MTH 203 Mathematics III (Multivariate Calculus)

Tutorial Sheet 2

1) Examine the following functions for local maxima, local minima and saddle points: (i) $4xy - x^4 - y^4$, (ii) $x^3 - 3xy^2$

2) Find the absolute maxima of $f(x,y) = xy$ on the unit disc $\{(x,y) : x^2 + y^2 \leq 1\}$

3) Find the maximum and minimum values of the function $f(x,y,z) = x^2 + y^2 + z^2 - 4(x+y+z)$ on $D = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 16, z \geq 0\}$

4) (a) For $n \geq 2$, let $f : \mathbb{R}^n \to \mathbb{R}$ be given by $f(x_1, x_2, \ldots, x_n) = x_1^2 x_2^2 \cdots x_n^2$ and let $c \in \mathbb{R}$. Find the maximum value of f subject to the constraint $g(x_1, x_2, \ldots x_n) = x_1^2 + x_2^2 + \cdots + x_n^2 - c^2 = 0$.

(b) Using (a), prove the inequality $(a_1 a_2 a_3 \ldots a_n)^{1/n} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}$ for any positive real numbers $a_1, a_2, \ldots, a_n$.

5) Let $f(x,y) = x^2 + y^3$ and $g(x,y) = x^4 + y^6 - 2$. Find the points of maxima and minima of the function f subject to the constraint $g(x,y) = 0$.