

HAMILTONIAN:  $H_0$

SMALL PERTURBATION } :  $\Delta H \rightarrow 0$   
 LARGE PERTURBATION } :  $\Delta H \gg 0$

~~EXCITED~~  
 ENERGY +  $\Delta E$   
 EXTRA ENERGY

$$H_0 + \Delta H$$

$$H_0 + H_P$$

$H_P = \Delta H$

PERTURBATIVE PART OF THE HAMILTONIAN

# MANY-BODY SYSTEMS

## → THERMODYNAMICS

•  $P, V, T, n$ ; HERE  $n$  → NUMER OF MOLES

• FIXED  $n$ ;  $P = f(V, T)$   
     $\Downarrow$   
    EQUATION OF STATE

- ALL PROPERTIES CAN BE EXPRESSED AS FUNCTIONS OF  $P, V, T, n$
- THE CONCEPTS OF ATOMS, MOLECULES, ELECTRONS ETC... ARE NOT NEEDED
- DIFFERENT ENERGY FUNCTIONS :
  - HEAT
  - WORK
  - ENTHALPY
  - INTERNAL ENERGY
  - ENTROPY
  - HELMHOLTZ FREE ENERGY
  - GIBBS FREE ENERGY
  - CHEMICAL POTENTIAL



# MANY-BODY SYSTEMS

## ⇒ CLASSICAL STATISTICAL MECHANICS:

- SYSTEM CONSISTS OF  $N$  PARTICLES

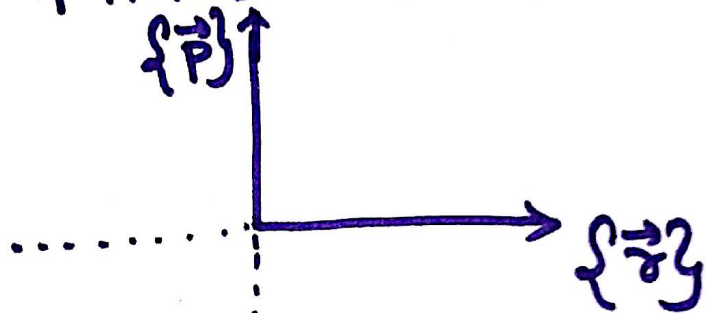
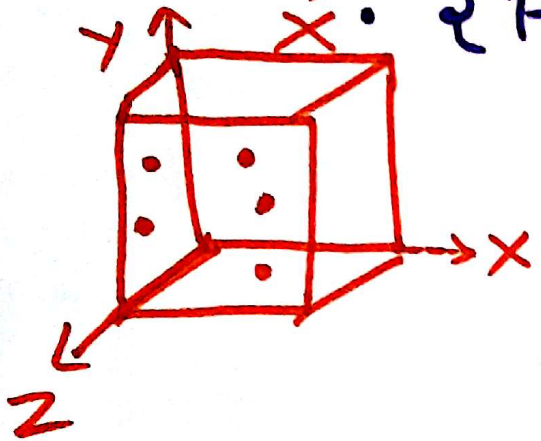
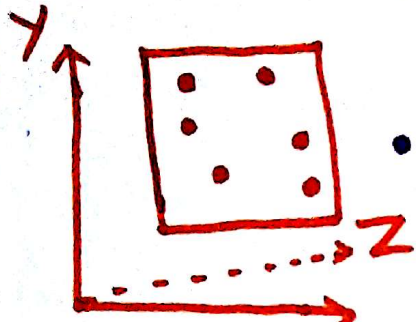
$$\cdot \{\vec{r}\} = (\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N)$$

SET OF POSITIONS

$$\cdot \{\vec{p}\} = (\vec{p}_1, \vec{p}_2, \dots, \vec{p}_N)$$

SET OF MOMENTA

- PHASE SPACE



- DEFINE HAMILTONIAN

$$H(\{\vec{r}\}, \{\vec{p}\}) = U(\{\vec{r}\}) + K(\{\vec{p}\})$$

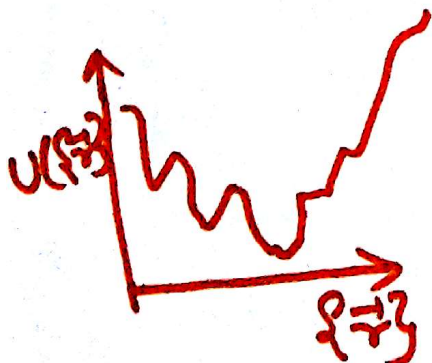
TOTAL ENERGY OF THE SYSTEM →

POTENTIAL ENERGY

KINETIC ENERGY

INTERATOMIC INTERACTIONS

THERMAL ENERGY



- $U(\{\vec{r}\})$  DEFINES THE POTENTIAL ENERGY SURFACE

- SOLVE HAMILTON'S EQUATIONS OF MOTION TO UNDERSTAND HOW  $\{\vec{r}\}$  AND  $\{\vec{p}\}$  VARY WITH TIME. (DYNAMICS)

FORCE ON ATOM  $i$

$$\frac{d\vec{p}_i}{dt} = - \frac{\partial H}{\partial \vec{r}_i} = - \frac{\partial U}{\partial \vec{r}_i} \quad \forall i$$

HERE  $\frac{\partial}{\partial \vec{r}_i} = \left( \frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_i}, \frac{\partial}{\partial z_i} \right)$   
 GRADIENT

$$\frac{d\vec{r}_i}{dt} = \frac{\partial H}{\partial \vec{p}_i} \quad ; \quad \forall i$$

- INITIAL CONDITIONS:

AT TIME  $t=0$ ;

$\{\vec{r}(t=0)\}$

$\{\vec{p}(t=0)\}$

X-RAY  
DIFFRACTION

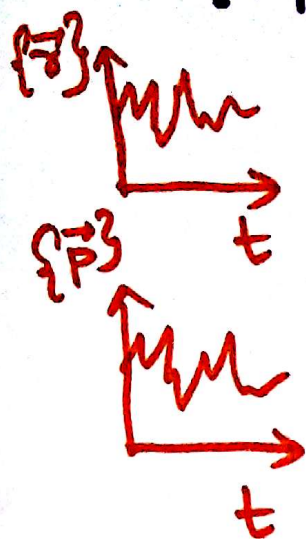
NMR

RANDOM

MAXWELL-  
BOLTZMANN  
DISTRIBUTION



## • TIME EVOLUTION

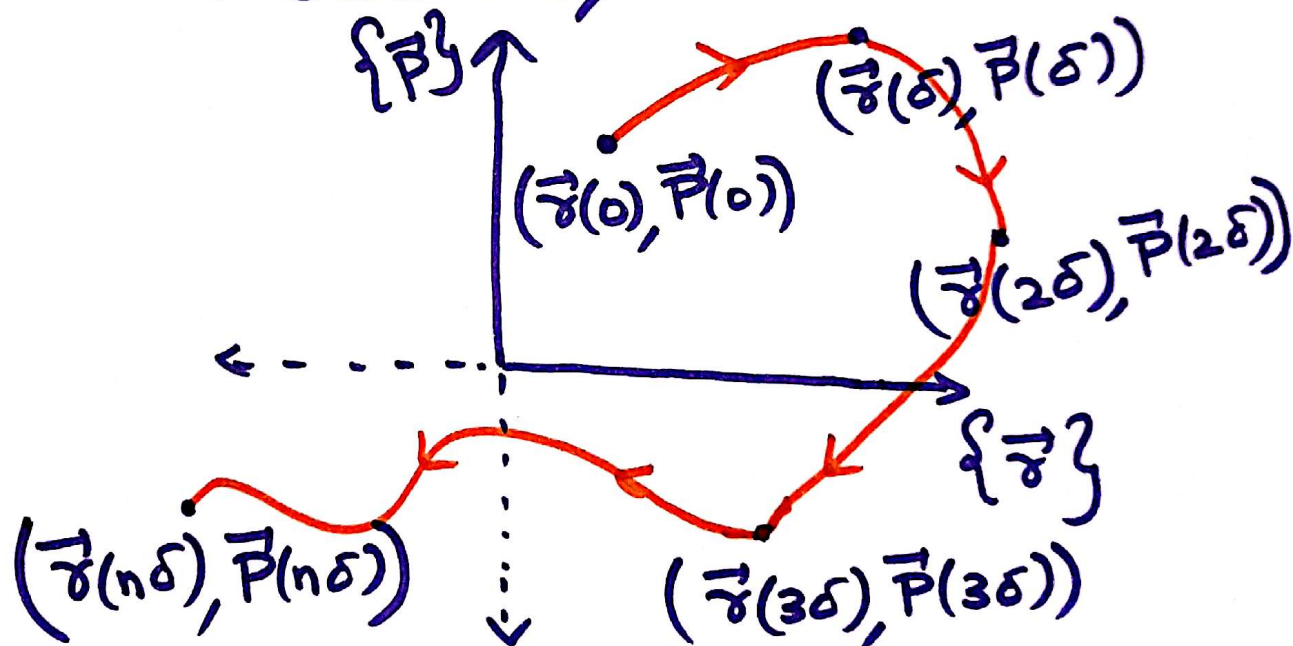


$$\Rightarrow \{\vec{q}(t=0)\} \rightarrow \{\vec{q}(t=\delta)\} \rightarrow \{\vec{q}(t=2\delta)\} \\ \{\vec{q}(t=n\delta)\} \leftarrow \dots \leftarrow \{\vec{q}(t=4\delta)\} \leftarrow \{\vec{q}(t=3\delta)\}$$

HERE  $\delta \Rightarrow$  TIME STEP  
 $n \Rightarrow$  A LARGE +VE INTEGER

$$\Rightarrow \{\vec{P}(t=0)\} \rightarrow \{\vec{P}(t=\delta)\} \rightarrow \{\vec{P}(t=2\delta)\} \\ \{\vec{P}(t=n\delta)\} \leftarrow \dots \leftarrow \{\vec{P}(t=4\delta)\} \leftarrow \{\vec{P}(t=3\delta)\}$$

## • TRAJECTORY ON PHASE SPACE



• ALL PROPERTIES CAN BE WRITTEN AS FUNCTIONS OF  $\{\vec{q}\}$  AND  $\{\vec{P}\}$



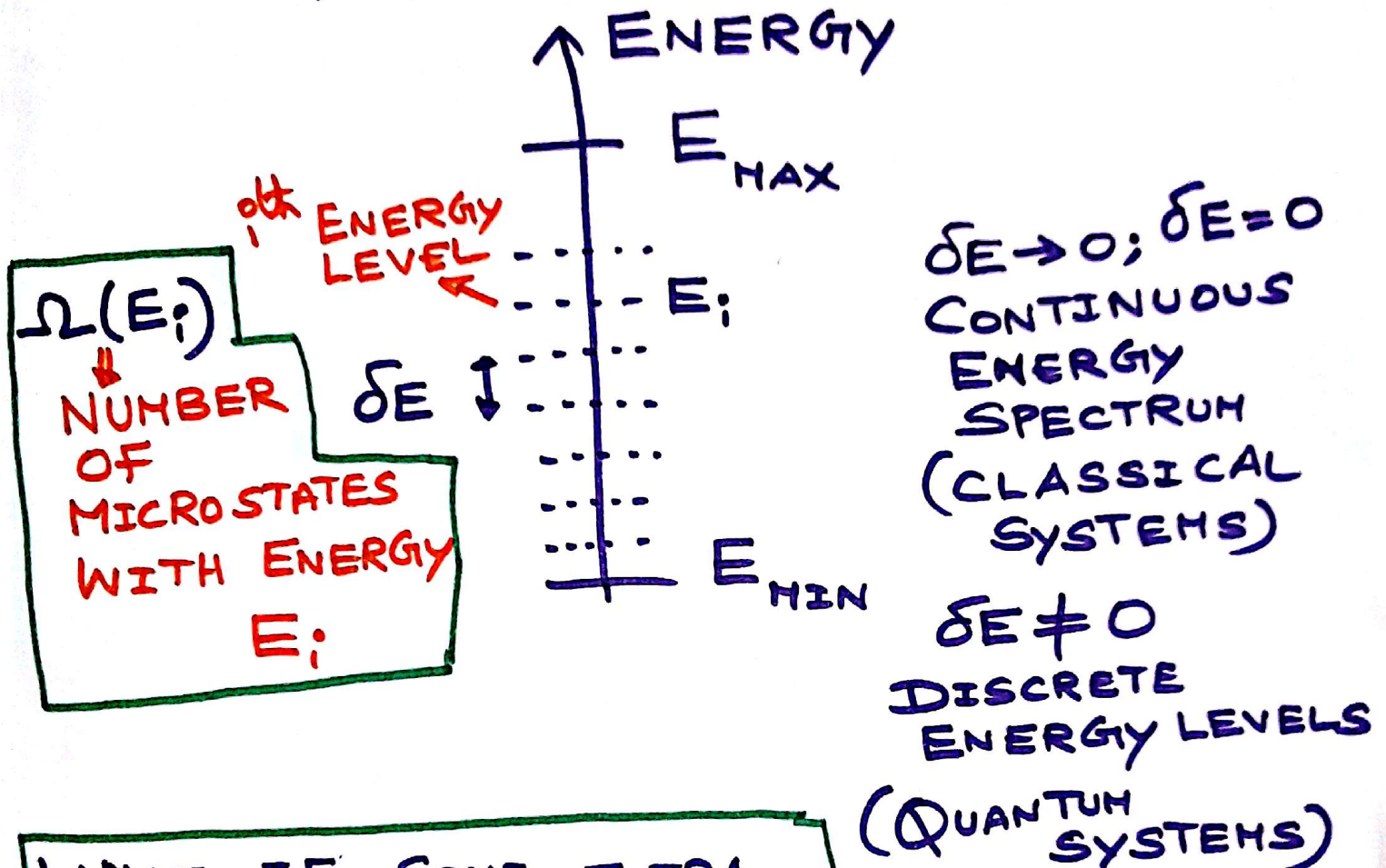
$$A(t) = A(\{\vec{q}(t)\}, \{\vec{P}(t)\})$$

TIME EVOLUTION OF "A" CAN BE STUDIED

# ENERGY-LEVEL DIAGRAM

$E_{\min}$  → LOWEST ENERGY OF THE SYSTEM

$E_{\max}$  → HIGHEST ENERGY



WHAT IF SOME EXTRA ENERGY  $E_p$  IS ADDED TO THE SYSTEM?

→  $E_{\max} - E_{\min}$  CAN VARY  
→ SYSTEM CAN TRANSIT BETWEEN DIFFERENT ENERGY LEVELS

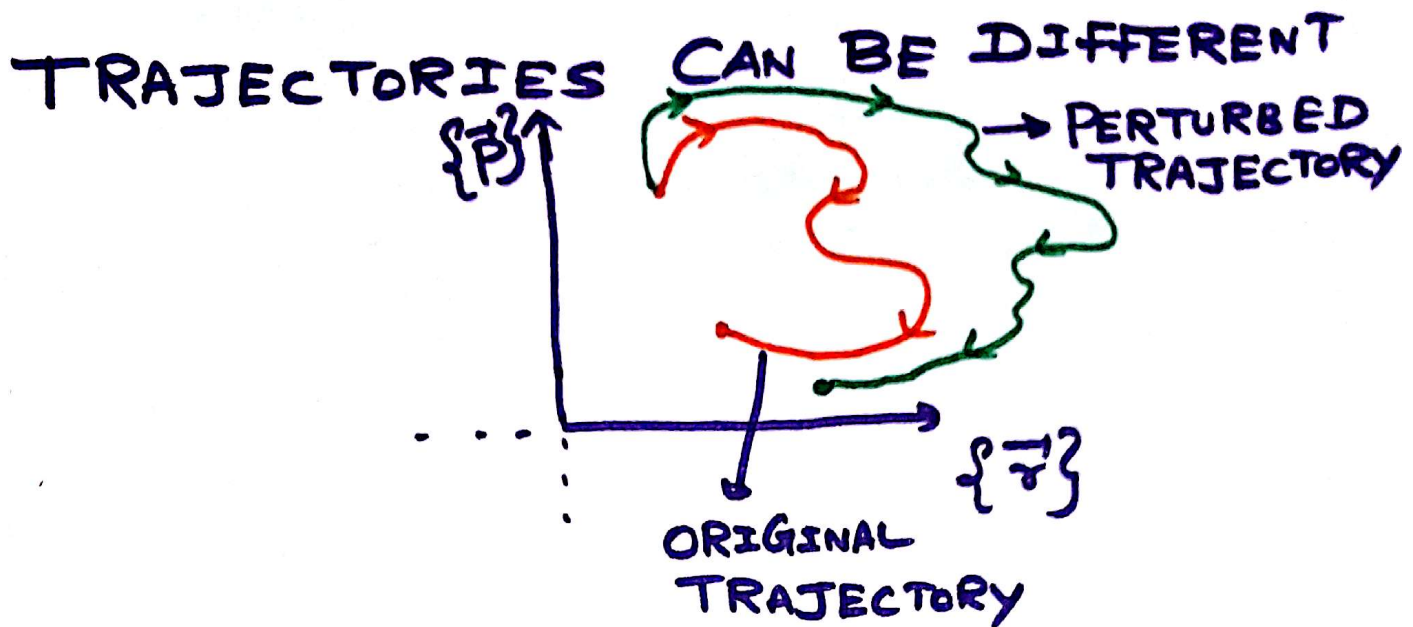
COMPARE ORIGINAL AND PERTURBED SYSTEMS



# COMPARE ORIGINAL AND PERTURBED SYSTEMS

ORIGINAL SYSTEM:  $H_0(\{q\}, \{p\})$

PERTURBED SYSTEM:  $H_0(\{q\}, \{p\}) + H_p$



EXAMPLE: ONE-DIMENSIONAL SIMPLE HARMONIC OSCILLATOR (CLASSICAL)



REDUCED MASS

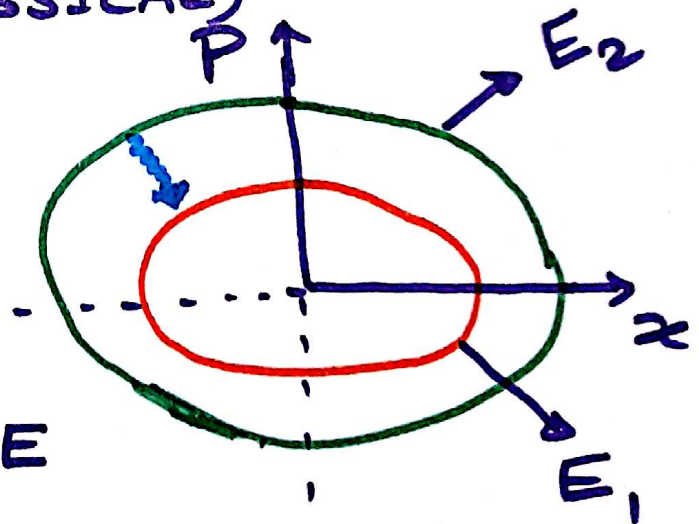
$$\mu = \frac{Mm}{M+m} \approx m$$

$$H_0(x, p) = \frac{p^2}{2m} + \frac{1}{2}kx^2 = E$$

SET  $H_0 = E$  INITIALLY

$$\frac{x^2}{\left(\frac{\sqrt{2E}}{k}\right)^2} + \frac{p^2}{(\sqrt{2mE})^2} = 1$$

ELLIPSE



$$E_2 > E_1$$

$$E_2 = E_1 + E_p$$