CBSE Board
Class X Mathematics
Sample Paper 3 (Standard) – Solution

Section A

1. **Correct Option:** D

Explanation:

We know that,

lcm (a, b) × hcf (a, b) = a × b

$\Rightarrow 36 \times 2 = a \times 18$

$\Rightarrow a = \dfrac{36 \times 2}{18}$

$\Rightarrow a = 4$

2. **Correct Option:** A

Explanation:

$\text{Mean} = \dfrac{\sum x_i}{n}$

$\Rightarrow 9 = \dfrac{6 + 7 + x + 8 + y + 14}{6}$

$\Rightarrow x + y = 54 - 35$

$\Rightarrow x + y = 19$

3. **Correct Option:** B

Explanation:

Here the power of 2 is 2 and the power of 5 is 1.

$2 > 1$

Hence, $\dfrac{23}{2^2 \times 5}$ has terminating decimal expansion which terminates after 2 places of decimals.

4. **Correct Option:** D

Explanation:

Given system of equations has infinitely many solutions.

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\Rightarrow \dfrac{2}{4} = \dfrac{3}{k} = \dfrac{-5}{-10}$

$\Rightarrow k = \dfrac{4 \times 3}{2} = 6$

5. Correct Option: A

Explanation:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} \qquad \text{.....} \ (a-b)(a+b)=a^2-b^2$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}$$

$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$$=\sec\theta+\tan\theta$$

6. Correct Option: C

Explanation:

$$b^2x^2 - a^2y^2 = b^2(a\sec\theta)^2 - a^2(b\tan\theta)^2$$

$$= b^2a^2(\sec^2\theta - \tan^2\theta)^2$$

$$= b^2a^2\left(\sec^2\theta - \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= b^2a^2\sec^2\theta(1-\sin^2\theta) \qquad \text{......} \ \frac{1}{\cos^2\theta}=\sec^2\theta$$

$$= b^2a^2\sec^2\theta\cos^2\theta \qquad \text{......} \ \cos^2\theta=\frac{1}{\sec^2\theta}$$

$$= a^2b^2$$

7. Correct Option: A

Explanation:

ΔABC is a right angled triangle, right angled at C.

We know that, sum of all the angles of a triangle is $180°$

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B = 180° - 90° = 90°$

$\therefore \cos(A+B) = \cos 90° = 0$

8. **Correct Option:** D

Explanation:

The given point is A(5, -12), and let O(0,0) be the origin.

$$\text{Then, } AO = \sqrt{(5-0)^2 + (-12-0)^2}$$
$$= \sqrt{5^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169}$$
$$= 13 \text{ units}$$

9. **Correct Option:** A

Explanation:

The point (3, a) lies on the line 2x – 3y = 5.

Substituting the values of x and y in the given equation:

$2 \times 3 - 3 \times a = 5$

$\therefore 6 - 3a = 5$

$\therefore\ 3a = 1 \Rightarrow a = \frac{1}{3}$

10. **Correct Option:** C

Explanation:

Let A(0, 4), B(0, 0) and C(3, 0).

AB = $\sqrt{(0-0)^2 + (0-4)^2} = 4$

BC = $\sqrt{(3-0)^2 + (0-0)^2} = 3$

AC = $\sqrt{(3-0)^2 + (0-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Perimeter of a ΔABC = AB + BC + AC = 4 + 3 + 5 = 12

11. If r, h and *l* denote respectively the radius of base, height and slant height of a right circular cone, then total surface area is $\underline{\pi r^2 + \pi r l}$.

12. The graph of y = p(x) is given in the following figure for some polynomial p(x). The number of zeroes of p(x) is $\underline{1}$.

Explanation:

The graph of p(x) intersects the x-axis at only 1 point.
So, the number of zeroes is 1.

OR

A quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively $\frac{1}{4}, -1$ is $\underline{k\,(4x^2 - x - 4)}$.

Explanation:

Let the required polynomial be ax^2+bx+c, and let its zeroes be α and β.

$$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$$

$$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$$

If $a=4k$, then $b=-k$, $c=-4k$

Therefore, the quadratic polynomial is $k(4x^2 - x - 4)$, where k is a real number.

13. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ}=\frac{4}{9}$ then $\frac{ar\Delta ABC}{ar\Delta PQR}=$ $\frac{\boxed{16}}{\boxed{81}}$

Explanation:

The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR}=\frac{AB^2}{PQ^2}=\frac{16}{81}$$

14. $\underline{\frac{n(n+1)}{2}}$ is the sum of the first n natural numbers.

Explanation:

Sum of n natural numbers = 1 + 2 + 3 + ... + n

Here, a = 1, d = 2 − 1 = 1

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$\therefore \text{Sum of natural numbers}=\frac{n}{2}\left[2\times 1+(n-1)d\right]$$

$$=\frac{n}{2}\left[2+(n-1)\right]=\frac{n(n+1)}{2}$$

15. The probability of getting at most one head when two coins are tossed simultaneously is $\underline{\frac{3}{4}}$.

Explanation:

When two coins are tossed simultaneously, all possible outcomes are HH, HT, TH, TT.

Total number of possible outcomes = 4

Let E_2 be the event of getting at the most one head.

So, the favourable outcomes are HT, TH, TT.

Number of favourable outcomes = 3

$\therefore$ P (getting at the most 1 head) = P (E_2) = $\frac{3}{4}$

16. It is given that α and β are the zeros of the quadratic polynomial $f(x) = x^2 + 2x + 1$

$$\therefore \alpha+\beta=-\frac{2}{1}=-2 \text{ and } \alpha\beta=\frac{1}{1}=1$$

$$\text{Now, } \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{-2}{1}=-2$$

17. In ΔABC, DE $\parallel$ BC.

Then, by Basic Proportionality Theorem,

$$\frac{AD}{DB}=\frac{AE}{EC}$$

$$\Rightarrow \frac{2}{3}=\frac{AE}{4}$$

$$\Rightarrow AE=\frac{2\times 4}{3}=\frac{8}{3}=2.67 \text{ cm}$$

OR

Let ΔABC and ΔDEF be similar.

$$\frac{A(\Delta ABC)}{A(\Delta DEF)}=\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}=\frac{25}{64}$$

$$\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}=\frac{5^2}{8^2}$$

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{5}{8}$$

18. Circumference of first circle = $C_1 = 2\pi \times 19 = 38\pi$

Circumference of second circle = $C_2 = 2\pi \times 9 = 18\pi$

$C_1 + C_2 = 56\pi$

Let R be the radius of the new circle.

$\therefore$ $2\pi R = 56\pi$

$\therefore$ $2R = 56$

$\therefore$ R = 28 cm

19. The given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$ or $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2}\ldots$

Common difference d = $2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Term next to $3\sqrt{2} = 3\sqrt{2} + d = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2} = \sqrt{16 \times 2} = \sqrt{32}$

20. Given equation is $px^2 + 6x + 1 = 0$

Here, a = p, b = 6 and c = 1

The given equation will have real roots, if $b^2 - 4ac \geq 0$.

$\Rightarrow (6)^2 - 4(p)(1) \geq 0$

$\Rightarrow 36 - 4p \geq 0$

$\Rightarrow 36 \geq 4p$

$\Rightarrow p \leq 9$

Section B

21. $x^2 + 7x + 12$

$= x^2 + 3x + 4x + 12$

$= x(x + 3) + 4(x + 3)$

$= (x + 3)(x + 4)$

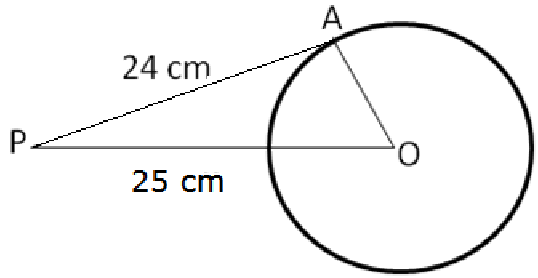
$\therefore$ −3 and −4 are the zeroes of the given polynomial.

Sum of zeroes = $-3 - 4 = -7 = \dfrac{-7}{1} = \dfrac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes = $(-3)(-4) = 12 = \dfrac{12}{1} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

22. PA is the tangent to the circle with centre O and radius, such that PO = 25 cm, PA = 24 cm

In ΔPAO, $\angle A = 90°$……….since tangent $\perp$ radius,



By Pythagoras' theorem,

$$PO^2 = PA^2 + AO^2$$
$$\Rightarrow OA^2 = PO^2 - PA^2$$
$$\Rightarrow OA^2 = (25)^2 - (24)^2 \text{ cm}$$
$$= (25+24)(25-24) \text{ cm}$$
$$= 49 \text{ cm}$$
$$\therefore \ OA = 7\text{cm}$$

Hence, the radius of the circle is 7 cm.

23. It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both triangles are proportional.

So, $\dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \dfrac{AB}{PQ}$

Let, $AB = x$ cm

Then, $\dfrac{x}{12} = \dfrac{32}{24}$

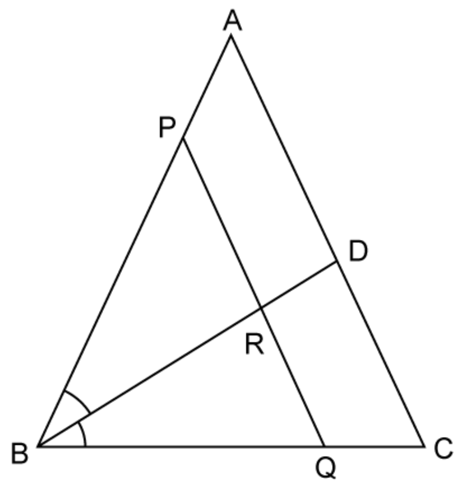
$$x = \frac{32 \times 12}{24} = 16 \text{ cm}$$

Hence, AB = 16 cm.

OR

Given ΔABC, the bisector of $\angle B$ meets AC at D, line PQ || AC meets AB, BC and BD at P, Q and R, respectively.

To prove: $PR \times BQ = QR \times BP$



Proof: In ΔBQP,

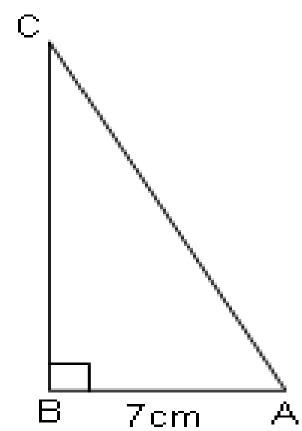
BR is the bisector of $\angle B$.

Therefore, by the angle bisector theorem,

$$\frac{BQ}{BP} = \frac{QR}{PR}$$

$$\Rightarrow PR \times BQ = QR \times BP$$

24. In ABC, we have



$AC^2 = BC^2 + AB^2$

$(1 + BC)^2 = BC^2 + AB^2$

$\Rightarrow 1 + BC^2 + 2BC = BC^2 + AB^2$

$\Rightarrow 1 + 2BC = 7^2$

$\Rightarrow 2BC = 48$

$\Rightarrow BC = 24$ cm

$\Rightarrow AC = 1 + BC = 1 + 24 = 25$ cm

Hence, $\sin C = \frac{AB}{AC} = \frac{7}{25}$ and $\cos C = \frac{BC}{AC} = \frac{24}{25}$

25. Let the number of red balls be x. Then,

P (drawing a white ball) $= \frac{4}{4+x}$, and

P (drawing a red ball) $= \frac{x}{4+x}$

$$\therefore \quad \frac{x}{4+x} = 2\left(\frac{4}{4+x}\right)$$

$$\Rightarrow \quad x(4+x) = 8(4+x)$$

$$\Rightarrow \quad (x-8)(4+x) = 0$$

$$\Rightarrow x = -4 \text{ or } x = 8$$

$x \neq -4$........no. of balls can't be negative

$$\therefore \quad x = 8$$

Hence, the number of red balls is 8.

OR

There are 18 cards having numbers 1, 3, 5, ..., 35 kept in a bag.

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13.
There are 5 numbers.

$\therefore$ Probability that a card drawn bears a prime number less than 15 = $\frac{5}{18}$

(ii) There is 1 number 15, which is divisible by both 3 and 5.

$\therefore$ Probability of drawing a card bearing a number divisible by both 3 and 5 is $\frac{1}{18}$.

26. Radius of the cone = 12 cm and its height = 24 cm

Volume of the cone = $\frac{1}{3}\pi R^2 h = \left(\frac{1}{3} \times \pi \times 12 \times 12 \times 24\right) cm^3 = (48 \times 24)\pi \text{ cm}^3$

$\text{Volume of each ball} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3 \times 3 \times 3 = (36\pi) cm^3$

$$\text{Number of balls formed} = \frac{\text{Volume of solid cone}}{\text{Volume of each ball}} = \frac{(48 \times 24)\pi}{36\pi} = 32$$

Section C

27. To find the minimum number of rooms required, first find the maximum number of participants which can be accommodated in each room such that the number of participants in each room is the same.
This can be determined by finding the HCF of 60, 84 and 108.

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

$\text{H.C.F.} = 2^2 \times 3 = 12$

So, the minimum number of rooms required

$$= \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12}$$

$$= 21$$

OR

Maximum number of columns in which the army contingent and the band can march will be given by HCF (616, 32).

We can use Euclid's algorithm to find the HCF.

Since 616 > 32, so applying Euclid's division lemma to a = 616 and b = 32, we get integers q and r as 32 and 19.

i.e. 616 = 32 × 19 + 8

Since the remainder r = 8 ≠ 0. So again, by applying Euclid's lemma to 32 and 8, we get integers 4 and 0 as the quotient and remainder.

i.e. 32 = 8 × 4 + 0

Step 3: Since the remainder is zero, the divisor at this stage will be the HCF.

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

28. Let the first term of the given AP = a and common difference = d

then, $T_n = a + (n-1)d$

$\Rightarrow T_4 = a + (4-1)d$, $T_{25} = a + (25-1)d$, and $T_{11} = a + (11-1)d$

$\Rightarrow T_4 = a + 3d$, $T_{25} = a + 24d$, and $T_{11} = a + 10d$

Now, $T_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

$\therefore T_{25} = a + 24d = (-3d + 24d) = 21d$

And $T_{11} = a + 10d = -3d + 10d = 7d$

$\therefore T_{25} = 21d = 3 \times (7d) = 3 \times T_{11}$

Hence, the 25^{th} term is triple its 11^{th} term.

29. Let the length and breadth of the rectangle be x and y respectively.

So the original area of the rectangle = xy

According to the question,

$(x + 2)(y - 2) = xy - 28$

$\Rightarrow xy - 2x + 2y - 4 = xy - 28$

$\Rightarrow 2x - 2y = 24$ .....(i)

And, $(x - 1)(y + 2) = xy + 33$

$\Rightarrow xy + 2x - y - 2 = xy + 33$

$\Rightarrow 2x - y = 35$ ....(ii)

Subtracting (i) from (ii), we get

$y = 11$

Substituting this value in (ii), we get

$2x - 11 = 35$

$\Rightarrow 2x = 46$

$\Rightarrow x = 23$

Thus, the length and breadth of the rectangle are 23 metres and 11 metres, respectively.

OR

Let the larger number be x and smaller be y. We know that
Dividend = Divisor × Quotient + Remainder
According to the question,
$3x = 4y + 3$
$3x - 4y = 3$........(i)
Also,
$7y = 5x + 1$
$5x - 7y = -1$.......(ii)
Solving (i) and (ii)
we get $x = 25$ and $y = 18$
The required numbers are 25 and 18.

30. Since a − b, a and a + b are the zeros of f(x) = $x^3 - 3x^2 + x + 1$.

$$\therefore\ (a-b)+a+(a+b) = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow 3a = -\frac{-3}{1}$$

$$\Rightarrow 3a = 3 \Rightarrow a = 1$$

$$\text{And, } (a-b)\times a\times(a+b) = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow a(a^2-b^2) = -\frac{1}{1}$$

$$\Rightarrow 1(1-b^2) = -1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

31. Given, the point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3.

$$\text{Co-ordinates of the point P} \equiv \left(\frac{2\times(-3)+3\times2}{2+3}, \frac{2\times6+3\times1}{2+3}\right) \equiv \left(\frac{-6+6}{5}, \frac{12+3}{5}\right) \equiv (0,3)$$

Now, the given equation is x − 5y + 15 = 0.

Substituting x = 0 and y = 3 in this equation, we have

L.H.S. = 0 − 5(3) + 15 = −15 + 15 = 0 = R.H.S.

Hence, the point P lies on the line x − 5y + 15 = 0.

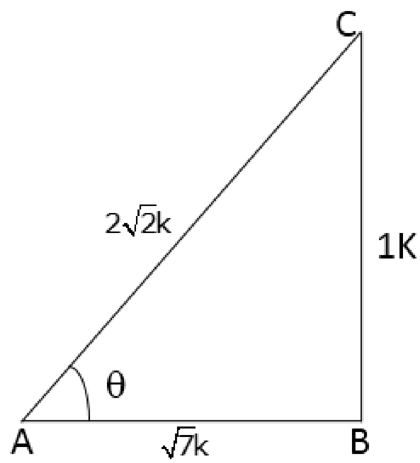
OR

$$\left.\begin{aligned}
AB &= \sqrt{(5-0)^2+(0-0)^2} = \sqrt{25+0} = 5\\
BC &= \sqrt{(8-5)^2+(4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5\\
CD &= \sqrt{(8-3)^2+(4-4)^2} = \sqrt{25+0} = 5\\
DA &= \sqrt{(0-3)^2+(0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5\\
AC &= \sqrt{(8-0)^2+(4-0)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}\\
BD &= \sqrt{(3-5)^2+(4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}
\end{aligned}\right\}$$

Now, AB = BC = CD = DA and AC $\neq$ BD.
Therefore, ABCD is a rhombus.

$$\text{Area} = \frac{1}{2}\times AC\times BD = \frac{1}{2}\times 4\sqrt{5}\times 2\sqrt{5} = \frac{1}{2}\times 40 = 20 \text{ sq.units}$$

32. Let us draw a ΔABC in which $\angle B = 90^\circ$ and $\angle A = \theta$.



$$\text{Given: } \tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{7}}$$

Let $BC = 1k$ and $AB = \sqrt{7}k$

where k is positive

By pythagoras theorem, we have

$$AC^2 = \left(AB^2 + BC^2\right)$$

$$\Rightarrow AC^2 = \left[\left(\sqrt{7}k\right)^2 + (1k)^2\right]$$

$$= 7k^2 + 1k^2 = 8k^2$$

$$\Rightarrow AC = 2\sqrt{2}k$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{1k} = 2\sqrt{2}$$

$$\sec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$$

$$\Rightarrow \frac{\left(\operatorname{cosec}^2\theta - \sec^2\theta\right)}{\left(\operatorname{cosec}^2\theta + \sec^2\theta\right)} = \left[\frac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}\right]$$

$$= \frac{\left(8 - \frac{8}{7}\right)}{\left(8 + \frac{8}{7}\right)} = \frac{\left(\frac{48}{7}\right)}{\left(\frac{64}{7}\right)} = \frac{48}{64} = \frac{3}{4}$$

Hence, $\left(\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}\right) = \frac{3}{4}$

33. 1 m of fencing costs Rs. 24.

Hence for Rs. 5280, the length of fencing = $\frac{5280}{24}$ = 220 m

$\Rightarrow$ Circumference of the field = 220 m

$\therefore 2\pi r = 220$

$\Rightarrow 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow r = \frac{220 \times 7}{44} = 35$ m

Area of the field = $\pi r^2 = \pi(35)^2 = 1225\pi$ m^2

Cost of ploughing = Rs. 0.50 per m^2

Total cost of ploughing the field = Rs. 1225 $\pi \times 0.50 = \frac{1225 \times 22 \times 1}{7 \times 2}$ = Rs. 1925

34. From the data given as above we may observe that maximum class frequency is 12 belonging to class interval 65 – 75.

So, modal class = 65 – 75

Lower class limit (l) of modal class = 65

Frequency (f_1) of modal class = 12

Frequency (f_0) of class preceding the modal class = 11

Frequency (f_2) of class succeeding the modal class = 9

Class size (h) = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 65 + \left(\frac{12 - 11}{2(12) - 11 - 9}\right) \times 10$$

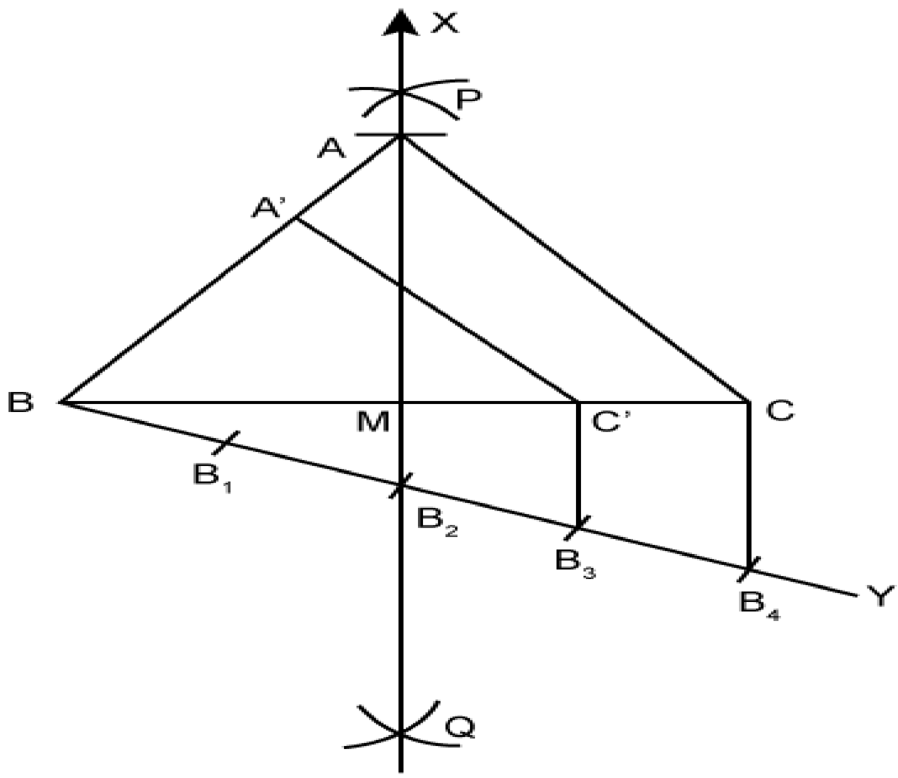
$$= 65 + \frac{1}{4} \times 10$$

$$= 65 + 2.5$$

$$= 67.5$$

Section D

35.



Steps of construction:

Step 1: Draw a line segment BC = 9 cm

Step 2: With centre B and radius more than $\frac{1}{2}BC,$ draw arcs on both sides of BC

Step 3: With centre C and the same radius, draw other arcs on both sides of BC intersecting the previous arcs at P and Q

Step 4: Join PQ and produce it to a point X such that PQ meets BC at M

Step 5: With centre M and radius 5 cm, draw an arc intersecting MX at A

Step 6: Join AB and AC

Δ ABC is the required triangle.

Step 7: Draw a line BY below BC

Step 8: Cut off 4 equal distances from BY so that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

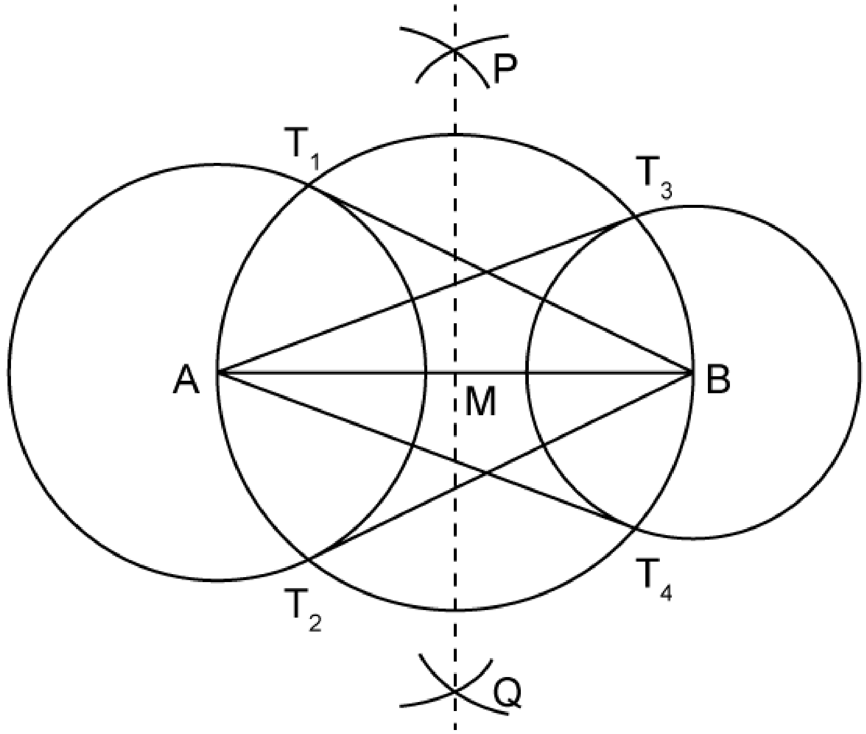
Step 9: Join CB_4

Step 10: Draw $C'B_3$ parallel to CB_4

Step 11: Draw C'A' parallel to CA, through C' intersecting BA at A'

ΔA'BC' is the required similar triangle.

OR



Steps of construction:

(i) Draw a line segment AB = 7 cm.

(ii) Taking A as the centre and radius 3 cm, a circle is drawn.

(iii) With centre B and radius 2.5 cm, another circle is drawn.

(iv) With centre A and radius more than $\frac{1}{2}AB$, arcs are drawn on both sides of AB.

(v) With centre B and the same radius [as in step (iv)], arcs are drawn on both sides of AB intersecting the previous arcs at P and Q.

(vi) Join PQ which meets AB at M.

(vii) With centre M and radius AM, a circle is drawn which intersects the circle with centre A at T_1 and T_2 and the circle with centre B at T_3 and T_4

(viii) Join AT_3, AT_4, BT_1 and BT_2

Thus, AT_3, AT_4, BT_1, BT_2 are the required tangents.

36. It is known that the tangents drawn from an external point to the circle are equal in length.

$\therefore$ AB = AC, PB = PD and QC = QD

$\therefore$ Perimeter of Δ APQ = AP + PQ + AQ

$$= (AB - BP) + (PD + DQ) + (AC - QC)$$

$$= (AB - BP) + (BP + QC) + (AB - QC)$$

$= 2AB$

$= 10$ m

Thus, the perimeter of the triangular garden is 10 m.

Qualities encouraged: Creativity, Social Consciousness, Environmental Friendliness, Problem Solving, Team work.

37. Let the width of the path be x metres.

Then

Area of the path = 16 × 10 – (16 – 2x) (10 – 2x) = 120

$$\Rightarrow 16\times10-\left(160-32x-20x+4x^2\right)=120$$

$$\Rightarrow 160-160+32x+20x-4x^2=120$$

$$\Rightarrow -4x^2+52x-120=0$$

$$\Rightarrow 2x^2-26x+60=0$$

$$\Rightarrow x^2-13x+30=0$$

$$\Rightarrow x^2-10x-3x+30=0\Rightarrow x(x-10)-3(x-10)=0$$

$$\Rightarrow (x-10)(x-3)=0$$

$$\Rightarrow x-10=0 \text{ or } x-3=0$$

$$x=10 \text{ or } x=3$$

Hence, the required width is 3 metres as x cannot be 10 m since the width of the path cannot be greater than or equal to the width of the field.

OR

Let the faster pipe take x minutes to fill the cistern.

Then the other pipe takes (x + 3) minutes.

$$\frac{1}{x}+\frac{1}{(x+3)}=\frac{13}{40}\Rightarrow\frac{(x+3)+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow 40(2x+3)=13\left(x^2+3x\right)$$

$$\Rightarrow 13x^2-41x-120=0$$

$$\Rightarrow 13x^2-65x+24x-120=0$$

$$\Rightarrow 13x(x-5)+24(x-5)=0$$

$$\Rightarrow (x-5)(13x+24)=0$$

$\Rightarrow x = 5 \quad \text{or} \quad x = \frac{-24}{3}$

$x = 5$ (Time cannot be negative)

If the faster pipe takes 5 minutes to fill the cistern, then the other pipe takes (5 + 3) minutes = 8 minutes

38. Let the radius of the base of the cone be x cm.

So, diameter = 2x cm

$\Rightarrow$ Height of the cone = 2(diameter) = 2(2x) = 4x

Volume of one ice cream

= volume of conical portion + volume of hemispherical portion

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 [h + 2r]$

$= 2\pi x^3$

Diameter of the cylindrical container = 12 cm

Radius of cylindrical container = 6 cm

Height of cylindrical container = 15 cm

Volume of cylindrical container $= \pi r^2 h = \pi(6)^2(15)$

We have:

Number of children $= \frac{\text{volume of cylindrical container}}{\text{volume of one ice cream cone}}$

$\Rightarrow 10 = \frac{\pi(6)^2(15)}{2\pi x^3}$

$\Rightarrow x^3 = 27$

$\Rightarrow$ x = 3 cm

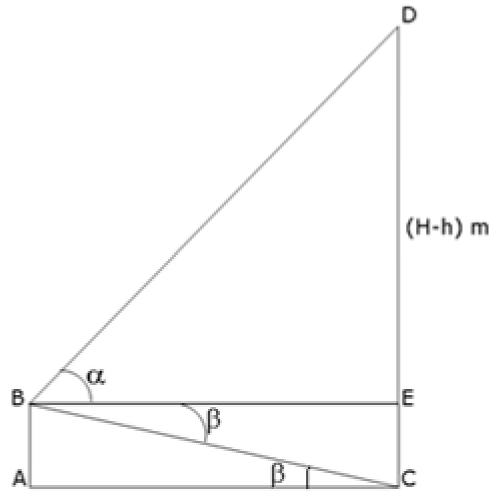
Thus, diameter of the base of the cone is 2x = 6 cm.

39. Let B be the window of a house AB and let CD be the other house.

Then, AB = EC = h metres.

Let CD = H metres.

Then, ED = (H − h) m



In ΔBED,

$\cot \alpha = \dfrac{BE}{ED}$

$BE = (H - h)\cot \alpha$ ... (a)

In ΔACB,

$\dfrac{AC}{AB} = \cot \beta$

$AC = h.\cot \beta$ .... (b)

But BE = AC

$\therefore (H - h) \cot \alpha = h\cot \beta$

$H = h \dfrac{(\cot\alpha + \cot\beta)}{\cot\alpha}$

$H = h(1 + \tan\alpha \cot\beta)$

Thus, the height of the opposite house is $h(1 + \tan\alpha.\cot\beta)$ meters.

40. We may prepare the less than series and the more than series.

(i) Less than series:

Height in (cm)	Frequency
Less than 140	0
Less than 144	3
Less than 148	12
Less than 152	36
Less than 156	67
Less than 160	109
Less than 164	173
Less than 168	248
Less than 172	330
Less than 176	416
Less than 180	450

Now, on the graph paper, plot the points (140, 0), (144, 3), (148, 12), (152, 36), (156, 67), (160, 109), (164, 173), (168, 248), (172, 330), (176, 416) and (180, 450).

(ii) More than series:

Height in cm	C.F.
More than 140	450
More than 144	447
More than 148	438
More than 152	414
More than 156	383
More than 160	341
More than 164	277
More than 168	202

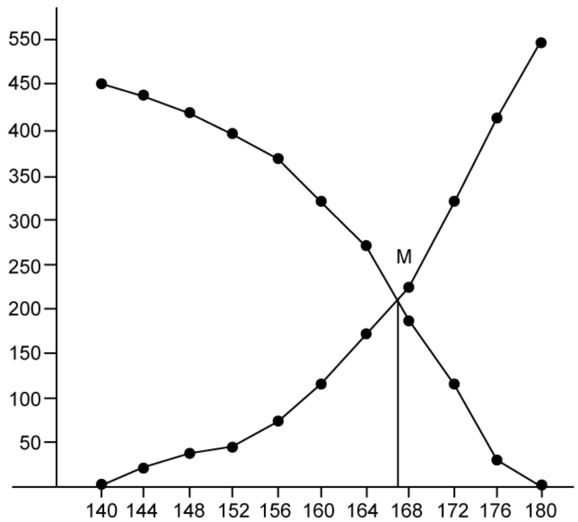
More than 172	120
More than 176	34
More than 180	0

Now, on the same graph paper, plot the points (140, 450), (144, 447), (148, 438), (152, 414), (156, 383), (160, 341), (164, 277), (168, 202), (172, 120), (176, 34) and (180, 0).

Scale:

X-axis: 1 cm = 4 cm

Y-axis: 1 cm = 50 people



The curves intersect at (166, 225).

Hence, 166 is the median.

OR

Let the assumed mean be 35, h = 10. Now, we have

Class	Frequency f_i	Mid-value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	C.F	$f_i u_i$
0–10	5	5	-3	5	-15
10–20	10	15	-2	15	-20
20–30	18	25	-1	33	-18
30–40	30	35 = A	0	63	0
40–50	20	45	1	83	20
50–60	12	55	2	95	24
60–70	5	65	3	100	15
	N = 100				$\Sigma f_i u_i = 6$

(i) Mean $\bar{x} = A + h\left(\frac{\Sigma f_i u_i}{N}\right)$

$$= 35 + 10 \times \left(\frac{6}{100}\right) = 35 + 0.6 = 35.6$$

(ii) N = 100, $\frac{N}{2} = 50$

Cumulative frequency just after 50 is 63.

$\therefore$ Median class is 30–40.

$\therefore$ l = 30, h = 10, N = 100, c = 33, f = 30

$$\therefore \text{Median } M_e = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 30 + 10\left(\frac{50 - 33}{30}\right) = 30 + 10\left(\frac{17}{30}\right) = 30 + 5.67 = 35.67$$

(iii) Mode $= 3 \times \text{median} - 2 \times \text{mean}$

$= 3 \times 35.67 - 2 \times 35.6$

$= 107.01 - 71.2$

$= 35.81$

Thus, mean = 35.6, median = 35.67 and mode = 35.81.