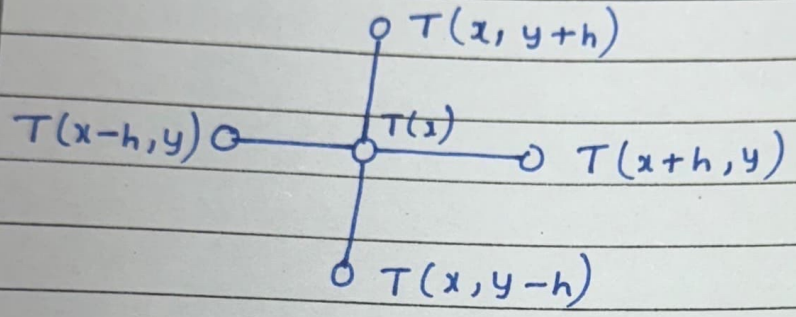
25-1-24

Assignment -3

* 2-D conduction Heat Transfer

$$\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\alpha \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha \frac{\partial T}{\partial y}\right)$$

$$\frac{\partial T}{\partial t} \approx \frac{\Delta T}{\Delta t} \Rightarrow \boxed{\frac{T'(x,y) - T(x,y)}{\Delta t} = \frac{\partial T}{\partial t}}$$



$$T(x+h) = T(x) + h\frac{\partial T}{\partial x} + \frac{h^2}{2}\frac{\partial^2 T}{\partial x^2}$$

$$T(x-h) = T(x) - h\frac{\partial T}{\partial x} + \frac{h^2}{2}\frac{\partial^2 T}{\partial x^2}$$

$$T(y+h) = T(y) + h\frac{\partial T}{\partial y} + \frac{h^2}{2}\frac{\partial^2 T}{\partial y^2}$$

$$T(y-h) = T(y) - h\frac{\partial T}{\partial y} + \frac{h^2}{2}\frac{\partial^2 T}{\partial y^2}$$

$$\therefore T(x+h,y) + T(x-h,y) = 4[T(x,y)] + h^2\frac{\partial^2 T}{\partial x^2} + h^2\frac{\partial^2}{\partial y}$$
$$+ T(x,y+h) + T(x,y-h)$$

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{T(x+h,y) + T(x-h,y) + T(x,y+h) + T(x,y-h)}{h^2}$$

$$\therefore \frac{T'(x,y) - T(x,y)}{\Delta t} = \frac{\alpha}{h^2}\left[\begin{array}{l} T(x+h,y) + T(x-h,y) + \\ T(x,y+h) + T(x,y-h) - 4T(x,y) \end{array}\right]$$

$$\therefore \boxed{T'(x,y) = T(x,y) + \frac{\Delta t \alpha}{h^2}\left[\begin{array}{l} T(x+h,y) + T(x-h,y) + T(x,y+h) \\ + T(x,y-h) - 4T(x,y) \end{array}\right]}$$