CBSE Board
Class X Mathematics
Sample Paper 4 (Basic) – Solution

Section A

1. **Correct Option:** C
Explanation:
Prime factorisation of 306 and 657 is given by,
$306 = 2 \times \underline{\mathbf{3 \times 3}} \times 17$
$657 = \underline{\mathbf{3 \times 3}} \times 73$
HCF (306, 657) = $3 \times 3 = 9$

2. **Correct Option:** B
Explanation:
Empirical relationship between the three measures of central tendency is
Mode = 3 Median – 2 Mean
$\Rightarrow$ Mode = 3 (20) – 2 (22. 5)
= 60 – 45
= 15

3. **Correct Option:** B
Explanation:
The denominator is of the form $2^n 5^m$.
Here, n = 2 > m = 1
$\therefore$ The decimal expansion of the rational number $\frac{23}{2^2 \times 5}$ will terminate after 2 decimal places.

4. **Correct Option:** A
Explanation:
$425 = 5 \times 5 \times 17 = 5^2 \times 17$

5. **Correct Option:** A
Explanation:
Total number of cards = 52
Total number of king cards = 4
P(getting a king card) = $\frac{\text{Total number of king cards}}{\text{Total number of cards}} = \frac{4}{52} = \frac{1}{13}$

6. **Correct Option:** A

Explanation:

$P(x) = 2x^2 - 8x + 6$

Here a = 2, b = –8 and c = 6

Product of the zeroes i.e. $\alpha\beta = \frac{c}{a} = \frac{6}{2} = 3$

7. **Correct Option:** B

Explanation:

The decimal expansion of a rational number is 0.15 which is terminating.

⇒ 0.15 is a rational number

8. **Correct Option:** A

Explanation:

Here, the graph y = p(x) cuts the x – axis at exactly one point.

⇒ The number of zeroes of p(x) is 1.

9. **Correct Option:** D

Explanation:

The distance of a point A(x, y) from the origin = $\sqrt{x^2 + y^2}$

Here the point is P(6, –8).

⇒ The distance of the point P(6, –8) form the origin

$= \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ units}$

10. **Correct Option:** B

Explanation:

The mid-point of the line segment joining the points (–5, 7) and (–1, 3)

$= \left(\frac{-5-1}{2}, \frac{7+3}{2}\right)$

$= (-3, 5)$

11. The point which divides the line segment joining the point X(–1, 7) and Y(4, –3) internally in the ratio 2 : 3 is <u>(1, 3).</u>

Explanation:

We know that,

The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2) internally in the ratio m_1: m_2 are

$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

By using the above formula, we get

$$\left(\frac{2\times 4+3\times -1}{2+3},\frac{2\times -3+3\times 7}{2+3}\right)=\left(\frac{8-3}{5},\frac{21-6}{5}\right)=(1,\ 3)$$

12. The pair of lines represented by the equation 2x + y + 3 = 0 and 4x + ky + 6 = 0 will be parallel if value of k is <u>2</u>.

Explanation:

We know that,

The given two lines are parallel only when $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

Here $b_1 = 1$ then $b_2 = k$ has to be 2.

OR

The quadratic equation $2x^2 - 6x + 3 = 0$ has <u>real and distinct</u> roots.

Explanation:

Here a = 2, b = –6 and c = 3

$b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

⇒ the quadratic equation $2x^2 - 6x + 3 = 0$ has real and distinct roots.

13. The value of $2\tan^2 45° + \cos^2 30° - \sin^2 60$ is <u>2</u>.

Explanation:

$$2\tan^2 45° + \cos^2 30° - \sin^2 60$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2$$

14. Value of $\frac{\tan 26°}{\cot 64°} = \underline{1}$

Explanation:

$$\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$$

15. The areas of two similar triangles are 9 cm^2 and 16 cm^2. The ratio of their corresponding sides is <u>3: 4</u>.

Explanation:

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{9}{16} = \frac{(\text{side of 1st triangle})^2}{(\text{side of 2nd triangle})^2}$$

Taking square root on both the sides, we get

$$\Rightarrow \frac{3}{4} = \frac{\text{side of 1st triangle}}{\text{side of 2nd triangle}}$$

$$\Rightarrow (\text{side of 1st triangle}) : (\text{side of 2nd triangle}) = 3 : 4$$

16. $\sin(2A - 40^\circ) = \cos 90^\circ$

$\Rightarrow \sin(2A - 40^\circ) = 0 = \sin 0^\circ$

$\Rightarrow 2A - 40^\circ = 0$

$\Rightarrow 2A = 40^\circ$

$\Rightarrow A = 20^\circ$

OR

$$\begin{aligned}\cos^2 54^\circ - \sin^2 36^\circ &= [\cos(90^\circ - 36^\circ)]^2 - \sin^2 36^\circ \\ &= \sin^2 36^\circ - \sin^2 36^\circ \\ &= 0\end{aligned}$$

17. Area of the sector of a circle

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7}\ \text{cm}^2$$

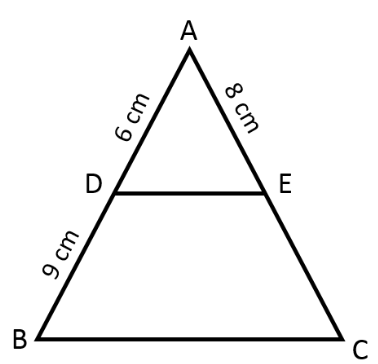
18. Let A be the event that the selected digit is an odd number.

$A = \{1, 3, 5, 7, 9\}$

Here, $n(S) = 9$ and $n(A) = 5$

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

19.



In the given figure, DE || BC and DE is intersecting AB and AC at D and E respectively.

By Basic proportionality theorem, we get

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{6}{9} = \frac{8}{EC}$$

$$\Rightarrow EC = \frac{8 \times 9}{6} = 12 \text{ cm}$$

20. Since the given three numbers are in A.P.

$\Rightarrow 2(x + 10) = (2x) + (3x + 2)$

$\Rightarrow 2x + 20 = 5x + 2$

$\Rightarrow 3x = 18$

$\Rightarrow x = 6$

Section B

21. When two coins are tossed simultaneously then the sample space = {HH, HT, TH, TT}

$n(S) = 4$

Let A be the event of getting exactly one tail.

A= {HT, TH) $\Rightarrow n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

22. Here, $n(S) = 6 + 8 + 4 = 18$

Let A be the event that the ball drawn is black.

$n(A) = 8$

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{8}{18} = \frac{4}{9}$$

OR

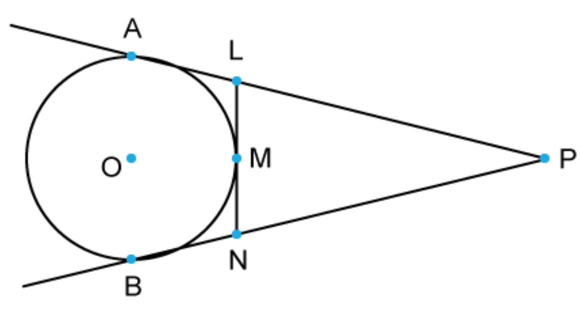
Here $n(S) = 6 \times 6 = 36$

Let A be the event of getting a doublet.

$n(A) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

23.



Since tangents drawn from an external point to a circle are equal.

$\therefore$ AL = LM .... (i)

BN = MN .... (ii)

PA = PB .... (iii)

$\Rightarrow$ PL + LA = PN + NB

$\Rightarrow$ PL + LM = PN + MN ... from (i) and (ii)

Hence proved.

24. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{3+1}{4}=1$

OR

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$=\frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$=\frac{\cos^2\theta}{\sin^2\theta}=\left(\frac{\cos\theta}{\sin\theta}\right)^2=(\cot\theta)^2=\left(\frac{7}{8}\right)^2=\frac{49}{64}$$

25. Circumference of a circle = $2\pi r$

$$\Rightarrow 22 = 2\times\frac{22}{7}\times r$$

$$\Rightarrow r = \frac{22\times 7}{2\times 22}=\frac{7}{2}=3.5\,\text{cm}$$

Area of a circle = $\pi r^2 = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}=\frac{77}{2}=38.5\,\text{cm}^2$

26. (i) 1^{st} student has drawn the correct graph.

Reason: If the graph of the polynomial p(x) doesn't cuts the x – axis then the polynomial has no zero.

(ii) 5 students gets incorrect graphs.

Section C

27. $x^2 - 17x + 66$

$= x^2 - 11x - 6x + 66$

$= x(x - 11) - 6(x - 11)$

$= (x - 11)(x - 6)$

$\Rightarrow$ Zeroes of the polynomial are 11 and 6.

So, $\alpha = 11$ and $\beta = 6$

Here, a = 1 , b = −17 and c = 66

$$\alpha + \beta = 11 + 6 = 17 = \frac{-(-17)}{1} = \frac{-b}{a}$$

$$\alpha\beta = 11 \times 6 = 66 = \frac{66}{1} = \frac{c}{a}$$

28. The steps of construction are as follows:

i. Draw a line segment AB of 7.6 cm and draw a ray AX making an acute angle with side AB.

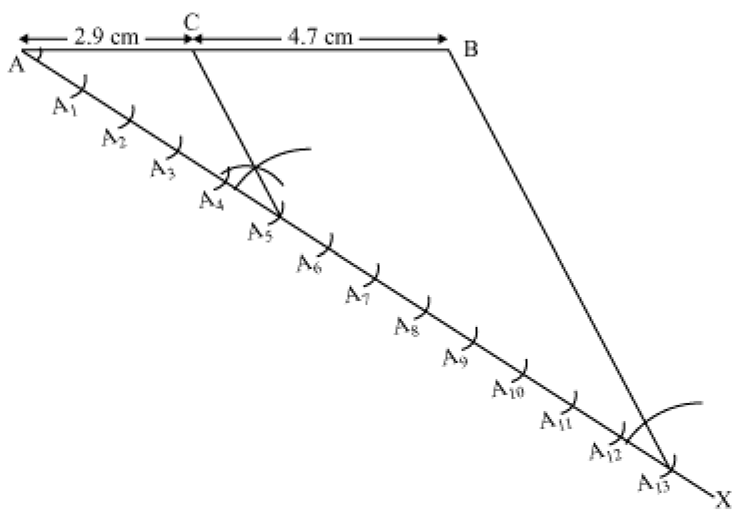
ii. Locate 13 (= 5 + 8) points A_1, A_2, A_3, A_4……..A_{13} on AX such that $AA_1 = A_1A_2 = A_2A_3$ … $= A_{12}A_{13}$

iii. Join BA_{13}.

iv. Through the point A_5 draw a line parallel to BA_{13} (by making an angle equal to $\angle AA_{13}B$ at A_5) intersecting AB at point C.

Now C is the point dividing line segment AB of 7.6 cm in the required ratio of 5: 8.

We can measure the lengths of AC and CB. The lengths of AC and CB comes to 2.9 cm and 4.7 cm respectively.

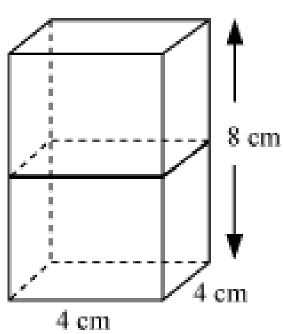


29. Given that:

Volume of each cube = 64 cm^3

$\Rightarrow (\text{Edge})^3 = 64$

$\Rightarrow$ Edge = 4 cm



If cubes are joined end to end, dimensions of the resulting cuboid will be 4 cm, 4 cm and 8 cm.

$$
\begin{aligned}
\therefore \text{surface area of the cuboid } &= 2(lb+bh+lh)\\
&= 2(4\times4+4\times8+4\times8)\\
&= 2(16+32+32)\\
&= 2(16+64)\\
&= 2\times80 = 160\text{ cm}^2
\end{aligned}
$$

30.

$$
\begin{aligned}
&\sqrt{\left(\frac{1}{1-\sin^2\theta}+\frac{1}{1+\tan^2\theta}+2\sec\theta\cos\theta\right)}\\
&=\sqrt{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sec^2\theta}+2\sec\theta\cos\theta\right)}\\
&=\sqrt{\sec^2\theta+\cos^2\theta+2\sec\theta\cos\theta}\\
&=\sqrt{\sec\theta+\cos\theta^{\;2}}\\
&=\sec\theta+\cos\theta
\end{aligned}
$$

OR

$$
\begin{aligned}
&\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\\
&=\sqrt{\frac{1-\sin\theta\ \ 1+\sin\theta}{1+\sin\theta\ \ 1+\sin\theta}}\\
&=\sqrt{\frac{1-\sin^2\theta}{1+\sin\theta^{\;2}}}\\
&=\sqrt{\frac{\cos^2\theta}{1+\sin\theta^{\;2}}}\\
&=\frac{\cos\theta}{1+\sin\theta}
\end{aligned}
$$

31.

2	336
2	168
2	84
2	42
3	21
	7

2	240
2	120
2	60
2	30
3	15
	5

2	96
2	48
2	24
2	12
2	6
	3

$336 = 2\times2\times2\times2\times3\times7 = 2^4\times3\times7$

$240 = 2\times2\times2\times2\times3\times5 = 2^4\times3\times5$

$96 = 2\times2\times2\times2\times2\times3 = 2^5\times3$

$\text{H.C.F} = 2^4\times3 = 16\times3 = 48$

Each stack will contain 48 books.

Number of stacks of the same height

$= \frac{240}{48} + \frac{336}{48} + \frac{96}{48} = 5+7+2 = 14$

OR

Let us assume on the contrary that $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b (b ≠ 0) such that

$3+2\sqrt{5} = \frac{a}{b}$

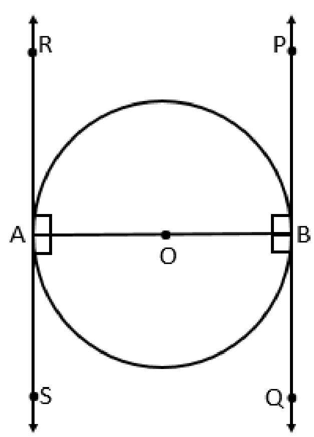
$2\sqrt{5} = \frac{a}{b} - 3$

$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational, and therefore $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

32.



Let AB be the diameter of a circle. Two tangents PQ and RS are drawn at the end points of the diameter AB.

It is known that the radius is perpendicular to tangent at the point of contact.

$\therefore$ $\angle OAR = 90°$, $\angle OAS = 90°$, $\angle OBP = 90°$ and $\angle OBQ = 90°$

$\angle OAR = \angle OBQ$ .... (alternate interior angles)

$\angle OAS = \angle OBP$ .... (alternate interior angles)

Since, alternate interior angles are equal, lines PQ and RS will be parallel.

33. From the figure, coordinates of points A, B, C and D are

A = (3, 4), B = (6, 7), C(9, 4), D = (6, 1)

$$AB = \sqrt{(3-6)^2+(4-7)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(6-9)^2+(7-4)^2} = \sqrt{(-3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(9-6)^2+(4-1)^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2+(4-1)^2} = \sqrt{(-3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(3-9)^2+(4-4)^2} = \sqrt{(-6)^2+(0)^2} = 6$$

$$BD = \sqrt{(6-6)^2+(7-1)^2} = \sqrt{(0)^2+(6)^2} = 6$$

Here, AB = BC = CD = AD = $3\sqrt{2}$ and AC = BD = 6

Means distance between A, B, C and D are not the same.

⇒ Seema is correct.

34. Let the income of the first person be 9x and that of the second person be 7x.

So, the expense is 4y and 3y, respectively.

$9x - 4y = 200$

$7x - 3y = 200$

$$\frac{x}{800-600} = \frac{-y}{-1800+1400} = \frac{1}{-27+28}$$

$$\frac{x}{200} = \frac{y}{400} = \frac{1}{1}$$

$x = 200$ and $y = 400$

Hence, their monthly incomes are 9x = 1800 and 7x = 2800, respectively.

SECTION D

35. Let x and x + 4 be the lengths of the other two sides.

By the Pythagoras theorem, we get

$(x + 4)^2 + x^2 = 20^2$

$\therefore x^2 + 8x + 16 + x^2 = 400$

$\therefore 2x^2 + 8x + 16 = 400$

$\therefore 2x^2 + 8x - 384 = 0$

$\therefore x^2 + 4x - 192 = 0$

$\therefore (x - 12)(x + 16) = 0$

$\therefore x = 12$ or $x = -16$

$x = -16$ is not acceptable, because length cannot be negative.

If x = 12, then x + 4 = 12 + 4 = 16.
Hence, the other two sides are 12 cm and 16 cm.

36. Here, $a_5 = a + 4d = 30$ and $a_{12} = a + 11d = 65$
a + 4d = 30 … (i)
a + 11d = 65 … (ii)
Solving both equations, we get
a = 10 and d = 5

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

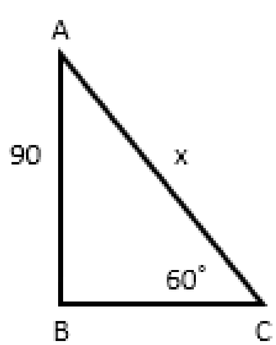
$$\therefore S_{20} = \frac{20}{2}\left[2\times10 + (20-1)5\right]$$

$$\therefore S_{20} = 10\left[20 + (19\times5)\right] = 10 \times 115 = 1150$$

OR

According to the question,
$T_9 = 0$
a + 8d = 0 … (i)
$2T_{19} = 2(a + 18d)$
= 2a + 36d … (ii)
$T_{29} = a + 28d$ … (iii)
$2T_{19} - T_{29} = 2a + 36d - (a + 28d)$… from(ii) and (iii)
= 2a + 36d – a – 28d
= a + 8d
= 0 … from(i)

37.



Let A be the position of the kite and C be the point on the ground.
In ΔABC,

$$\frac{AB}{AC} = \sin 60^\circ$$

$$\therefore \frac{90}{x} = \frac{\sqrt{3}}{2}$$

$$\therefore x = \frac{180}{\sqrt{3}}$$

$$\therefore x = \frac{180}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore x = 60\sqrt{3} = 103.92$$

The length of the string is 103.92 m.

38. From the figure, DE || BC.

$$\frac{AD}{DB} = \frac{AE}{EC} \because BPT$$

$$\therefore \frac{x+10}{4x+10} = \frac{x+6}{x+10}$$

$$\therefore (x+10)^2 = (x+6)(4x+10)$$

$$\therefore x^2 + 20x + 100 = 4x^2 + 34x + 60$$

$$\therefore 3x^2 + 14x - 40 = 0$$

$$\therefore 3x^2 + 20x - 6x - 40 = 0$$

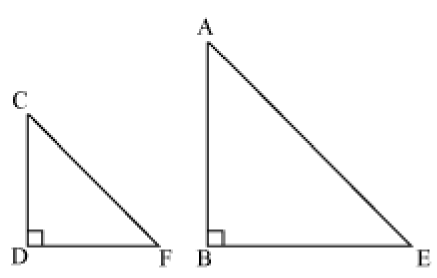
$$\therefore x(3x+20) - 2(3x+20) = 0$$

$$\therefore (3x+20)(x-2) = 0$$

$\therefore$ x cannot be negative; hence, x = 2.

For x = 2, DE || BC using the converse of the basic proportionality theorem.

OR



Let AB be a tower and CD be a pole

Shadow of AB is BE

Shadow of CD is DF

The light rays from sun will fall on tower and pole at same angle and at the same time.

So, $\angle$DCF = $\angle$BAE

And $\angle$DFC = $\angle$BEA

$\angle$CDF = $\angle$ABE (tower and pole are vertical to ground)

Therefore $\Delta ABE \sim \Delta CDF$

Therefore $\dfrac{AB}{CD} = \dfrac{BE}{DF}$

$$\frac{AB}{6} = \frac{28}{4}$$

$$AB = 42$$

So, height of tower will be 42 meters.

39.

Coins are cylindrical in shape

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\dfrac{1.75}{2}$ = 0.875 cm

Let n coins were melted to form the required cuboid.

Volume of n coins = Volume of cuboid

$$n \times \pi \times r^2 \times h_1 = l \times b \times h$$

$$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$$

$$n = \frac{5.5 \times 10 \times 3.5 \times 7}{(0.875)^2 \times 0.2 \times 22} = 400$$

So, the number of coins melted to form such a cuboid is 400.

OR

Radius (r_1) of 1st sphere = 6

Radius (r_2) of 2nd sphere = 8

Radius (r_3) of 3rd sphere = 10

Let the radius of resulting sphere be r

The object formed by recasting these spheres will be equal (in volume) to the sum of volumes of these spheres.

Volume of 3 spheres = volume of resulting sphere

$$\frac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\left[6^3 + 8^3 + 10^3\right] = \frac{4}{3}\pi r^3$$

$$r^3 = 216 + 512 + 1,000 = 1,728$$

$$r = 12 \text{ cm.}$$

So, radius of the sphere so formed will be 12cm.

40. We can obtain cumulative frequency distribution of more than type as follows:

Production yield (lower class limits)	**Cumulative frequency**
more than or equal to 50	100
more than or equal to 55	100 – 2 = 98
more than or equal to 60	98 – 8 = 90
more than or equal to 65	90 – 12 = 78
more than or equal to 70	78 – 24 = 54
more than or equal to 75	54 – 38 = 16

Now taking lower class limits on *x*-axis and their respective cumulative frequencies on y-axis we can obtain its ogive as follows: