KushagraT-TML-W2

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1 Problem 1

1.1 Sol:

Let $X^{-1}(\Phi) = \omega$. Naturally this implies $X(\omega) = \Phi$. We now look at ω:

$$\omega = [\xi \in \Omega | X(\xi) = \Phi]$$

Naturally, such set ω must be the null set. Hence:

$$X^{-1}\Phi = \Phi$$

1.2 Sol:

Since X is defined as a function from its sample set 'Ω' to R Hence, obviously:

$$X(\Omega) = R$$

$$\mathrm{X}^{-1}(R) = \Omega$$

2 Porblem 2

Support of X implies all values of x where $f(x) > 0$ where $f(x)$ signifies the PDF of X Here,We have

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

Thus, to obtain PDF, we take $\dfrac{dF_X(x)}{dx}$ we get:

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \lambda e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

This implies $f(x) \geq 0$ for all $x \geq 0$. Hnece support of X is the set $[0, \infty)$.

3 Problem 3

Let

$$Y = \beta_0 + \beta_1 x + \epsilon$$

The loss function for L2-norm is:

$$L(\beta_0, \beta_1) = \sum_{i=1}^{N} (y_i - (\beta_1 x_i + \beta_0))^2$$

Naturally,

$$L(\beta_0, \beta_1) = \sum_{i=1}^{N} (y_i^2 - 2y_i(\beta_1 x_i + \beta_0) + (\beta_1 x_i + \beta_0)^2)$$

Now, to minimize $L(\beta_0, \beta_1)$, the partial derivatives with respect to β_0 and β_1 must be zero:

$$\frac{d\mathbf{L}}{d\begin{bmatrix}\beta_0 \\ \beta_1\end{bmatrix}} = 0$$

Thus:

$$\frac{\partial L}{\partial \beta_1} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \beta_0} = 0$$

Solving these equations, we get the expressions for β_1 and β_0.

If these β_1 and β_0 minimize the loss function for the L2 norm, the second derivative of $L(\beta_0, \beta_1)$ with respect to β_1 and β_0 must be positive.

On simplifying the first-order differential equations, we get:

$$\beta_1 = \frac{N(\sum xy) - (\sum x)(\sum y)}{N(\sum x^2) - (\sum x)^2}$$

$$\beta_0 = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{N(\sum x^2) - (\sum x)^2}$$