CBSE Board
Class X Mathematics
Sample Paper 6 (Standard) – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1.

Correct option : (b)

Explanation :

$a = 2^2 \times 3^3 \times 5^4$

$b = 2^3 \times 3^2 \times 5$

$HCF(a,b) = 2^2 \times 3^2 \times 5 = 180$

2. Correct option: (b)

Explanation:

While computing the mean of the grouped data, we assume that the frequencies are centered at the class marks of the classes.

3.

Correct option : (a)

Explanation :

70 and 125 are divided by a largest number leaving remainders 5 and 8 respectively.

Now, $70 - 5 = 65$

$125 - 8 = 117$

So, 65 and 117 are exactly divisible by their HCF

$HCF(65,117) = 13$

4.

Correct option: (c)

Explanation :

$x - y = 2$ ....(i)

$$\frac{2}{x+y} = \frac{1}{5}$$

$\Rightarrow x + y = 10$ ....(ii)

Adding equations (i) and (ii), we get

$2x = 12$

$\Rightarrow x = 6$

Substituting $x = 6$ in (ii), we get $y = 4$.

5.

Correct option: (c)

Explanation :

$$\frac{\tan 35^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan 12^\circ}=\frac{\tan\left(90^\circ-55^\circ\right)}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan\left(90^\circ-78^\circ\right)}$$

$$=\frac{\cot 55^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\cot 78^\circ}$$

$$=1+1$$

$$=2$$

6.

Correct option: (d)

Explanation :

$\tan\ 10^\circ \tan\ 15^\circ \tan\ 75^\circ \tan\ 80^\circ$

$=\tan 10^\circ \tan 15^\circ \tan(90^\circ-15^\circ)\tan(90^\circ-10^\circ)$

$=\tan 10^\circ \tan 15^\circ \cot 15^\circ \cot 10^\circ$

$=(\tan 10^\circ \cot 10^\circ)(\tan 15^\circ \cot 15^\circ)$

$=1\times 1$

$=1$

7.

Correct option: (c)

Explanation :

$\sin\ 47^\circ \cos\ 43^\circ+\cos\ 47^\circ \sin\ 43^\circ$

$=\sin\ 47^\circ \cos\ (90^\circ-47^\circ)+\cos\ 47^\circ \sin\ (90^\circ-47^\circ)$

$=\sin\ 47^\circ \sin 47^\circ+\cos\ 47^\circ \cos 47^\circ$

$=\sin^2 47^\circ+\cos^2 47^\circ$

$=1$

8.

Correct option: (c)

Explanation :

By Section Formula,

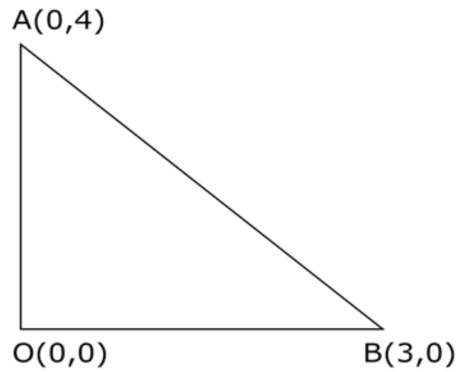
The x-coordinate of $C=\dfrac{2(5)+3(2)}{2+3}$

$\Rightarrow k=\dfrac{16}{5}$

9.

Correct option: (d)

Explanation:



$AO = 4$ units

$BO = 3$ units

Using Distance formula, we get

$AB = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

So, the perimeter of the triangle

$= AB + AO + BO$

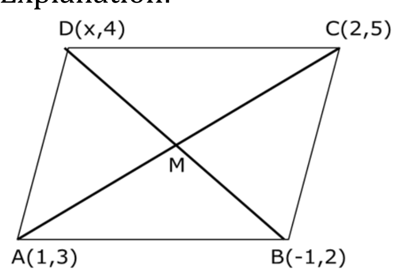
$= 5 + 4 + 3$

$= 12$ units

10.

Correct option: (b)

Explanation:



Since ABCD is a ||gm, the diagonals bisect each other.

So, M is the mid-point of BD as well as AC.

$$\frac{1+2}{2} = \frac{x-1}{2}$$

$\Rightarrow 1 + 2 = x - 1$

$\Rightarrow x = 4$

11.

A rectangular ground 80 m x 50 m has a path 1 m wide outside around it. The area of the path is <u>264 m²</u>

Explanation:

Area of the path

$= \text{area of the outer rectangle} - \text{area of the inner rectangle}$

Now, length of the outer rectangle $= 80 + 2 = 82$ m

and breadth of the outer rectangle $= 50 + 2 = 52$ m

So, area of the path

$= (82 \times 52) - (80 \times 50)$

$= (4264) - (4000)$

$= 264 \text{ m}^2$

12.

If α and β are the zeroes of $x^2 + 5x + 8$ then the value of $(\alpha + \beta)$ is <u>-5</u>

Explanation:

Since α and β be the zeros of $x^2 + 5x + 8$.

Then, we have

$$\alpha + \beta = -\frac{b}{a} = -\frac{5}{1} = -5$$

OR

If the sum of the zeros of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to the product of its zeros then <u>k = -2/3</u>

Explanation:

Let α and β be the zeros of $kx^2 + 2x + 3k$.

Then, we have

$$\alpha + \beta = \alpha\beta$$

$$\Rightarrow -\frac{2}{k} = \frac{3k}{k}$$

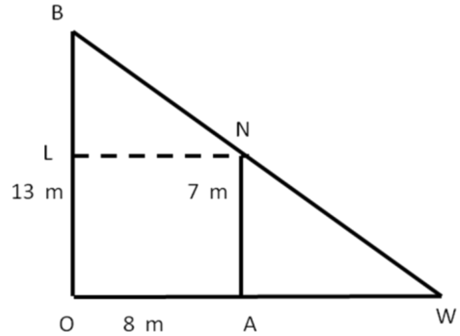
$$\Rightarrow -\frac{2}{k} = 3$$

$$\Rightarrow k = -\frac{2}{3}$$

13.

Two poles of height 13 m and 7 m respectively stand vertically on a plane ground at a distance of 8 m from each other. The distance between their tops is <u>10m</u>

Explanation:



OB and AN are the two poles.

We have to find the distance between their tops

that is, BN.

Construction : Draw $NL \perp OB$

OANL is a rectangle....(Since all the angles are right angles)

$LN = OA = 8 \text{ m}$

$OL = AN = 7 \text{ m}$

$\Rightarrow BL = OB - OL = 13 \text{ m} - 7 \text{ m} = 6 \text{ m}$

ΔBLN forms a right-angled triangle.

By Pythagoras theorem,

$BN^2 = LN^2 + BL^2$

$BN^2 = 8^2 + 6^2$

$BN^2 = 64 + 36$

$BN^2 = 100$

$BN = 10 \text{ m}$

So, the distance between their tops is 10 m.

14.

If 4, x_1, x_2, x_3, 28 are in AP then x_3 = 22

Explanation:

Given that 4, x_1, x_2, x_3, 28 are in AP.

Let d be the common difference.

Since 28 is the 5th term,

$28 = 4 + 4d$

$\Rightarrow 4d = 24$

$\Rightarrow d = 6$

$x_3 = a + (3)d$ ...(x_3 is the fourth term)

$\Rightarrow x_3 = 4 + 3(6)$

$\Rightarrow x_3 = 22$

15.

The probability that a number selected at random from the numbers 1, 2, 3,....15 is a multiple of 4, is <u>1/5</u>

Explanation:

The selected numbers would be 4, 8, and 12.

So, there are 3 numbers

P(number is a multiple of 4)

$$= \frac{\text{number of multiples of 4}}{\text{Total numbers}}$$

$$= \frac{3}{15}$$

$$= \frac{1}{5}$$

16.

A number is a terminating decimal, if the denominator

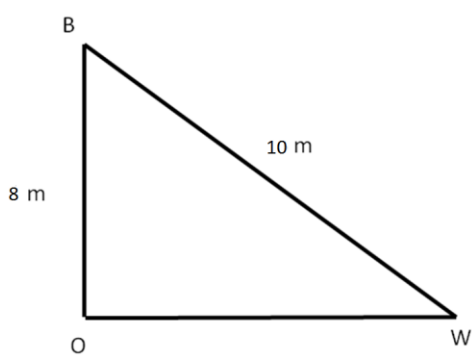
is of the form $2^m \times 5^n$, where m and n are non-negative integers.

$$\frac{17}{30} = \frac{17}{2 \times 3 \times 5}$$

Clearly, $\frac{17}{30}$ is a not a terminating decimal,

since its denominator is not of the form $2^m \times 5^n$.

17.



Let BW be the ladder and OB be the house.

ΔBOW forms a right-angled triangle.

By Pythagoras theorem,

$$BW^2 = OW^2 + OB^2$$

$$OW^2 = BW^2 - OB^2$$

$$OW^2 = 10^2 - 8^2$$

$$OW^2 = 100 - 64$$

$$OW = 6 \text{ m}$$

Thus, the distance of the foot of the ladder from the house is 6 m.

18.

Construction : Join OT.

$PT = 24$ cm

$OP = 26$ cm

Since PT is a tangent to the circle at T.

$\angle PTO = 90°$ ....(tangent is perpendicular to the radius of a circle)

In ΔPTO,

By Pythagoras theorem,

$OP^2 = PT^2 + OT^2$

$\Rightarrow OT^2 = OP^2 - PT^2$

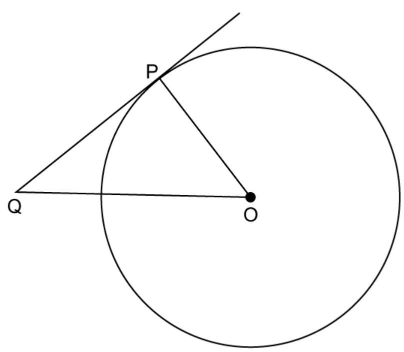
$\Rightarrow OT^2 = 26^2 - 24^2$

$\Rightarrow OT^2 = 676 - 576$

$\Rightarrow OT^2 = 100$

$\Rightarrow OT = 10$ cm

OR



Given that ΔPQO is an isosceles triangle.

Since PQ is a tangent to the circle at P.

$\angle OPQ = 90°$ ....(tangent is perpendicular to the radius of a circle)

In ΔOPQ,

$OP = PQ$

$\Rightarrow \angle OQP = \angle POQ$

Using Angle Sum Property,

$\angle OQP + \angle POQ + \angle OPQ = 180°$

$\Rightarrow \angle OQP + \angle OQP + 90° = 180°$

$\Rightarrow 2\angle OQP = 90°$

$\Rightarrow \angle OQP = 45°$

19.

The given AP is 9, 13, 17, 21, ….

$a = 9$ and $d = 13 - 9 = 4$

$a_n = a + (n-1)d$

$\Rightarrow a_{20} = 9 + 19(4)$

$\Rightarrow a_{20} = 85$

So, the 20th term is 85.

20.

$9x^2 - 3x - 2 = 0$

$\Rightarrow 9x^2 - 6x + 3x - 2 = 0$

$\Rightarrow 3x(3x-2) + 1(3x-2) = 0$

$\Rightarrow (3x-2)(3x+1) = 0$

$\Rightarrow 3x - 2 = 0$ or $3x + 1 = 0$

$\Rightarrow x = \frac{2}{3}$ or $x = -\frac{1}{3}$

Section B

21.

If possible, let $(2+\sqrt{3})$ be rational.

Then 2 and $\sqrt{3}$ are rational.

$\Rightarrow 2+\sqrt{3}-2$ is rational ....(Since difference of two rationals is rational)

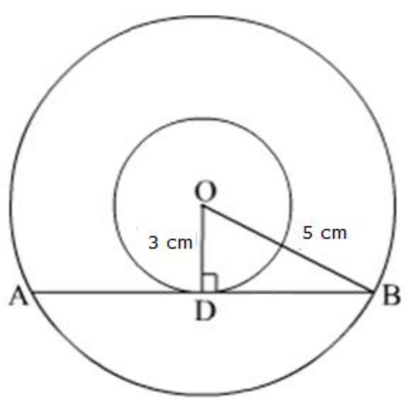
$\Rightarrow \sqrt{3}$ is rational

This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction arises by assuming that $(2+\sqrt{3})$ is rational.

Hence, $2+\sqrt{3}$ is irrational.

22.



Since AB is a tangent to the inner circle.

$\angle ODB = 90°$ ....(tangent is perpendicular to the radius of a circle)

AB is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

So, $AB = 2DB$.

In ΔODB,

By Pythagoras theorem,

$OB^2 = OD^2 + DB^2$

$\Rightarrow 5^2 = 3^2 + DB^2$

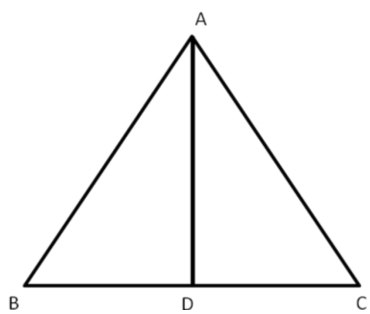
$\Rightarrow DB^2 = 5^2 - 3^2$

$\Rightarrow DB^2 = 25 - 9$

$\Rightarrow DB = 4$ cm

$AB = 2DB = 2(4) = 8$ cm

23.



Let ΔABC be an equilateral triangle.

We know that,

In an equilateral triangle the altitude is same as the median.

So, $BD = DC = a$ cm

By Pythagoras theorem,

$AC^2 = AD^2 + DC^2$

$\Rightarrow AD^2 = AC^2 - DC^2$

$\Rightarrow AD^2 = (2a)^2 - a^2$

$\Rightarrow AD^2 = 4a^2 - a^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = \sqrt{3}a$ cm

So, length of the altitude is $\sqrt{3}a$ cm.

OR

Given $\Delta ABC \sim \Delta DEF$ and $2AB = DE$ and $BC = 6$ cm

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{EF}$$

$$\Rightarrow EF = 3 \text{ cm}$$

24.

Let the length of a shadow of 12.5 m high tree $= x$ m

Now,

Ratio of lengths of objects $=$ Ratio of lengths of their shadow

$$\Rightarrow \frac{5}{12.5} = \frac{2}{x}$$

$$\Rightarrow x = \frac{12.5 \times 2}{5} = 5 \text{ m}$$

25.

There are 26 letters in the English alphabet.

Total number of outomes $= 26$

The vowels are A, E, I, O and U

So, there are $26 - 5 = 21$ consonants

$$\text{P(getting a consonant)} = \frac{21}{26}$$

OR

There are 200 electric bulbs in total.

Out of this, 16 are defective

So, the remaining 184 are non defective.

(i) $\text{P(getting a defective bulb)} = \frac{16}{200} = \frac{2}{25}$

(ii) $\text{P(getting a non defective bulb)} = \frac{184}{200} = \frac{23}{25}$

26.

Given radius of the top of the bucket $(R) = 28$ cm,

radius of the bottom of the bucket $(r) = 7$ cm and

slant height of the bucket $(l) = 45$ cm

$\because$ The bucket will be in the form of a frustum

$\therefore$ Curved surface area of the bucket $= \pi(r + R)l$

$$= \frac{22}{7} \times (28 + 7) \times 45$$

$$= 22 \times 5 \times 45$$

$$= 4950 \text{ cm}^2$$

Thus, Curved surface area of the bucket is 4950 cm^2.

Section C

27.

On dividing n by 3, let q be the quotient and r be the remainder.

Then, n=3q+r, where $0 \le r < 3$

$\Rightarrow$ n=3q+r, where r=0,1 or 2

$\Rightarrow$ n=3q or n=3q+1 or n=3q+2

Case 1: If n=3q, then n is clearly divisible by 3.

Case 2: If n=3q+1, then (n+2)=(3q+3)=3(q+1),

which is clearly divisible by 3.

In this case, (n+2) is divisible by 3.

Case 3: If n=3q+2, then (n+4)=(3q+6)=3(q+2),

which is clearly divisible by 3.

In this case, (n+4) is divisible by 3.

Hence, one and only one out of n, (n+2) and (n+4)

is divisible by 3.

OR

Let a be the given positive odd integer.

On dividing a by 4, let q be the quotient and r be the remainder.

Then, by Euclid's algorithm, we have

$a = 4q + r$, where $0 \le r < 4$

$\Rightarrow$ a=4q+r, where r=0,1,2,3

$\Rightarrow$ a=4q or a=4q+1 or a=4q+2 or a=4q+3

But, a=4q and a=4q+2=2(2q+1) are clearly even.

Thus, when a is odd, it is of the form a=(4q+1) or (4q+3)

for some integer q.

28.

Students of first section of class 1 will plant 2 trees.

Students of second section of class 1 will plant 2 trees.

Thus, students of class 1 will plant 4 trees.

Students of first section of class 2 will plant 4 trees.

Students of second section of class 2 will plant 4 trees.

Thus, students of class 2 will plant 8 trees.

Students of first section of class 3 will plant 6 trees.

Students of second section of class 3 will plant 6 trees.

Thus, students of class 3 will plant 12 trees.

Thus, the number of trees planted by the students,

form an AP: $4,\ 8,\ 12,\ ....$

Thus, $a = 4$ and $d = 4$

Let us find the number of trees planted in total.

$$\Rightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{12} = \frac{12}{2}\left[2\times 4 + (12-1)4\right]$$

$$\Rightarrow S_{12} = 6\left[8 + 44\right]$$

$$\Rightarrow S_{12} = 312$$

Thus, the total number of trees is 312.

We should conserve the nature around us and bring about awareness to save trees.

29.

Let each pencil cost Rs. x and each pen cost Rs. y.

According to the first condition,

$5x + 7y = 195 \quad ....(i)$

According to the second condition,

$7x + 5y = 153 \quad ....(ii)$

Adding (i) and (ii), we get

$12x + 12y = 348$

$\Rightarrow x + y = 29 \quad .....(iii)$

Subtract (i) from (ii), we get

$2x - 2y = -42$

$\Rightarrow x - y = -21 \quad .....(iv)$

Adding (iii) and (iv), we get

$2x = 8$

$\Rightarrow x = 4$

Substituting $x = 4$ in (iii), we get $y = 25$.
Hence, the cost of each pencil is Rs. 4 and the cost of each pen is Rs. 25.

OR

Given that in cyclic quadrilateral ABCD,

$\angle A = (4x + 20)°$, $\angle B = (3x - 5)°$,

$\angle C = (4y)°$ and $\angle D = (7y + 5)°$

We know that,

opposite angles of a quadrilateral sum upto 180°.

$\Rightarrow \angle B + \angle D = 180°$

$\Rightarrow (3x - 5)° + (7y + 5)° = 180°$

$\Rightarrow 3x + 7y = 180 \quad ....(i)$

Similarly, $\angle A + \angle C = 180°$

$\Rightarrow (4x + 20)° + (4y)° = 180°$

$\Rightarrow 4x + 4y = 160$

$\Rightarrow x + y = 40 \quad ....(ii)$

Multiply (ii) by 7 and subtract from (i).

$\Rightarrow 3x + 7y = 180$ and $7x + 7y = 280$

$\Rightarrow -4x = -100$

$\Rightarrow x = 25$

Substituting $x = 25$ in (ii), we get $y = 15$.

Hence, the angles of ABCD are

$\angle A = 120°$, $\angle B = 70°$, $\angle C = 60°$ and $\angle D = 110°$.

30.

$p(x) = x^3 - 6x^2 + 11x - 6$

Since 3 is a zero of $p(x)$, so $(x-3)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x-3)$, we get

$$
\begin{array}{r l}
 & x^2 - 3x + 2 \\
x-3 \,\big) & x^3 - 6x^2 + 11x - 6 \\
 & (-)\; x^3 - 3x^2 \\
 & \quad - \quad + \\
\hline
 & -3x^2 + 11x - 6 \\
 & (-)\; -3x^2 + 9x \\
 & \quad + \quad - \\
\hline
 & 2x - 6 \\
 & (-)\; 2x - 6 \\
 & \quad - \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore f(x) = (x^2 - 3x + 2)(x-3)$

$= (x^2 - 2x - x + 2)(x-3)$

$= [x(x-2) - 1(x-2)](x-3)$

$= (x-1)(x-2)(x-3)$

$\therefore f(x) = 0$

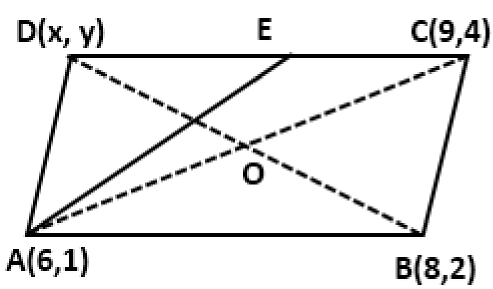
$\Rightarrow (x-1)(x-2)(x-3) = 0$

$\Rightarrow x-1 = 0$ or $x-2 = 0$ or $x-3 = 0$

$\Rightarrow x = 1$ or $x = 2$ or $x = 3$

Thus, the other two zeros are 1 and 2.

31.



Let D(x,y) be the fourth vertex of parallelogram ABCD.

$$\text{Mid-point of AC} = \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right) = \text{Coordinates of O}$$

$$\text{Mid-point of BD} = \left(\frac{8+x}{2}, \frac{2+y}{2}\right) = \text{Coordinates of O}$$

$$\Rightarrow \frac{8+x}{2} = \frac{15}{2} \text{ and } \frac{2+y}{2} = \frac{5}{2}$$

$$\Rightarrow x = 7 \text{ and } y = 3$$

$$\therefore \text{Coordinates of D} = (7,3)$$

$$\therefore \text{Coordinates of E} = \left(\frac{7+9}{2}, \frac{3+4}{2}\right) = \left(8, \frac{7}{2}\right)$$

$$\therefore \text{Area of } \Delta ADE = \frac{1}{2}\left|6\left(3-\frac{7}{2}\right)+7\left(\frac{7}{2}-1\right)+8(1-3)\right|$$

$$= \frac{1}{2}\left|6\left(-\frac{1}{2}\right)+7\left(\frac{5}{2}\right)+8(-2)\right|$$

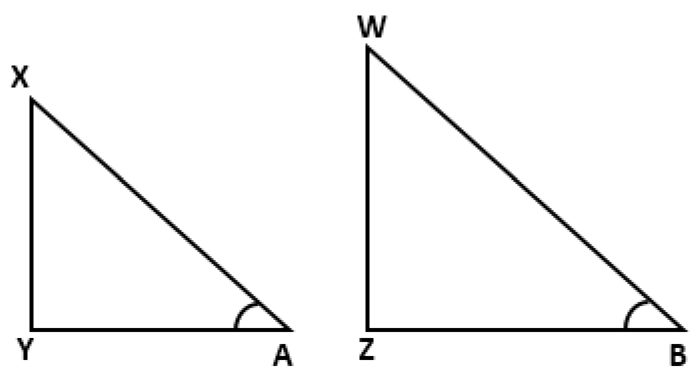
$$= \frac{1}{2}\left|-3+\frac{35}{2}-16\right|$$

$$= \frac{1}{2}\left|\frac{35}{2}-19\right|$$

$$= \frac{3}{4} \text{ sq. units}$$

32.

Consider two right triangles XAY and WBZ such that sin A = sin B



We have,

$$\sin A = \frac{XY}{XA} \text{ and } \sin B = \frac{WZ}{WB}$$

$\text{Since } \sin A = \sin B$

$$\Rightarrow \frac{XY}{XA} = \frac{WZ}{WB}$$

$$\Rightarrow \frac{XY}{WZ} = \frac{XA}{WB} = k(\text{say}) \quad \text{....(i)}$$

$$\Rightarrow XY = k \times WZ \text{ and } XA = k \times WB \quad \text{....(ii)}$$

Using Pythagoras theorem in triangles XAY and WBZ, we have

$$XA^2 = XY^2 + AY^2 \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow AY = \sqrt{XA^2 - XY^2} \quad \text{and} \quad BZ = \sqrt{WB^2 - WZ^2}$$

$$\Rightarrow \frac{AY}{BZ} = \frac{\sqrt{XA^2 - XY^2}}{\sqrt{WB^2 - WZ^2}} = \frac{\sqrt{k^2 WB^2 - k^2 WZ^2}}{\sqrt{WB^2 - WZ^2}} = \frac{k\sqrt{WB^2 - WZ^2}}{\sqrt{WB^2 - WZ^2}}$$

$$\Rightarrow \frac{AY}{BZ} = k \quad \text{....(iii)}$$

From (i), (ii) and (iii), we get

$$\frac{XY}{WZ} = \frac{XA}{WB} = \frac{AY}{BZ}$$

$$\Rightarrow \Delta XYA \sim \Delta WZB$$

$$\Rightarrow \angle A = \angle B$$

OR

$$\text{R.H.S.} = \frac{p^2 - 1}{p^2 + 1}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2 - 1}{(\operatorname{cosec}\theta + \cot\theta)^2 + 1}$$

$$= \frac{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta + 1}$$

$$= \frac{(\operatorname{cosec}^2\theta - 1) + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta + (\cot^2\theta + 1)}$$

$$= \frac{\cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta + \operatorname{cosec}^2\theta}$$

$$= \frac{2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta}{2\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta}$$

$$= \frac{2\cot\theta(\cot\theta + \operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{\cot\theta}{\operatorname{cosec}\theta}$$

$$= \frac{\cos\theta / \sin\theta}{1 / \sin\theta}$$

$$= \cos\theta$$

$$= \text{L.H.S.}$$

33.

Inner circumference of a racetrack $= 352$ m

$\Rightarrow 2\pi r = 352$

$\Rightarrow 2 \times \frac{22}{7} \times r = 352$

$\Rightarrow r = \frac{352 \times 7}{2 \times 22} = 56$ m

Outer circumference of a racetrack $= 396$ m

$\Rightarrow 2\pi R = 396$

$\Rightarrow 2 \times \frac{22}{7} \times R = 396$

$\Rightarrow R = \frac{396 \times 7}{2 \times 22} = 63$ m

$\therefore$ Width of the track $= R - r = 63 - 56 = 7$ m

Area of the track $= \pi(R^2 - r^2)$

$$= \frac{22}{7}(63^2 - 56^2)$$

$$= \frac{22}{7}(63+56)(63-56)$$

$$= \frac{22}{7} \times 119 \times 7$$

$$= 2618 \text{ m}^2$$

34.

Class interval	Frequency	Cumulative frequency
$85-100$	10	10
$100-115$	4	14
$115-130$	7	21
$130-145$	9	30

Here, $N = 30 \Rightarrow \frac{N}{2} = 15$

The cumulative frequency just greater than 15 is 21.

Hence, median class is $115-130$.

$\therefore l = 115,\ h = 15,\ f = 7,\ cf = cf$ of preceding class $= 14$

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$$

$$= 115 + \left\{ 15 \times \frac{(15-14)}{7} \right\}$$

$$= 115 + \left\{ 15 \times \frac{1}{7} \right\}$$

$$= 115 + 2.1$$

$$= 117.1$$

Thus, the median bowling speed is 117.1 km/hr.

Section D

35.

Steps of construction:

1) Draw a line segment $PQ = 6$ cm
2) With P as centre and radius 8 cm, draw an arc.
3) With Q as centre and radius 7 cm, draw another arc intersecting previous arc at R.

4) Join PR and QR to obtain ΔPQR

5) Below PQ, make an acute $\angle QPX$.

6) Along PX, mark off 5 points $\left(\text{greater of 4 and 5 in } \frac{4}{5}\right)$

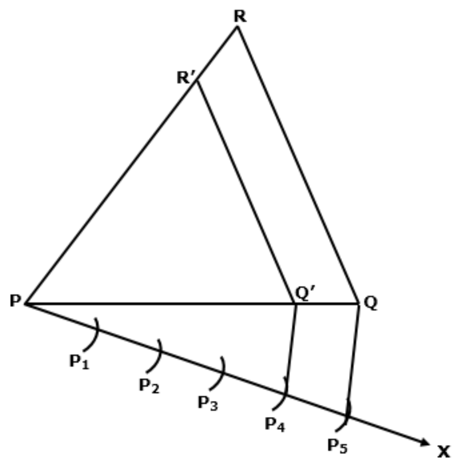
P_1, P_2, P_3, P_4, P_5 such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5$

7) Join P_5Q

8) From point P_4, draw a line parallel to P_5Q intersecting PQ at Q'

9) From point Q', draw a line parallel to QR intersecting PR at R'

Thus, $\Delta PQ'R'$ is the required triangle.



OR

Steps of construction:

1) Draw a line segment $BC = 4$ cm

2) AT B, construct $\angle MBC = 90°$

3) Cut-off $BA = 3$ cm from BM.

4) Join AC.

Thus, right-angled ΔABC is obtained.

5) Below BC, make an acute $\angle CBX$.

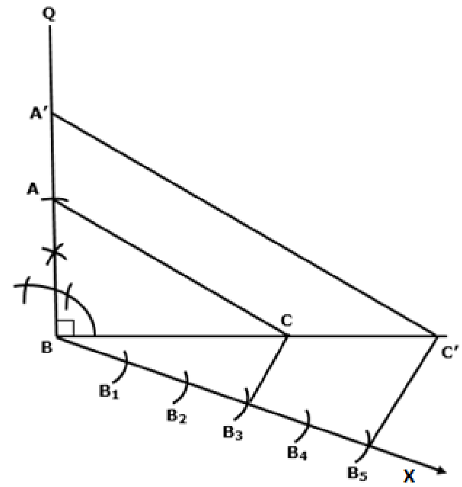
6) Along BX, mark off 5 points B_1, B_2, B_3, B_4, B_5 such that $BB_1 = B_1B_2 = ... = B_4B_5$

7) Join B_3C

8) From B_5, draw $B_5C' \| B_3C$, meeting BC produced at C'.

9) From C', draw $C'A' \| CA$, meeting BA produced at A'.

Then, $\Delta A'BC'$ is the required triangle.



36.

In ΔABC and ΔXBY,

$\angle ABC = \angle XBY$ .....(common angle)

$\angle BXY = \angle BAC$ .....(corresponding angles)

$\Delta ABC \sim \Delta XBY$ ....(AA criterion for similarity)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \frac{AB^2}{BX^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \frac{AB^2}{(AB - AX)^2} \quad \text{....(i)}$$

Given that XY divides ΔABC into two regions.

$$\text{So,} \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \frac{2}{1}$$

$$\Rightarrow \frac{AB^2}{(AB - AX)^2} = \frac{2}{1}$$

$$\Rightarrow \frac{AB}{AB - AX} = \frac{\sqrt{2}}{1}$$

$$\Rightarrow \frac{AB - AX}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1 - \frac{AX}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

On rationalising the denominator, we get

$$\Rightarrow \frac{AX}{AB} = \frac{2 - \sqrt{2}}{2}$$

Hence proved.

37.

Let the time taken by the smaller pipe to fill the tank be x hours.

Then, the time taken by the larger pipe $= (x - 9)$ hours.

Part of tank filled by smaller pipe in 1 hour $= \frac{1}{x}$

Part of tank filled by larger pipe in 1 hour $= \frac{1}{x - 9}$

It is given that the tank can be filled in 6 hours by both the pipes together.

$$\Rightarrow \frac{1}{x} + \frac{1}{x - 9} = \frac{1}{6}$$

$$\Rightarrow \frac{x - 9 + x}{x^2 - 9x} = \frac{1}{6}$$

$$\Rightarrow 6(2x - 9) = x^2 - 9x$$

$$\Rightarrow 12x - 54 = x^2 - 9x$$

$$\Rightarrow x^2 - 21x + 54 = 0$$

$$\Rightarrow x^2 - 18x - 3x + 54 = 0$$

$$\Rightarrow x(x - 18) - 3(x - 18) = 0$$

$$\Rightarrow (x - 18)(x - 3) = 0$$

$$\Rightarrow x - 18 = 0 \text{ or } x - 3 = 0$$

$$\Rightarrow x = 18 \text{ or } x = 3$$

Since time cannot be less than 9, $x \neq 3$.

$$\Rightarrow x = 18$$

$$\Rightarrow x - 9 = 18 - 9 = 9$$

Hence, the smaller pipe takes 18 hours to fill the tank

and the larger pipe takes 9 hours to fill the tank.

OR

Let the usual speed of the passenger train be x km/hr.

Time taken to cover 300 km $= \frac{300}{x}$ hours

Time taken to cover 300 km when the speed

is increased by 5 km/hr $= \frac{300}{x+5}$ hours

It is given that the time taken to cover 300 km is decreased by 2 hours.

$$\therefore \frac{300}{x} - \frac{300}{x+5} = 2$$

$$\Rightarrow \frac{300x + 1500 - 300x}{x^2 + 5x} = 2$$

$$\Rightarrow 1500 = 2x^2 + 10x$$

$$\Rightarrow 2x^2 + 10x - 1500 = 0$$

$$\Rightarrow x^2 + 5x - 750 = 0$$

$$\Rightarrow x^2 + 30x - 25x - 750 = 0$$

$$\Rightarrow x(x+30) - 25(x+30) = 0$$

$$\Rightarrow (x+30)(x-25) = 0$$

$$\Rightarrow x + 30 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = -30 \text{ or } x = 25$$

Since the speed cannot be negative, $x \neq -30$.

$$\Rightarrow x = 25$$

Thus, the usual speed of the train is 25 km/hr.

38.

Given Internal radius of the hemispherical bowl $(R) = 9$ cm

Amount of the liquid in the bowl = Capacity of the bowl

$$= \frac{2}{3}\pi R^3$$

$$= \frac{2}{3}\pi (9)^3$$

$$= 486\pi \text{ cm}^3$$

Now, liquid from the bowl is to be emptied into cylindrical bottles.

Diameter of each cylindrical bottle $(d) = 3$ cm

$\Rightarrow$ Radius of each cylindrical bottle $(r) = \frac{3}{2}$ cm

Height of each cylindrical bottle $(h) = 4$ cm

$\therefore$ Capacity of each cylindrical bottle $= \pi r^2 h$

$$= \pi\left(\frac{3}{2}\right)^2 \times 4$$

$$= 9\pi \text{ cm}^3$$

Number of cylindrical bottles filled $= \dfrac{\text{Capacity of the bowl}}{\text{Capacity of each cylindrical bottle}}$

$$= \frac{486\pi}{9\pi}$$

$$= 54$$

Thus, 54 cylindrical bottles can be filled with the liquid available in the bowl.

OR

We know that,

Surface area of a sphere $= 4\pi r^2$ and

Surface area of a cube $= 6a^2$

Given Surface area of sphere $=$ Surface area of cube

$$\Rightarrow 4\pi r^2 = 6a^2$$

$$\Rightarrow \frac{r^2}{a^2} = \frac{6}{4\pi}$$

$$\Rightarrow \left(\frac{r}{a}\right)^2 = \frac{6}{4\pi}$$

$$\Rightarrow \frac{r}{a} = \sqrt{\frac{6}{4\pi}} \qquad \text{...(i)}$$

Now,

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3}\pi\left(\frac{r}{a}\right)^3$$

$$= \frac{4}{3}\pi\left(\sqrt{\frac{6}{4\pi}}\right)^3 \qquad \text{....(From (i))}$$

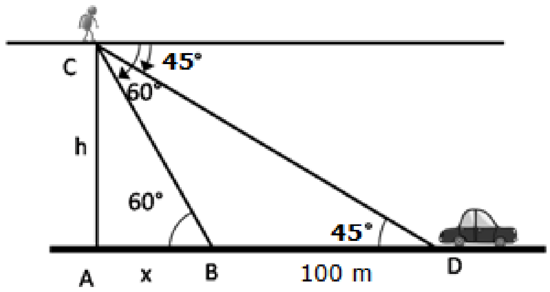
$$= \frac{4}{3} \times \pi \times \sqrt{\frac{6}{4\pi}} \times \frac{6}{4\pi}$$

$$= 2 \times \sqrt{\frac{6}{4\pi}} = \sqrt{\frac{6}{\frac{22}{7}}}$$

$$= \sqrt{\frac{6 \times 7}{22}}$$

$$= \sqrt{\frac{21}{11}}$$

39.



Let AC be the tower of height, h metres.

In right ΔDAC,

$$\cot 45° = \frac{AD}{AC}$$

$$\Rightarrow 1 = \frac{x+100}{h}$$

$$\Rightarrow x + 100 = h$$

$$\Rightarrow x = h - 100 \quad .....(i)$$

In right ΔBAC,

$$\cot 60° = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{h}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \quad .....(ii)$$

From (i) and (ii),

$$h - 100 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow h - \frac{h}{\sqrt{3}} = 100$$

$$\Rightarrow h\left(1 - \frac{1}{\sqrt{3}}\right) = 100$$

$$\Rightarrow h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 100$$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}-1}$$

On rationalising we get,

$$h = \frac{100\sqrt{3}}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$\Rightarrow h = \frac{300 + 100\sqrt{3}}{2}$$

$$= 100\left(\frac{3 + \sqrt{3}}{2}\right)$$

$$= 50\left(3 + \sqrt{3}\right)$$

$$= 50(3 + 1.73)$$

$$= 236.5 \text{ m}$$

Hence, the height of the tower is 236.5 m.

40.

We prepare the following table:

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$	Cumulative frequency
$0-10$	6	5	30	6
$10-20$	8	15	120	14
$20-30$	10	25	250	24
$30-40$	15	35	525	39
$40-50$	5	45	225	44
$50-60$	4	55	220	48
$60-70$	2	65	130	50
	$\sum f_i = 50$		$\sum f_i x_i = 1500$	

<u>Mean</u>:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1500}{50} = 30$$

<u>Median</u>:

$$N = 50 \Rightarrow \frac{N}{2} = 25$$

The cumulative frequency just greater than 25 is 39.

Hence, median class is $30-40$.

$\therefore l = 30,\ h = 10,\ f = 15,\ cf = cf$ of preceding class $= 24$

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\} = 30 + \left\{ 10 \times \frac{25 - 24}{15} \right\} = 30 + 0.67 = 30.67$$

<u>Mode</u>:

Maximum frequency $= 15$

Hence, modal class is $30-40$

$$\text{Now, Mode} = x_k + h\left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\} = 30 + 10\left\{\frac{15-10}{2(15)-10-5}\right\} = 30 + 3.33 = 33.33$$