Lab Test-Control Systems

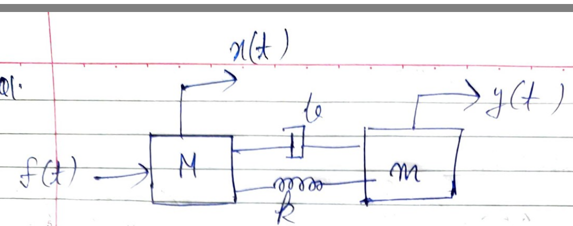
RollNo-190020021

Kushagra Khatwani

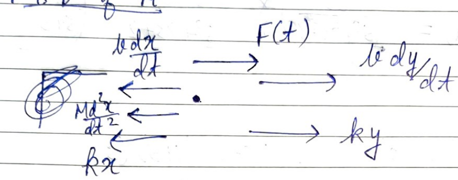
Answers-

Q1

Q1.

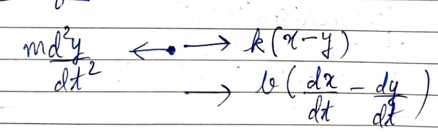


Ans (1) F.B.D of M -



$$F(t) = b\left(\frac{dx}{dt} - \frac{dy}{dt}\right) + k(x-y) + M\frac{d^2x}{dt^2} \quad -(1)$$

F.B.D of m -



$$m\frac{d^2y}{dt^2} = k(x-y) + b\left(\frac{dx}{dt} - \frac{dy}{dt}\right) \quad -(2)$$

From (1), (2) system differential eqns are -

$$F(t) = b\left(\frac{dx}{dt} - \frac{dy}{dt}\right) + k(x-y) + M\frac{d^2x}{dt^2}$$

$$m\frac{d^2y}{dt^2} = k(x-y) + b\left(\frac{dx}{dt} - \frac{dy}{dt}\right)$$

(2) Taking Laplace transform of (1), (2) -

$$F(s) = bs(X(s) - Y(s)) + k(X(s) - Y(s)) + Ms^2 X(s)$$

$$ms^2Y(s) = k(X(s) - Y(s)) + bs(X(s) - Y(s))$$

$$Y(s)(ms^2 + bs + k) = (k + bs)X(s)$$

$$X(s) = \frac{Y(s)(ms^2 + bs + k)}{(k + bs)}$$

$$F(s) = X(s)(bs + k + Ms^2) - Y(s)(bs + k)$$

so

$$F(s) = \frac{Y(s)(ms^2 + bs + k)(bs + k + Ms^2)}{(k + bs)} - \frac{Y(s)(k + bs)^2}{k + bs}$$

$$\frac{Y(s)}{F(s)} = \frac{k + bs}{Mms^4 + ms^2(bs + k) + Ms^2(bs + k)}$$

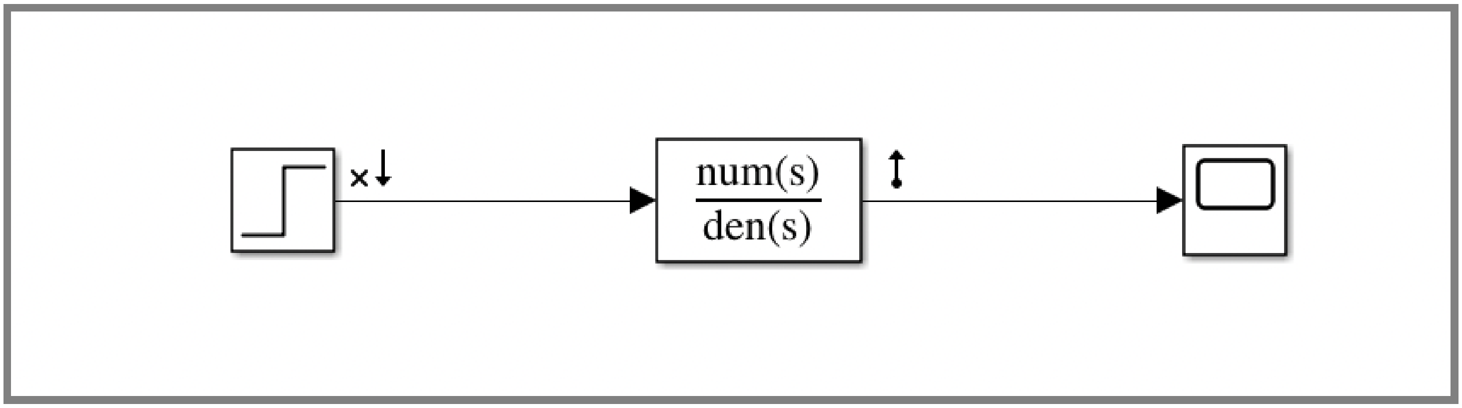
$$\boxed{T(s) = \frac{Y(s)}{F(s)} = \frac{bs + k}{s^2(Mms^2 + s(mb + Mb) + mk + Mk)}}$$

put $M = 50\,kg$, $m = 4\,kg$, $b = 2\,Ns/m$, $k = 5\,N/m$

$$T(s) = \frac{Y(s)}{F(s)} = \frac{2s + 5}{s^2(200s^2 + 108s + 270)}$$

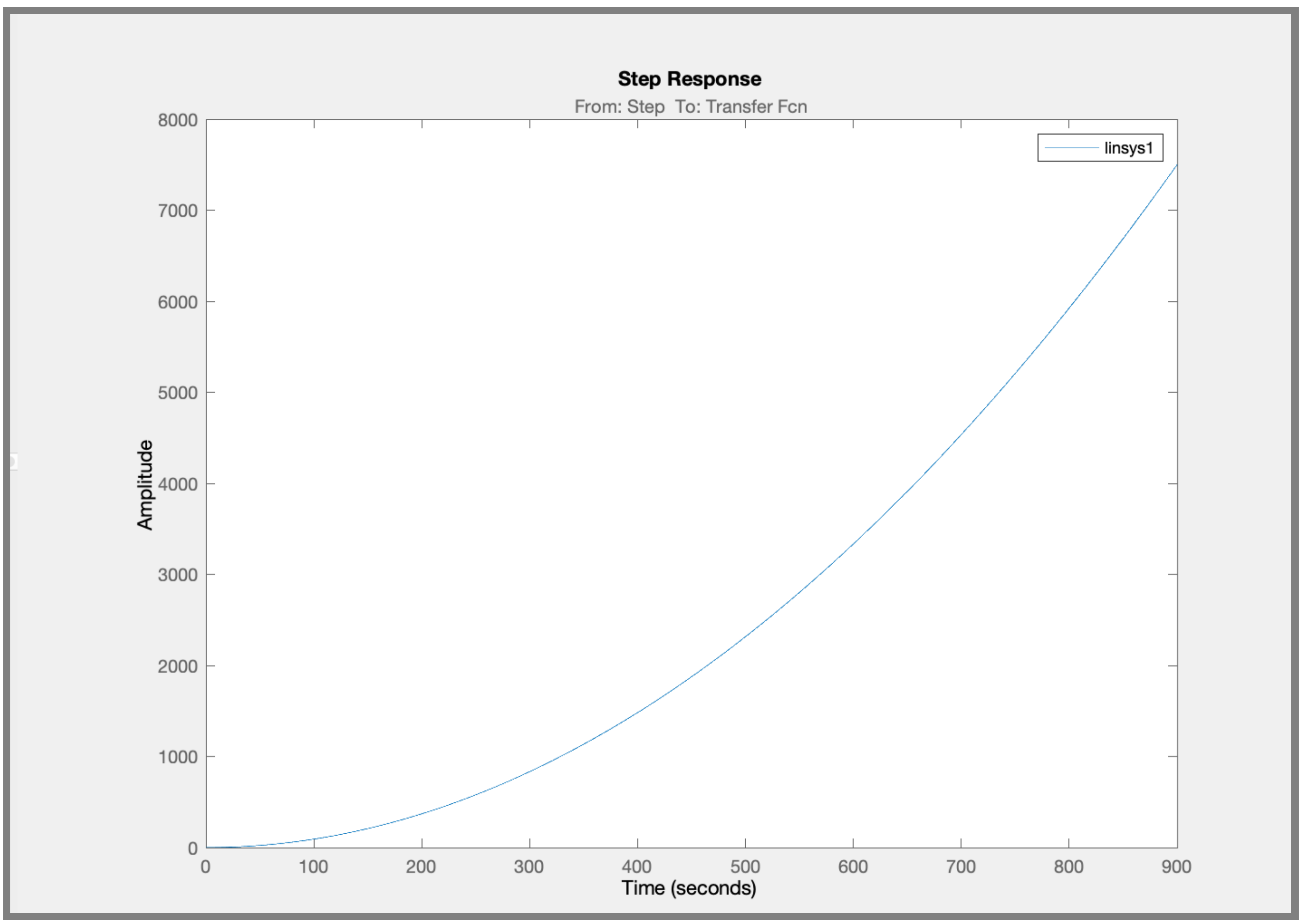
$$= \frac{2s + 5}{200s^4 + 108s^3 + 270s^2}$$

Simulink Model-



We can see that system was of high order.
Settling Time = NA , as graph keeps on increasing.

Plot-



Q2-

Q2. Routh Table for -

$$T(s) = \frac{2s+5}{200s^4+108s^3+270s^2}$$

s^4	200	270	0
s^3	108	0	0
s^2	270	0	0
s^1	$\cancel{0}$ 540	$\cancel{0}$ 0	$\cancel{0}$ 0
s^0	0 (≈ 270)	0	0

$$P(s) = 270 s^2$$

$$\frac{dP(s)}{ds} = 540 s$$

Total sign changes befor s^2 - = 0

so poles on LHP = 2

After s^2 -

sign changes = 0

so poles on jw axis = 2

so

Total poles = 4

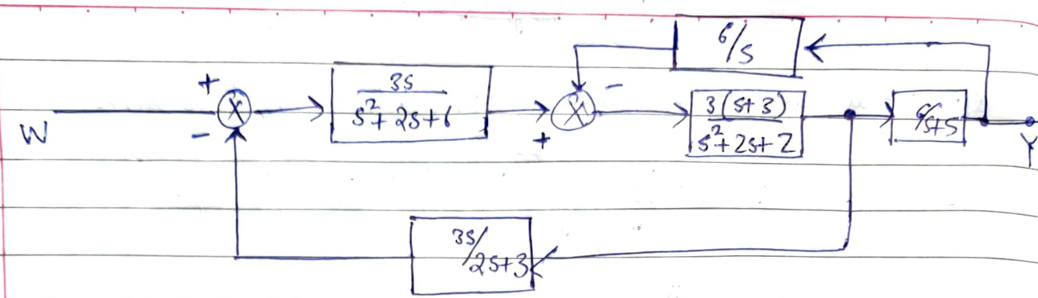
LHP = 2

jw axis = 2

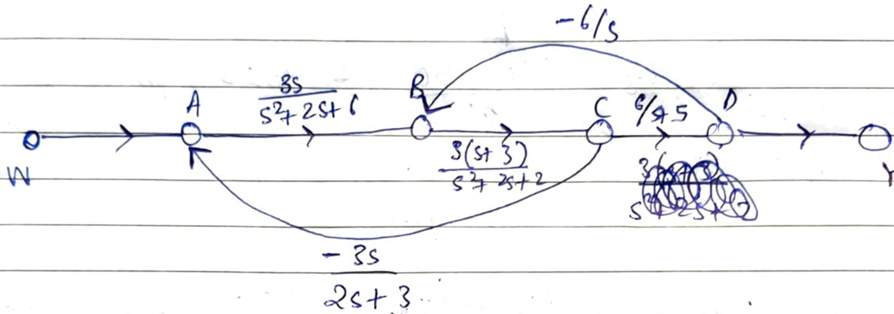
system ⇒ marginally stable.

Q3-

Q3.



Ans



Forward path -

WABCDY -

$$T_1 = \left(\frac{3s}{s^2+2s+6}\right) \times \frac{3(s+3)}{s^2+2s+2} \times \left(\frac{6}{s+5}\right)$$

Loop gain -

① CABC -

$$\left(\frac{3s}{s^2+2s+6}\right) \times \frac{3(s+3)}{s^2+2s+2} \times \frac{(-3s)}{2s+3}$$

② DBCD -

$$\frac{6}{s+5} \times \left(\frac{-6}{s}\right) \times \frac{3(s+3)}{s^2+2s+2}$$

$$\Delta = 1 - (\text{loop gain})$$

(non touching loop gain = 0)

$$\Delta = 1 - \left\{\left(\frac{3s}{s^2+2s+6}\right) \times \frac{3(s+3)}{s^2+2s+2} \times \left(\frac{-3s}{2s+3}\right) + \frac{6}{s+5} \times \left(\frac{-6}{s}\right) \times \frac{3(s+3)}{(s^2+2s+2)}\right\}$$

$\Delta_k = 1$

By Mason's Rule -

$$G(s) = \frac{T_1}{\Delta} \qquad \left(\frac{\sum_k T_k \Delta_k}{\Delta} \right)$$

$$G(s) = \frac{\left(\frac{3s}{s^2+2s+6}\right) \times \frac{3(s+3)}{s^2+2s+2} \times \left(\frac{6}{s+5}\right)}{\Delta}$$

$$\Delta = \frac{(s^2+2s+6)(s^2+2s+2)(2s+3)(s+5)(s) + (27)(s^2)(s+3)(s+5)(s) + (108)(s+3)(s^2+2s+6)(2s+3)}{(s^2+2s+6)(s^2+2s+2)(2s+3)(s+5)(s)}$$

$$\boxed{G(s) = \frac{54(s^2)(s+3)(2s+3)}{(s^2+2s+6)(s^2+2s+2)(2s+3)(s+5)(s) + 27(s^3)(s+3)(s+5) + 108(s+3)(s^2+2s+6)(2s+3)}}$$