For Thursday 20150205

(this is a simplified and tightened version of what was done in the class).

Proof of Theorem 3 (König - Egerváry):-

Let G be a fixed X-Y bipartite graph, and let M be a maximum matching in G. We will construct a vertex cover Q with $|Q| = |M|$. In view of earlier remark, the result follows.

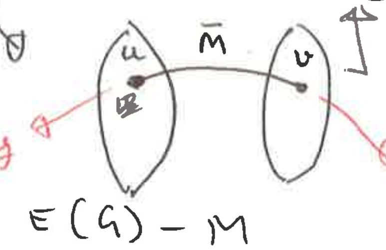
For each edge $xy \in M$, select one of its end-points :- select y if y is the end-point of an alternating path starting in X s.t. its first edge (from X to Y) is not in M; i.e. if $P = x_1 y_1 \cdots x_k y$ and $x_1 y_1 \notin M$. Else, select x.

Put Q = set of selected vertices, so $|Q| = |M|$. We need to show Q is a vertex cover.

So let $uv \in E(G)$, $u \in X$, $v \in Y$.

If $uv \in M$, then either u or $v \in Q$ and we are done. So we may assume $uv \notin M$.

But, either $uy \in M$ for some $y \in Y$ or $xv \in M$ for some $x \in X$; if not, then $M \cup \{uv\}$ is a matching larger than M $\Rightarrow\Leftarrow$



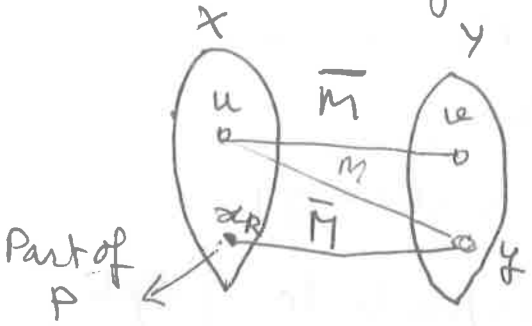
no edges of M ending in u

no edges of M ending in v

$\bar{M} = E(G) - M$

(PTO)

Case 1. $uy \in M$ for some $y \in Y$.



Then either $u \in \varphi$ or $y \in \varphi$. If $u \in \varphi$, we are done, so ~~ass~~ suppose $y \in \varphi$.

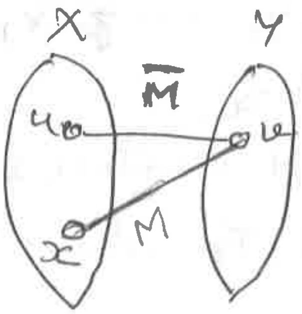
$\therefore$ y is the end-point of an alternating path $P: x_1 y_1 \cdots x_k y$.

If $x_k \neq u$, then $w u y x_k$ is an M-augmenting path $\Rightarrow\Leftarrow$

~~$x_k = u$~~

But if $x_k = u$, then since P is an alternating path with $x_1 y_1 \notin M$, it follows that ~~$x_k y \notin M$~~ $x_k y = uy \notin M$ $\Rightarrow\Leftarrow$

Case 2. $xw \in M$ for some $x \in X$.



Then w being an end-point of the alternating path $P: uw$ has to be the vertex for φ selected on behalf of the edge $xw \in M$, i.e. $w \in \varphi$.