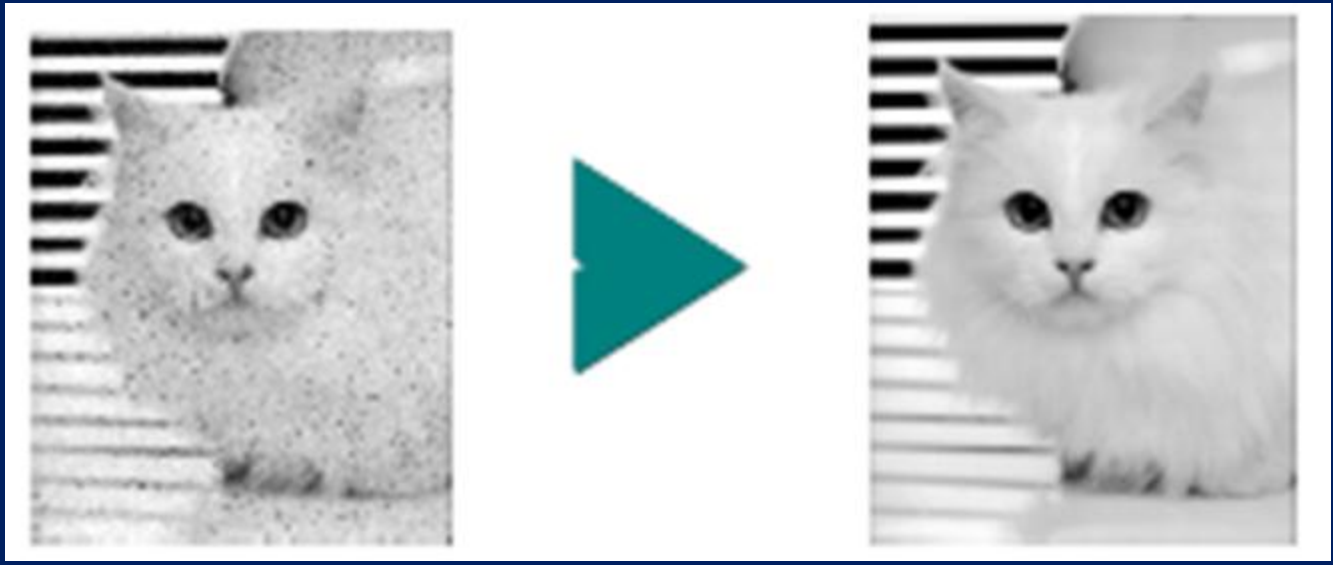
Image Enhancement in the Spatial Domain



Dr A S Jalal

Image is NOT Perfect Sometimes

What Is Image Enhancement?

Image enhancement is the process of making images more useful

The reasons for doing this include:

- Highlighting interesting detail in images
- Removing noise from images
- Making images more visually appealing

Why do we need Image Enhancement?

- The goal of image enhancement is to process a digital image and make it more "suitable" than the original image for a specific application
- Images may suffer from the following degradations:
  - Poor contrast due to poor illumination or finite sensitivity of the imaging device
  - Electronic sensor noise or atmospheric disturbances leading to broad band noise
  - Aliasing effects due to inadequate sampling
  - Finite aperture effects or motion

After Image Enhancement

Image Enhancement

- There are Two broad categories of Image Enhancement Approaches
  - **Spatial Domain methods**
    - Based on direct manipulation of pixels in an image
  - **Frequency Domain methods**
    - Based on modifying the Fourier transform of an image

Spatial Domain

- **Intensity Transformations (Point processing)**
  - Operate on single pixels of an image
    - e.g., image averaging; logic operation; contrast stretching ...
- **Spatial Filtering (Mask processing)**
  - Working in a neighbourhood of every pixel in an image
    - e.g., blurring, median

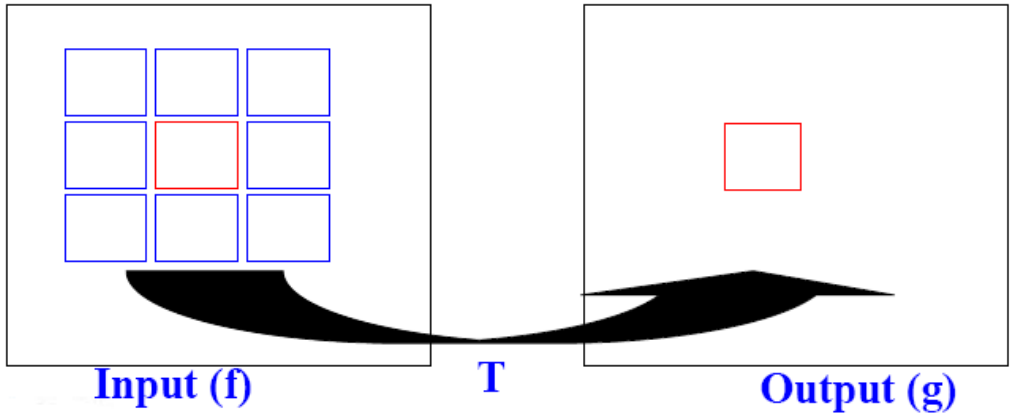
Spatial Domain

- Spatial domain can be denoted by:

$$g(x,y) = T[f(x,y)]$$

where

- $f(x,y)$ is the input image
- $g(x,y)$ is the processed image
- T is an operator on f defined over some neighborhood of (x,y)



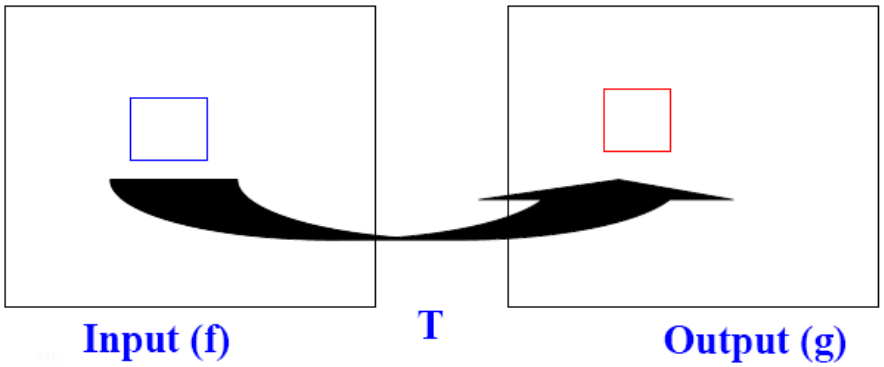
Intensity Transformations (Point processing)

Intensity Transformation

- **Simplest T** : operates on neighborhood of 1x1
- Then, it becomes **intensity transformation**
- g depends on only the value of f at (x,y)
- T = gray level (or intensity or mapping) transformation function

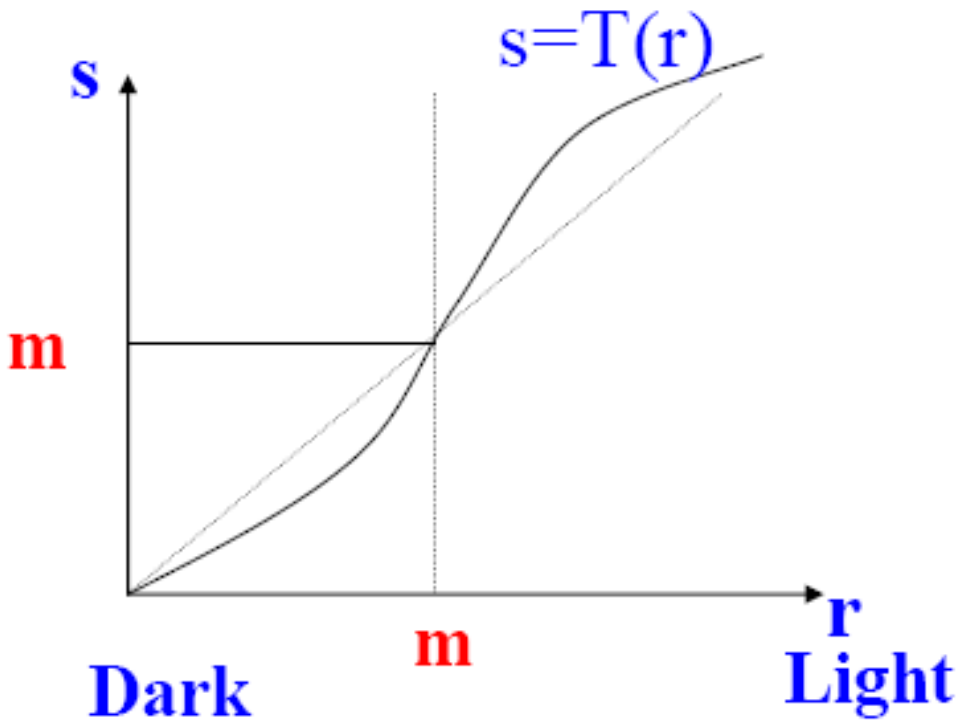
$$s = T(r)$$

- Where
  - r = gray level of $f(x,y)$
  - s = gray level of $g(x,y)$



Intensity Transformation …

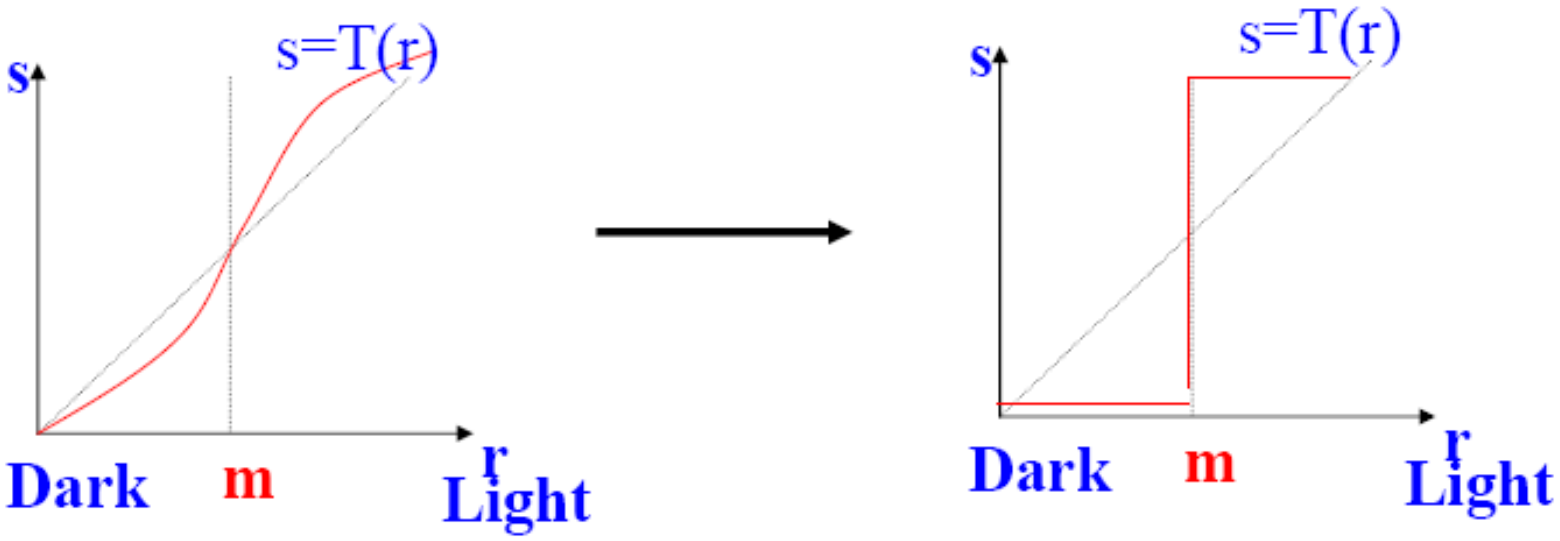
- The gray level below **m** are darkened and the levels above **m** are brightened.



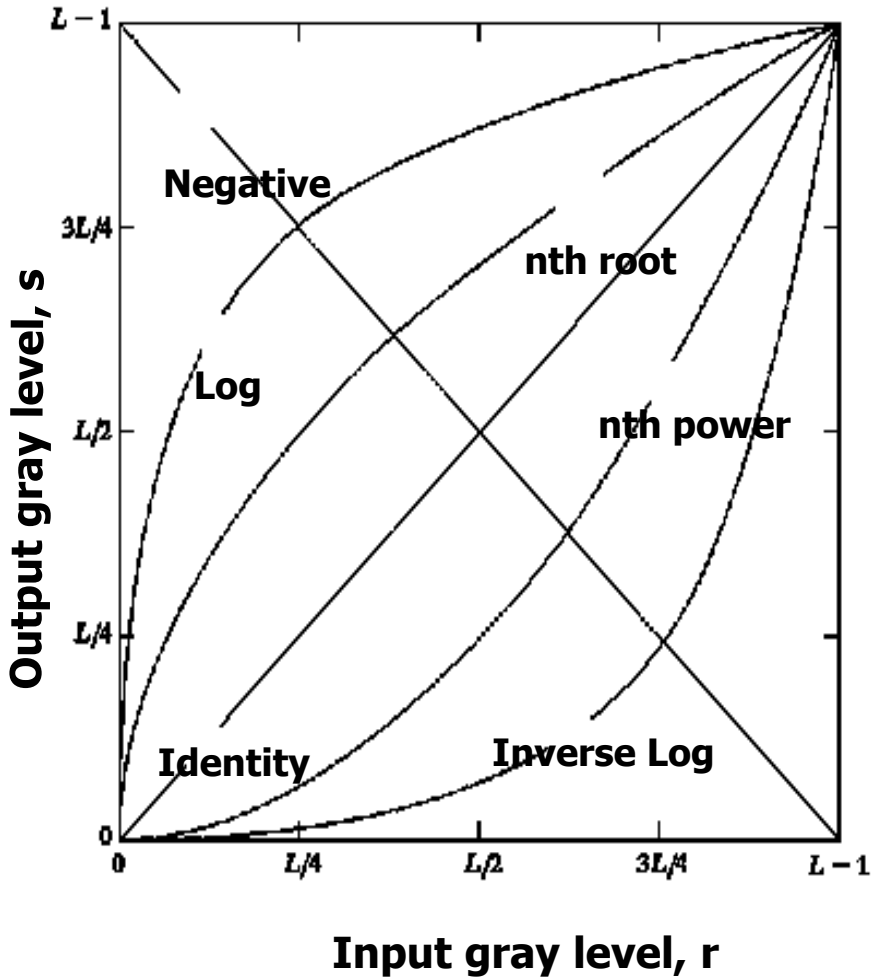
Intensity Transformation …

- **Limiting case:** Produces a binary image (two level) from input image and the function is known as Thresholding function

$$g(x,y) = \begin{cases} L & \text{if } f(x,y) > m, \\ 0 & \text{otherwise} \end{cases}$$



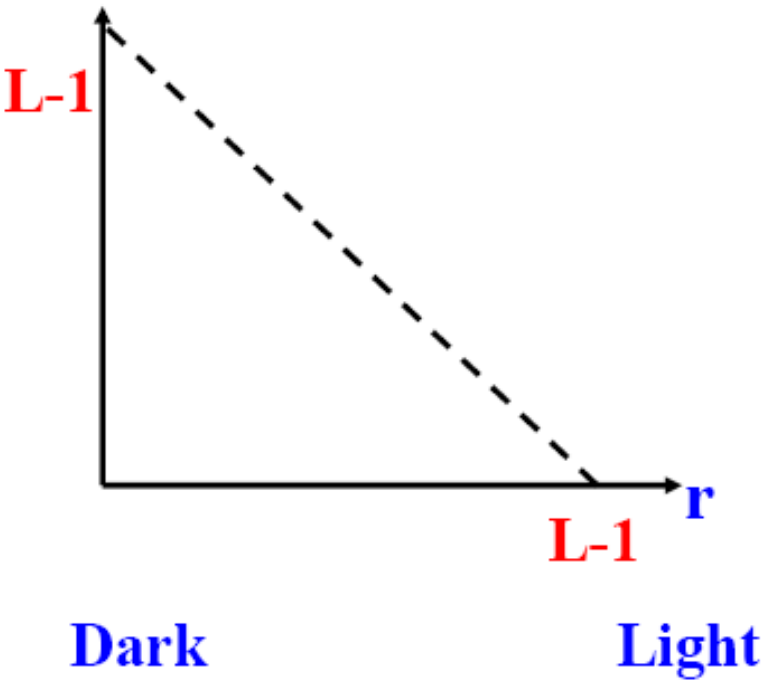
Basic Intensity Transformation Functions



- **Linear function**
  - Negative and identity transformations
- **Logarithm function**
  - Log and inverse-log transformation
- **Power-law function**
  - n^{th} power and n^{th} root transformations

Identity function: Output intensities are identical to input intensities.

Image Negatives



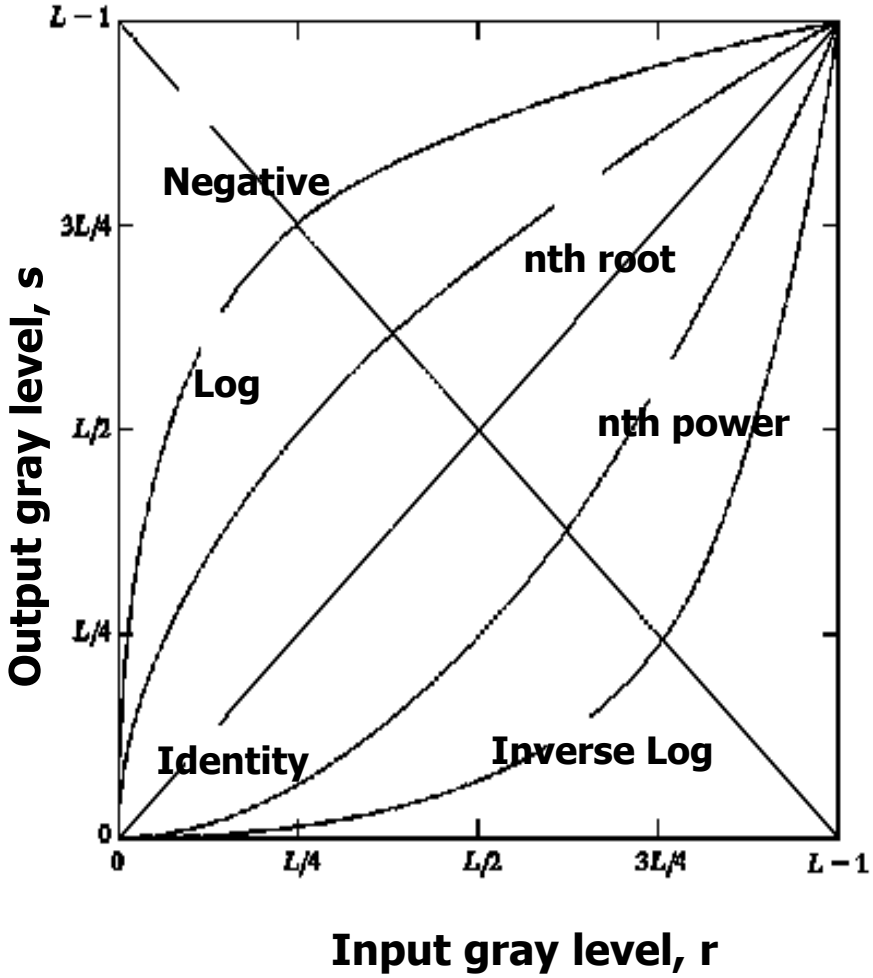
- An image with gray level in the range $[0, L-1]$ where $L = 2^n$; n = 1, 2…
- Negative transformation :

$$s = L - 1 - r$$

- Reversing the intensity levels of an image.
- Suitable for enhancing white or gray detail embedded in dark regions of an image, especially when the black area dominant in size.

Image Negatives

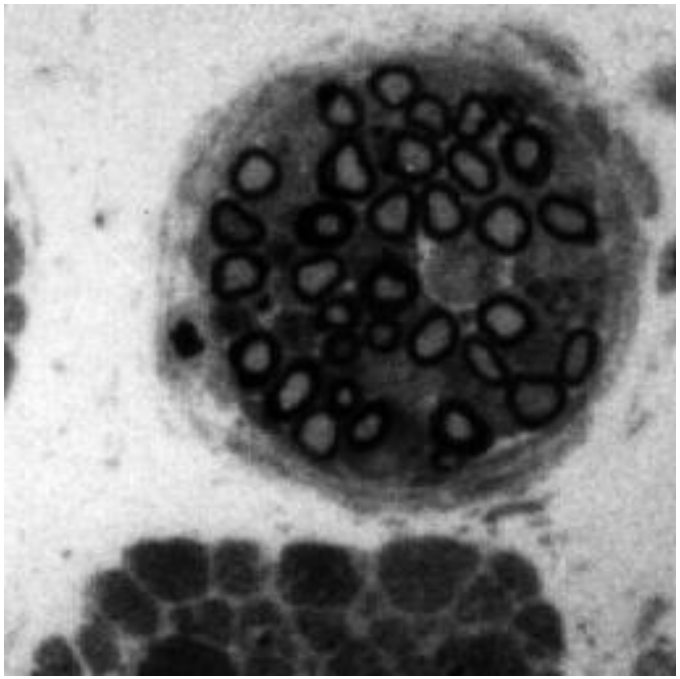
Log Transformations



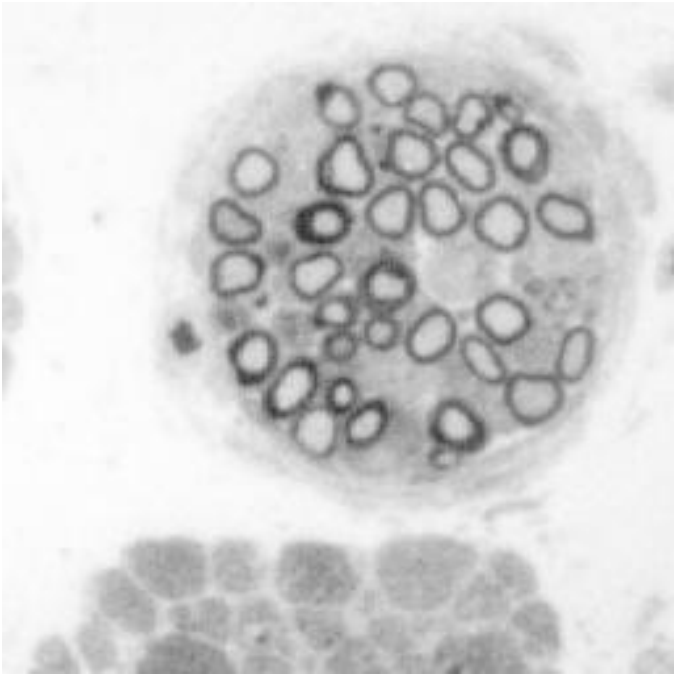
$$s = c \log (1+r)$$

- c is a constant and $r \geq 0$
- Log curve maps a narrow range of low gray-level values in the input image into a wider range of output levels.
- Used to expand the values of dark pixels in an image while compressing the higher-level values.

Log Transformations: Example

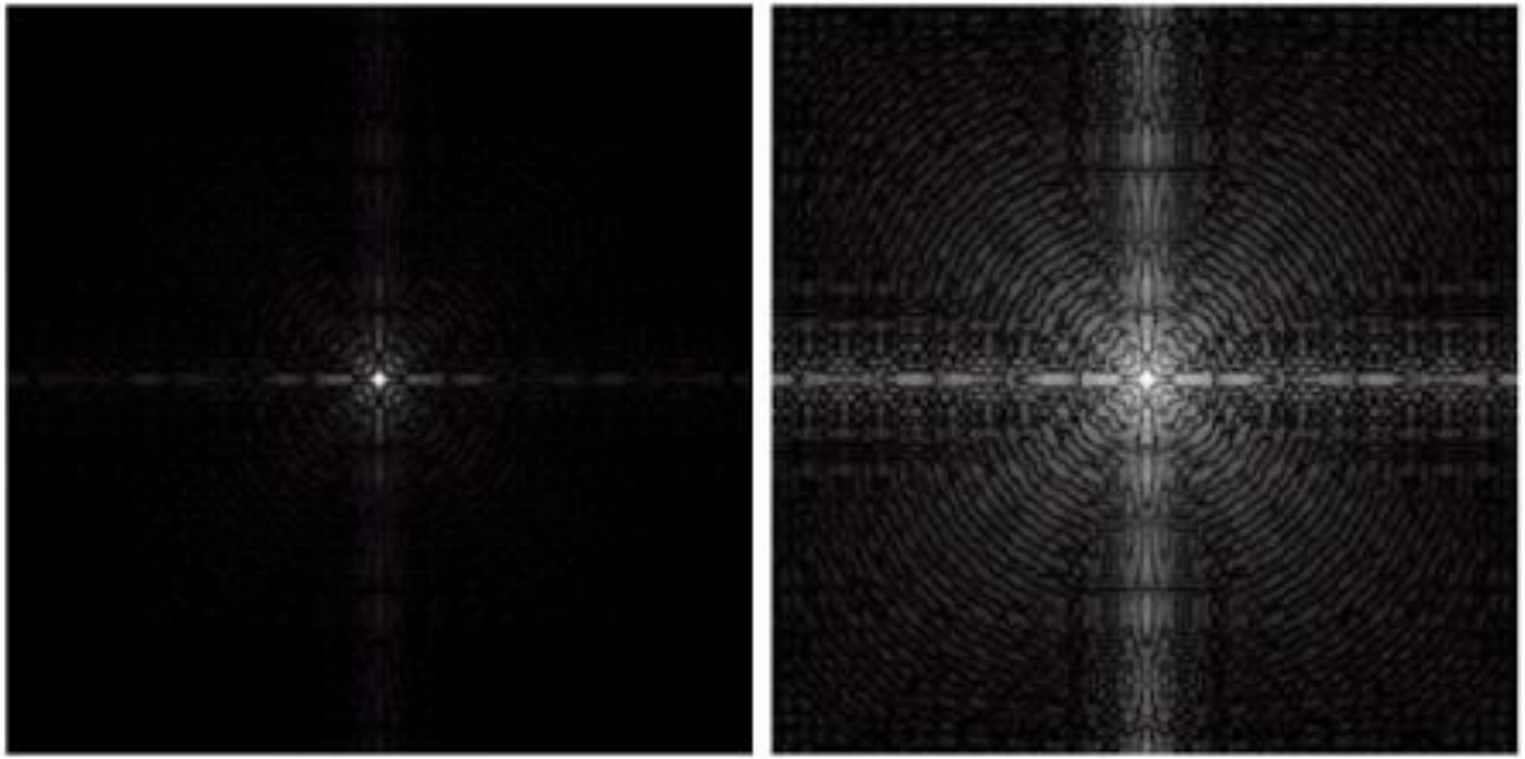


Original Image



Mapped image

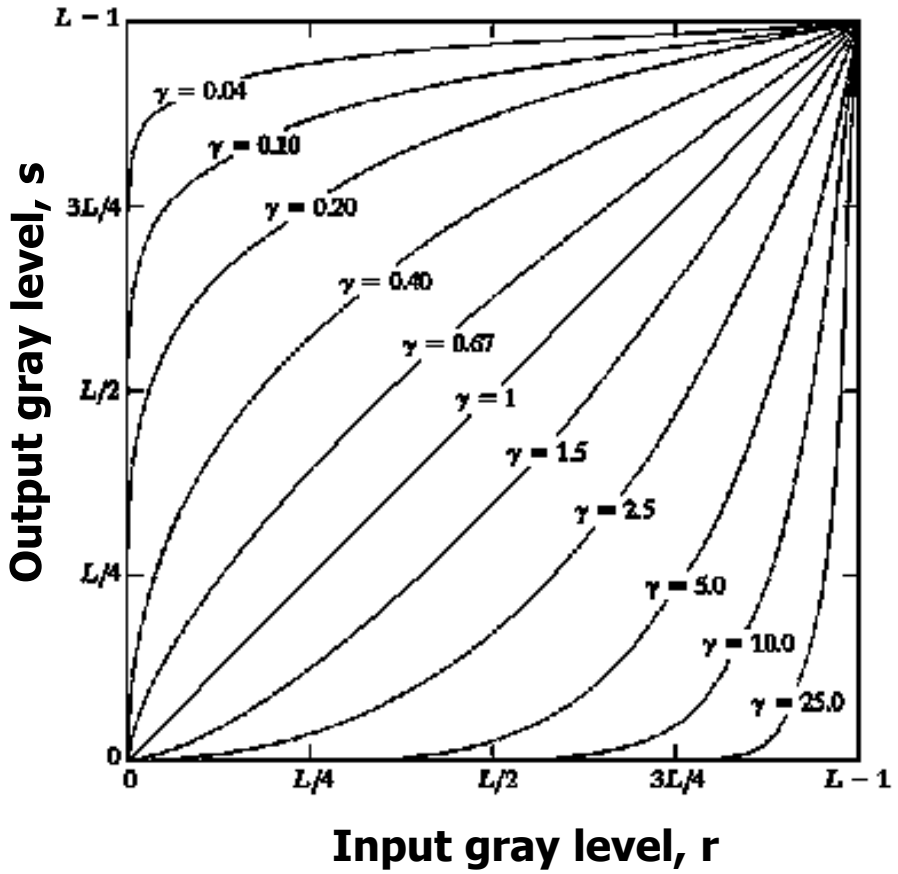
Example of Logarithm Image



Fourier Spectrum with range = 0 to 1.5 x 10^6

Result after apply the log transformation with c = 1, range = 0 to 6.2

Power-Law Transformations

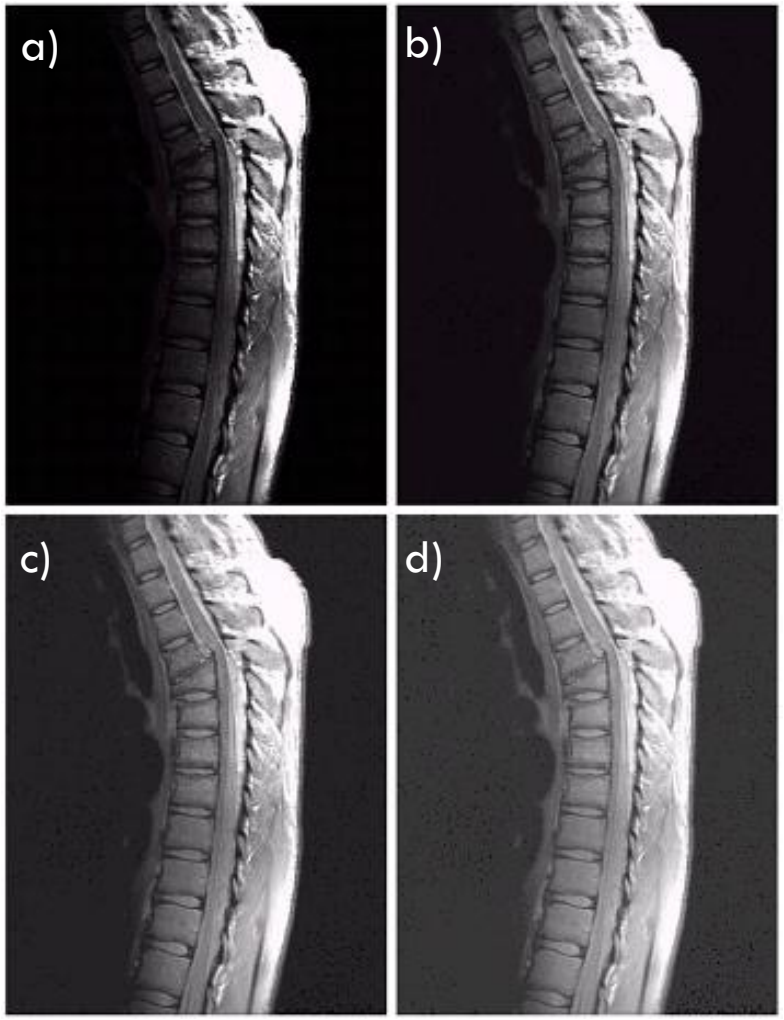


Plots of $s = cr^{\gamma}$ for various values of γ (c = 1 in all cases)

$$s = cr^{\gamma}$$

- c and γ are positive constants
- Power-law curves with fractional values of γ map a narrow range of dark input values into a wider range of output values, with the opposite being true for higher values of input levels.
- $c = \gamma = 1$ ⇨ Identity function

Example : MRI



(a) a magnetic resonance image of an upper human spine with a fracture dislocation and spinal cord impingement

- The picture is predominately dark
- An expansion of gray levels are desirable ⇨ needs $\gamma < 1$

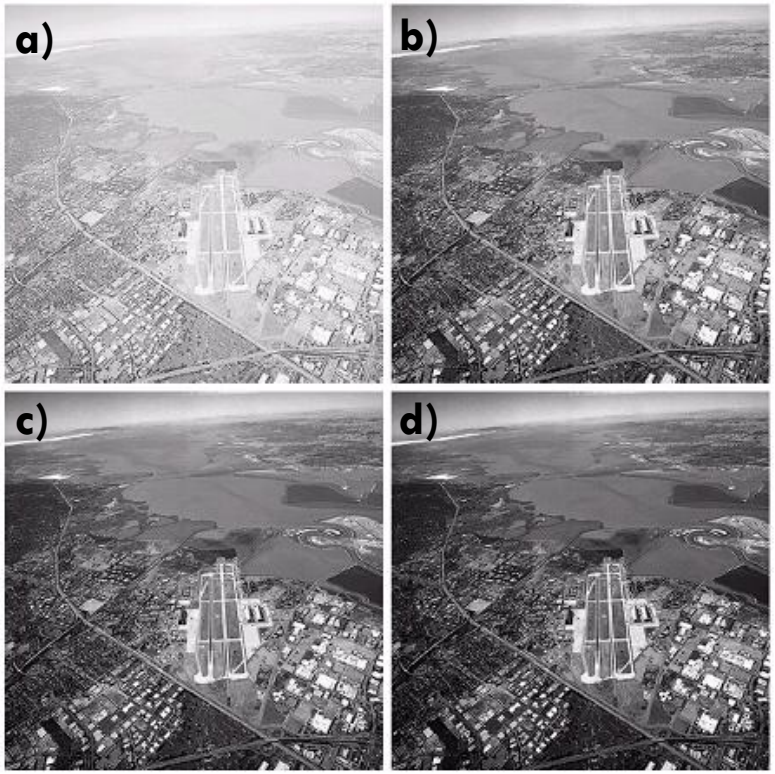
(b) result after power-law transformation with $\gamma = 0.6$, c=1

(c) transformation with $\gamma = 0.4$ (best result)

(d) transformation with $\gamma = 0.3$ (under acceptable level)

Effect of decreasing gamma

- **When the γ is reduced too much, the image begins to reduce contrast to the point where the image started to have very slight "wash-out" look, especially in the background**



(a) image has a washed-out appearance, it needs a compression of gray levels ⇨ needs $\gamma > 1$

(b) result after power-law transformation with $\gamma = 3.0$ (suitable)

(c) transformation with $\gamma = 4.0$ (suitable)

(d) transformation with $\gamma = 5.0$ (high contrast, the image has areas that are too dark, some detail is lost)

Piecewise-Linear Transformation Functions

- A complementary approach to the previous methods
- The form of piecewise functions can be arbitrarily complex
- Practical implementation of some important transformations can be formulated only as piecewise functions
- Their specification requires considerably more user input

Contrast Stretching

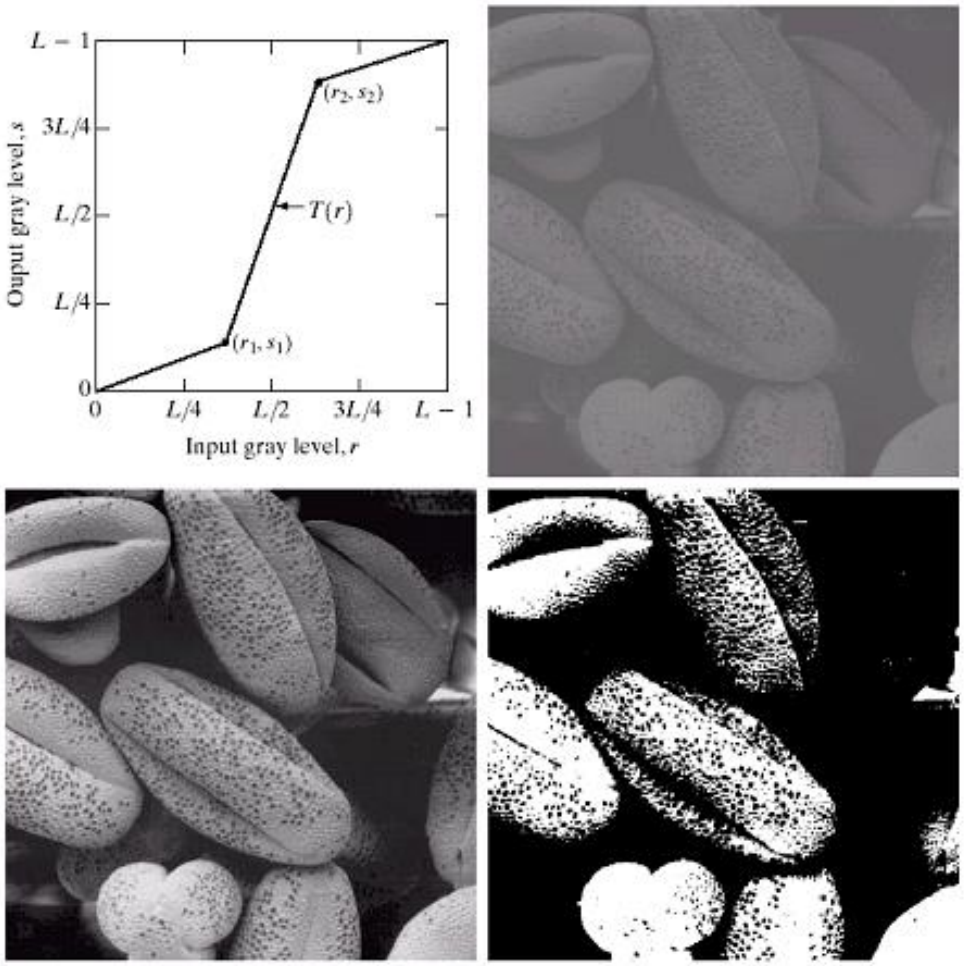
- The Simplest of piecewise linear functions.
- **Basic idea** – to increase the dynamic range of the gray levels in the image being processed.
- During image acquisition, low contrast images may result due to
  - Poor illumination
  - Lack of dynamic range in image sensor
  - Wrong setting of the lens aperture

Contrast Stretching

- Contrast transform result

Original **Enhanced**

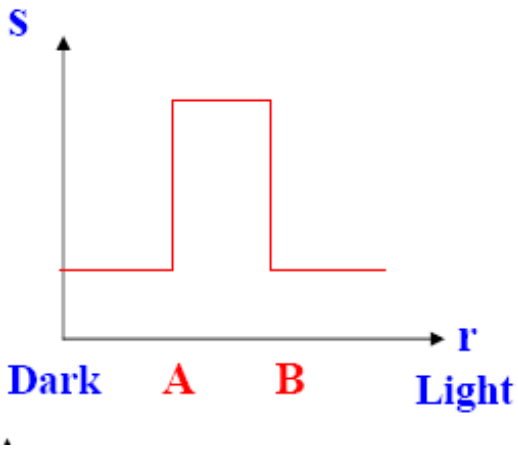
Contrast Stretching: Example

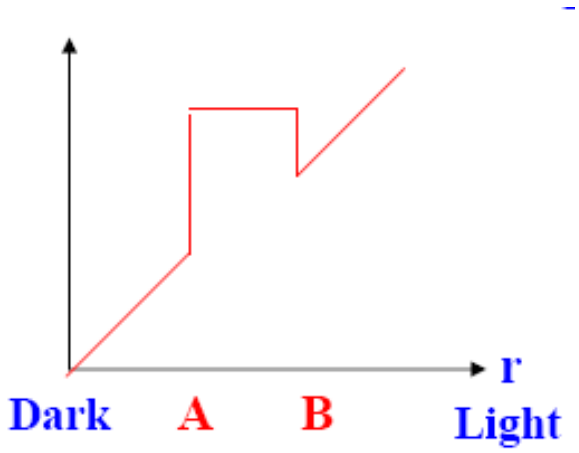


- increase the dynamic range of the gray levels in the image
- (b) a low-contrast image
- (c) result of contrast stretching: $(r_1,s_1) = (r_{min},0)$ and $(r_2,s_2) = (r_{max},L-1)$
- (d) result of thresholding

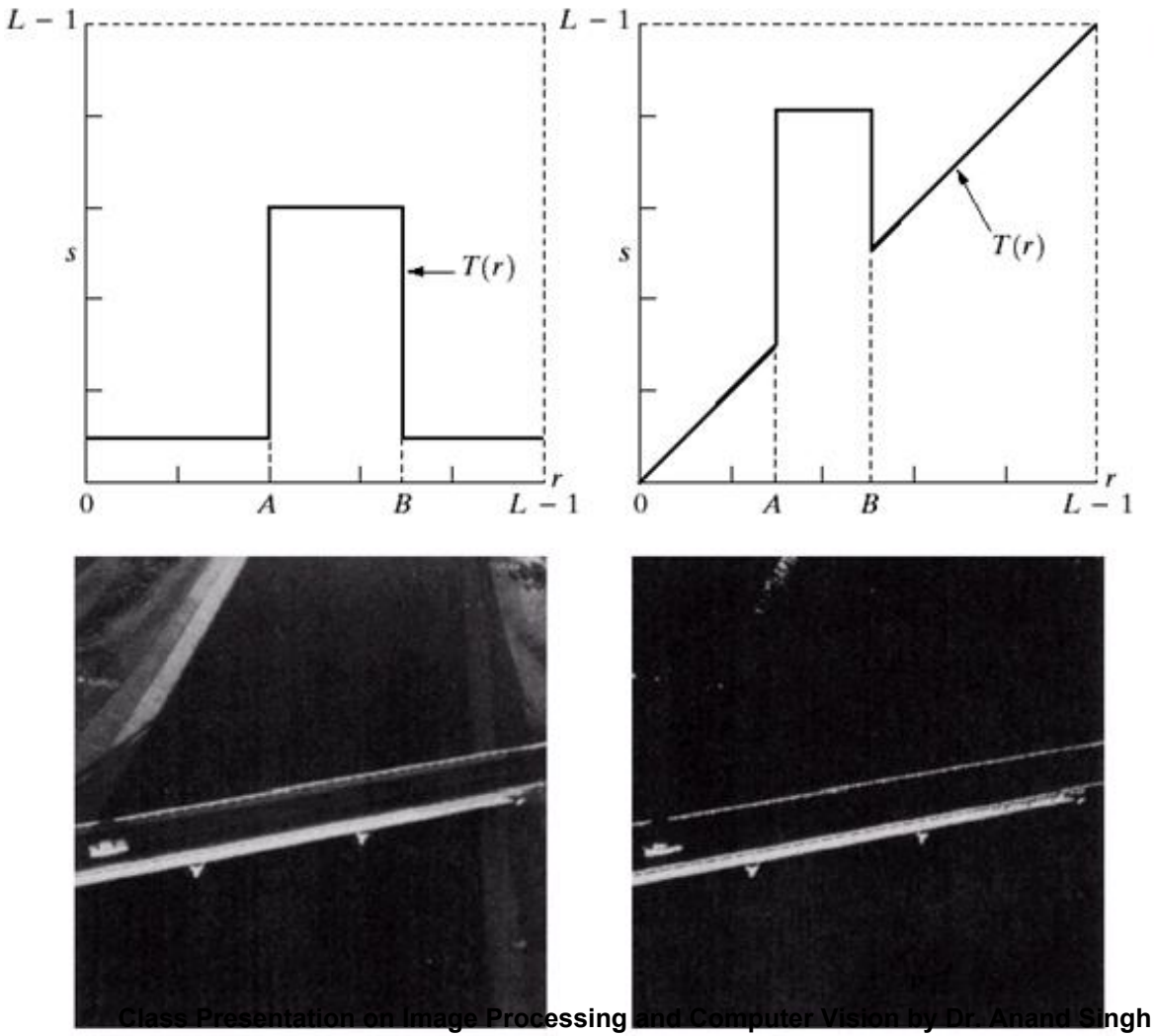
Gray-level slicing

- Highlight a specific range of gray values
- Two basic Methods
  - Display a high value for all gray levels in the range of interest and a low value for all other
  - Brighten the desired range of gray levels but preserve all other levels





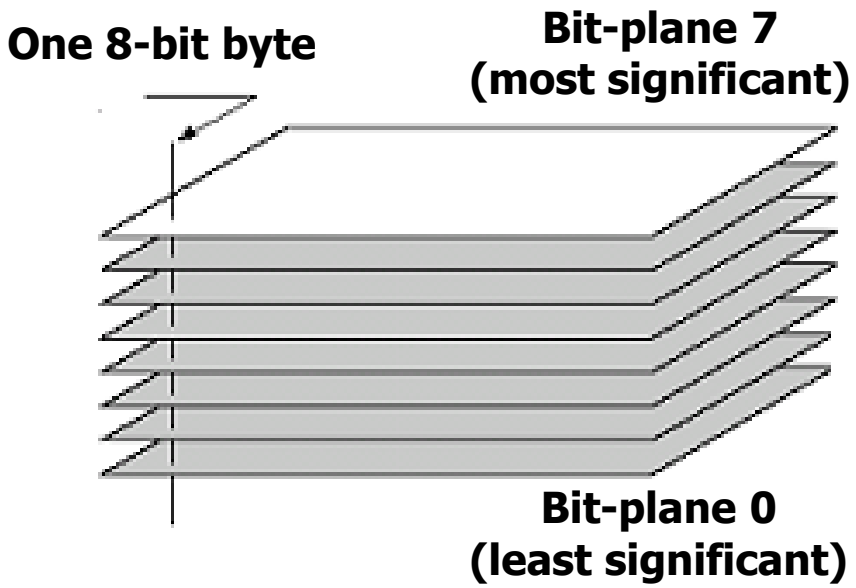
Gray-level slicing: Example



a b
c d

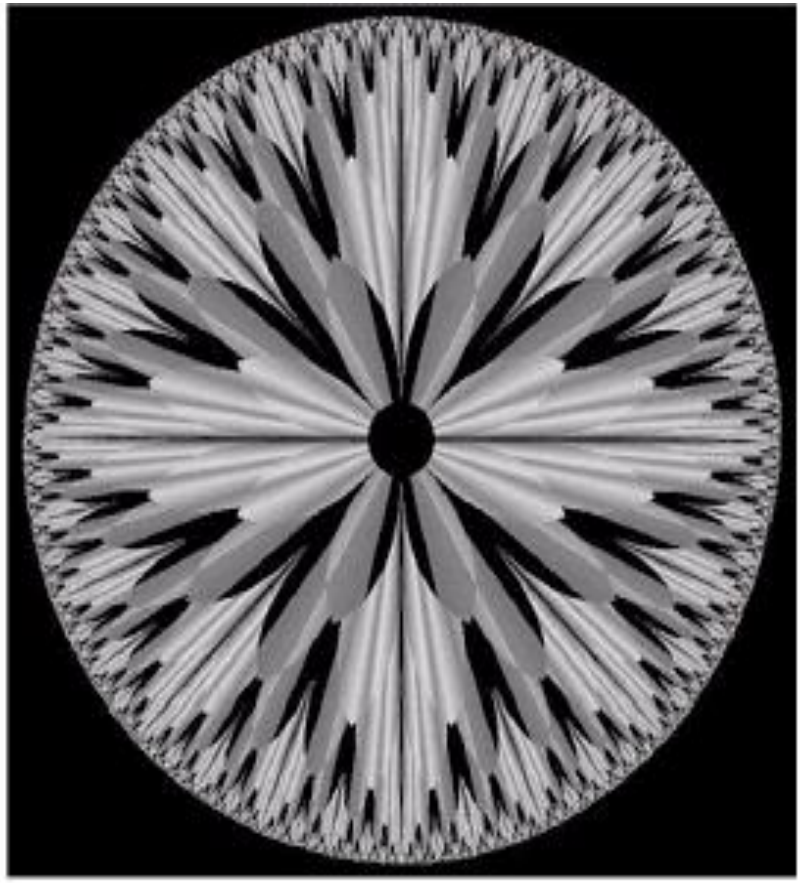
(a) This transformation highlights range $[A, B]$ of gray levels and reduces all others to a constant level. (b) This transformation highlights range $[A, B]$ but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Bit-plane slicing



- Highlighting the contribution made to total image appearance by specific bits
- Suppose each pixel is represented by 8 bits
- Higher-order bits contain the majority of the visually significant data
- Useful for analyzing the relative importance played by each bit of the image

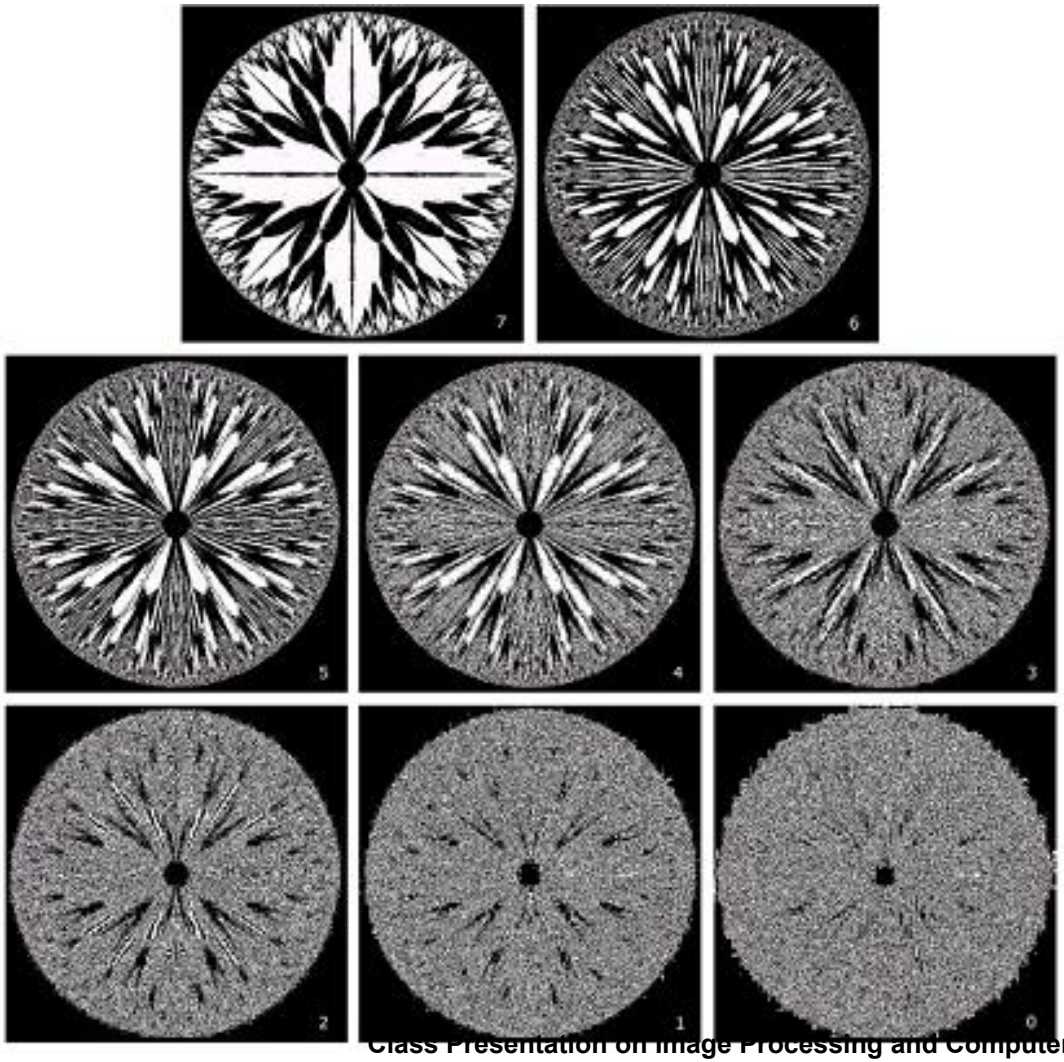
Bit-plane slicing: Example



An 8-bit fractal image

- Higher order bit planes of an image carry a significant amount of visually relevant details.
- Lower order planes contribute more to fine (often imperceptible) details.
- The (binary) image for bit-plane 7 can be obtained by processing the input image with a thresholding gray-level transformation.
  - Map all levels between 0 and 127 to 0
  - Map all levels between 129 and 255 to 255

Bit-plane slicing: Example (8 bit planes)



Bit-plane 7	Bit-plane 6	
Bit-plane 5	Bit-plane 4	Bit-plane 3
Bit-plane 2	Bit-plane 1	Bit-plane 0

Bit-plane slicing: Example

a b c
d e f
g h i

(a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-plane slicing: Example

a b c

Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Histogram Processing

- Histogram is a discrete function formed by counting the number of pixels that have a certain gray level in the image .
- Histogram of a digital image with gray levels in the range [0,L-1] is a discrete function

$$h(r_k) = n_k$$

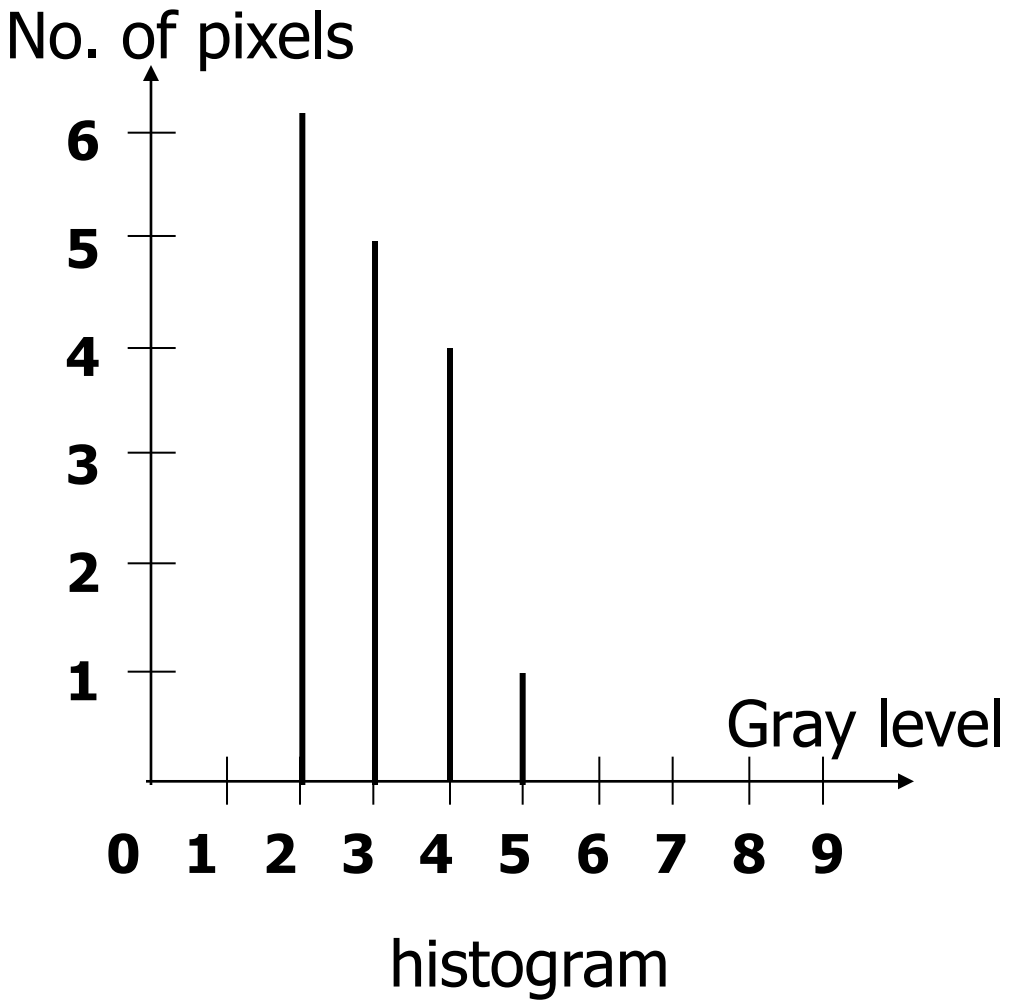
- Where
  - r_k : the k^{th} gray level
  - n_k : the number of pixels in the image having gray level r_k
  - $h(r_k)$: histogram of a digital image with gray levels r_k

Histogram Processing: Example

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



histogram

Normalized Histogram

- Dividing each of histogram at gray level r_k by the total number of pixels in the image, n

$$p(r_k) = n_k / n$$

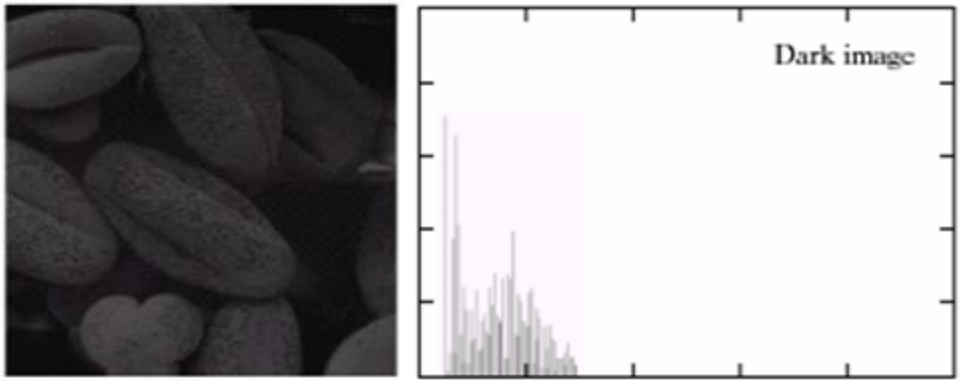
- For k = 0,1,…,L-1
- $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k
- The sum of all components of a normalized histogram is equal to 1

Histogram Processing: Example

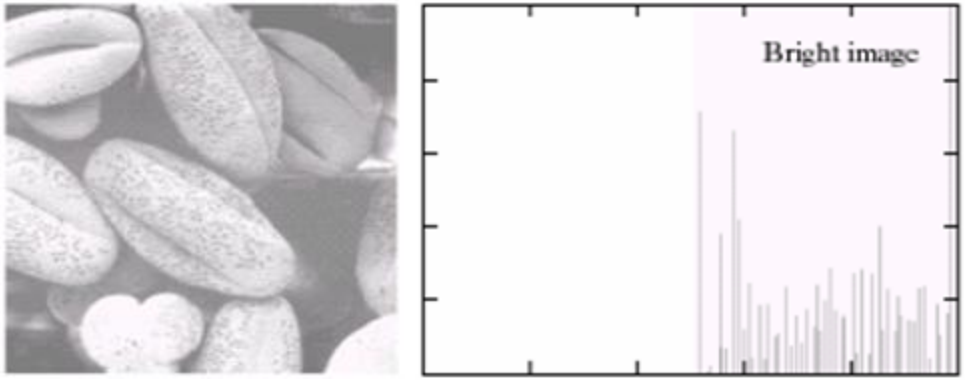
- Problem: an image with gray levels between 0 and 7 is given below. Find the histogram of the image

1	6	2	2
1	3	3	3
4	6	4	0
1	6	4	7

Histogram Processing: Example

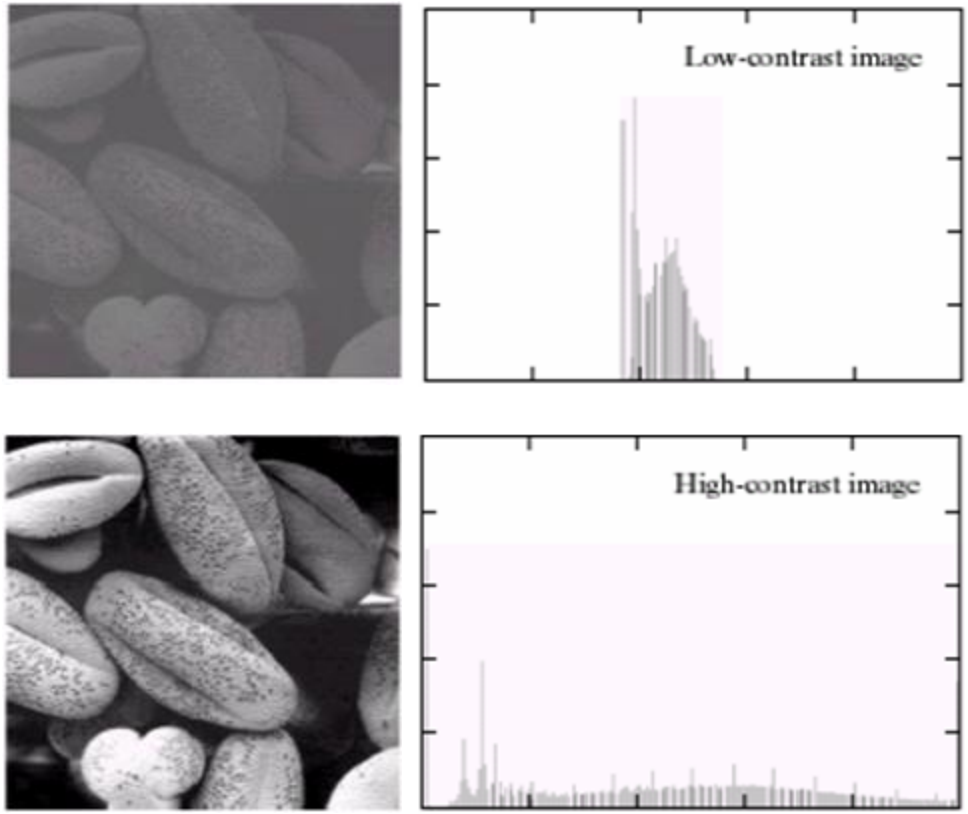


- In the dark images, components of the histogram are concentrated on the low (dark) side of the gray scale.



- In bright images, the histogram is biased towards the high side of the gray scale.

Histogram Processing: Example



- In image with low contrast, the histogram will be narrow & centred towards the middle of the gray scale.
- An image with high contrast & a large variety of gray tones will have pixels that occupy the entire range of possible gray levels & are uniformly distributed.

Histogram Equalization OR Histogram Linearization

- It is the process that transforms the intensity values so that the histogram of the output image approximately matches the flat (uniform) histogram.
- The aim is to create an image with equally distributed brightness levels over the whole brightness scale.

Histogram Equalization OR Histogram Linearization

- As the low-contrast image's histogram is narrow and centered toward the middle of the gray scale, if we distribute the histogram to a wider range the quality of the image will be improved.
- We can do it by adjusting the probability density function of the original histogram of the image so that the probability spread equally.
- Histogram equalization (HE) results are similar to contrast stretching but offer the **advantage of full automation**, since HE automatically determines a transformation function to produce a new image with a uniform histogram.

Histogram Equalization

- Goal: find a transform **s=T(r)** such that the transformed image has a flat (equalized) histogram.
- Where **T(r)** satisfies the following conditions:
  - **T(r)** is single-valued and monotonically increasing in interval [0,1];
  - $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$.

Histogram Equalization …

For discrete values, the probability of occurrence of gray level r_k in an image is given by:

$$P_r(r_k) = n_k / n$$

where k = 0, 1, …, L-1,

- n is the total no. of pixels in the image
- n_k is the no. of pixels with gray level r_k
- L is the total no. of possible gray levels in the image

Histogram Equalization …

The discrete transformation function is given by:

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_k)$$

$$= \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0, 1, \ldots, L-1$$

The o/p image is obtained by mapping each pixel with level r_k in the i/p image into a corresponding pixel with level s_k in the o/p image with the help of the above equation.

Histogram Equalization …

The following equations bring back the gray levels in the range [0, L-1]

Discrete values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= (L-1)\sum_{j=0}^{k} \frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad \text{k=0,1,..., L-1}$$

M – no. of rows

N – no. of columns

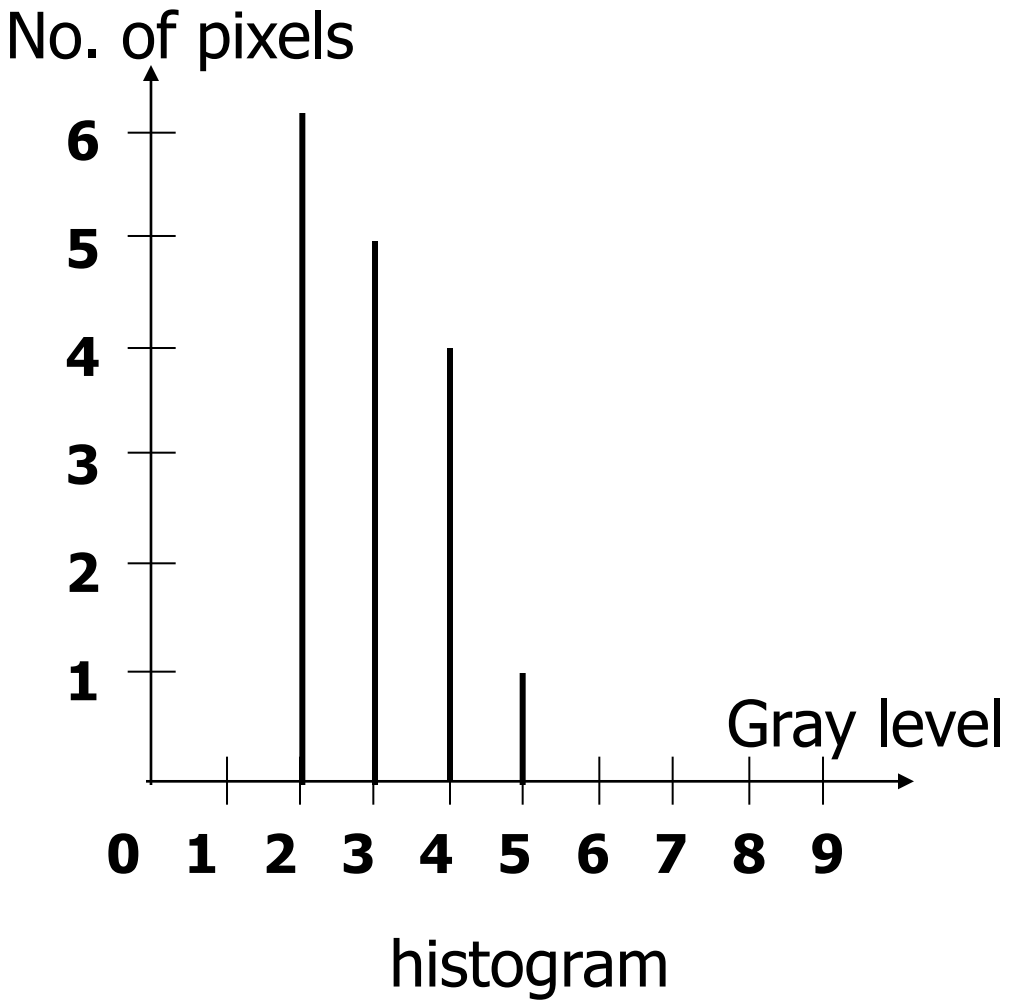
MN – total no. of pixels in the image

Histogram Equalization: Example 1

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



histogram

Histogram Equalization: Example 1…

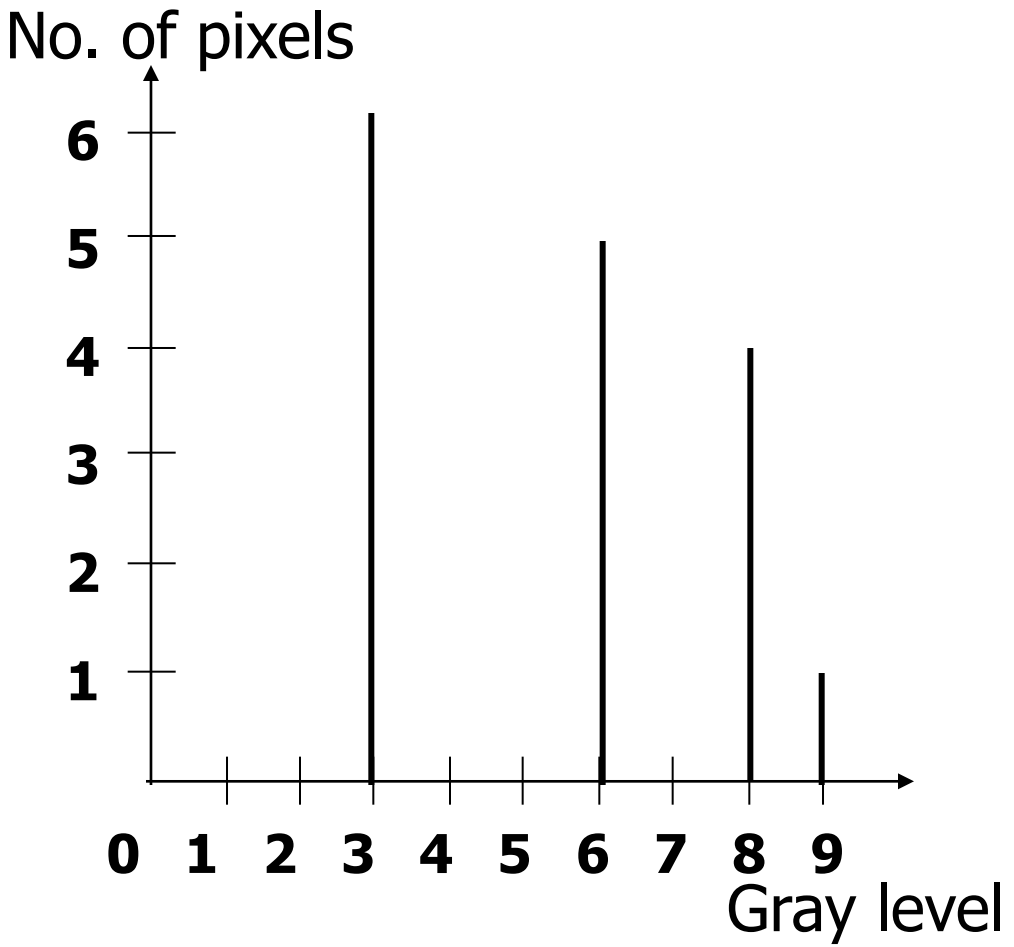
Gray Level(j)	0	1	2	3	4	5	6	7	8	9
No. of pixels	0	0	6	5	4	1	0	0	0	0
$\sum_{j=0}^{k} n_j$	0	0	6	11	15	16	16	16	16	16
$s = \sum_{j=0}^{k} \frac{n_j}{n}$	0	0	6 / 16	11 / 16	15 / 16	16 / 16	16 / 16	16 / 16	16 / 16	16 / 16
$s \times 9$	0	0	3.3 ~3	6.1 ~6	8.4 ~8	9	9	9	9	9

Histogram Equalization: Example 1…

3	6	6	3
8	3	8	6
6	3	6	9
3	8	3	8

Output image

Gray scale = [0,9]



Histogram equalization

Histogram Equalization: Example 2

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in following table. Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

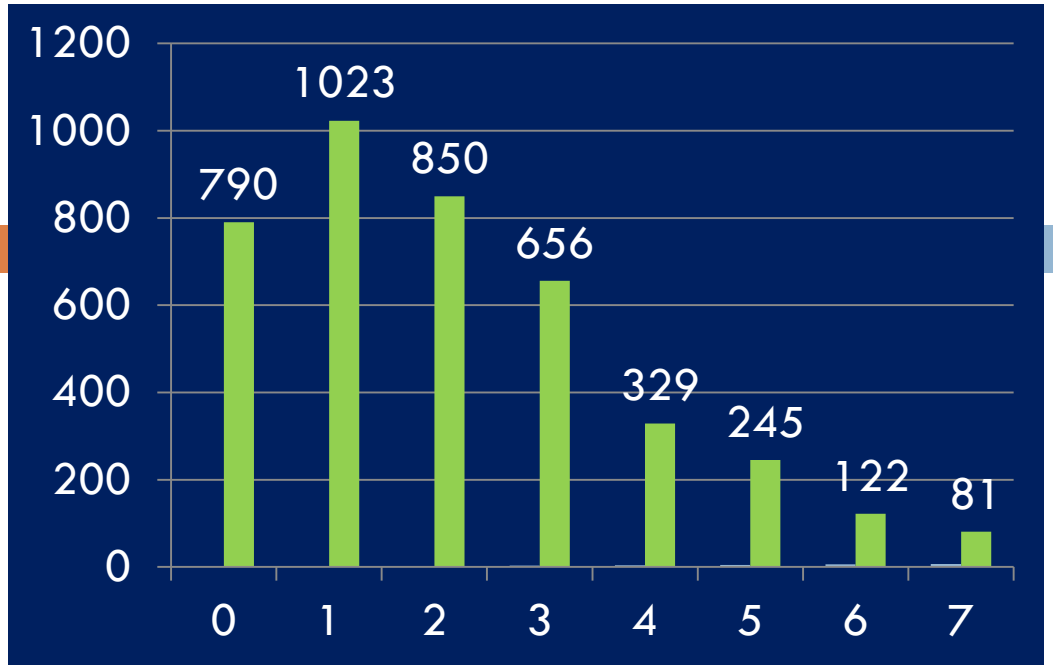
Histogram Equalization: Example 2 …

I/p Gray Level (r_k)	no. of pixels (n_k)	$p(r_k) = n_k/MN$	Σ	(L-1)Σ	O/p Gray Level (s)
0	790	0.19			
1	1023	0.25			
2	850	0.21			
3	656	0.16			
4	329	0.08			
5	245	0.06			
6	122	0.03			
7	81	0.02			

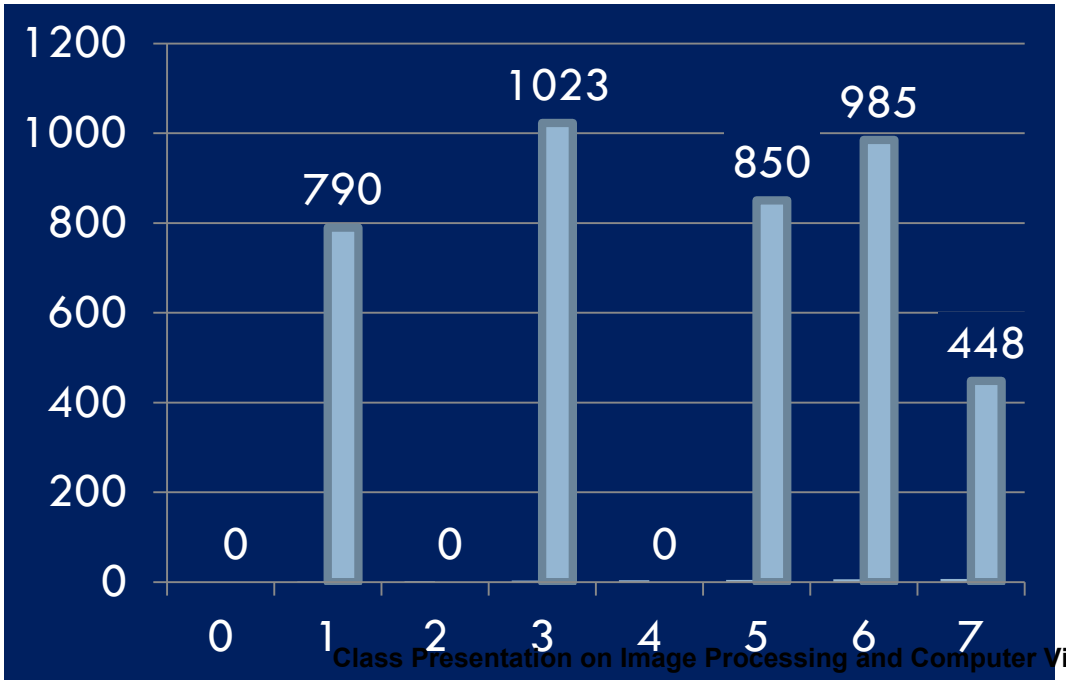
Grey level	n_k	$p_r(r_k)$	s_k
0	790	0.19	1
1	1023	0.25	3
2	850	0.21	5
3	656	0.16	6
4	329	0.08	6
5	245	0.06	7
6	122	0.03	7
7	81	0.02	7

What will be the histogram of the new image?

Equalized Grey level	n_k
0	0
1	790
2	0
3	1023
4	0
5	850
6	656 + 245 = 985
7	245 + 122 + 81 = 448

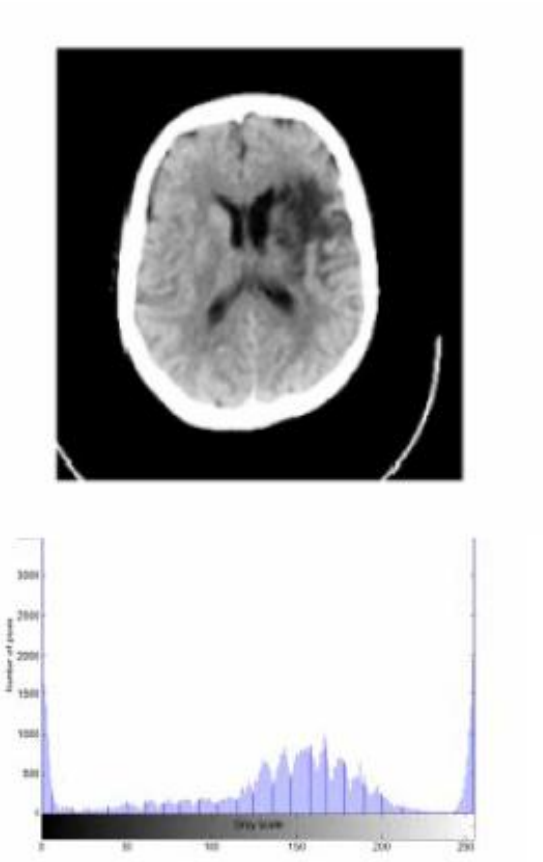


Histogram of the dark image

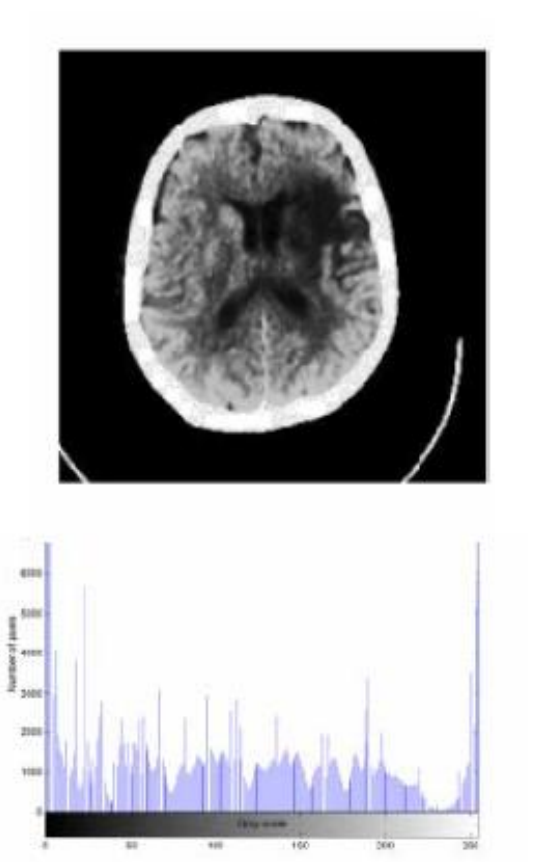


Equalized Histogram of the image

Histogram Equalization: Example

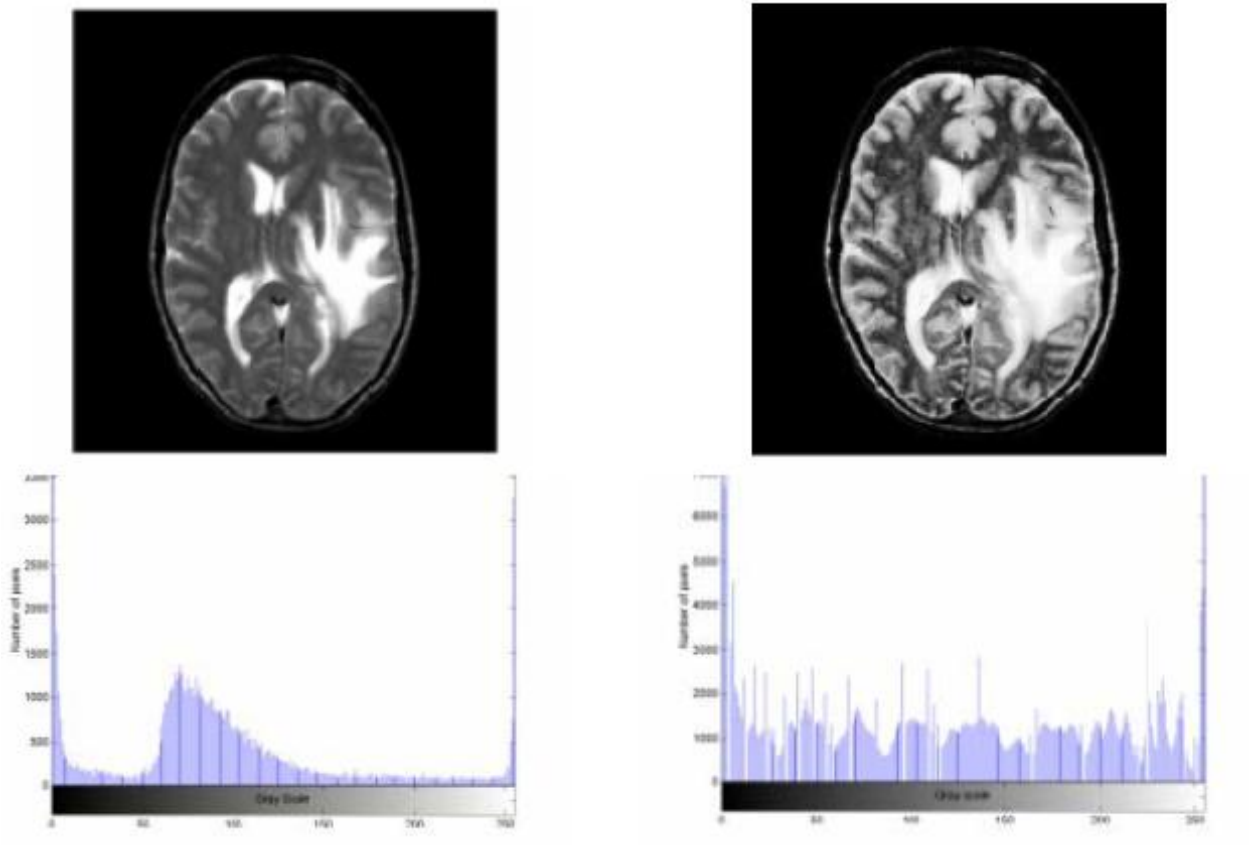


Before HE



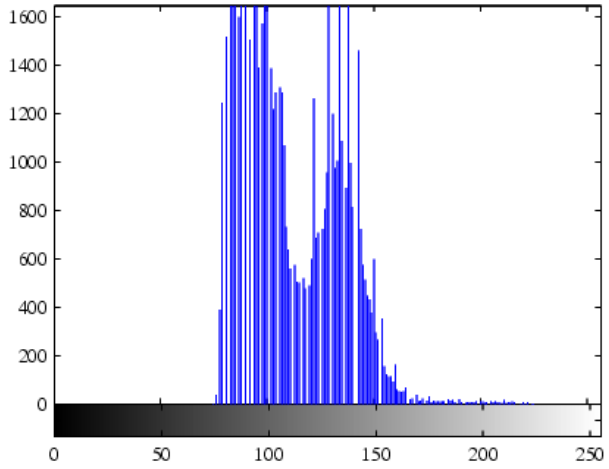
After HE

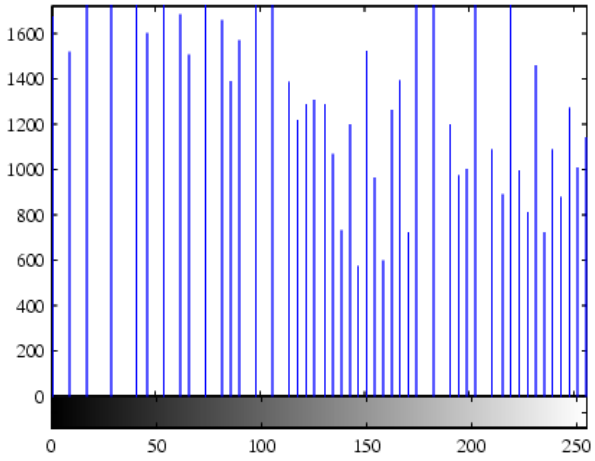
Histogram Equalization: Example



Before HE **After HE**

Histogram Equalization: Example





Histogram Matching (Specification)

- Histogram equalization has a disadvantage which is that it can generate only one type of output image.
- Sometimes it is useful to be able to specify the shape of the histogram that we wish the processed image to have.
- With Histogram Specification, we can specify the shape of the histogram that we wish the output image to have.
- It doesn't have to be a uniform histogram

Histogram Matching (Specification) …

For an image, whose enhancement is to be done, we are given an histogram, $G(z_k)$, that actually shows how the processed image's histogram should look after applying the transformation function to the i/p image.

$$s_k = T(r_k)$$

$$G(z_k) = \sum_{i=0}^{k} p_k(z_i) = s_k$$

$$z_k = G^{-1}(s_k) = G^{-1}[T(r_k)]$$

$k = 0, 1, \ldots, L-1$

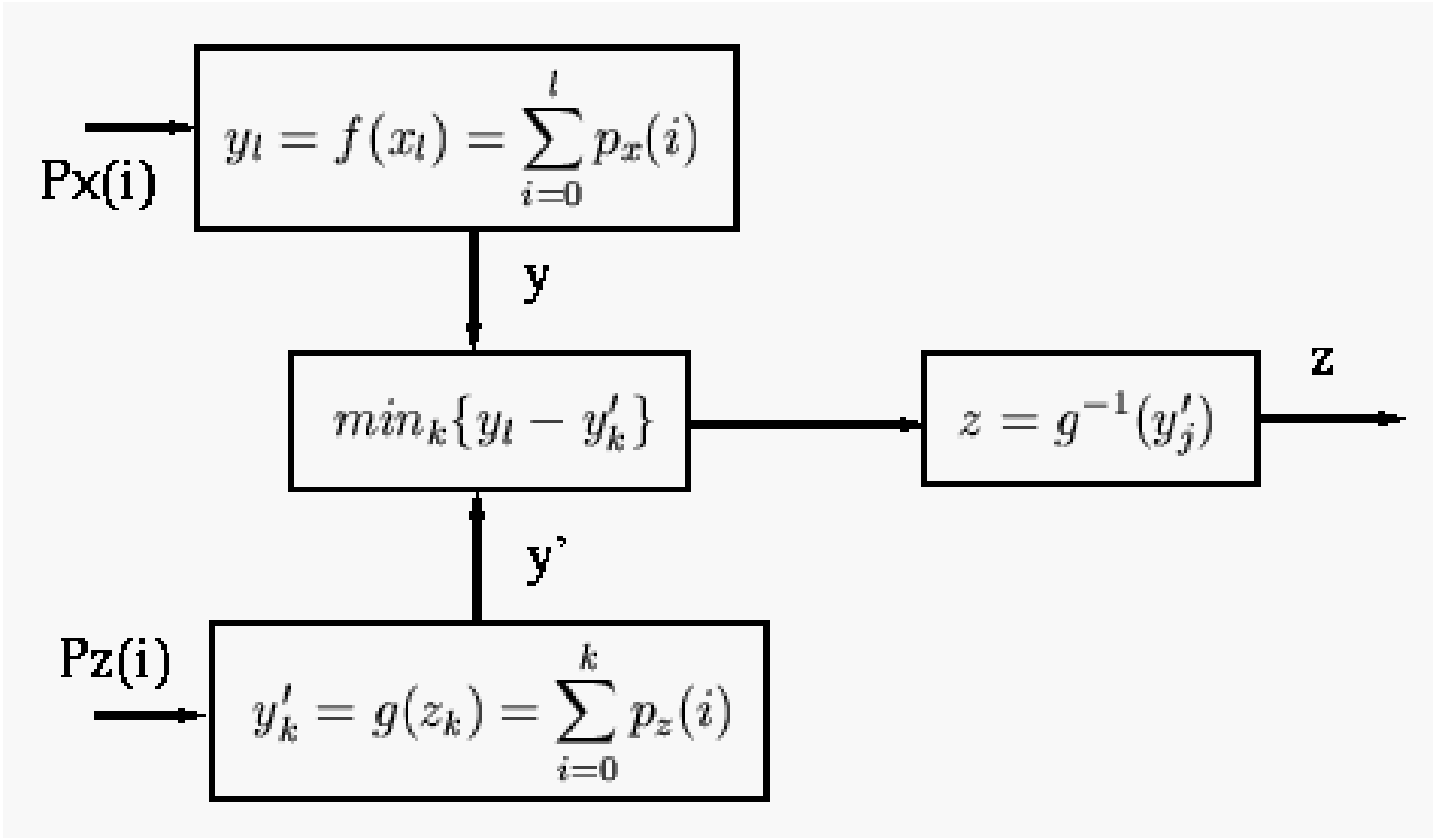
Histogram Matching (Specification) …

- Step 1: Find histogram of input image $p_r(r_j)$, and find histogram equalization mapping.
- Step 2: Specify the desired histogram, and find histogram equalization mapping.
- Step 3: Build lookup table:
  - For each gray level k, find s_k and then find a 'l' level so that s_k best matches z_l:

$$\min |s_k - z_l|$$

  - setup a lookup entry lookup[k]=l.

Histogram Matching: Discrete Cases



Histogram Matching: Example

- Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

We have already equalized the first histogram.

I/p Gray Level (r_k)	no. of pixels (n_k)	$p(r_k) = n_k/MN$	Σ	$(L-1)\Sigma$	O/p Gray Level (s)
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1.00	7.00	7

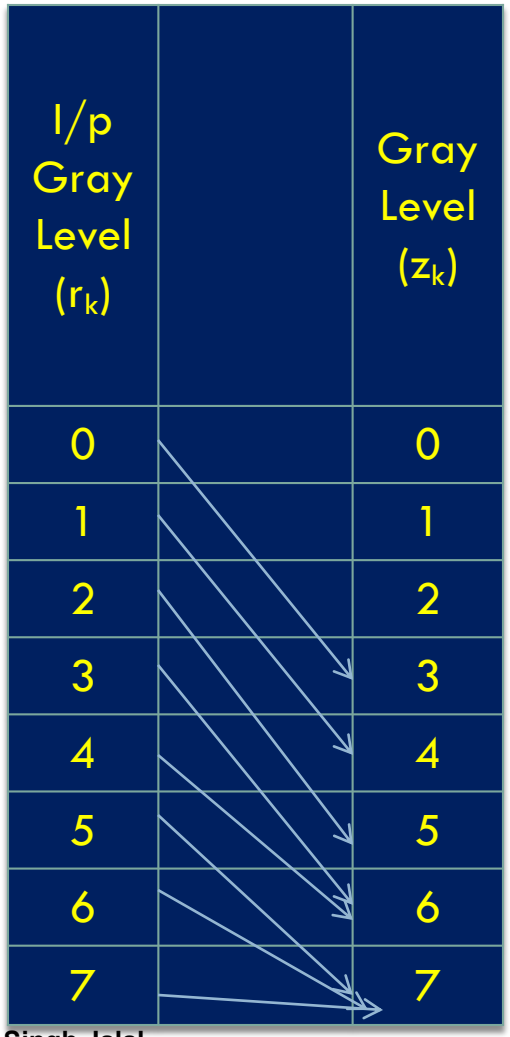
Now equalize the second histogram.

Gray Level (z_k)	$p(z_k) = n_k/MN$	Σ	$(L-1)\Sigma$	O/p Gray Level $G(z_k)$
0	0.00	0	0	0
1	0.00	0	0	0
2	0.00	0	0	0
3	0.15	0.15	1.05	1
4	0.20	0.35	2.45	2
5	0.30	0.65	4.55	5
6	0.20	0.85	5.95	6
7	0.15	1	7	7

And now we do the matching

I/p Gray Level (r_k)	O/p Gray Level (s)
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

O/p Gray Level $G(z_k)$	Gray Level (z_k)
0	0
0	1
0	2
1	3
2	4
5	5
6	6
7	7



Histogram Matching: Example

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

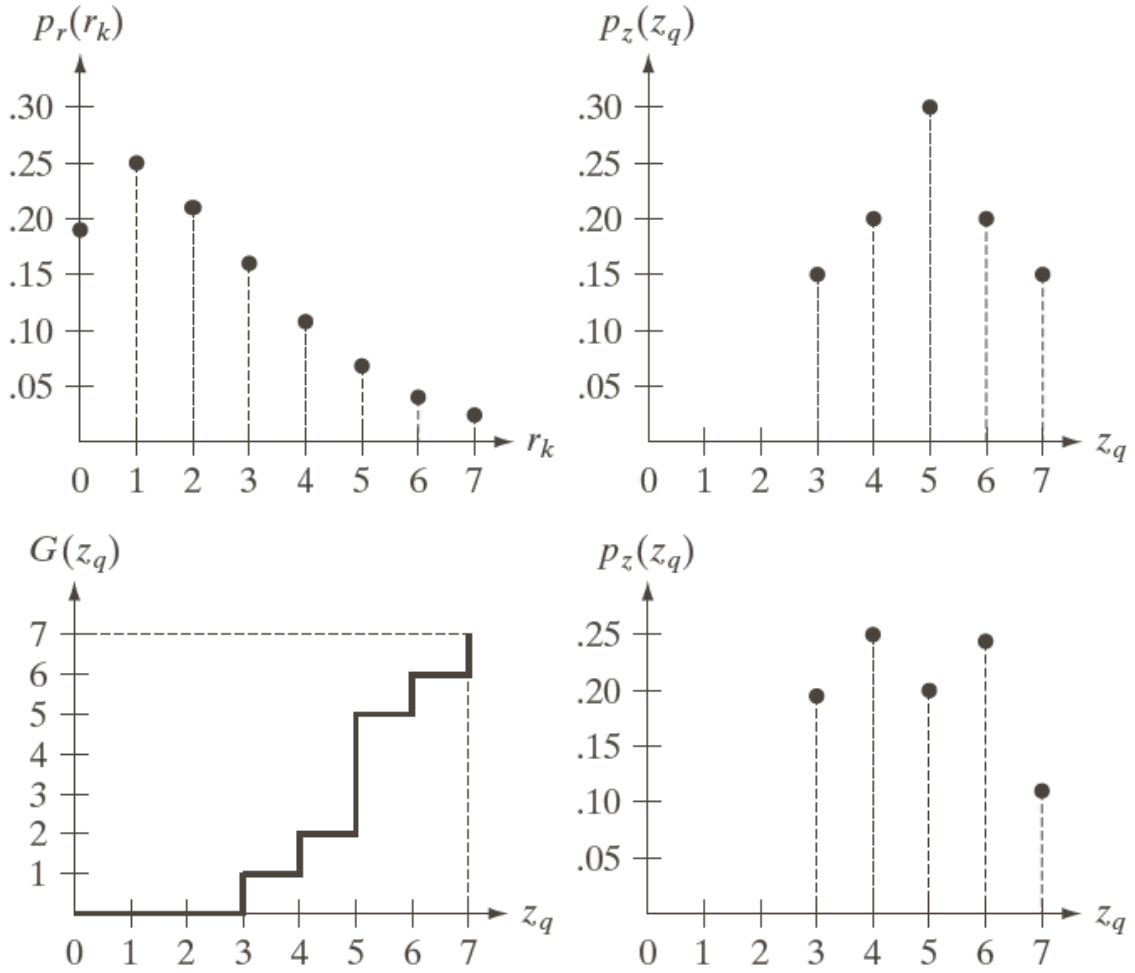
$$4 \rightarrow 6$$

$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

$$7 \rightarrow 7$$

Histogram Matching: Example

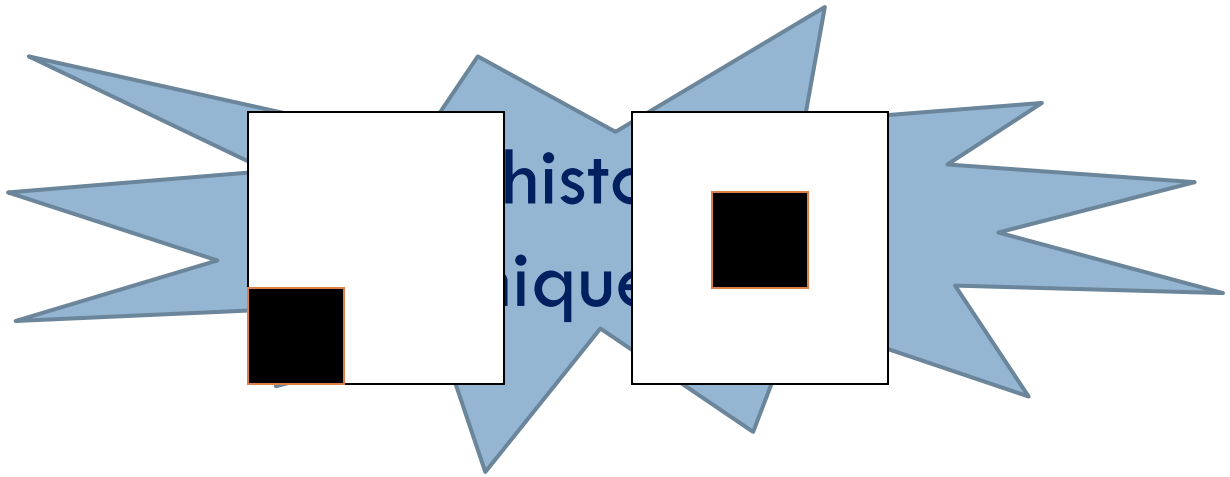


a b
c d

(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Local Histogram Processing

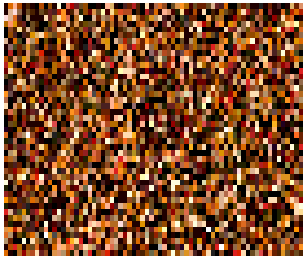
Histogram processing yields robust image processing results



Histograms of both the images are same !!

Local Histogram Processing …

- Q: What happens if I reshuffle all pixels within the image?



- A: It's histogram won't change. No point processing will be affected…
- Need spatial information to capture this.

Local Histogram Processing …

- Transformation should be based on gray-level distribution in the neighborhood of every pixel.
- Local histogram processing:
  - At each location the histogram of the points in the neighborhood is computed and a histogram equalization or histogram specification transformation function is obtained
  - The gray level of the pixel centered in the neighborhood is mapped
  - The center of the neighborhood is moved the next pixel and the procedure repeated

Assignment

- Construct the histogram of the following image.

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15

- What will be the effect on the histogram if we set to zero the
  - Higher order bit plane?
  - Lower order bit plane ?
- Take your grey scale image & construct its histogram.

Assignment

Q. Equalize the following histogram and give the resultant histogram.

Gray levels (rk)	0	1	2	3	4	5	6	7
Number of pixels (nk)	50	100	100	300	200	200	50	0

Q. Perform histogram specification on the 8x8 image. The gray level distribution of the image is given below:

Gray levels (rk)	0	1	2	3	4	5	6	7
Number of pixels (nk)	8	10	10	2	12	16	4	2

Gray levels (rk)	0	1	2	3	4	5	6	7
Number of pixels (nk)	0	0	0	0	20	20	16	8

Assignment

Match the histogram A to the histogram B.

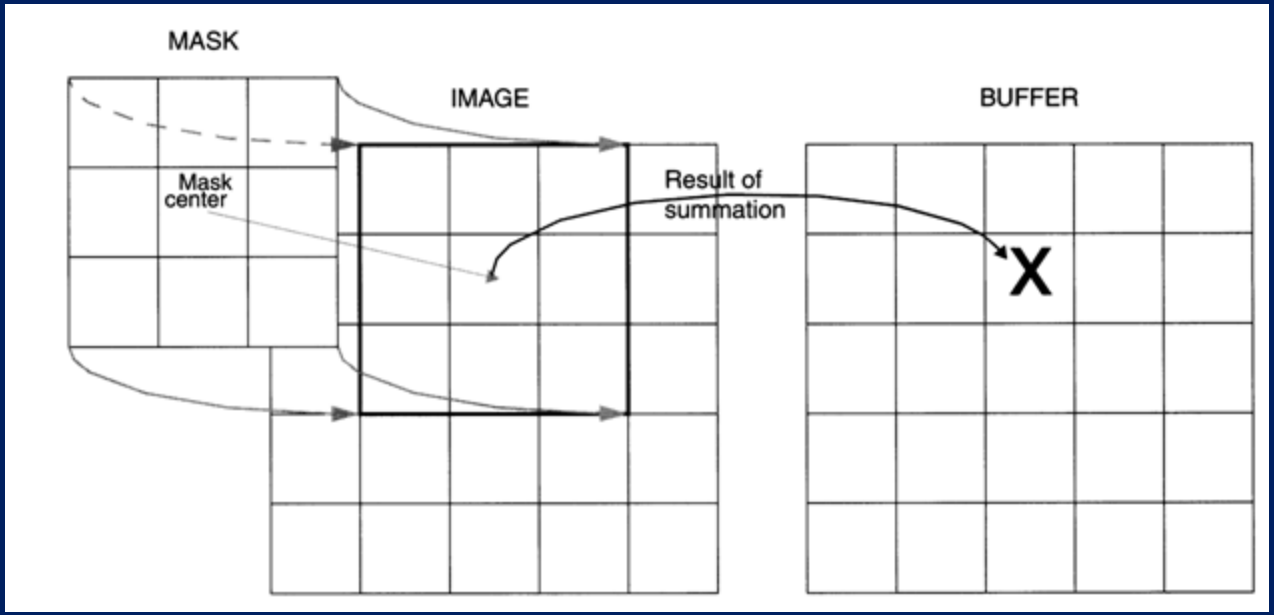
Histogram A

Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	4	17	15	18	24	12	0	10

Histogram B

Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	36	24	12	8	20

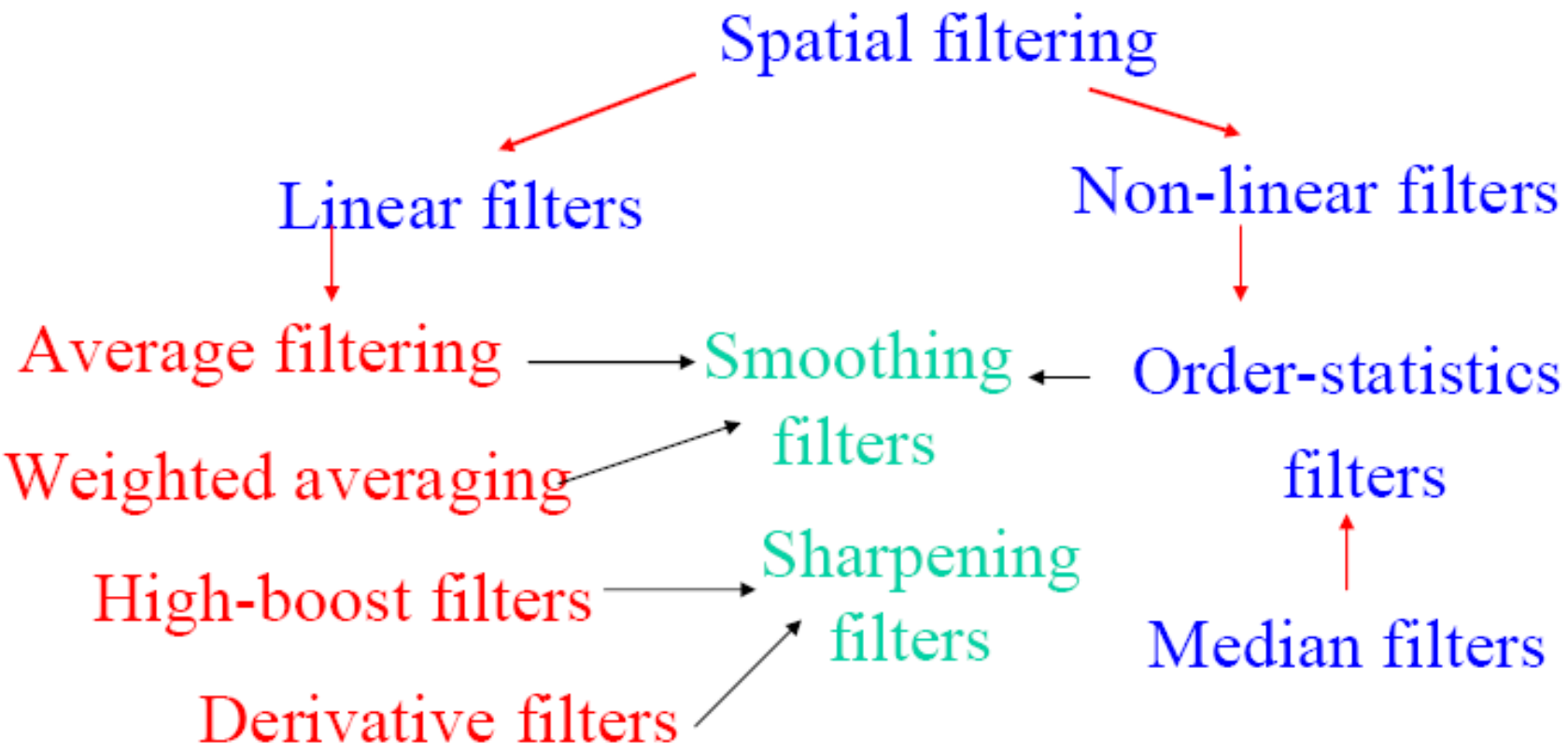
Spatial Filtering



Spatial Filtering

- Enhancement techniques based on the pixels of an image are often referred to as Spatial filtering
  - Working in a neighbourhood of every pixel in an image
- Filter term in "Digital image processing" is referred to the subimage
- There are others term to call subimage such as mask, kernel, template, or window
- The value in a filter subimage are referred as coefficients, rather than pixels.

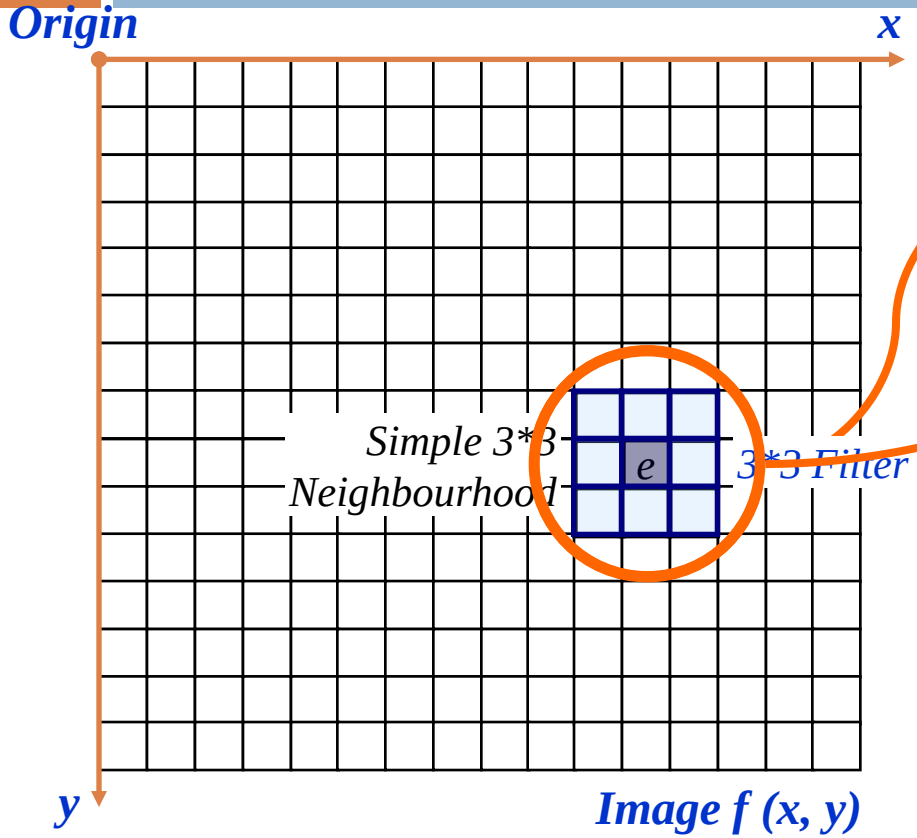
Spatial Filtering



Mechanics of Spatial filtering

- The process consists simply of moving the filter mask from point to point in an image.
- At each point (x, y) the response of the filter at that point is calculated using a predefined relationship.
- If the operation performed on the pixel is linear, then the filter is called **linear spatial filter**.

Mechanics of Spatial filtering



a	b	c
d	e	f
g	h	i

Original Image Pixels

*

r	s	t
u	v	w
x	y	z

Filter

$$e_{processed} = v*e + r*a + s*b + t*c + u*d + w*f + x*g + y*h + z*i$$

The above is repeated for every pixel in the original image to generate the filtered image

Vector Representation of Linear Filtering

- Simply move the filter mask from point to point in an image.
- At each point (x,y), the response of the filter at that point is calculated using a predefined relationship.

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$

$$= \sum_{i=i}^{mn} w_i z_i$$

Spatial Filtering Process …

In general, linear filtering of an image f of size M x N with a filter mask of size m x n is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

where (as per the previous discussion)

a = (m – 1) / 2 & b = (n – 1) / 2

To generate a complete filtered image, the above equation must be applied for

x = 0, 1, …, M-1 & y = 0, 1, …, N-1

Correlation Vs Convolution

Two closely related concepts

- Correlation

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

- Convolution

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x-s, y-t)$$

Find the result of Correlation & Convolution for the following image f and 1x5 mask w.

f

0	0	0	1	0	0	0	0

w

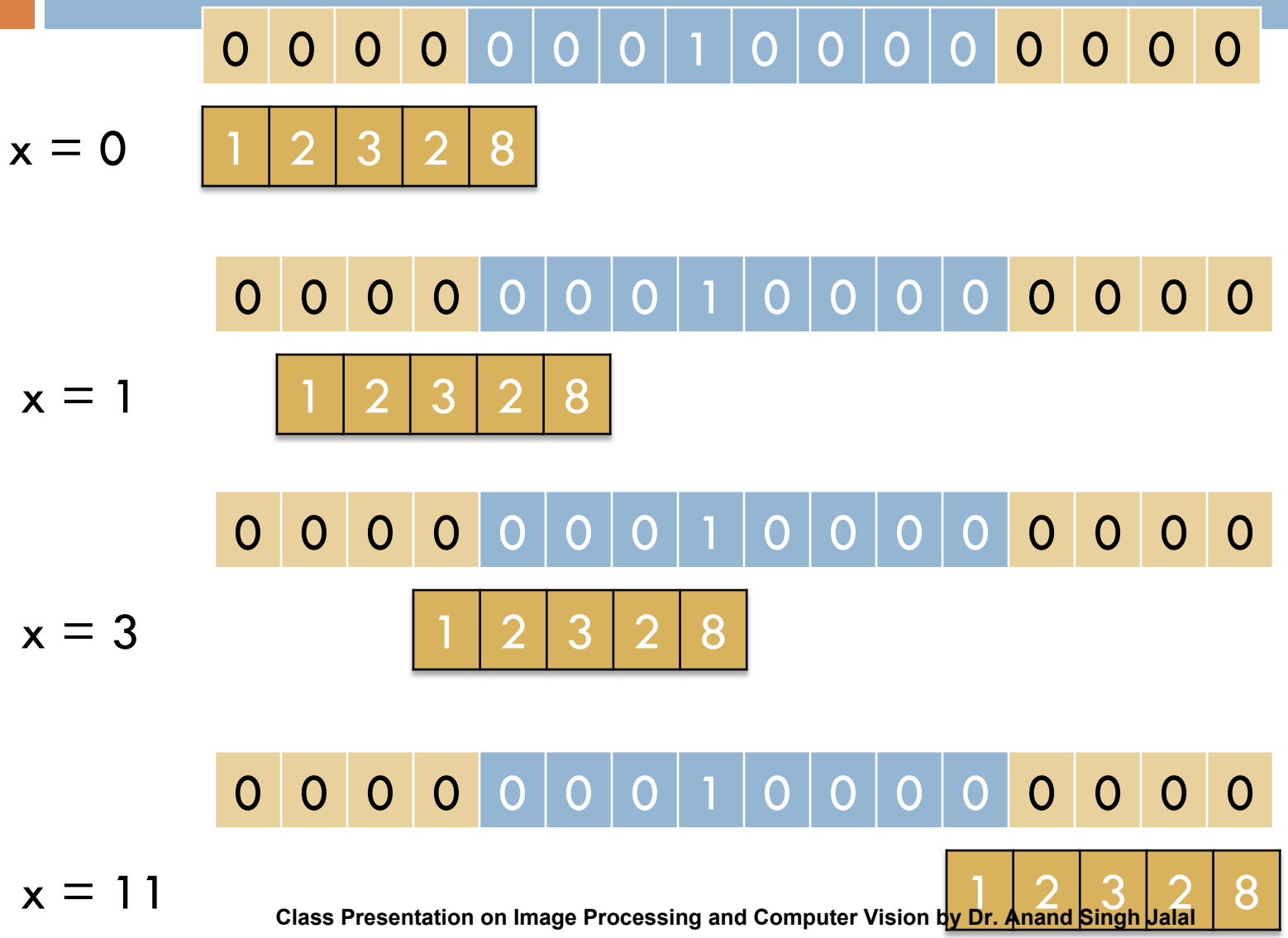
1	2	3	2	8

Origin

Note:
To allow every pixel in w to visit every pixel in f, pad f with enough 0s on either side.
If the filter is of size m, then the no. of 0s to be added are ???

m-1

Correlation



Correlation

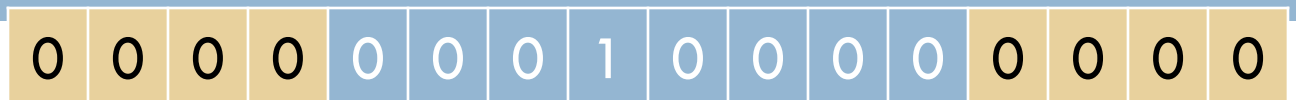
Result of full correlation

0	0	0	8	2	3	2	1	0	0	0	0	0	0	0	0

Result of cropped correlation

0	8	2	3	2	1	0	0

Convolution



x = 0

x = 1

Result of full convolution

Result of cropped convolution

Smoothing Spatial Filters

- Used for blurring and for noise reduction
- Blurring is used in preprocessing steps, such as
  - removal of small details from an image prior to object extraction
  - bridging of small gaps in lines or curves
- noise reduction can be accomplished by blurring with a linear filter and also by a nonlinear filter
- There are 2 types of smoothing spatial filters
  - Smoothing Linear Filters
  - Order-Statistics Filters

Smoothing Linear Filters

- Output is simply the average of the pixels contained in the neighborhood of the filter mask.
- Also Called **Averaging filters** or **Lowpass filters**.
- Replacing the value of every pixel in an image by the average of the gray levels in the neighborhood will reduce the "sharp" transitions in gray levels.
- Sharp transitions
  - **random noise in the image**
  - **edges of objects in the image**

Thus, smoothing can reduce noises (desirable) and blur edges (undesirable)

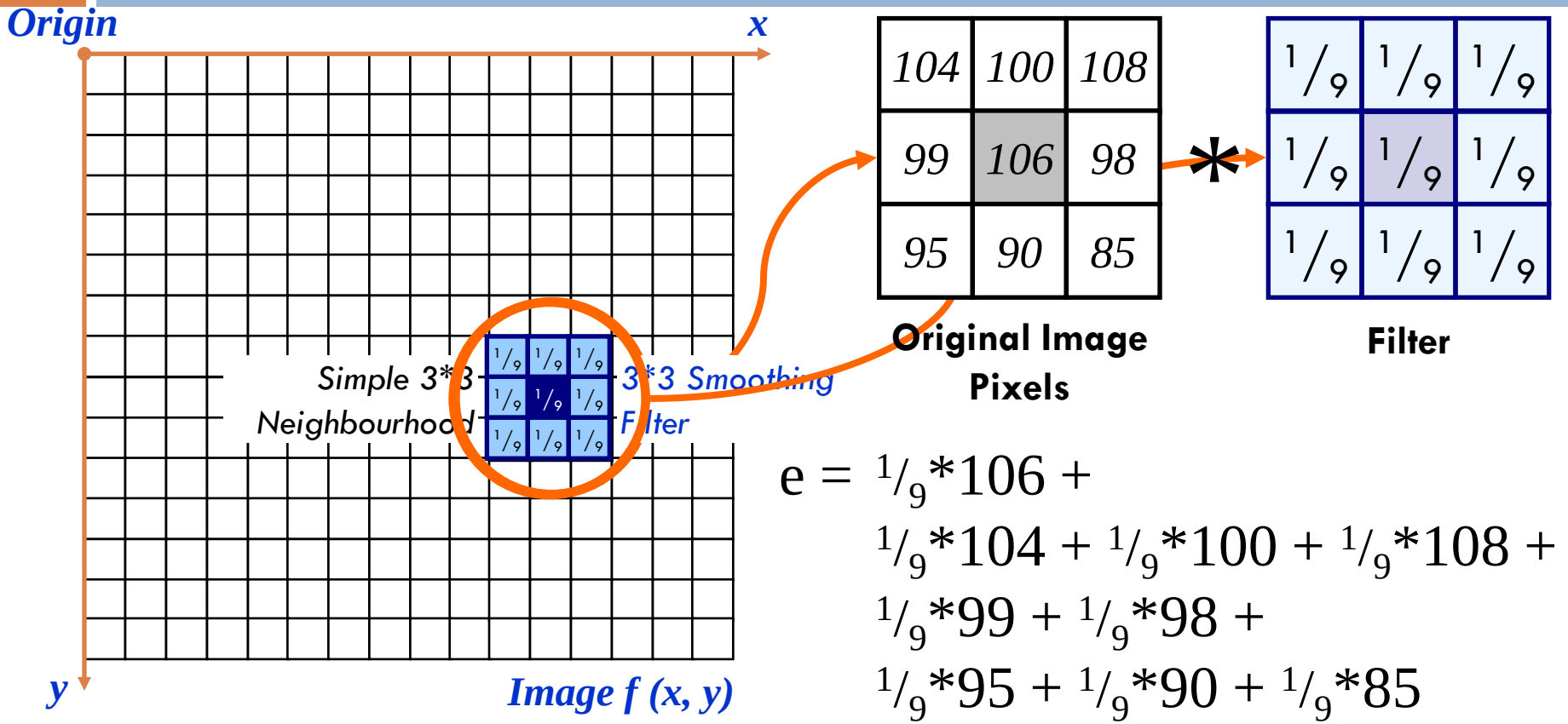
Smoothing Linear Filters …

- One of the simplest spatial filtering operations we can perform is a smoothing operation
  - Simply average all of the pixels in a neighbourhood around a central value
  - Especially useful in removing noise from images
  - Also useful for highlighting gross detail

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

Simple averaging filter

Smoothing Linear Filters …



$$e = \tfrac{1}{9}*106 + \tfrac{1}{9}*104 + \tfrac{1}{9}*100 + \tfrac{1}{9}*108 + \tfrac{1}{9}*99 + \tfrac{1}{9}*98 + \tfrac{1}{9}*95 + \tfrac{1}{9}*90 + \tfrac{1}{9}*85$$

$$= 98.3333$$

The above is repeated for every pixel in the original image to generate the smoothed image

3x3 Smoothing Linear Filters

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

Standard Average

weighted average

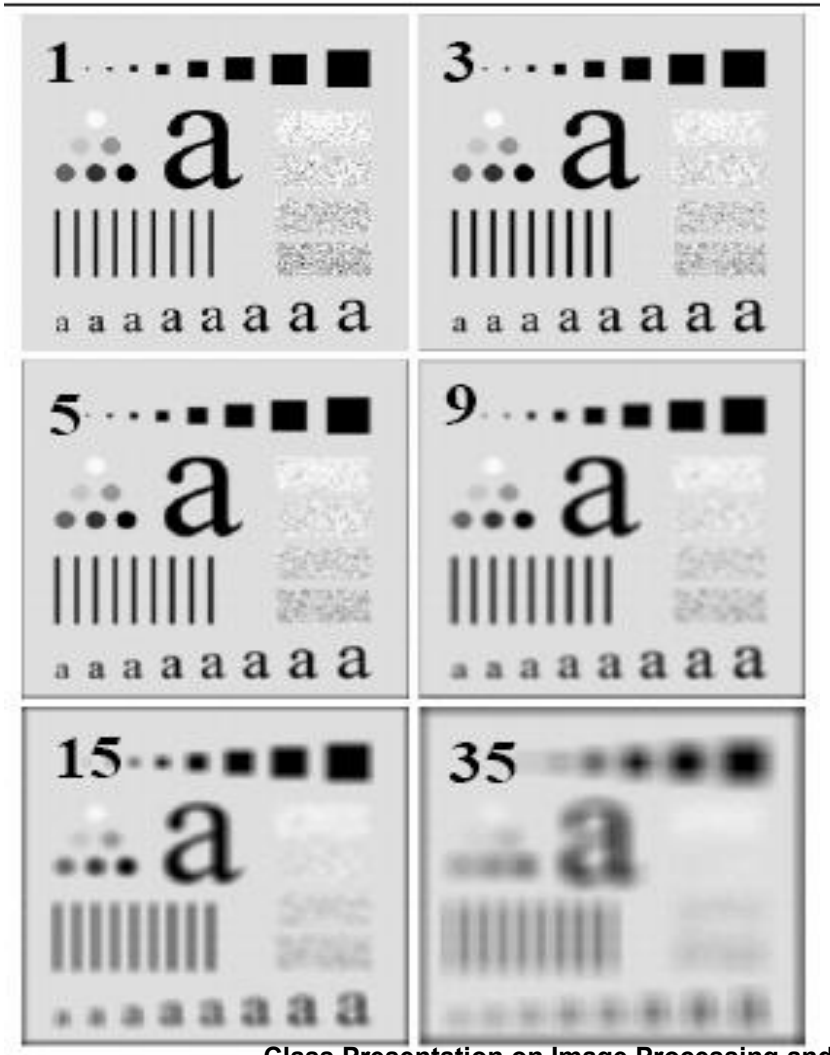
the center is the most important and other pixels are inversely weighted as a function of their distance from the center of the mask

Smoothing Linear Filters

- The general implementation for filtering an M x N image with a weighted averaging filter of size m x n is given by the expression

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

Smoothing Linear Filters: Example



- Original image of 500 x 500 pixels
- Results of smoothing with square averaging filter masks of size m=3, 5, 9,15 and 35 respectively
- Note:
  - **big mask is used to eliminate small objects from an image.**
  - **the size of the mask establishes the relative size of the objects that will be blended with the background.**

Weighted Smoothing Filters

- More effective smoothing filters can be generated by allowing different pixels in the neighbourhood different weights in the averaging function
  - Pixels closer to the central pixel are more important
  - Often referred to as a *weighted averaging*

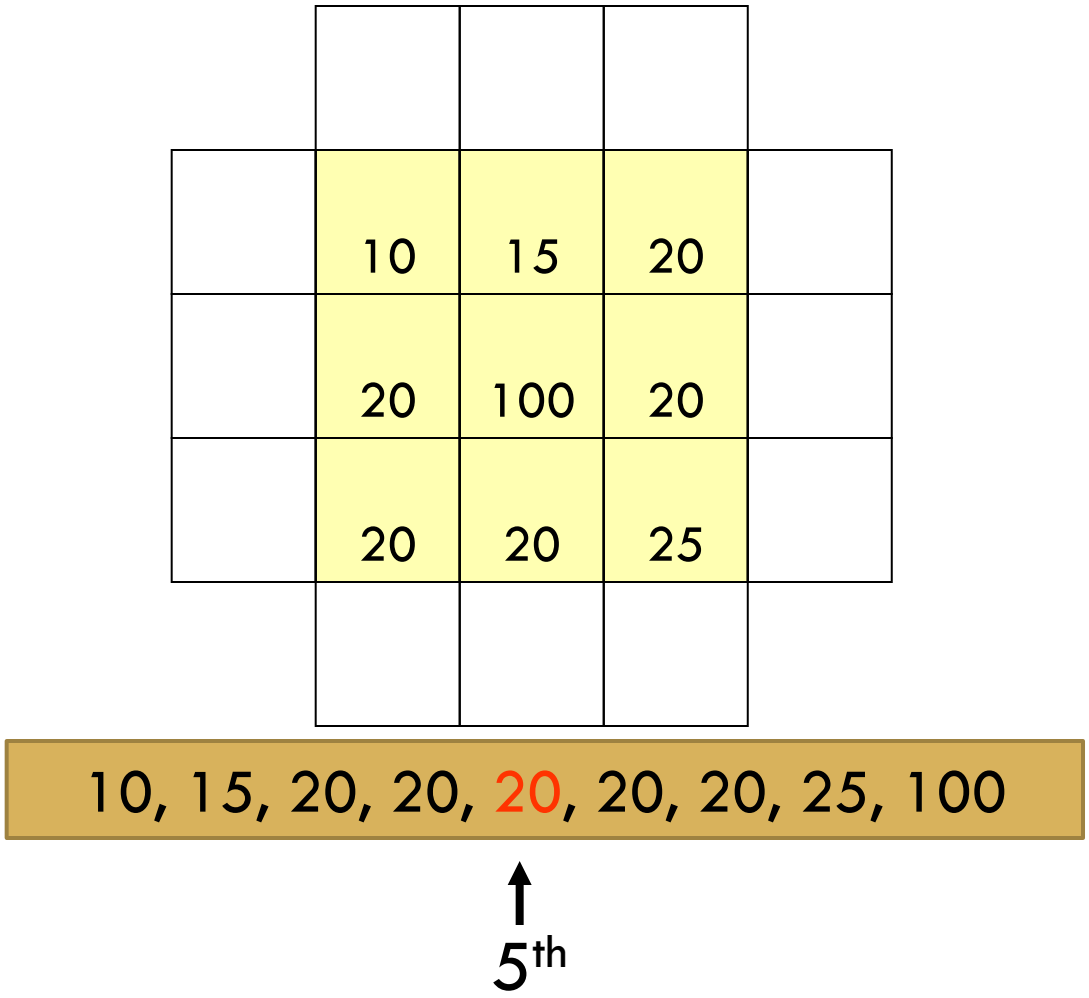
$^1/_{16}$	$^2/_{16}$	$^1/_{16}$
$^2/_{16}$	$^4/_{16}$	$^2/_{16}$
$^1/_{16}$	$^2/_{16}$	$^1/_{16}$

Weighted averaging filter

Order-Statistics Filters

- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter,
- Example
  - median filter : $R = \text{median}\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - max filter : $R = \max\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - min filter : $R = \min\{z_k \mid k = 1,2,\ldots,n \times n\}$
- note: n x n is the size of the mask

Process of Median filter

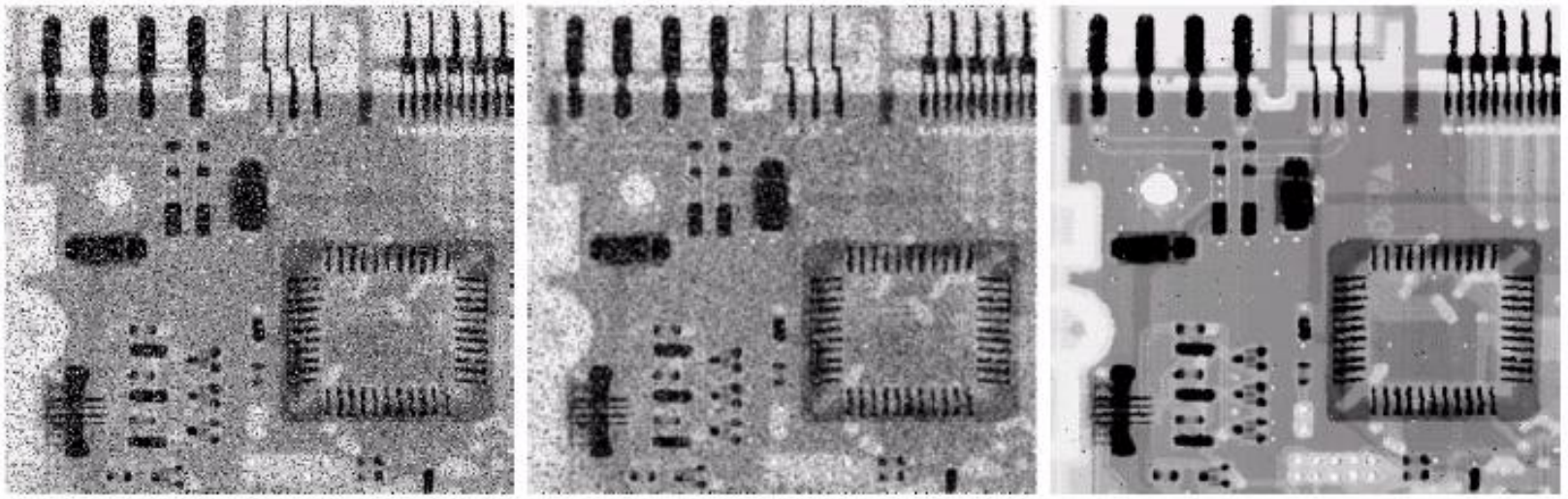


- Crop region of neighborhood
- Sort the values of the pixel in the region
- In the M x N mask the median is (M x N div 2) +1

Median Filtering

- Median filtering is particularly effective in the presence of impulse noise (salt and pepper noise).
- Unlike average filtering, median filtering does not blur too much image details.
- Advantages:
  - Removes impulsive noise
  - Preserves edges
- Disadvantages:
  - performance poor when # of noise pixels in the window is greater than 1/2 # in the window
  - performs poorly with Gaussian noise

Median Filtering: Example



a) X-ray image of circuit board corrupted by salt-pepper noise
b) Noise reduction with a 3 x 3 averaging mask
c) Noise reduction with a 3 x 3 median filter

Median Filtering: Example

Noise reduction with a 3 x 3 median filter

Sharpening Spatial Filters

- The principal objective of sharpening is to **highlight fine detail in an image** or **to enhance detail that has been blurred**, either in error or as a natural effect of a particular method of image acquisition.
- First and second order derivatives are commonly used for sharpening:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Blurring vs. Sharpening

- As we know that blurring can be done in spatial domain by **pixel averaging** in a neighbors
- since **averaging is analogous to integration**
- Therefore, logically it can be concluded that the sharpening must be accomplished by **spatial differentiation.**

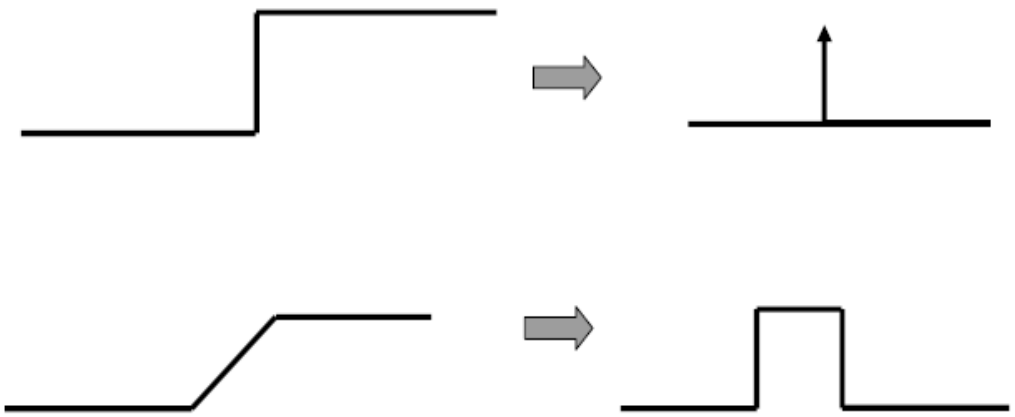
Derivative operator

- The strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.
- thus, image differentiation
  - enhances edges and other discontinuities (noise)
  - deemphasizes area with slowly varying gray-level values.

First-order derivative

- a basic definition of the first-order derivative of a one-dimensional function f(x) is the difference

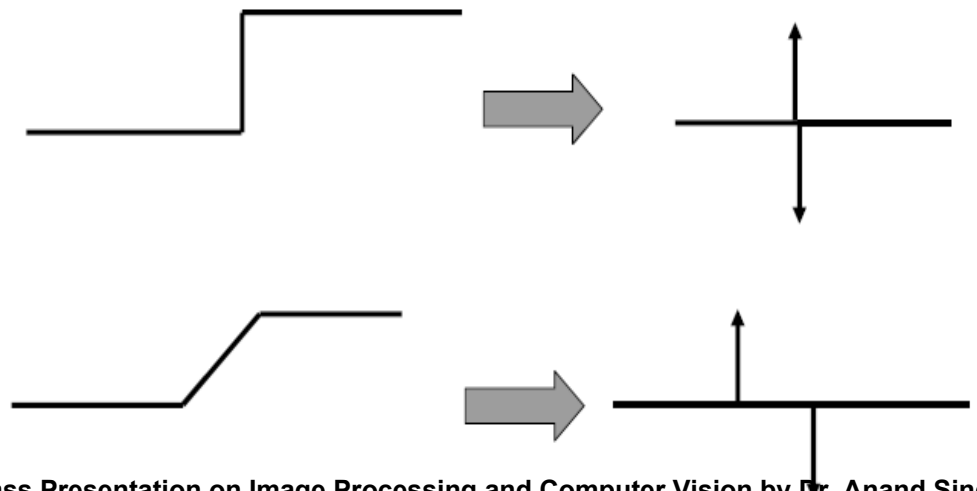
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$



Second-order derivative

- similarly, we define the second-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



Response of First and Second order derivatives

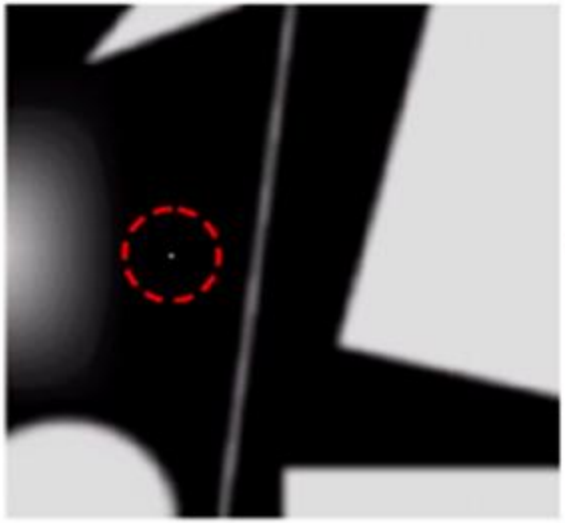
Response of first order derivative is:

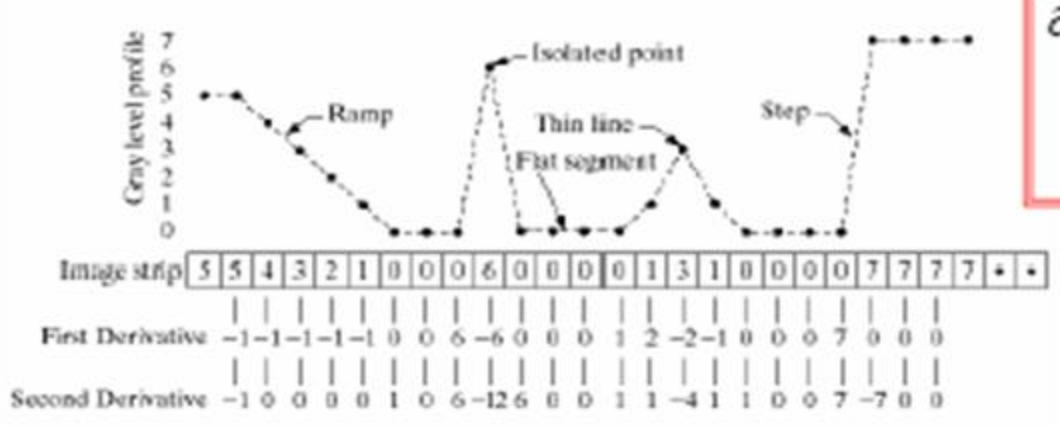
- zero in flat segments (area of constant grey values)
- Non zero at the onset of a grey level step or ramp
- Non zero along ramps

Response of second order derivative is:

- Zero in flat areas
- Non zero at the onset of a grey level step or ramp
- Zero along ramps of constant slope

First and Second Order Derivatives: Example

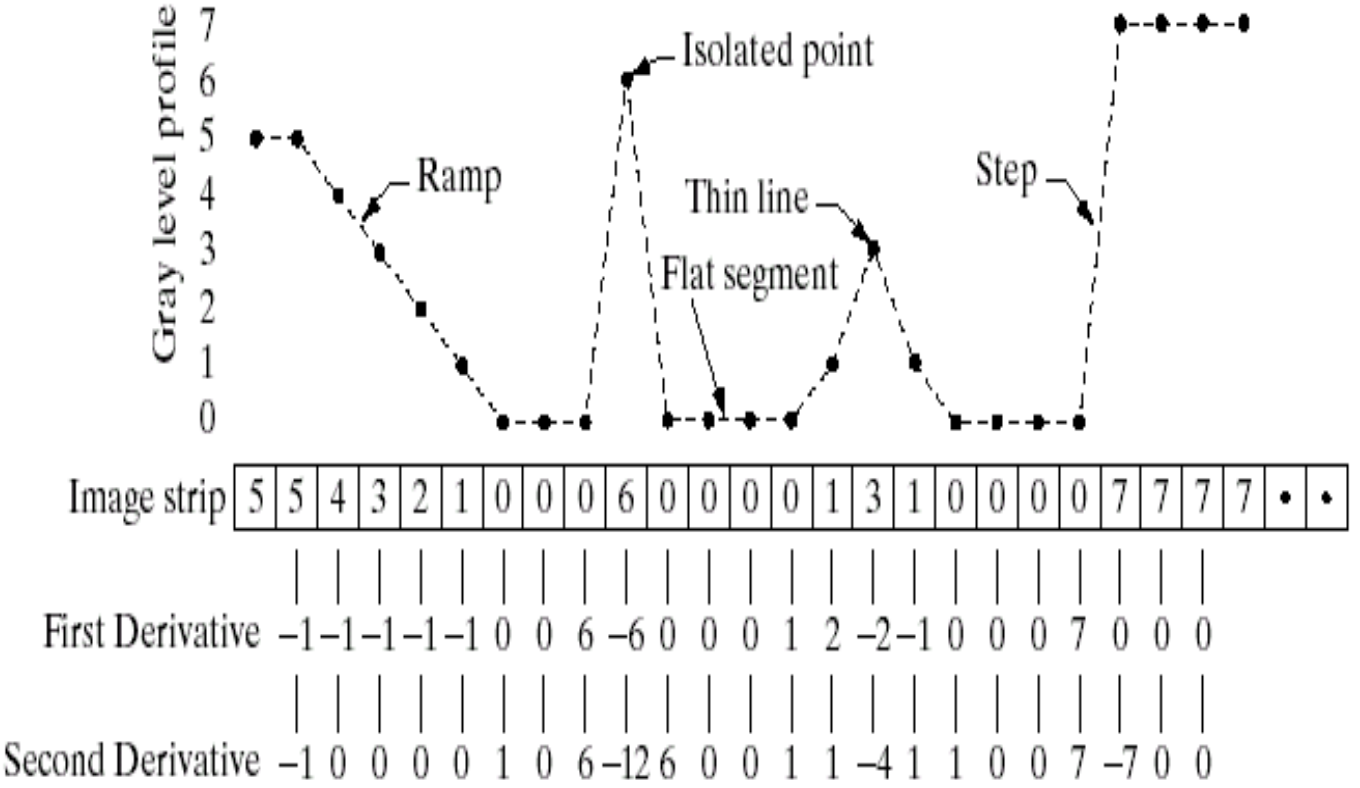




$$\frac{\partial f}{\partial x} = f_x(x) = f(x+1) - f(x)$$

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= f_x(x) - f_x(x-1) \\ &= \big(f(x+1) - f(x)\big) - \big(f(x) - f(x-1)\big) \\ &= f(x+1) + f(x-1) - 2\cdot f(x)\end{aligned}$$

First and Second Order Derivatives: Example …



First and Second Order Derivatives …

- **Analysis**

1) The 1st-order derivative is nonzero along the entire ramp, while the 2nd-order derivative is nonzero only at the onset and end of the ramp.
2) Edges in an image represent this type (ramp) of transition. Therefore,
   1st make thick edge and 2nd make thin, much finer edges
3) The response at and around the point is much stronger for the 2nd- than for the 1st-order derivative.

1st and 2nd Derivative Comparison

- First-order derivatives generally produce thicker edges in an images.
- Second-order derivatives have a stronger response to fine detail (e.g. thin lines or isolated points).
- First-order derivatives generally have a stronger response to a gray-level step.
- Second-order derivatives produce a double response at step changes in gray level

Derivative Operator …

f(x-1,y)	f(x,y)	f(x+1,y)

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

f(x,y-1)
f(x,y)
f(x,y+1)

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

The Laplacian

(Use of Second derivative for Enhancement)

- The filter is expected to be isotropic: response of the filter is independent of the direction of discontinuities in an image.
- Simplest 2-D isotropic second order derivative is the Laplacian:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

2-Dimensional Laplacian

The digital implementation of the 2-Dimensional Laplacian is obtained by summing 2 components

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

Laplacian Masks

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ 90° *isotropic*

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ 45° *isotropic*

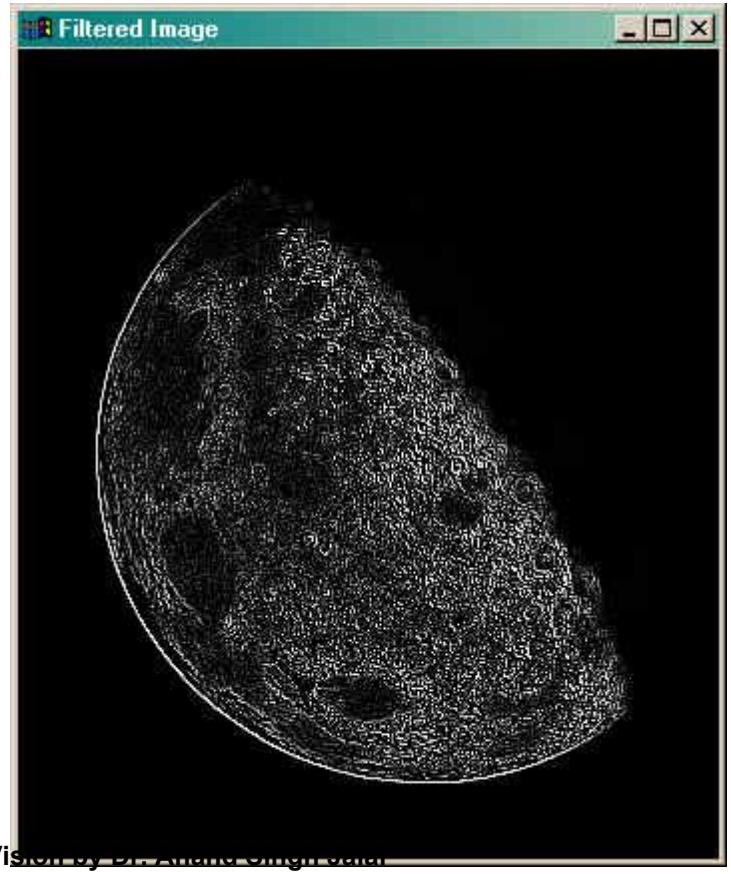
0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

Effect of Laplacian Operator

- **as it is a derivative operator,**
  - it highlights gray-level discontinuities in an image
  - it deemphasizes regions with slowly varying gray levels
- **tends to produce images that have**
  - grayish edge lines and other discontinuities, all superimposed on a dark,
  - featureless background.

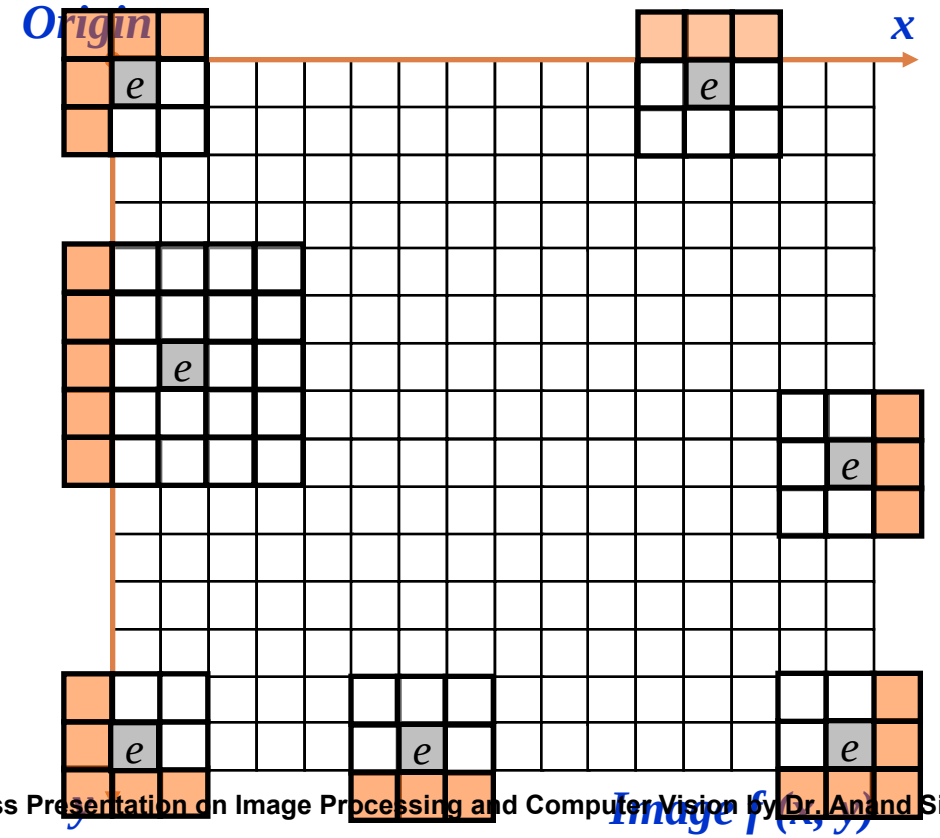
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Image of the north pole of the moon



Strange Things Happen At The Edges!

At the edges of an image we are missing pixels to form a neighbourhood



Strange Things Happen At The Edges …

- There are a few approaches to dealing with missing edge pixels:
  - Omit missing pixels
    - Only works with some filters
    - Can add extra code and slow down processing
  - Pad the image
    - Typically with either all white or all black pixels
  - Replicate border pixels

Using Fuzzy techniques for intensity transformations and spatial filtering

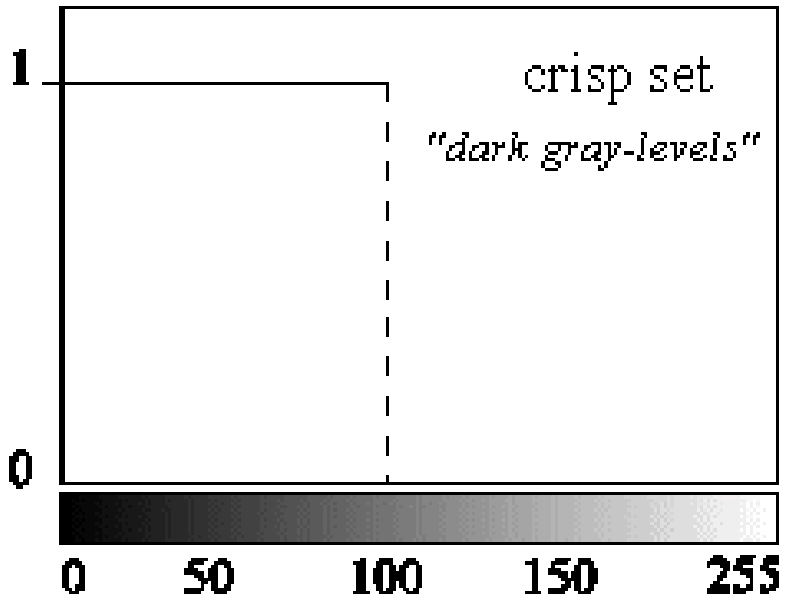
What is Fuzzy Set Theory?

- Fuzzy set theory is the extension of conventional (crisp) set theory.
- It handles the concept of partial truth (values between 1 (completely true) and 0 (completely false)).
- It was introduced by **Prof. Lotfi A. Zadeh** of UC/Berkeley in 1965 as a means to model the vagueness and ambiguity in complex systems.

Fuzzy Set …

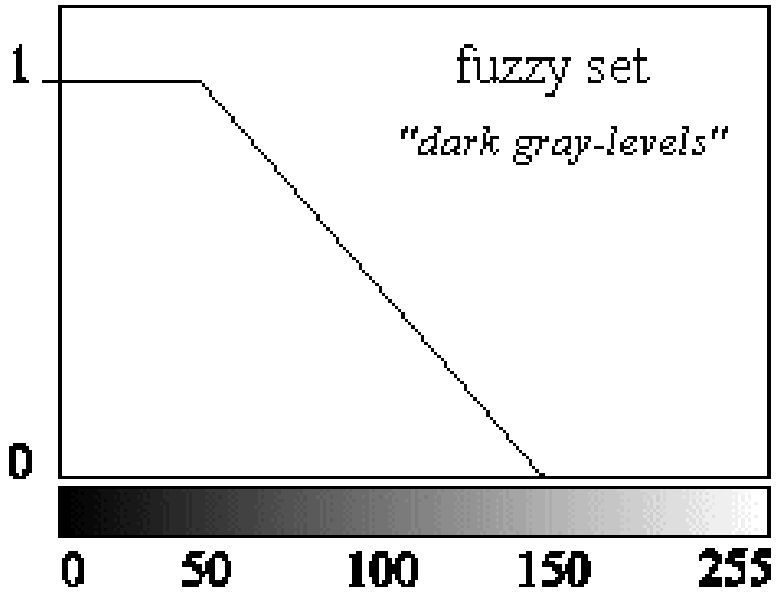
- The idea of fuzzy sets is simple and natural. For instance, we want to define a set of gray levels that share the property dark.

In classical set theory, we have to determine a threshold, say the gray level 100. All gray levels between 0 and 100 are element of this set, the others do not belong to the set



Fuzzy Set …

- But the darkness is a matter of degree.
- So, a fuzzy set can model this property much better. To define this set, we also need two thresholds, say gray levels 50 and 150.
- All gray levels that are less than 50 are the full member of the set,
- All gray levels that are greater than 150 are not the member of the set.
- The gray levels between 50 and 150, however, have a partial membership in the set

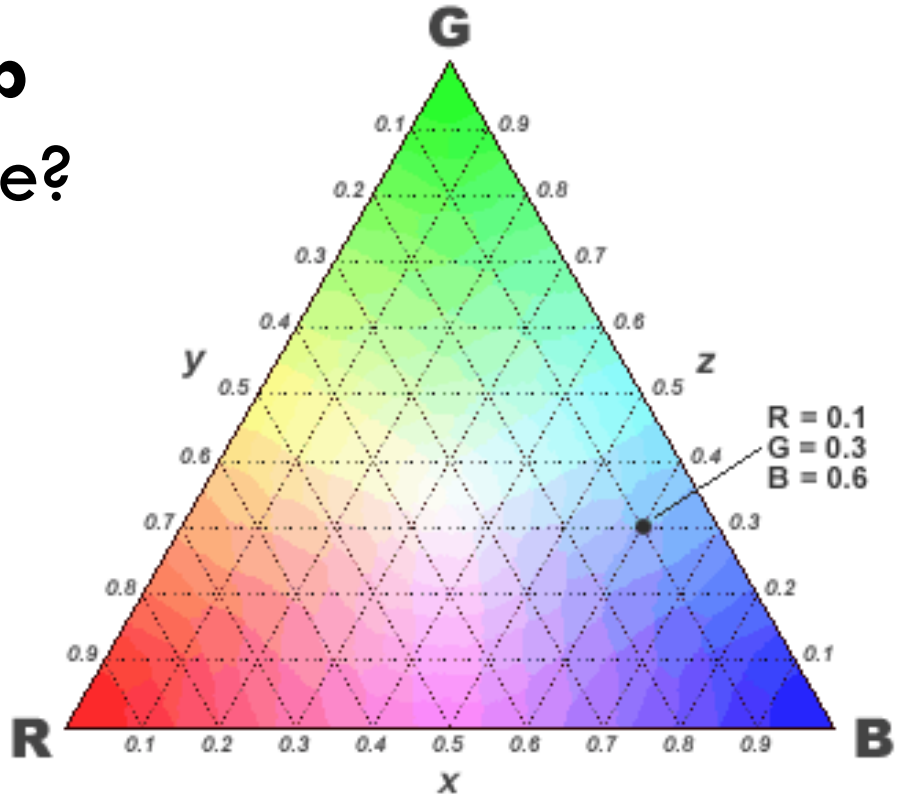


What is Fuzzy Logic?

- Definition of fuzzy
  - Fuzzy – "not clear, or imprecise or blurred"
- Definition of fuzzy logic
  - A form of knowledge representation suitable for notions that cannot be defined precisely, but which depend upon their contexts.

Fuzzy Logic …

Can we give a **crisp** definition to light blue?



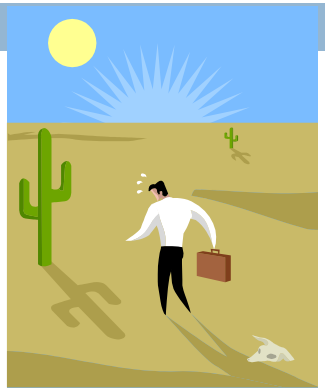
Probability vs Fuzzy logic

Probability, which ranges from 0.0 to 1.0, is used to gauge the likelihood of some particular, well-defined state, under conditions of ignorance or chance.

For instance: a fair coin has a 50% probability of coming up heads. Note:

1. we do not know the outcome ahead of time, due to chance and
2. there are only two, clearly-defined states: "heads" and "tails".

Suppose a person is dying of thirst in the desert. He finds two bottles of water.



Bottle's label says that it has a 0.9 membership in the class of fluids known as non-poisonous drinking water.

Bottle's label states that it has a 0.9 probability of being pure drinking water (i.e. only a 0.1 probability of being poison.)

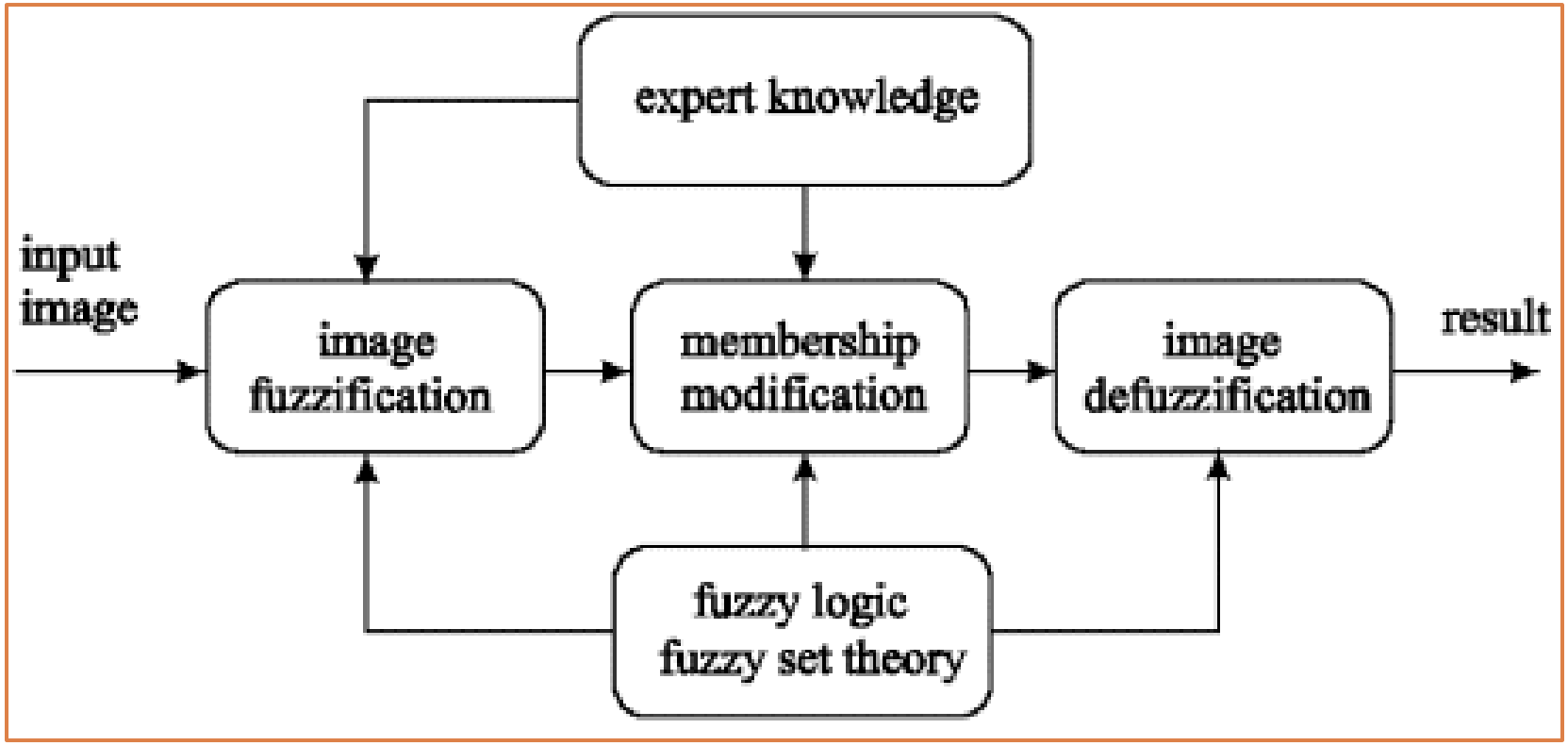
Which bottle should he choose?

Fuzzy Image Processing

It is a collection of different fuzzy approaches to image processing

Fuzzy image processing is the collection of all approaches that understand, represent and process the images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and on the problem to be solved.
(From: Tizhoosh, Fuzzy Image Processing, Springer, 1997)

Fuzzy image processing has three main stages: image fuzzification, modification of membership values, and, if necessary, image defuzzification

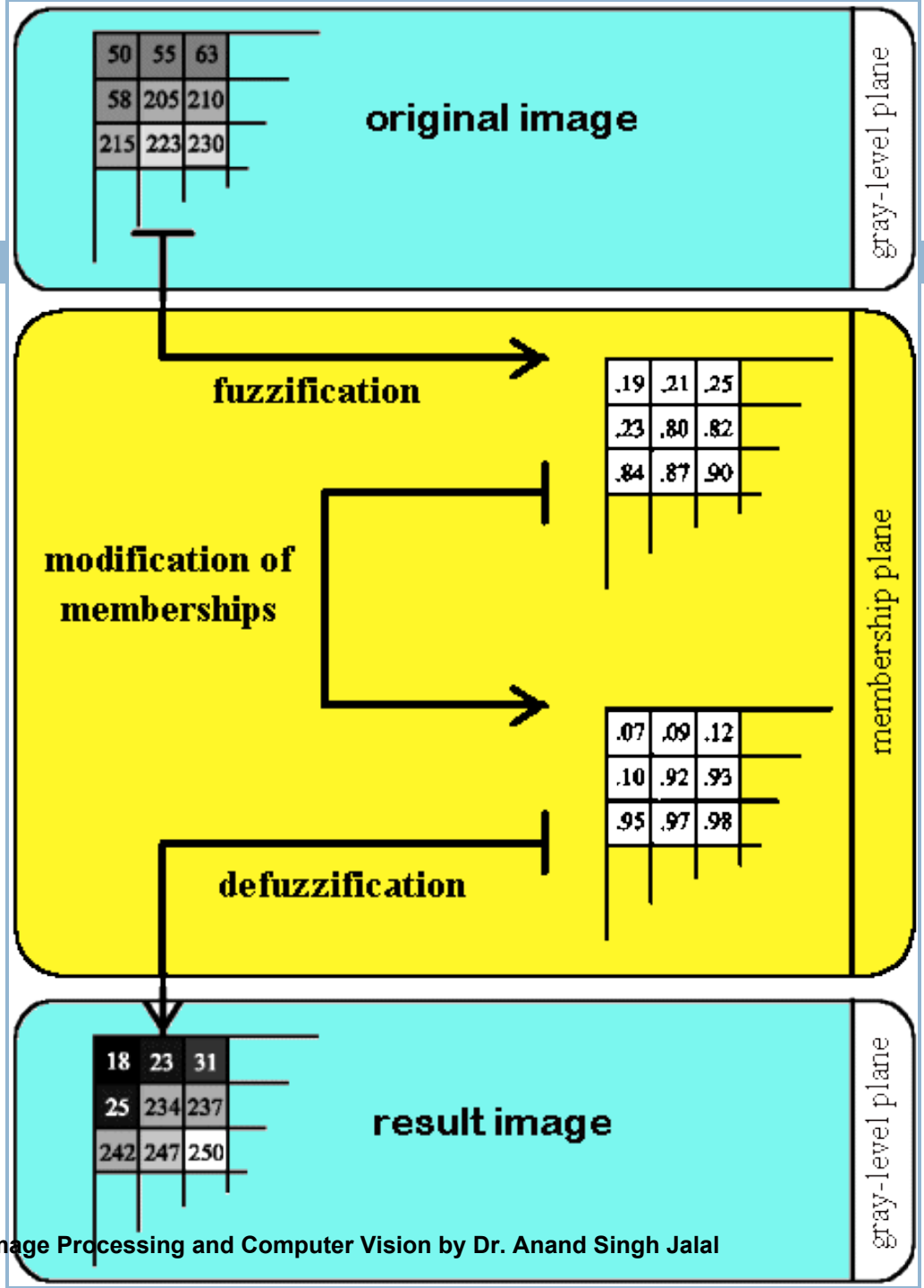


The general structure of fuzzy image processing

Steps of fuzzy image processing

After the image data are transformed from gray-level plane to the membership plane (fuzzification), appropriate fuzzy techniques modify the membership values.

This can be a fuzzy clustering, a fuzzy rule-based approach, a fuzzy integration approach and so on



Advantage of Fuzzy Image Processing

- Fuzzy techniques are powerful tools for knowledge representation and processing.
- Fuzzy techniques can manage the vagueness and ambiguity efficiently

In many image processing applications, we have to use expert knowledge to overcome the difficulties (e.g. object recognition, scene analysis).

Fuzzy set theory and fuzzy logic offer us powerful tools to represent and process human knowledge in form of fuzzy if-then rules.

Any Questions ?