ASSIGNMENT 3

COMPUTATIONAL FLUID DYNAMICS

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M.Tech Aerodynamics & Propulsion

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1D Linear Hyperbolic Wave Equation

FTCS Scheme computation code (Page 2-12)

```
# include <stdlib.h>

# include <stdio.h>

# include <math.h>

# include <time.h>

# include <string.h>


int main ( );

int i4_modp ( int i, int j );

int i4_wrap ( int ival, int ilo, int ihi );

double *initial_condition ( int nx, double x[] );

double *r8vec_linspace_new ( int n, double a, double b );

void timestamp ( );


/******************************************************************************/


int main ( )


/******************************************************************************/

/*

  Purpose:


    FD1D_ADVECTION_FTCS solves the advection equation using the FTCS method.


  Discussion:


    The FTCS method is unstable for the advection problem.
```

```
    Given a smooth initial condition, successive FTCS approximations will
    exhibit erroneous oscillations of increasing magnitude.

*/
{
  double a;
  double b;
  double c;
  char command_filename[] = "advection_commands.txt";
  FILE *command_unit;
  char data_filename[] = "advection_data.txt";
  FILE *data_unit;
  double dt;
  double dx;
  int i;
  int j;
  int jm1;
  int jp1;
  int nx;
  int nt;
  int nt_step;
  double t;
  double *u;
  double *unew;
  double *x;

  timestamp ( );
  printf ( "\n" );
  printf ( "FD1D_ADVECTION_FTCS:\n" );
  printf ( "  C version\n" );
  printf ( "\n" );
  printf ( "  Solve the constant-velocity advection equation in 1D,\n" );
  printf ( "    du/dt = - c du/dx\n" );
```

```
  printf ( "  over the interval:\n" );
  printf ( "    0.0 <= x <= 1.0\n" );
  printf ( "  with periodic boundary conditions, and\n" );
  printf ( "  with a given initial condition\n" );
  printf ( "    u(0,x) = 0.0 for 0.5 <= x\n" );
  printf ( "           = 1.0 elsewhere.\n" );
  printf ( "\n" );
  printf ( "  We use a method known as FTCS:\n" );

  nx = 201;
  dx = 1.0 / ( double ) ( nx - 1 );
  a = 0.0;
  b = 1.0;
  x = r8vec_linspace_new ( nx, a, b );
  nt = 50;
  float lambda[] ={0.2,0.8,0.9,1.0,1.1};
  float lmb;
  dt = 1.0 / ( double ) ( nt );
  c = 1.0;

  u = initial_condition ( nx, x );
/*
  Open data file, and write solutions as they are computed.
*/
  data_unit = fopen ( data_filename, "wt" );

  t = 0.0;
  fprintf ( data_unit, "%10.4f  %10.4f  %10.4f\n", x[0], t, u[0] );
  for ( j = 0; j < nx; j++ )
  {
    fprintf ( data_unit, "%10.4f  %10.4f  %10.4f\n", x[j], t, u[j] );
  }
  fprintf ( data_unit, "\n" );
```

```
  nt_step = 10;

  printf ( "\n" );
  printf ( "  Number of nodes NX = %d\n", nx );
  printf ( "  Number of time steps NT = %d\n", nt );
  printf ( "  Constant velocity C = %g\n", c );

  unew = ( double * ) malloc ( nx * sizeof ( double ) );

  for ( i = 0; i < nt; i++ )
  { lmb=lambda[i];
    for ( j = 0; j < nx; j++ )
    {
      jm1 = i4_wrap ( j - 1, 0, nx - 1 );
      jp1 = i4_wrap ( j + 1, 0, nx - 1 );
      unew[j] = u[j] - lmb / 2.0 * ( u[jp1] - u[jm1] );
    }
    for ( j = 0; j < nx; j++ )
    {
      u[j] = unew[j];
    }
    if ( i == nt_step - 1 )
    {
      t = ( double ) ( i ) * dt;
      for ( j = 0; j < nx; j++ )
      {
        fprintf ( data_unit, "%10.4f  %10.4f  %10.4f\n", x[j], t, u[j] );
      }
      fprintf ( data_unit, "\n" );
      nt_step = nt_step + 15;
    }
  }
/*
  Close the data file once the computation is done.
```

```
*/
  fclose ( data_unit );

  printf ( "\n" );
  printf ( "  Plot data written to the file \"%s\"\n", data_filename );

  return 0;
}
/******************************************************************************/

int i4_modp ( int i, int j )

/******************************************************************************/
/*
  Purpose:

    I4_MODP returns the nonnegative remainder of I4 division.

  Discussion:

    If
      NREM = I4_MODP ( I, J )
      NMULT = ( I - NREM ) / J
    then
      I = J * NMULT + NREM
    where NREM is always nonnegative.

    The MOD function computes a result with the same sign as the
    quantity being divided.  Thus, suppose you had an angle A,
    and you wanted to ensure that it was between 0 and 360.
    Then mod(A,360) would do, if A was positive, but if A
    was negative, your result would be between -360 and 0.

    On the other hand, I4_MODP(A,360) is between 0 and 360, always.
```

```
  Example:

       I        J     MOD  I4_MODP   I4_MODP Factorization

     107       50       7       7    107 =  2 *  50 + 7
     107      -50       7       7    107 = -2 * -50 + 7
    -107       50      -7      43   -107 = -3 *  50 + 43
    -107      -50      -7      43   -107 =  3 * -50 + 43

  Parameters:

    Input, int I, the number to be divided.

    Input, int J, the number that divides I.

    Output, int I4_MODP, the nonnegative remainder when I is
    divided by J.
*/
{
  int value;

  if ( j == 0 )
  {
    fprintf ( stderr, "\n" );
    fprintf ( stderr, "I4_MODP - Fatal error!\n" );
    fprintf ( stderr, "  I4_MODP ( I, J ) called with J = %d\n", j );
    exit ( 1 );
  }

  value = i % j;

  if ( value < 0 )
  {
```

```
    value = value + abs ( j );
  }

  return value;
}
/******************************************************************************/

int i4_wrap ( int ival, int ilo, int ihi )

/******************************************************************************/
/*
  Purpose:

    I4_WRAP forces an I4 to lie between given limits by wrapping.
  Parameters:

    Input, int IVAL, an integer value.

    Input, int ILO, IHI, the desired bounds for the integer value.

    Output, int I4_WRAP, a "wrapped" version of IVAL.
*/
{
  int jhi;
  int jlo;
  int value;
  int wide;

  if ( ilo < ihi )
  {
   jlo = ilo;
   jhi = ihi;
  }
  else
```

```
  {
    jlo = ihi;
    jhi = ilo;
  }

  wide = jhi + 1 - jlo;

  if ( wide == 1 )
  {
    value = jlo;
  }
  else
  {
    value = jlo + i4_modp ( ival - jlo, wide );
  }

  return value;
}
/******************************************************************************/

double *initial_condition ( int nx, double x[] )

/******************************************************************************/
/*
  Purpose:

    INITIAL_CONDITION sets the initial condition.

  Parameters:

    Input, int NX, the number of nodes.

    Input, double X[NX], the coordinates of the nodes.
```

```
    Output, double INITIAL_CONDITION[NX], the value of the initial condition.
*/
{
  int i;
  double *u;

  u = ( double * ) malloc ( nx * sizeof ( double ) );

  for ( i = 0; i < nx; i++ )
  {
    if  ( 0.5 <= x[i] )
    {
      u[i] = 0.0;
    }
    else
    {
      u[i] = 1.0;
    }
  }
  return u;
}
/******************************************************************************/

double *r8vec_linspace_new ( int n, double a, double b )

/******************************************************************************/
/*
  Purpose:

    R8VEC_LINSPACE_NEW creates a vector of linearly spaced values.

  Discussion:

    An R8VEC is a vector of R8's.
```

```
    4 points evenly spaced between 0 and 12 will yield 0, 4, 8, 12.

    In other words, the interval is divided into N-1 even subintervals,
    and the endpoints of intervals are used as the points.

  Parameters:

    Input, int N, the number of entries in the vector.

    Input, double A, B, the first and last entries.

    Output, double R8VEC_LINSPACE_NEW[N], a vector of linearly spaced data.
*/
{
  int i;
  double *x;

  x = ( double * ) malloc ( n * sizeof ( double ) );

  if ( n == 1 )
  {
    x[0] = ( a + b ) / 2.0;
  }
  else
  {
    for ( i = 0; i < n; i++ )
    {
      x[i] = ( ( double ) ( n - 1 - i ) * a
             + ( double ) (         i ) * b )
             / ( double ) ( n - 1     );
    }
  }
  return x;
```

```
}

/****************************************************************************************/
```

Lax Wendroff Scheme (Page 13 – 22)

```
# include <stdlib.h>

# include <stdio.h>

# include <math.h>

# include <time.h>

# include <string.h>


int main ( );

int i4_modp ( int i, int j );

int i4_wrap ( int ival, int ilo, int ihi );

double *initial_condition ( int nx, double x[] );

double *r8vec_linspace_new ( int n, double a, double b );

void timestamp ( );


/******************************************************************************/


int main ( )


/******************************************************************************/

/*

  Purpose:


    FD1D_ADVECTION_LAX solves the advection equation using the Lax method.


  Discussion:


    The Lax method is stable for the advection problem, if the time step

    satisifies the Courant-Friedrichs-Levy (CFL) condition:


      dt <= dx / c


*/
```

```
{
  double a;
  double b;
  double c;
  char command_filename[] = "advection_commands.txt";
  FILE *command_unit;
  char data_filename[] = "advection_data.txt";
  FILE *data_unit;
  double dt;
  double dx;
  int i;
  int j;
  int jm1;
  int jp1;
  int nx;
  int nt;
  int nt_step;
  double t;
  double *u;
  double *unew;
  double *x;

  printf ( "\n" );
  printf ( "FD1D_ADVECTION_LAX:\n" );
  printf ( "  C version\n" );
  printf ( "\n" );
  printf ( "  Solve the constant-velocity advection equation in 1D,\n" );
  printf ( "    du/dt = - c du/dx\n" );
  printf ( "  over the interval:\n" );
  printf ( "    0.0 <= x <= 1.0\n" );
  printf ( "  with periodic boundary conditions, and\n" );
  printf ( "  with a given initial condition\n" );
```

```
printf ( "    u(0,x) = 0.0 for 0.5 <= x \n" );
printf ( "            = 1.0 elsewhere.\n" );
printf ( "\n" );
printf ( "  We modify the FTCS method using the Lax method:\n" );

nx = 201;
dx = 1.0 / ( double ) ( nx - 1 );
a = 0.0;
b = 1.0;
x = r8vec_linspace_new ( nx, a, b );
nt = 50;
float lambda[] ={0.2,0.8,0.9,1.0,1.1};
float lmb;
dt = 1.0 / ( double ) ( nt );
c = 1.0;

u = initial_condition ( nx, x );
data_unit = fopen ( data_filename, "wt" );

t = 0.0;
fprintf ( data_unit, "%10.4f  %10.4f  %10.4f\n", x[0], t, u[0] );
for ( j = 0; j < nx; j++ )
{
  fprintf ( data_unit, "%10.4f  %10.4f  %10.4f\n", x[j], t, u[j] );
}
fprintf ( data_unit, "\n" );

nt_step = 10;

printf ( "\n" );
printf ( "  Number of nodes NX = %d\n", nx );
printf ( "  Number of time steps NT = %d\n", nt );
```

```
  printf ( "  Constant velocity C = %g\n", c );

  unew = ( double * ) malloc ( nx * sizeof ( double ) );

  for ( i = 0; i < nt; i++ )
  { lmb=lambda[i];
    for ( j = 0; j < nx; j++ )
    {
      jm1 = i4_wrap ( j - 1, 0, nx - 1 );
      jp1 = i4_wrap ( j + 1, 0, nx - 1 );
      unew[j] = 0.5 * u[jm1] + 0.5 * u[jp1]
        - lmb * ( u[jp1] - u[jm1] ) + (lmb*lmb)/2.0*(u[jp1] -2*u[ceil((jpl+jml)/2.0)] +u[jm1]);
    }
    for ( j = 0; j < nx; j++ )
    {
      u[j] = unew[j];
    }
    if ( i == nt_step - 1 )
    {
      t = ( double ) ( i ) * dt;
      for ( j = 0; j < nx; j++ )
      {
        fprintf ( data_unit, "%10.4f  %10.4f  %10.4f\n", x[j], t, u[j] );
      }
      fprintf ( data_unit, "\n" );
      nt_step = nt_step + 15;
    }
  }
/*
  Close the data file once the computation is done.
*/
  fclose ( data_unit );
```

```
  printf ( "\n" );
  printf ( "  Plot data written to the file \"%s\"\n", data_filename );

  return 0;
}
/******************************************************************************/

int i4_modp ( int i, int j )

/******************************************************************************/
/*
  Purpose:

    I4_MODP returns the nonnegative remainder of I4 division.

  Discussion:

    If
      NREM = I4_MODP ( I, J )
      NMULT = ( I - NREM ) / J
    then
      I = J * NMULT + NREM
    where NREM is always nonnegative.

    The MOD function computes a result with the same sign as the
    quantity being divided.  Thus, suppose you had an angle A,
    and you wanted to ensure that it was between 0 and 360.
    Then mod(A,360) would do, if A was positive, but if A
    was negative, your result would be between -360 and 0.

    On the other hand, I4_MODP(A,360) is between 0 and 360, always.
```

```
  Example:

        I         J     MOD  I4_MODP   I4_MODP Factorization

      107        50       7       7    107 =  2 *  50 + 7
      107       -50       7       7    107 = -2 * -50 + 7
     -107        50      -7      43   -107 = -3 *  50 + 43
     -107       -50      -7      43   -107 =  3 * -50 + 43

  Parameters:

    Input, int I, the number to be divided.

    Input, int J, the number that divides I.

    Output, int I4_MODP, the nonnegative remainder when I is
    divided by J.
*/
{
  int value;

  if ( j == 0 )
  {
    fprintf ( stderr, "\n" );
    fprintf ( stderr, "I4_MODP - Fatal error!\n" );
    fprintf ( stderr, "  I4_MODP ( I, J ) called with J = %d\n", j );
    exit ( 1 );
  }

  value = i % j;
```

```
  if ( value < 0 )
  {
    value = value + abs ( j );
  }

  return value;
}
/******************************************************************************/

int i4_wrap ( int ival, int ilo, int ihi )

/******************************************************************************/
/*
  Purpose:

    I4_WRAP forces an I4 to lie between given limits by wrapping.

  Parameters:

    Input, int IVAL, an integer value.

    Input, int ILO, IHI, the desired bounds for the integer value.

    Output, int I4_WRAP, a "wrapped" version of IVAL.
*/
{
  int jhi;
  int jlo;
  int value;
  int wide;
```

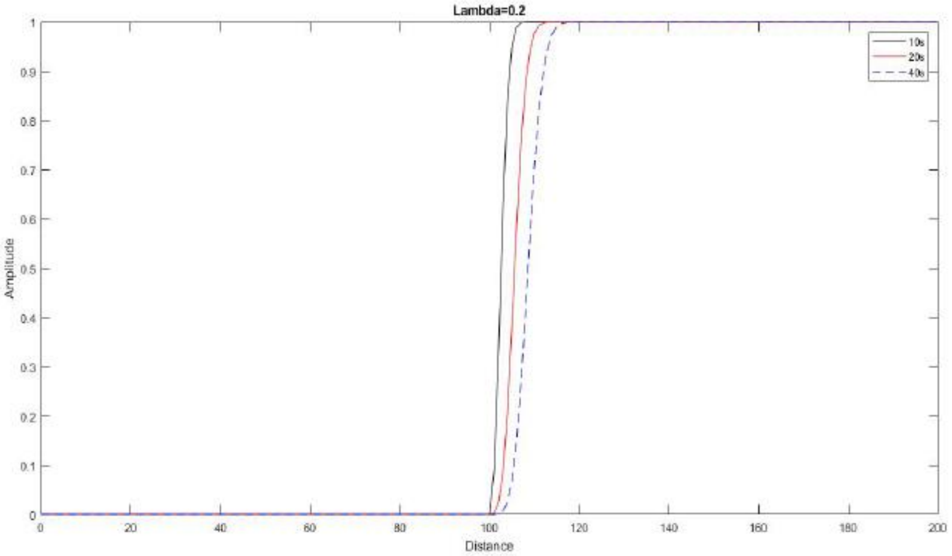
```
  if ( ilo < ihi )
  {
   jlo = ilo;
   jhi = ihi;
  }
  else
  {
    jlo = ihi;
    jhi = ilo;
  }

  wide = jhi + 1 - jlo;

  if ( wide == 1 )
  {
    value = jlo;
  }
  else
  {
    value = jlo + i4_modp ( ival - jlo, wide );
  }

  return value;
}
/******************************************************************************/

double *initial_condition ( int nx, double x[] )

/******************************************************************************/
/*
  Purpose:
```

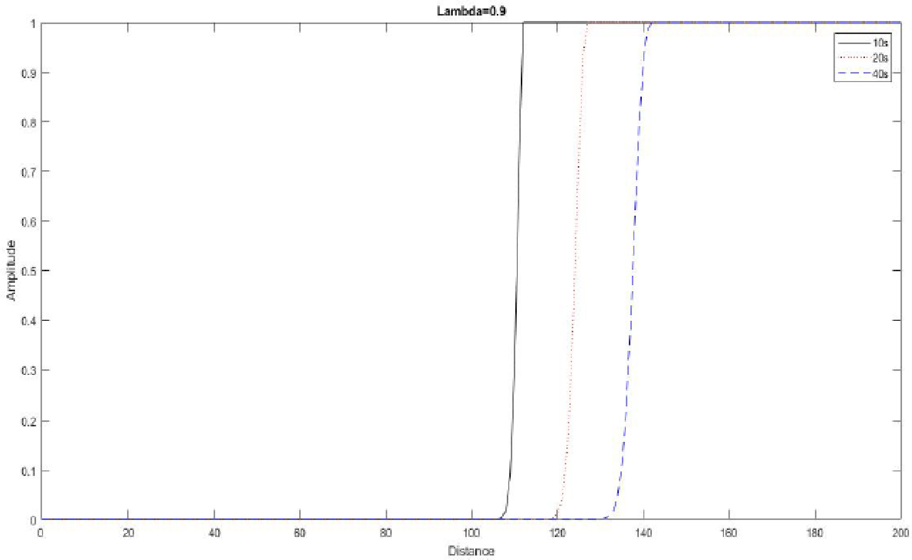
```
    INITIAL_CONDITION sets the initial condition.



  Parameters:

    Input, int NX, the number of nodes.

    Input, double X[NX], the coordinates of the nodes.

    Output, double INITIAL_CONDITION[NX], the value of the initial condition.
*/
{
  int i;
  double *u;

  u = ( double * ) malloc ( nx * sizeof ( double ) );

  for ( i = 0; i < nx; i++ )
  {
    if  ( 0.5 <= x[i] )
    {
      u[i] = 0.0;
    }
    else
    {
      u[i] = 1.0;
    }
  }
  return u;
}
/******************************************************************************/
```

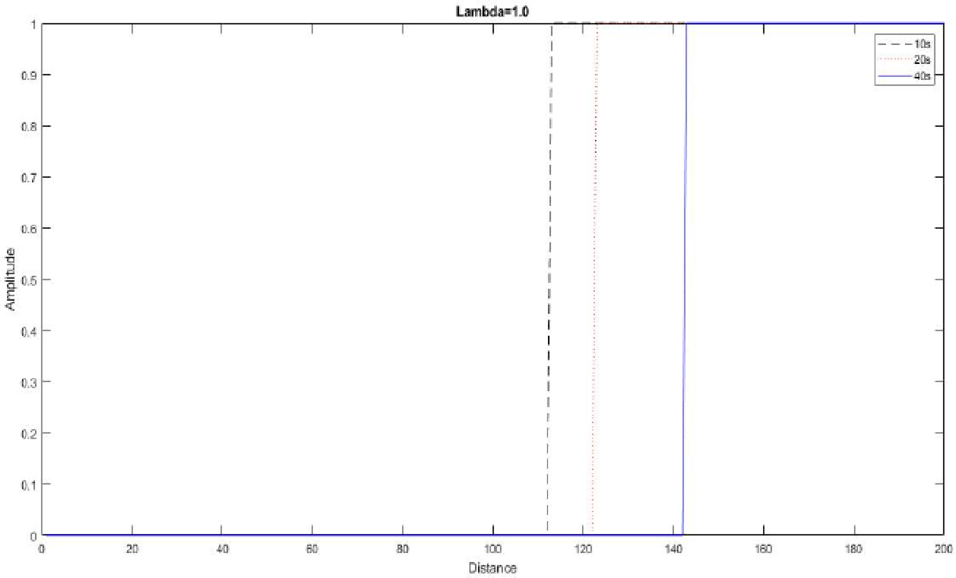
```
double *r8vec_linspace_new ( int n, double a, double b )

/******************************************************************************/

{
  int i;
  double *x;

  x = ( double * ) malloc ( n * sizeof ( double ) );

  if ( n == 1 )
  {
    x[0] = ( a + b ) / 2.0;
  }
  else
  {
    for ( i = 0; i < n; i++ )
    {
      x[i] = ( ( double ) ( n - 1 - i ) * a
             + ( double ) (         i ) * b )
             / ( double ) ( n - 1     );
    }
  }
  return x;
}
```

Plots

FTCS Scheme




Lambda=0.9
10s
20s
40s
Amplitude
Distance


Lambda=1.0
10s
20s
40s
Amplitude
Distance