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QUIZ – 15 minutes – open notes

1. Prove that every simple planar graph with fewer than 12 vertices has a vertex of degree at most four.
2. Give an example of the following or expalin why no such example exists: a nonplanar graph with chromatic number 3.

Each question worth 5 marks.

Saturday, April 11, 2015

QUIZ. THURSDAY 20150409

SOLUTION

1. Assume BWOC that there exists a planar graph G of order $n \leq 11$ s.t. $\delta(G) \geq 5$.

$$\therefore \; 2|E(G)| = \sum_{v \in V(G)} d(v) \geq 5n \qquad (1)$$

Putting $|E(G)| = m$, we also have

$m \leq 3n - 6$ since G is planar

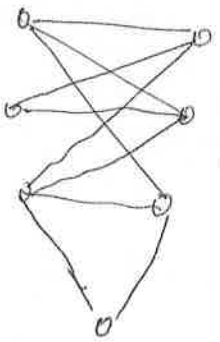
$$\therefore \; 5n \leq 2m \leq 2(3n-6) = 6n - 12$$

$$\Rightarrow 12 \leq n \qquad \Rightarrow \Leftarrow$$

(PTO)

Q2. There can be non-planar graphs with ~~correct~~ chromatic number 3

Example 1: G consists of a $K_{3,3}$ and an additional vertex v which is joined to precisely one vertex of the partite sets of the $K_{3,3}$.



Since G contains a $K_{3,3}$, it is nonplanar by Kuratowski's Theorem.
Since G is not bipartite, $\chi(G) > 2$.
OTOH, we can properly ~~color~~ color G with 3 colors by giving v a color different from ~~those~~ used to color the $K_{3,3}$ subgraph.

$\therefore\ 2 < \chi(G) \leq 3 \Rightarrow \chi(G) = 3$

as required.

Example 2: The Petersen Graph, P

Since P is 3-regular, $\chi(G) \leq 3$ by Brooks' Theorem.

OTOH, P contains a 5-cycle so it is not bipartite.

$\therefore\ \chi(G) > 2$.

Hence, again $\chi(G) = 3$.

Since P contains a subdivision of K_5, it is nonplanar by Kuratowski's Theorem.