NM Assignment2

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Evaluating $e^{-0.25}$ using $x_0 = 0$ and $x_1 = 0.5$.
The function values are

$$f(x_0) = 1$$

$$f(x_1) = 0.6065$$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} * f(x_0) = \frac{x - 0.5}{0 - 0.5} * 1 = -2x + 1$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} * f(x_1) = \frac{x - 0}{0.5 - 0} * 0.6065 = 1.1213x$$

$$p_1(x) = L_0(x) + L_1(x) = 1 - 0.7869x$$

$$\Rightarrow p_1(0.25) = 1 - 0.7869 * 0.25 = 0.8024$$

Evaluating $e^{-0.75}$ using $x_0 = 0.5$ and $x_1 = 1$.
The function values are

$$f(x_0) = 0.6065$$

$$f(x_1) = 0.3679$$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} * f(x_0) = \frac{x - 1}{0.5 - 1} * 0.6065 = -1.2131x + 1.2131$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} * f(x_1) = \frac{x - 0.5}{1 - 0.5} * 0.3679 = 0.7358x - 0.3679$$

$$p_1(x) = L_0(x) + L_1(x) = 0.8452 - 0.4773x$$

$$\Rightarrow p_1(0.75) = 0.8542 - 0.4773 * 0.75 = 0.4872$$

Now, using quadratic interpolation using $x_0 = 0$ and $x_1 = 0.5$ and $x_2 = 1$.

$$f(x_0) = 1$$

$$f(x_1) = 0.6065$$

$$f(x_2) = 0.3679$$

$$L_0(x) = \frac{(x-x_1)*(x-x_2)}{(x_0-x_1)*(x_0-x_2)} * f(x_0) = \frac{(x-0.5)*(x-1)}{(0-0.5)*(0-1)} * 1 = 2x^2 - 3x + 1$$

$$L_1(x) = \frac{(x-x_0)*(x-x_2)}{(x_1-x_0)*(x_1-x_2)} * f(x_1) = \frac{(x-0)*(x-1)}{(0.5-1)*(0.5-1)} * 0.6065 = -2.426x^2 + 2.426x$$

$$L_0(x) = \frac{(x-x_0)*(x-x_1)}{(x_2-x_0)*(x_2-x_1)} * f(x_2) = \frac{(x-0)*(x-0.5)}{(1-0)*(1-0.5)} * 0.3679 = 0.7358x^2 - 0.3679x$$

$$p_2(x) = L_0(x) + L_1(x) + L_2(x) = 0.3098x^2 - 0.9419x + 1$$

$$p_2(0.25) = 0.0194 - 0.2287 + 1 = 0.7840$$

$$p_2(0.75) = 0.1743 - 0.6862 + 1 = 0.4772$$

Error approximation :
For linear interpolation, $\epsilon_{1a}(x) = |(x-0)*(x-0.5)*e^{-t}/2|$, now $x = 0.25, 0 \le t \le 0.5$
$\Rightarrow 0.0189 \le \epsilon_{1a}(0.25) \le 0.0313$ $\epsilon_{1b}(x) = |(x-0.5)*(x-1)*e^{-t}/2|$, now $x = 0.75, 0.5 \le t \le 1$
$\Rightarrow 0.0115 \le \epsilon_{1a}(0.75) \le 0.0189$

Actual error :
$f(0.25) = 0.7788 \Rightarrow \epsilon(0.25) = 0.8024 - 0.7788 = 0.0236$ which is within the range predicted.
$f(0.75) = 0.4724 \Rightarrow \epsilon(0.75) = 0.4872 - 0.4724 = 0.0148$ which is within the range predicted.

For quadratic interpolation, $\epsilon_2(x) = |(x-0)*(x-0.5)*(x-1) - e^{-t}/6|$, now $x = 0.25, 0.75; 0 \le t \le 1$
$\Rightarrow 0.0029 \le \epsilon_{1a}(0.25) \le 0.0061$

Actual error :
$\epsilon(0.25) = 0.7840 - 0.7788 = 0.0052$ which is within the range predicted.
$\epsilon(0.75) = 0.4772 - 0.4724 = 0.0048$ which is within the range predicted.

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Code in python

Figure 1: $L_0(x)$

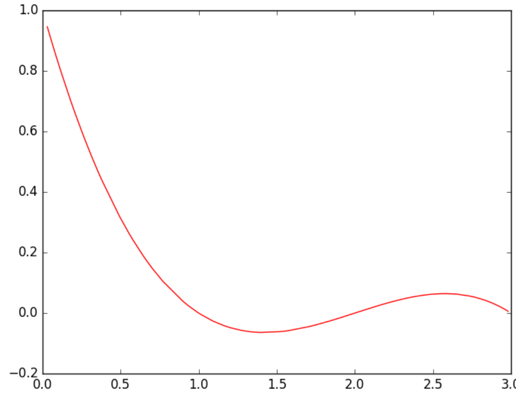


Figure 2: $L_1(x)$

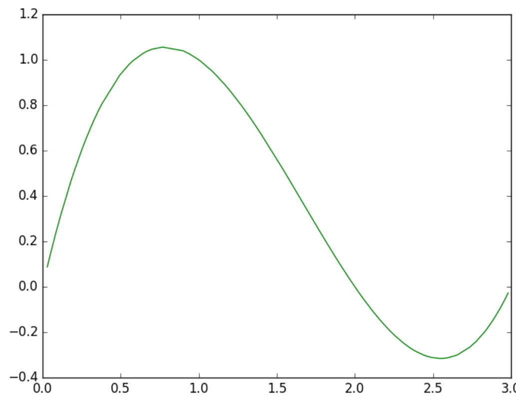


Figure 3: $L_2(x)$

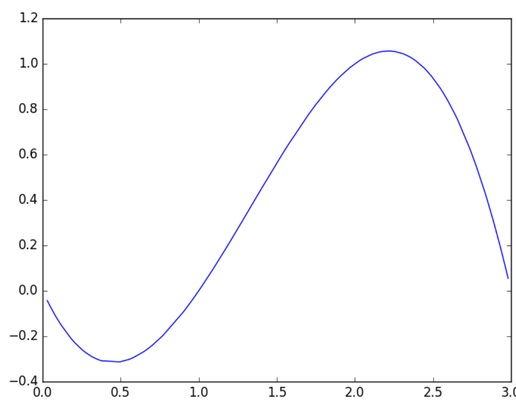


Figure 4: $L_3(x)$

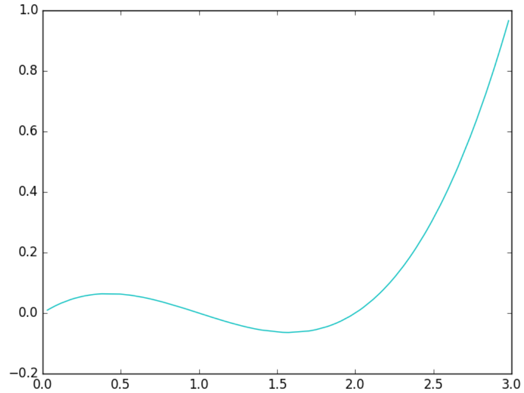


Figure 5: $p_3(x)$

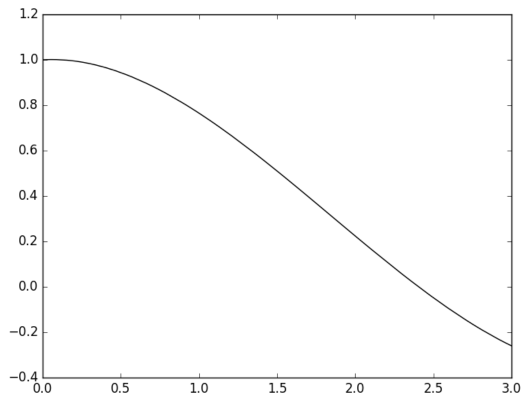
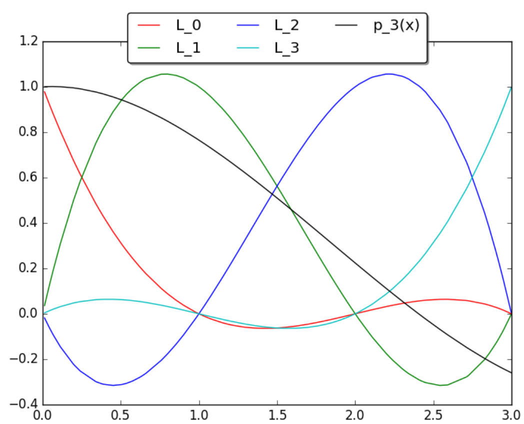


Figure 6: $Comparsion of all graphs$



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$f(x) = x^4$
Data points:

$$x_0 = -1 \qquad f_0 = 1$$
$$x_1 = 0 \qquad f_1 = 0$$
$$x_2 = 1 \qquad f_2 = 1$$

Using newton's divided difference.

x_j	f_j	$f[x_j, x_{j+1}]$	$f[x_j, x_{j+1}, x_{j+2}]$
-1	1	-1	1
0	0	1	
1	1		

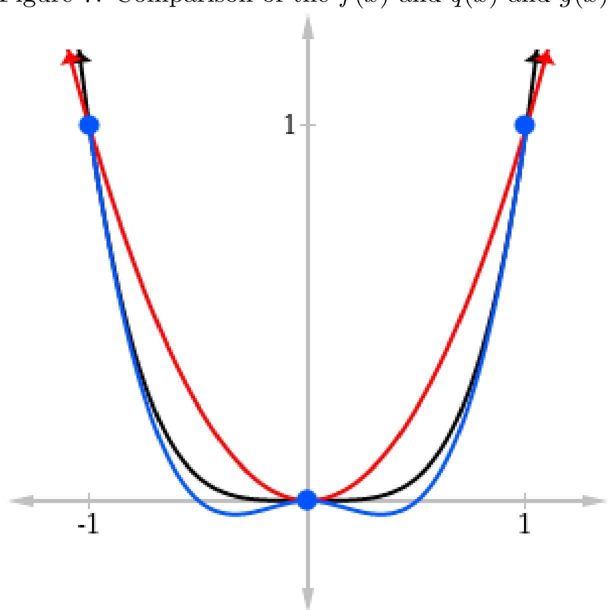
$$p_2(x) = f_0 + (x - x_0)f[x_0, x_1] + (x - x_1)(x - x_0)f[x_0, x_1, x_2]$$

$$p_2(x) = 1 - x - 1 + x^2 + x = x^2$$

As done in class, spline interpolation polynomial :

$$q_0(x) = -x^2 + 2x^3 \qquad -1 \le x \le 0$$
$$q_1(x) = -x^2 - 2x^3 \qquad 0 \le x \le 1$$

Figure 7: Comparison of the $f(x)$ and $q(x)$ and $g(x)$



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Cubic spline interpolation :
Data points:

$$\begin{aligned} x_0 &= 0 \qquad & f_0 &= 1 \\ x_1 &= 2 \qquad & f_1 &= 9 \\ x_2 &= 4 \qquad & f_2 &= 41 \\ x_3 &= 4 \qquad & f_3 &= 41 \\ k_0 &= 0 \qquad & k_3 &= -12 \end{aligned}$$

First, let us determine k_1, k_2.
We see that the nodes are equidistant, thus we can say

$$\begin{aligned} k_{j-1} + 4k_j + k_{j+1} &= \frac{3}{h}[f_{j+1} - f_{j-1}] \\ k_0 + 4k_1 + k_2 = 60 &\Rightarrow 4k_1 + k_2 = 60 \\ k_1 + 4k_2 + k_3 = 48 &\Rightarrow k_1 + 4k_2 = 60 \\ &\Rightarrow k_1 = 12, k_2 = 12 \end{aligned}$$

$$q_j(x) = a_{j0} + a_{j1}(x - x_j) + a_{j2}(x - x_j)^2 + a_{j3}(x - x_j)^3$$

$$a_{j0} = f_j \tag{1}$$
$$a_{j1} = k_j \tag{2}$$
$$a_{j2} = \frac{3}{h^2}(f_{j+1} - f_j) - \frac{k_{j+1} + 2k_j}{h} \tag{3}$$
$$a_{j3} = \frac{2}{h^3}(f_j - f_{j+1}) + \frac{k_{j+1} + k_j}{h} \tag{4}$$

$$a_{00} = 1 \qquad a_{01} = 0 \qquad a_{02} = 0 \qquad a_{03} = 1$$

$$\Rightarrow q_0(x) = 1 + x^3$$

$$a_{10} = 9 \qquad a_{11} = 12 \qquad a_{12} = 6 \qquad a_{13} = -2$$

$$\Rightarrow q_1(x) = -2x^3 + 18x^2 - 36x + 25$$

$$a_{20} = 41 \qquad a_{21} = 12 \qquad a_{22} = -6 \qquad a_{23} = 0$$

$$\Rightarrow q_0(x) = -6x^2 + 60x - 103$$

Figure 8: Cubic Spline interpolation

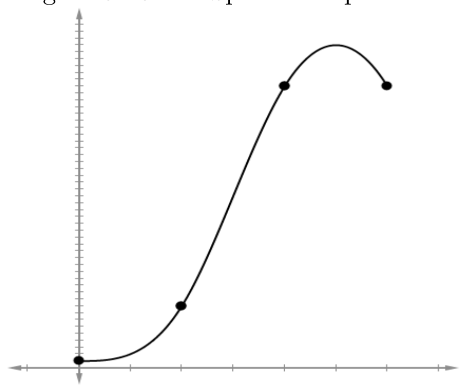

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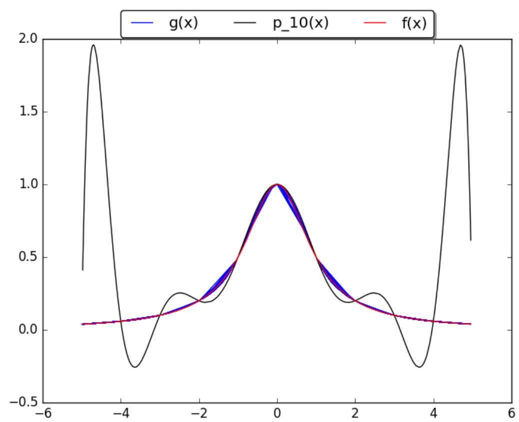
Figure 9: Comparison of $f(x)$, $g(x)$ and $p(x)$



$$Max deviation of g(x) from f(x) = 0.021970363202157617 \tag{5}$$
$$Max deviation of p(x) from f(x) = 1.9156484836078913 \tag{6}$$