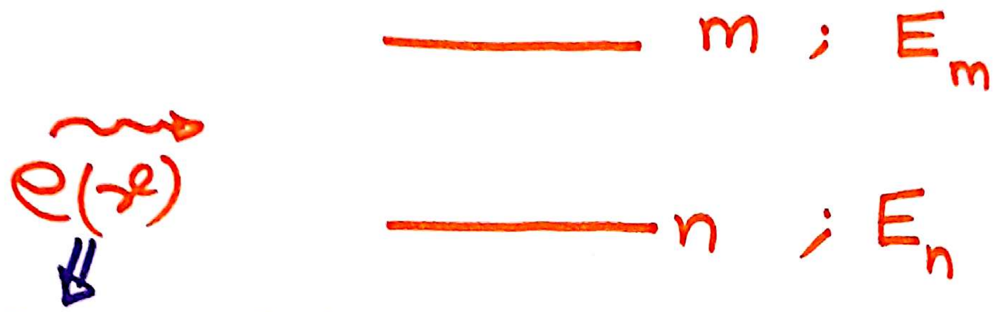
QUANTUM THEORY OF RADIATION
- A. EINSTEIN (1917)

- ABSORPTION
- EMISSION → SPONTANEOUS
  → STIMULATED
- TWO-LEVEL SYSTEM



$\boxed{E_m > E_n}$

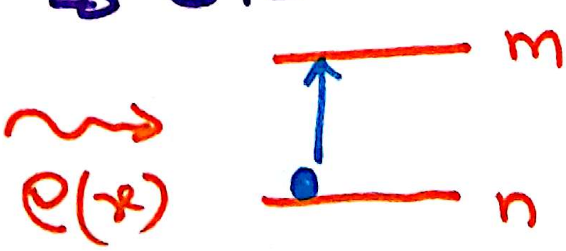
$\Downarrow$
RADIATION FIELD
(OR)
DENSITY OF PHOTONS WITH FREQUENCY ν

$N_n \Rightarrow$ NUMBER OF ATOMS IN THE 'n' STATE

$N_m \Rightarrow$ NUMBER OF ATOMS IN THE 'm' STATE

$\tau_n, \tau_m \Rightarrow$ LIFETIMES

- ATOMS CAN ABSORB ENERGY FROM THE RADIATION AND UNDERGO $n \rightarrow m$ TRANSITION
  (IN THE PRESENCE OF EXTERNAL RADIATION)
  $\Rightarrow$ STIMULATED ABSORPTION



$$\boxed{E_m - E_n = h\nu_{nm}}$$

- SIGNATURE OF ABSORPION

N_n SHOULD DECREASE

- RATE OF ABSORPION

$$-\frac{dN_n}{dt} \propto N_n \rho(\nu_{nm})$$

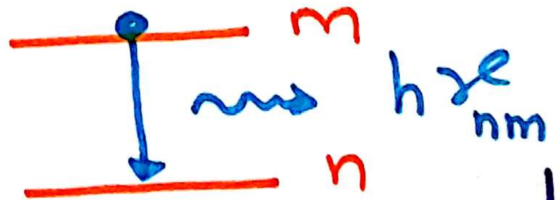
$$\boxed{\frac{dN_n}{dt} = -B N_n \rho(\nu_{nm})}$$

B → PROPORTIONALITY CONSTANT

- SPONTANEOUS EMISSION:

⇒ AN EXITED ATOM CAN SPONTANEOUSLY JUMP FROM THE UPPER LEVEL TO THE LOWER LEVEL (IN THE ABSENCE OF EXTERNAL RADIATION)

h → PLANCK'S CONSTANT



⇒ LIGHT OF ENERGY $h\nu_{nm}$ IS EMITTED

- RATE OF SPONTANEOUS EMISSION:

$$\boxed{\frac{dN_m}{dt} = -A N_m}$$

A → PROPORTIONALITY CONSTANT

A, B ⇓ EINSTEIN COEFFICIENTS

- AT EQUILIBRIUM (at TEMPERATURE T)

$$N_n \propto e^{-\frac{E_n}{k_B T}}$$

(BOLTZMANN'S IDEA)

$k_B \rightarrow$ BOLTZMANN CONSTANT

$$N_m \propto e^{-\frac{E_m}{k_B T}}$$

$$\boxed{\frac{N_m}{N_n} = e^{-\frac{(E_m - E_n)}{k_B T}}}$$

- USING BOHR'S IDEA

$$\boxed{\frac{N_m}{N_n} = e^{-\frac{h\nu_{nm}}{k_B T}}}$$

- AT EQUILIBRIUM

$$\frac{dN_n}{dt} = \frac{dN_m}{dt}$$

$$\rho(\nu_{nm}) = \frac{A\ N_m}{B\ N_m}$$

$$\boxed{\rho(\nu_{nm}) = \left(\frac{A}{B}\right) e^{-\frac{h\nu_{nm}}{k_B T}}}$$

- WIEN'S ~~DISPLACEMENT~~ DISTRIBUTION LAW

$$\boxed{\rho(\nu_{nm}) = \underbrace{\alpha}_{\text{CONSTANT}}\, \nu_{nm}^{3}\, e^{-\frac{h\nu_{nm}}{k_B T}}}$$

$\Downarrow$

ACCURATELY DESCRIBES THE HIGH-ν REGION OF THE SPECTRUM OF THERMAL OR BLACKBODY RADIATION.

$$\Rightarrow \boxed{\frac{A}{B} = \alpha\, \nu_{nm}^{3}}$$

- PLANCK'S DISTRIBUTION LAW DESCRIBES THE BLACKBODY RADIATION SPECTRUM WELL.

$$\boxed{\rho(\nu_{nm}) = \underbrace{\Gamma}_{\text{CONSTANT}}\, \nu_{nm}^{3} \left(\frac{1}{e^{\frac{h\nu_{nm}}{k_B T}} - 1}\right)}$$

- EINSTEIN MODIFIED HIS THEORY. HE INTRODUED THE PROCESS OF STIMULATED EMISSION.

$m \to n$ TRANSITION INDUCED BY EXTERNAL RADIATION

PROPORTIONALITY CONSTANT

$$\boxed{\frac{dN_m}{dt} = -\underbrace{A N_m}_{\text{SPONTANEOUS EMISSION}} - \underbrace{C N_m \rho(\nu_{nm})}_{\text{STIMULATED EMISSION}}}$$

• AT EQUILIBRIUM

$$\frac{dN_m}{dt} = \frac{dN_n}{dt}$$

$$\left(A + C\rho(\nu_{nm})\right) N_m = B N_n \rho(\nu_{nm})$$

$$\rho(\nu_{nm}) = \frac{(A/B)}{\frac{N_n}{N_m} - \frac{C}{B}}$$

$$\boxed{\rho(\nu_{nm}) = \frac{(A/B)}{e^{\frac{h\nu_{nm}}{k_B T}} - (C/B)}}$$

COMPARE THIS EQUATION WITH THE PLANCK'S LAW

$$\boxed{\frac{A}{B} = \vdash \nu_{nm}^{3}}$$

$$\boxed{\frac{C}{B} = 1}$$

WHEN $h\nu_{nm} \gg k_B T$, $e^{\frac{h\nu_{nm}}{k_B T}} \gg 1$;

$$\boxed{\rho(\nu_{nm}) \sim \left(\frac{A}{B}\right) e^{-\frac{h\nu_{nm}}{k_B T}}}$$

WIEN'S LAW

• WHEN $h\nu_{nm} \gg k_B T$,

$$E_m - E_n \gg k_B T$$

$\Rightarrow \boxed{N_m \ll N_n}$ $\Rightarrow$ STIMULATED EMISSION IS NEGLIGIBLE

THE NUMBER OF ATOMS IN THE LOWER LEVEL IS MUCH GREATER THAN THE NUMBER OF ATOMS IN THE UPPER LEVEL

ELECTROMAGNETIC FIELD

- VECTOR POTENTIAL $\vec{A}(\vec{r}, t)$
- SCALAR POTENTIAL $\phi(\vec{r}, t)$
- ELECTRIC AND MAGNETIC FIELDS CAN BE WRITTEN IN TERMS OF $\vec{A}$ AND ϕ

$$\Rightarrow \boxed{\vec{E}(\vec{r}, t) = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi}$$

$$\Rightarrow \boxed{\vec{B}(\vec{r}, t) = \nabla \times \vec{A}}$$

$$\vec{E}(\vec{r}, t) = E_x(\vec{r}, t)\,\hat{i} + E_y(\vec{r}, t)\,\hat{j} + E_z(\vec{r}, t)\,\hat{k}$$

$$\vec{B}(\vec{r}, t) = B_x(\vec{r}, t)\,\hat{i} + B_y(\vec{r}, t)\,\hat{j} + B_z(\vec{r}, t)\,\hat{k}$$

$(E_x, E_y, E_z, B_x, B_y, B_z)$

SIX VARIABLES ARE NEEDED TO DEFINE AN ELECTROMAGNETIC WAVE AT A GIVEN $\vec{r}$ AND t.

WITH $\vec{A}$ AND ϕ, ONLY FOUR VARIABLES ARE SUFFICIENT

(A_x, A_y, A_z, ϕ)

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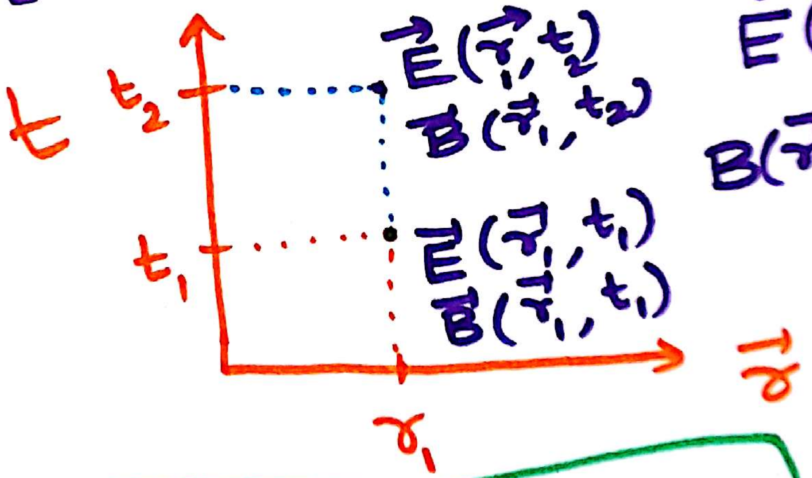
A CLASSICAL DESCRIPTION OF SPECTROSCOPY

- SYSTEM COMPOSED OF CHARGED PARTICLES
- CHARGES ARE BOUND TO THE ATOMS/MOLECULES IN THE SYSTEM
- CONSIDER A SINGLE CHARGE Q LOCATED AT THE ORIGIN ; MASS m

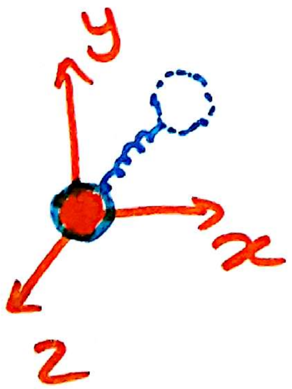
$$\langle \vec{r} \rangle = (0, 0, 0)$$

- TURN ON AN ELECTROMAGNETIC WAVE OF FREQUENCY ω
- CHARGE WILL OSCILLATE AROUND THE ORIGIN (HARMONIC OSCILLATOR)



$$\vec{E}(\vec{r}_1, t_1) \neq \vec{E}(\vec{r}_1, t_2)$$

$$B(\vec{r}_1, t_1) \neq B(\vec{r}_1, t_2)$$



$$\boxed{U_{HAR}(\vec{r}, t) = \frac{1}{2} \underbrace{k}_{\text{SPRING CONSTANT}} \vec{r} \cdot \vec{r}}$$

IGNORE THE EFFECT OF THE MAGNETIC FIELD (NOT A MOVING CHARGE)

- DIPOLE MOMENT OF THE SYSTEM

$$\boxed{\vec{\mu} = Q\vec{r}}$$

FOR A SYSTEM OF DISCRETE CHARGES :

$$\vec{\mu} = \sum_{i=1}^{N} Q_i \vec{r}_i$$

- INTERACTION ENERGY OF THE CHARGE WITH THE ELECTRIC FIELD

$$\boxed{U_{EXT}(\vec{r},t) = -\vec{\mu}\cdot\vec{E}(\vec{r},t)}$$

AT A GIVEN TIME, MAGNITUDE OF $\vec{E}$ IS CONSTANT WITH IN THE VICINITY OF THE CHARGE.

(ONLY TIME DEPENDENCE)

- EQUATION OF MOTION

$$\boxed{m\frac{d^2\vec{r}}{dt^2} = -k\vec{r} - b\frac{d\vec{r}}{dt} + F_0\cos\omega t}$$

POSITIVE

HARMONIC RESTORING FORCE

DAMPING FORCE

EXTERNAL FORCE DUE TO LIGHT

NOTE:- $\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r})\cos\omega t$

$$F_0 = \frac{d\vec{\mu}}{d\vec{r}}\cdot\vec{E}_0$$

(E_0 DOESN'T CHANGE WITH X, Y, Z)

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• $$\boxed{\frac{d^2\vec{r}}{dt^2} + 2\Gamma \frac{d\vec{r}}{dt} + \omega_0^2 \vec{r} = \frac{F_0}{m}\cos\omega t}$$

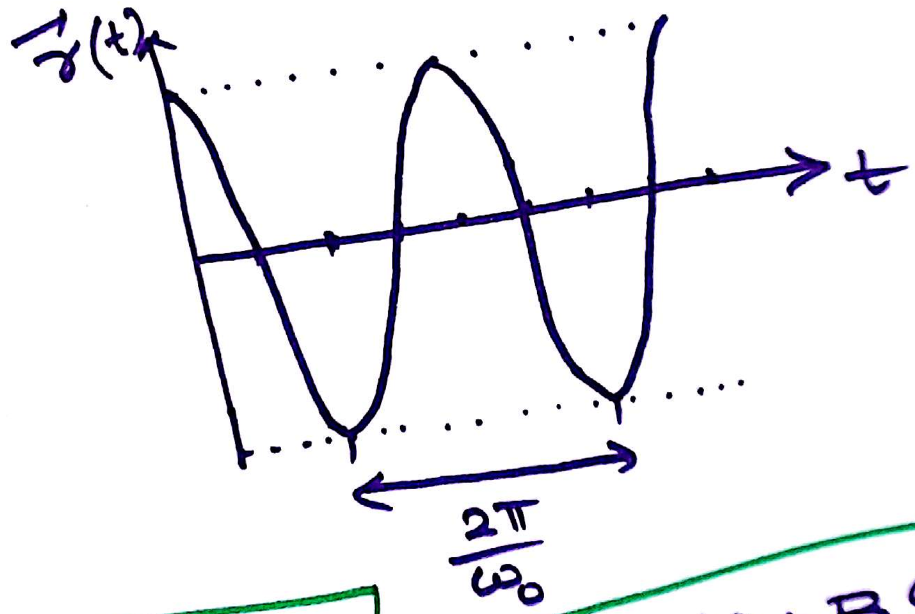
HERE $\Gamma = \dfrac{b}{2m}$ (DAMPING COEFFICIENT)

$\omega_0 = \sqrt{k/m}$ (NATURAL FREQUENCY OF THE OSCILLATOR)

DRIVEN HARMONIC OSCILLATOR

• CASE I : $\Gamma = 0$ ⇒ NO DAMPING FORCE

$F_0 = 0$ ⇒ NO LIGHT



$$\boxed{\vec{r}(t) = A\sin(\omega_0 t) + B\cos(\omega_0 t)}$$

$$\vec{r}(t) = \vec{r}_0\, e^{-i\omega_0 t}$$

- CASE II : $\Gamma \neq 0$ DAMPING FORCE

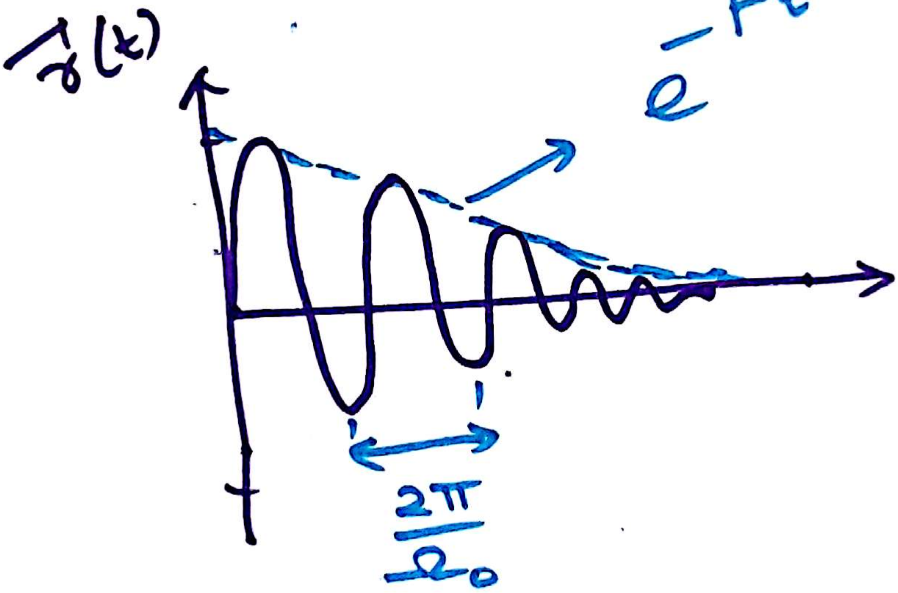
$f_0 = 0 \Rightarrow$ NO LIGHT

$$\vec{\gamma}(t) = \vec{\gamma}_0 \, e^{-i\Omega_0 t} \, e^{-\Gamma t}$$

REDUCED FREQUENCY $\Leftarrow$ $\Omega_0 = \sqrt{\omega_0^2 - \Gamma^2}$

WEAK DAMPING : $\Gamma \to 0$

$$\Omega_0 \simeq \omega_0$$



- CASE III : $\Gamma \neq 0$

$f_0 \neq 0$

WHEN $\omega = \omega_0$; RESONANCE