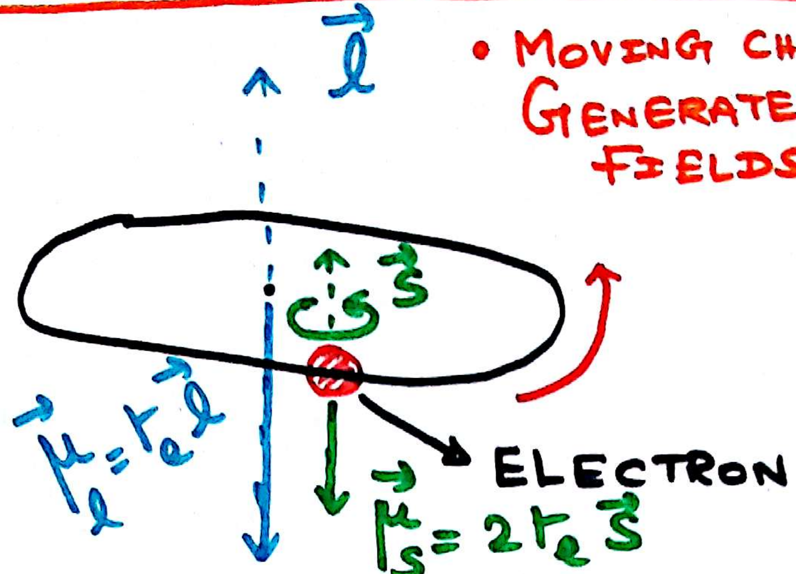
MAGNETIC MOMENT AND ANGULAR MOMENTUM

- MOVING CHARGES GENERATE MAGNETIC FIELDS



$\vec{l}$ = ORBITAL ANGULAR MOMENTUM

$\vec{\mu}_l = \gamma_e \vec{l}$ ⇒ MAGNETIC MOMENT DUE TO ORBITAL MOMENTUM

$\vec{S}$ = SPIN ANGULAR MOMENTUM

$\vec{\mu}_s = 2\gamma_e \vec{S}$ ⇒ MAGNETIC MOMENT DUE TO ELECTRON SPIN

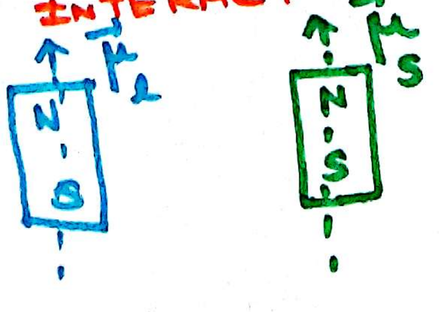
- **SPIN-ORBIT COUPLING:**

THE INTERACTION OF THE SPIN MAGNETIC MOMENT WITH THE MAGNETIC FIELD ARISING FROM THE ORBITAL ANGULAR MOMENTUM IS CALLED SPIN-ORBIT COUPLING

$\vec{\mu}_l$ ACTS LIKE A BAR MAGNET

$\vec{\mu}_S$ ACTS LIKE ANOTHER BAR MAGNET

THEY INTERACT WITH EACH OTHER



- TOTAL ANGULAR MOMENTUM

$\vec{J} = \vec{l} + \vec{S}$ (VECTOR SUM)

- TOTAL ANGULAR MOMENTUM

$$\vec{j} = \vec{l} + \vec{s}$$ (VECTOR SUM; RELATIVE ORIENTATION OF $\vec{l}$ AND $\vec{S}$ IS IMPORTANT)

$\vec{\mu}_l \uparrow \uparrow \vec{\mu}_s$ — $\vec{\mu}_l$ AND $\vec{\mu}_s$ ARE PARALLEL TO EACH OTHER

$$j = l + s$$

$\vec{\mu}_l \uparrow \downarrow \vec{\mu}_s$ — $\vec{\mu}_l$ AND $\vec{\mu}_s$ ARE ANTI-PARALLEL

$$j = l - s$$

- SPIN-ORBIT COUPLING ENERGY

$$E_{l,s,j} \propto \Big[\underbrace{j(j+1)}_{\text{ENERGY WHEN } \vec{l} \text{ AND } \vec{s} \text{ ARE COUPLED}} - \underbrace{l(l+1) - s(s+1)}_{\text{ENERGY WHEN } \vec{l} \text{ AND } \vec{s} \text{ ARE UNCOUPLED}}\Big]$$

QUANTUM MECHANICS

$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{1}{2}\left[\langle j^2 \rangle - \langle l^2 \rangle - \langle s^2 \rangle\right]$$

- NOTE: $\vec{j}$, $\vec{l}$, AND $\vec{s}$ ARE QUANTIZED.

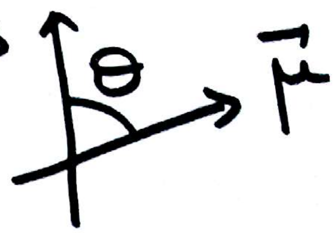
- ADDITION OF ANGULAR MOMENTA

- $$\vec{j} \cdot \vec{j} = (\vec{l} + \vec{s}) \cdot (\vec{l} + \vec{s})$$
$$j^2 = \vec{l} \cdot \vec{l} + \vec{s} \cdot \vec{s} + 2\, \vec{l} \cdot \vec{s}$$
$$j^2 = l^2 + s^2 + 2\, \vec{l} \cdot \vec{s}$$
$$\vec{l} \cdot \vec{s} = \frac{1}{2}\left(j^2 - l^2 - s^2\right)$$

MAGNETIC MOMENT IN A MAGNETIC FIELD

$\vec{\mu} \Rightarrow$ MAGNETIC MOMENT

$\vec{B} \Rightarrow$ EXTERNAL MAGNETIC FIELD



INTERACTION ENERGY

$$E = -\vec{\mu} \cdot \vec{B}$$

$$E = -\mu B \cos\theta$$

$$\theta = 0 \Rightarrow E = -\mu B \quad \text{(MINIMUM ENERGY)}$$

- FOR AN ELECTRON POSSESSING ORBITAL ANGULAR MOMENTUM $(\vec{l})$ AND SPIN ANGULAR MOMENTUM $(\vec{S})$ IN EXTERNAL MAGNETIC FIELD $\vec{B}$:

$\Rightarrow$ ORBITAL ANGULAR MOMENTUM PART:

$$E_l = -(\gamma_e \vec{l}) \cdot \vec{B}$$

$$E_l = -\gamma_e \vec{B} \cdot \vec{l}$$

$\gamma_e = -\dfrac{|e|}{2m_e}$; $m_e \rightarrow$ MASS OF ELECTRON

$\gamma_e \rightarrow$ MAGNETOGYRIC RATIO (OR) GYROMAGNETIC RATIO OF ELECTRON

IF UNIFORM MAGNETIC FIELD IS APPLIED ALONG Z-AXIS $\vec{B} = (0, 0, B_0)$

$$\Rightarrow E_l = -\gamma_e B_0 \hat{l}_z \; ; \; \hat{l}_z \Rightarrow \text{Z-COMPONENT OF } \vec{l}$$

RECALL: $\hat{l}_z \psi = m_l \hbar \psi \quad m_l = -l, -l+1, \ldots, l-1, l$

$(2l+1)$ DEGENERATE STATES $(B_0 = 0)$ SPLIT INTO DIFFERENT LEVELS $(B_0 \neq 0)$:

$m_l = l$
$\vdots$
$m_l = -l+1$
$m_l = -l$

$$E_l = -\gamma_e m_l \hbar B_0$$

$$\mu_{l,z} = \gamma_e m_l \hbar = -\frac{e}{2m_e} m_l \hbar$$

$$\mu_{l,z} = -\mu_B m_l$$

HERE $\mu_B = \dfrac{e\hbar}{2m_e}$ BOHR MAGNETON

FUNDAMENTAL QUANTUM OF MAGNETIC MOMENT

⇒ SPIN ANGULAR MOMENTUM PART :

SPIN QUANTUM NUMBER OF AN ELECTRON

$$S = 1/2$$

$$\vec{\mu}_s = g_e \gamma_e \vec{S}$$

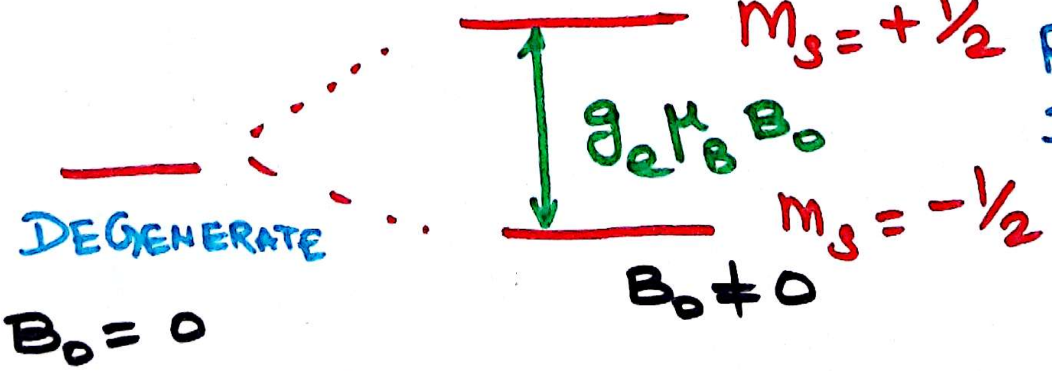
$$E_s = -(g_e \gamma_e \vec{S}) \cdot \vec{B}$$

$g_e = 2.002319$ g-factor or g-value of the electron

IF $\vec{B}$ IS ALONG Z-AXIS $\vec{B} = (0, 0, B_0)$

$$E_s = -g_e \gamma_e \hat{S}_z B_0 \; ; \; \hat{S}_z \Rightarrow \text{Z-COMPONENT OF } \vec{S}$$

$$\hat{S}_z \psi = m_s \hbar \psi \quad m_s = \pm 1/2$$



REMOVAL OF DEGENERACY

NUCLEUS IN MAGNETIC FIELDS

→ SPIN QUANTUM NUMBER OF A NUCLEUS I

→ $\hat{I}\psi = \sqrt{I(I+1)}\,\hbar\,\psi$

→ $\hat{I}_z\psi = m_I\hbar\psi$; $m_I = -I, -I+1, \ldots, I$

→ $I \Rightarrow$ INTEGER OR HALF-INTEGER, BUT POSITIVE

NUMBER OF PROTONS	NUMBER OF NEUTRONS	I
EVEN	EVEN	0
ODD	ODD	INTEGER (1, 2, 3, ...)
EVEN	ODD	HALF-INTEGER ($\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$)
ODD	EVEN	HALF-INTEGER ($\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$)

EXAMPLES:

INTEGER SPINS:

$^{2}H \Rightarrow I = 1$

$^{6}Li \Rightarrow I = 1$

$^{10}B \Rightarrow I = 3$

$^{14}N \Rightarrow I = 1$

$^{50}V \Rightarrow I = 6$

HALF-INTEGER SPINS:

$^{1}H \Rightarrow I = \frac{1}{2}$

$^{13}C \Rightarrow I = \frac{1}{2}$

$^{15}N \Rightarrow I = \frac{1}{2}$

$^{31}P \Rightarrow I = \frac{1}{2}$

$^{11}B \Rightarrow I = \frac{3}{2}$

$^{17}O \Rightarrow I = \frac{5}{2}$

$^{43}Ca \Rightarrow I = \frac{7}{2}$

$^{209}Bi \Rightarrow I = \frac{9}{2}$

- FOR A NUCLEUS

$$\vec{\mu} = \gamma \hat{I}$$

$$E_I = -\gamma \vec{B} \cdot \hat{I}$$

$$\gamma = \frac{g_I}{\hbar}\left(\frac{|e|\hbar}{2m_p}\right) = \frac{g_I}{\hbar}\mu_N$$

g_I: NUCLEAR g-FACTOR

m_p: PROTON MASS

γ: MAGNETO GYRIC RATIO OF THE NUCLEUS

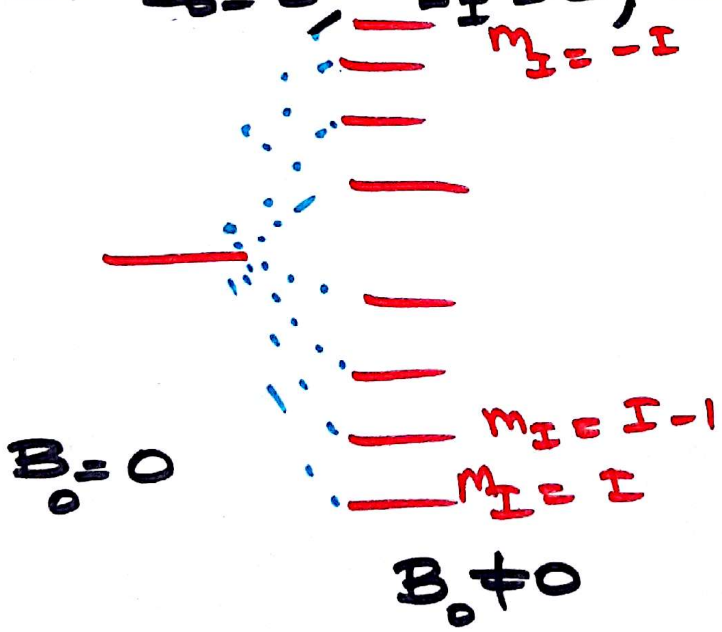
μ_N: NUCLEAR MAGNETON

IF $\vec{B}$ IS ALONG Z-AXIS $\vec{B} = (0, 0, B_0)$

$$E_I = -\gamma \hat{I}_z B_0$$

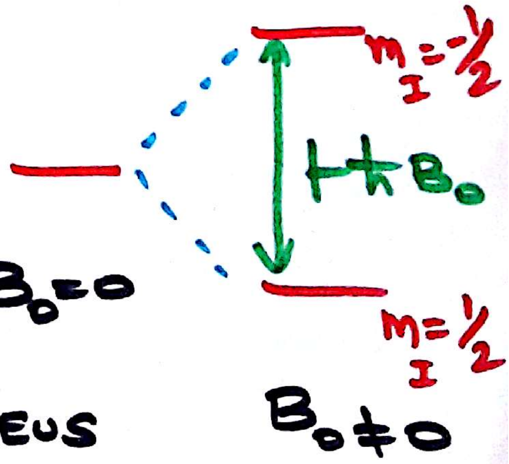
$$E_I = -\gamma m_I \hbar B_0 \; ; \; m_I = I, I-1, \ldots, -I$$

IF $B_0 = 0$, $E_I = 0$; BUT $E_I \neq 0$, WHEN $B_0 \neq 0$



EXAMPLE:

$I = 1/2$



FOR ELECTRON

$$\Delta E = g_e \mu_B B_0$$

$$h\nu = g_e \mu_B B_0$$

ELECTRON SPIN RESONANCE (ESR)
(MICROWAVE REGION)

FOR NUCLEUS

$$\Delta E = \gamma \hbar B_0$$

$$h\nu = \gamma \hbar B_0$$

NUCLEAR MAGNETIC RESONANCE (NMR)
(RADIOFREQUENCY REGION)