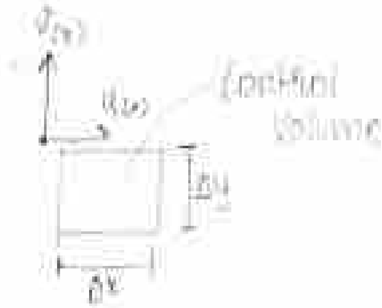
Computational Engineering

Case Study - 02

[Kush Patel
20BME081]

· Derivation of Equation →



* 2D convection diffusion equation:

$$\rho \frac{\partial}{\partial x}(uT) + \rho \frac{\partial}{\partial y}(vT) = \frac{\partial}{\partial x}\left(\Gamma_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma_y \frac{\partial T}{\partial y}\right)$$

$$\underbrace{\rho \int_s^n\!\!\int_w^e \frac{\partial}{\partial x}(uT)\,dx\,dy + \rho \int_s^n\!\!\int_w^e \frac{\partial}{\partial y}(vT)\,dx\,dy}_{\text{convection}} = \underbrace{\int_s^n\!\!\int_w^e \frac{\partial}{\partial x}\left(\Gamma_x \frac{\partial T}{\partial x}\right)dx\,dy + \int_s^n\!\!\int_w^e \frac{\partial}{\partial y}\left(\Gamma_y \frac{\partial T}{\partial y}\right)dx\,dy}_{\text{diffusion}}$$

· Convection:

$$= \int_s^n \rho(uT)_w^e\,dy + \int_w^e \rho(vT)_s^n\,dx$$

$$= \int_s^n \rho(u_eT_e - u_wT_w)\,dy + \int_s^n \rho(v_nT_n - v_sT_s)\,dx$$

$$= \underbrace{\rho[u_eT_e\Delta y - u_wT_w\Delta y]}_{(i)} + \underbrace{\rho[v_nT_n\Delta x - v_sT_s\Delta x]}_{(ii)}$$

using central difference scheme,

$$① = \left[\Delta y\, u_e\left(\frac{T_E+T_P}{2}\right) - \Delta y\, u_w\left(\frac{T_P+T_W}{2}\right)\right]\rho$$

$$② = \left[\Delta x\, v_n\left(\frac{T_N+T_P}{2}\right) - \Delta x\, v_s\left(\frac{T_P+T_S}{2}\right)\right]\rho$$

• Diffusion:

$$= \Gamma_x \int_s^n \left(\frac{\partial T}{\partial y}\right)_w^e dy + \Gamma_y \int_w^e \left(\frac{\partial T}{\partial x}\right)_s^n dx$$

$$= \Gamma_x \int_s^n \left[\left(\frac{\partial T}{\partial y}\right)_e - \left(\frac{\partial T}{\partial y}\right)_w\right] dy + \Gamma_y \int_w^e \left[\left(\frac{\partial T}{\partial x}\right)_n - \left(\frac{\partial T}{\partial x}\right)_s\right] dx$$

$$= \frac{\Gamma_e (T_E - T_P)}{2\Delta x_e} - \frac{\Gamma_w (T_P - T_W)}{2\Delta x_w} \longrightarrow ③$$

Equating ①, ②, ③

$$\left[\Delta y\, V_e(T_E+T_P) - \Delta y\, U_w(T_P+T_W)\right]\rho + \left[\Delta x\, V_n(T_N+T_P) - \Delta x\, V_S(T_P+T_S)\right]\rho =$$

$$\frac{\Gamma_e (T_E - T_P)}{\Delta x} - \frac{\Gamma_w (T_P - T_W)}{\Delta x}$$

Let us assume - $\underbrace{F = \rho u}_{\text{Strength of convection}}$ and $\underbrace{D = \frac{\Gamma}{\Delta x}}_{\text{Diffusion conductance}}$

So, we have:

$F_e = \rho u_e \Delta y$ $\quad F_w = \rho u_w \Delta y$ $\quad F_n = \rho v_n \Delta x$ $\quad F_s = \rho v_s \Delta x$

$D_e = \frac{\Gamma_e}{\Delta x}$ $D_w = \frac{\Gamma_w}{\Delta x}$ $D_n = \frac{\Gamma_n}{\Delta y}$ $D_s = \frac{\Gamma_s}{\Delta y}$

$$F_e(T_E+T_P) - F_w(T_P+T_W) + F_n(T_N+T_P) - F_S(T_P+T_S) =$$
$$D_e(T_E-T_P) - D_w(T_P-T_W) + D_n(T_N-T_P) - D_s(T_P-T_S)$$

Now,
defining:

$a_E = D_E - \frac{F_E}{2}$, $a_W = D_W + \frac{F_W}{2}$, $a_n = D_N - \frac{F_N}{2}$, $a_S = D_S + \frac{F_S}{2}$

$$a_P = a_e + a_w + a_s + a_n - S_P \Delta x \Delta y$$

$\rightarrow (S_P \Delta x \Delta y) = 0$ for this case

$$\therefore \boxed{a_P T_P = a_E T_E - a_W T_W + a_N T_N + a_S T_S}$$

*2D Convection Diffusion Equation for porous media-
We have:

$$\rho \frac{\partial}{\partial x}(uT) + \rho \frac{\partial}{\partial y}(vT) = \frac{\partial}{\partial x}\left(\Gamma_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma_y \frac{\partial T}{\partial y}\right) + \frac{\mu}{K}u + \frac{\mu}{K}v$$

On solving like above

$$\boxed{a_P T_P = a_E T_E - a_W T_W + a_N T_N + a_S T_S + \frac{\mu}{K}u + \frac{\mu}{K}v}$$

• The Matlab Codes and their plots are attached in the following pages:

Result -

We conclude that on adding porous media in the whole chanel, there is a change in thermal field and more heat is transferred by convection-diffusion. The heat transfer is comparably less through the channel when porous media is absent. The porous media changes the flow field conditions and causes the frontal layer to thicken.