Nitika Singh
20009101139006.

Date

Unit-1

Assignment-1

1. Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$

Solution. The given equation is

$(D^3 - 3D^2 D' + 4D'^3) = e^{x+2y}$ where

$D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$

Auxilary equation is

$m^3 - 3m^2 + 4 = 0$

$m^2(m+1) - 4m(m+1) + 4(m+1) = 0$

$(m-2)^2(m+1) = 0$

$m = 2, 2, -1$

two roots are equal and one is different so C.F

$CF = f_1(y-x) + f_2(y+2x) + x f_3(y+2x)$ —— (1)

$P.I = \frac{1}{D^3 - 3D^2 D' + 4D'^3} e^{x+2y}$

$= \frac{1}{1^3 - 3(1)^2(2) + 4(2)^3} \int\int\int e^u \, du\, du\, du$

$= \frac{1}{27} e^x + 2y$ —— (2)

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Hence the complete solution is (eq^n ① and ②)

$$U = C.F + P.I = f_1(y-x) + f_2(y+2x) + x f_3(y+2x) + \frac{1}{27} e^{x+2y}$$

Where f_1, f_2 and f_3 are arbitrary functions.

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2) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^3 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$

Sol. $(D^2 + 2DD' + D'^2)z = \sin(2x-3y)$

$$D = \frac{\partial}{\partial x}, \quad D' = \frac{\partial}{\partial y}$$

Auxilary equation is

$$m^2 + 2m + 1 = 0$$
$$m^2 + m + m + 1 = 0$$
$$(m+1)^2 = 0$$
$$m = -1, -1$$

$$C.F = (C_1 + C_2 x)e^{-x}$$

$$P.I = \frac{\sin(2x+3y)}{(D^2 + 2DD' + D'^2)}$$

put $D^2 = -a^3 = -4$
$D'^2 = -b^2 = -9$
$DD' = -ab = -6$

$$= \frac{\sin(2x+3y)}{(-4-6-9)}$$

$$= -\frac{1}{19}\sin(2x+3y)$$

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$$z = C.F + PI$$

$$z = (C_1 + C_2x)e^x - \frac{1}{19}\sin(2x+3y).$$

3) Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^3 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x-y} + e^{x+y}$
$+ \cos(x+2y).$

Sol. For CF
$$D^2 - 3DD' + 2D'^2 = 0$$
Auxilary eq^n

$$m^2 - 3m + 2 = 0$$
$$m^2 - 2m - m + 2 = 0$$
$$m(m-2) - 1(m-2) = 0$$
$$(m-1)(m-2) = 0$$
$$m = 1, 2$$

$$C.F = C_1 e^x + C_2 e^{2x}$$

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$$P.I = \frac{e^{2x-y}}{(D^2-3DD'+2D'^2)} + \frac{e^{x+y}}{(D^2-3DD'-2D'^2)} + \frac{\cos(x+2y)}{(D^2-3DD'+2D')}$$

$$= \frac{e^{2x-y}}{(4-3(2)(-1)+2(-1)^2} + \frac{e^{x+y}}{(1-3\cdot1\cdot1+2\cdot1^2)} + \frac{\cos(x+2y)}{(-1+6-8)}$$

$$= \frac{e^{2x-y}}{(4+6+2)} + \frac{e^{x+y}}{(1-3+2)} + \frac{\cos(x+2y)}{(-3)}$$

$$= \frac{e^{2x-y}}{12} - \frac{\cos(x+2y)}{3}$$

$$Z = CF + PI$$

$$Z = C_1 e^{x} + C_2 e^{2x} + \frac{e^{2x-y}}{12} - \frac{1}{3}\cos(x+2y).$$

5) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

Solution $(D^2 + DD' - 6D'^2)z = y\cos x$

Auxilary equation is

$m^2 + m - 6 = 0$

$m = 2, -3$ roots are different, so

$C.F = f_1(y+2x) + f_2(y-3x)$ --- (1)

$$P.I = \frac{1}{(D^2 + DD' - 6D'^2)} y\cos x$$

$$= \frac{1}{(D-2D')(D+3D')} y\cos x$$

$$= \frac{1}{(D-2D')} \int (c+3x)\cos x \, dx \quad y = (c+3x)$$

$$= \frac{1}{(D-2D')} \left[\int c\cos x \, dx + \int 3x \, dx \right]$$

$$= \frac{1}{(D-2D')} \left[c\sin x + 3\left[x\sin x - \int 1 \cdot \sin x \, dx \right] \right]$$

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$$= \frac{1}{(D-2D')}\left[(c+3x)\sin x + 3\cos x\right]$$

$$= \frac{1}{(D-2D')}\left[y\sin x + 3\cos x\right] \quad \text{where } y = (c+3x)$$

$$= \int (b-2x)\sin x\,dx + 3\sin x \qquad y = (b-2x)$$

$$= \left(-b\cos x - 2\left\{x(-\cos x) - \int(-\cos x)\,dx\right\} + 3\sin x\right.$$

$$= -b\cos x + 2x\cos x - 2\sin x + 3\sin x$$

$$= -(b-2x)\cos x + \sin x$$

$$= -y\cos x + \sin x \quad \text{where } y = (b-2x)$$

Hence the complete solution is

$$C.S = C.F + P.I$$

$$z = f_1(y+2x) + f_2(y-3x) - y\cos x + \sin x$$

Where f_1 and f_2 are arbitrary function.

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Ques 6) Solve $(D+D'-1)(D+2D'-2)z=0$

Sol. $CF = e^{x}\phi(y-x)+e^{2x}\phi(y-2x)$.

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7) Solve $(D^2-D'^2-3D+3D')z = xy + e^{x+2y}$

Solution Given PDE is non homogenous, to find C.F, put

$$(D^2-D'^2-3D+3D')z=0$$

$$= ((D+D')(D-D')-3(D-D'))z=0$$

$$= (D-D')(D+D'-3)z=0$$

$$C.F = f_1(y+x)+e^{3x}f_2(y-x)$$

$$PI = \frac{1}{(D^2-D'^2-3D+3D')} xy + e^{x+2y}.$$

$$P.I = \frac{1}{(D^2-D'^2-3D+3D')} xy + \frac{1}{(D^2-D'^2-3D+3D')} e^{x+2y}$$

$$= P1 + P2$$

$$\therefore P1 = \frac{1}{(D^2-D'^2-3D+3D')} xy$$

$$\Rightarrow P_1 = \frac{1}{(D-D')(D+D'-3)} xy$$

$$= \frac{1}{D\left[1-\frac{D'}{D}\right] \cancel{\times} -3\left[1-\frac{D+D'}{3}\right]} xy$$

$$\because \left[1-x\right]^{-1} = 1+x+x^2+x^3+ \cdots$$

$$= -\frac{1}{3D}\left[1+\frac{D'}{D}+\left(\frac{D'}{D}\right)^2+\cdots\right]\left[1+\frac{D+D'}{3}+\left(\frac{D+D'}{3}\right)^2+\cdots\right]xy$$

$$= -\frac{1}{3D}\left(1+\frac{D+D'}{3}+\frac{2DD'}{9}+\frac{D'}{D}+\frac{D'}{3}\right)xy$$

$$= -\frac{1}{3D}\left[xy+\frac{y}{3}+\frac{x}{3}+\frac{2}{9}+\frac{1}{D}x+\frac{x}{3}\right]$$

$$= -\frac{1}{3D}\left[xy+\frac{y}{3}+\frac{x}{3}+\frac{2}{9}+\frac{x^2}{2}+\frac{x}{3}\right]$$

$$= -\frac{1}{3}\left[\frac{x^2y}{3}+\frac{xy}{3}+\frac{x^2}{6}+\frac{2x}{9}+\frac{x^3}{6}+\frac{x^2}{6}\right]$$

$$P_1 = -\frac{1}{3}\left[\frac{x^2y}{3}+\frac{xy}{3}+\frac{x^2}{3}+\frac{2x}{9}+\frac{x^3}{6}\right]$$

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$$\because \quad P_2 = \frac{1}{(D^2 - D'^2 - 3D + 3D')} e^{2x+y}$$

$$= P_2 = \frac{1}{(D-D')(D+D'-3)} e^{2x+y}$$

$$= P_2 = \frac{1}{(2-1)(D+D'-3)} e^{2x+y}$$

$$= P_2 = \frac{1}{(D+D'-3)} \cdot e^{2x+y} \cdot 1$$

$$P_2 = e^{2x+y} \frac{1}{(D+2+D'+1-3)} \times 1$$

$$P_2 = e^{2x+y} \frac{1}{(D+D')} \times 1$$

$$= P_2 = e^{2x+y} \frac{1}{D\left[1+\frac{D'}{D}\right]} \times 1$$

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$$= P_2 = e^{2x+y} \times \frac{1}{D}\left[1-\frac{D'}{D}\right]^{-1} \times 1$$

$$= P_2 = e^{2x+y} \times \frac{1}{D}\left[1+\frac{D'}{D}+\left(\frac{D'}{D}\right)^2+\cdots\right]$$

$$= P_2 = x e^{2x+y}$$

So general solution is given by

$$z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{3}\left[\frac{x^2 y}{3} + \frac{xy}{3} + \frac{x^2}{3} + \frac{2x}{9} + \frac{x^3}{6}\right] + x e^{2x+y}.$$

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Q 8) Solve $(D^2 - DD' + D' - 1)z = \cos(x-2y) + e^y$

Sol. $D^2 - 1 + D' - DD' = 0$

$(D^2-1) - D'(D-1) = 0$

$(D+1)(D-1) = D'(D-1) = 0$

$(D-1)\{D+1-D'\} = 0$

$(D+0D'-1)(D-D'+1) = 0$

$CI = e^x \phi(y) + e^{-x} \phi(y+x)$

$$P.I = \frac{\cos(x+2y)}{(D^2 - DD' + D' - 1)} + \frac{e^y}{(D^2 - DD' + D' - 1)}$$

$D^2 = -a^2,\ D'^2 = -b^2,\ DD' = -ab$ in cos terms

$$= \frac{\cos(x+2y)}{(-1+2+D'-1)} + \frac{e^y}{(D^2 - DD' + D' - 1)}$$

In e^y term put $[D=0] \cdot D' = b = 1$

$$= \frac{\cos(x+2y)}{D'} + \frac{e^y}{(0-0+1-1)}$$

$P.I = \frac{1}{2}\sin(x+2y)$

$z = CF + PI$

$z = e^x\phi(y) + e^{-x}\phi(y+x) + \frac{1}{2}\sin(x+2y)$

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Que 9) Solve $(x^2D^2 + 4xyDD' + 4y^2D'^2 + 6yD')z = x^3y^4$

Sol. In given PDE, coefficient are variables so convert in into PDE with constant coefficient by substituting.

$x = e^X \Rightarrow X = \log x$
$y = e^Y = Y = \log y$
$xyDD' = DD'$
$yD' = D'$

$x^2D^2 = D(D-1)$
$y^2D'^2 = D'(D'-1)$
Here right hand side D, D' is partial variable derivative w.r.t X and Y respectively.

Put these values
$[D(D-1) - 4DD' + 4D'(D'-1) + 6D']z = e^{3x} \cdot e^{4y}$

$= [D^2 - 4DD' + 4D'^2 - D + 2D']z = e^{3X+4y}$

$= [(D-2D')^2 - D + 2D']z = e^{3X+4Y}$

$= (D-2D')(D-2D'-1)z = e^{3x+4Y}$

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Now, equation is non homogeneous
To find C.f put

$$(D-2D')(D-2D'-1)z=0$$

$CF = f_1(y+2x) + e^x f_2(y+2x)$

$CF = f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$

$C.F = f_1(\log yx^2) + x f_2(\log yx^2)$

$$P.I = \frac{1}{(D-2D')(D-2D'-1)} e^{3x+4y}$$

$$P.I = \frac{1}{(3-2\times4)(3-2\times4-1)} e^{3x+4y}$$

$$P.I = \frac{1}{30} e^{3x+4y}$$

$$P.I = \frac{1}{30} e^{3x} \cdot e^{4y}$$

$$P.I = \frac{1}{30} x^3 \cdot y^4 z$$

$$f_1(\log yx^2) + x f_2(\log yx^2) + \frac{1}{30} x^3 \cdot y^4$$