CBSE Board
Class X Mathematics
Sample Paper 4 (Standard) – Solution

Section A

1. **Correct option:** C
Explanation:-
Let x = $0.\overline{8}$
10x = $8.\overline{8}$
10x – x = 8
9x = 8
x = 8/9

2. **Correct option:** B
Explanation:-
A real number is an irrational number when it has a non-terminating non-repeating decimal representation.
Thus, 0.1011001253……. is an irrational number.

3. **Correct option:** c
Explanation:-
We may find cumulative frequencies with their respective class intervals as below

Weight (in kg)	**Frequency (f_i)**	**Cumulative frequency**
40 – 45	2	2
45 – 50	3	2 + 3 = 5
50 – 55	8	5 + 8 = 13
55 – 60	6	13 + 6 = 19
60 – 65	6	19 + 6 = 25
65 – 70	3	25 + 3 = 28
70 – 75	2	28 + 2 = 30
Total (n)	30	

Cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e.} \frac{30}{2}=15\right)$ is 19,

belonging to class interval 55 – 60.

Median class = 55 – 60

Lower limit (l) of median class = 55
Hence, the lower limit is 55.

4. Correct option: B
Explanation:-
For the two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ to be coincident, we must have

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

The given equation of lines are 3x + 6y – 15 = 0 and 9x + 18y – m = 0

As $\frac{a_1}{a_2}=\frac{3}{9}=\frac{1}{3}, \frac{b_1}{b_2}=\frac{6}{18}=\frac{1}{3}$

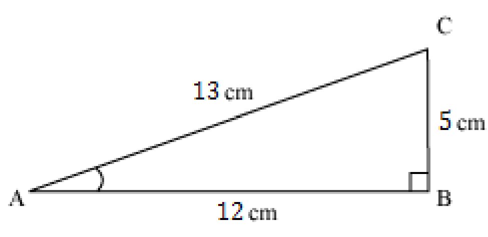
Therefore, $\frac{c_1}{c_2}=\frac{1}{3} \Rightarrow \frac{-15}{-m}=\frac{1}{3} \Rightarrow m=45$

Hence, the value of m is 45.

5. Correct option: A
Explanation:-
In ΔABC, right angled at B, AB = 12 cm and BC = 5 cm.
Applying Pythagoras theorem, we have
$AC^2 = AB^2 + BC^2$
$= (12)^2 + (5)^2$

$= 144 + 25 = 169$

$AC = \sqrt{169} = 13$ cm



$\sin A=\frac{\text{side opposite to } \angle A}{\text{Hypotenuse}}$

$\therefore \sin A=\frac{BC}{AC}=\frac{5}{13}$

6. Correct option: C

Explanation:-

Given sin 2x = 1

$\Rightarrow 2x = \sin^{-1} 1 = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{4}$

Also, cos y = $\frac{\sqrt{3}}{2}$

$\Rightarrow y = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\Rightarrow y = \frac{\pi}{6}$

$\therefore x - y = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

Hence, the value of x – y is $\frac{\pi}{12}$.

7. Correct option: C

Explanation:-

sin 55° cos 35° + cos 55° sin 35°

= sin 55° {cos(90 – 55)°} + cos 55° {sin (90 – 55)°}

= sin 55° sin 55° + cos 55° cos 55° ... (∵ sin(90 – x) = cos x and cos(90 – x) = sin x)

= $\sin^2 55°$ + $\cos^2 55°$

= 1 .... (∵ $\sin^2 x + \cos^2 x = 1$)

Hence, sin 55° cos 35° + cos 55° sin 35° = 1.

8. Correct option: D

Explanation:-

Let P(x, y) be the point which divides the line segment AB in the ratio 1 : 3

Coordinates of point P, dividing the line segment joining A(x_1, y_1) & B(x_2, y_2) internally in the ratio m:n is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Therefore, $x = \frac{1(5)+3(-3)}{1+3} = -1$ and

$y = \frac{1(2)+3(6)}{1+3} = 5$

Hence, the coordinates of P are (–1, 5)

9. **Correct option:** A

Explanation:-

Origin is O(0, 0) and given point is P(−24, 7)

Now, distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between point P and origin is

$$\sqrt{(0+24)^2 + (0-7)^2} = \sqrt{24^2 + 7^2} = \sqrt{576+49} = 25$$

Hence, the distance of (−24, 7) from the origin is 25 units.

10. **Correct option:** D

Explanation:-

Vertices of a triangle are (7, −2), (5, 1) and (3, 2)

Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

Area = $\frac{1}{2}\{7(1-2) + 5(2-(-2)) + 3(-2-1)\}$

$$= \frac{1}{2}\{-7 + 20 - 9\}$$

$$= \frac{1}{2}\{4\} = 2$$

Hence, the area of the triangle is 2 sq. units.

11. If 4/5, a, 2 are three consecutive terms of an AP then the value of a is $\underline{\frac{7}{5}}$.

Explanation:-

According to the question,

$$a - \frac{4}{5} = 2 - a$$

$$2a = 2 + \frac{4}{5}$$

$$2a = \frac{14}{5}$$

$$a = \frac{7}{5}$$

12. A number x is chosen from the numbers −3, −2, −1, 0, 1, 2, 3. Then the probability that $|x| < 2$ is $\underline{\frac{3}{7}}$.

Explanation:-
Total number of outcomes = 7
From the given list, numbers which satisfy $|x| < 2$ are −1, 0, 1.

Number of favourable outcomes = 3

Required probability = $\frac{3}{7}$

OR

The probability for a student to get pass marks in an examination is $\underline{\frac{1}{2}}$.

Explanation:-
All possible outcomes = {Pass, Fail}
n(S) = 2
Possible outcomes of passing = A = {Pass}
n(A) = 1

Required probability = $\frac{n(A)}{n(S)} = \frac{1}{2}$

13. If the polynomial $p(x) = 3x^2 + 7x - 3$ is divided by another polynomial $x^2 - 2$ then the remainder will be <u>7x + 3</u>.
Explanation:-
$p(x) = 3x^2 + 7x - 3$ is divided by polynomial $x^2 - 2$

$$\begin{array}{r|l} & 3 \\ \hline x^2-2 & 3x^2+7x-3 \\ & 3x^2 \quad\quad -6 \\ & - \quad\quad\quad + \\ \hline & \quad\quad 7x+3 \end{array}$$

Hence, the remainder is 7x + 3.

14. 2 cubes each with side 4 cm are joined to form a cuboid. The surface area of the resulting cuboid is 160 cm^2.

Explanation:-

Dimensions of the resulting cuboid will be 4 cm, 4 cm, 8 cm.

Surface area of the cuboid

$= 2(lb + bh + lh)$

$= 2(4 \times 4 + 4 \times 8 + 4 \times 8)$

$= 2(16 + 32 + 32)$

$= 2(16 + 64)$

$= 2 \times 80 = 160\ cm^2$

Hence, the surface area of the resulting cuboid is 160 cm^2

15. Length of PT is 3 cm.

Explanation:-

Let PT = x cm

Given: In ΔPQR, ST || QR

Using Basic proportionality theorem, we have

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

$$\frac{2.5}{5} = \frac{x}{6}$$

$\therefore x = 3$

Hence, the length of PT is 3 cm.

16. Since $\Delta ABC \sim \Delta DEF$,

$$\frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow BC^2 = \frac{64}{121} \times (15.4)^2$$

$$\Rightarrow BC = \frac{8}{11} \times 15.4 = 11.2\ cm$$

17. $x^2 - 3\sqrt{3}x + 6 = 0$

$\Rightarrow x^2 - \sqrt{3}x - 2\sqrt{3}x + 6 = 0$

$\Rightarrow x\left(x - \sqrt{3}\right) - 2\sqrt{3}\left(x - \sqrt{3}\right) = 0$

$\Rightarrow \left(x - \sqrt{3}\right)\left(x - 2\sqrt{3}\right) = 0$

$\Rightarrow x - \sqrt{3} = 0 \text{ or } x - 2\sqrt{3} = 0$

$\Rightarrow x = \sqrt{3} \text{ or } x = 2\sqrt{3}$

OR

Given equation is $x^2 - 2x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$

$a = 1$, $b = -2$ and $c = 1$

Discriminant $= b^2 - 4ac = (-2)^2 - 4 = 0$

18. Given, AR = 5 cm, BR = 4 cm and AC = 11 cm

We know that the tangents drawn from a point outside a circle are equal in length.

$\Rightarrow$ AR = AQ = 5 cm and BR = BP = 4 cm

And PC = QC = AC – AQ = 11 – 5 = 6 cm

Therefore, BC = BP + PC = 4 cm + 6 cm = 10 cm

19. Given that the first and last terms of an A.P. are 1 and 11, i.e., $a = 1$ and $l = 11$.

Sum of its n terms $= S_n = 36$

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow 36 = \frac{n}{2}(1 + 11)$$

$$\Rightarrow n = \frac{36}{6} = 6$$

Thus, the number of terms in the A.P. is 6.

20. 17 and 8 are co-prime and denominator $= 8 = 2^3$

Hence, it is a terminating decimal expansion.

Section B

21. $4^n = (2^2)^n = 2^{2n}$

The only prime in the factorisation of 4^n is 2.

There is no other primes in the factorisation of $4^n = 2^{2n}$

[By uniqueness of the Fundamental Theorem of Arithmetic]

$\Rightarrow$ 5 does not occur in the prime factorisation of 4^n for any n.

$\Rightarrow$ 4^n does not end with the digit 0 for any natural number n.

OR

$455 = 84 \times 5 + 35$

$\Rightarrow 84 = 35 \times 2 + 14$

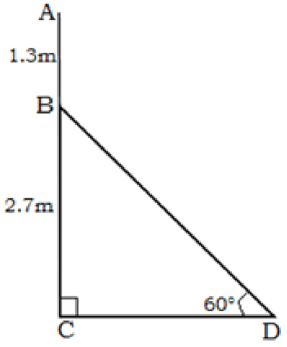
$\Rightarrow 35 = 14 \times 2 + 7$

$\Rightarrow 14 = 7 \times 2 + 0$

Therefore, H.C.F. = 7

22. Let AC be the electric pole of height 4 m. Let B be a point 1.3 m below the top A of the pole AC.

Then, BC = AC − AB = 4 − 1.3 = 2.7 m



Let BD be the length of ladder inclined at an angle of 60°.

$\sin 60° = \frac{BC}{BD}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{2.7}{BD}$

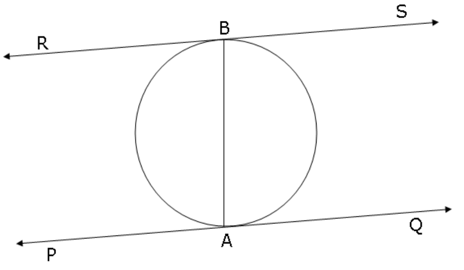
$\Rightarrow BD = \frac{5.4}{\sqrt{3}} = 1.8\sqrt{3} \text{ or } \frac{9\sqrt{3}}{5}$

Thus, the length of the ladder is $\frac{9\sqrt{3}}{5}$ m.

23. Let AB be the diameter of the given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively.

Since tangent at a point to a circle is perpendicular to the radius through the point of contact.

Therefore, AB is perpendicular to both PQ and RS.



$\Rightarrow \angle PAB = 90°$ and $\angle ABS = 90°$

$\Rightarrow \angle PAB = \angle ABS$

But, these are a pair of alternate interior angles.

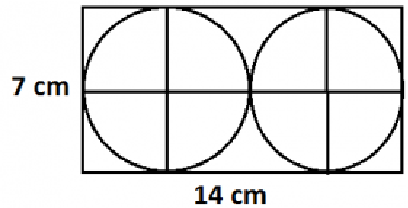
Therefore, PQ is parallel to RS.

OR

Dimension of the rectangular cardboard = 14 cm × 7 cm

Since, two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore,

the diameter of each circular piece is 14/ 2 = 7 cm.



$$\therefore \text{ Area of each circle} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2$$

$$\Rightarrow \text{Area of two circles} = 2 \times \frac{77}{2} = 77 \text{ cm}^2$$

$$\text{Area of rectangular cardboard} = 14 \times 7 = 98 \text{ cm}^2$$

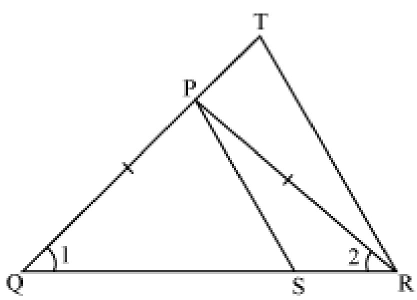
Thus, area of remaining cardboard

= Area of rectangular cardboard – Area of two circles

$= (98 - 77) \text{ cm}^2$

$= 21 \text{ cm}^2$

24.



In ΔPQR, $\angle PQR = \angle PRQ$

Therefore PQ = PR ...(i)

Given, $\dfrac{QR}{QS} = \dfrac{QT}{PR}$

Using (i), we have

$\dfrac{QR}{QS} = \dfrac{QT}{QP}$.....(ii)

In ΔPQS and ΔTQR,

$\frac{QR}{QS} = \frac{QT}{QP}$.....[using (ii)]

$\angle Q = \angle Q$

Therefore, ΔPQS ~ ΔTQR .... [SAS rule]

25. Given: Height of the cylindrical part (h) = 10cm

Radius (r) of cylindrical part = radius (r) of hemispherical part = 3.5cm

Surface area of article = CSA of cylindrical part + 2×CSA of hemispherical part

$$= 2\pi rh + 2 \times 2\pi r^2$$
$$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$$
$$= 70\pi + 49\pi$$
$$= 119\pi$$
$$= 17 \times 22 = 374\ cm^2$$

Hence, the surface area of the article is 374 cm^2.

26. Out of 52 cards, one card can be drawn in 52 ways.

So, total number of outcomes = 52

i. There are 6 red face cards, 3 each from diamonds and hearts.

Out of these 6 red face cards, one card can be chosen in 6 ways.

So, favorable number of elementary events = 6

Hence, required probability = $\frac{6}{52} = \frac{3}{26}$

ii. There are two suits of black cards, viz., spades and clubs.

Each suit contains one card bearing number 10.

So, favorable number of elementary events = 2

Hence, required probability = $\frac{2}{52} = \frac{1}{26}$

Section C

27. Given system of equations are

$$7x-2y-3=0$$

$$11x-\frac{3}{2}y-8=0$$

By cross multification, we have

$$\therefore \frac{x}{\left[(-2)(-8)-\left(\frac{-3}{2}\right)\times(-3)\right]}=\frac{-y}{\left[7\times(-8)-(-3\times 11)\right]}=\frac{1}{\left[7\times\left(\frac{-3}{2}\right)-11\times(-2)\right]}$$

$$\Rightarrow \frac{x}{16-\frac{9}{2}}=\frac{-y}{-56+33}=\frac{1}{\frac{-21}{2}+22}$$

$$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)}=\frac{y}{23}=\frac{1}{\frac{23}{2}}$$

$$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)}=\frac{1}{\frac{23}{2}} \text{ and } \frac{y}{23}=\frac{1}{\frac{23}{2}}$$

$$\Rightarrow x=1 \text{ and } y=2$$

Hence, the solution of the given system of equations is x = 1 and y = 2.

OR

$$2x+3y-7=0$$

$$(a-b)x+(a+b)y-(3a+b-2)=0$$

Here, $a_1=2,\ b_1=3,\ c_1=-7$

$a_2=(a-b),\ b_2=(a+b),\ c_2=-(3a+b-2)$

$$\frac{a_1}{a_2}=\frac{2}{a-b},\quad \frac{b_1}{b_2}=\frac{3}{a+b},\quad \frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$$

For the equations to have infinitely many solutions, we have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$$6a+2b-4 = 7a-7b$$

$$a-9b = -4 \qquad \ldots(1)$$

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$2a+2b = 3a-3b$$

$$a-5b = 0 \qquad \ldots(2)$$

Subtracting (1) from (2), we obtain:

$$4b = 4$$

$$b = 1$$

Substituting the value of b in equation (2), we obtain:

$$a - 5\times 1 = 0$$

$$a = 5$$

Thus, the values of a and b are 5 and 1 respectively.

28. Let the fraction be $\frac{x}{y}$.

According to the question,

$x + y = 8 \quad ....(1)$

$$\frac{x+3}{y+3} = \frac{3}{4}$$

$$\Rightarrow 4x+12 = 3y+9$$

$$\Rightarrow 4x-3y = -3 \qquad ....(2)$$

Multiplying (1) by 3, we get

$3x + 3y = 24 \quad ....(3)$

Adding (2) and (3), we get

$7x = 21$

$\Rightarrow x = 3$

$\Rightarrow y = 8 - x = 8 - 3 = 5$

Thus, the fraction is $\frac{3}{5}$.

29. A and B are complementary angles.

$\Rightarrow A + B = 90°$

$$
\begin{aligned}
\text{L.H.S.} &= \cot B + \cos B \\
&= \cot(90° - A) + \cos(90° - A) \\
&= \tan A + \sin A \\
&= \frac{\sin A}{\cos A} + \sin A \\
&= \frac{\sin A + \sin A \cos A}{\cos A} \\
&= \frac{\sin A(1 + \cos A)}{\cos A} \\
&= \sec A \sin A(1 + \cos A) \\
&= \sec A \sin(90° - B)\left[1 + \cos(90° - B)\right] \\
&= \sec A \cos B(1 + \sin B) \\
&= \text{R.H.S.}
\end{aligned}
$$

OR

$$\text{L.H.S.} = \frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$$

$$\begin{bmatrix} \text{Use}: a^3 + b^3 = (a+b)(a^2 - ab + b^2) \\ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \end{bmatrix}$$

$$= \frac{(\cos A + \sin A)(\cos^2 A - \cos A \sin A + \sin^2 A)}{\cos A + \sin A} +$$

$$\frac{(\cos A - \sin A)(\cos^2 A + \cos A \sin A + \sin^2 A)}{\cos A - \sin A}$$

$$= (\cos^2 A - \cos A \sin A + \sin^2 A) + (\cos^2 A + \cos A \sin A + \sin^2 A)$$

$$= 1 - \cos A \sin A + 1 + \cos A \sin A$$

$$= 2$$

$$= \text{R.H.S.}$$

30. Let the first term of these APs be a_1 and a_2 respectively and the common difference of these APs be d.

For first AP,

$a_{100} = a_1 + (100 - 1)\, d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)\, d$

$a_{1000} = a_1 + 999d$

For second AP,

$a_{100} = a_2 + (100 - 1)\, d$

$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)\, d$

$= a_2 + 999d$

Given that, difference between 100^{th} terms of these APs = 50

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 50$

$a_1 - a_2 = 50 \qquad (1)$

Difference between 1000^{th} terms of these APs

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 50$

Hence, the difference between 1000^{th} terms of these APs will be 50.

31. Suppose the line $3x + y - 9 = 0$ divides the line segment joining the points A(1, 3) and B(2, 7) in the ratio k : 1 at point C.

Then, the co-ordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

But, C lies on $3x + y - 9 = 0$.

Therefore,

$$\left[3\left(\frac{2k+1}{k+1}\right)\right] + \left[\frac{7k+3}{k+1}\right] - 9 = 0$$

$\Rightarrow 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow 4k - 3 = 0$

$\Rightarrow k = \frac{3}{4}$

So, the required ratio is 3 : 4

OR

Given, the point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3.

Co-ordinates of the point P $\equiv \left(\frac{2\times(-3)+3\times2}{2+3}, \frac{2\times6+3\times1}{2+3} \right) \equiv \left(\frac{-6+6}{5}, \frac{12+3}{5} \right) \equiv (0,3)$

Now, the given equation is x − 5y + 15 = 0.

Substituting x = 0 and y = 3 in this equation, we have

L.H.S. = 0 − 5(3) + 15 = −15 + 15 = 0 = R.H.S.

Hence, the point P lies on the line x − 5y + 15 = 0.

32.

Class Interval	Frequency	c.f.
0 - 20	7	7
20 - 40	8	15
40 - 60	12	27
60 - 80	10	37
80 - 100	8	45
100 - 120	5	50

N = 50

$\frac{N}{2} = \frac{50}{2} = 25$

Median class is 40 - 60

l = 40, f = 12, cf = 15, h = 20

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

$$= 40 + \left(\frac{25-15}{12} \right) \times 20$$

$$= 40 + \frac{200}{12}$$

$$= 40 + 16.67$$

$$= 56.67$$

33. Dividend, $p(x) = x^3 - 3x^2 + x + 2$

Quotient = (x – 2)

Remainder = (– 2x + 4)

Let g(x) be the divisor.

According to the division algorithm,

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$
$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$$
$$x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$$

Now, g(x) is the quotient when $x^3 - 3x^2 + 3x - 2$ is divided by $x-2$

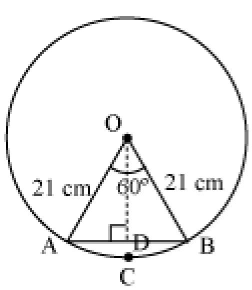
$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x-2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

$\therefore g(x) = x^2 - x + 1$

34. Radius (r) of circle = 21 cm

Angle subtended by given arc = 60°

Length of an arc of a sector of angle θ = $\frac{\theta}{360^\circ} \times 2\pi r$



A. Length of arc ACB = $\frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

$$= \frac{1}{6} \times 2 \times 22 \times 3 \ = 22 \text{ cm}$$

B. Area of sector OACB $= \frac{60^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

C. Now in ΔOAB

$\angle$OAB = $\angle$OBA (as OA = OB)

$\angle$OAB + $\angle$AOB + $\angle$OBA = 180°

2$\angle$OAB + 60° = 180°

$\angle$OAB = 60°

So, ΔOAB is an equilateral triangle.

Area of ΔOAB $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$$

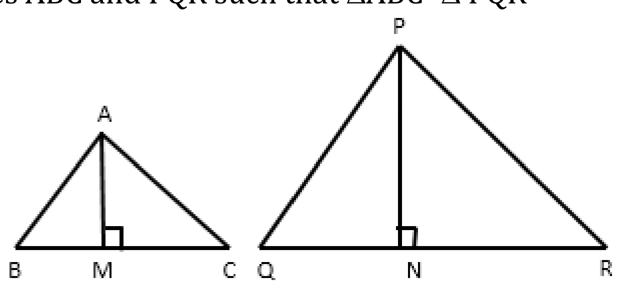
Area of segment ACB = Area of sector OACB – Area of ΔOAB

$$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{ cm}^2$$

Section D

35. **Statement :** Ratio of the areas of two similar triangles is equal to the ration of the squares of their corresponding sides.

Given : Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$



To prove : $\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Proof : For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles.

Now, $ar(\triangle ABC) = \frac{1}{2} BC \times AM$

And $ar(\triangle PQR) = \frac{1}{2} QR \times PN$

So, $\dfrac{ar(\triangle ABC)}{ar(\triangle PQR)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \dfrac{BC \times AM}{QR \times PN}$ ....(1)

Now, in ΔABM and ΔPQN.

	$\angle B = \angle Q$	(As $\Delta ABC \sim \Delta PQR$)
And	$\angle M = \angle N$	(Each = 90°)
So,	$\Delta ABM \sim \Delta PQN$	(AA similarity criterion)
Therefore,	$\dfrac{AM}{PN} = \dfrac{AB}{PQ}$	....(2)
Also,	$\Delta ABC \sim \Delta PQR$	
So,	$\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$	....(3)

$\dfrac{ar(ABC)}{ar(PQR)} = \dfrac{AB}{PQ} \times \dfrac{AM}{PN}$ [from(1) and (3)]

$= \dfrac{AB}{PQ} \times \dfrac{AB}{PQ}$ [from(2)]

$= \left(\dfrac{AB}{PQ}\right)^2$

Now using (3), we get

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB^2}{PQ^2}$$

Similarly

$$\frac{ar(ABC)}{ar(PQR)} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

36. Let B be the window of a house AB and let CD be the other house.

Then, AB = EC = h metres.

Let CD = H metres.

Then, ED = (H − h) m

In ΔBED,

$\cot \alpha = \dfrac{BE}{ED}$

$BE = (H - h) \cot \alpha$ ... (a)

In ΔACB,

$\dfrac{AC}{AB} = \cot \beta$

$AC = h. \cot \beta$ .... (b)

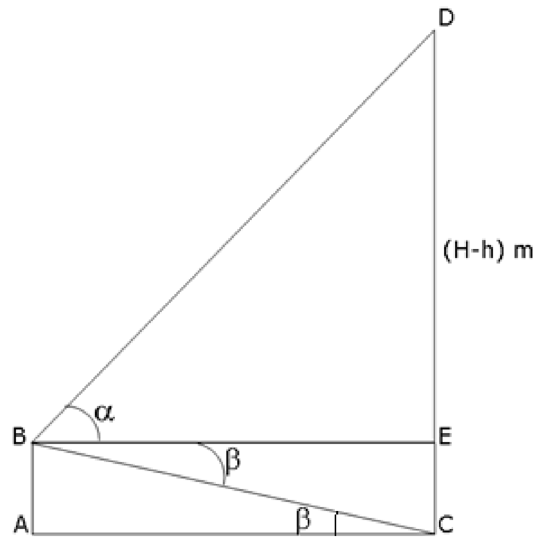
But BE = AC

$\therefore (H - h) \cot \alpha = h \cot \beta$ ....[From (a) and (b)]

$$H = h\,\frac{(\cot\alpha + \cot\beta)}{\cot\alpha}$$

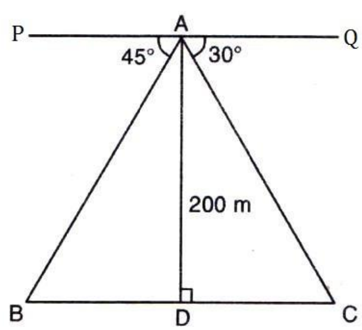
$H = h(1 + \tan\alpha \cot \beta)$

Thus, the height of the opposite house is $h(1 + \tan\alpha.\cot\beta)$ metres



OR

Let AD represent the light house.
Let the points B and C denote the ships based on the opposite sides of the light house.



$\angle ABD = \angle PAB = 45°$ (interior alternate angle)
$\angle ACD = \angle QAC = 30°$ (interior alternate angle)

$$\therefore \tan 45° = \frac{AD}{BD} \Rightarrow 1 = \frac{200}{BD} \Rightarrow BD = 200 \text{ m}$$

Also, $\tan 30° = \dfrac{AD}{DC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{200}{DC} \Rightarrow DC = 200\sqrt{3}$ m

$\Rightarrow$ DC = 200 × 1.732 = 346.4 m

$\therefore$ BC = BD + DC = (200 + 346.4) $\Rightarrow$BC = 546.4 m

Distance between two ships=546.4 m

37. Length of the cylinder = 24 cm

Diameter of copper wire = 4 mm = 0.4 cm

Therefore, the number of rounds required for a wire to cover the length of cylinder

$= \frac{\text{Length of cylinder}}{\text{Thickness of wire}}$

$= \frac{24 \text{ cm}}{0.4 \text{ cm}}$

$= 60$

Now, diameter of cylinder = 20 cm

Therefore, length of the wire required to complete one round = circumference of base of the cylinder = πd $= \frac{22}{7} \times 20 = \frac{440}{7}$ cm

Length of wire for covering the whole surface of cylinder

= length of wire in completing 60 rounds

$= 60 \times \frac{440}{7} = 3771.428$ cm

Radius of copper wire = $\frac{0.4}{2}$ cm $= 0.2$ cm

Therefore, volume of wire = $\pi r^2 h$ = $\frac{22}{7} \times (0.2)^2 \times 3771.428$ = 474.122 cm^3

Weight of wire = volume × density

= 474.122 × 8.68 gm

= 4115.38 gm

= 4.11538 kg

$\approx$ 4.12 kg

38. Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = (x – 10) hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8}=\frac{75}{8}$ hours by both the pipes together.

Therefore,

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$

$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$

$$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$$

$$\Rightarrow 75(2x-10)=8x^2-80x$$

$$\Rightarrow 150x-750=8x^2-80x$$

$$\Rightarrow 8x^2-230x+750=0$$

$$\Rightarrow 8x^2-200x-30x+750=0$$

$$\Rightarrow 8x(x-25)-30(x-25)=0$$

$$\Rightarrow (x-25)(8x-30)=0$$

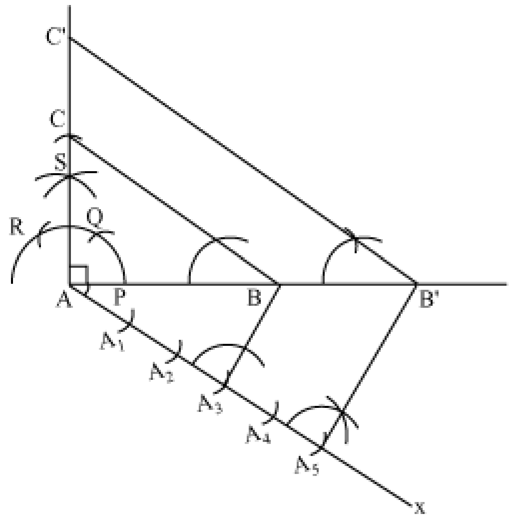
$$\text{i.e.}, x=25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 =15 hours respectively.

39. The steps of construction are as follows:

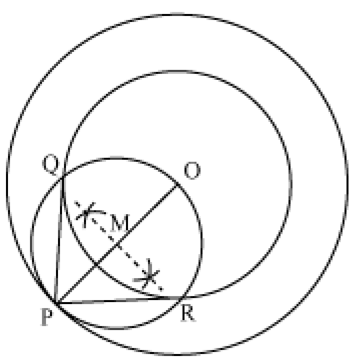
A. Draw a line segment AB = 4 cm, draw a ray SA making 90° with it.
B. Draw an arc of 3 cm radius while taking A as its centre to intersect SA at C. Join BC. ΔABC is required triangle.
C. Draw a ray AX making an acute angle with AB, opposite to vertex C.
D. Locate 5 points (as 5 is greater in 5 and 3) A_1, A_2, A_3, A_4, A_5 on line segment AX.
E. Join A_3B. Draw a line through A_5 parallel to A_3B intersecting extended line segment AB at B'.
F. Through B' draw a line parallel to BC intersecting extended line segment AC at C'. ΔAB'C' is the required triangle.



OR

The steps of construction are as follows:

1. Draw a circle of 4 cm radius with centre as O on the given plane.
2. Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.
3. Bisect OP. Let M be the midpoint of PO.
4. Taking M as its centre and MO as its radius draw a circle. Let it intersects the given circle at the points Q and R.
5. Join PQ and PR. PQ and PR are the required tangents.



Now, PQ and PR are of length 4.47 cm each.

In ΔPQO, since PQ is tangent, $\angle PQO = 90°$.

PO = 6 cm

QO = 4 cm

Applying Pythagoras theorem in ΔPQO,

$PQ^2 + QO^2 = PO^2$

$PQ^2 + (4)^2 = (6)^2$

$PQ^2 = 20$

$PQ = 2\sqrt{5} = 4.47$ cm

40.

Class Interval	Frequency
0 – 15	6
15 – 30	7
30 – 45	p
45 – 60	15
60 – 75	10
75 – 90	q
Total	51 = 38 + p + q

We have: $38 + p + q = 51$

$p + q = 51 - 38$

$\Rightarrow p + q = 13$ ....(1)

Mode = 55 (Given)

Thus, Modal Class is 45 – 60

Lower limit, $l = 45$,

Frequency of the modal class, $f_1 = 15$,

Frequency of the class preceding the modal class, $f_0 = p$,

Frequency of the class following the modal class $f_2 = 10$,

Class width, $h = 15$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\Rightarrow 55 = 45 + \left(\frac{15 - p}{2 \times 15 - p - 10}\right) \times 15$$

$$\Rightarrow 10 = \frac{15-p}{20-p} \times 15$$

$$\Rightarrow \frac{10}{15} = \frac{15-p}{20-p}$$

$$\Rightarrow \frac{2}{3} = \frac{15-p}{20-p}$$

$$\Rightarrow 40-2p = 45-3p$$

$$\Rightarrow p = 5$$

$$\Rightarrow q = 13-p = 13-5 = 8$$

OR

	x	f	cf
100-120	110	12	12
120-140	130	14	26
140-160	150	8	34
160-180	170	6	40
180-200	190	10	50