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IT-1
Math - IV
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Part-B

8) Let

X is a function of x only and T is a function of t only

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}(XT) = X\frac{dT}{dt} = XT' \text{ (say)}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(XT) = T\frac{dX}{dx} = TX' \text{ (say)}$$

From the given eq:-

$$4XT' + TX' = 3XT$$

$$\frac{4T'}{T} + \frac{X'}{X} = 3$$

$$\frac{4T'}{T} - 3 = \frac{-X'}{X} = p^2 \text{ (say)}$$

i) $\frac{4T'}{T} = p^2 + 3$

$$\frac{dT}{T} = \left(\frac{3+p^2}{4}\right) dt$$

$$\log T = \left(\frac{3+p^2}{4}\right)t + \log C_1 \Rightarrow T = C_1 e^{\left(\frac{3+p^2}{4}\right)t}$$

ii) $\frac{-X'}{X} = p^2 \Rightarrow \frac{X'}{X} = -p^2 \Rightarrow \frac{dX}{X} = -p^2 dx$

$$\log X = -p^2 x + \log C_2$$

$$X = C_2 e^{-p^2 x}$$

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From (1), we get

$$U = XT = C_1 C_2 e^{-p^2 x + \left(\frac{3+p^2}{4}\right)t}$$

$$u(x,t) = b_n e^{-p^2 x + \left(\frac{3+p^2}{4}\right)t}$$

Most general soln is :-

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-p^2 x + \left(\frac{3+p^2}{4}\right)t}$$

When $t=0$, $u(x,0) = 3e^{-x} - e^{-5x} = \sum_{n=1}^{\infty} b_n e^{-p^2 x}$

comparing when $p^2 = 1$, $b_1 = 3$ and when $p^2 = 5$, $b_2 = -1$

$\therefore$ From (5) general soln is :-

$$u(x,t) = 3e^{-x+t} - e^{-5x+2t}$$

6) $z(x+y)p + z(x-y)q = x^2 + y^2$

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$$

Using multiplier $x, -y, -z$ as multiplier we get :-

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$$= \frac{x\,dx - y\,dy - z\,dz}{z(x^2+xy)+z(-xy+y^2)-z(x^2-y^2)}$$

$$\frac{x^2}{2} - \frac{y^2}{2} - \frac{z^2}{2} = \frac{C_1}{2}$$

$$\boxed{x^2 - y^2 - z^2 = C_1} \quad -①$$

Using $y, x, -z$ as multiplier -:

$$\frac{y\,dx + x\,dy - z\,dz}{z(xy+y^2)+z(x^2-yx)-z(x^2+y^2)} = 0$$

$$y\,dx + x\,dy - z\,dz = 0$$

Integrating -

$$xy - \frac{z^2}{2} = \frac{C_2}{2}$$

$$\boxed{2xy - z^2 = C_2} \quad -②$$

from ① & ② the general soln -:

$$\phi(x^2-y^2-z^2,\ 2xy - z^2) = 0$$

9) Since R & G are negligible, we use radio eq-

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2}$$

Since the ends are suddenly bounded, we have the boundary conditions -:

$$v(0, t) = 0, \; v(l, t) = 0$$

Also, the initial conditions are -:

$$i(x, 0) = i_0$$
$$v(x, 0) = e_1 \sin \frac{\pi x}{l} + e_5 \sin \frac{5\pi x}{l}$$

$$\frac{\partial i}{\partial x} = -C \frac{\partial v}{\partial t}$$ gives

$$\left(\frac{\partial v}{\partial t}\right)_{t=0} = 0$$

Let $v = XT$ be the soln. of 1, where X is a function of x only and T is a function of t only -

$$\therefore \quad TX'' = LCXT''$$
$$\frac{X''}{X} = LC \frac{T''}{T} = -p^2 \text{ (say)}$$

$$X'' + p^2 X = 0 \Rightarrow X = C_1 \cos px + C_2 \sin px$$

$$T'' + \left(\frac{p^2}{LC}\right) T = 0$$

$$T = C_3 \cos \frac{p}{\sqrt{LC}} t + C_4 \sin \frac{p}{\sqrt{LC}} t$$

$$V = XT = \left(C_1 \cos px + C_2 \sin px\right)\left(C_3 \cos \frac{pt}{\sqrt{LC}} + C_4 \sin \frac{pt}{\sqrt{LC}}\right) \quad \text{—(5)}$$

Using the boundary condition (2) :-

$$C_1 = 0 \text{ and } p = \frac{n\pi}{l}, \; n \in I$$

$$V = \sin \frac{n\pi x}{l}\left(a_n \cos \frac{n\pi t}{l\sqrt{LC}} + b_n \sin \frac{n\pi t}{l\sqrt{LC}}\right)$$

Using initial condition (4) :- $b_n = 0$

$$V = a_n \sin \frac{n\pi x}{l} \cos \frac{n\pi t}{l\sqrt{LC}}$$

$\therefore$ The most general soln. :-

$$\boxed{V = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{l} \cos \frac{n\pi t}{l\sqrt{LC}}}$$

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Part-A

4) $(1+x^2)\frac{\partial^2 z}{\partial x^2} + (5+2x^2)\frac{\partial^2 z}{\partial y \partial x} + (4+x^2)\frac{\partial^2 z}{\partial y^2} = \sin(x+y)$

$A = 1+x^2$, $B = 5+2x^2$, $C = 4+x^2$

Now, $B^2 - 4AC = (5+2x^2)^2 - 4(1+x^2)(4+x^2)$

$= 25 + 4x^4 + 20x^2 - 16 - 16x^2 - 4x^2 - 4x^4$

$= 25 - 16$

$= 9$

$B^2 - 4AC > 0$

∴ The equation is hyperbolic.

3) $xp + yq = z$

A.E

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

(i) (ii) (iii)

from eq (i) & (ii)

$$\frac{dx}{x} = \frac{dy}{y}$$

$\log x = \log y + \log C1$

$\frac{x}{y} = C1$ — (1)

from (ii) & (iii)

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\log y = \log z + \log C_2$$

$$\frac{y}{z} = C2 \quad \text{— (ii)}$$

from (i) & (ii)

$$f\left(\frac{x}{y}, \frac{y}{z}\right)$$

1) Eq. of sphere -:

$$x^2+y^2+(z-c)^2 = r^2 \quad \text{— (1)}$$

Partially differentiating eq. wrt x & y

$$2x + 2(z-c)p = 0$$
$$2(z-c)p = -2x$$
$$(z-c)p = -x \quad \text{— (2)}$$

$$2y + 2(z-c)q = 0$$

$$2(z-c)q = -2y$$
$$(z-c)q = -y \quad \text{— (3)}$$

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Divide eq (1) by (3)

$$\frac{p}{q} = \frac{x}{y}$$

$$py = xq$$

$$py - xq = 0$$

5) $V = v(x, t)$

$I = v(x, T)$

$$\frac{\partial v}{\partial x} = -L\frac{\partial I}{\partial t}$$

$$= -C\frac{\partial v}{\partial t}$$

On differentiating wave eq. of voltage & current

$$\frac{\partial^2 v}{\partial t^2} - \frac{u^2 \partial^2 v}{\partial x^2} = 0$$

$$\boxed{\frac{\partial^2 v}{\partial x^2} = RC\frac{\partial v}{\partial t}}$$

$$\frac{\partial^2 I}{\partial t^2} - \frac{u^2 \partial^2 I}{\partial x^2} = 0$$

$$\boxed{\frac{\partial^2 v}{\partial x^2} = \frac{RC\,\partial I}{\partial t}}$$ are telegraph eq.

2) $\frac{\partial^4 z}{\partial x^4} = 0$

Intigrating wrt x

$$\frac{\partial^3 z}{\partial x^3} = C_1 = f_1(y)$$

Again-:

$$\frac{\partial^2 z}{\partial x^2} = x f_1(y) + f_2(y)$$

$$\frac{\partial z}{\partial x} = x^2 f_1(y) + x f_2(y) + f_3(y)$$

$$z = x^3 f_1(y) + x^2 f_2(y) + x f_3(y) + f_4(y)$$

Part-C

11)

a) $x^2 r - y^2 t + px - qy = \log x$

Let $x = e^X$, $y = e^Y$ so that $X = \log x$ & $Y = \log y$

$D = \frac{\partial}{\partial X}$ and $D' = \frac{\partial}{\partial X}$, —

$$[D(D-1) - D'(D'-1) + D - D']z = X$$

$$z(D^2 - D'^2) = X$$

Which is a homogenous linear partial differential eq., with const. coefficient.

$$CF = \phi_1(Y+X) + \phi_2(Y-X)$$

$$PI = \frac{1}{D^2 - D'^2}(X) = \frac{1}{1^2 - (0)^2} \iint u\,du\,du$$

where $X = u$

$$= \int \frac{u^2}{2} du = \frac{u^3}{6} = \frac{X^3}{6}$$

$$z = \phi_1(Y + X) + \phi_2(Y - X) + \frac{X^3}{6}$$

$$= \phi_1(\log y + \log x) + \phi_2(\log y - \log x) + \frac{(\log x)^3}{6}$$

$$\therefore z = \phi_1(\log xy) + \phi_2\left(\log \frac{y}{x}\right) + \frac{1}{6}(\log a)^3$$

$$z = f_1(xy) + f_2\left(\frac{y}{x}\right) + \frac{1}{6}(\log x)^3$$

where f_1 and f_2 are arbitrary functions

11) b) Two dimension heat flow eq. in steady state :-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Its soln is $u(x, y) = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py})$

putting $x = 0$ & $u = 0$

$$0 = c_1 (c_3 e^{py} + c_4 e^{-py})$$
$$c_1 = 0$$

Putting the value of c_1 :-

$$u = c_2 \sin px (c_3 e^{py} + c_4 e^{-py})$$

Again putting $x = 8$ & $u = 0$

$$0 = c_2 \sin 8p (c_3 e^{py} + c_4 e^{-py})$$

$$\sin 8p = 0 = \sin n\pi \Rightarrow p = \frac{n\pi}{8} \ (n \in I)$$

Putting value of P in (2) :-

$$u(x, y) = c_2 \sin \frac{n\pi}{8} x \left(c_3 e^{\frac{n\pi y}{8}} + c_4 e^{\frac{-n\pi y}{8}}\right)$$

Putting $y = \infty$ and $u = 0$ in (3) -

$$0 = C_2 \sin\frac{n\pi x}{8}\, C_3 e^{\frac{n\pi y}{8}}$$

$$C_3 = 0$$

Putting $C_3 = 0$:-

$$U = C_2 C_4 \sin\frac{n\pi x}{8}\, e^{\frac{-n\pi y}{8}}$$

$$U = b_n \sin\frac{n\pi x}{8}\, e^{\frac{-n\pi y}{8}} \quad -(4)$$

Putting $y = 0$ & $U = 100 \sin\frac{\pi x}{8}$ in (4)

$$100 \sin\frac{\pi x}{8} = b_n \sin\frac{n\pi x}{8}$$

$$b_n = 100,$$

Putting the value of b_n in (4)

$$\boxed{U = 100 \sin\left(\frac{\pi x}{8}\right) e^{\frac{-\pi y}{8}}}$$