86 marks in University Exam

UNIT-1

10 marks guaranty

1. Prove that following matrices are unitary: $A = \frac{1}{2}\begin{bmatrix} i & \sqrt{3} \\ \sqrt{3} & i \end{bmatrix}$.

(i) $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ (.

2. The Eigen values of matrix A are 2,3,1, then eigen values of $A^{-1} + A^2$.

3. Find inverse employing elementary transformation of the matrix

(i) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

4. Find rank and nullity of following matrices reducing them in to normal form

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -4 & 3 & -1 \\ 1 & -2 & -1 & -4 \\ 0 & 1 & -1 & 3 \\ 4 & -7 & 4 & -4 \end{bmatrix}$ (iii)

5. (i) Find value of λ for which the vectors $(1,-2,\lambda),(2,-1,5)$ and $(3,-5,7\lambda)$ are linearly dependent.

10 marks guaranty

6. Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

7. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.

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8. Investigate for what values of λ and μ, the system of equations $x+y+z=6,\ x+2y+3z=10$ and $x+2y+\lambda z=\mu$ has

   (i) no solution (ii) unique solution (iii) infinite solution.

9. For what values of λ, the equations $x+y+z=1, x+2y+4z=\lambda, x+4y+10z=\lambda^2$ have a solution? Solve completely in each case.

UNIT-2

10 marks any one of Five

1. Verify Lagrange's mean value theorem for following function, $f(x)=x^3$ in $[-1,1]$.

2 State: (i) Cauchy's mean value theorem (ii) Lagrange,s mean value theorem

   (iii) Rolle's theorem

3. Verify Rolle's theorem for the function $f(x)=e^x(\sin x-\cos x)$ in $\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$.

4. Verify Rolle's theorem for following function $f(x)=x(x+3)e^{-\frac{x}{2}}$ in $[-3,0]$

5. Find 'c' of Cauchy's mean value theorem for the following pairs functions in $[a,b]$: $f(x)=e^x,\ g(x)=e^{-x}$.

10 marks any one of six

6. If $\mu=\sin nx+\cos nx$, prove that $\mu_r=n^r\left[1+(-1)^r\sin 2nx\right]^{\frac{1}{2}}$, where μ_r denotes the r^{th} differential coefficient of μ w.r.t. x. Hence show that $\mu_8(\pi)=\left(\frac{1}{2}\right)^{\frac{31}{2}}$ when $n=\frac{1}{4}$.

7. If $x=\cos\left(\frac{1}{m}\log y\right)$, find value of y_n at $x=0$.

8. $y=(sin^{-1}x)^2$ *or* If $x=\sin\sqrt{y}$, find the value of y_n at $x=0$.

9. If $y=\sin\left(m\sin^{-1}x\right)$, find the value of y_n at $x=0$.

10. If $y=\left(x+\sqrt{1+x^2}\right)^m$, prove that $(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0.$ Hence find y_n at $x=0.$

11. If $y^{1/m}+y^{-1/m}=2x$, Prove that $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$

<u>UNIT-3</u>

10 marks any one of these

1. If $\boldsymbol{u}=\boldsymbol{f}(\mathbf{2}\boldsymbol{x}-\mathbf{3}\boldsymbol{y},\mathbf{3}\boldsymbol{y}-\mathbf{4}\boldsymbol{z},\mathbf{4}\boldsymbol{z}-\mathbf{2}\boldsymbol{x})$, prove that $\frac{\mathbf{1}}{\mathbf{2}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}+\frac{\mathbf{1}}{\mathbf{3}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{y}}+\frac{\mathbf{1}}{\mathbf{4}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{z}}=\mathbf{0}.$

2. If $\boldsymbol{u}=\boldsymbol{f}(\boldsymbol{r})$ where $\boldsymbol{r}^2=\boldsymbol{x}^2+\boldsymbol{y}^2$, prove that $\frac{\partial^2\boldsymbol{u}}{\partial\boldsymbol{x}^2}+\frac{\partial^2\boldsymbol{u}}{\partial\boldsymbol{y}^2}=\boldsymbol{f}''(\boldsymbol{r})+\frac{\mathbf{1}}{\boldsymbol{r}}\boldsymbol{f}'(\boldsymbol{r})$

3. If $\boldsymbol{u}=\cos\boldsymbol{ec}^{-1}\left(\frac{\boldsymbol{x}^{1/2}+\boldsymbol{y}^{1/2}}{\boldsymbol{x}^{1/3}+\boldsymbol{y}^{1/3}}\right)^{1/2}$, evaluate $\boldsymbol{x}^2\frac{\partial^2\boldsymbol{u}}{\partial\boldsymbol{x}^2}+\mathbf{2}\boldsymbol{xy}\frac{\partial^2\boldsymbol{u}}{\partial\boldsymbol{x}\partial\boldsymbol{y}}+\boldsymbol{y}^2\frac{\partial^2\boldsymbol{u}}{\partial\boldsymbol{y}^2}.$

4. Find approximate value of $\left[(\mathbf{0.98})^2+(\mathbf{2.01})^2+(\mathbf{1.94})^2\right]^{\frac{1}{2}}.$

5. Compute an approximate value of $(2.98)^3+(1.01)^2$.

6. Expand $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})=\boldsymbol{x}^2+\mathbf{3}\boldsymbol{y}^2-\mathbf{9}\boldsymbol{x}-\mathbf{9}\boldsymbol{y}+\mathbf{26}$ as Taylor's series expansion about the point (1,2).

7. Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighborhood of $(1,1)$ up to and inclusive of second degree terms. Hence compute $f(1.1,0.9)$.

8. Find the stationary point of $f(x,y)=x^3+y^3+3axy, a>0$

10 marks any one of Four

Maximize $sinA\ sinB\ sinC$ A,B,C is angle of Triangle

9. Find the dimensions of a rectangular box of maximum capacity whose surface area is given when (i) box is open at the top (ii) box is closed.

10. Find the maximum and minimum distance of the point (1,2,-1) from the sphere $\boldsymbol{x}^2+\boldsymbol{y}^2+\boldsymbol{z}^2=\mathbf{24}.$

11. Divide a number into three parts such that the product of first, square of second and cube of third is maximum.

10 marks any one of three

12. If $\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}$ are the roots of equations $(\boldsymbol{x}-\boldsymbol{\lambda})^3+(\boldsymbol{y}-\boldsymbol{\lambda})^3+(\boldsymbol{z}-\boldsymbol{\lambda})^3=\mathbf{0}$, then find $\frac{\partial(\boldsymbol{u},\boldsymbol{v},\boldsymbol{w})}{\partial(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})}.$

13. If $u_1 = \frac{x_2 x_3}{x_1}, u_2 = \frac{x_3 x_1}{x_2}, u_3 = \frac{x_1 x_2}{x_3}$ find the value of $\frac{\partial(u_1, u_2, u_3)}{\partial(x_1, x_2, x_3)}$

14. If $\boldsymbol{u = x + y + z, v = x^2 + y^2 + z^2, w = x^3 + y^3 + z^3 - 3xyz}$, prove that $\boldsymbol{u, v, w}$ are not independent and hence find the relation between them.

UNIT-5

2 marks any two of three

1. If $\boldsymbol{\phi = 3x^2 y - y^3 z^2}$, find grad $\boldsymbol{\phi}$ at point (2,0,-2).
2. For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3} + \frac{z^2}{4}$, find the greatest directional derivative at (1,2,1).
3. State: (i) Gauss divergence theorem (ii) Green's theorem (iii) Stoke,s Theorem.

10 marks any one of Four

4. Find the value of *'b'* for a Solenoidal vector $\boldsymbol{\vec{F} = (bx)\hat{i} - (5y)\hat{j} + (2z)\hat{k}}$.
5. Show that the vector field (y + z) i+(z + x) j + (x + y) k is irrotational and find velocity potential
6. A vector field is given by $\boldsymbol{\vec{A} = \left(x^2 + xy^2\right)\hat{i} + (y^2 + x^2 y)\hat{j}}$. Show that the field is irrotational and find the scalar potential.
7. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, r = \left|\vec{r}\right|, \phi$ be any scalar then show that $div(grad r^n) = \nabla^2 r^n = n(n+1) r^{n-2}$. Hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$.
8. Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal.

10 marks any one of three

9. Find the directional derivative of $\phi = (x^2 + y^2 + z^2)^{-1/2}$ at the point $P(3,1,2)$ in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.
10. Find the directional derivative of $\boldsymbol{\varphi(x, y) = 5x^2 y - 5y^2 z + \frac{5}{2} z^2 x}$ at the point $\boldsymbol{(1,1,1)}$ in the direction of line $\boldsymbol{\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}}$.
11. Find the directional derivative of $xy^2 + zy^2 + xz^2$ at the point *(2, 0, 3)* in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point (3, 2, 1).