Assignment 1: Probability review

CSE-319/519:Modern Algorithm Design

Due: Tuesday, August 8 2017, by 11:55pm

Assignments are to be submitted in groups of *two*. Upload the solutions on backpack as one PDF file with names of collaborators on the first page. If you had used external sources (websites, other books) please mention the same at the top of the page. Only one copy needs to be submitted per group.

Problem 1 (2 points).
Given events A, B, C, such that the events are pairwise independent, prove or disprove $P(A \cap B \cap C) = P(A)P(B)P(C)$.

Problem 2 (2 points).
For random variables $X_1, \ldots, X_n$, let the random variable $X = \sum_{i=1}^{n} c_i X_i$, where $c_i \in \mathbb{R}$, $i = 1, \ldots, n$. Prove that

$$\mathbb{E}[X] = \sum_{i=1}^{n} c_i \mathbb{E}[X_i]$$

Is it true that $Var[X] = \sum_{i=1}^{n} c_i^2 Var[X_i]$? If so, give proof, if not, a counter-example.

Problem 3 (2 points).
In a TV game show, you and your opponent have a choice of picking either a gold, silver, or plastic coin. Each shows up heads 60% of the time. However, their values are different. With a gold coin, a head is worth 10 points, while a tail is worth 2 points. With a silver coin you get 4 points for both head and tail; while a plastic coin gives you 3 points for a head, and 20 points for a tail.
You and your opponent can choose a coin one after the other. You can not choose the same coin. Each of you toss your chosen coin once, and the winner is the one with higher points, and wins the winner gets to take home a pet elephant. Of course, you want a pet elepant badly. Is it better for you to go first, or second ?

Problem 4 (2 points).
You receive TCP packets of a movie in your browser. However, the packets do not arrive in order. For some reason known only to yourself, you want to keep a copy of only the packet with the smallest number. Suppose n packets arrive in random order, and you keep a copy C of the smallest labeled packet seen thus far. How many times do you expect to change the value of C?

Problem 5 (2 points).
Suppose you come into possession of n bar magnets. One bored afternoon you decide to play a game. You place the bar magnets in a line, each in random orientation. Of course, you

know that like poles repel, while opposite poles attract. Placing the magnets in a row in this manner creates several disjoint chains of magnets. What is the expected number of such chains? What is the variance?