Assignment 1

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1 Question 1

Let A be

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Let X be represented by

$$X^T = [x_1, x_2, \dots, x_n]$$

Then, AX equals:

$$(AX)^T = [\sum a_{1i}x_i, \sum a_{2i}x_i, \dots, a_{mi}x_i]$$

Thus

$$\frac{dA}{dX} = \begin{bmatrix} \frac{d\sum a_{1i}x_i}{dx_1}, \frac{d\sum a_{1i}x_i}{dx_2}, \dots, \frac{d\sum a_{1i}x_i}{dx_n} \\ \frac{d\sum a_{2i}x_i}{dx_1}, \frac{d\sum a_{2i}x_i}{dx_2}, \dots, \frac{d\sum a_{2i}x_i}{dx_n} \\ \vdots \ddots \vdots \\ \frac{d\sum a_{mi}x_i}{dx_1}, \frac{d\sum a_{mi}x_i}{dx_2}, \dots, \frac{d\sum a_{mi}x_i}{dx_n} \end{bmatrix}$$

Naturally since

$$\frac{dcx}{dx} = c$$

$$\frac{dA}{dX} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$= \text{A}$$

$$Hence, \frac{dA}{dX} = A$$

2 Question 3

2.1 subpart (a)

$$\frac{d}{d\begin{bmatrix} x \\ y \end{bmatrix}} \begin{bmatrix} 2\sin^2 x \cos y \\ x^2 + 3e^y \end{bmatrix} = \begin{bmatrix} 4\sin(x)\cos(x)\cos(y) & 2x + 3e^y \\ -2\sin^2(x)\sin(y) & 3e^y \end{bmatrix}$$

2.2 subpart(b)

$$\frac{d}{d\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}} \begin{bmatrix} 3x^2y + xywz \\ \sin(x^2 + wy + xz) \end{bmatrix}$$

$$= \begin{bmatrix} 6xy + ywz & 3x^2 + xwz & xyw & xyz \\ (2x+z)\cos(x^2 + wy + xz) & w\cos(x^2 + wy + xz) & x\cos(x^2 + wy + xz) & y\cos(x^2 + wy + xz) \end{bmatrix}$$

3 Question 2

The required dimension is $m * k * n$, this is because if we consider a $m * n$ matrix as a linear combination of vectors, then each vector yields a $m * k$ matrix, n times over, (I think?)