Assignment-4

Que-1 find context free grammer for the following language (with $n>=0, m>=0$)

(a) $L = \{a^n b^m : n<m+3\}$

Solve

$S \to AAAB$
$A \to a \mid \varepsilon$
$B \to aBb \mid Bb \mid \varepsilon$

(B) $L = \{a^n b^m c^k : n = m \text{ or } m<=k\}$

$S \to A \mid B$
$A \to Ac \mid aAb \mid \varepsilon$
$B \to aB \mid bBc \mid Bc \mid \varepsilon$.

Que-2 find a CFG for each of the language defined by the following expressions

a) ab^*

Solve

$S \to aA$
$A \to b \mid \varepsilon$

b) a^*b^*

Solve $L = \{ab, aabb, aaabbb \ldots\}$

$S \to aSb \mid \varepsilon$

Qu-3 find CFG for the following language over the alphabet $\{a, b\}$

a) All words in which the letter b is never tripled.

Solve

$S \rightarrow aS \mid bA \mid \varepsilon$
$A \rightarrow aS \mid bB \mid \varepsilon$
$B \rightarrow aS \mid \varepsilon$

B) All words that have exactly two or three b's.

c) All words that have different first and last letter.

Solve

$S \rightarrow aAb \mid bAa \mid$
$A \rightarrow aA \mid bA \mid \varepsilon$

Qu-5 find CFG for each of the following language

a) $\{0^n 1^n : n \geq 0\}$

Solve

$S \rightarrow 0S1 \mid \varepsilon$

B) $\{a^n b^n c^m : n \geq 0, m \geq 0\}$

$S \rightarrow aSbA \mid \varepsilon$
$A \rightarrow cA \mid \varepsilon$

Qu-6 (a) Give the CFG to produce balanced parenthesis.

b) Construct CFG to generate String with aa.

Solve

$S \rightarrow aS \mid \varepsilon$

Reg $= a^*$
$L = \{a, aa, aaa, aaa \cdots\}$

Q-7 (a) Find a CFG for $\{n : n_a(n) = n_b(n)\}$ where $n \in (a|b)^*$ and $n_i(n)$ is the no of i's in the String s

$S \rightarrow aSb \mid bSa \mid \varepsilon$

b) Find CFG for $\{n : n_a(n) > n_b(n)\}$ where $n \in \{a^* b^*\}$.

Solve

$S \rightarrow aAb \mid bAA \mid \varepsilon$
$A \rightarrow a \mid \varepsilon$

Que-8 (a) Show that $\{ww^R : w \in (a,b)^*\}$ is context free.

Solve

$S \rightarrow aSb \mid bSa \mid \varepsilon$

Section - B

1) Show a derivation tree for the String aabbbb with the gramma.

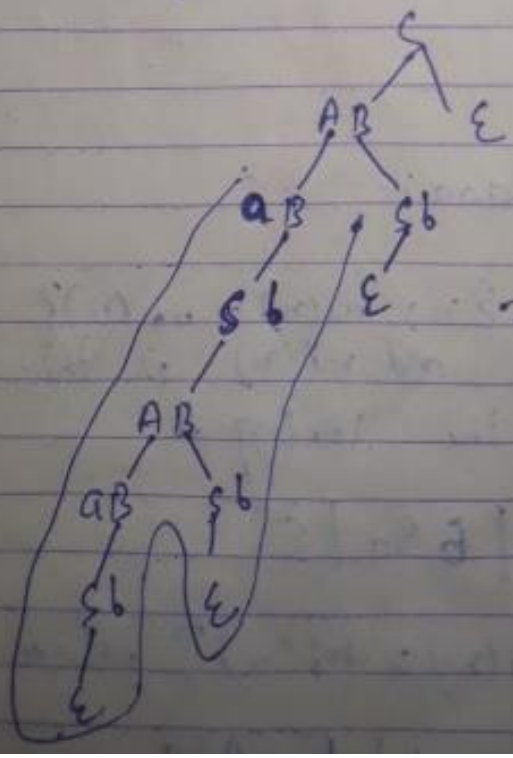
Sove

$S \rightarrow AB \mid \varepsilon$

$A \rightarrow aB$

$B \rightarrow Sb$

left most Derivation tree.



= aabbbb

Section-D

(1) Eliminate ε production from the following CFGs

(a) $S \to AB \mid ABC$
$A \to BA \mid BC \mid \varepsilon$ ha
$B \to AC \mid CB \mid \varepsilon$ ha
$C \to BC \mid AB \mid A \mid C$.

Nullable variable are $\{A, B\}$
$\{A, B, C\}$ (Production $C \to AB$)
$\{A, B, C, S\}$ (Production $S \to AB$)

Remove all ε production -
$S \to AB \mid Ac \mid BC \mid A \mid B \mid C$.
$A \to BA \mid Bc \mid AC \mid a \mid A \mid B \mid C$
$B \to AC \mid CB \mid AB \mid b \mid A \mid B \mid C$.
$C \to BC \mid AB \mid Ac \mid A \mid B \mid C$.

(B) $S \to XY$
$X \to Zb$
$Y \to bW$
$Z \to AB$
$W \to Z$

Solve there is no Nullable variable so the grammar correct

$S \to XY \mid WsZ$
$X \to Zb$
$Y \to bW$

(C) $A \to aA \mid bA \mid \varepsilon$
$B \to Ba \mid Bb \mid \varepsilon$

Solve Nullable variable are (A, B)

after Removing Epsilon. the Grammer is -

$A \to aA \mid bA \mid a \mid b \mid A$
$B \to Ba \mid Bb \mid B \mid a \mid b$

Qu-2 Eliminating unit production from the given grammer.

(B) $S \to A \mid bb$
$A \to B \mid b$
$B \to S \mid b$.

Solve unit pairs are/production (P') are

(S, S)
(A, A)
(B, B)
(S, A)
(S, B)
(A, B)
(B, A)

~~production p' are~~

unit pair	production in P'
(S, S)	$S \to bb$
(A, A)	$A \to b$
(B, B)	$B \to b$
(S, A)	$S \to b$
(S, B)	$S \to b$
(A, B)	$A \to b$
(B, A)	$B \to b$

Grammer after Removing unit production.

$S \to b \mid bb$
$A \to b \mid b$
$B \to bb \mid b$

(4) Remove all unit production all useless production and all ε production from the Grammer.

S → aA | aBB
A → aaA | ε
B → bB | bbC
C → B

Solve

first Eliminate ε production

Nullable variable are A and S

After Removing ε.

S → aA | aBB | ABB | A | B | B
A → aaA | aA | a | A
B → bB | bbC
C → B.

unit pair are:

(S, S)
(A, A)
(B, B)
(C, C)
(S, A)
(S, B)
(S, C)
(A, B)
(A, C)
(B, A)
(B, C)
(C, A)
(C, B)

production (P') are.	
(S, S)	aA | aBB'
(A, A)	aaA | aA | a
(B, B)	bB | bbC
(S, B)	bB | bbC
(S, A)	aaA | aA | a
(A, B)	bB | bbC
(B, A)	aaA | aA | a
(C, A)	aaA | aA | a
(C, B)	bB | bbC.

Final Grammer without unit production is.

S → aA | aBB | aaA | aA | a | bB | bb
A → aaA | aA | a | aaA | aA | a.
B → bB | bb |

there is no unreachable State.
after the Grammer is.

S → aA | aBB | aaA | aA | a | bB | bb
A → aaA | aA | a | aaA | aA | a.
B → bB | bb

Qu-5 Convert the Grammer in to CNF

S → aSb | Sab | ab

Solve

S → AD | DA | AB
A → a
B → b
D → SB

(6) Tranfer in to CNF Form.

S → bAB
A → bAB | ε
B → BAa | A | ε.

Solve

first Simplify the Grammer
(1) Elimination of ε
the Nullable variable are (A, B)

The Grammer after eliminet ε as

S → baAB | baA | baB | A | B
A → bAB | bA | bB | A | B | b
B → BAa | Ba | Aa | B | A | a

(2) Eliminate Unit production.

(S, S)	S → baAB | baA | baB
(A, A)	A → bAB | bA | bB | b
(B, B)	B → BAa | Ba | Aa | a.
(S, A)	S → b
(S, B)	S → a
(A, B)	A → a.
(B A)	B → b

After the Grammer is:-

S → baAB | baa | bab | ab
A → bAB | ba | bb | a | b
B → BAa | ba | aa | b | a

CNF form.

S → CD | CA | CB | a | b
A → BD | AB | BB | a | b
B → EA | BA | AA | b | a.
A → a
B → b
C → ba

Qu-8 Convert the Grammer-

$S \rightarrow ABb \mid a \mid b$
$A \rightarrow aaA \mid B$
$B \rightarrow bAb$

Solve Convert it in to CNF

unit pair are-

(S, S)	$S \rightarrow ABb \mid a \mid b$
(S, A)	$S \rightarrow aaA$
(S, B)	$S \rightarrow bAb$
(A, A)	$A \rightarrow aaA$
(A, B)	$A \rightarrow bAb$
(B, B)	$B \rightarrow bAb$
(B, A)	$B \rightarrow aaA$

$S \rightarrow ABb \mid a \mid b \mid aaA \mid bAb$
$A \rightarrow aaA \mid bAb \mid aaA$
$B \rightarrow bAb \mid aaA \mid$

Qu-9 Convert the following Grammer in to Greibach Normal form.

a) $S \rightarrow AB$
$A \rightarrow aA \mid bB \mid b$
$B \rightarrow b$

Solve Convert it into CNF

$S \rightarrow AB$
$A \rightarrow CA \mid BB \mid b$
$B \rightarrow$

Replace non terminal with some another variable. Ø

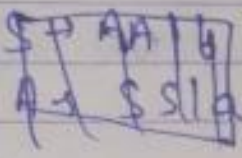
$S \to A_1$
$A \to A_2$
$B \to B_3$
$C \to A_4$

$A_1 \to A_2 A_3$
$A_2 \to A_4 A_2 \mid A_3 A_3 \mid b$
$A_3 \to b$
$A_4 \to a.$

GNF Forms

$A_1 \to b A_3$
$A_2 \to a A_2 \mid b A_3 \mid b$
$A_3 \to b$
$A_4 \to a.$

b)



$S \to AB \mid a$
$A \to BC \mid b$
$B \to CD \mid C$
$C \to DS \mid d.$

Solve the Grammer is already in form of CNF

$S \to AB \mid a$
$A \to BC \mid b$
$B \to CD \mid C$
$C \to DS \mid d$

$S \to AB \mid a$
$A \to B \mid b$
$B \to C$

Replace Non terminal with some another variable.

$S \to A_1$
$A \to A_2$
$B \to A_3$

$A_1 \to A_2 A_3 \mid a.$
$A_2 \to A_3 \mid b$
$A_3 \to C$

$A_1 \rightarrow A_2 A_3 | a$.
$A_2 \rightarrow A_3 | b$.
$A_3 \rightarrow c$.

CNF Form

$A_1 \rightarrow b A_3 | a$.
$A_2 \rightarrow c | b$
$A_3 \rightarrow c$.

B) $S \rightarrow AA | b$
$A \rightarrow SS | a$.

Solve

Convert in to CNF Form.

(S, S)	$S \rightarrow b$
(S, A)	$A \rightarrow a$.
(A, A)	$A \rightarrow a$
(A, S)	$S \rightarrow b$.

$S \rightarrow AA | b | aa$.
$A \rightarrow SS | a | bb$.

define Some New variable.

$A_1 \rightarrow S$	$A_1 \rightarrow A_2 A_2 | b | aa$.
$A_2 \rightarrow A$	$A_2 \rightarrow A_1 A_1 | a | bb$.

$A_1 \rightarrow a A_2 | b | aa$.
$A_2 \rightarrow A_1 A_1 | a | bb$.

left Recursion.

$Z \rightarrow A_1 Z$

$Z \rightarrow bZ$.

GNF forms

$A_1 \rightarrow aA_2 \mid b \mid aa$.

$A_2 \rightarrow bZ \mid aZ \mid bbZ$

$Z \rightarrow bZ$