QUIZ – 20 minutes – open notes

1. Prove that the complement of a simple disconnected graph G must be connected
2. Let v be a cut vertex in G, prove or disprove: G' – v is connected, where G' indicates the complement of G.

Each question worth 5 marks.

Thursday, January 22, 2015

Q 1. Let G' denote the complement of G.

Consider $u, v \in V(G)$.

Case 1. u and v are in different components of G.

Then there is no uv edge in $E(G)$.

Hence, there is a uv edge in $E(G')$;

and so there is a ~~path~~ uv- path in G'

Case 2. u and v are in the same component, say K_1, in G.

Now, since G is disconnected, it has a component K_2 different from K_1.

Let x be a vertex in K_2.

Then, by case 1, we have edges ux and vx in G'.

Hence, there is a path uxv from u to v in G'.

Since there is a uv-path in G' for any vertices u, v, G' is connected.

Q2, $G' - v$ is connected.

- i.e. the statement is to proved; here v is a cut vertex of G.

Proof: Let $K_1, K_2, \ldots, K_m$, $m \geq 2$, be the components of ~~[illegible]~~ $G - v$ and let x, y be any two vertices of G.

We need to show there is an xy-path in $G' - v$

Case 1. x and y are in different components of $G - v$.

$\therefore$ there is no xy-edge in $E(G)$.

$\therefore$ the edge $xy \in E(G')$

$\therefore$ the edge $xy \in E(G' - v)$

Case 2. ~~Sa~~ x and y belong to the same component of $G - v$, say K_i.

Then, there is at least one other ~~c~~ component, say K_j.

Let $z \in K_j$.

As shown in Case 1, the edges xz and yz $\in E(G' - v)$.

Hence, we get ~~an~~ ~~alternate~~ path xzy between x and y in $G' - v$.

Q2. Alternate Method,

We ~~show~~ claim that $G' - v = (G - v)'$

Now, since $G - v$ is a disconnected graph (since v is given to be a cut-vertex), $(G - v)'$ is connected by Q 1, and we are done.

Proof of Claim :-

~~We~~ Put $H = G' - v$

and $K = (G - v)'$.

Then, $V(H) = V(K) = V(G) - v$

It remains to show that $E(H) = E(K)$.

a) Suppose $e \in E(H)$

$\therefore$ $e \in E(G')$ and v is not incident with e

$\therefore$ $e \notin E(G)$ and v is not incident with e

$\therefore$ $e \notin E(G - v)$

$\therefore$ $e \in E(G - v)' = E(K)$.

b) Suppose $e \in E(K)$

~~$\therefore$ $e \notin E(G - v)$~~

$\therefore$ e is an edge ~~of the~~ on vertex set of $G - v$, i.e. v is not incident with e

But e is not an edge of $G - v$, i.e. $e \notin E(G)$ and ~~e~~ v is not incident with v,

i.e. $e \in E(G' - v) = E(H)$

$\therefore E(H) = E(K)$