{7} \text{ cm}^2$$

$$= 538.56 \text{ cm}^2$$

Further, the volume of the cylinder = $\pi r^2 h = 4.2 \times 4.2 \times 12\pi \text{ cm}^2$

$$= 211.68\pi \text{ cm}^2$$

Volume of two hemispheres = $2 \times \frac{2}{3}\pi r^3 \text{ cu.units}$

$$= \frac{4}{3}\pi \times 4.2 \times 4.2$$

$$= 98.784\pi \text{ cm}^3$$

Volume of wood left = $(211.68 - 98.784)\pi \text{ cm}^3$

$$= 112.896\pi \text{ cm}^3$$

$$= 112.896 \times \frac{22}{7} \text{ cm}^3$$

$$= 354.816 \text{ cm}^3$$

Chapter 14: Statistics

1. We have

Class	Frequency f_i	Class Mark x_i	$f_i x_i$
10-20	11	15	165
20-30	15	25	375
30-40	20	35	700
40-50	30	45	1350
50-60	14	55	770
60-70	10	65	650
	$\Sigma f_i = 100$		$\Sigma f_i x_i = 4010$

$$\therefore \text{Mean } \bar{x} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i} = \frac{4010}{100} = 40.10$$

2.

- i. Curve 1 is more than type and curve 2 is less than type.
- ii. Median rainfall is 4.
- iii. Median = 4 and mean = 5
3 median = Mode + 2 mean
 $\Rightarrow 3 \times 4 = \text{Mode} + 2 \times 5$
 $\Rightarrow \text{Mode} = 2$

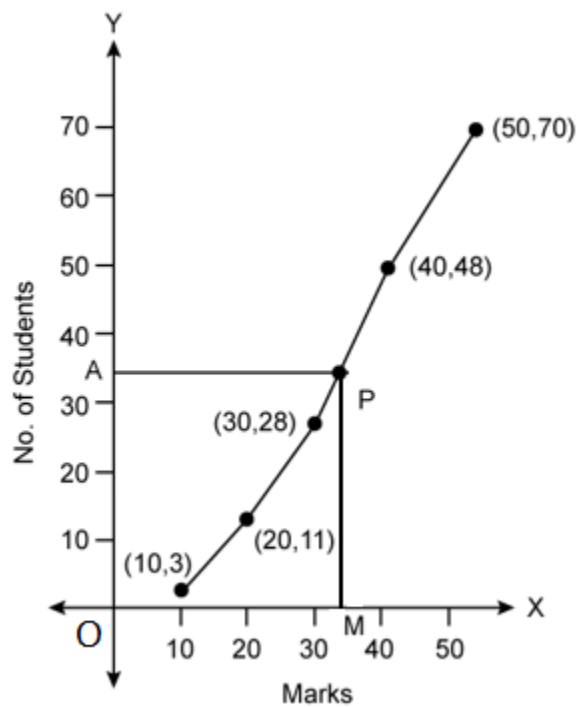
3. Cumulative frequency table is as follows:

Marks	C.F.
Marks less than 10	3
Marks less than 20	11
Marks less than 30	28
Marks less than 40	48
Marks less than 50	70

Scale:

X-axis: 1 cm = 10 marks

Y-axis: 1 cm = 10 students



The points (10, 3), (20, 11), (30, 28), (40, 48) and (50, 70) are plotted and joined as shown above. This is the required cumulative curve.

$$N = 70, \therefore \frac{N}{2} = \frac{70}{2} = 35$$

On the vertical line OY, take OA = 35

Through A, a horizontal line AP is drawn meeting the graph at P.

Through P, a vertical line PM is drawn.

Now, OM = 34 \Rightarrow Median = 34

4. We have

$$17 + f_1 + 32 + f_2 + 19 = 120$$

$$\Rightarrow f_2 = 52 - f_1$$

Class	Frequency f_i	Mid Value x_i	$f_i x_i$
0-20	17	10	170
20-40	f_1	30	$30 f_1$
40-60	32	50	1600
60-80	$52 - f_1$	70	$3640 - 70 f_1$
80-100	19	90	1710
	$\Sigma f_i = 120$		$\Sigma f_i x_i = 7120 - 40 f_1$

$$\therefore \text{Mean, } \bar{x} = \frac{\sum(f_i \times x_i)}{\sum f_i} = \frac{7120 - 40f_1}{120} = 50$$

$$\Rightarrow 7120 - 40f_1 = 6000 \Rightarrow 40f_1 = 1120 \Rightarrow f_1 = 28$$

$$\text{Thus, } f_1 = 28 \text{ and } f_2 = (52 - 28) = 24$$

5. More than series:

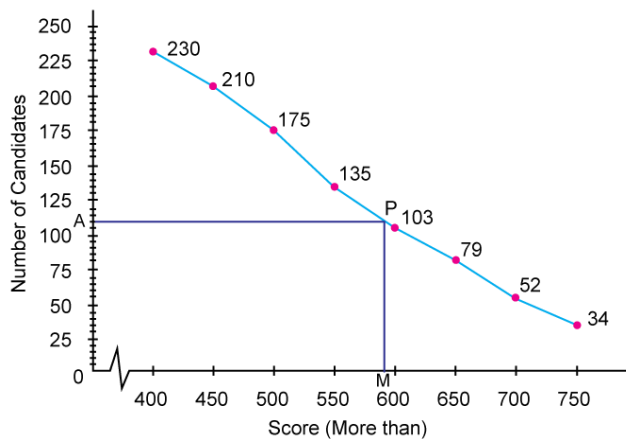
Score	No. of candidates
More than 400	230
More than 450	210
More than 500	175
More than 550	135
More than 600	103
More than 650	79
More than 700	52
More than 750	34

We plot the points (400, 230), (450, 210), (500, 175), (550, 135), (600, 103), (650, 79), (700, 52) and (750, 34).

Scale:

X-axis: 10 cm = 25 units of score

Y-axis: 1 cm = 25 candidates



Here M is at 675..... correct the graph

$$\text{Hence, } N = 230 \Rightarrow \frac{N}{2} = 115$$

Take a point A(0, 115) on the Y-axis and draw AP || X-axis meeting the curve at P. Draw PM \perp X-axis intersecting the X-axis at M.

Then, OM = 590

Hence, the median is 590.

6. We prepare the frequency table as given below:

Class	Frequency f_i	C.F
5-10	5	5
10-15	6	11
15-20	15	26
20-25	10	36
25-30	5	41
30-35	4	45
35-40	2	47
40-45	2	49
	$\Sigma f_i = 49$	

$$\text{Now, } N = 49 \Rightarrow \frac{N}{2} = \frac{49}{2} = 24.5$$

The cumulative frequency just greater than and nearest to 24.5 is 26 and the corresponding class is 15-20.

Thus, the median class is 15-20.

$$\therefore l = 15, h = 5, f = 15$$

$$c = \text{CF preceding median class} = 11 \text{ and } \left(\frac{N}{2}\right) = 24.5$$

$$\begin{aligned} \text{Median } m_e &= l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f} \right] = 15 + \left(5 \times \frac{(24.5 - 11)}{15} \right) \\ &= 15 + \left(5 \times \frac{13.5}{15} \right) = 15 + 4.5 = 19.5 \end{aligned}$$

The median of the frequency distribution is 19.5.

Chapter 15: Probability

1. Total number of tickets sold = 250

Number of prizes = 5

Let E be the event of getting a prize.

Number of favourable outcomes = 5

$$\therefore P(\text{getting a prize}) = P(E) = \frac{5}{250} = \frac{1}{50}$$

2. A one rupee coin is tossed 3 times then the total number of possible outcomes = $2^3 = 8$

$S = \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$n(S) = 8$

Let A be the event that Harsh will lose the game.

$A = \{HTH, THH, TTH, HHT, HTT, THT\}$

$n(A) = 6$

$$\text{Required probability} = \frac{n(A)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

3. Face cards in a pack of cards are jacks, queens and kings.

\therefore The number of face cards = $4 \times 3 = 12$

Total number of cards = 52

(i) Probability of getting a face card = $\frac{12}{52} = \frac{3}{13}$

(ii) Number of king cards which are not red = 2

Number of red cards = 26

$$\therefore \text{Probability of getting a red card or a black king card} = \frac{28}{52} = \frac{7}{13}$$

$$\text{Probability of getting neither a red card nor a king card} = 1 - \frac{7}{13} = \frac{6}{13}$$

4. Two dice are thrown simultaneously.

Total number of outcomes = $6 \times 6 = 36$

(i)

Favourable cases are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (3, 1), (3, 2),

(3, 3), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 6)

Number of favourable cases = 25

$$\therefore \text{Probability that 5 will not come upon either die} = \frac{25}{36}$$

(ii) Favorable cases are

(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) = 11

$$\text{Probability that 5 will come at least once} = \frac{11}{36}$$

(iii) 5 will come up on both dice in 1 case = (5, 5)

$$\therefore \text{Probability that 5 will come on both dice} = \frac{1}{36}$$

5. Total number of tickets = 100

(i) Even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100.

Total number of even numbers = 50

$$P(\text{getting an even number}) = \frac{50}{100} = \frac{1}{2}$$

(ii) Numbers less than 16 are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.

Total number of numbers less than 16 is 14.

$$P(\text{getting a number less than 16}) = \frac{14}{100} = \frac{7}{50}$$

(iii) Numbers which are a perfect square are 4, 9, 16, 25, 36, 49, 64, 81, 100.

Total number of perfect squares = 9

$$P(\text{getting a perfect square}) = \frac{9}{100}$$

(iv) Prime numbers less than 40 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37.

Total number of prime numbers = 12

$$P(\text{getting a prime number less 40}) = \frac{12}{100} = \frac{3}{25}$$