Section A

1. Correct option : B

Explanation:

$3.\overline{27}$ is an rational number. Since, it cannot be represented in the form of $\frac{p}{q}$ where $q \neq 0$.

2. Correct option : B

Explanation:

The mean of 6, 7, x, 8, y, 14 is 9.

$\Rightarrow \bar{x} = 9$

$\Rightarrow \frac{6+7+x+8+y+14}{6} = 9$

$\Rightarrow x + y + 35 = 54$

$\Rightarrow x + y = 19$

3. Correct option : A

Explanation:

OT is perpendicular to PT by tangent radius theorem.

In Δ OTP using Pythagoras theorem,

$\Rightarrow OT^2 + PT^2 = OP^2$

$\Rightarrow PT^2 = OP^2 - OT^2$

$\Rightarrow PT^2 = 10^2 - 6^2$

$\Rightarrow PT^2 = 100 - 36$

$\Rightarrow PT^2 = 64$

$\Rightarrow PT = 8$ cm

4. Correct option : C

Explanation:

Consider, $\sqrt{27} \times \sqrt{3} = \sqrt{27 \times 3} = \sqrt{81} = 9$

9 is a rational number.

5. Correct option : B

Explanation:

Since, probability neither negative nor greater than zero.

6. Correct option : C

Explanation:

Given a quadratic polynomial, the sum of whose zeroes is 0 and product is 3.

A quadratic polynomial is $x^2 - 0x + 3 = x^2 + 3$

7. Correct option : B

Explanation:

The denominator is of the form $2^2 \times 5^1$ where m = 2 and n = 1.

Here 2 > 1

$\Rightarrow$ The decimal expansion of the rational number $\frac{33}{2^2 \times 5}$ will terminates after 2 places of decimals.

8. Correct option : D

Explanation:

$kx^2 + 2x + 3k$

a = k, b = 2, c = 3k

$\alpha + \beta = \frac{-2}{k}$ and $\alpha\beta = \frac{3k}{k} = 3$

Let α, β be the zeros of a polynomial.

According to the question,

$\alpha + \beta = \alpha\beta$

$\frac{-2}{k} = 3 \Rightarrow k = \frac{-2}{3}$

9. Correct option : B

Explanation:

The distance between the points P(-1, 1) and Q(5, -7) is

PQ = $\sqrt{(-1-5)^2 + (1+7)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units

10. Correct option : B

Explanation:

The centroid of the triangle formed by (7, x), (y, -6) and (9, 10) is at (6, 3).

$\Rightarrow \left(\frac{7+y+9}{3}, \frac{x-6+10}{3}\right) = (6,3)$

$\Rightarrow \left(\frac{y+16}{3}, \frac{x+4}{3}\right) = (6,3)$

$\Rightarrow \frac{y+16}{3} = 6$ and $\frac{x+4}{3} = 3$

$\Rightarrow y = 2$ and $x = 5$

The point is (5, 2).

11. The coordinates of the point which divides the join of A(-1, 7) and B(4, -3) in the ratio 2:3 are $\left(\frac{2\times4+3\times(-1)}{2+3},\frac{2\times(-3)+3\times7}{2+3}\right)=(1,3)$

12. If x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ then k is -11.

x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$

$\Rightarrow 3^3 + 3(k - 1) + 9 = 0$

$\Rightarrow 27 + 3(k - 1) + 9 = 0$

$\Rightarrow 3(k - 1) = -36$

$\Rightarrow k - 1 = -12$

$\Rightarrow k = -11$

OR

The system of equations 3x – 2y = 0 and kx + 5y = 0 has infinitely many solutions. Then $k = \frac{-15}{2}$

$a_1 = 3$, $b_1 = -2$, $a_2 = k$, $b_2 = 5$

The system of equations 3x – 2y = 0 and kx + 5y = 0 has infinitely many solutions.

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}\Rightarrow\frac{3}{k}=\frac{-2}{5}\Rightarrow k=\frac{-15}{2}$$

13. The value of sin 40° – cos 50° is 0.

sin 40° – cos 50°

= sin (90° – 50°) – cos 50°

= cos 50° – cos 50°

= 0

14. If $2\sin 2\theta = \sqrt{3}$ then find $\theta = 30°$

$2\sin 2\theta = \sqrt{3}$

$$\Rightarrow \sin 2\theta=\frac{\sqrt{3}}{2}$$

$$\Rightarrow 2\theta=60°\Rightarrow\theta=30°$$

15. In a triangle ABC, AD is the bisector of ∠ A. If AB = 5.6 cm, AC = 4 cm and DC = 3 cm then BD = 4.2 cm.

In a triangle ABC, AD is the bisector of ∠ A.

$$\Rightarrow\frac{AB}{BD}=\frac{AC}{DC}$$

$$\Rightarrow\frac{5.6}{BD}=\frac{4}{3}\Rightarrow BD=\frac{5.6\times3}{4}=4.2\text{ cm}$$

16. $(1 - \cos^2 A)\sec^2 A$

$= \sin^2 A \sec^2 A$

$= \sin^2 A \times 1/\cos^2 A$

$= \tan^2 A$

OR

$\sin 3A = \cos (A - 10°)$

$\Rightarrow \cos (90° - 3A) = \cos (A - 10°)$ $\quad\because \cos (90° - A) = \sin A$

$\Rightarrow 90° - 3A = A - 10°$

$\Rightarrow 4A = 100°$

$\Rightarrow A = 25°$

17. $2\pi r = 88$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14 \text{ m}$$

The diameter of a wheel is 28 m.

18. The probability of winning a game is 0.4, the probability of losing it is 1 – 0.4 = 0.6.

19. 5, 2, -1, -4, -7,...

a = 5, d = 2 – 5 = -3, n = 8

$$a_n = a + (n-1)d$$

$$a_8 = 5 + (8-1)(-3) = 5 - 21 = -16$$

20. A die is rolled then sample space S = {1, 2, 3, 4, 5, 6}

n(S) = 6

Let A be the event that appearing a number less than 3.

A = {1, 2}

n(A) = 2

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

21. S is the sample space.

n(S) = 2 red + 2 white + 2 green = 6

Given that C be the event that getting the ball is not green.

There are 2 green balls.

Remaining balls are 6 – 2 = 4.

n(C) = 4

Required probability = $\dfrac{n(C)}{n(S)} = \dfrac{4}{6} = \dfrac{2}{3}$

22. The word 'mathemestics' has 11 letters.

n(S) = 11

Let A be the event that getting a card bears the letter 'm'.

n(A) = 2

Required probability = $\frac{n(A)}{n(S)} = \frac{2}{11}$

OR

The die is rolled once.

S = {A, B, C, D, E, A}

n(S) = 6

i. Let 'A' appear on the upper face

Then n(A) = 2

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

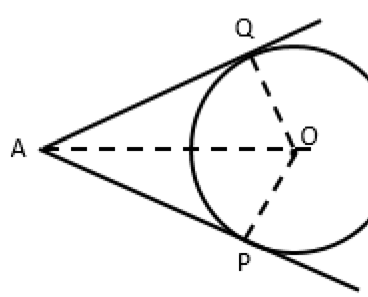
ii. Let 'D' appear on the upper face.

Then n(D) = 1

$P(D) = \frac{n(D)}{n(S)} = \frac{1}{6}$

23. Given : A circle with centre O and a point A outside it. AP and AQ are two tangents to the circle.

To prove : ∠AOP = ∠AOQ and ∠OAP = ∠OAQ



In ΔAOP and ΔAOQ,

AP = AQ	∵ tangents from an external point
OP = OQ	∵ radii of the same circle
OA = OA	∵ common
ΔAOP ≅ ΔAOQ	∵ SSS congruence
∠ AOP = ∠ AOQ and ∠ OAP = ∠ OAQ	∵ c. a. c. t.

24. L.H.S = $\dfrac{1-\tan^2 A}{\cot^2 A-1}$

$=\dfrac{1-\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A}{\sin^2 A}-1}$

$=\dfrac{\dfrac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A-\sin^2 A}{\sin^2 A}}$

$=\dfrac{\cos^2 A-\sin^2 A}{\cos^2 A}\times\dfrac{\sin^2 A}{\cos^2 A-\sin^2 A}$

$=\dfrac{\sin^2 A}{\cos^2 A}=\tan^2 A$ = R.H.S

$\Rightarrow \dfrac{1-\tan^2 A}{\cot^2 A-1}=\tan^2 A$

OR

L.H.S = $\dfrac{\tan\theta}{\sec\theta-1}+\dfrac{\tan\theta}{\sec\theta+1}$

$=\dfrac{\tan\theta(\sec\theta+1)+\tan\theta(\sec\theta-1)}{\sec^2\theta-1}$

$=\dfrac{\tan\theta\sec\theta+\tan\theta+\tan\theta\sec\theta-\tan\theta}{\tan^2\theta}$

$=\dfrac{2\tan\theta\sec\theta}{\tan^2\theta}$

$=\dfrac{2\sec\theta}{\tan\theta}$

$=\dfrac{2\times\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}$

$=2\operatorname{cosec}\theta$ = R.H.S

$\Rightarrow \dfrac{\tan\theta}{\sec\theta-1}+\dfrac{\tan\theta}{\sec\theta+1}=2\operatorname{cosec}\theta$

25. Circumference of a circle = 22 cm

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2\times\dfrac{22}{7}\times r=22$

$\Rightarrow r=3.5$ cm

$\Rightarrow$ Area of a quadrant of a circle = $\dfrac{1}{4}\pi r^2=\dfrac{1}{4}\times\dfrac{22}{7}\times 3.5\times 3.5=9.625\ m^2$

26. 1. The linear polynomials are x + 2, x – 1.

Hence, 2 students wrote linear polynomial.

2.

$$\begin{array}{r|l} & x-1 \\ \hline x+2 & x^2+x+1 \\ & \underline{x^2+2x} \\ & \quad -x+1 \\ & \quad \underline{-x-2} \\ & \qquad 3 \end{array}$$

Section C

27. $x^2 - 4kx + k + 3 = 0$

$x^2 - 4kx + (k + 3) = 0$

$\Rightarrow a = 1, b = -4k$ and $c = k + 3$

Let α, β are the zeros of the quadratic polynomial.

Sum of zeros = $\alpha+\beta=\frac{-b}{a}=4k$ ...(i)

Product of zero = $\alpha\beta=\frac{c}{a}=k+3$ ...(ii)

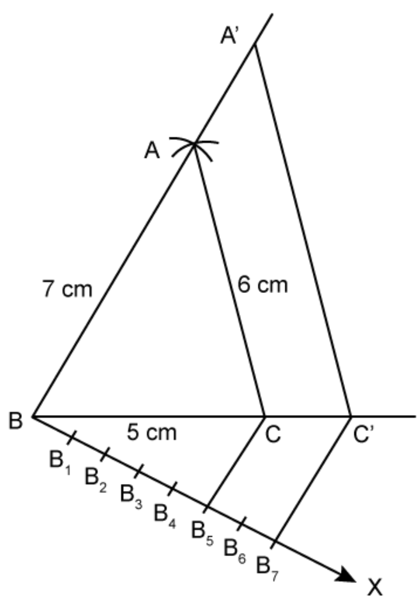
Also, $\alpha+\beta=2\alpha\beta$

$4k=2(k+3)$

$\Rightarrow 4k=2k+6$

$\Rightarrow 2k=6 \Rightarrow k=3$

28. Solution:



Steps of construction:

Step 1: Draw a line segment BC = 5 cm

Step 2: With B as the centre and radius 7 cm, an arc is drawn

Step 3: With C as the centre and radius 6 cm, another arc is drawn intersecting the previous arc at A

Step 4: Join AB and AC

Step 5: ΔABC is the given triangle

Step 6: Draw a line BX below BC

Step 7: Cut off equal distances from DX such that

$BB_1 = B_1B_2 = B_2B_3 = B_4B_5 = B_5B_6 = B_6B_7$

Step 8: Join B_5C

Step 9: Draw a line through B_7 parallel to B_5C cutting BC produced at C’

Step 10: Through C’, draw a line parallel to CA, cutting BA produced at A’

Step 11: ΔA’BC’ is the required triangle

29. Area of the first circle = $\pi r^2 = 962.5 \text{ cm}^2$

$$r^2 = \left(962.5 \times \frac{7}{22}\right) \text{cm}$$

$$r^2 = 306.25$$

$$r = 17.5 \text{ cm}$$

Area of the second circle = $\pi R^2 = 1386 \text{cm}^2$

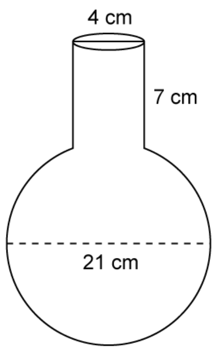
$$R^2 = \left(1386 \times \frac{7}{22}\right) \text{cm}$$

$$R^2 = 441$$

$$\Rightarrow R = 21 \text{cm}$$

Width of ring R - r = (21 - 17.5) cm = 3.5 cm

OR



Diameter of the spherical part of vessel = 21 cm

Its radius $= \frac{21}{2}$ cm

Its volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 11 \times 21 \times 21 \text{ cm}^3 = 4851 \text{ cm}^3$$

Volume of cylindrical part of vessel

$$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 7 \text{ cm}^3$$

$$= 88 \text{ cm}^3$$

$\therefore$ Volume of whole vessel $= (4851 + 88)\text{cm}^3 = 4939 \text{ cm}^3$

30. L.H.S = $\sec\theta\,(1 - \sin\theta)(\sec\theta + \tan\theta)$

$$= \sec\theta(1 - \sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \sec\theta(1 - \sin\theta)\left(\frac{1 + \sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\cos^2\theta} \times \left(1 - \sin^2\theta\right)$$

$$= \frac{\cos^2\theta}{\cos^2\theta}$$

= 1

= R.H.S

$\Rightarrow \sec\theta\,(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

OR

L.H.S= $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + 2\sin\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta + \cos^2\theta + 2\cos\theta\sec\theta + \sec^2\theta$

$= \sin^2\theta + 2 + \operatorname{cosec}^2\theta + \cos^2\theta + 2 + \sec^2\theta$

$= \sin^2\theta + \cos^2\theta + 2 + 2 + \operatorname{cosec}^2\theta + \sec^2\theta$

$= 1 + 4 + 1 + \cot^2\theta + 1 + \tan^2\theta$ $\quad \because \sin^2\theta + \cos^2\theta = 1, \operatorname{cosec}^2\theta = 1 + \cot^2\theta,$

$\sec^2\theta = 1 + \tan^2\theta$

$= 7 + \cot^2\theta + \tan^2\theta$

31. Let us assume that $\sqrt{5}+\sqrt{3}$ be a rational equal to $\frac{a}{b}$.

Then $\sqrt{5}+\sqrt{3}=\frac{a}{b}$

$\Rightarrow \sqrt{5}=\frac{a}{b}-\sqrt{3}$

$\Rightarrow \left(\sqrt{5}\right)^2=\left(\frac{a}{b}-\sqrt{3}\right)^2$

$\Rightarrow 5=\frac{a^2}{b^2}-\frac{2a\sqrt{3}}{b}+3$

$\Rightarrow 2=\frac{a^2}{b^2}-\frac{2a\sqrt{3}}{b}$

$\Rightarrow \frac{a^2}{b^2}-2=\frac{2a\sqrt{3}}{b}$

$\Rightarrow \frac{2a\sqrt{3}}{b}=\frac{a^2-2b^2}{b^2}$

$\Rightarrow \sqrt{3}=\frac{a^2-2b^2}{2ab}$

$\Rightarrow$ $\sqrt{3}$ is rational. It is a contradiction.

Also, $\sqrt{5}$ is irrational number.

Hence, $\sqrt{5}+\sqrt{3}$ is irrational.

OR

Let us assume that $\frac{2\sqrt{2}}{3}$ be rational number.

Let its simplest form be $\frac{2\sqrt{2}}{3}=\frac{a}{b}$ where a and b are integers having no common factor other than 1. Then,

$\sqrt{2}=\frac{3a}{2b}$ ....(i)

Since, 3a and 2b are non-zero integers, so $\frac{3a}{2b}$ is rational.

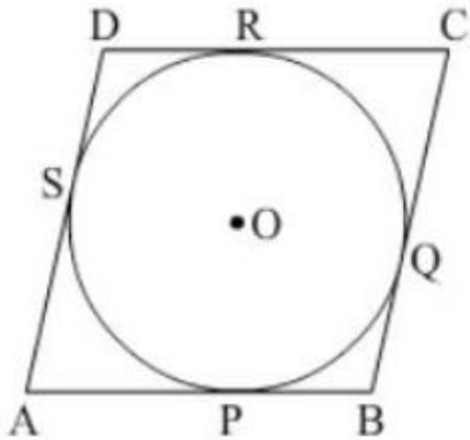
From (i) it follows that $\sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is an irrational.

Hence, $\frac{2\sqrt{2}}{3}$ is an irrational number.

32. Given: ABCD be a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus.



We know that the tangents drawn to circle from an exterior point are equal in length.

Therefore, AP = AS, BP = BQ, CR = CQ and DR = DS

Adding the above equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

2AB = 2BC ABCD is a parallelogram so AB = DC and AD = BC

AB = BC

Therefore, AB = BC = DC = AD

Hence, ABCD is a rhombus.

33. $3x + y = 1$ and $kx + 2y = 5$

$a_1 = 3, b_1 = 1, a_2 = k, b_2 = 2, c_1 = -1$ and $c_2 = -5$

(i) For a unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{3}{k} \neq \frac{1}{2} \Rightarrow k \neq 6$$

(ii) For no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \frac{3}{k} = \frac{1}{2} \Rightarrow k = 6$$

34. A(2, 1), B(5, 1), C(5, 4) and D(2, 4)

i. Teacher tells E to sit in the middle of the students B and D. Find the coordinates of the position where E can sit.
The coordinates of E are the mid-point of B(5, 1) and D(2, 4) i. e.
$\left(\frac{2+5}{2},\frac{4+1}{2}\right)=(3.5,2.5)$

ii. The distance between A(2, 1) and C(5, 4) is $\sqrt{(2-5)^2+(1-4)^2}=\sqrt{9+9}=3\sqrt{2}$

iii. The distance between B(5, 1) and D(2, 4) is $\sqrt{(5-2)^2+(1-4)^2}=\sqrt{9+9}=3\sqrt{2}$

Section D

35. Let the assumed mean = 225 and h = 50.

Class	Frequency f_i	Mid-value x_i	$u_i=\left(\frac{x_i-A}{h}\right)$	f_iu_i	C.F.
100–150	6	125	-2	-12	6
150–200	7	175	-1	-7	13
200–250	12	225	0	0	25
250–300	3	275	1	3	28
300–350	2	325	2	4	30
	N = 30			$\sum f_iu_i=-12$	

(i) Mean = $A+h\left(\frac{\sum f_iu_i}{N}\right)=225+50\left(\frac{-12}{30}\right)=225-20=205$

(ii) $\frac{N}{2}=\frac{30}{2}=15$

Cumulative frequency just after 15 is 25.

$\therefore$ Corresponding class interval is 200–250.

$\therefore$ Median class is 200–250.

Cumulative frequency c just before this class = 13

So, $l=200, f=12, \frac{N}{2}=15, h=50, c=13$

$$\therefore \text{Median} = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 200 + 50\left(\frac{15-13}{12}\right)$$

$$= 200 + \frac{50 \times 2}{12} = 200 + \frac{25}{3} = 200 + 8.33 = 208.33$$

Hence, mean = 205 and median = 208.33.

OR

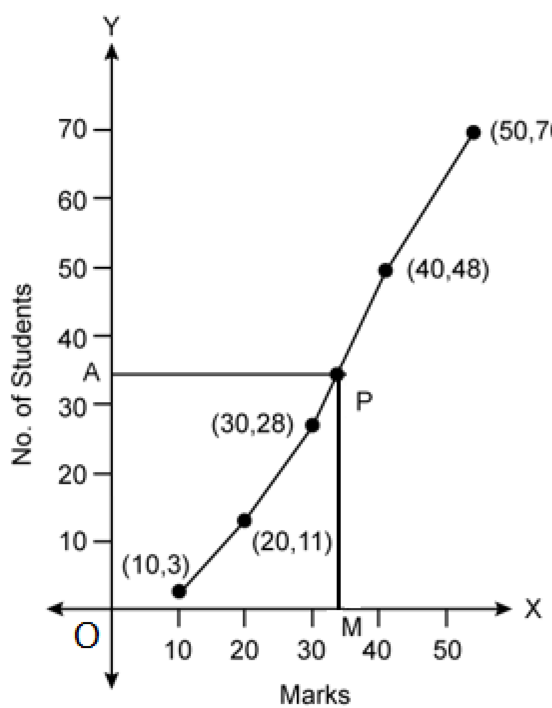
Cumulative frequency table is as follows:

Marks	C.F.
Marks less than 10	3
Marks less than 20	11
Marks less than 30	28
Marks less than 40	48
Marks less than 50	70

Scale:

X-axis: 1 cm = 10 marks

Y-axis: 1 cm = 10 students



The points (10, 3), (20, 11), (30, 28), (40, 48) and (50, 70) are plotted and joined as shown above. This is the required cumulative curve.

$$N = 70, \therefore \frac{N}{2} = \frac{70}{2} = 35$$

On the vertical line OY, take OA = 35

Through A, a horizontal line AP is drawn meeting the graph at P.

Through P, a vertical line PM is drawn.

Now, OM = 34 $\Rightarrow$ Median = 34

36. Let the two numbers be x and y.
According to the question,
$x + y = 8$ ...(i)
$\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$ ...(ii)
Consider, $x + y = 8 \Rightarrow y = 8 - x$ Put it in (ii)

$$\Rightarrow \frac{1}{x} + \frac{1}{8-x} = \frac{8}{15}$$

$$\Rightarrow \frac{8-x+x}{x(8-x)} = \frac{8}{15}$$

$$\Rightarrow \frac{8}{x(8-x)} = \frac{8}{15}$$

$$\Rightarrow \frac{1}{x(8-x)} = \frac{1}{15}$$

$$\Rightarrow x(8-x) = 15$$

$$\Rightarrow 8x - x^2 - 15 = 0$$

$$\Rightarrow x^2 - 8x + 15 = 0$$

$$\Rightarrow (x-5)(x-3) = 0$$

$$\Rightarrow x = 5 \text{ or } x = 3$$

If x = 5 then y = 3 or x = 3 then y = 5
Hence, the two numbers are 5 and 3.

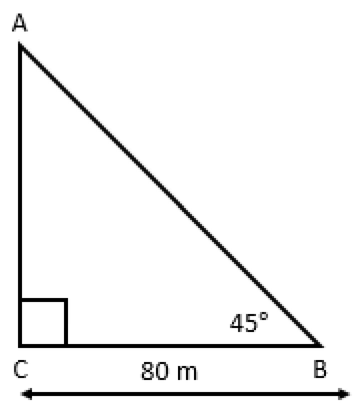
37. According to the question,
$a_9 = 75$ and $t_{21} = 183$
$\Rightarrow a + 8d = 75$ ...(i)
and $a + 20d = 183$...(ii)
Subtracting (i) from (ii)
$\Rightarrow 12d = 108$

$\Rightarrow d = 9$
Put it in (i)
$\Rightarrow a + 72 = 75$
$\Rightarrow a = 3$
We have a = 3, d = 9, n = 81
$\Rightarrow a_{81} = a + 80d = 3 + 80(9) = 723$

OR

3, 5, 7, 9, 11,...
Given sequence is in AP.
$\Rightarrow a = 3$ and $d = 2$
i. $a_n = a + (n - 1)d$
$\Rightarrow a_n = 3 + 2(n - 1)$
$\Rightarrow a_n = 3 + 2n - 2$
$\Rightarrow a_n = 1 + 2n$
ii. $a_{16} = a + 15d$
$\Rightarrow a_{16} = 3 + 15 \times 2 = 33$

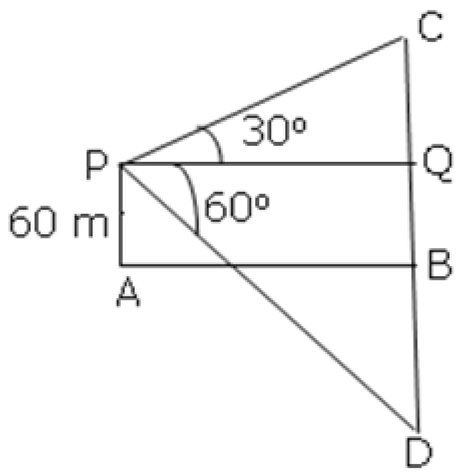
38. Let AC represent the church. B is the position of the observer.



∠ABC is the angle of elevation.
BC = 80 m, ∠ABC = 45°
In right angle triangle ACB,
$\tan 45° = \frac{AC}{BC}$
$1 = \frac{AC}{80}$
$\Rightarrow AC = 80$ m.
The height of the church is 80 m.

OR

Let C be the cloud and D be its reflection. Let the height of the cloud be H metres.



BC = BD = H

BQ = AP = 60 m.

Therefore CQ = H − 60 and DQ = H + 60

In ΔCQP,

$$\frac{PQ}{CQ} = \cot 30^\circ$$

$$\Rightarrow \frac{PQ}{H-60} = \sqrt{3}$$

$\Rightarrow PQ = (H - 60)\sqrt{3}$ m ....(i)

In ΔDQP,

$$\frac{PQ}{DQ} = \cot 60^\circ$$

$$\Rightarrow \frac{PQ}{H+60} = \frac{1}{\sqrt{3}}$$

$\Rightarrow PQ = \dfrac{H+60}{\sqrt{3}}$ ....(ii)

From (i) and (ii),

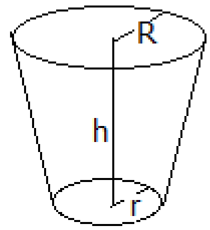
$$(H-60)\sqrt{3} = \frac{H+60}{\sqrt{3}}$$

$\Rightarrow 3H - 180 = H + 60$

$\Rightarrow H = 120$

Thus, the height of the cloud is 120 m.

39.



Here, R = 28 cm and r = 21 cm, we need to find h.

Volume of frustum = 28.49 L = 28.49×1000 cm³ = 28490 cm³

Now, Volume of frustum = $\frac{\pi h}{3}\left(R^2+Rr+r^2\right)$

$\Rightarrow \frac{22h}{7\times 3}\left(28^2+28\times 21+21^2\right)=28490$

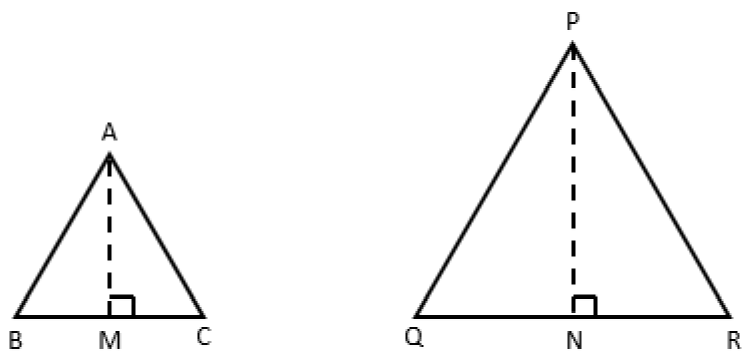
$\Rightarrow \frac{22}{21}h\times 1813=28490$

$\Rightarrow h=\frac{28490\times 21}{22\times 1813}=15\text{ cm}$

Hence, the height of bucket is 15 cm.

40. **Statement :** Ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given : Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$



To prove : $\frac{ar(ABC)}{ar(PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{CA}{RP}\right)^2$

Construction: Draw perpendicular AM and PN in the triangles ABC and PQR respectively.

Proof : For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles.

Now, $ar(ABC)=\frac{1}{2}BC\times AM$

And $\quad ar(PQR) = \frac{1}{2} QR \times PN$

So, $$\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN} \quad ....(1)$$

Now, in ΔABM and ΔPQN.

$\angle B = \angle Q \quad$ (As $\Delta ABC \sim \Delta PQR$)

And $\quad \angle M = \angle N \quad$ (Each = 90°)

So, $\quad \Delta ABM \sim \Delta PQN \quad$ (AA similarity criterion)

Therefore, $$\frac{AM}{PN} = \frac{AB}{PQ} \quad ....(2)$$

Also, $\quad \Delta ABC \sim \Delta PQR$

So, $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad ....(3)$$

Therefore,

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN} \quad [\text{from}(1)\text{and}(3)]$$

$$= \frac{AB}{PQ} \times \frac{AB}{PQ} \quad [\text{from}(2)]$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Now using (3), we get

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB^2}{PQ}\right) = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Hence proved.