Major Project 2

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One of the simplest ways of obtaining a polynomial approximation of degree n to a given continuous function f(x)on[a,b] is to interpolate between the values of f(x) at n+1 suitably selected distinct points in the interval. The approach used for the probelm is Chebyshev interpolation.
The Chebyshev Polynomials (of the first kind) are defined by as

$$T_n(x) = cos[n * arccos(x)] \tag{1}$$

The infinite continuous Chebyshev series expansion is

$$f(x) \approx \Sigma_0^N \alpha_n T_n(x) \tag{2}$$

$$\alpha_n = 2/\pi \int \sqrt{(1-x^2)} f(x) T_n(x) dx \tag{3}$$

When the integral in (3) can not be evaluated exactly, we can introduce a discrete grid and use a numerical quadrature (integration) formula. Several possible grids, and related quadrature formulas exist. The Chebyshev-Gauss-Lobatto (CGL) points

$$x_k = -cos(k\pi/N) \tag{4}$$

for k=0,1....N
are a popular choice of quadrature points. The CGL points are where the n-1 extrema of $T_n(x)$ occur plus the endpoints of the interval [-1:1].

$$\alpha_n = 2/N \Sigma_0^N f(x_k) T_n(x_k) \tag{5}$$

we get the interpolating partial sum

$$I_N(x) = \Sigma_0^N \alpha_n T_n(x) \tag{6}$$

For the inteval [a,b], x is shifted as

$$x \to (2x - b - a)/(b - a) \tag{7}$$