Lab5-Q1

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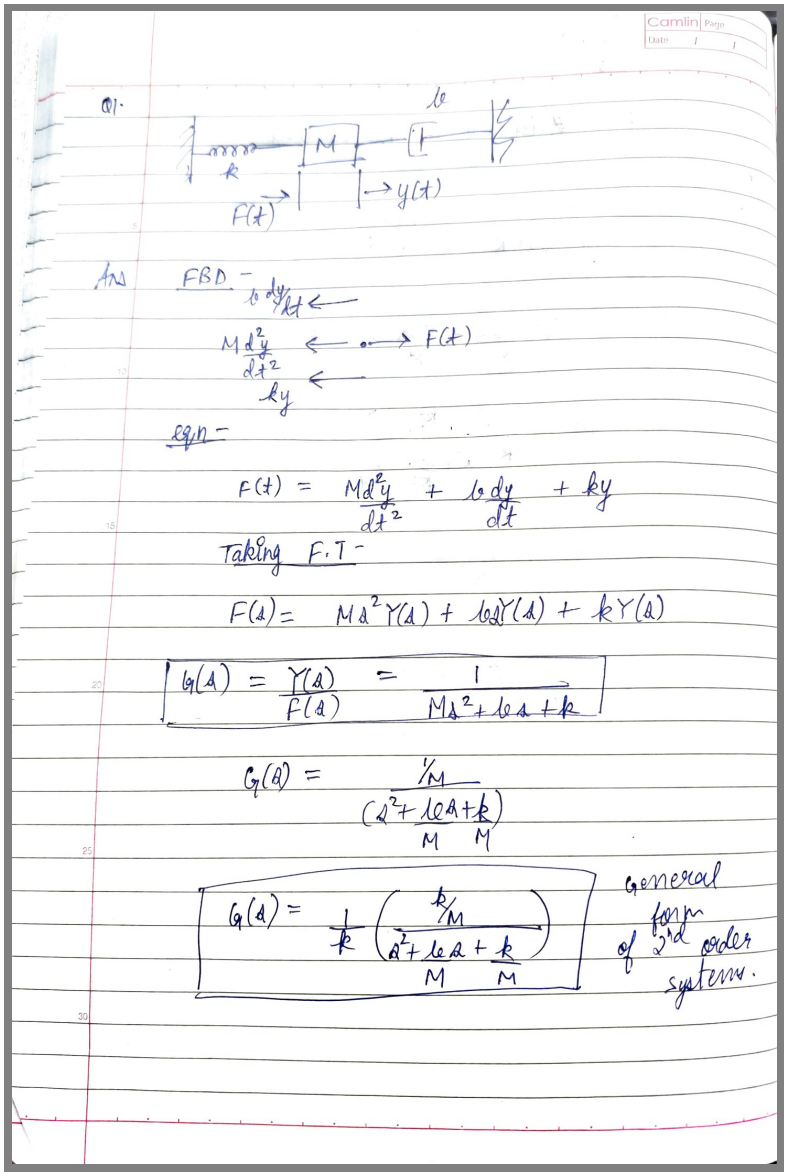
Answers-

Q1



Case I: $M = 10\ kg$, $b = 1.5\ Ns/m$, $k = 1\ N/m$

$$G(s) = \frac{1/10}{\left(s^2 + 0.15s + \frac{1}{10}\right)} \qquad \left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$$

$$\omega_n = \sqrt{\frac{1}{10}}$$

$$2\zeta\omega_n = 0.15$$

$$\zeta = \frac{\sqrt{10} \times 0.15}{2} \approx 0.237$$

$0 < \zeta < 1$

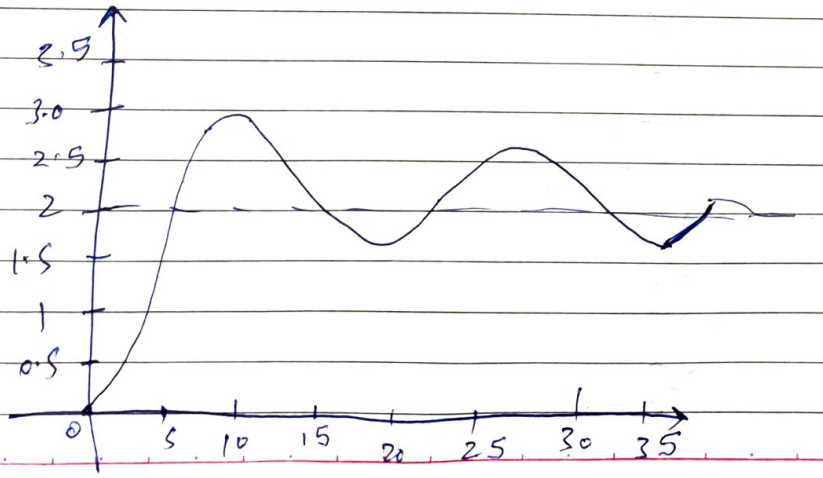
underdamped case

$Y(s) \rightarrow$

$$Y(s) = F(s) \times \left(\frac{1/10}{s^2 + 0.15s + \frac{1}{10}}\right)$$

$$Y(s) = \frac{(2/10)}{s\left(s^2 + 0.15s + \frac{1}{10}\right)} = \frac{1}{5}\left(\frac{1}{s(s^2 + 0.15s + 0.1)}\right)$$

→ I used grapher tool for $y(t)$.



Case II: $m = 2kg$, $b = 4 Ns/m$, $k = 2N/m$

$$G(s) = \frac{\frac{1}{2}\left(\frac{2}{2}\right)}{(s^2+2s+1)}$$

$$G(s) = \frac{1}{2}\left(\frac{1}{s^2+2s+1}\right)$$

$\omega_n^2 = 1$

$2\zeta\omega_n = 2$

$\zeta = 1$

critically damped.

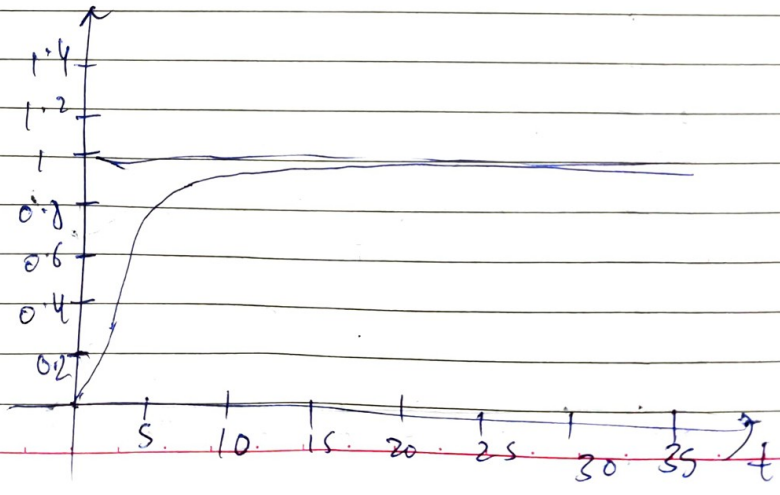
$Y(s) \rightarrow$

$$Y(s) = F(s) \times \frac{\left(\frac{1}{2}\right)}{(s^2+2s+1)}$$

$$Y(s) = \frac{1}{s(s^2+2s+1)} = \frac{1}{s(s+1)^2}$$

$$\frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{Cs}{(s+1)^2}$$

$$= \frac{1}{s} - \frac{2}{s+1} + \frac{s}{(s+1)^2}$$



Case III: $m = 5kg$, $b = 7.4\ Ns/m$, $k = 1.2\ N/m$

$$G(s) = \frac{\frac{1}{1.2}\left(\frac{1.2}{5}\right)}{\left(s^2 + \frac{7.4}{5}s + \frac{1.2}{5}\right)}$$

$$G(s) = \frac{1}{1.2}$$

$$\omega_n = \sqrt{\frac{1.2}{5}}, \quad 2\omega_n\zeta = \frac{7.4}{5}$$

$$\zeta = \frac{7.4}{10} \times \sqrt{\frac{5}{1.2}}$$

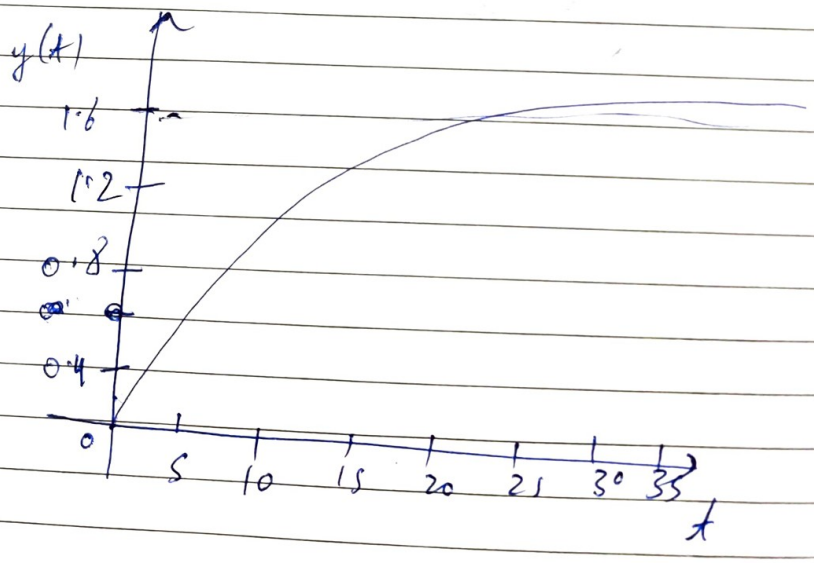
$$= 1.51 > 1$$

So ~~under~~ damped

$Y(s) \rightarrow$

$$Y(s) = F(s) \times \frac{1}{5s^2 + 7.4s + 1.2}$$

$$Y(s) = \frac{2}{s(5s^2 + 7.4s + 1.2)}$$



For underdamped case -

- Time Domain characteristics -

Peak Time (T_p) -

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{\frac{1}{\sqrt{10}}\sqrt{1-(0.237)^2}}$$

$$= 9.87$$

Percentage overshoot -

$$\%OS = e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

$$= 46.32\%$$

Settling time -

$$T_s \simeq \frac{4}{\zeta\omega_n} = \frac{4}{0.075}$$

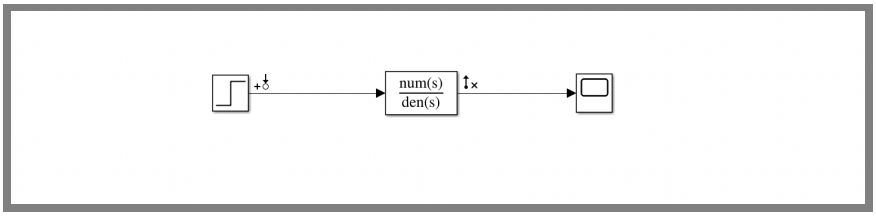
$$T_s \approx 53.34$$

steady state -

$$t \to \infty$$

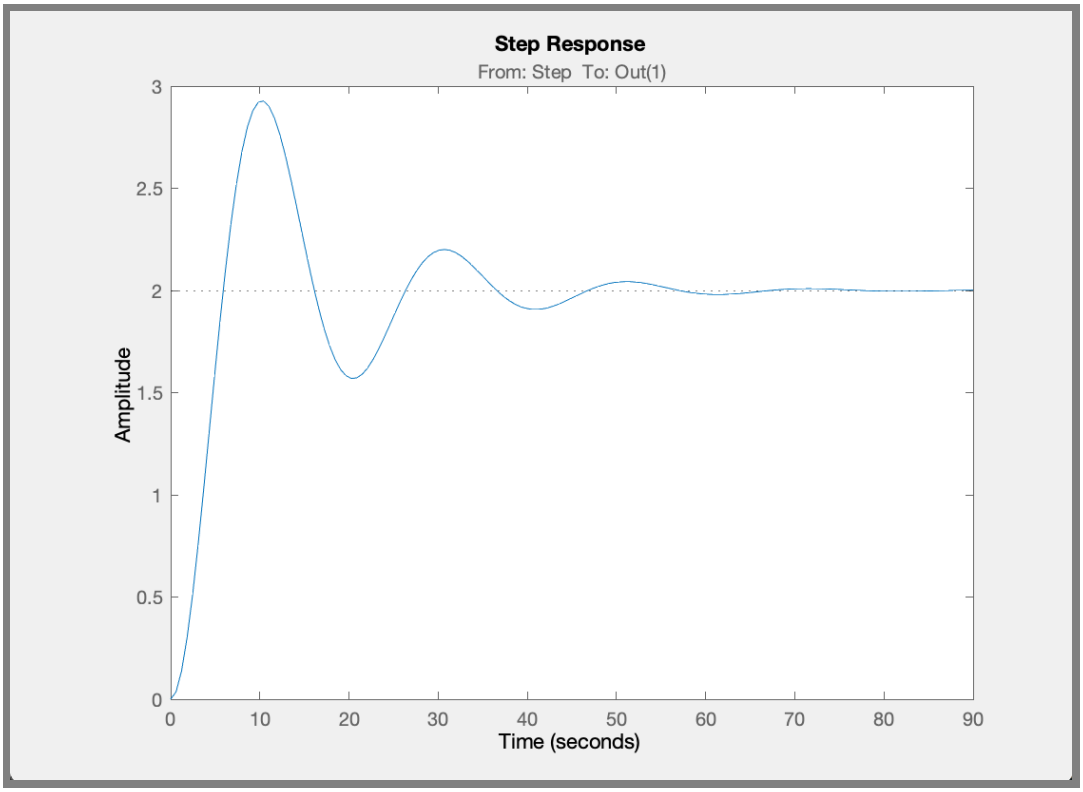
$$y = 2$$

Simulink model of underdamped-



Code to get plot and time domain characterstics-

```
%% Exact linearization of the Simulink model Q1_simulink
%
% This MATLAB script is the command line equivalent of the exact
% linearization tab in linear analysis tool with current settings.
% It produces the exact same linearization results as hitting the Linearize button.

% MATLAB(R) file generated by MATLAB(R) 9.9 and Simulink Control Design (TM) 5.6.
%
% Generated on: 05-Feb-2021 13:06:48

%% Specify the model name
model = 'Q1_simulink';

%% Specify the analysis I/Os
% Get the analysis I/Os from the model
io = getlinio(model);

%% Specify the operating point
% Use the model initial condition
op = operpoint(model);


%% Linearize the model
sys = linearize(model,io,op);

%% Plot the resulting linearization
stepinfo(2*sys)
step(2*sys)
```



Output-
RiseTime: 3.9632
SettlingTime: 52.3711
SettlingMin: 1.5694
SettlingMax: 2.9268
Overshoot: 46.3382
Undershoot: 0
Peak: 2.9268
PeakTime: 10.4384

By comparing the time-domain characterstics we can see for ourselves that they are almost same.