**Ques-1 Prove or disprove:-**

* 1. **If every vertex of a simple graph G has a degree 2, then G is a cycle.**

**Ans-1.1** **False:-** We can disprove it by giving a counter example. Consider there are 2 components of a graph G each one being a triangle making n=6 and e=6 in total. Every vertex has degree=2 and there are cycles within both components but G itself is not a cycle.

* 1. **If every vertex of a connected simple graph G has a degree 2, then G is a cycle.**

**Ans-1.2** **True**. Assume, for contradiction, that G has no cycle, and consider the longest path P in G (one must exist, since the graph is ﬁnite). Let v be the ﬁnal vertex in P — since v has degree 2, it must have two edges e1 and e2 incident on it, of which one, say e1, is the last edge of the path P. Then e2 cannot be incident on any other vertex of P since that would create a cycle (v, e2, [section of P ending in e1], v). So e2 and its other endpoint are not part of P, and can be appended to P to give a strictly longer path, which contradicts our choice of P. Hence G

must contain a cycle C. Now, if u is a vertex of G not belonging to C, then since G is connected, there must be a path from u to a vertex v on C. But then this would contradict the fact that d(v) = 2. Thus, G = C, as desired.

* 1. **The complement of a simple disconnected graph must be connected.**

**Ans-1.3 True.** Let G' denote the complement of G. Consider any two vertices u, v in G. If u and v are in different connected components in G, then no edge of G connects them, so there will be an edge {u, v} in G'. If u and v are in the same connected component in G, then consider any vertex w in a different connected component (since G is disconnected, there must be at least one other connected component). By our ﬁrst argument, the edges {u, w} and {v, w} exist in G', so u and v are connected by the path (u, w, v). Hence any two vertices are connected in G', so the whole graph is connected.

* 1. **If G is a non trivial graph and has no cycle, then G has a vertex of degree 1.**

**Ans-1.4** G is a non-trivial graph with no cycles. If it has no vertex of degree 1, then it has a component C in which every vertex has degree at least 2. But by Proposition 6, C must contain a cycle.

* 1. **A vertex of degree 1 cannot be a cut vertex.**

**Ans-1.5 True.** Let G be any graph and u∈V(G), d(u)=1 and u be a cut vertex. Also let w belong to V(G) such that (u,w)belongs to E(G). Now if we remove u from G, we will remove (u,w), its sole edge. Thus we would be left with the same number of components as before – which contradicts our assumption that u is a cut vertex. Hence there can be no cut vertex of degree 1.

**Ques-2 For a simple connected graph G, prove that a vertex v has a neighbor in every component of G-v.**

**Ans-2:** Let *G* be a graph and let *v* be a vertex of *G*. Let *H* be a component of *G – v* and let *u* be any vertex of *H*. Since *H* is connected, there is a *vu*-path, the first vertex on this path is a neighbor of *v* in *H*.

**Ques-3 Let G be the graph whose vertex set is the set of k-tuples with coordinates in {0,1} with x adjacent to y when x and y differ in exactly one position. Determine whether G is bipartite.**

**Ans-3:** For example, k = 3, V(G) = {000, 001, 010, 011, 100, 101, 110, 111},

E(G) = {{000, 001}, {000, 010}, {000, 100}, {001, 011}, {001, 101}, {010, 011}, {010, 110},

{011, 111}, {100, 101}, {100, 110}{101, 111}, {110, 111}}.

Let X = {001, 010, 100, 111}, Y = {000, 011, 101, 110} be partite sets of G.

Then, adjacent vertices differ in exactly one position, no edges in X or Y, and G is a bipartite graph. In general, let X be the set of k-tuples with odd numbers of 1’s and Y be the set of k-tuples with even numbers of 1’s.Then, adjacent vertices have opposite parity, no edges in X or Y and G is a bipartite graph.

**Ques – 4 Let v be a cut vertex in G, prove that G’-v is connected.**

**Ans-4:** Since v is a cut vertex of G, let (G'-v) have components C1,C2,.....,Ck, k > 1.

Consider any two vertices x∈ Ci and y∈ Cj, i≠j. Then (x,y)∉ E(G). Therefore (x,y) belongs to E(G'). Thus any two vertices in two different components of G-v are connected by a direct edge in G'-v. Now consider any two vertices (x,y) in the same component Ci. If (x,y) belongs to E(G) then (x,y) belongs to E(G'). If (x,y) belongs to E(G), then (x,y) belongs to E(G'). But then, consider any node z belongs to Cj, i≠j. We have already

shown that (x,z) belongs to E(G'-v) and (y,z) belongs to E(G'-v). Hence there is a path between x and y in (G'-v). Thus, there is a path between any two vertices in (G'-v). hence (G'-v) is connected.