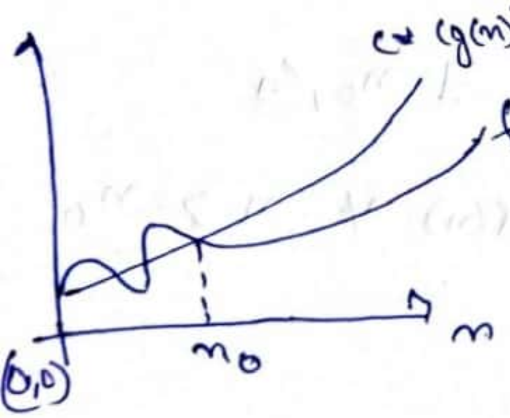
Q.1) Asymptotic Notation: There are language to express the required time & space by an algorithm to solve a given problem.

(1) Big - O Notation: It is notation for the worst case analysis of an algorithm. (Upper bound)

According to it for a two func $f(n)$ & $g(n)$

$f(n) = O(g(n))$, if and only if there exist $n_0$ & $c$ such that.

$0 \leq f(n) \leq c * g(n)$ for all $n \geq n_0$




Ans => $n+n^2 = O(n^2)$

here $f(n) = n+n^2$, $g(n) = n^2$.

$n+n^2 \leq n^2 + n^2$ ($\because n < n^2, n^2 = n^2$)

$n+n^2 \leq 2n^2$ (here $c=2$) for $n_0 = 1$

so $f(n) = O(g(n))$

or $n+n^2 = O(n^2)$

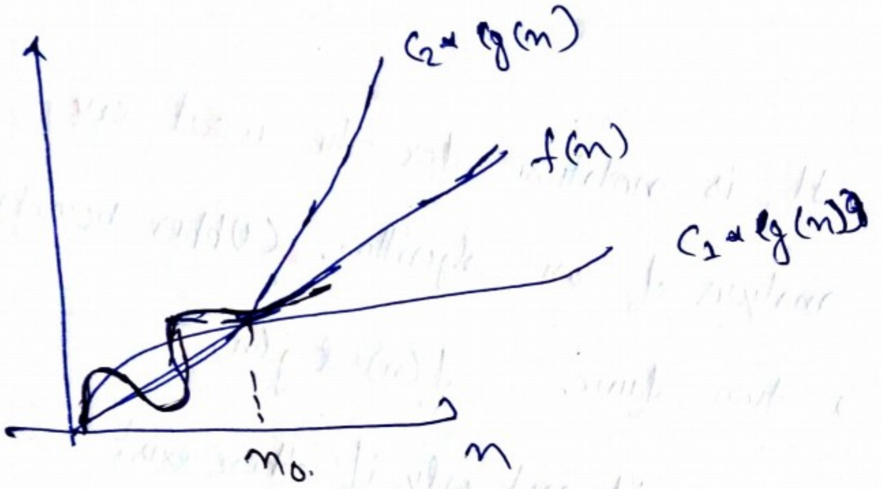
**⊙ Big theta ($\Theta$)**: For avg case time complexity (tighly bound)

for any two function $f(n)$ & $g(n)$

$f(n) = \Theta(g(n))$ if and only if there exists $n_0, c_1, c_2$

such that $0 \le c_1 * g(n) \le f(n) \le c_2 * g(n)$

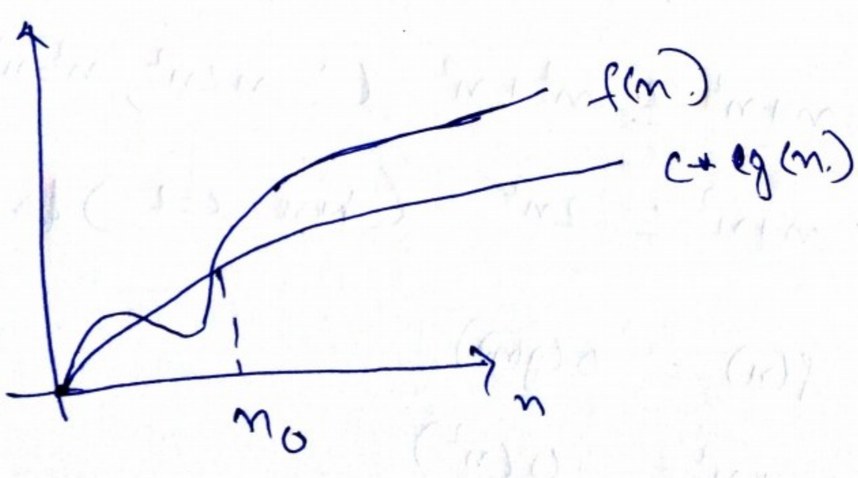
[ for $n \ge n_0$ ]




**⊙ Big Omega ($\Omega$)**: for best case complexity (lower bound)

$f(n) = \Omega(g(n))$ iff $\exists\ n_0, c_1$

$\ni\ 0 \le c_1 * g(n) \le f(n)\ \forall\ n \ge n_0$



Q.2 ⇒ T.C. of for (i=1 to n) { i = i * 2 }

Series ⇒ 1, 2, 4, 8, 16 - - - - n (G.P.)

$a = 1$ , $r = 2$

$t_k = ar^{k-1} \Rightarrow n = a * 2^{k-1}$

$\Rightarrow n = 2^{k-1}$

$\Rightarrow 2^k = 2n$

$\Rightarrow k = 2 \log_2 n$

so T.C. ⇒ $O(\log_2 n)$

Q.3 ⇒ $T(n) = \{ 3T(n-1) \}$ if $n>0$, otherwise 1 }

$T(n) = 3T(n-1)$ - - - (i)

let $n = n-1$ , $T(n-1) = 3T(n-2)$

$T(n) = 3^2 T(n-2)$

or $T(n) = 3^3 T(n-3)$

or $T(n) = 3^n T(n-n)$

$T(n) = 3^n T(0) = 3^n$

so T.C. ⇒ $O(3^n)$

Q.4) $T(n) = \{ 2T(n-1) - 1$ if $n>0$, otherwise $1 \}$

$T(n) = 2T(n-1) - 1$

let $n = n-1$, $T(n-1) = 2T(n-2) - 1$

so $T(n) = 2(2T(n-2) - 1) - 1$

$= 2^2 T(n-2) - 2 - 1$

let $n = n-2$, $T(n-2) = 2T(n-3) - 1$

so $T(n) = 2^2(2T(n-3) - 1) - 2 - 1$

$= 2^3 T(n-3) - 2^2 - 2 - 1$

or

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \dots - 2^1 - 2^0$

$T(0) = 1$, let $n-k=0$ so $k=n$

$T(n) = 2^n T(n-n) - 2^{n-1} - 2^{n-2} - \dots - 2^1 - 2^0$

$= 2^n - 2^{n-1} - 2^{n-2} - \dots - 2^1 - 2^0$

$= 2^n - (2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0) \}$ G.P

$T(n) = 2^n - \frac{1(2^n - 1)}{2-1} = \cancel{2^n} - \cancel{2^n} + 1$

so T.C. ⇒ $O(1)$

Q.5 ⇒

```
int i=1, s=1;
while (s<=n) {
        i++; s=s+i;
        printf("#");
};
```

Series ⇒ 1, 3, 6, 10, 15, 21, 28 ------ n

1st iteration ⇒ s = s + 1

2nd iteration ⇒ s = s + 1 + 2

till ⇒ $1+2+3+ \dots + k <= n$

$$\frac{k*(k+1)}{2} <= n$$

or $O(k^2) <= n$

or $k = O(\sqrt{n})$

so T.C. = $O(\sqrt{n})$

Q.6 ⇒ for (i=1 ; i*i <= n ; i++)

count ++

let loop run till k i=k

$k^2 <= n$

$k <= \sqrt{n}$

so T.C. ⇒ $O(\sqrt{n})$

Q.7 ⇒ for (i=n/2 ; i<=n ; i++) $O(n)$

for (j=1 ; j<=n ; j=j*2) $O(\log n)$

for (k=1 ; k<=n ; k=k*2) $O(\log n)$

so T.C. ⇒ $O(n \log^2 n)$

Q.8 ⇒

```
function (int n) {
    if (n==1) return;
    for (i=1 to n) {
        for (j=1 to n) {
            print ("*");
        }
    }
    function (n-3);
}
```

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Recurrence Relation $\Rightarrow$ $T(n) = T(n-3) + n^2$

or $T(n) = T(n-6) + 2*n^2$

$T(n) = T(n-9) + 3n^2$

or $T(n) = T(n-3k) + kn^2$

$T(1) = 0$ , $n-3k = 1$ $\Rightarrow k = \frac{n-1}{3}$

so $T(n) = T(1) + \frac{(n-1)}{3} n^2$

so T.C. $\Rightarrow$ $O(n^3)$

Q.9 $\Rightarrow$

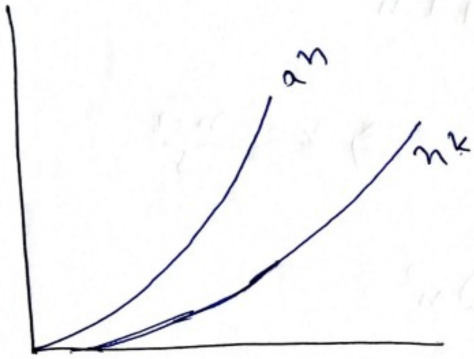
```
for (i=1 to n){
    for (j=1 ; j<=n ; j=j+i)
        printf("*");
}
```

| $i$ | $j$ | times |
| --- | --- | --- |
| 1 | $1 \to n$ | $n$ |
| 2 | $1 \to n$ | $\frac{n}{3}$ |
| 3 | $1 \to n$ | $n/3$ |
| ⋮ | ⋮ | ⋮ |
| $n$ | $1 \to n$ | 1 |
| | | $n \log n$ |

T.C. = $\cancel{O(n^2)}$ $O(n \log n)$

Q.10) Find asymptotic relation btw $n^k$ & $a^n$, $k >= 1$ & $a > 1$ are constants, find $c$ & $n_0$ for which relations holds.

Sol)




$n^k = O(a^n)$

$n^k \leq a^n$, $c \;\forall\; c > 0$ & $n \geq n_0$

let $n = n_0$

$n_0^k \leq c \cdot a^{n_0}$

[so let $k = a = 3$]

[ $n_0^3 \leq c\, 3^{n_0}$ so $c \geq 1$ & $n_0 \geq 1$ ]

Q.11 ⇒ void fun (int n){

int i=0, j=1;

while (i<n){

i = i+j;

j++;

}}

Series ⇒ 0, 1, 3, 6, 10, 15 --

let at ~~kth~~ last iteration :

$n = 0 + 1 + 2 + 3 + 4 + 5 + \cdots + k$

$n = \frac{k(k+1)}{2}$

$n = \frac{k^2+1}{2}$

$n \cong k^2$

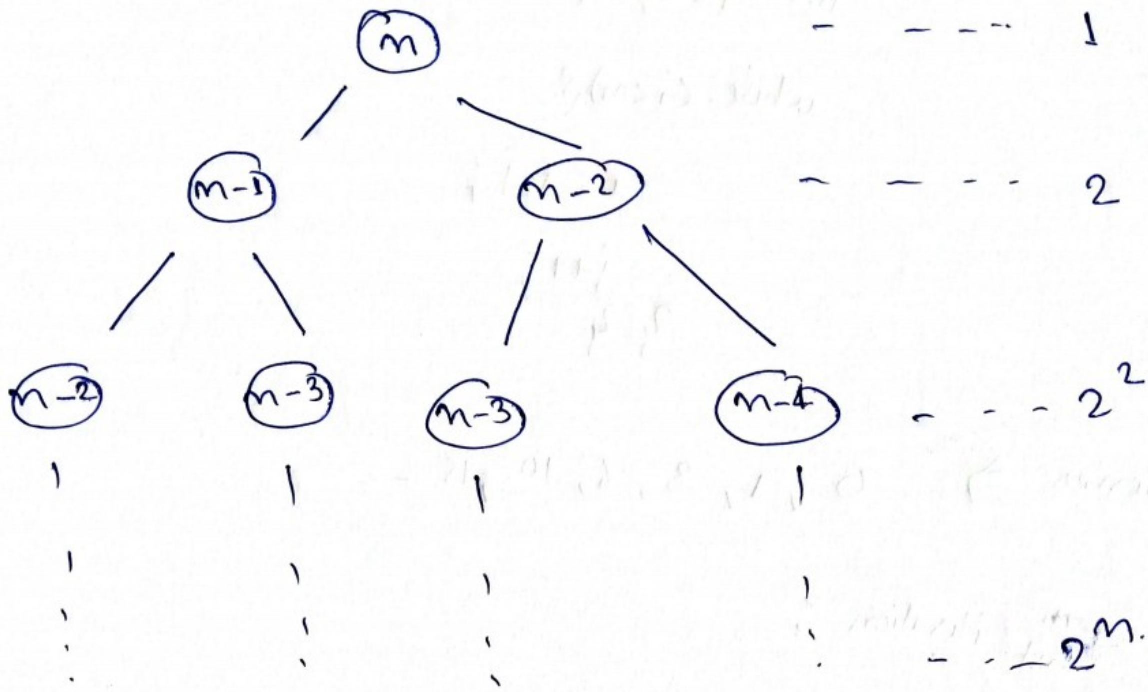
$k \cong \sqrt{n}$

so T.C. ⇒ $O(\sqrt{n})$.

Q.12 ⇒ Recurrence relation for fibonacci series ..

$T(n) = T(n-1) + T(n-2) + 1$

using Recurrence tree method :




$$T.C. = 1 + 2 + 4 + \cdots + 2^n = 1\,\frac{(2^{n+1}-1)}{2-1} = 2^{n+1}-1$$

so $T.C. = O(\cancel{n}\, 2^n)$

Space Complexity : Space complexity of fibonacci series using recursion is proportional to height of recurrence tree.

so $S.C. \rightarrow O(n)$

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Q.13 ⇒ Write code for complexity.

(i) $n \log n$

```
for (i to n)
{
        for (j=1 , j<=n, j*=2)
                O(1) statements
}
```

(ii) $n^3$

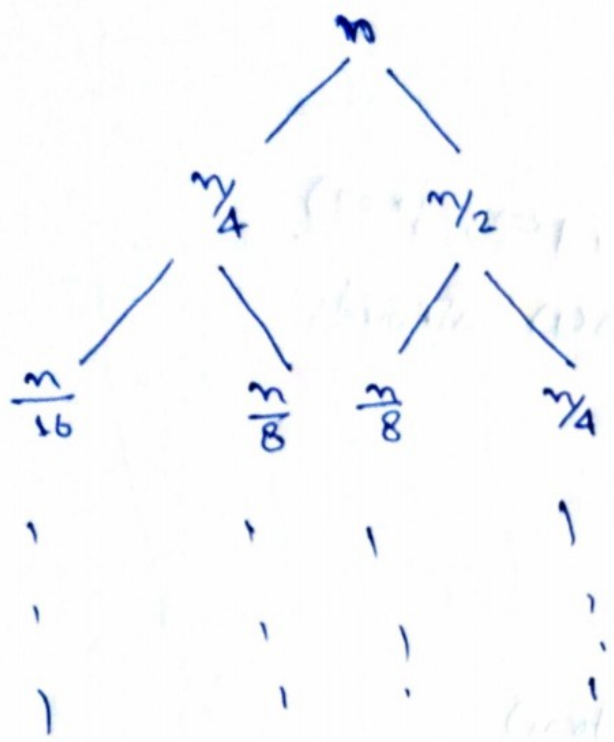
```
for (i to n)
    for (j to n)
        for (k to n)
            O(1) statements
```

(iii) $\log(\log n)$

~~for ( int i=0; i<n ;~~

```
int i=n;
while (i>0)
{   - - -}
    - - -}
        i = √i ;
}
```

Q.14) $T(n) = T(n/4) + T(n/2) + cn^2$




$\rightarrow cn^2$

$\rightarrow \frac{cn^2}{16} + \frac{cn^2}{4} = \frac{5cn^2}{16}$

$\rightarrow \frac{cn^2}{256} + \frac{cn^2}{64} + \frac{cn^2}{64} + \frac{cn^2}{16}$

$= \frac{25cn^2}{256}$

so $T(n) = (cn^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \cdots)$

here $r = \frac{5}{16}$ so $S_n = \frac{1}{1-r}$

$T(n) = cn^2(1 + \frac{5}{16} + \frac{25}{256} + \cdots)$

$= cn^2\left(\frac{1}{1-\frac{5}{16}}\right) = cn^2 \times \frac{16}{11}$

so T.C. $\Rightarrow \Theta(n^2)$

Q.15 ⇒ int fun (int n)

{

for (i to n)

for (j=1 ; j<n ; j+=i) {

O(1) task

}

}

| i | j | times |
|---|---|---|
| 1 | 1→n | n−1 |
| 2 | 1→n | (n−1)/2 |
| 3 | 1→n | (n−1)/3 |
| ⋮ | ⋮ | ⋮ |
| n | 1→n | n−1/n |
| | | n log n |

[ T.C. ⇒ O(n log n) ]

Q.16 ⇒ for (i=2 ; i<=n ; j= pow (i,k))

{

O(1)

}

sol ⇒ Series = $2, 2^k, 2^{2k}, 2^{3k}, \dots,$

let last term be $2^{x*k}$

$n = 2^{k^x}$

$\log n = x \log 2^k$

Q.16 ⇒ for (int i = 2 ; i <= n ; i = pow (i, k))

{ O(1);

}

$$i = 2, 2^k, 2^{k^2}, 2^{k^3} \ldots, 2^{k^x}$$

$$n = 2^{k^x}$$

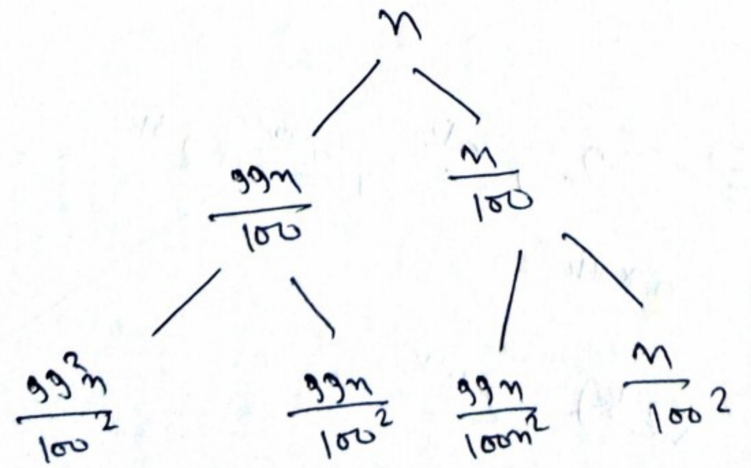
$$\log n = k^x \log 2$$

$$\frac{\log \log n}{\log 2} = x \log k$$

$$x = \frac{\log \log n}{\log 2 + \log k}$$

So T.C. ⇒ $O(\log \log n)$

Q.17 ⇒ ~~F(n) = T(n-1) + n~~ $T(n) = T\left(\frac{99n}{100}\right) + T\left(\frac{n}{100}\right)$



If we take longer branch i.e. $\frac{99n}{100}$

T.C. $\Rightarrow \log_{\frac{100}{99}} n \cong \log n$

$n = \left(\frac{99}{100}\right)^k$  $k = \log_{\frac{100}{99}} n$

$T(n) = n \left(\frac{\log \frac{100}{99}}{100}\right)^n = O(n \log_{99} n)$

Q18 ⇒ Increasing of growth.

(a) $100 < \log\log n < \log n < \sqrt{n} < n < n\log n < n^2 < 2^n < 2^{2n} < 4^n < n!$

(b) $1 < \log\log n < \sqrt{\log(n)} < \log n$ ~~$< 2n < 4n < 2(2^n) < \log$~~ $< \log 2n < 2\log n < n < 2n < 4n < n^2\log n < n^2 < \log(n!) < 2^{2n} < n!$

(c) $96 < \log_8 n <$ ~~$n\log$~~ $\log_2 n < 5n < n\log_8(n) < n\log_2 n < 8n^2 < 7n^3 <$ $\boxed{\text{~~}8^n(2n) < \log(n!)\text{~~} < n!}$

$\log n! < 8^{2n} < n!$

Q.19 ⇒ Linear Search :

```
for (i=0 to k-1)
{    if (ar[i] = key)
        {  return i ;
        }
    return -1 ;
}
```

Q.20 ⇒ Iterative Insertion Sort :

~~void insertion_sort ( arr , n)~~

~~loop from i=1 to n-1~~

~~pick element ar[i] & insert it into sorted into~~

~~sorted sequence.~~

```
void insertion_sort ( int arr[] , int n)
{   int i, temp, j ;
    for  i ← 1 to n
    {      temp ← arr[i] ;
           j ← i-1;
           while (j>=0  AND arr[j] > temp)
           {    arr[j+1] ← arr[j] ;
```

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```
            j ← j-1;
        }
        arr[j+1] ← temp;
    }
}
```

## Recursive Insertion sort :-

```
void recursive_insertion_sort (int arr[], int n)
{
    if (n <= 1)
        return
    recursive_insertion_sort (arr, n-1)
    val = arr[n-1]
    pos = n-2
    while (pos >= 0 && arr[pos] > val) {
        arr[pos+1] = arr[pos]
        pos = pos-1
    }
    arr[pos+1] = val
}.
```

It is called online sorting because it provided ~~one~~ one sorted element at a time & ~~sequence of sorted as consider~~ produces a partial solution without considering future elements.

Q.21 ⇒

| Algorithm | Time complexity: Best case | Average Case | Worst Case |
|---|---|---|---|
| ① Bubble sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ |
| ② Selection sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ |
| ③ Merge sort | $O(n\log n)$ | $O(n\log n)$ | $O(n\log n)$ |
| ④ Insertion sort | $O(n)$ | $O(n^2)$ | $O(n^2)$ |
| ⑤ Quick sort | $O(n\log n)$ | $O(n\log n)$ | $O(n^2)$ |
| ⑥ Heap sort | $O(n\log n)$ | $O(n\log n)$ | $O(n\log n)$ |

Q.22 ⇒

| Algorithm | Inplace | Stable | Online Sorting |
|---|---|---|---|
| Bubble Sort | ✓ | ✓ | ✗ |
| Selection Sort | ✓ | ✗ | ✗ |
| Merge Sort | ✗ | ✓ | ✗ |
| Insertion Sort | ✓ | ✓ | ✓ |
| Quick Sort | ✗ | ✗ | ✗ |
| Heap Sort | ✓ | ✗ | ✗ |

Q.23 ⇒ Recursive Binary Search :

```
int b_search ( int arr [ ], int l, int r, int x).

{    if (l > r)
         return -1;
     int m = (l+r) /2
     if (arr[m] = x)
          return m;
     else if (arr [m] < x)
          ~~return~~ b_search (arr, m+1, r, x);
     else
          b_search (arr, l, m-1, x
}
```

Iterative Binary Search :

```
int binarysearch (int arr [ ], int n, int x)
{
    l = 0 , r = n-1 ;
    while ( l < r)
    {   m = (l+r) /2
        if (arr [m] = x)        return m;
        else if ( arr [m] < x)      l = m+1;
        else    r = m-1;
    }
```

return -1;

}

Time & Space Complexity of Iterative Binary search ⇒ $O(\log n)$ & $O(1)$

Time & Space Complexity of Recursive Binary search ⇒ $O(\log n)$, $O(\log n)$

Q24 ⇒ Recurrence Relation for Binary Search ⇒

$$T(n) = T(n/2) + 1$$