Lab10-Report

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Answers-

Q1-

Q1. $G(s) = \dfrac{K}{(s+2)(s+3)(s+7)}$ % overshoot = 10%

1. $\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$

$$\frac{10}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$0.1 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

calculating- (from software)

$$\zeta \approx 0.59$$

$G(s)$ is 0^{th} order system.

$$K_p = \lim_{s \to 0} G(s)$$

static error constant $\rightarrow K_p = \dfrac{K}{2 \times 3 \times 7} = \dfrac{K}{42}$ —①

so static error ~~constant~~ $= \dfrac{1}{1+K_p} = \dfrac{1}{1+K/42}$

$$\boxed{e(\infty) = \frac{42}{42+K}}$$

2. fixing $K_p = 4$

so ~~from ①~~ $\cancel{\dfrac{K}{42} = 4}$ $\cancel{K =}$

$\cos(\theta) = \zeta$

$\cos(\theta) = 0.59$

$\theta = \cos^{-1}(0.59)$

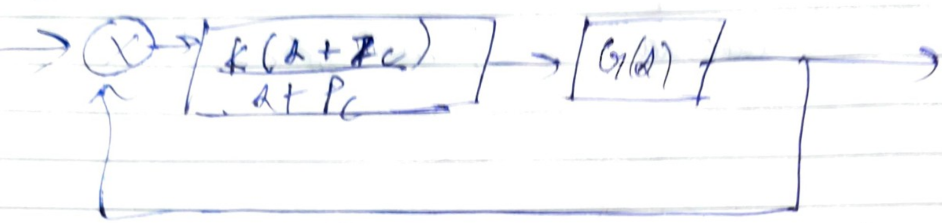
$\theta = \pm 53.843°$

From graph from MATLAB -

$k = 41.1$

intersection poles: $-1.87 + 2.56j$
$-1.87 - 2.56j$

Lag compensator system -



Given: $P_c = 0.1$

$$\frac{Z_c}{P_c} = \frac{(K_p)_N}{(K_p)_o}$$

$(K_p)_N = 4$

$\frac{(K)_N}{42} = 4$

$K = 168$

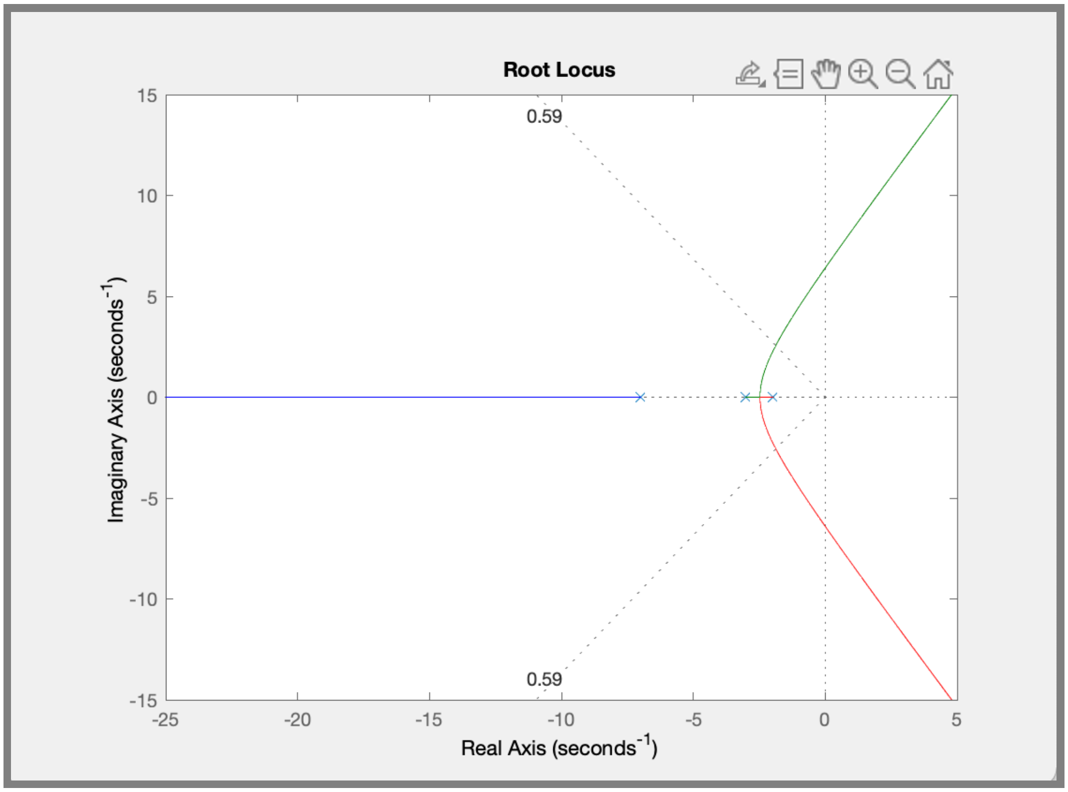
$$\frac{Z_c}{0.1} = \frac{168}{41.1}$$

$$\boxed{Z_c = 0.409}$$

so lag compensator -

$$\frac{K(s + 0.409)}{s + 0.1} \quad (K = 168)$$

Rootlocus from MATLAB-



Given $P_c = 0.1$, $K_P = 4$

Kp for compensated system -

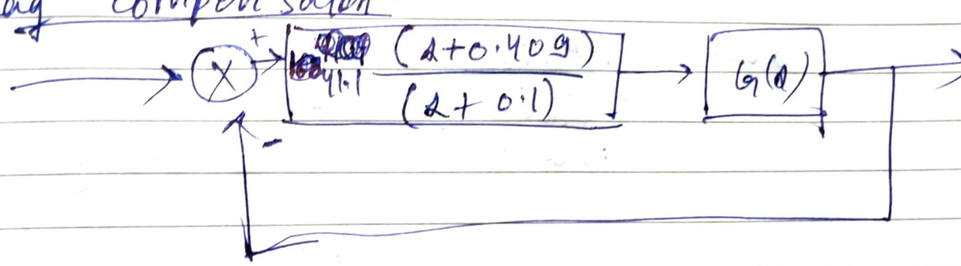
$$\frac{41.1\,(Z_c)}{0.1} \times \frac{1}{42} = 4$$

$$Z_c = \frac{4 \times 42 \times 0.1}{41.1}$$

$$Z_c = 0.409$$

So

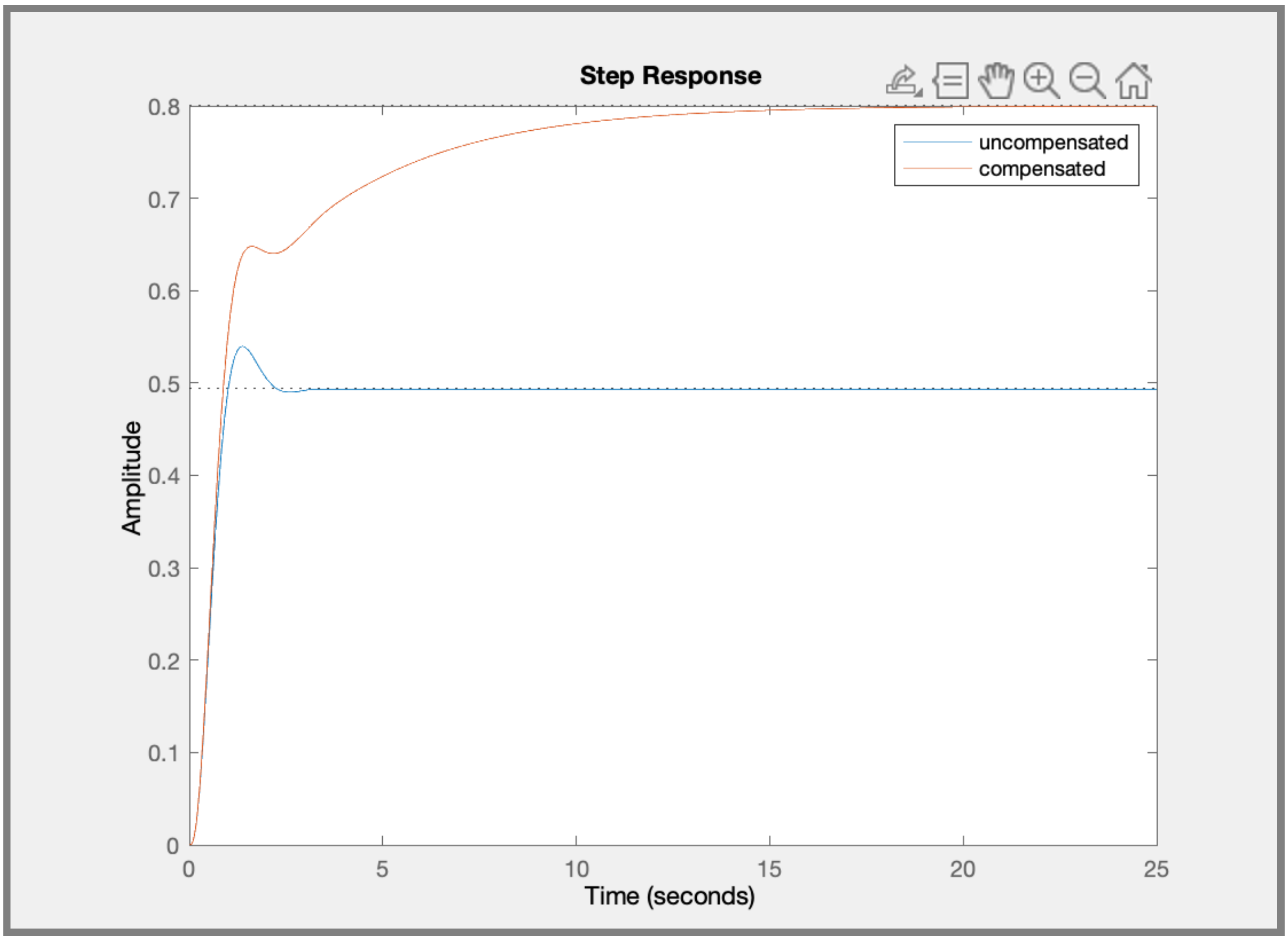
Lag compensator -



Code for plotting response-

```
%Q1
clear all;
close all;
%uncompensated system
sys = tf([41.1],[1,12,41,42]);
%compensated system
sys1 = tf([41.1,16.81],[1,12.1,42.2,46.1,4.2]);

hold on;
step(sys/(sys+1));
step(sys1/(sys1+1));
legend('uncompensated','compensated');
hold off;
```



We can see error from graph - 1-0.8 =0.2.
which will give

$$\frac{1}{1+K_p}. = 0.2$$

Hence

$$K_p = 4$$

So it matches and hence it is correct.

Q2-

Q2 · Given - $G(s) = \frac{K}{s(s+10)(s+20)}$

1. PD controller so that T_s, reduces by a factor of 4 ; %OS = 20%

An $\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

$0.2 = \frac{20}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

calculating -

$\zeta \simeq 0.456$

From MATLAB -

Gain = 1080

Poles = $-3.28 \pm 5.94j$

closed loop system poles :-

$\frac{G}{1+G} \rightarrow \frac{1080}{s^3 + 30s^2 + 200s + 1080}$

Poles → $s \rightarrow -23.432$

As it is 8 times away from $j\omega$ axis from the dominant pair so second order approx. valid.

→ Settling time, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma} = \frac{4}{3.28}$

$T_s = 1.22$

$$(T_s)' = \frac{T_s}{4} = \frac{1.22}{4} = 0.305$$

$$0.305 = \frac{4}{(\zeta\omega_n)_N} = \frac{4}{\sigma_N}$$

$$\sigma_N = 13.11$$

Since, for $\% OS = 20\%$, $\zeta = 0.456$

$$\theta = \cos^{-1}(\zeta) = 62.87°$$

So,

$$\tan(180 - 117.13) = \frac{\omega_{dN}}{\sigma_N}$$

$$\tan(62.87) = \frac{\omega_{dN}}{13.11}$$

$$25.59 = \omega_{dN}$$

Desired closed loop dominant poles -

$$P = -13.11 \pm 25.59j$$

Now root locus should pass through P. So

$$\angle P = 180°$$

current angle at P -

$$\theta = -(\theta_0 + \theta_{10} + \theta_{20})$$

$$\theta = -(117.13 + 132.09 + \ldots)$$

$$\theta = -(96.93 + 117.13 + 74.93)$$

$$\theta = -289$$

So

$$\theta_Z + \theta = -180°$$

$$\theta_Z = 109°$$

so <u>location of zero</u> -

$$\frac{y_1 - y_2}{x_1 - x_2} = \tan(109^\circ)$$

$$\frac{25.59 - 0}{-13.11 - z_c} = \tan(109)$$

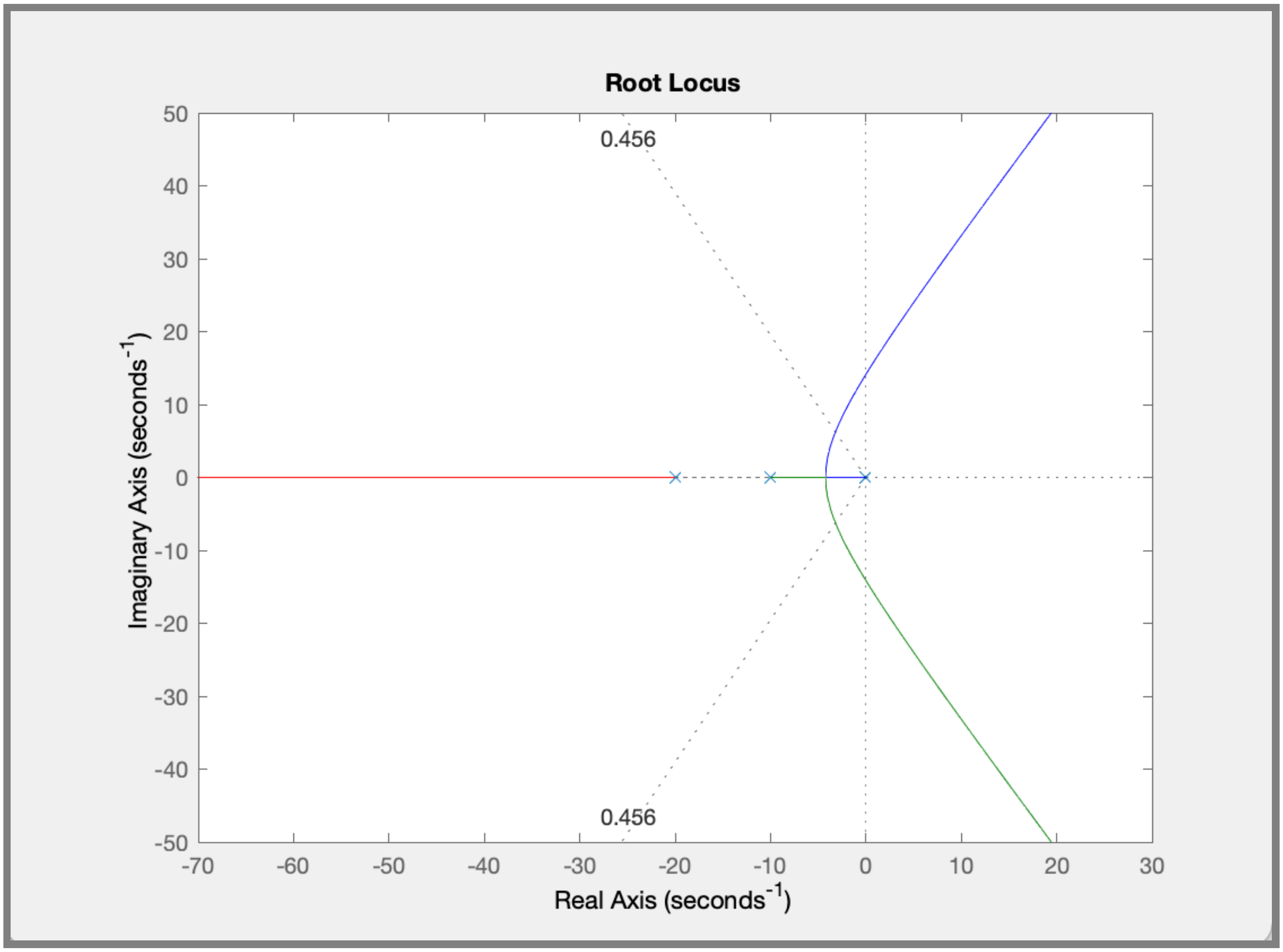
$$\frac{25.59}{-13.11 - z_c} = -2.90$$

$$-8.811 = -13.11 - z_c$$

$$\boxed{z_c = -4.30}$$

so add extra pole at $z_c = -4.30$

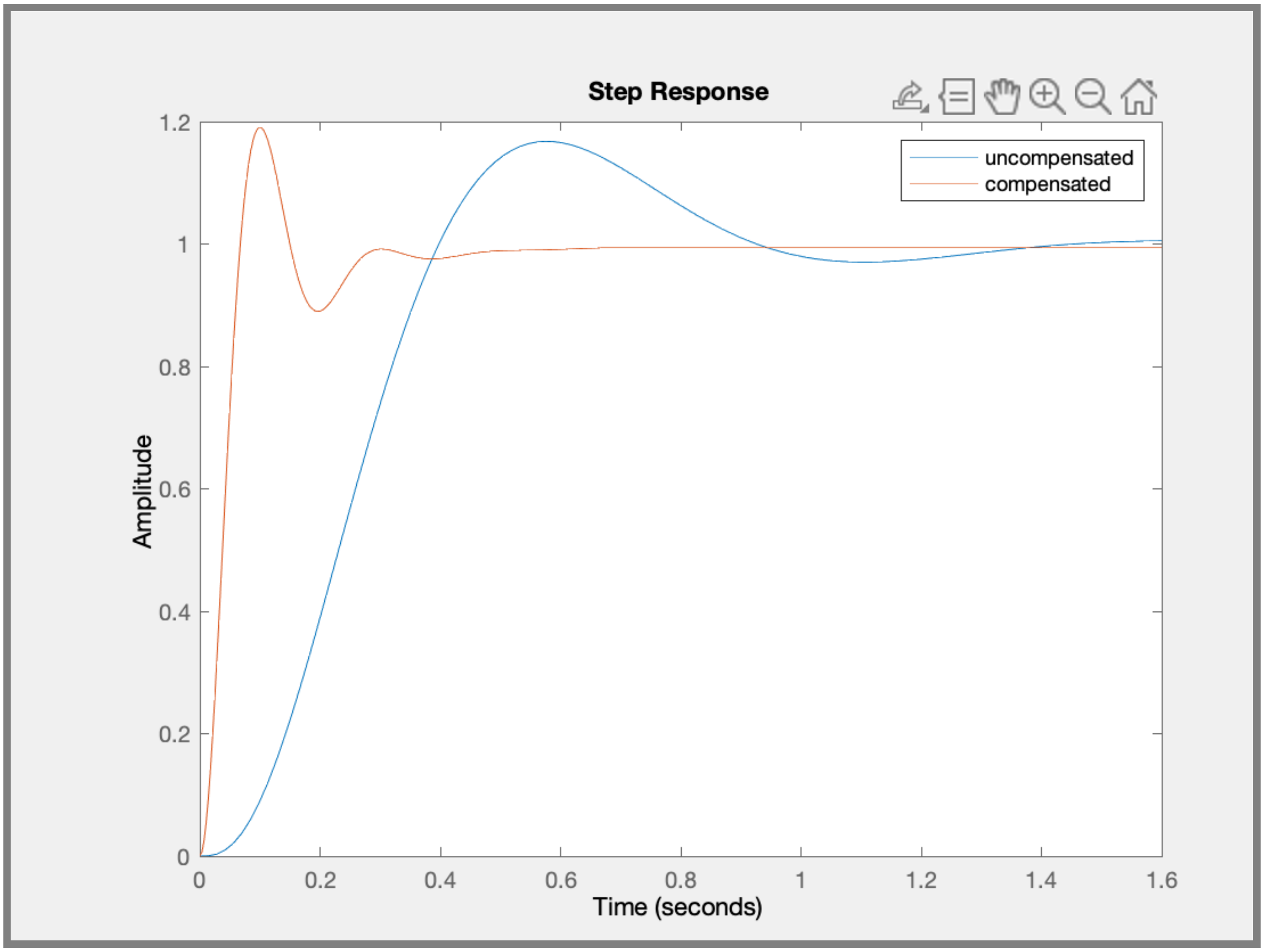
Root Locus-



Code for plotting response-

```
%Q2
clear all;
close all;
%uncompensated system
 sys = tf([1080],[1,30,200,0]);
%compensated system
 sys1 = tf([1080,4644],[1,30,200,0]);

hold on;
step(sys/(sys+1));
step(sys1/(sys1+1));
legend('uncompensated','compensated');
hold off;
```


Step Response
uncompensated
compensated
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

Q3-

~~Q2:~~ $\cancel{G(s)} = \cancel{\frac{K}{s(s+10)(s+20)}}$

Q3: $G(s) = \frac{K}{(s+15)(s^2+6s+13)}$, $\%OS = 30\%$

Ans

$\zeta \rightarrow$

$\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$0.3 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$\zeta = 0.358$

From MATLAB – (for $\%OS \rightarrow 30\%$)

Gain = 383

Poles = $-2.03 \pm 5.48j$

$T_S = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma} = \frac{4}{2.03} = 1.97$

$(T_S)' = \frac{T_S}{2} = \frac{1.97}{2}\ 0.985$

$(T_S)' = \frac{4}{(\zeta\omega_n)'} = 0.985$

$\sigma' = \frac{4}{0.985} = 4.061$

$\theta = \cos^{-1}(0.358) = 69^\circ$

$\tan(69) = \frac{\omega_N}{4.061}$

$\omega_N = 10.58$

so desired pole location – (for $T_S' = T_S/2$)

$-4.06 \pm 10.58j$

$z_c = -7$ (given)

so if desired poles to lie on root locus $\angle P = 180^\circ$

due to z_c and other poles

$\theta = \theta_{z_c} - \theta_1 - \theta_2 - \theta_3$

$= 69.57 - 41.54 - 97.82 - 97.05$

$\theta = -163.84$

so

angle contribution due to P_c -

$\theta_{P_c} = -180 + 163.84$

$= -16.16$

$\tan(-16.16) = \dfrac{10.58}{P_c - 4.06}$

$P_c - 4.06 = -36.51$

$P_c = -32.45$

so TF of lead compensator -

$TF = \dfrac{383(s+7)}{(s+32.45)}$

Code for plotting response-

```
%Q1
clear all;
close all;
%uncompensated system
sys = tf([383],[1,21,103,195]);
%compensated system
sys1 = tf([383,2681],[1,61.57,954.97,4731.71,7911.15]);

hold on;
step(sys/(sys+1));
step(sys1/(sys1+1));
legend('uncompensated','compensated');
hold off;
```