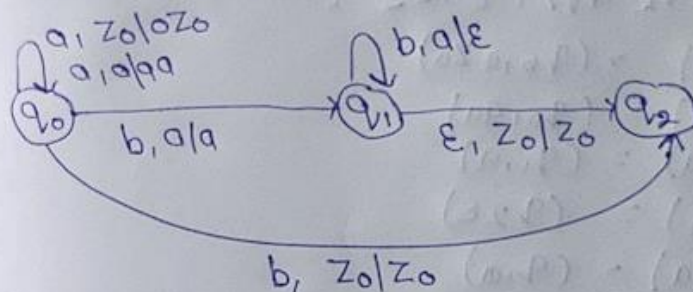


Tutorial-9

Que-1 Construct PDAs for following language over $\Sigma = \{a, b\}$

(a) $L = a^n b^{n+1}, n \geq 0$



$$\delta(q_0, a, Z_0) = (q_0, a, Z_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

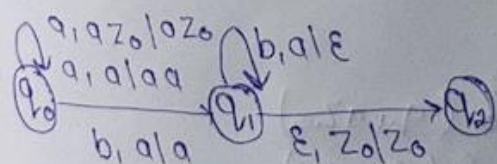
$$\delta(q_0, b, a) = (q_1, a)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$$

$$\delta(q_0, b, Z_0) = (q_2, Z_0) (q_0)$$

(b) $L = a^n b^{n+1}, n \geq 1$



$$\delta(q_0, a, Z_0) = (q_0, a, Z_0)$$

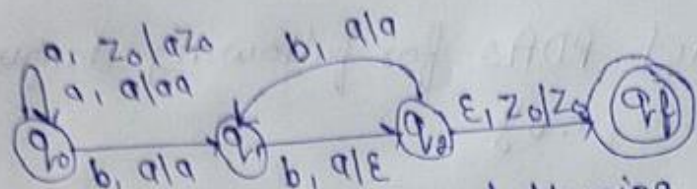
$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, a)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$$

(c) $L = a^n b^n, n \geq 1$



We have PDA N as following

$$N = \{ q_0, q_1, q_2, q_f \}, \{ a, b \}, \{ Z_0, x \} \delta (q_0, Z_0, q_0)$$

$$\delta (q_0, a, Z_0) = (q_0, aZ_0)$$

$$\delta (q_0, a, a) = (q_0, aa)$$

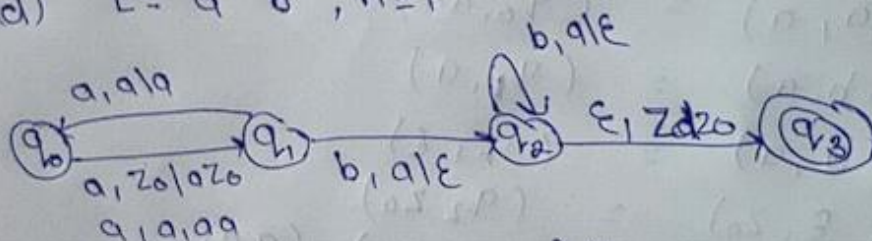
$$\delta (q_0, b, a) = (q_1, a)$$

$$\delta (q_1, b, a) = (q_2, \epsilon)$$

$$\delta (q_2, b, a) = (q_1, a)$$

$$\delta (q_2, \epsilon, Z_0) = (q_f, Z_0)$$

(d) $L = a^n b^n, n \geq 1$



We have PDA N as following

$$N = \{ q_0, q_1, q_2, q_3 \}, \{ a, b \}, \{ Z_0, x \} \delta (q_0, Z_0, q_0)$$

δ is given by

$$\delta (q_0, a, Z_0) = \delta (q_1, aZ_0)$$

$$\delta (q_0, a, a) = \delta (q_1, aa)$$

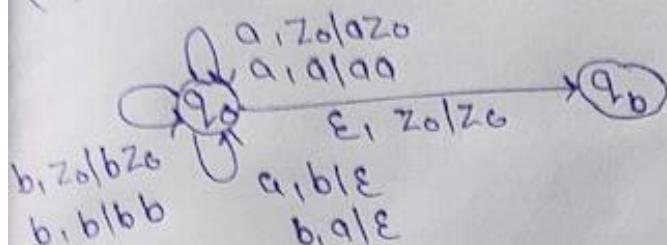
$$\delta (q_1, a, a) = \delta (q_0, a)$$

$$\delta (q_1, b, a) = \delta (q_2, \epsilon)$$

$$\delta (q_2, b, a) = \delta (q_2, \epsilon)$$

$$\delta (q_2, \epsilon, Z_0) = \delta (q_3, Z_0)$$

(e) $L = \{w \mid a(w) = nb(w)\}$



$$\delta(q_0, a, Z_0) = \delta(q_0, aZ_0)$$

$$\delta(q_0, b, Z_0) = \delta(q_0, bZ_0)$$

$$\delta(q_0, a, a) = \delta(q_0, aa)$$

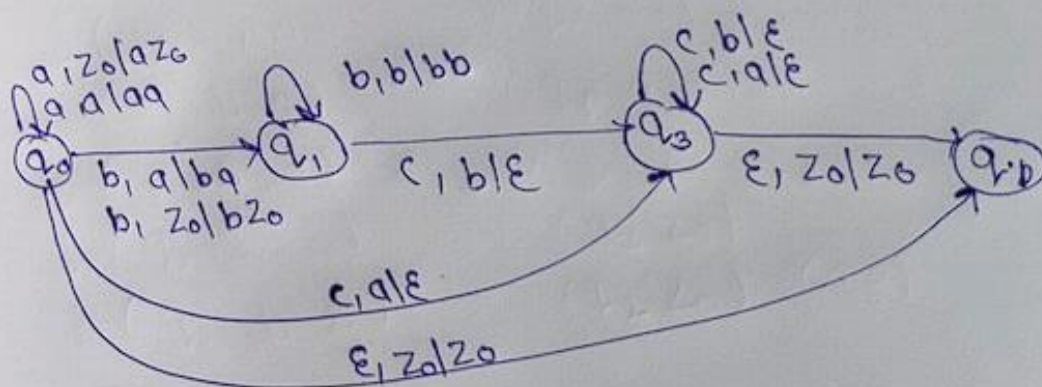
$$\delta(q_1, b, b) = \delta(q_0, bb)$$

$$\delta(q_0, a, b) = \delta(q_0, \epsilon)$$

$$\delta(q_0, b, a) = \delta(q_0, \epsilon)$$

$$\delta(q_0, \epsilon, Z_0) = \delta(q_1, Z_0)$$

(f) $L = a^n b^m c^{n+m} \quad n, m \geq 0$



$$\delta(q_0, a, Z_0) = \delta(q_0, aZ_0)$$

$$\delta(q_0, a, a) = \delta(q_0, aa)$$

$$\delta(q_0, b, a) = \delta(q_1, ba)$$

$$\delta(q_1, b, a) = \delta(q_1, bZ_0) \quad (n=0)$$

$$\delta(q_1, b, b) = \delta(q_1, bb)$$

$$\delta(q_2, c, b) = \delta(q_2, \epsilon)$$

$$\delta(q_2, c, b) = \delta(q_2, \epsilon) \quad (\text{pop } b)$$

$$\delta(q_2, c, a) = \delta(q_2, \epsilon) \quad (\text{pop } a)$$