classmate

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# MAD Assignment 1

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1. Experiment → A fair coin is tossed 3 times.
Let us define three events as follows:

$E_1$ : ~~Heads~~ same on ~~coin~~ toss 1 and 2
$E_2$ : " " " 2 and 3
$E_3$ : " " " 1 and 3

$P(E_1) = \frac{1}{2}$ $\quad P(E_2) = \frac{1}{2}$ $\quad P(E_3) = \frac{1}{2}$

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\Rightarrow E_1 \cap E_2$ : same on coins 1, 2, 3 $= E_2 \cap E_3 = E_1 \cap E_3$

$\Rightarrow P(E_1 \cap E_2) = \frac{1}{4} = P(E_1) \cdot P(E_2)$

$P(E_2 \cap E_3) = \frac{1}{4} = P(E_2) \cdot P(E_3)$

$P(E_1 \cap E_3) = \frac{1}{4} = P(E_1) \cdot P(E_3)$

$\Rightarrow$ $E_1$, $E_2$ and $E_3$ are pairwise independent.

$P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \neq P(E_1) P(E_2) P(E_3)$

$\Rightarrow$ $E_1, E_2, E_3$ are not mutually independent.

2. If we can prove that

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

We can generalise it to

$$\mathbb{E}[\sum_i X_i] = \sum \mathbb{E}[X_i] \quad (\because \text{addition is associative})$$

Moreover we know that $\mathbb{E}[cX] = c\,\mathbb{E}[X]$

when $c$ is a constant

$$\Rightarrow \mathbb{E}[\sum_i c_i X_i] = \sum c_i \mathbb{E}[X_i]$$

Now, we need to prove $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$

By definition we know that

$$\mathbb{E}[X] = \sum_x x P(X=x)$$

$$\Rightarrow \mathbb{E}[X+Y] = \sum_x \sum_y (x+y) P(X=x, Y=y)$$

$$= \sum_x \sum_y xP(X=x, Y=y) + yP(X=x, Y=y)$$

$$= \sum_x x \sum_y P(X=x, Y=y) + \sum_y y \sum_x P(X=x, Y=y)$$

$$= \sum_x x P(X=x) + \sum_y y P(Y=y) \quad \left[\because \sum_x P(X=x, Y=y) = P(Y=y)\right]$$

$$= \mathbb{E}[X] + \mathbb{E}[Y]$$

3. There are six possible scenario's

1. GP
2. GS
3. PG
4. PS
5. SP
6. SG

Let us consider the probability of winning of 1st person.

i) GP. Win → HH, $P_{(Win)} = \frac{6}{10} \times \frac{6}{10} = 36\%$

Loss = 1-P(Win) = 64%

⇒ Second person can take plastic coin

ii) GS Win → HT & HH $P(Win) = \frac{6}{10} = 60\%$

⇒ Second person will never take silver coin if gold is taken

iii) PG Win → TH & TT & HT

⇒ $P(Win) = \frac{4}{10} \times \frac{6}{10} + \frac{4}{10} \times \frac{4}{10} + \frac{6}{10} \times \frac{4}{10}$

$= \frac{24 + 16 + 24}{100} = 64\%$

P(Loss) = 36%

iv) PS Win → TH & TT

⇒ $P(win) = \frac{4}{10} \times \frac{6}{10} + \frac{4}{10} \times \frac{4}{10} = 40\%$

⇒ Second person will take silver coin if plastic taken

v) SG Win : HT & TT $\Rightarrow P(\text{win}) = \frac{6}{10} \times \frac{4}{10} + \frac{4}{10} \times \frac{4}{10}$

$= 40\%$

vi) SP Win : HH & TH $\Rightarrow P(\text{win}) = \frac{6}{10} \times \frac{6}{10} + \frac{4}{10} \times \frac{4}{10}$

$= 52\%$

$\Rightarrow$ Second person will take gold coin if plastic coin is taken

We can see that second person always has a better chance of winning.

Ans 4. $X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ packet causes a change in value of } C \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow X_i = 1$ if $\forall j < i \quad A[j] > A[i]$

$P(X_i = 1) = \frac{(i-1)!}{i!} = \frac{1}{i}$ [$\because$ I can fix the $i^{th}$ position to be smallest amongst 1st $i$ elements]

$\Rightarrow X = \sum X_i$
$=$ Total number changes

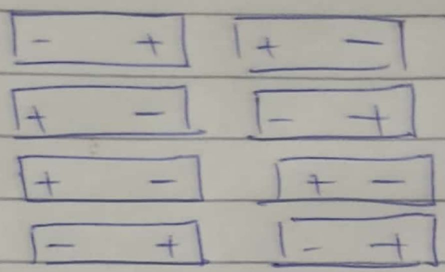
$E[X] = E[\sum X_i] = \sum E[X_i]$

$= \sum \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$

$\simeq \ln n$

5. $X_i = \begin{cases} 1 & \text{if there is a break after } i^{th} \text{ magnet} \\ 0 & \text{otherwise} \end{cases}$

$P(X_i = 1) = \frac{1}{2}$



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$X = \sum X_i$ = Total number of breaks in arrangement

$\Rightarrow E[X] = E[\sum X_i] = \sum E[X_i]$

$= \frac{n}{2}$

$\Rightarrow$ Expected number of ~~breaks~~ segments

$= 1 +$ expected number of breaks

$= 1 + \frac{n}{2}$