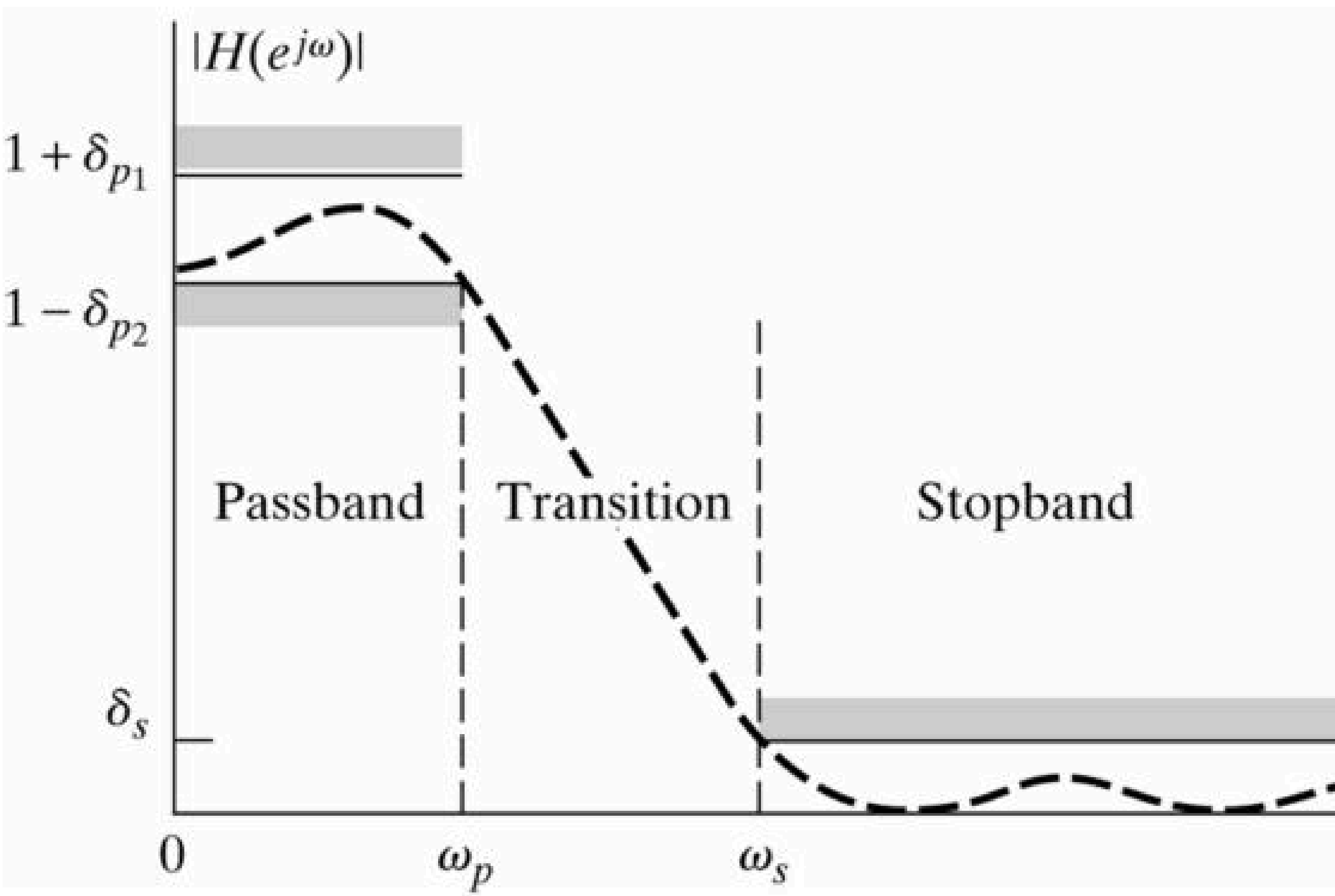
Design of FIR Filters (100 Points)

Welcome to Lab 3!

In this lab, you will design FIR filters. Refer to the below diagram for the lab.

Figure 7.1 Lowpass filter tolerance scheme.



Discrete-Time Signal Processing, Third Edition
Alan V. Oppenheim • Ronald W. Schafer

We will design FIR filters satisfying the following specifications:

- $\omega_p = 0.4\pi$
- $\omega_s = 0.6\pi$
- $\delta_{p_1} = \delta_{p_2} = 0.01, \ \delta_s = 0.01$
-

Windowing Method (50 Points)

Now, using the Kaiser window, come up with a window-based FIR filter with the above specifications (presented just after the first image of the notebook)

For theoretical understanding, see (i) page 499, Example 7.8 of the Second Edition of the textbook, or (ii) page 569, Section 6.1 of Third Edition of your textbook.

- From your theoretical understanding, what is the filter order (approx)? *ENTER YOUR ANSWER HERE*

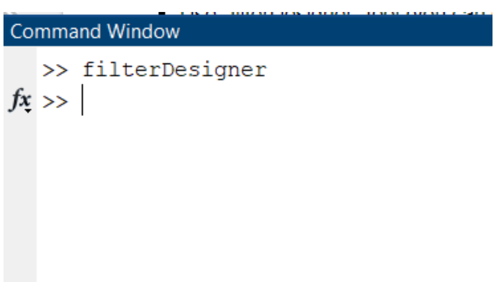
$$M = \frac{A-8}{2.285\Delta\omega} = \frac{(-20log_{10}\delta) - 8}{2.285(\omega_s - \omega_p)}$$

substituting in the values we get-

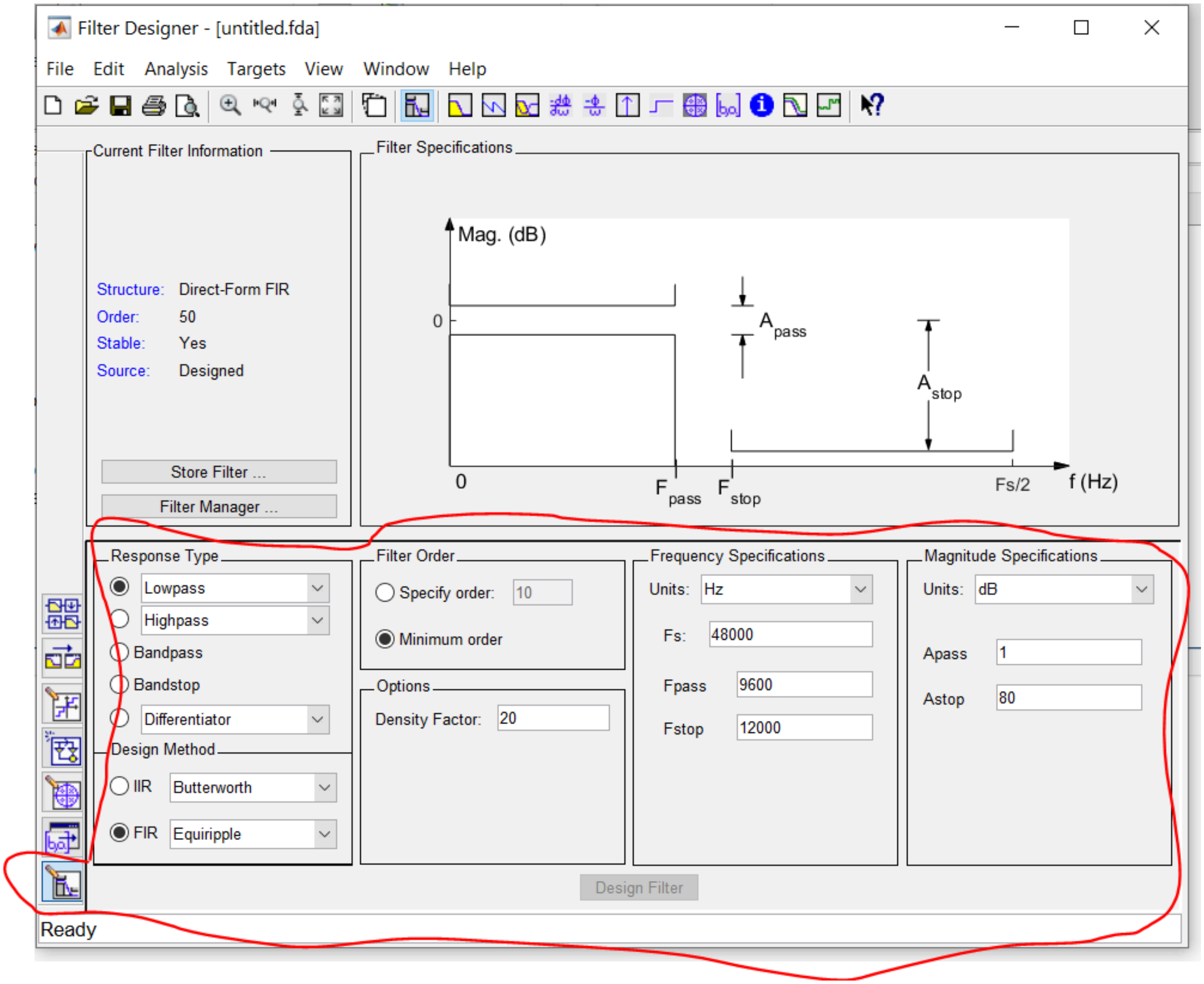
$M \approx 22.29$

We always take ceil(M) so M = 23 so expected order from theoritical calculations comes out to be 23.

- Open "filterDesigner" tool (you can open it by typing *filterDesigner* in the Matlab command window as shown below.)



Open the below view. Here you can specify frequency and magnitude specifications.



- Obtain the coefficients of the filter. Write down the coefficients below.

```
% ASSIGN THE FILTER COEFFICIENTS TO "b".
b = [-0.002907211955384275854641362712982299854;
-0.004903665350658096937219720956591118011;
 0.007556835867128868303166377984325663419;
 0.011022695479796449855025564090738043888;
-0.015522540177083104726007967144596477738;
-0.021399197607360729428416235009535739664;
 0.029235890996987257128836290576145984232;
 0.040134591714861472810937925714824814349;
-0.056471920114176024640961770728608826175;
-0.084468388232154023587128222061437554657;
 0.147031071648377920091377291100798174739;
 0.450691837729664368517035200056852772832;
 0.450691837729664368517035200056852772832;
 0.147031071648377920091377291100798174739;
-0.084468388232154023587128222061437554657;
-0.056471920114176024640961770728608826175;
```

```
 0.040134591714861472810937925714824814349;
```

- Does the number of entries in b correspond to the above theoretically obtained filter order?

ANSWER

Yes,

Order comes out to be 23 so filter coefficients from the filterDesigner toolbox comes out to be 24 and also from theoritical calculation the coefficients are 23. So,both results match each other.

- Observe the pole-zero plot, magnitude and phase response, group delay plot and make your comments.

ANSWER

Pole zero plot observations-

There are 23 zeros and 1 pole.

All zeros come in conjugate pair and the obtained system is inverse.

11 of the zeros lie on unit circle.

Magnitude Response observations-

As we have use window function so the the filter tries to minimize mean squared error.Also , it matches as the figure given

above.

Phase Response observations-

Phase response is generalized linear as it was expected from theoritical knowledge.

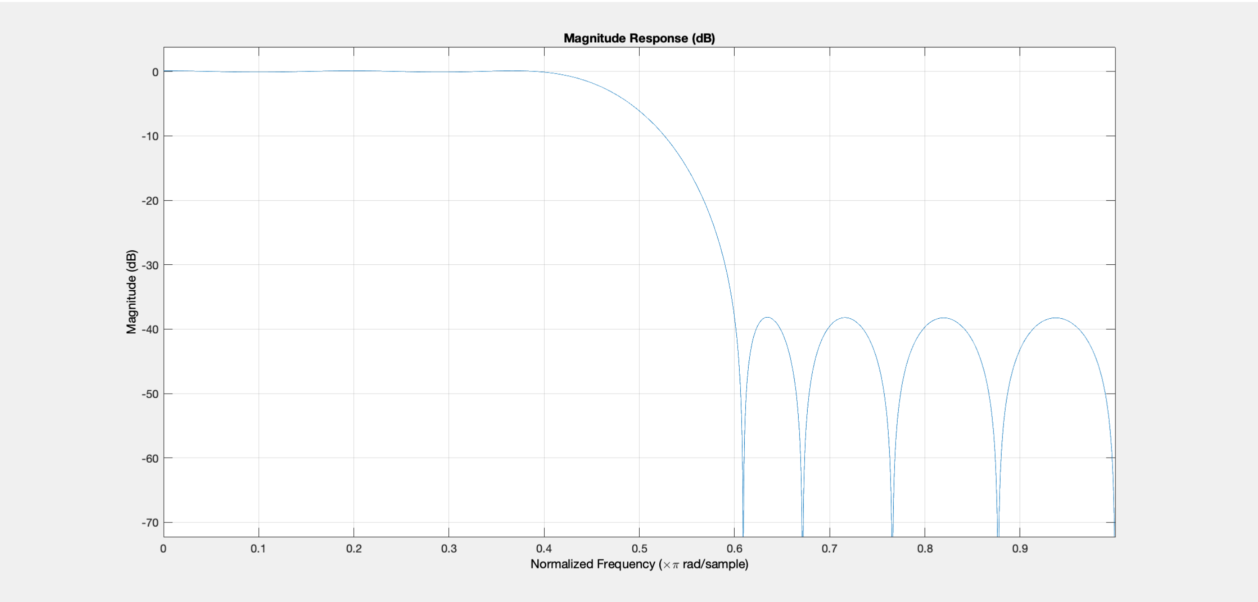
Group Delay-

Group delay comes out to be constant.(GD = 11.5)

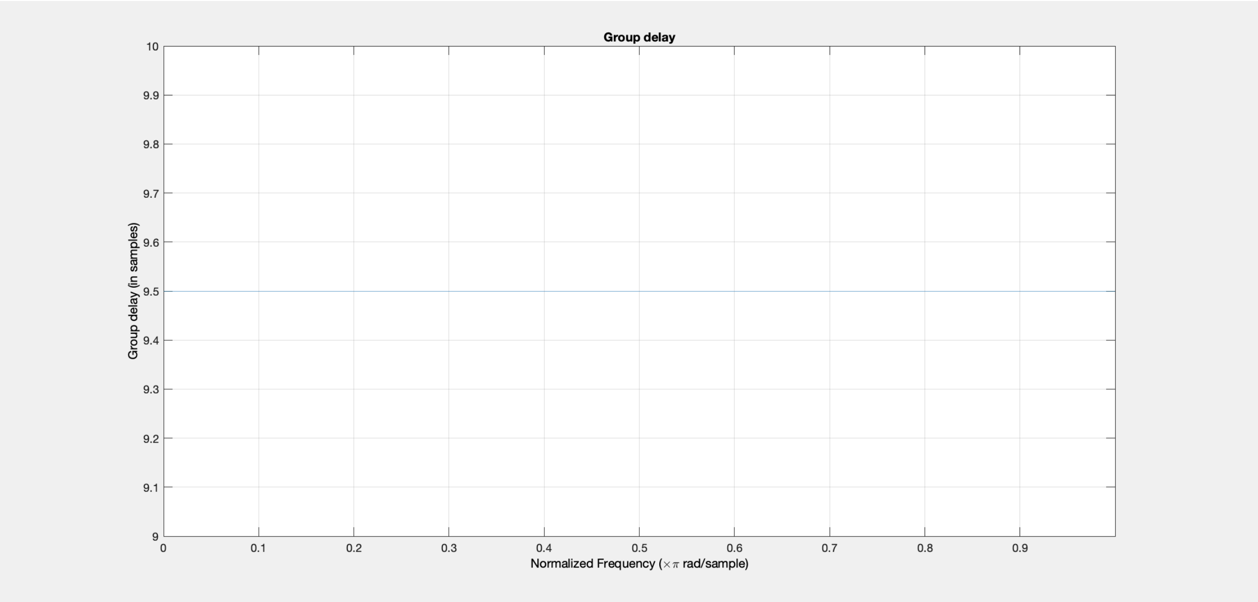
Min-Max Method (50 Points):

You already know that the the FIR filter designed using the windowing method has minimimum mean squared error with respect to the ideal filter response. Now, we will design a filter which minimizes the maximum error with respect to the ideal filter. Now watch optionally watch https://drive.google.com/file/d/1oIeN7SOJp_EZcXk_god2Ze9BxSz7iUyr/view?usp=sharing to understand the theory behind the optimal min-max FIR filter design method. Design an FIR filter satisfying the specifications presented just after the first image of the notebook using firpm.

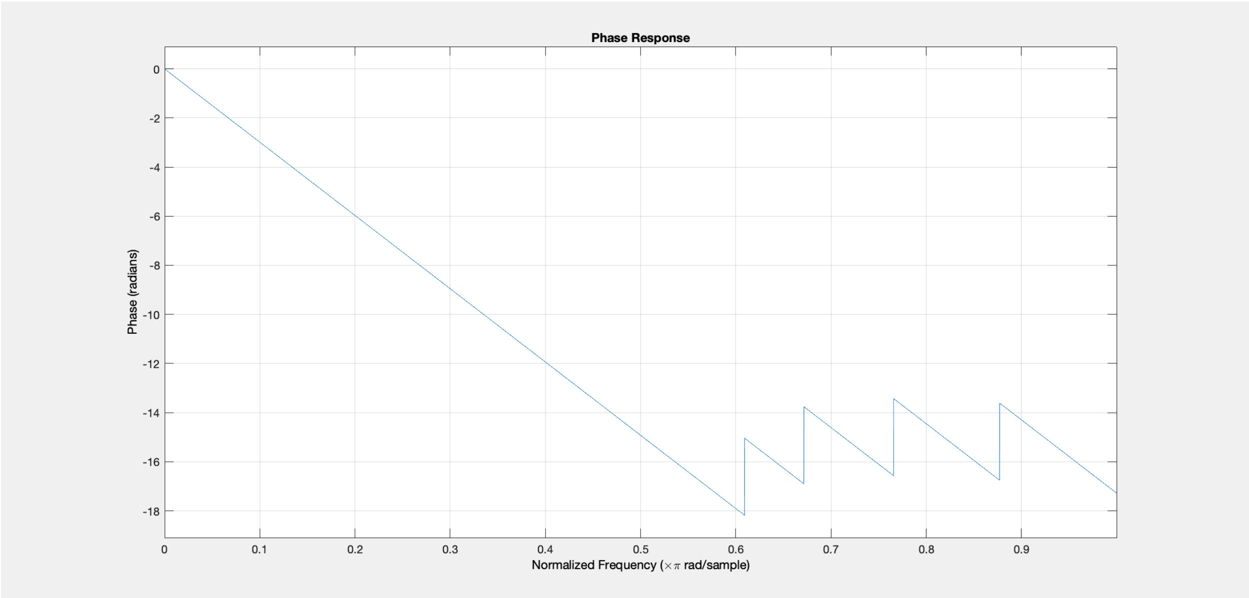
```
% WRITE YOUR CODE HERE
[n,fo,ao,w] = firpmord([9600 14400],[1 0],[0.01 0.01],48000);
b = firpm(n,fo,ao,w);
fvtool(b,1,'Magnitude')
```



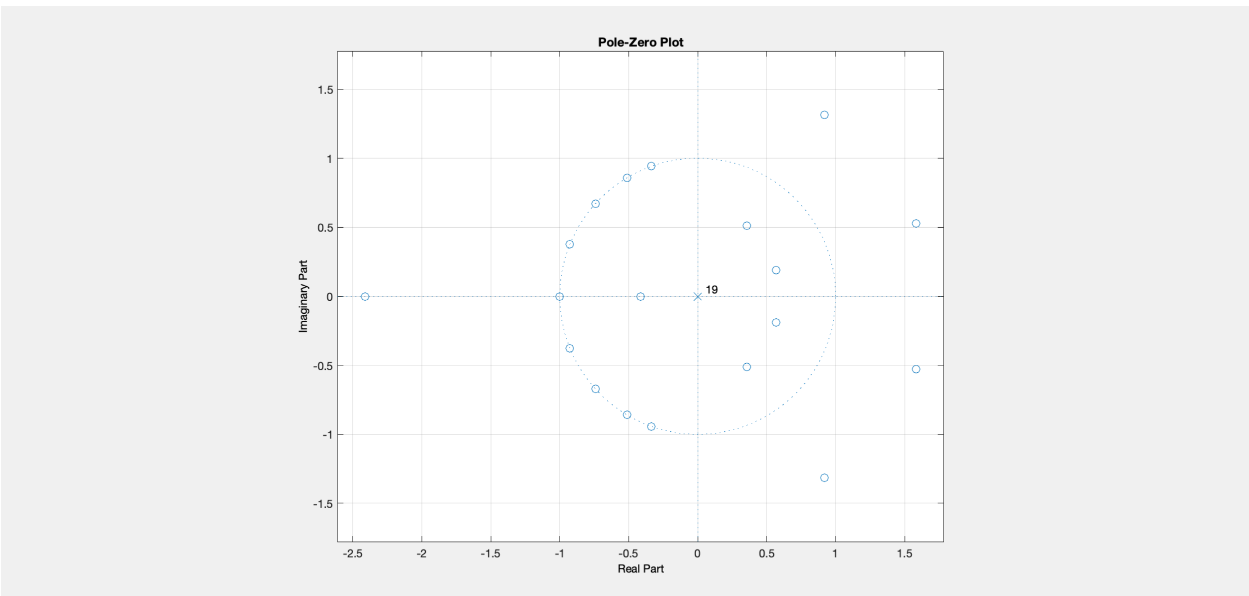

```
fvtool(b,1,'grpdelay')
```




```
fvtool(b,1,'phase')
```




```
fvtool(b,'polezero')
```



- Observe the pole-zero plot, magnitude and phase response, group delay plot and make your comments.

ANSWER

Pole-Zero plot-

We get 19 zeros and 1 pole as we can see from the plot above.

Magnitude Response observations-

This time the transition band was narrower and there is more mean squared error as we can see the ripples are larger.

Phase Response observations-

Phase response is generalized linear as it was expected from theoritical knowledge.

Group Delay-

Group delay comes out to be constant.(GD = 9.5)

- Give a comparison with the filter obtained using the windowing method above.

ANSWER

- *Parks-McClellan filter gave us lesser number of coefficients than the kaiser window hence the order of filter obtained is lower in Parks-McClellan.*
- Parks-McClellan algorithm which is based on polynomial approximations.
- The Kaiser windows are a family of near optimal windows that allow controlled trade-offs between the sidelobe amplitudes and mainlobe widths.
- The transition band was narrower in case of Parks-McClellan in comparison to Kaiser.
- Mean squared error will be less in Kaiser and ripples are larger in Parks McClellan.

Thank you!