

Control Systems Lab4

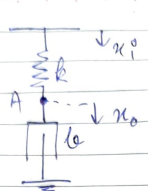
RollNo-190020021

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Answers-

Q1-

Q1.



$k = 6 \text{ N/m}$
 $b = 1 \text{ N s/m}$

Free body diagram of the mass:

Upward force: $b \frac{dx_0}{dt}$
Downward force: $k(x_i - x_0)$

$$k(x_i - x_0) = b \frac{dx_0}{dt}$$
$$6(x_i - x_0) = \frac{dx_0}{dt}$$

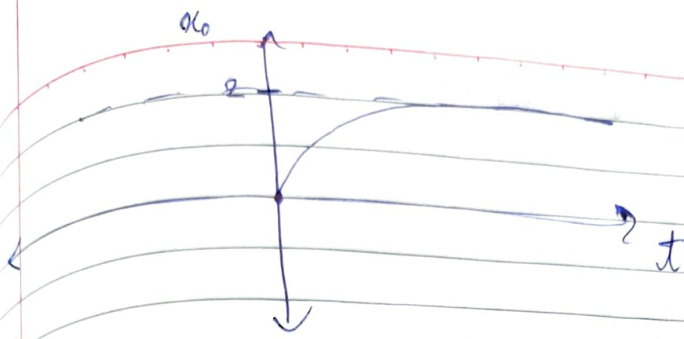
As step input of magnitude 2 units is applied so

$$x_i = 2, \quad t \geq 0$$
$$x_i = 0, \quad t < 0$$

so,

$$6(2 - x_0) = \frac{dx_0}{dt}$$
$$6 \int dt = \int \frac{dx_0}{2 - x_0}$$
$$6t = -\ln(2 - x_0) \Big|_0^{x_0}$$
$$-6t = \ln(2 - x_0)$$
$$\boxed{x_0 = 2(1 - e^{-6t})}$$

$t \geq 0$



Time constant -

$$\tau = \frac{1}{\text{initial slope}} = \frac{1}{\left. \frac{d(x_0)}{dt} \right|_{t=0}}$$

$$\tau = \frac{1}{0.6} \text{ sec}$$

Rise time -

$$x_0(t_1) = 0.1, \quad x_0(t_2) = 0.9$$

$$0.1 \times 2 = 2(1 - e^{-6t_1}) \Rightarrow e^{-6t_1} = 1.8$$

$$0.9 \times 2 = 2(1 - e^{-6t_2}) \Rightarrow e^{-6t_2} = 0.2$$

$$e^{6(t_2 - t_1)} = 9$$

$$t_2 - t_1 = \frac{1}{6} \ln(9)$$

$$\text{Rise time} \rightarrow \boxed{t_2 - t_1 = 0.366}$$

Settling time -

$$x_0(T_s) = 0.98 = 1 - e^{-6T_s}$$

$$e^{-6T_s} = 0.02$$

$$\boxed{T_s = \frac{3.91}{6} = 0.652 \text{ sec}}$$

steady state value -

$$t \rightarrow \infty$$

$$x_0 \rightarrow 2$$

so

$$\boxed{x_{s.t} = 2}$$

from ① -

$$6(x_i - x_0) = \frac{dx_0}{dt}$$

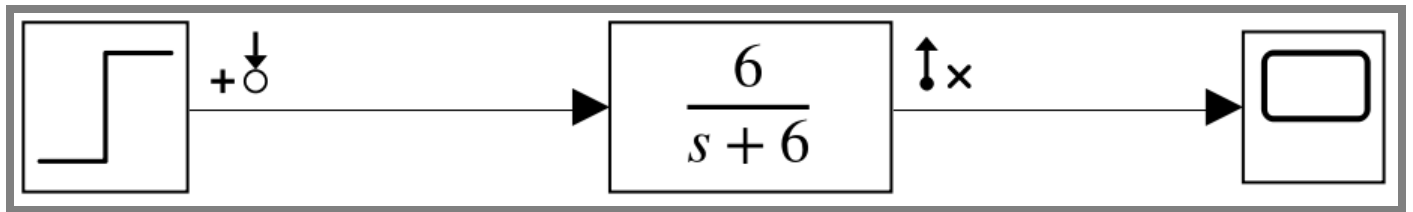
taking laplace transform -

$$6(X_i(s) - X_0(s)) = sX_0(s)$$

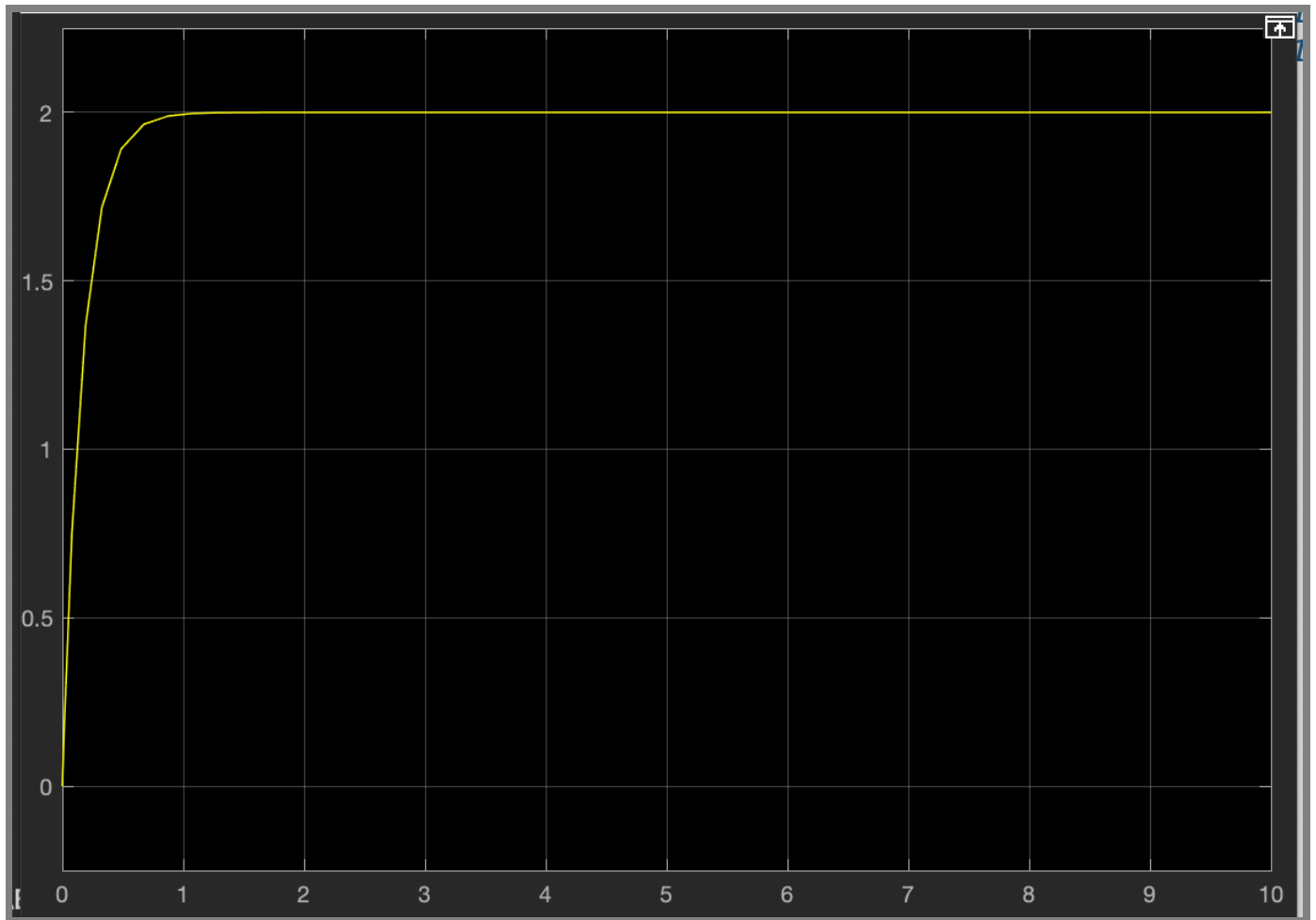
$$6X_i(s) = (s+6)X_0(s)$$

$$T(s) = \boxed{\frac{X_0(s)}{X_i(s)} = \frac{6}{s+6}}$$

Simulink Model-



Plot from scope-

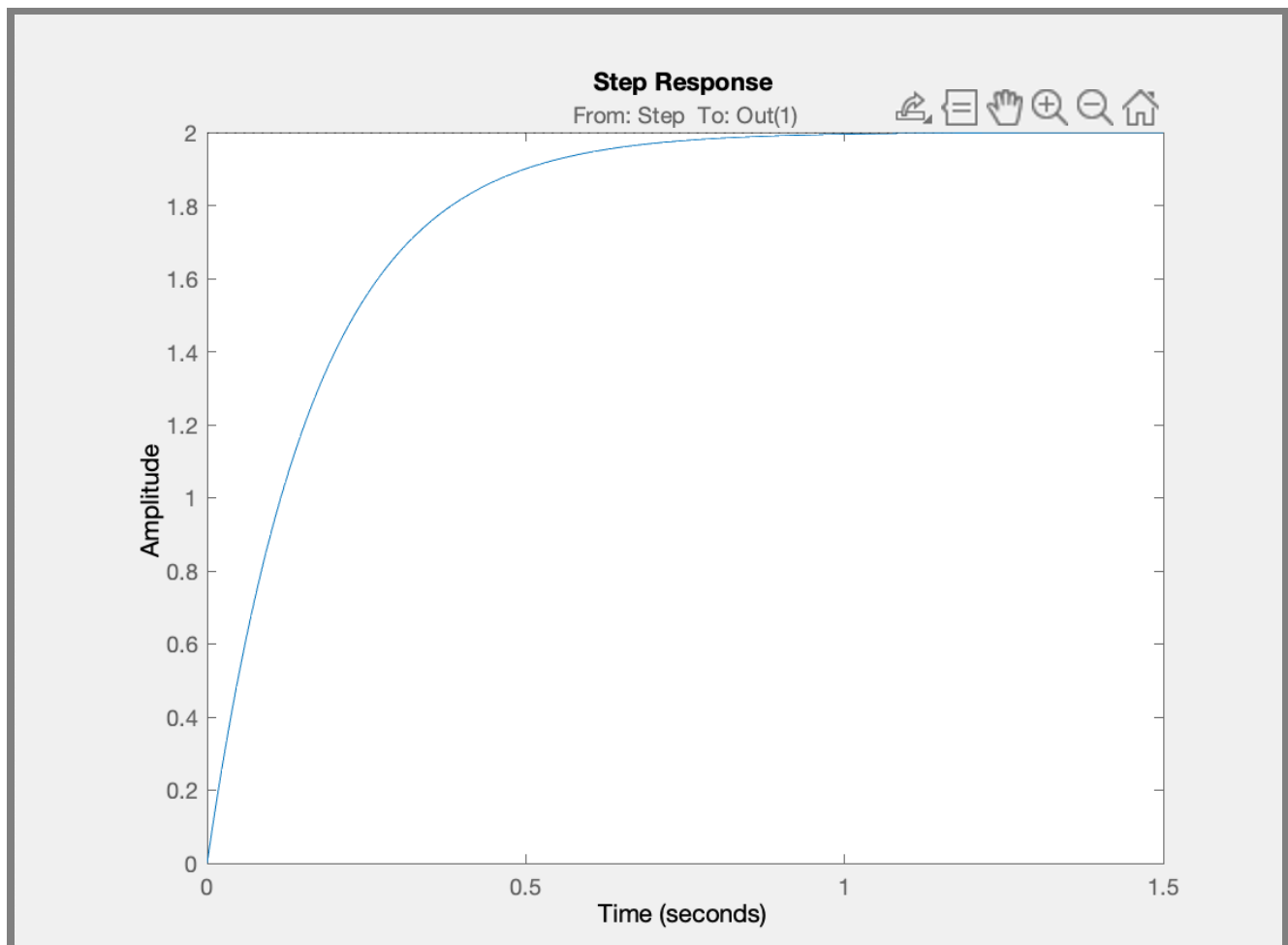


Time Domain Characteristics-

Code-

1	<code>%% Exact linearization of the Simulink model untitled</code>	
2	<code>%</code>	
3	<code>% This MATLAB script is the command line equivalent of the exact</code>	
4	<code>% linearization tab in linear analysis tool with current settings.</code>	
5	<code>% It produces the exact same linearization results as hitting the Linearize button.</code>	
6		
7	<code>% MATLAB(R) file generated by MATLAB(R) 9.9 and Simulink Control Design (TM) 5.6.</code>	
8	<code>%</code>	
9	<code>% Generated on: 29-Jan-2021 15:36:52</code>	
10		
11	<code>%% Specify the model name</code>	
12	<code>model = 'Lab4_Q1';</code>	
13		
14	<code>%% Specify the analysis I/Os</code>	
15	<code>% Get the analysis I/Os from the model</code>	
16	<code>io = getlinio(model);</code>	
17		
18	<code>%% Specify the operating point</code>	
19	<code>% Use the model initial condition</code>	
20	<code>op = operpoint(model);</code>	
21		
22		
23	<code>%% Linearize the model</code>	
24	<code>sys = linearize(model,io,op);</code>	
25		
26	<code>%% Plot the resulting linearization</code>	
27	<code>stepinfo(2*sys)</code>	
28	<code>step(2*sys)</code>	

Output-



Characterstics from plot-

RiseTime: 0.3662

SettlingTime: 0.6520

SettlingMin: 1.8090

SettlingMax: 1.9999

Overshoot: 0

Undershoot: 0

Peak: 1.9999

PeakTime: 1.7576

We can see that plots match and also time-domain characterstics calculated match.