Tutorial-6

Use the pumping lemma to prove that the following language are not Regular (assume $\Sigma = \{a,b\}$.

1) $L = \{w \in \{a,b\}^* \mid n_a(w) = n_b(w)\}$.

Solve.

① Assume L is Regular language. Since L is infinite we can apply the pumping Lemma.

Let m be the Critical length for L.

picks a String w such that: $w \in L$. and length $|w| \geq m$.

$w = a^m b^m$

from the pumping lemma -
we write $w = a^m b^m = xyz$.
with length: $|xy| \leq m$, $|y| \geq 1$

Suppose $m = 4$

$w = \underbrace{aa}_{x}\underbrace{a}_{y}\underbrace{abbbb}_{z}$

$w_1 = xy^i z$.

$w_2 = xy^2 z =$ aa aaaa bbbb.

$w_1 \neq w$

(2) $L = ww$ where $w \in \Sigma^*$

Solve

Assume L is Regular. Let m be the Critical Length.

Choose w s.t $|w| \geq m$.
where m is the critical length.

$w = ab\,ab$.

$w = a^m b^m a^m b^m \in L$

Divide w in to xyz. Subjects to the Condition $y \neq \epsilon$ and $|xy| < m$

$~~w_i = a^{m-k} b^i~~$

$w_i = a^{m-k} a^{ik} b^m a^m b^m$

$w_0 = a^{m-k} b^m a^m b^m$

Since $k \geq 1$.

Contradiction of pumping lemma.
Hence proved.

(3) $L = \{a^p \mid p \text{ is prime}\}$ $(L = \{w \mid |w| \text{ is prime}\})$

Solve

Assume L is Regular & m is Critical length.

Choose $w = a^k$ s.t p is the Smallest prime no. that is equal to or greater than m.

Divide $w = xyz$ s.t $|y| \neq \epsilon$ and $|xy| < m$

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$w^* = a^{p-k} \; a^{k}$.
$w_i = a^{p-k} \; a^{ik}$.

Now $|w_i| = p-k+ik$
$= p+k(i-1)$

Choose i s.t $= i-1 = p$
than $|w_i| = p+kp = p(k+1)$
which can not be prime.
Hence $w_i \in L$

(4) $L = \{ (ab)^n b^p \mid n > p \text{ and } p \geq 0 \}$.

Solve Assume L is Regular Let m be the Critical Length.

Choose w .s.t $|w| \geq m$
$w_2 \; (ab)^{m+1} \; b^m \in L$

Divide w in to xyz s.t. $y \neq \epsilon$
and $|xy| \leq m$.

$y = (ab)^k$.
$w_2 = (ab)^{m-k+1} \; (ab)^{ik} \; b^m$
$w_2 = (ab)^{m+k+1} \; b^m \notin L$ Since $k > 1$.

Qu-6 $L = \{ a^m b^k \mid n \neq k \}$

Solve

Let $L_2 = L(a^* b^*)$

$L_3 = a^n b^n \mid n \geq 0.$

Than $\bar{L_1} \cap L_2 = L_3.$

We know L_2 is Regular. and L_3 is not Regular (proven earlier).

if L_1 was Regular, than $\bar{L_1}$ would have been Regular and $\bar{L_1} \cap L_2$ would also be Regular. However we know that L_3 (which is $\bar{L_1} \cap L_2$) is not Regular. Hence, L_1 can not be Regular.

Qu-7 $L = \{ w \in \{a,b\}^* \mid n_a(w) \neq n_b(w) \}.$

Solve

Assume L is Regular. Let m be the critical Length.

choose w s.t $|w| \geq m.$

~~w= aa b — w= a^m a^m b^m~~

$w = abba.$

$w = a^m b^m b^m a^m$

Divide w in to xyz s.t $y \neq \epsilon$ and $|xy| < m.$

$y = (b)^k.$

$w_i = a^m b^{m-k} b^{k} a^m.$

$w_2 = a^m b^{m-k} b^{2k} m.$

$= a^m b^{m+k} m \notin L$

Contradiction Occurs.

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Que Q $L = \{a^n b^{2n} \mid n \geq 1\}$.

Solve

Assume L is Regular language. and m is the Critical. length.

Choose w . s.t $|w| \geq m$
$w = a^m b^{2m} \in L$

Divide w in to xyz s.t $y \neq \epsilon$ and $|xy| \leq m$

$y = a^k$.
$w_i = a^{m-k} a^{ik} b^{2m}$
$w_0 = a^{m-k} b^{2m} . \notin L$. Since $k > 0$

Contradiction of pumping lemma.
hence prove.