Hw1

1.

a) $A[0 \ldots 4] = \langle 10, 7, 0, 3, 8 \rangle$

$I(A) = \{(0,1), (0,2), (0,3), (0,4), (1,2), (1,3)\}$

b)

```
1  for i:=0 to n-1:              a1, b1, c1
2      j:=i                      c2
3      while j+1>0:              c3
4          if A[j]>A[i]:         c4
5              add (j,i) to the set   c5
6          j:=j-1                c6
```

Let t_i be the number of times line 5 runs. Thus, $t_i \leq i-1$

Lines	
1,2	$a_1 + nb_1 + (n-1)(C_1 + C_2)$
3	iC_3
4,6	$(i-1)(C_4 + C_6)$
5	t_i

Runtime

$$|I(A)| = a_1 + nb_1 + (n-1)(C_1 + C_2) + \left(\sum_{i=0}^{n-1} i\right) C_3 + \left(\sum_{i=0}^{n-1} (i-1)\right)(C_4 + C_6) + \sum_{i=0}^{n-1} t_i$$

$$= (a_1 + b_1) + (n-1)(b_1 + C_1 + C_2) + \frac{n(n-1)}{2}(C_3 + C_4 + C_6) - n(C_4 + C_6) + \sum_{i=0}^{n-1} i-1$$

$$= (a_1 + b_1) + (n-1)(b + C_1 + C_2 - (C_4 + C_6 + 1)) + \frac{n(n-1)}{2}(C_3 + C_4 + C_6) - (C_4 + C_6 + 1)$$

$$= A + (n-1)(B) + \frac{n(n-1)}{2} C$$ <u>Qudratic</u>

$O(|I(A)|) = n^2$

c) $|I(A)|$ is minimized when there are no elements in A greater than any subsequent elements. This makes $t_i = 0$

$|I(A)|$ is maximized when all elements are greater than all subsequent elements. This makes $t_i = i-1$

2.

- selection sort pseudo-code

```
1  for i:=0 in n-1              a1, b1, c1
2  |  j:=i+1                    c2
3  |  a:=A[i]                   c3
4  |  min:=a                    c4
5  |  while j<n-1:              c5
6  |  |  if A[j] < min:         c6
7  |  |  |  min:=A[j]           c7
8  |  |  j:=j+1                 c8
9  |  A[i]:=min                 c9
10 |  min:=a                    c10
```

Lines

$1\dots4, 9, 10: \; a_1 + nb_1 + (n-1)(C_1+C_2+C_3+C_4+C_9+C_{10})$

$5: \; (n-i+1)\,C_5$

$6, 8: \; (n-i)(C_6+C_8)$

$7: \; t_i C_7$

worst case for t_i

$$|I(A)| = a_1 + nb_1 + (n-1)(C_1+C_2+C_3\dots) + \left(\frac{n(n+1)}{2}+n\right)C + \left(\frac{n(n+1)}{2}C_6 C_8\right)\left(\frac{n(n+1)}{2}C_7\right)$$

$A = a_1+b_1+C_5 \quad B = b_1+C_5+C_1+C_2+C_3+C_4+C_9+C_{10} \quad C = C_5+C_6+C_7+C_8$

$$|I(A)| = A + (n-1)B + \frac{n(n+1)}{2}C$$

- Quadratic time; $O=n^2$