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Highlights

- Generalization of the classical tailored base-surge dual sourcing inventory model to support firms' modal split transport decision.
- Structural properties of the model, which allows for optimal solutions using dynamic programming and bisection search.
- Sensitivity analysis.

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A dual sourcing inventory model for modal split transport: Structural properties and optimal solution

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ABSTRACT

Shifting freight volumes from road to rail transport increases the economic performances of freight logistics. However, compared to road transport, rail transport generally lacks the flexibility in delivery quantity and frequency, and exhibits economies of scale in its shipment volume. This often leads to high inventory levels in the destination after deliveries. We generalize the tailored base-surge dual sourcing inventory model by introducing a fixed cost in rail transport, adding an extra decision in its delivery frequency, and relaxing the assumption of the base stock control of road transport, to support firms' modal split transport optimization. The objective is to optimize the controls of the two transport modes and the corresponding inventory management at the destination, which minimize the combined average transport and inventory costs per period in the steady state. Using stochastic dynamic programming, we find that when the delivery quantity and frequency of rail transport is fixed, the optimal shipment volume via the road transport indeed follows a base stock control. This allows to solve the relevant Bellman equation via an efficient policy iteration approach. We also find that the total cost is convex in the delivery quantity of rail transport, and a bi-section search can be applied. Finally, we analyze the sensitivity and robustness of our model using values suggested by a consumer goods firm.

1. Introduction

In order to obtain a more sustainable freight system, firms seek to develop strategies that shift transport volume from road to rail. Compared to road transport, rail transport, in general, incurs a lower unit transport cost, but it is also less responsive due to longer lead time and rather rigid with respect to delivery quantity and frequency. To capture the classical cost and responsiveness trade-off of the transport service, a simultaneous use of both transport modes, or so-called modal split transport (MST), is encouraged. This study is motivated by a problem from a fast-moving consumer goods firm that plans to ship freight volume from its plant to one of its distribution centers (DC), split between the two transport modes rail and road.

A distinct feature of rail transport, compared to road, is its economies of scale. In order to maximize this benefit, the firm seeks to partner with a rail operator to run a dedicated train, instead of outsourcing its transport to a third-party logistics service provider (LSP). When working with LSPs, the firm only pays a variable cost per container shipped, while in the dedicated train project, the firm will pay a significantly lower unit transport cost, but bear the fixed cost incurred from locomotive, crane, infrastructure, etc. (European Intermodal Association, 2012). As a comparison, the firm wants to keep the current operations of road transport: It pays a variable cost per container shipped but no fixed cost. Other advantages of the "in-house" transport management are better controls of the entire transport processes, alignment between its transport decisions and other supply chain metrics, increase of reliability and punctuality, etc. (CapGemini, 2016).

Rail transport in general lacks the flexibility in delivery quantity and requires stable volumes (Newman, Nozick, & Yano, 2002; Reis, Meier, Pace, & Palacin, 2013). An LSP can aggregate demand from several shippers and smooth the total volume on board, but this dedicated train cannot. In order to better manage the train capacity for the lowest cost, the firm plans to commit a constant delivery quantity every time the dedicated train operates over a mid-term time horizon. For example, every Monday in the year 2019, the train delivers 30 containers from the plant to the DC. Benefits from using a fixed-quantity commitment are: (1) it is much easier to manage the train capacity and therefore keep the variable transport cost as low as possible. (2) A fixed flow of the shipment allows the firm to level and smooth its production, and

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therefore acquires savings from stabilized material flow and labor requirements (Dong, Boute, McKinnon, & Verelst, 2018a). (3) A fixed-quantity shipment allows the firm to better manage its inventory at the DC (Baumol & Vinod, 1970; Janakiraman, Seshadri, & Sheopuri, 2015). The use of rail to ship a stable volume is described as a well-accepted principle in transport industry (Groothedde, Ruijgrok, & Tavasszy, 2005). While rail transport exhibits lower unit cost due to economies of scale, the firm naturally faces a substantial fixed cost when using rail transport, e.g., locomotive and infrastructure spendings (European Intermodal Association, 2012; The World Bank, 2011). Therefore, operating rail services with the same frequency as truck is not economically feasible, which raises the question on its optimal delivery frequency and quantity, when road transport is simultaneously used.

Our general research question is: what is the optimal split between a flexible but expensive fast transport mode (such as road) and a more rigid but cheaper slow transport mode (such as rail) that minimize the long-run average total costs, being transport and inventory holding or backorder penalty cost. More specifically, this involves determining the optimal delivery frequency (represented by the number of days between two train deliveries) and the number of containers that are shipped in every train delivery, as well as determining the flexible volume that is shipped via road transport on a daily basis.

We develop a mathematical model, whose mathematical structure is an extension of the periodic-review tailored base-surge (TBS) model known from the dual-sourcing literature (e.g., Janakiraman et al., 2015). The TBS model examines an international sourcing instead of transportation problem. As stated in Allon and Van Mieghem (2010), a US firm purchases a constant “base” quantity every period from a slow but cheap supplier (analogous to the rail transport in MST), and simultaneously a flexible “surge” quantity subjective to a base stock control from a fast but expensive supplier in Mexico (analogous to the road transport in MST). Compared to the TBS models in the literature, our MST model incorporates economies of scale of rail transport by including its fixed cost as an additional cost parameter, and its delivery frequency as an additional decision variable. We therefore generalize the periodic-review TBS model by allowing different delivery frequencies of both modes. In addition, previous TBS models (Allon & Van Mieghem, 2010; Dong, Transchel, & Hoberg, 2018b; Janakiraman et al., 2015; Janssen & de Kok, 1999) exclusively consider a base stock control of the fast mode (source) and approximate the base stock level as a decision variable. We relax this base stock assumption and seek to obtain optimal shipment quantities.

In the model analysis, we find that the MST problem is a two-stage optimization problem, where the firm first tactically commits the delivery quantity and frequency of rail transport over a mid-term, and then adjusts its operational decisions in road transport every period. As a consequence, the solution process of the MST problem consists of two stages, but in inverse order: first road and then rail. Following this logic, we show that when the decision of the rail transport is committed, the optimal control of the fast mode orders indeed has a base stock structure. This is interesting because the base stock control was an assumption in the TBS models that helps to simplify the problem, and we show that this assumption can actually be relaxed. The optimal base stock level in the TBS model remains constant in the steady state of a TBS model. However, in the steady state of our MST model, there is a repeating, time-dependent pattern of base stock levels within a cycle of two consecutive slow mode shipments. The fast mode usage is therefore characterized from a unique base stock level in the TBS model to a “base stock vector” in the MST model. Additionally, we show that the total cost per period in the steady state is convex in the slow mode quantity. Given these properties, we use stochastic dynamic programming (SDP) to obtain the optimal base stock control of road transport in the steady state, and bisection search to calculate the optimal rail transport decisions. This solution algorithm avoids examining all possible shipment volumes of both transport modes in a complex SDP algorithm, and significantly reduces the computing effort.

Equipped with the aforementioned solution technique, we then additionally contribute to the dual-sourcing TBS (MST) literature by offering a numerical study using the numbers suggested by a consumer goods firm. We find that the fixed cost is the main driver of its delivery frequency, which further impacts its constant delivery quantity per delivery cycle. The optimal delivery frequency and quantity of the rail transport then impact the firm’s daily adjustment in road transport deliveries. The sensitivity and robustness of the MST model are also presented.

The rest of the paper is structured as follows: Section 2 reviews the literature. Section 3 formulates the model and Section 4 analyzes the optimal solution of the MST policy. Section 5 reports the numerical validation and Section 6 concludes the paper.

2. Literature

Over the years, an extensive body of literature (see, e.g., Bontekoning, Macharis, & Trip, 2004; McKinnon, 2015) has discussed the shift of freight volume from road to rail transport, and its corresponding economic and environmental importance. These discussions establish the basis of our study. Furthermore, two streams of literature are of immediate relevance to our study: the transport literature that studies freight mode choice based on the total logistics (transport and inventory) costs approach, and the inventory literature on dual-sourcing.

In the first stream, the pioneering work from Baumol and Vinod (1970) studies a model minimizing a firm’s combined transport and inventory costs and shows that the use of a slow transport mode increases firms’ inventory. Sheffi, Eskandari, and Koutsopoulos (1988) solve the same model numerically and show how it is implemented to support a firm’s freight mode choice decisions. Blumenfeld, Burns, and Diltz (1985) use simple plots to illustrate how firms’ inventory controls are interconnected with shipment size and frequency in different transport networks, including direct shipping, shipping via a terminal, and a hybrid network. Speranza and Ukovich (1994) study a multi-product model and find that under the objective of minimizing total transport and inventory costs, firms could consolidate the freight volume of different products. This way the freight mode is changed from partial truckload to full truckload, and the average delivery frequency is reduced. Lloret–Battle and Combes (2013) examine more than 1000 shipments data from a French shipper survey, and confirm with an empirical study that firms improve their freight mode choices when considering inventory requirements.

Although the general problem context of this stream of literature is similar to ours, i.e., determining the transport mode to minimize the total transport and inventory costs, these studies do not consider a simultaneous use of more than one transport mode in the same transport corridor. Our MST model allows firms to effectively exploit the greater modal split flexibility by a simultaneous usage of both road and rail transport in a certain corridor.

The second stream of literature studies dual sourcing inventory problems, where a firm simultaneously orders from two sources, a fast but expensive supplier (equivalent to road transport in MST) and a slow but cheap supplier (equivalent to rail transport in MST), and minimizes total sourcing and inventory costs (equivalent to transport and inventory costs). Although this vast body of literature studies sourcing instead of transport problems, the dual-sourcing models have similar mathematical structures to ours. The general dual sourcing problem is known to be complex and an analytical solution can only be obtained when the lead time difference between the two suppliers is one (Fukuda, 1964). Otherwise, the optimal solution depends on the entire orders in the pipeline (Whittemore & Saunders, 1977) and it remains unknown for over 50 years (Sun & Van Mieghem, 2019). Recent dual sourcing models therefore incorporate additional assumptions and analyze practical policies, such as base stock policy (Feng, Sethi, Yan, & H., 2006), single index policy (Scheller-Wolf, Veeraraghavan, & van Houtum, 2007), dual index policy(Sheopuri, Janakiraman, & Seshadri, 2010; Song, Li Xiao, Zhang, & Zipkin, 2017), tailored base-surge policy (Allon & Van Mieghem, 2010; Janakiraman et al., 2015), order splitting policy (Thomas & Tyworth, 2006), etc. A neat and extensive review is referred to Minner (2003) and Yao and Minner (2018).

Of all the practical dual sourcing policies, the tailored base-surge (TBS) is the closest to our MST problem. The two major assumptions of this policy are (1) the slow supplier always delivers a constant "Base" quantity, and (2) the fast supplier responds to the demand "Surge" and delivers variable quantities following a base stock control (Allon & Van Mieghem, 2010). The main reason for the constant "Base" delivery is that it allows focusing on cost efficiency of the slow supplier with a stable flow. The TBS policy are studied using continuous-review (e.g., Allon & Van Mieghem, 2010) and periodic-review (e.g., Janakiraman et al., 2015) inventory models in the literature. Even though the TBS problem is technically easier than the generalized dual sourcing problem, it is still not amenable for simple analysis. The major complexity arises from the assessment of "overshoot": because the slow supplier always pushes a constant quantity to the firm, it is possible that the inventory position, after the delivery from the slow supplier, exceeds the base stock control level of the fast supplier. The excessive inventory is defined as overshoot. An early dual sourcing model with TBS settings from Rosenshine and Obee (1976) allows the firm to sell part of the excessive inventory back to the supplier so as to avoid the overshoot. They propose a heuristic approach to solve the model. Chiang (2007) studies the same model and obtains the optimal solution using dynamic programming. Later studies on TBS models relax the inventory sell-off assumption and focus on overshoot assessment. Janssen and de Kok (1999) find that the overshoot is analogous to the waiting time of a GI/G/1 queue, and estimate it numerically. Combes (2011) uses simulations to estimate the mean and standard deviation of overshoot, which further results in approximate numerical solutions of the model. Allon and Van Mieghem (2010) and Janakiraman et al. (2015) obtain approximate analytic expressions of overshoot in continuous- and discrete-time models respectively, and find approximate solutions of their TBS models with simple formulae. Boute and Van Mieghem (2015) circumvent the determination of the overshoot by using a linear control policy to replenish both the slow and fast mode, which is analytically tractable with normally distributed demand.

Recently, Dong et al. (2018b) allow non-identical delivery frequencies (The fast mode delivers exactly twice as frequent as the slow mode) in a periodic-review TBS model and obtain approximate analytical solutions. It should be noted that an extended periodic-review TBS policy with non-identical delivery frequencies share similarities with an emergency replenishment model: A firm uses a slow supplier to ship regular orders, and, between two regular shipments, uses a fast but expensive supplier to expedite emergency orders (e.g., Chiang & Gutierrez, 1996; Moinzadeh & Nahmias, 1988). This way the firm could use the emergency order to satisfy demand before the next regular order arrives. The regular orders are usually controlled by long review periods (low delivery frequency), and the expedited orders can be flexibly triggered in any period when necessary (high delivery frequency) at a higher cost. Tagaras and Vlachos (2001) analyze an emergency shipment problem where only a single emergency order is allowed between two regular shipments, and Teunter and Vlachos (2001) extend this model to the case where multiple (a finite number of) emergency orders are allowed. Both studies assume that orders from both modes are periodically controlled by a separate order-up-to policy. Johansen and Thorstenson (2014) study a model where the regular mode is controlled by a re-order policy but with a fixed delivery quantity. Under a TBS policy (and our MST policy), however, the regular supplier (mode) always "pushes" a constant quantity (the "base") whenever it operates, without controls from any re-order or order-up-to points. The constant "push", which is driven by simple practical operations, is a key assumption of TBS and MST models.

The mathematical model studied in this paper is a further generalization of the TBS models, as well as a generalization of the emergency shipment models proposed in the literature so far. While keeping the "Base" and "Surge" characteristics of the TBS problem, and the non-identical delivery frequencies of the emergency shipment problems, our model (1) incorporates a fixed cost in the slow supplier and adds an extra decision in its delivery frequency, and (2) allows the fast supplier to ship in every period between two slow-supplier shipments and relaxes the assumption of the base stock control on it shipment quantity. These generalizations are driven by the characteristics of the real-world modal split transport problems. In terms of solution methodology, we apply an exact approach instead of the approximations widely used in the literature. We obtain properties of the problem, which allow calculating optimal solutions using stochastic dynamic programming with realistic computing effort.

3. Model formulation

We consider a distribution center (D) that periodically orders from a manufacturing plant (P) with unlimited capacity. The demand at the distribution center is denoted by the random variable ξ and is assumed to follow an i.i.d. distribution with mean μ and standard deviation σ. The cumulative distribution function (CDF) $\Phi(\cdot)$ and probability density function (PDF) $\phi(\cdot)$ of ξ are both known.

Two transport modes are available: a *fast mode* (road transport) with short delivery notice and a *slow mode* (rail transport) that requires a longer lead time. We restrict the lead time of the fast mode to zero because of model tractability, and assume the lead time of the slow mode is deterministic. Fast mode transport incurs a variable cost c^f per delivery unit. When the same unit is shipped via the slow mode, a lower variable cost c^s with $c^s \leq c^f$ is incurred, with an additional volume-independent fixed cost K per delivery. Note, both c^f and c^s represent not only the traditional transport fee paid to the logistics operators, but also account for all costs incurred in the end-to-end delivery process, such as customs, duties, etc. Especially, the unit slow mode cost c^s should incorporate the so-called "cost of capital" linked to the pipeline inventory kept on the train and influence the allocation. We assume that the rate of the cost of capital is fixed. Given that the lead time of the slow mode is deterministic, the pipeline inventory on the slow mode and the corresponding cost of capital are all fixed terms included in c^s.

While the fast mode has the flexibility to deliver any unit z_t as required in t, the slow mode delivers a constant quantity q every n periods[1]. To cope with the frequency of the slow mode, we define a delivery *cycle* as the number of periods between two consecutive slow mode deliveries, i.e., a cycle consists of n periods with q arriving in the first period. Let x_t denote the net starting inventory at the beginning of period t. After the deliveries from the fast

[1] For example, $n = 1$ means the slow mode delivers every day, and $n = 7$ means the slow mode delivers every week.

and slow modes, the demand is realized and an order with the fast mode is placed. At the end of the period, excessive inventory is kept at a unit holding cost h, and unmet demand is backordered to the next period at a unit backorder cost b. We assume $c^f < b$ so that it is not a feasible solution to order nothing and always pay backorder penalties. Denote $f_t(x_t)$ the optimal value function in t given the starting inventory is x_t. When the slow mode controls (q, n) are committed, the Bellman equation solving for the optimal fast mode control z_t can be expressed as follows:

$$f_t(x_t|q,n) = \begin{cases} \min\limits_{z_t \geqslant 0} \{c^f z_t + c^s q + K + L(x_t + z_t + q) \\ \quad + E[f_{t+1}(x_t + q + z_t - \xi|q,n)]\} & q \text{ arrives in } t, \\ \min\limits_{z_t \geqslant 0} \{c^f z_t + L(x_t + z_t) \\ \quad + E[f_{t+1}(x_t + z_t - \xi|q,n)]\} & \text{otherwise}, \end{cases} \tag{1}$$

where $L(y) = h\int_0^y (y-\xi)\phi(\xi)d\xi + b\int_y^\infty (\xi - y)\phi(\xi)d\xi$, the one period expected inventory mismatch cost and $E[f_{t+1}(\cdot|q,n)]$ is the expected value-to-go function of period $t+1$. For simplicity, it is assumed that at the end of the horizon, any excessive/backorder inventory in period $T+1$ is discarded, i.e., $f_{T+1} = 0$.

Obviously, the formation and value of f_t depend on whether or not the slow mode quantity is delivered in t. This indicates that the optimal fast mode order, z_t^*, is time-dependent. In addition, the optimal order quantity z_t^* might also depend on the starting inventory position x_t in period t: With a minor inventory position at the beginning of period t, for example, the company tends to respond with larger fast mode shipment in order to avoid unmet demand. As a result, the total computing effort spent on obtaining $z_t^*(x_t)$ might be substantial.

After solving for $z_t^*(x_t)$, the company then decides the optimal q and n that minimizes the average total costs per period in the steady state. The cost in the steady state can be obtained by truncating the number of periods in the infinite-horizon problem to T and subsequently letting $T \to \infty$. The objective function of MST policy is:

$$C^{q,n}(x_1) = \lim_{T\to\infty} \frac{1}{T} f_1(x_1|q,n). \tag{2}$$

The decision variables are the optimal delivery quantity q^* and frequency $1/n^*$ (or the optimal delivery cycle n^*) of the slow mode, and the optimal delivery quantities z_t^* of the fast mode in t.

The problem setting of MST (and also the TBS policy) reveals a two-stage optimization problem: The company first makes the tactical decision by committing the delivery quantity q and frequency n of the slow mode, and then it makes operational decisions and determines the optimal delivery quantity of the fast mode.

4. Analysis of the optimal policy

Given that the slow mode always incurs a fixed cost when used, if its variable cost is even more expensive than the fast mode, it is optimal for the firm to use the fast mode only. The MST problem is then degenerated to a single-sourcing problem and it is already known that a base stock policy is optimal with the order-up-to level $S^B = \Phi^{-1}(\frac{b}{b+h})$ (Arrow, Karlin, & Scarf, 1958). If only the slow mode is used, the firm needs to ship the average demand in each cycle, i.e., $q = n\mu$, in order to balance total supply with total demand in the steady state. Since demand is stochastic, significant inventory is needed to maintain a sufficient service level. The inventory requirement grows to infinite when t approaches to infinity because the variance of the accumulated demand increases in t. As a result, it is not cost-efficient for the firm to use the slow mode only if it does have any flexibility in its delivery quantity.

When both the fast and the slow modes are simultaneously used, the MST is a two-stage decision problem: the tactical decision stage in which the delivery quantity and frequency of rail transport are committed over a mid-term time horizon, and the operational decision stage in which the delivery quantity of road transport is adjusted on a daily basis. Consequently, the solution process of the MST problem can then be decoupled into the following three steps:

1. Given the fixed delivery cycle n and quantity q of the slow mode, obtain the corresponding optimal delivery quantities $z_1^*(q,n), z_2^*(q,n), \ldots$ of the fast mode by minimizing $f_1(x_1|q,n)$.
2. Given the fixed slow mode delivery cycle n, obtain the corresponding optimal slow mode quantity $q^*(n)$ by minimizing $f_1(x_1|q)$.
3. Find the optimal delivery cycle n^* of the slow mode so as to minimize $f_1(x_1)$.

Step 1 represents the operational decision stage and steps 2 and 3 solve for the optimal controls of the tactical decision stage. We next illustrate the solutions of the two stages separately.

4.1. The optimal operational decision of the fast mode

The operational decision stage solves for the optimal policy of the fast mode deliveries, given fixed delivery quantity q and delivery cycle n of the slow mode.

Theorem 1 (Properties of the operational decision stage)**.** *For any given slow mode delivery policy (n, q), the optimal controls of the fast mode have the following properties:*

(a) *(Base stock structure) A base stock policy is optimal in each period t.*
(b) *(Bounds on base stock levels) The base stock level in period t, denoted as Y_t, is upper–bounded by $S^B = \Phi^{-1}(\frac{b}{b+h})$, the classical order-up-to level in the fast-mode-only single sourcing problem.*
(c) *(Property across cycles) If $n \geq 2$ and t is the last period of a cycle, the base stock level in the first period of the next cycle satisfies the inequality: $Y_{t+1} \leqslant Y_t + q$.*
(d) *(Property inside a cycle) If $n \geq 2$ and t is the last period of a cycle, the base stock levels within the cycle satisfy the following inequalities: $Y_{t-n+1} \geqslant \ldots \geqslant Y_{t-1} \geqslant Y_t$.*

Appendix A provides detailed proofs to the theorem. Here we offer first explanations of the theorem. When q and n are both fixed, the MST model is analogous to the classical single sourcing (with the fast mode only), dynamic inventory model with deterministic changes every cycle as follows: In the first period of each cycle, a fixed term of $(c^f - c^s)q - K$ (the transport cost savings from using the slow mode) is subtracted from the objective function, and a fixed term of q (because the slow mode already delivers q units of the product, the use of the fast mode is aimed to satisfy a smaller demand) is subtracted from the random demand ξ. These two deterministic changes are not relevant for the optimal fast mode controls. The optimality of a base stock policy remains valid in the case of a general distribution of demand (Arrow et al., 1958, p. 110). As a result, the optimal policy should remain a base stock type. This explains *(a)* of Theorem 1. The size of the base stock level is an indicator of demand uncertainty. In an MST problem, the firm can use two modes simultaneously to satisfy stochastic demand, and therefore needs less safety stock (a base stock level smaller than S^B) to buffer demand uncertainty. This explains *(b)* of Theorem 1.

The cyclic delivery of q acts as the cycle stock of the inventory system. When the cycle stock arrives in $t+1$, the probability of replenishing extra inventory from the fast mode is low. This is the

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reason why the base stock control Y_{t+1} of the fast mode should be upper-bounded by a value positively related to q, which is illustrated in *(c)*. If the company is at the end of a cycle (period t) and knows that q will arrive in the next period, it prefers to place a small order in the fast mode because any excessive inventory at the end of a cycle tops up the cycle stock of the next cycle, and possibly increases the holding cost of all periods of the next cycle. An excessive inventory at the end of the previous period $t-1$ could also increase the cycle stock in the next cycle, but not so directly as that in period t. This clarifies why Y_{t-1} should be no less than Y_t, and thereafter, *(d)* of Theorem 1.

The existence of an optimal base stock policy allows for a solution of the SDP from “value iteration”, i.e., solving for $z_t^*(x_t)$, to “policy iteration”, i.e., solving for Y_t. Recall the Bellman equation (1), the state space of the SDP problem in period t consists of all possible values of inventory position x_t at the beginning of t, and the action space includes all possible fast mode delivery quantities z_t in t. The solution of the SDP requires a search for $z_t^*(x_t)$ for all x_t, with t ranging from 1 till T. Significant computing effort is needed. With the results from Theorem 1, the solution of the SDP problem shifts from finding numerous values of $z_t^*(x_t)$ to a single target: the base stock level Y_t in t. The more general notation indicating this policy iteration of the SDP problem is then:

$$f_t(x_t|q,n) = \inf_{Y_t}\{r_t(x_t, Y_t) + E[f_{t+1}(x_{t+1}|q,n)]\}, \tag{3}$$

where $r_t(x_t,\ Y_t)$, the cost-per-stage function, represents the total transport and inventory costs in period t when applying policy Y_t and the later term is the value-to-go function whereas the initial inventory of period $t+1$ depends on whether q arrived in period t. Comparing (1) with (3), the cost-per-stage function is then:

$$r_t(x_t, Y_t) = \begin{cases} c^f(Y_t - x_t - q)^+ + c^s q + K + L(x_t & q \text{ arrives in} t, \\ \quad + (Y_t - x_t - q)^+ + q) & \\ c^f(Y_t - x_t)^+ + L(x_t + (Y_t - x_t)^+) & \text{otherwise,} \end{cases} \tag{4}$$

where $(Y_t - x_t - q)^+$ and $(Y_t - x_t)^+$ indicate the fast mode delivery quantities by following the base stock decision policy Y_t.

Apparently, depending on whether or not the slow mode delivers in t, r_t does change from stage (a review period) to stage. This violates the condition mentioned in (Bertsekas, 2005, p. 402), and the corresponding infinite-horizon periodic review inventory system is therefore not stationary. In pursuit of a stage-independent cost-per-stage function, we study the same MST model as a cyclic review inventory system. Denote τ the starting period of a cycle, and $\widetilde{Y}_\tau$ the decision policy (base stock levels) of the cycle with n periods, i.e., $\widetilde{Y}_\tau = (Y_\tau, Y_{\tau+1}, \ldots, Y_{\tau+n-1})$, (3) can then be rewritten as:

$$f_\tau(x_\tau|q,n) = \inf_{\widetilde{Y}_\tau}\left\{\widetilde{r}_\tau\left(x_\tau, \widetilde{Y}_\tau\right) + E[f_{\tau+n}(x_{\tau+n}|q,n)]\right\}. \tag{5}$$

One stage of the SDP problem is then one cycle consisting of n periods, and the corresponding cost-per-stage function for an initial inventory level x_τ is:

$$\begin{aligned} \widetilde{r}_\tau(x_\tau, \widetilde{Y}_\tau) = {} & K + c^s q + c^f(Y_\tau - x_\tau - q)^+ + L(x_\tau + (Y_\tau - x_\tau - q)^+ + q) \\ & + E\left(\sum_{j=1}^{n-1}\left[c^f(Y_{\tau+j} - X_{\tau+j})^+ + L(X_{\tau+j} + (Y_{\tau+j} - X_{\tau+j})^+)\right]\right), \end{aligned} \tag{6}$$

where $X_{\tau+1} = x_\tau + (Y_\tau - x_\tau - q)^+ + q - \xi$, $X_{\tau+2} = x_{\tau+1} + (Y_{\tau+1} - x_{\tau+1})^+ - \xi, \ldots$ define the random initial inventory levels of period $\tau+1$, ..., $\tau+n$. According to Bellman's “Principle of Optimality”, an optimization of the SDP problem over the entire time horizon can be broken down into a sequential optimization of subproblems. In Eq. (3), a subproblem is a periodic problem, whereas in (5), a subproblem represents a cyclical problem. The solution of the SDP will then focus on cyclic rather than periodic results.

Now the system Eq. (5), the cost-per-stage function (6), and the random disturbance, represented by the i.i.d. demand ξ, do not change from one stage (a review cycle) to the next. Therefore, according to Bertsekas (2005) p.402, the cyclic-review inventory system is stationary. We denote the “base stock vector” in the steady state of the infinite-horizon problem $\mathbb{S} = (S_1, S_2, \ldots, S_n)$. The optimal MST decision in the operational decision stage is then simplified to find the optimal $\mathbb{S}$, for any given $(n,\ q)$.

Until now, the analysis only secures the existence of an optimal base stock policy of the fast mode. In order to obtain the exact values of the base stock levels, stochastic dynamic programming (SDP) will be applied. The state space of the SDP is defined as all possible values of inventory position x_t at the beginning of period t and the action space is defined as all possible fast mode order quantity z_t in this period. With the knowledge obtained from Theorem 1, the objective of the SDP is to obtain the optimal decision rule in the steady state cycle, denoted as the base stock vector $\mathbb{S}(S_1, S_2, \ldots, S_n)$. This policy iteration approach could save significant computing effort compared to the value iteration.

4.2. The optimal tactical decision of the slow mode

The optimal solution of the slow mode includes two decision variables: the optimal delivery frequency of the slow mode, represented by its delivery cycle n^*, and the optimal volume delivered by the slow mode q^*. We first present the following Theorem:

Theorem 2 (Property of the tactical decision stage)**.** *For any given slow mode delivery cycle n, the average total cost per period in the steady state, stated in (2), is convex in the slow mode delivery quantity q.*

The proof is in Appendix A and here we show an explanation. The utilization of the slow mode has two trade-off impacts. On the one hand, it reduces the total transport cost for each unit of product shifted to the slow mode; on the other hand, the fixed delivery quantity weakens the flexibility of the MST policy and brings extra inventory cost into the system. This trade-off drives the convexity of f in q. Based on this theorem, a bisection search over q (q is upper-bounded by $n\mu$ because the slow mode can not supply more than the expected demand in a cycle) is possible to reduce the computing effort of the numerical solution.

4.3. Solution algorithm

On the basis of Theorems 1 and 2, we propose the following algorithm to solve for the optimal decisions. (1) In a inner loop with any given $(n,\ q)$, we first backward iterate the inventory system using SDP and obtain the optimal decision policy of the fast mode, i.e., $\mathbb{S}(S_1, S_2, \ldots, S_n)$, and then forward simulate the inventory system from period 1 till T and calculate $C^{q,n}(x_1)$ in Eq. (2); (2) in a middle loop with a given n, we bisectionally search over q to minimize $C^{q,n}(x_1)$ and obtain $q^*(n)$; (3) In a outer loop, we search for the optimal n^*. For simplicity, we assume the system starts with zero inventory, i.e., $x_1 = 0$. Because the delivery cycle of a rail transport is practically a small integer, e.g., the rail transport delivers once every two days ($n=2$), or at least once per week ($n=7$), an enumeration of all possible integers is sufficient to search for the optimal n. The calculations can be summarized in Algorithm 1.

Algorithm 1 Compute the optimal decisions of the MST policy.

```
initialization
for n = 1 to 7 do
    qmin = 1, qmax = nμ
    while qmin ≠ qmax do
        q1 = (qmin + qmax)/2 − 1
        q2 = (qmin + qmax)/2
        initialize the base stock vector 𝕊
        while the base stock vector 𝕊 does not converge do
            given n and q1, from the ending period T backward
              iterate 𝕊
        end while
        given n, q1, and 𝕊, compute C^{q1,n}(x_1)
        initialize the base stock vector 𝕊
        while the base stock vector 𝕊 does not converge do
            given n and q2, from the ending period T backward
              iterate 𝕊
        end while
        given n, q2, and 𝕊, compute C^{q2,n}(x_1)
        if C^{q1,n}(x_1) < C^{q2,n}(x_1) then
            qmax = q1
        else
            qmin = q2
        end if
    end while
    q = qmax
    compute C^{q,n}(x_1)
end for
find n* to minimize C^{q,n}(x_1)
```

Table 1
A list of the parameters used in the model with normalized values.

Notation	Description	Value	Unit
μ	Mean of demand	30	FCL
σ	Standard deviation of demand	10	FCL
α	Service level	98%	pct
h	Unit holding cost at the DC	68	EUR per FCL per day
b	Unit backorder cost at the DC	3332	EUR per FCL per day
c^f	Unit transport cost via the fast mode	550	EUR per FCL
c^s	Unit transport cost via the slow mode	224	EUR per FCL
K	Fixed transport cost via the slow mode	8170	EUR per train

Table 2
The impact of backorder cost on the optimal controls of rail and road transport

Unit backorder cost	n^*	q^*	S_1	S_2	S_3	C
102	3	81	30	29	8	12,023
159	3	81	34	33	14	12,430
272	3	81	38	37	22	12,736
612	3	81	43	43	32	13,339
1292	3	81	48	48	38	13,681
3332	3	81	54	54	45	14,082
6732	3	81	58	58	49	14,317
13,532	3	81	62	62	54	14,544

5. Numerical analysis

In this section, we conduct a numerical study to illustrate the modal split transport problem based on our solution algorithm discussed before. The objective of this numerical study is to answer two questions: (1) how can our model and algorithm be implemented to support companies' modal split transport optimizations? (2) how sensitive/robust is our MST policy against the uncertainties of the parameters?

5.1. Numerical design

We use modified industry-level data suggested by a consumer goods firm. These numbers allow to obtain realistic results of the MST policy without revealing confidential information of any firm. The volume of the freight is measured and its cost is paid at the industrial standard unit of full container load (FCL). One FCL is equivalent to the volume loaded in a standard 45-foot container, which fits a standard truck- or railcar-trailer. The assumption of the demand being stationary is mainly motivated by the fact that the plant sends products to a distribution center that serves all retailers within a large area (e.g., a country). Even if one retailer runs a sales campaign, the aggregated demand from all retailers will not be largely affected. The daily demand on a product level is consolidated to a demand for FCL's for which we consider that it follows a Gamma distribution with mean $\mu = 30$ and standard deviation $\sigma = 10$. Note, both TBS and MST models are not subject to any specific demand distribution and other types of distribution can also be applied.

The firm pays $c^f = 550$ EUR to transport one FCL from the plant to the distribution center via road transport. Based on data from European Intermodal Association (2012), to transport one FCL using rail transport incurs a variable cost of $c^s = 224$ EUR, and the fixed cost of the train operation, regardless of the volume delivered, is $K = 8170$ EUR. The inventory holding cost includes the storage and handling spending incurred in the warehouse, as well as the cost of capital, i.e., by holding the inventory on hand, the company loses the opportunity to use the capital linked to the inventory for other investments. The general industrial "rule of thumb" is that the annual inventory holding cost is 25% of the stock value. Assuming that the average value of an FCL cargo is about 100,000 EUR, $h = 100,000/365 \cdot 25\% = 68$ EUR. The firm assumes that the unit backorder cost is 3332 EUR per container per period. This results in a non-stockout probability of $b/(b+h) = 98\%$ at the distribution center. A list of the parameters is shown in Table 1.

On the basis of the calibrated parameters, we validate our algorithm and illustrate the drivers of the optimal road and rail transport decisions. We then apply sensitivity analyses of the MST policy, in which the impact of demand volatility, service level, cargo value, and transport cost savings are analyzed. Finally, we show if the MST policy is already being applied, how robust is the whole system against exogenous noises. The noises are categorized into two types: (1) on the operational level, the misspecifications of base stock controls of road transport. (2) on the tactical level, the mismatch of fixed rail transport decisions over time and the fluctuating demand volatility.

5.2. Numerical results

On the basis of the benchmark parameters, our algorithm finds that the optimal modal split transport operates as follows: A train delivers a fixed quantity of $q^* = 81$ FCLs every $n^* = 3$ days, and trucks deliver to bring the inventory level to $S_1 = 54$, $S_2 = 54$ and $S_3 = 45$ in the three periods of a cycle. Note, the base stock levels are also measured in the unit of FCL, representing the corresponding amount of inventory that can be loaded into the containers. Under this policy, the optimal volume split in rail transport is $\frac{q}{n\mu} = \frac{81}{3 \cdot 30} = 90\%$. The large ratio aligns with the findings in Allon and Van Mieghem (2010) and Janakiraman et al. (2015) that the volume delivered by the slow mode (supplier) is high due to a so-called "heavy-traffic" phenomenon.

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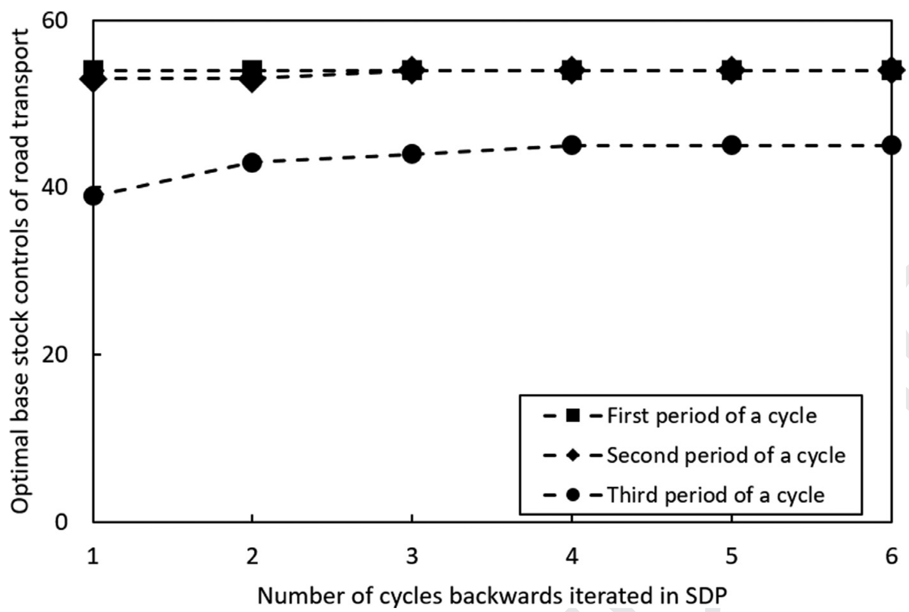

Fig. 1. The base stock controls of road transport converge after six cycles of SDP iterations.

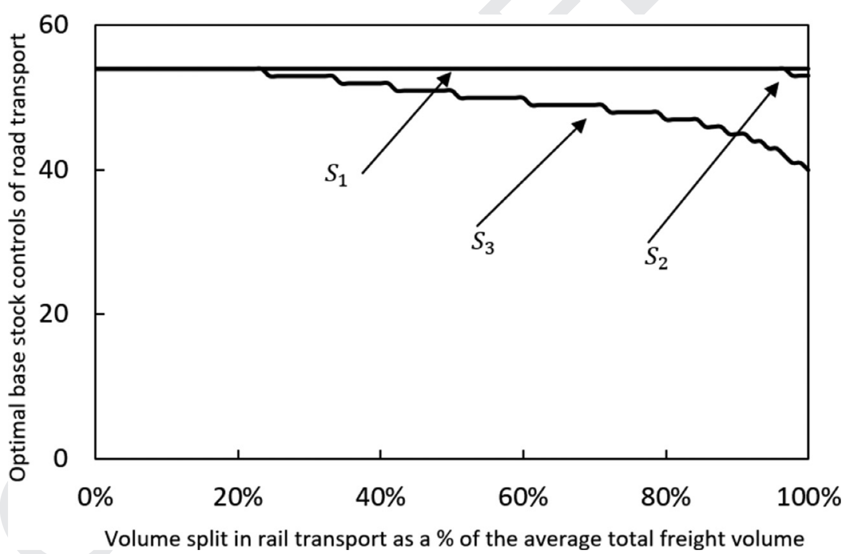

Fig. 2. The base stock controls of road transport decrease when more freight is shifted to rail transport.

Fig. 1 shows the values of S_1, S_2, and S_3 calculated by the SDP algorithm in the first six cycles of iteration, when q and n are fixed. The optimal policy, i.e., the base stock vector (S_1, S_2, S_3), converges and the convergence process is surprisingly short.

The impact of q on the base stock controls is shown in Fig. 2. When more volume is shifted into rail transport, the road transport will be less utilized via reduced base stock control levels. Nevertheless, the base stock level in the last period of a cycle, in this case S_3, is more sensitive to q compared to the previous periods. This is because the cyclical arrival of q acts as cycle stock for the inventory system. Any excessive inventory at the last period of a previous cycle (in this case period 3) will top up the cycle stock of the next cycle, and possibly leads to higher inventory holding costs for every period of the next cycle.

The delivery cycle n of rail transport impacts the modal shift. The curve in Fig. 3 indicates that when n increases, the freight shift to rail transport in % decreases. This is because the larger the cycle stock linked to q is, the more inventory holding cost will be generated in the cycle, which will offset the transport cost savings from the use of rail transport. Nevertheless, the percentage still remains at a rather high level, indicating that rail transport will still be heavily used.

The above-mentioned optimal decision of both transport modes are only based on the demand with gamma distribution. We have also examined normal and uniform distributions with exactly the same mean and standard deviation. Interestingly, the optimal decisions of rail transport, i.e., the delivery quantity q^* and cycle n^*, remain unchanged, whereas the optimal control of road transport, i.e., the base stock levels, vary. This indicates that the uncertainty in demand distribution can be fully adjusted by the road transport decisions and the inflexible rail transport luckily remains unaffected. The optimal base stock vector is (50,50,43) with normal distribution and (62, 62, 57) with uniform distribution, and both remain the decreasing pattern. This finding aligns with the approximate analytic solution reported in Janakiraman et al. (2015) and Dong et al. (2018b): The base stock levels are calculated by the demand distribution function and the slow mode decisions are distribution-free.

Thus far, we have validated the impact of the rail transport decisions on the road transport controls. Fig. 4 shows that the main

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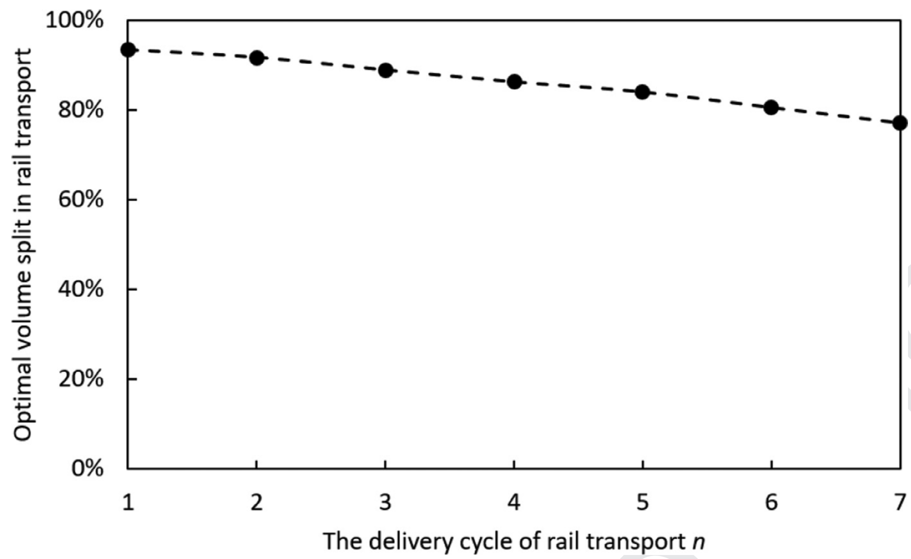

Fig. 3. When the delivery cycle of rail transport increases, the freight shift to rail transport in % decreases.

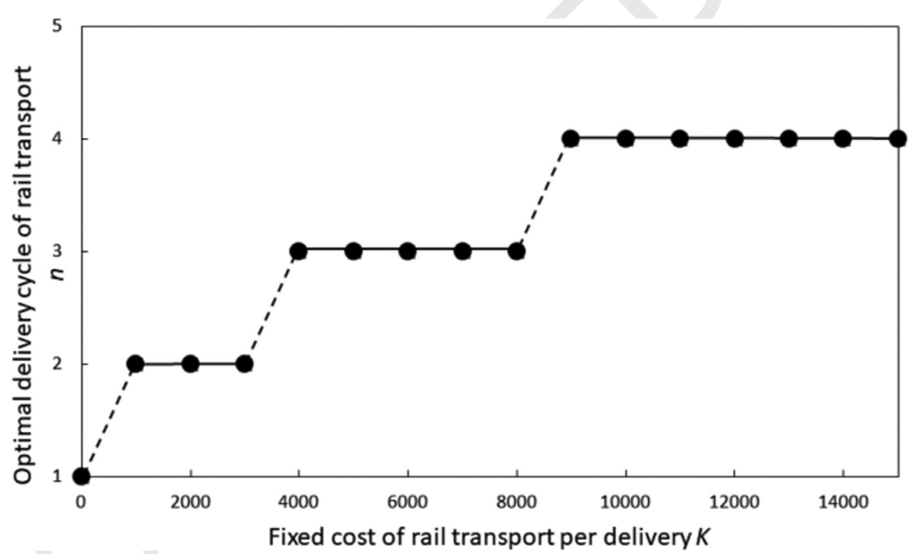

Fig. 4. The optimal delivery cycle of the rail transport increases when its fixed cost increases. When $K = 0$, $n^* = 1$.

driver of the optimal rail transport decision is its fixed cost K. The larger the fixed cost is, the less frequent the company wants to use rail transport. Fig. 5 shows when K increases, the volume shifted to rail transport increases, but with a decreasing margin. The three-stage optimization problem addressed in Section 4 is therefore numerically validated: The fixed cost of rail transport drives its delivery cycle and quantity, and the delivery cycle and quantity of rail transport impact the base stock controls of road transport. In practice the company could collaborate with other shippers to fill in the empty wagons of the train on the same corridor, the fixed cost of the slow mode is shared and K in the MST model is reduced. On the other hand, the company could also expect an increase in K due to e.g., extra maintenance or toll charges of railway infrastructures. Our analysis reveals that the change in K is particularly important because it will tactically impact the entire decision of both rail and road transport controls. Especially, in the case of horizontal collaboration where multiple firms load the train together and share the fixed cost K, rail transport will be more frequently used.

In the classical inventory theory with one single supplier, the introduction of a fixed cost changes the optimal base stock policy to an (s, S) policy (see, e.g., Arrow et al., 1958). In the general dual sourcing inventory models, the introduction of fixed costs makes the problem even more complex. Even though the optimal solution is not an (s, S)-type in general (Porteus, 1971), it can be approximated by an (s, S) policy (Chiang, 2003; Jain, Groenevelt, & Rudi, 2011). So far, the impact of a fixed cost has not been discussed in a TBS policy. We find, as shown in Fig. 4, the introduction of a fixed cost of rail transport increases its delivery cycle. Even though the setting of our model is different from Chiang (2003) and Jain et al. (2011), the managerial insights share similarities: The fixed cost encourages the economies of scale so that the company will decrease the number of orders and increase the average quantity ordered.

A possible question from the firm is to understand the cost advantages of the MST policy compared to the fast-mode-only (the current baseline situation), or even the slow-mode-only case. Fig. 6 shows the average total costs per period as a function of the fixed cost K of the slow mode. The solid line represents the current baseline situation, where only the fast mode is used. It remains obviously stable. The dotted line shows the cost of the optimal MST policy in K, which indicates clear cost savings compared to the

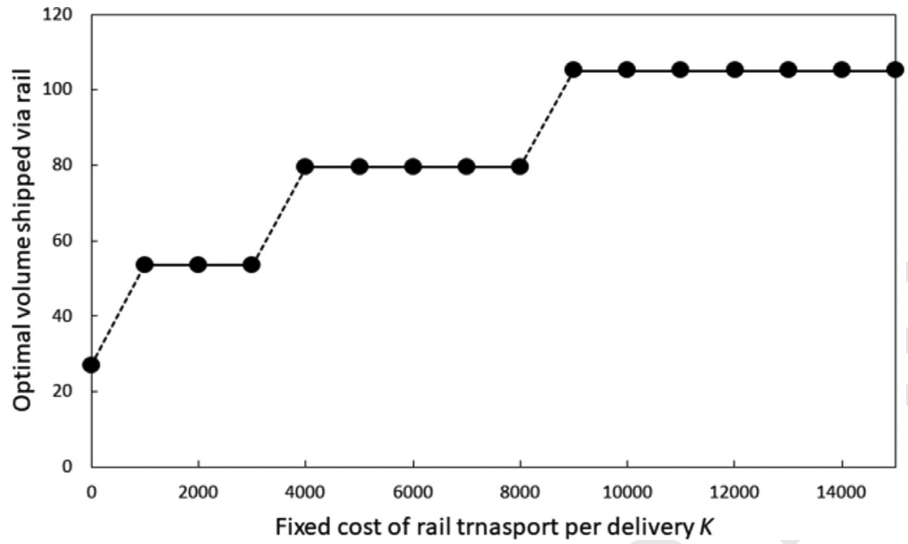

Fig. 5. The volume shipped via rail transport increases when its fixed cost increases. The curve has a similar shape to the one in the left.

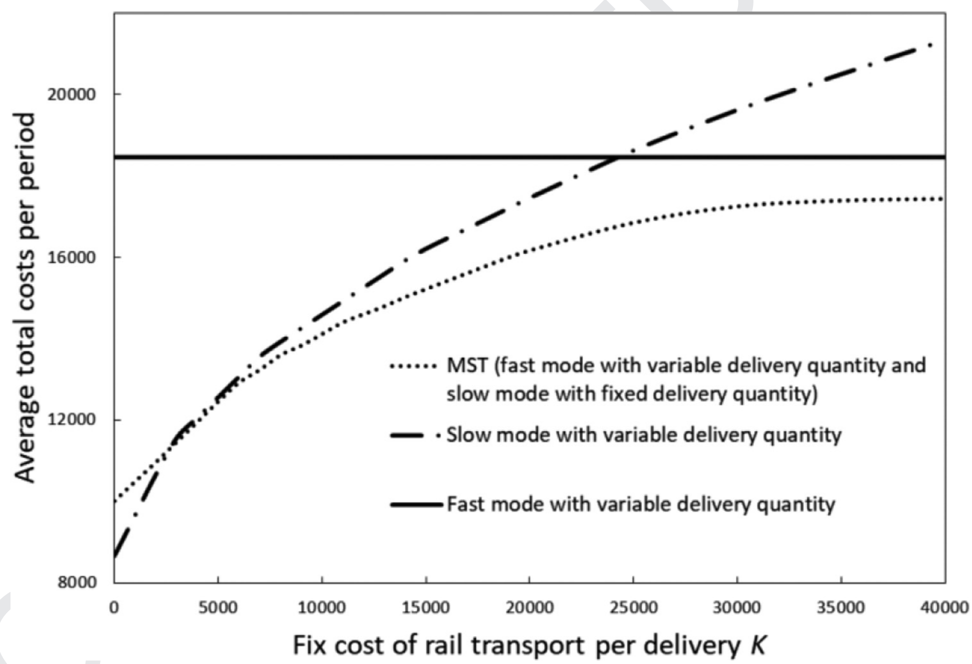

Fig. 6. The MST policy outperforms the current baseline situation (only the fast mode is used). Even if we allow the slow mode to have the same flexibility in delivery quantity as the fast mode, it incurs a higher cost when K is larger than about 3000.

current baseline situation. In order to better understand the impact of the slow mode, we plot the dash-dot line to show the total cost of a special case with slow mode only. In the special case, we allow the slow mode to deliver any variable quantity as required (just as the fast mode). Its solution is an (n, S) policy, where n indicates the periods of a delivery cycle and S the base stock control of the slow mode orders. Interestingly, the total cost exceeds that of the optimal MST policy when K is larger than about 3000 EUR, and exceeds that of the baseline situation when K is larger than 25,000 EUR. Recall from Table 1 that $K = 8170$ is a realistic number, the optimal MST policy would most probably outperform the (n, S) policy in practice. In addition, the (n, S) policy is a special case subject to two strong assumptions: (1) the slow mode (rail) has the same flexibility in delivery quantity as the fast mode (road), and (2) even with this flexibility, (since all other parameters remain unchanged) the firm does not pay a high variable cost for the slow mode. As a result, it is highly likely that in reality, the MST policy will always outperform the slow-mode-only case.

5.3. Sensitivity analysis

We conduct four different analyses to understand how sensitive the MST results are against the coefficient of variation (CV) of demand, the unit backorder cost, the value of the cargo shipped (associated with the unit holding cost h), and the variable transport cost saving of rail over road ($c^f - c^s$). The benefit of the performance evaluation is twofold: (1)the optimal delivery cycle n^* of rail transport, because it is already shown in Section 5.2 that the delivery cycle of rail transport is the major driver of its delivery quantity, and consequentially impacts the base stock controls of road transport. (2) The modal split ratio in rail transport, i.e., $q^*/(n^*\mu)$, because shifting volume from road to rail transport is companies' key objective of implementing MST.

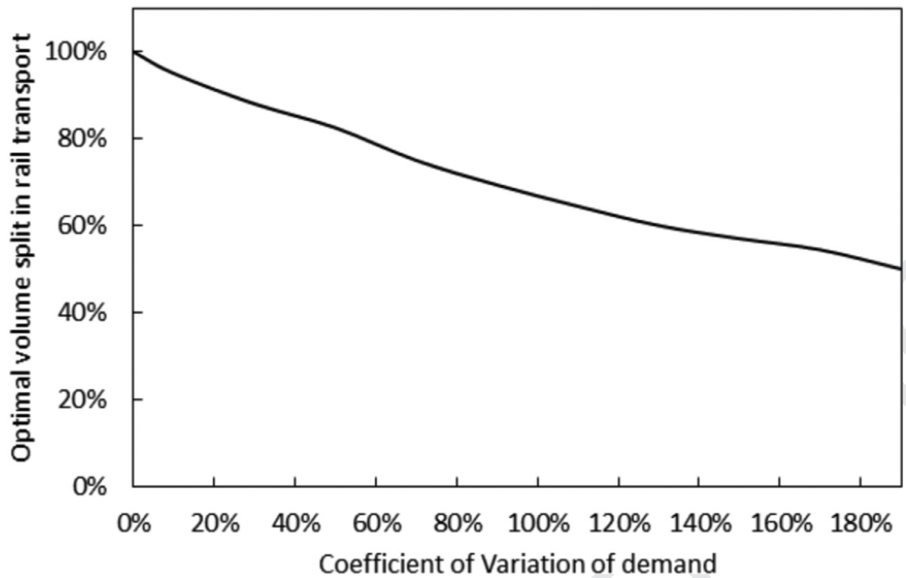

Fig. 7. When the demand is more volatile, the freight shift to rail transport in % decreases.

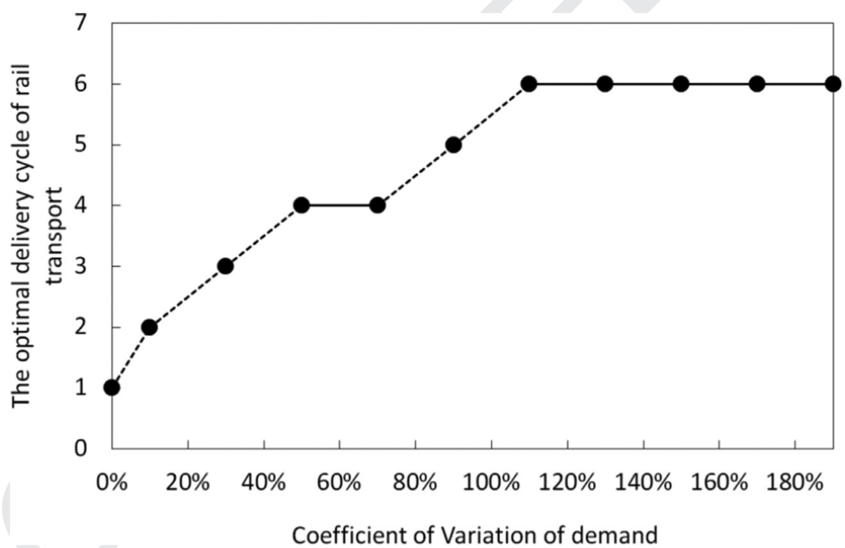

Fig. 8. When the demand is more volatile, rail transport will be used less frequent.

The previous baseline model was based on the demand with CV 0.3, which is a typical ratio of a fast-moving SKU in the industry. A transfer of the knowledge in MST requires analyses for other classes of SKUs, and the SKUs are typically classified under the criteria of CV (see, e.g., Van Kampen, Akkerman, & Van Donk, 2012). Fig. 7 shows that when the demand is more volatile, the modal split ratio in rail transport decreases because the company needs to increase the flexibility of the MST system by a higher use of road transport. Fig. 8 shows that the optimal delivery cycle increases in CV, indicating that rail transport is less frequently used.

The unit back order cost might vary across different products, and hence impacts companies' decision in MST. We have evaluated it from 102 to 13,532, which can roughly indicate an increase of service level calculated by the classical newsvendor ratio $\frac{b}{b+h}$. Interestingly, the optimal delivery cycle and delivery quantity of rail transport remain unchanged, and the company merely needs to increase its optimal base stock controls of road transport to avoid high backorder penalties. The results again address the advantage of the setup of our MST policy: It allows companies to capture the responsiveness of the transport system by using road transport, and to focus on cost savings of the transport system by using rail transport. To avoid more backorders, the firm simply needs to change decisions on the road transport.

The product value might influence the companies' MST decisions. In our model, the product value indirectly impacts the MST decisions via its unit holding cost h, i.e., the annual average holding cost is measured as 25% of the product value. Fig. 9 shows that when the product value increases, the company prefers to use more road transport compared to rail transport. A transport mode with a higher service level (in this case, road transport), is favored for deliveries of high-value products. Fig. 10 shows that the higher the product value, the less frequent the usage of the "low service" transport mode (in this case, rail transport).

Whereas the cost of road transport is rather straightforward, the cost of rail transport is often lumpy and highly dependent on the specific corridors (European Intermodal Association, 2012). We therefore fix the unit cost of road transport c^f, and observe how the MST decisions will behave when the unit cost of rail transport c^s changes. Fig. 11 shows that when c^s approaches zero, the optimal ratio in rail transport is close to 100%; when $c^s = 0$, the

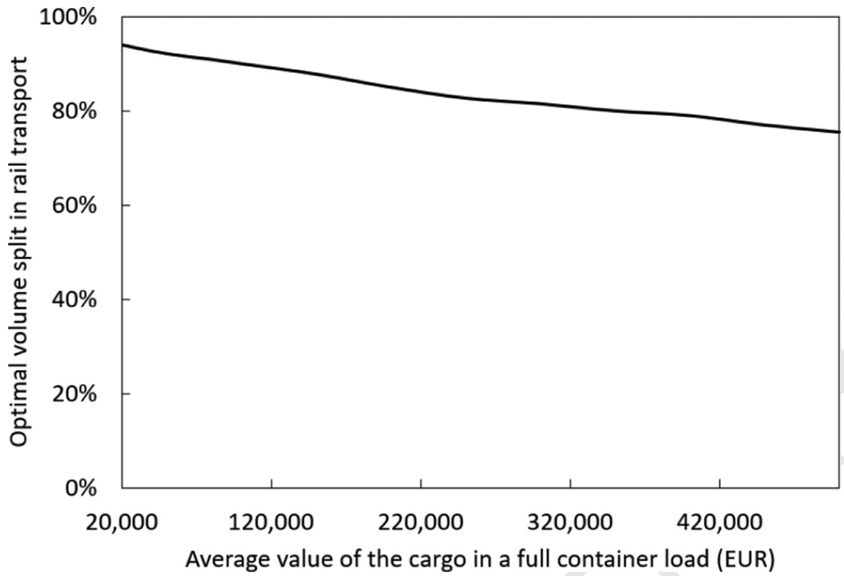

Fig. 9. The more expensive the product is, the less it will be shifted to rail transport.

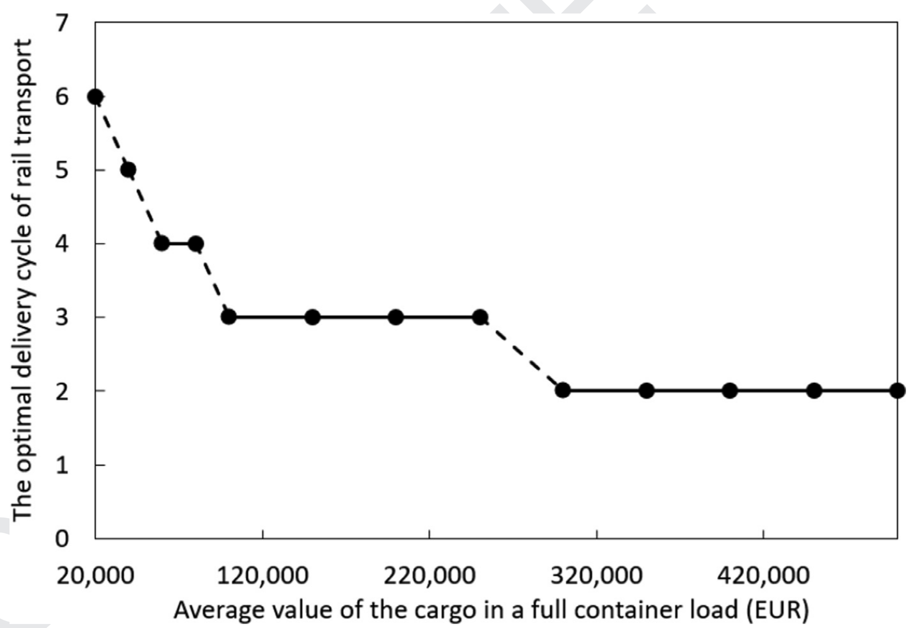

Fig. 10. The more expensive the product is, the more frequent it will be shipped by rail.

corresponding ratio is zero; and the curve decreases in c^s. However, the curve is flat until a certain threshold and then quickly drops to zero. In this specific example, this interesting threshold is around c^s being approximately 70% of c^f. To the left of this point, even if the cost savings increase substantially, the incremental modal split into rail transport is moderate; to the right of this point, the modal split ratio will decrease significantly when c^s increases. The identification of this point could help companies better utilize the MST policy and promote the modal shift into rail transport. Fig. 12 shows that the optimal delivery cycle of rail transport remains stable until approximately $c^s = 70\%$ of c^f, and then surges to infinite. When the cost savings of rail transport is subtle, the company needs to have a very large (approaches to ∞) delivery cycle to secure the economies of scale of rail transport, and the optimal split in rail would approach zero.

5.4. The robustness of MST policy

When the MST policy is already implemented by the company, exogenous noises might impact its performance. We distinguish between two types of noise: (1) on the operational level, managers might misspecify the base stock controls of road transport. (2) On the tactical level, the pre-fixed delivery cycle and quantity of rail transport over a mid-term period might not be able to match the posterior changes in demand uncertainties. These noises impact the robustness of the MST policy. The performance measure of the robustness is the error in the average total cost of a steady state period.

Since the base stock controls in a steady state cycle differ from each other, it is reasonable to presume that the exact position of the base stock misspecification in a cycle will impact the

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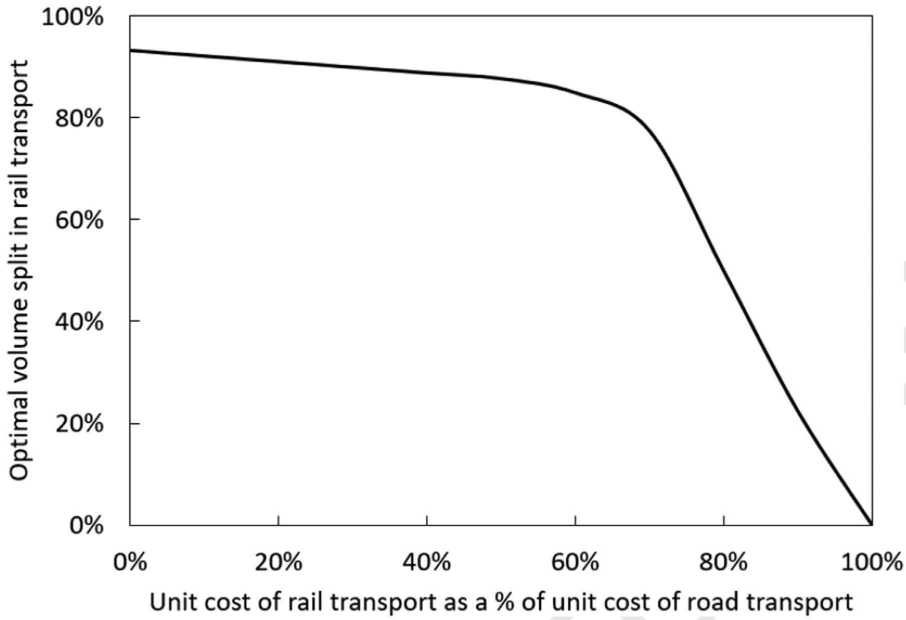

Fig. 11. The more expensive the rail transport is, the less volume will be shifted to it.

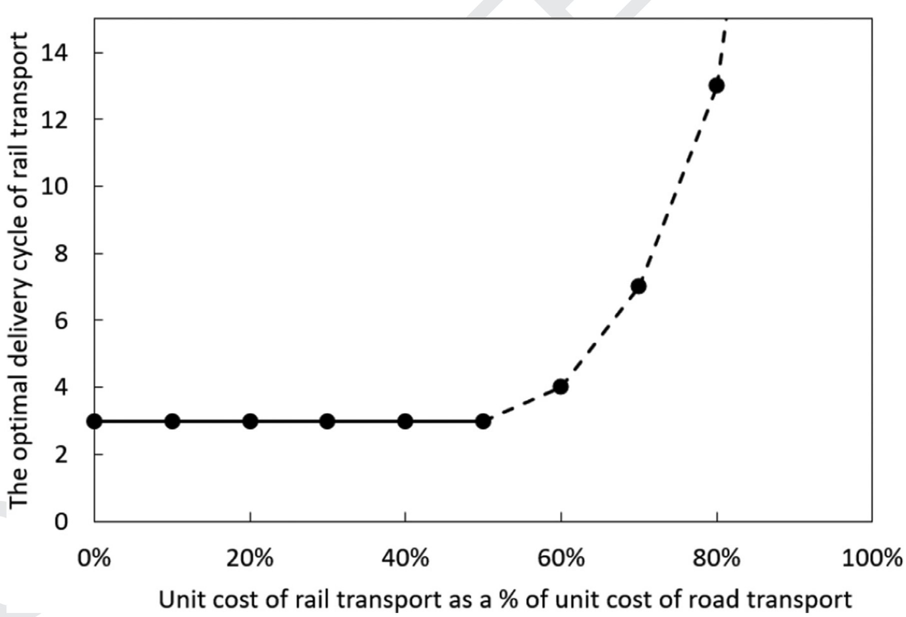

Fig. 12. The more expensive the rail transport is, the less frequent it will be used.

performance of the MST policy. In order to better understand the impact of the position, we consider a large cycle with seven periods, i.e., $n = 7$. Fig. 13 illustrates how the misspecifications of the seven base stock levels lead to errors in average total costs per period. When a misspecification of 100% (Misspecification here is measured as the difference as a percentage of the optimal base stock level. In this example, a 100% misspecification means the base stock level used is the double of the optimality.) happens in the first period of a delivery cycle, the inventory system is hardly affected. However, if the misspecification happens in the seventh period of a cycle, the cost error could be close to 10%.

After the delivery of quantity q from the rail transport in the first period of a cycle, the company is less likely to replenish its inventory via the fast mode to satisfy the demand in this period. This is the reason that a misspecification of the base stock control of the road transport hardly impacts the total costs of the MST system. The fixed delivery quantity of the rail transport q acts as cycle stock in the inventory system. In the later periods of a cycle when the cycle stock is depleted, more and more volume is hence needed from road transport to satisfy the demand. This explains the finding that when a misspecification happens in later periods, the consequence is more severe.

This result inspires us to think whether or not road transport is really necessary in the beginning periods of a rail transport cycle. We therefore simulate the average volume shifted via the fast mode in a steady-steady rail transport cycle, when the optimal decisions of both road and rail transport are taken (no misspecifications). In the optimal scenario with $n = 7$, the rail delivers 169 FCLs and the road transport delivers $7 \times 30 - 169 = 41$ FCLs every cycle. These 41 FCLs are on average distributed in the seven periods of a cycle shown in Fig. 14. The result reassures that road transport is indeed not used in the first three periods of a

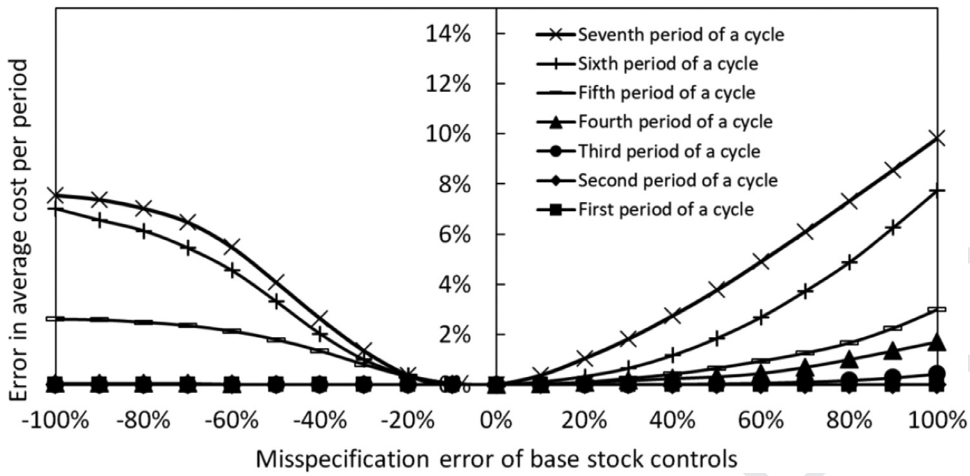

Fig. 13. If a misspecification of road transport control happens in an early period of a rail transport cycle, it only slightly increases the total cost of MST. However, if the misspecification happens at the end of a cycle, the consequence is much severe.

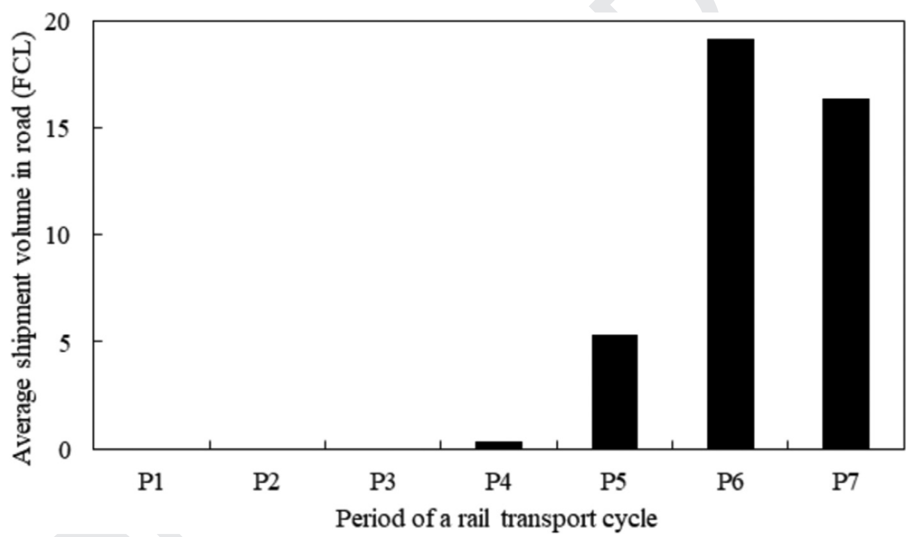

Fig. 14. Road transport is hardly used (on average) in the first three periods of a rail transport cycle. And it is most heavily used (on average) in the last but second period of the cycle.

seven-period cycle. Interestingly, road transport is mostly used in period six, instead of the last period of a cycle. The reason might be that a large volume will soon be delivered after period seven via rail transport and the system therefore prefers to order less beforehand. Practically speaking, operational managers could simply save their effort on road transport in the first few periods after a rail delivery. However, the exact boundary should depend on the set of parameter and needs to be calculated case by case.

This result could provide additional help in the technical solution of the MST problem. One of the reasons for the considerable computing effort of the model is that there are n different base stock targets to be solved. If we already know that the base stock targets from the first few periods (in this example, the first three periods) only have little impact on the results of the MST policy, the solution algorithm can drop them and focus on fewer decision variables. This way the entire computing effort of the problem will be reduced.

A follow-up question from the firm might be, could the solution be further reduced to a simple (q, S) policy, where q represents the constant shipment quantity of the slow mode, and S denotes the time-independent base stock control of the fast mode over the entire cycle? To exploit it, we fix S to the classical newsvendor base stock level, i.e., $S = \Phi^{-1}(\frac{b}{b+h}) = 54$, and compare the average total cost per period of this policy to the optimal MST policy in Fig. 15. The two curves are very close to each other, and the simplified (q, S) policy only incurs about 4% additional total cost. The result demonstrates a proper approximation of the MST policy by simply using the newsvendor base stock level. The approximation requires the computation of only one base stock level instead of a long base stock vector, with the trade-off of a moderate penalty on the total logistics costs.

Thus far, the analysis on the MST has been based on the assumption that the demand can be described by a probability distribution function with definite mean and standard deviation. In a practical problem, the two parameters will be forecasted on the basis of historical data. It is reasonable to suspect, however, that future demand will be realized with different parameters. Even if a company can closely forecast the mean of the demand over a certain time horizon, which is relatively simple, the forecasting error in the variance of the demand could be unavoidably high. Considering that the optimal control of the rail transport in MST is a commitment fixed over a future mid-term horizon, it is very likely that the coefficient of variation of the demand will fluctuate during this time horizon and the pre-defined MST policy will deviate from

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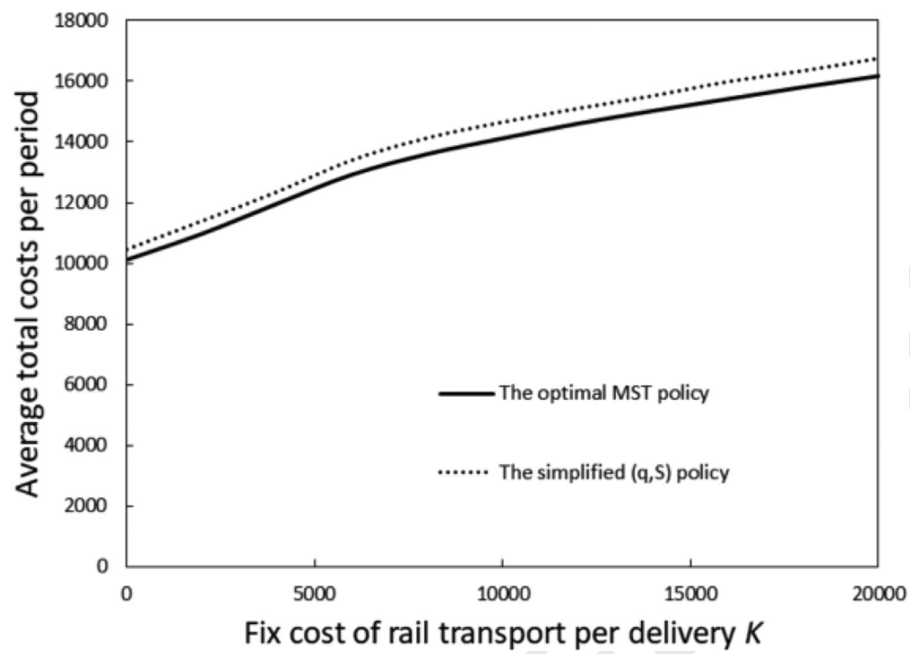

Fig. 15. The performance of a simplified (q, S) policy (The firm always uses the newsvendor base stock level to control the shipment in the fast mode) is close to that of the optimal MST policy.

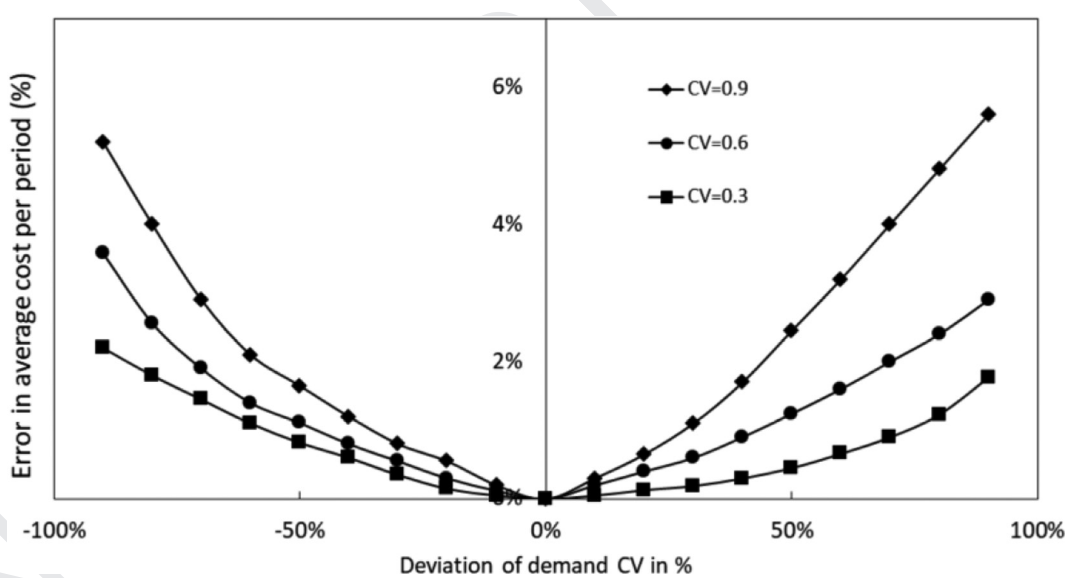

Fig. 16. Even with a substantial unexpected change of CV of demand, the total cost of the MST model only slightly deviates from the original optimal value.

optimality. Nevertheless, the company is still able to make daily adjustments of the shipment volume via the flexible road transport deliveries. Fig. 16 shows that by allowing adjustments from road transport deliveries, the commitment to rail transport is rather robust against CV changes. For example, if the original MST policy is decided based on a forecast CV of 0.6 and in reality, the CV deviates from the forecast value with an error of 50%, the average cost per period is only affected by about 1%.

Managers tend to have the concern that the fixed mid-term commitment to rail transport controls lacks the flexibility to respond to demand fluctuations. We show that by the simultaneous usage of the flexible road transport, the commitment of rail transport decisions is robust. This result supports the long term viability of MST policy against demand uncertainties.

6. Summary

In this paper, we contribute to two streams of literature: the freight mode choice literature and the dual sourcing literature. In the first stream, we develop a dual sourcing inventory model to support firms' modal split transport (MST) optimization, i.e., how to optimally split freight volume between road and rail transport. In the second stream, we extend the tailored base-surge (TBS) dual sourcing model by incorporating the economies of scale of the slow source (mode), and therefore allow the slow mode to deliver less frequently. This extension is driven by the nature of rail and road transport. In addition, previous studies in TBS models exclusively impose a base stock control on the fast source (mode). We prove that, when the slow mode decisions are pre-defined, the base stock policy is indeed optimal. The base stock levels cannot be obtained analytically and previous TBS literature uses approximations. We apply an exact approach and propose an efficient algorithm to calculate the optimal results based on stochastic dynamic programming. Using this algorithm, we further provide a detailed sensitivity and robustness analysis with respect to the impact of fixed cost, demand uncertainty, backorder cost, etc., with numbers suggested by a multinational company.

Several extensions could be applied to further support companies' modal split transport decisions. First, our model assumes that rail transport always delivers a constant quantity. Companies could have other options to manage the freight in trains. Other inventory replenishment policies, besides TBS, can also be applied to MST decisions. Second, we study a single-product dual sourcing model and it would be interesting to investigate a multi-product problem. A simple extension could be consolidating the volumes in rail transport from multiple products. A more realistic model is to consider the covariance of the demand of multiple products and the corresponding pooling effect on MST decisions. Third, the company might want to share the capacity of the train with other shippers, and collaborative modal split transport needs to be studied.

Appendix A

A1. Proof of Theorem 1

We introduce a decision variable y_t, defined as the net inventory position after deliveries from the two modes and before demand realization, i.e., if q arrives in t, $y_t = x_t + z_t + q$; if not, $y_t = x_t + z_t$. The Bellman equation (1) can then be rewritten as:

$$f_t(x_t|q,n) = \begin{cases} \min\limits_{y_t \geqslant x_t+q} \{-c^f x_t + c^f y_t - c^f q + c^s q + K \\ \quad + L(y_t) + E[f_{t+1}(y_t - \xi|q,n)]\} & q \text{ arrives in } t, \\ \min\limits_{y_t \geqslant x_t} \{-c^f x_t + c^f y_t + L(y_t) \\ \quad + E[f_{t+1}(y_t - \xi|q,n)]\} & \text{otherwise}, \end{cases} \tag{7}$$

We study the properties of the infinite-horizon problem by truncating the number of periods to T and subsequently letting $T \to \infty$. Since $f_{T+1} = 0$, the optimal value function in period T is:

$$f_T(x_T|(q,n)) = \begin{cases} \min\limits_{y_T \geqslant x_T+q} \{-c^f x_T + c^f y_T - c^f q & q \text{ arrives in } T, \\ \quad + c^s q + K + L(y_T)\} \\ \min\limits_{y_T \geqslant x_T} \{-c^f x_T + c^f y_T + L(y_T)\} & \text{otherwise}. \end{cases} \tag{8}$$

We then define G_t as a function of y_t:

$$G_t(y_t) = \begin{cases} c^f y_T + L(y_T) & t = T, \\ c^f y_t + L(y_t) + E[f_{t+1}(y_t - \xi|q,n)] & \text{otherwise}. \end{cases} \tag{9}$$

Note, $G_t(y_t)$ is independent on whether or not q arrives in t.

Proof of (a) of Theorem 1 using mathematical induction:

Period T: $L(y_T)$ is convex in y_T, G_T in (9) is also convex in y_T. Because $\lim_{|y_T|\to\infty} G_T = \infty$ (based on the assumption $c^f < b$), there exists a unique Y_T that minimizes G_T. Therefore, the optimality of a base stock policy in T exists. By setting $G_T'(Y_T) = 0$, we can obtain the base stock level:

$$Y_T = \Phi^{-1}\left(\frac{b - c^f}{b+h}\right). \tag{10}$$

Given the optimality of a base stock policy, f_T and f_T' can be written as:

If q arrives in T:

$$f_T(x_T|q,n) = \begin{cases} c^f(Y_T - x_T - q) + c^s q + K + L(Y_T) & x_T \leqslant Y_T - q, \\ c^s q + K + L(x_T + q) & x_T \geqslant Y_T - q, \end{cases} \tag{11}$$

and

$$f_T'(x_T|q,n) = \begin{cases} -c^f & x_T \leqslant Y_T - q, \\ L'(x_T+q) & x_T \geqslant Y_T - q. \end{cases} \tag{12}$$

otherwise:

$$f_T(x_T|q,n) = \begin{cases} c^f(Y_t - x_T) + L(Y_T) & x_T \leqslant Y_T, \\ L(x_T) & x_T \geqslant Y_T, \end{cases} \tag{13}$$

and

$$f_T'(x_T|q,n) = \begin{cases} -c^f & x_T \leqslant Y_T, \\ L'(x_T) & x_T \geqslant Y_T. \end{cases} \tag{14}$$

$f_T(x_T|q,n)$ is convex in x_T and $\lim_{|x_T|\to\infty} f_T = \infty$, and f_T' is no less than $-c^f$ and converges to a positive number when $x_T \to \infty$. These two properties are independent of the arrival of q in T.

Period $T-1$: According to (9), $G_{T-1}(y_{T-1}) = c^f y_{T-1} + L(y_{T-1}) + E[f_T(y_{T-1} - \xi|q,n)]$. Combining (11) and (13), we can derive that G_{T-1} is convex in y_{T-1} with $\lim_{|y_{T-1}|\to\infty} = \infty$. There must exist a unique Y_{T-1} with $G_{T-1}'(Y_{T-1}) = 0$, that minimizes G_{T-1}, i.e., the optimal delivery policy of the fast mode in $T-1$ is a base stock policy.

Given the optimality of a base stock policy in $T-1$, it can be further shown that $f_{T-1}(x_{T-1}|q,n)$ is convex in x_{T-1}, $\lim_{|x_{T-1}|\to\infty} f_{T-1} = \infty$, and f_{T-1}' is no less than $-c^f$ and converges to a positive number when $x_{T-1} \to \infty$. Both properties are independent of the arrival of q in $T-1$.

From any arbitrary period k to period $k-1$. We now suppose that the optimal fast mode policy in k is a base stock policy, the base stock level is Y_k with $G_k'(Y_k) = 0$, and $f_k(x_k|q,n)$ and $f_k'(x_k|q,n)$ are:

If q arrives in k:

$$f_k(x_k|q,n) = \begin{cases} c^f(Y_k - c_k - q) + c^s q + K + L(Y_k) & x_k \leqslant Y_k - q, \\ \quad + E[f_{k+1}(Y_k - \xi|q,n)] \\ c^s q + K + L(x_k + q) & x_k \geqslant Y_k - q, \\ \quad + E[f_{k+1}(x_k + q - \xi|q,n)] \end{cases} \tag{15}$$

and

$$f_k'(x_k|q,n) = \begin{cases} -c^f & x_k \leqslant Y_k - q, \\ L'(x_k+q) & x_k \geqslant Y_k - q. \\ \quad + E'[f_{k+1}(x_k + q - \xi|q,n)] \end{cases} \tag{16}$$

otherwise:

$$f_k(x_k|q,n) = \begin{cases} c^f(Y_k - x_k) + L(Y_k) & x_k \leqslant Y_k, \\ \quad + E[f_{k+1}(Y_k - \xi|q,n)] \\ L(x_k) + E[f_{k+1}(x_k - \xi|q,n)] & x_k \geqslant Y_k, \end{cases} \tag{17}$$

and

$$f_k'(x_k|q,n) = \begin{cases} -c^f & x_k \leqslant Y_k, \\ L'(x_k) + E'[f_{k+1}(x_k - \xi|q,n)] & x_k \geqslant Y_k. \end{cases} \tag{18}$$

$f_k(x_k|q,n)$ is convex in x_k with $\lim_{|x_k|\to\infty} f_k = \infty$, and f_k' is no less than $-c^f$ and converges to a positive number when $x_k \to \infty$. Both properties are independent of the arrival of q in k.

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In period $k-1$, according to (9), $G_{k-1}(y_{k-1}) = c^f y_{k-1} + L(y_{k-1}) + E[f_k(y_{k-1} - \xi|q,n)]$. Given (15) and (17), G_{k-1} is convex in y_{k-1} with $\lim_{|y_{k-1}|\to\infty} = \infty$. As a result, there must exist a unique Y_{k-1} with $G'_{k-1}(Y_{k-1}) = 0$ that minimizes G_{k-1}, and the fast mode shipment in $k-1$ follows a base stock policy with base stock level Y_{k-1}.

With the knowledge of a base stock policy in $k-1$, it can be further shown that $f_{k-1}(x_{k-1}|q,n)$ is convex in x_{k-1}, with $\lim_{|x_{k-1}|\to\infty} f_{k-1} = \infty$, and f'_{k-1} is no less than $-c^f$ and converges to a positive number when $x_{k-1} \to \infty$. Both properties independent of the arrival of q in $k-1$.

Eqs. (15), (16), (17), and (18) can be iterated to $k-1$, and furthermore, any arbitrary period t.

Allowing T to tend to infinity, the proof could be inducted to the infinite-horizon problem.

Proof of (b) of Theorem 1 using mathematical induction:

Period T: (10) secures $Y_T < S^B = \Phi^{-1}\left(\frac{b}{b+h}\right)$.

Period T-1: We already know from the proof of *(a)* of Theorem 1 that, G_{T-1} in is convex in y_{T-1} with $G'_{T-1}(Y_{T-1}) = 0$. In order to prove $Y_{T-1} < S^B$, we only need to show that $G'_{T-1}(S^B) > 0$ in the following:

If q arrives in T:

$$\begin{aligned} G'_{T-1}(S^B) &= c^f + \underbrace{L'(S^B)}_{=0} + E'[f_T(S^B - \xi|q,n)] \\ &= c^f + \int_0^{S^B - Y_T + q} \underbrace{f'_T(S^B - \xi|q,n)}_{\geqslant -c^f,\ \text{refer to (12)}} \phi(\xi)d\xi \\ &\quad + \int_{S^B - Y_T + q}^{\infty} \underbrace{f'_T(S^B - \xi|q,n)}_{=-c^f,\ \text{refer to (12)}} \phi(\xi)d\xi \\ &= \int_0^{S^B - Y_T + q} \underbrace{c^f + f'_T(S^B - \xi|q,n)}_{\geqslant 0} \phi(\xi)d\xi > 0, \end{aligned} \tag{19}$$

otherwise:

$$\begin{aligned} G'_{T-1}(S^B) &= c^f + \underbrace{L'(S^B)}_{=0} + E'[f_T(S^B - \xi|q,n)] \\ &= c^f + \int_0^{S^B - Y_T} \underbrace{f'_T(S^B - \xi|q,n)}_{\geqslant -c^f,\ \text{refer to (14)}} \phi(\xi)d\xi \\ &\quad + \int_{S^B - Y_T}^{\infty} \underbrace{f'_T(S^B - \xi|q,n)}_{=-c^f,\ \text{refer to (14)}} \phi(\xi)d\xi \\ &= \int_0^{S^B - Y_T} \underbrace{c^f + f'_T(S^B - \xi|q,n)}_{\geqslant 0} \phi(\xi)d\xi > 0. \end{aligned} \tag{20}$$

From any arbitrary period k to k − 1:

Assume that $Y_k < S^B$ and f_k is illustrated in (15) and (17), we want to derive that $Y_{k+1} < S^B$. We already know from the proof of *(a)* of Theorem 1 that, G_{k-1} is convex in y_{k-1} with $G'_{k-1}(Y_{k-1}) = 0$. In order to prove $Y_{k-1} < S^B$, we need to show that $G'_{k-1}(S^B) > 0$ in the following:

If q arrives in k:

$$\begin{aligned} G'_{k-1}(S^B) &= c^f + \underbrace{L'(S^B)}_{=0} + E'[f_k(S^B - \xi|q,n)] \\ &= c^f + \int_0^{S^B - Y_k + q} \underbrace{f'_k(S^B - \xi|q,n)}_{\geqslant -c^f,\ \text{refer to (16)}} \phi(\xi)d\xi \\ &\quad + \int_{S^B - Y_k + q}^{\infty} \underbrace{f'_k(S^B - \xi|q,n)}_{=-c^f,\ \text{refer to (16)}} \phi(\xi)d\xi \\ &= \int_0^{S^B - Y_k + q} \underbrace{c^f + f'_k(S^B - \xi|q,n)}_{\geqslant 0} \phi(\xi)d\xi > 0, \end{aligned} \tag{21}$$

otherwise:

$$\begin{aligned} G'_{k-1}(S^B) &= c^f + \underbrace{L'(S^B)}_{=0} + E'[f_k(S^B - \xi|q,n)] \\ &= c^f + \int_0^{S^B - Y_k} \underbrace{f'_k(S^B - \xi|q,n)}_{\geqslant -c^f,\ \text{refer to (18)}} \phi(\xi)d\xi \\ &\quad + \int_{S^B - Y_k}^{\infty} \underbrace{f'_k(S^B - \xi|q,n)}_{=-c^f,\ \text{refer to (18)}} \phi(\xi)d\xi \\ &= \int_0^{S^B - Y_k} \underbrace{c^f + f'_k(S^B - \xi|q,n)}_{\geqslant 0} \phi(\xi)d\xi > 0. \end{aligned} \tag{22}$$

Allowing T to tend to infinity, the proof could be inducted to the infinite-horizon problem.

Proof of (c) of Theorem 1:

Similar to the approach used in proving *(b)* of Theorem 1, in order to prove $Y_t > Y_{t+1} - q$ and given $G_t(y_t)$ is convex in y_t with $G'_t(Y_t) = 0$, we need to show $G'_t(Y_{t+1} - q) < 0$ as follows:

$$\begin{aligned} G'_t(Y_{t+1} - q) &= c^f + L'(Y_{t+1} - q) \\ &\quad + \int_0^{\infty} \underbrace{f'_{t+1}(Y_{t+1} - q - \xi|q,n)}_{=-c^f,\ \text{refer to (16)}} \phi(\xi)d\xi = L'(Y_{t+1} - q). \end{aligned} \tag{23}$$

Because L is convex with $L(S^B) = 0$ and $Y_{t+1} - q < S^B$ (from *(b)* of Theorem 1), $L'(Y_{t+1} - q) < 0$. End of proof of *(c)* of Theorem 1.

Proof of (d) of Theorem 1:

We firsta show that in the last cycle, $Y_{T-1} \geqslant Y_T$, and in any arbitrary cycle with t being the last period of the cycle, $Y_{t-1} \geqslant Y_t$. After that we iterate the inequalities in the following way: suppose that $k-1, k$, and $k+1$ are three sequential periods of the same cycle, and $Y_k \geqslant Y_{k+1}$, we then derive the inequality that $Y_{k-1} \geqslant Y_k$.

Recall from *(a)* that $G_{T-1}(y_{T-1})$ is convex in y_{T-1} with $G'_{T-1}(y_{T-1}) = 0$, we prove $Y_{T-1} \geqslant Y_T$ by showing $G'_{T-1}(Y_T) < 0$ as follows:

$$G'_{T-1}(Y_T) = \underbrace{c^f + L'(Y_T)}_{=0} + \int_0^{\infty} \underbrace{f'_T(Y_T - \xi|q,n)}_{=-c^f,\ \text{refer to (14)}} \phi(\xi)d\xi < 0. \tag{24}$$

If t is the last period of any arbitrary cycle (note, q arrives in $t+1$), we then need to prove $Y_{t-1} \geqslant Y_t$ by showing $G'_{t-1}(Y_t) \leqslant 0$

as follows:

$$\begin{aligned}
G_{t-1}^{'}(Y_t) &= c^f + L^{'}(Y_t) + \int_0^\infty f_t^{'}(Y_t-\xi|q,n)\phi(\xi)d\xi \\
&= \underbrace{c^f + L^{'}(Y_t) + \int_0^\infty f_{t+1}^{'}(Y_t-\xi|q,n)\phi(\xi)d\xi}_{=G_t^{'}(Y_t)=0} \\
&\quad + \int_0^\infty \underbrace{f_t^{'}(Y_t-\xi|q,n)}_{=-c^f,\ \text{refer to (18)}}\phi(\xi)d\xi \\
&\quad - \int_0^\infty f_{t+1}^{'}(Y_t-\xi|q,n)\phi(\xi)d\xi \\
&= -c^f - \int_0^{Y_t+q-Y_{t+1}} \underbrace{f_{t+1}^{'}(Y_t-\xi|q,n)}_{\geqslant -c^f,\ \text{refer to (16)}}\phi(\xi)d\xi \\
&\quad - \int_{Y_t+q-Y_{t+1}}^\infty \underbrace{f_{t+1}^{'}(Y_t-\xi|q,n)}_{=-c^f,\ \text{refer to (16)}}\phi(\xi)d\xi \\
&= -\int_0^{Y_t+q-Y_{t+1}} [c^f + \underbrace{f_{k+1}^{'}(Y_k-\xi|q,n)}_{\geqslant -c^f\ \text{refer to (16)}}]\phi(\xi)d\xi.
\end{aligned} \tag{25}$$

Because $Y_t + q - Y_{t+1} > 0$ is already proved in *(c)* of Theorem 1, (25) is non positive.

Then we want to show that, suppose $k-1$, k, $k+1$ are three sequential periods in a same cycle, and Y_{k-1}, Y_k, and Y_{k+1} are the optimal base stock levels in the three periods. If $Y_k \geqslant Y_{k+1}$, then $Y_{k-1} \geqslant Y_k$. Given the results from *(a)* of Theorem 1 that $G_{k-1}(y_{k-1})$ is convex in y_{k-1} with $G_{k-1}^{'}(Y_{k-1}) = 0$, we want to prove $Y_{k-1} \geqslant Y_k$ by showing that $G_{k-1}^{'}(Y_k) \leqslant 0$ as follows:

$$\begin{aligned}
G_{k-1}^{'}(Y_k) &= c^f + L^{'}(Y_k) + \int_0^\infty f_k^{'}(Y_k-\xi|q,n)\phi(\xi)d\xi \\
&= \underbrace{c^f + L^{'}(Y_k) + \int_0^\infty f_{k+1}^{'}(Y_k-\xi|q,n)\phi(\xi)d\xi}_{=G_k^{'}(Y_k)=0} \\
&\quad + \int_0^\infty \underbrace{f_k^{'}(Y_t-\xi|q,n)}_{=-c^f,\ \text{refer to (18)}}\phi(\xi)d\xi \\
&\quad - \int_0^\infty f_{k+1}^{'}(Y_k-\xi|q,n)\phi(\xi)d\xi \\
&= -c^f - \int_0^{Y_k-Y_{k+1}} \underbrace{f_{k+1}^{'}(Y_k-\xi|q,n)}_{\geqslant -c^f,\ \text{refer to (16)}}\phi(\xi)d\xi \\
&\quad - \int_{Y_k-Y_{k+1}}^\infty \underbrace{f_{k+1}^{'}(Y_k-\xi|q,n)}_{=-c^f,\ \text{refer to (16)}}\phi(\xi)d\xi \\
&= -\textstyle\int_0^{Y_k-Y_{k+1}} [c^f + \underbrace{f_{k+1}^{'}(Y_k-\xi|q,n)}_{\geqslant -c^f,\ \text{refer to (16)}}]\phi(\xi)d\xi.
\end{aligned} \tag{26}$$

Because $Y_k \geqslant Y_{k+1}$, (26) is non positive.

A2. Proof of Theorem 2

The proof is a non-trivial extension of that of Janakiraman et al. (2015)'s Theorem, since our model is an extension of theirs.

We drop n in the proof because it remains unchanged. Denote $C^{q,*}$ the average total cost per period of an optimal MST policy with cyclical slow mode delivery q and the corresponding optimal fast mode controls, we shall prove the theorem by demonstrating the following standard inequality for showing convexity:

$$\frac{C^{q_1,*} + C^{q_2,*}}{2} \geqslant C^{(q_1+q_2)/2,*} \qquad \text{for all } q_1 \text{ and } q_2. \tag{27}$$

Suppose when the company ships a fixed amount q_1 in the slow mode cyclically, the optimal shipment quantities of the fast mode are $z_1^{q_1}, z_2^{q_1}, \ldots, z_T^{q_1}$. We denote this MST policy θ_1. Similarly, when the company ships a fixed amount q_2 in the slow mode cyclically, the optimal shipment quantities of the fast mode are $z_1^{q_2}, z_2^{q_2}, \ldots, z_T^{q_2}$. And we denote this MST policy θ_2. Now we consider a new MST policy, denoted as θ_3: The company ships a fixed quantity $q_3 = (q_1+q_2)/2$ in the slow mode cyclically, and $(z_1^{q_1}+z_1^{q_2})/2, (z_2^{q_1}+z_2^{q_2})/2, \ldots, (z_T^{q_1}+z_T^{q_2})/2$ in the fast mode. Denote $C^{(q_1+q_2)/2}$ the average total cost per period of policy θ_3. Note, the new policy does not guarantee the optimal fast mode controls when the company ships $(q_1+q_2)/2$ in the slow mode cyclically. As a result, $C^{(q_1+q_2)/2} \geqslant C^{(q_1+q_2)/2,*}$.

In order to prove (27), we now need to show $\frac{C^{q_1,*}+C^{q_2,*}}{2} \geqslant C^{(q_1+q_2)/2}$, i.e., the mean of the average total cost per period under policy θ_1 and θ_2 is no smaller than the average total cost per period under policy θ_3. Recall that the total cost of a MST policy always has two parts: the transport cost and the inventory mismatch cost, we then investigate these two parts separately. Under the policy θ_3, by its definition, the firm always ships the average volume of those under policy θ_1 and θ_2 in the fast and the slow mode every period. As a result, the transport cost under θ_3 should be the same as the average transport cost under policy θ_1 and θ_2.

Let us now consider the realized mismatch cost every period. Assuming that all three policies start with the same inventory level in period 1, i.e., $x_1^{q_1} = x_1^{q_2} = x_1^{(q_1+q_2)/2}$, it is straightforward to see that $x_t^{(q_1+q_2)/2} = (x_t^{q_1} + x_t^{q_2})/2$ for any t. The realized one-period inventory mismatch cost is:

$$\mathcal{L}_t = \begin{cases} h(x_t + z_t + q - \xi)^+ + b(\xi - x_t - z_t - q)^+ & q \text{ arrives in } t, \\ h(x_t + z_t - \xi)^+ + b(\xi - x_t - z_t)^+ & \text{otherwise.} \end{cases} \tag{28}$$

It is convex in x_t for any t, independent on whether or not the slow mode delivers in that period. Considering the convexity of the mismatch cost and the equality of the transport cost explained before, the average total (mismatch and transport combined) costs per period satisfies: $\frac{C^{q_1,*}+C^{q_2,*}}{2} \geqslant C^{(q_1+q_2)/2}$. Therefore we obtain: $\frac{C^{q_1,*}+C^{q_2,*}}{2} \geqslant C^{(q_1+q_2)/2} \geqslant C^{(q_1+q_2)/2,*}$ This completes the proof.

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