



PRICE DISCRIMINATION IN TWO-SIDED MARKETS

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We examine the profitability and welfare implications of targeted price discrimination (PD) in two-sided markets. First, we show that equilibrium discriminatory prices exhibit novel features relative to discriminatory prices in one-sided models and uniform prices in two-sided models. Second, we compare the profitability of perfect PD, relative to uniform prices in a two-sided market. The conventional wisdom from one-sided horizontally differentiated markets is that PD hurts the firms and benefits consumers, prisoners' dilemma. We show that PD, in a two-sided market, may actually soften the competition. Our results suggest that the conventional advice that PD is good for competition based on one-sided markets may not carry over to two-sided markets.

1. INTRODUCTION

The aim of this paper is to study the implications of price discrimination (PD) in two-sided markets.¹ For example, TV stations target advertisers, the one side of the market,

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1. Two-sided (or multiple-sided) markets are markets that are organized around intermediaries or “platforms” with two (or multiple) sides who should join a platform in order for successful exchanges (trade) to take place, see Alexandrov et al. (2011), Armstrong (2006a), Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Spulber (1999). For example, videogame platforms (e.g., Nintendo, Sony, and Microsoft) need to attract both gamers and game developers. Newspapers need to attract advertisers and readers. Credit cards need merchants and users. Most media and advertising markets are two-sided markets, for example, Anderson and Coate (2005) and Anderson and Gabszewicz (2006). Other examples of two-sided markets include, newspapers, scholarly journals, magazines, shopping malls, dating services and Business-to-Business (B2B) markets. More formally, a two-sided market is defined as one where the volume of transactions between end-users depends on the structure of the fees and not only on the overall level of fees charged by platforms (Rochet and Tirole, 2006).

with different advertising fees and viewers, the other side, with different subscription fees (Gil and Grichton, 2010), or newspapers offer low introductory rates to new subscribers (Asplund et al., 2008) and different rates to advertisers.

There exists a relatively large literature on oligopolistic third-degree PD in “one-sided” markets, but this paper is among the first ones that examine this problem in the context of a two-sided market.² The main message of the one-sided literature is that PD intensifies price competition (prisoners’ dilemma) and therefore it is beneficial for the consumers (at least on average).³ The advice then given to policymakers and antitrust authorities is that they should not worry much about firms acquiring and using consumer information with the intention to customize prices, because after all firm competition for consumers dissipates profits and transfers most of the surplus to consumers.

Furthermore, it is well known that the presence of indirect externalities in two-sided markets can intensify competition, for example, Armstrong (2006a). Platforms have strong incentives to lower prices in order to sign-up more agents. Therefore, putting together the results from one-sided models with PD and from two-sided models with no PD, one would expect that price discrimination in a two-sided market will generate a very competitive environment with low prices and profits. This is true, but not always. We show that when the marginal cost is low relative to the cross-group network externalities (e.g., digital products), then PD increases platform profits and hurts consumer welfare.

The intuition for this result is as follows. In two-sided markets, PD has two effects on competition. First, and similar to one-sided markets, when platforms can price discriminate, in equilibrium, any two prices charged by two platforms to an agent always differ by the difference in transportation costs. This flexibility in pricing reduces profits, a *negative effect*. However, PD also has a second effect through the cross-group externality. In particular, it may render the cross-group externality irrelevant in equilibrium, and thus to improve profits relative to that under uniform pricing, a *positive effect*.⁴ This is because, uniform equilibrium prices depend on the cross-group externality. A stronger externality increases each platform’s incentives to cut prices and as a result equilibrium prices fall. Discriminatory prices, on the other hand, are, under certain conditions, independent of the cross-group externality.⁵ The presence of the indirect externality intensifies competition and discriminatory prices fall. Under the reasonable assumption that prices cannot become negative, each platform, in the symmetric equilibrium, will charge zero price to the agents that are located closer to the rival platform and to its own agents will charge a premium which only depends on the transportation cost. Due to the “limit price” nature of the problem under perfect PD and the assumption of nonnegative prices, the feedback effect disappears in equilibrium. Hence, strong externalities imply that uniform prices will fall whereas discriminatory prices do not change, which further implies that PD in such a case is more profitable. Price flexibility is a curse in one-sided

2. By “one-sided” markets we simply mean markets with no externalities. For a survey of the literature on oligopolistic PD in one-sided markets we refer the reader to Armstrong (2006b) and Stole (2007).

3. See, for example, Thisse and Vives (1988), Shaffer and Zhang (1995), Bester and Petrakis (1996), Chen (1997), Corts (1998), Fudenberg and Tirole (2000), and Liu and Serfes (2004). For spatial PD using nonlinear pricing see Spulber (1981, 1989).

4. In one-sided markets, cross-group externality is absent so only the first effect exists. Therefore, targeted PD intensifies competition relative to uniform pricing (under the standard assumptions of uniform distribution and linear transportation cost).

5. If, on the other hand, discriminatory prices depend on externality as well, then both effects are negative, and targeted PD intensifies competition even more than in one-sided markets.

markets, but it can be a blessing in a two-sided market. The result and intuition are similar even when we allow for imperfect PD or when we allow agents on both sides to multihome.

This is the first paper concerned with perfect PD in a two-sided market. As we show, the features of the perfect PD equilibrium are qualitatively very different from those in a one-sided market. For example, the prices a platform charges in its rival's turf are not constant (which is typically the case in one-sided models). This, in turn, has implications about the platform's prices in its own turf. Equilibrium prices are not distribution-free (as it is the case in one-sided perfect PD models). Moreover, equilibrium prices under perfect PD in a two-sided model may depend not only on the other-group externality (as it is the case under uniform pricing in a two-sided model), but also on the own-group externality. These new results have managerial implications that cannot be deduced from a one-sided perfect PD model.

Caillaud and Jullien (2003) and Armstrong (2006a) also allow for PD. In Caillaud and Jullien agents in each group are homogeneous and therefore PD means different prices charged to each group of agents, whereas within each group the price is constant. This is also the meaning of PD in Armstrong (2006a), although he allows for heterogeneous populations of agents. In contrast, we allow the prices within each group to vary.

A related paper that also deals with PD in a two-sided market is Jullien (2008). Jullien (2008), in a leader–follower model, focuses on the issue of pricing strategies, market power and barriers to entry. Relatedly, he investigates how PD can help a platform to coordinate the choices of consumers. In contrast, we focus on the effect of PD on the structure of prices and we perform a comparison with respect to uniform prices.

The result that targeted PD may relax competition can potentially apply to settings other than two-sided markets, where there is a binding price floor under PD but not under uniform pricing. For example, consider a two-period model with switching costs. In the second period, each firm has incentives to price discriminate between its own and the rival's customers. In particular, it may want to charge lower prices to its rival's customers to compensate for switching costs. Various studies (e.g., Chen, 1997) have shown that such PD can lead to lower profits. However, PD also makes the nonnegative price constraint more likely to be binding, relative to the case of uniform pricing where the single price applies to its own customers as well. That is, PD has a *positive* effect due to switching costs which is similar to the positive effect due to cross-group externality in our model.⁶

The rest of the paper is organized as follows. In Section 2 we present the benchmark model. In Section 3 we perform the analysis. In Section 4 we extend the benchmark model to allow agents to multihome. We conclude in Section 5.

2. THE DESCRIPTION OF THE BENCHMARK MODEL

There are two groups of agents $\ell = 1, 2$ and two horizontally differentiated platforms $k = A, B$.⁷ We will denote the “other” group of agents by m . We capture platform differentiation as follows. There is a continuum of agents of group ℓ that is distributed

6. One can also introduce consumer heterogeneity in addition to that due to switching costs, in which case the nonnegative price constraint is more likely to be binding under PD. Of course, it remains to be seen whether firms' discounted profits over the two periods are higher under PD or uniform pricing.

7. Our benchmark model follows closely the model in Armstrong (2006a).

on the $[0, 1]$ interval according to the distribution function $F_\ell(\cdot)$ with density f_ℓ . The distributions are independent across the two groups of agents and symmetric about $\frac{1}{2}$, that is, $F_\ell(\frac{1}{2}) = \frac{1}{2}$ and $f_\ell(x) = f_\ell(1 - x)$. The two platforms are located at the two end points of each interval, with platform A located at 0 and platform B located at 1.

The common per-unit transportation cost of both groups is denoted by $t > 0$. We assume that each agent joins only one platform (single-homing).⁸ Each member of a group who joins a given platform cares about the number of members from the other group who join the same platform. Denote by $n_{\ell k}$ the number of participants from group ℓ that platform k attracts. The maximum willingness to pay for a member of group ℓ if he joins platform k is given by $V + \alpha_\ell n_{mk}$, where V is a stand-alone benefit each agent receives independent of the number of participants from the other group on platform k .⁹ The parameter $\alpha_\ell > 0$ measures the cross-group externality for group ℓ participants. The indirect utility of an agent from group ℓ who is located at point $x \in [0, 1]$ is given by,

$$U_\ell = \begin{cases} V + \alpha_\ell n_{mA}^e - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha_\ell n_{mB}^e - t(1 - x) - p_{\ell B}(x), & \text{if he joins platform } B' \end{cases} \quad (1)$$

where $p_{\ell k}(x)$ is platform k 's lump-sum charge to a group ℓ participant who is located at point x and n_{mk}^e denotes the expectations agents from group ℓ have about how many agents from group m will join platform k . Under a uniform pricing rule prices are constant across all agents in the same group (prices are allowed to vary across groups), whereas under discriminatory pricing the price each agent pays depends on his preferences (location). We assume that V is high enough which ensures that the market is covered. Platforms have constant marginal cost $c \geq 0$. We assume that prices cannot be negative.¹⁰

The timing of the game is as follows. In stage 1, the two platforms make, simultaneously, their pricing decisions. In stage 2, the agents decide which platform to join.

3. ANALYSIS

We study two different price regimes. In the first regime each platform charges uniform prices to the agents of each group. In the second regime each platform can price discriminate perfectly the agents of each group. Then, we compare prices and profits between these two price regimes. We assume that each agent has rational expectations about how many agents from the other group will join each platform. Each agent observes all prices before he decides which platform to join (public prices) (e.g., Caillaud and Jullien, 2003; Armstrong, 2006a).¹¹

8. We relax this assumption by allowing agents to multihome in Section 4

9. Hence, our assumption is that there exists a membership externality. An interesting extension would be to also allow for a usage externality, whereby agents from one group also derive benefit from the number of units each agent from the other group consumes.

10. In many cases negative prices are unrealistic and create perverse incentives [see also Armstrong (2006a) for a discussion on this issue]. If negative prices were allowed, viewers could, for example, subscribe to a TV channel, never watched it and got paid for that. More generally, agents will have incentives, when they get paid, to make multiple purchases and it will be difficult for the platforms to prevent this from happening.

11. We consider private prices in Section 3.5.

3.1 NO PD (UNIFORM PRICES WITHIN EACH GROUP OF AGENTS)

The next proposition summarizes the main result when platforms cannot price discriminate within each group of agents with a general distribution of preferences.¹²

PROPOSITION 1: (Uniform prices) *If a symmetric equilibrium exists, then it is given by:*

$$p_{1A}^* = p_{1B}^* = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)} + c \text{ and } p_{2A}^* = p_{2B}^* = \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)} + c. \quad (2)$$

The equilibrium profits are,

$$\pi_A = \pi_B = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{2f_1\left(\frac{1}{2}\right)} + \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{2f_2\left(\frac{1}{2}\right)}. \quad (3)$$

Proof. See the Appendix. □

Each platform serves one half of the members of each group. The equilibrium prices depend positively on the differentiation parameter t , negatively on the strength of the cross-group externality α_ℓ and negatively on the number of marginal agents $f_\ell(\frac{1}{2})$. When the externality for group ℓ is stronger platforms offer lower prices to the members of group m , all else equal. Potentially, prices can be negative, but we do not allow for this possibility.

3.1.1 UNIFORM DISTRIBUTION

If we assume that the distribution is uniform ($f_1(x) = f_2(x) = 1$), then the equilibrium prices and profits are.¹³

$$p_{1A}^* = p_{1B}^* = t - \alpha_2 + c, \quad p_{2A}^* = p_{2B}^* = t - \alpha_1 + c \quad (4)$$

and

$$\pi_A = \pi_B = t - \frac{(\alpha_1 + \alpha_2)}{2}. \quad (5)$$

3.2 PERFECT PRICE DISCRIMINATION

Now we assume that platforms can price discriminate perfectly. Agent utility is given by (1) and platforms compete on an agent-by-agent basis. Each agent receives a targeted offer. Platform A 's own territory is the $[0, 1/2]$ interval and platform B 's own territory is the $[1/2, 1]$ interval. The next proposition summarizes the equilibrium. We focus on symmetric equilibria.

12. We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform (see below), then the profit functions are strictly concave provided that $2t > (\alpha_1 + \alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

13. The existence of a symmetric equilibrium is guaranteed if $2t > \alpha_1 + \alpha_2$, see Armstrong (2006).

PROPOSITION 2: (Perfect PD) *There are two distinct cases:*¹⁴

- (1) *High marginal cost and/or low cross group externality, $c \geq \max\{2\alpha_1 + \alpha_2, \alpha_1 + 2\alpha_2\}$. Suppose that*

$$t \geq 2(\alpha_1 + \alpha_2)f_\ell(x), \ell = 1, 2. \quad (6)$$

The equilibrium prices are

$$p_{\ell A}^*(x) = t(1 - 2x) + p_{\ell B}^*(x), p_{\ell B}^*(x) = c - 2\alpha_m(1 - F_m(x)) - \alpha_\ell(1 - 2F_m(x)),$$

$$\text{for } x \leq \frac{1}{2} \text{ and}$$

$$p_{\ell A}^*(x) = c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x) - 1), p_{\ell B}^*(x) = p_{\ell A}^*(x) + t(2x - 1), \text{ for } x \geq \frac{1}{2}. \quad (7)$$

- (2) *Low marginal cost and/or high cross-group externality, $c \leq \min\{\alpha_1, \alpha_2\}$. All prices in the rival platform's own territory are negative. As negative prices are not allowed, they are replaced by zero. Suppose that*

$$t > c + \max\{\alpha_1, \alpha_2\} \text{ and } t < (\alpha_1 + \alpha_2) \min\{f_1(x), f_2(x)\}. \quad (8)$$

The equilibrium prices are

$$p_{\ell A}^*(x) = t(1 - 2x), p_{\ell B}^*(x) = 0, \text{ for } x \leq \frac{1}{2} \text{ and}$$

$$p_{\ell A}^*(x) = 0, p_{\ell B}^*(x) = t(2x - 1), \text{ for } x \geq \frac{1}{2}. \quad (9)$$

Proof. The proof is long and is omitted. It can be found in the working paper version posted on SSRN and it is also available upon request. \square

The idea behind the equilibrium prices in case (i) is as follows. First, there is symmetry, that is, in equilibrium, $p_{\ell A}^*(x) = p_{\ell B}^*(1 - x)$, for all $x \in [0, 1]$, $\ell = 1, 2$. Second, the two platforms split both groups evenly. Third, each agent is indifferent between buying from either platform (and buys from the one closer to his location). Prices are constructed as follows. If platform A, say, deviates to $x > 1/2$ in group ℓ , its benefit from signing up an extra agent in group ℓ is $2\alpha_m F_m(x)$.¹⁵ The cost to make an extra sale at price below marginal cost is $p_{\ell A}^{dev}(x) - c$. For platform A not to have incentive to deviate, we need

$$2\alpha_m F_m(x) \leq -(p_{\ell A}^{dev}(x) - c) \Rightarrow p_{\ell A}^{dev}(x) \leq c - 2\alpha_m F_m(x). \quad (10)$$

$p_{\ell A}^{dev}(x)$ is decided by the indifference condition for the agents, that is,

$$V - p_{\ell A}^{dev}(x) - tx + \alpha_\ell F_m(x) = V - p_{\ell B}^*(x) - t(1 - x) + \alpha_\ell(1 - F_m(x)) \Rightarrow \quad (11)$$

14. There is also a third case that falls in between the two cases presented in this proposition, that is, $\min\{\alpha_1, \alpha_2\} < c < \max\{2\alpha_1 + \alpha_2, \alpha_1 + 2\alpha_2\}$. Relative to case (i) some prices in the rival platform's territory become negative. In equilibrium, these prices are replaced by zero and the prices charged by a platform in its own territory are equal to the transportation cost premium, as in (9). For the prices that are not negative the equilibrium is the same as in (7). We do not pursue this case further, as it does not add anything to our understanding of the problem.

15. It is multiplied by 2 because an extra agent increases platform A's share as well as decreases platform B's market share in group ℓ .

$$p_{\ell B}^*(x) = p_{\ell A}^{dev}(x) - \alpha_{\ell}(2F_m(x) - 1) + t(2x - 1) \\ \leq [c - 2\alpha_m F_m(x)] - \alpha_{\ell}(2F_m(x) - 1) + t(2x - 1).$$

The upper bound is the equilibrium price for platform *B* when $x \geq 1/2$, as stated in Proposition 2. Then, the upper bound for platform *A*'s equilibrium price in $x \geq 1/2$ is

$$p_{\ell A}^*(x) = p_{\ell B}^*(x) + t(1 - 2x) = c - 2\alpha_m F_m(x) - \alpha_{\ell}(2F_m(x) - 1). \quad (12)$$

When platform *A* deviates and chooses its deviation prices, there is a large multiplicity of consumer allocations due to the network effects and a selection needs to be made. Here, by assuming that agents join the deviating platform when they are indifferent, the selection leads to the maximum profit for the deviating platform. If platforms have no incentive to deviate under this selection, they have even less incentive to deviate under other selections. Therefore, the equilibrium stated in Proposition 2 is robust to the choice of selection.¹⁶ In Proposition 2, the upper bound of the prices is chosen and there is no symmetric equilibrium of this kind that supports higher prices.¹⁷

What remains is to confirm that indeed no platform has an incentive to deviate from the candidate equilibrium prices. (We do this in the proof of Proposition 2). As the externalities intensify, or the marginal cost decreases, some prices become negative. Negative prices are replaced by zero and we move to case (ii) in Proposition 2.

Case (i) of Proposition 2 presents novel results. There are two main differences between the perfect PD equilibrium in a one-sided market and in a two-sided market. In a one-sided market a firm charges prices equal to marginal cost in the rival firm's territory, whereas in its own territory prices reflect the transportation cost difference. This is not the case in a two-sided market. First, in a two-sided market, the prices a platform charges in the rival's market (and some of the prices in its own market) are below marginal cost. Second, they are not constant and they are not distribution-free (that is, they depend on F_{ℓ}). Notice, for example, from case (i) in Proposition 2, that when $x \geq 1/2$, $p_{\ell A}^*$ decreases in x and depends on the distribution of agent preferences.

The intuition behind these two differences is as follows. A platform may be willing to charge a price below marginal cost to an agent, because an extra agent from one group is valuable to *all* the agents in the other group. This is a direct consequence of the network externalities and a platform's ability to customize prices. The second difference we mentioned above is more subtle. For an equilibrium to exist, a platform should be losing increasingly more money as it tries to poach the rival platform's agents. The reason for this is that the benefit from signing up one more agent increases with the market share of a platform, that is, the more agents a platform has already signed up the higher the benefit of an additional agent. Thus, the price in the rival platform's territory should be decreasing in accordance to the mass of agents up to that point. Finally, equilibrium uniform prices depend on the externality in the other group (see (2)), whereas perfect PD prices depend on the externalities in *both* groups. These new results should also serve as a guidance to managers in two-sided markets.

16. We thank a referee for pointing out this notion of "robust" equilibrium and we acknowledge that there are most likely other nonrobust equilibria that can yield higher profits. This happens when the selection of consumer allocation leads to lower-than-maximum deviation profit (e.g., minimum deviation profit as in Caillaud and Jullien, 2003) which then should allow equilibria that support higher prices.

17. There is a continuum of (symmetric) price equilibria (when all prices are positive), that can be Pareto ranked (from the platforms' perspective) from the "highest price" one to the "lowest price" one. We assume that platforms can coordinate on the one that yields the highest prices and hence profits and this is the one we present in Proposition 2 (case (i)). More details can be found in the proof of Proposition 2.

An interesting and possibly empirically testable implication is the following. Our results indicate that a platform's discriminatory prices in its own turf exhibit less dispersion in a two-sided market than in a one-sided market. This is because discriminatory prices decrease slower when moving from loyal to less loyal agents in a two-sided market. From (7), we can infer that $dp_{\ell A}^*(x)/dx = -2t + 2\alpha_m f_m(x) + 2\alpha_\ell f_m(x)$ when $x \leq \frac{1}{2}$, which under our assumptions is negative but greater than $-2t$, the corresponding price decline in a one-sided market.¹⁸

When the marginal cost is high, as in case (i) in Proposition 2, the equilibrium prices depend on the cross-group externalities. Actually, the presence of cross-group network externalities intensifies the competition further under perfect PD relative to uniform pricing. Under uniform pricing, prices are reduced by α_m , see (2), whereas under perfect PD the price decline in group ℓ , according to (7), ranges from $2\alpha_m + \alpha_\ell$ for the agents located at the extremes to α_m for the agents located at $x = 1/2$.

Nevertheless, the above comparison fails when the marginal cost is low relative to network externalities as in case (ii) in Proposition 2. Precisely because perfect PD in a two-sided market generates a very competitive environment, prices fall so low that they reach the natural floor of zero. In this case, the network externality is priced-out of the equilibrium prices, see (9).

Therefore, when the marginal cost is high relative to network externalities, perfect PD unambiguously yields lower profits than uniform pricing. This result is in line with the comparison in one-sided markets, for example, Thisse and Vives (1988). The difference arises when the marginal cost c is low (e.g., digital products) and/or α is high, $c \leq \min\{\alpha_1, \alpha_2\}$, case (ii) in Proposition 2. (More on this comparison in the next section). After the next subsection, in the remaining of the paper we assume that we are in case (ii).

Nonnegative prices. The assumption that prices cannot become negative is key for our main result. Although it is a reasonable assumption, it does not always hold. In many instances, platforms can use in-kind subsidies or bundles to achieve implicit negative prices. Furthermore, the Internet allows platforms to target consumers with direct payments more effectively and hence minimize the concern of multiple purchases that a negative price would create.

3.2.1 UNIFORM DISTRIBUTION

Suppose that the distribution is uniform, that is, $f_1(x) = f_2(x) = 1$. The equilibrium profits when all prices are positive, as in case (i) in Proposition 2, are

$$\begin{aligned}\pi_A &= \int_0^{1/2} [t(1-2x) - 2\alpha_2(1-x) - \alpha_1(1-2x)]dx \\ &\quad + \int_0^{1/2} [t(1-2x) - 2\alpha_1(1-x) - \alpha_2(1-2x)]dx \\ &= \frac{t}{2} - \alpha_1 - \alpha_2 = \pi_B.\end{aligned}\tag{13}$$

Condition (6) guarantees that the above profits are nonnegative.

18. Our results also indicate that a platform's prices in its rival's turf exhibit more dispersion in a two-sided market than in a one-sided market. However, this is unlikely to be testable because one should not expect to observe platform's prices in its weak market where it makes no sales.

On the other hand, when all unconstrained prices in the rival platform's territory are negative (in which case they are replaced by zero), as in case (ii) in Proposition 2, the equilibrium profits are

$$\pi_A = \int_0^{1/2} [t(1-2x) - c]dx + \int_0^{1/2} [t(1-2x) - c]dx = \frac{t}{2} - c = \pi_B. \quad (14)$$

Condition (8) together with $c \leq \min\{\alpha_1, \alpha_2\}$ guarantee that the above profits are positive. Notice that the network externalities affect the equilibrium profits given by (13), whereas they do not affect the equilibrium profits given by (14).

3.3 PRICE AND PROFIT COMPARISON

We compare the equilibrium uniform prices given by (4) with the discriminatory prices given by (9). So, we assume that $c \leq \min\{\alpha_1, \alpha_2\}$.¹⁹ Discriminatory prices, as it is the case in one-sided markets that are characterized by horizontal differentiation, are decreasing in the degree of agent loyalty to a platform. Agents located very close to one or the other platform pay higher prices than those located in the middle. The highest price is t and the lowest is 0. If we compare these prices with the no discriminatory prices, $\frac{t - \alpha_\ell f_m(\frac{1}{2})}{2f_m(\frac{1}{2})} + c$, we will see that it is possible that nearly all agents pay higher prices under PD if c is small and α_ℓ is high enough. It then becomes obvious that there exists a threshold for the cross-group externality parameters above which perfect PD benefits the platforms (relative to uniform pricing). The next proposition summarizes the profit comparison.

PROPOSITION 3: (Profit comparison) *If the marginal cost is low relative to the cross-group externalities, then perfect PD is more profitable than uniform pricing.*

When the distribution is uniform, equilibrium profits increase with PD if and only if $t/2 - c > t - (\alpha_1 + \alpha_2)/2$, (we compare (14) with (5)). This is the case if and only if,

$$t < (\alpha_1 + \alpha_2 - 2c). \quad (15)$$

Furthermore, the necessary and sufficient condition for a market sharing equilibrium under uniform pricing to exist is $2t > (\alpha_1 + \alpha_2)$. Under perfect discrimination the condition we need, from Proposition 2 condition (8), is

$$t > c + \max\{\alpha_1, \alpha_2\} \quad \text{and} \quad t < (\alpha_1 + \alpha_2).$$

Therefore, if c is low and/or the externalities are strong, that is, $c \leq \min\{\alpha_1, \alpha_2\}$, there exists a range of parameters such that uniform equilibrium prices are given by (4), discriminatory prices are given by (9) and PD leads to higher profits. For example, if $\alpha_1 = \alpha_2 = \alpha$, for the above assertion to be true we need t to be in the nonempty interval $[c + \alpha, 2(\alpha - c)]$.

The main idea behind the price and profit comparison is that the externalities are priced in the equilibrium uniform prices, but not in the discriminatory prices. When platforms lower their prices below marginal cost due to the externality, the natural price floor of zero is reached before the price goes down all the way to $c - 2\alpha_m(1 - F_m(x)) - \alpha_\ell(1 - 2F(x))$, for $x \leq 1/2$, (which is negative). So, strong enough externalities combined

19. For this comparison, we focus on case (ii) of Proposition 2, because, as we mentioned above, under case (i) the standard ranking of profits between uniform and discriminatory prices from one-sided markets carries over to two-sided markets.

with low marginal cost (i.e., $c \leq \min\{\alpha_1, \alpha_2\}$) make perfect PD more profitable (relative to uniform prices). This intuition does not rely on specific modeling assumptions and it is likely to hold in more general models.

Armstrong (2006a) compares PD with uniform prices. In his model PD is defined as the uniform pricing rule in our model, that is, when platforms charge each group a different price. A uniform pricing rule in Armstrong's model is when a platform charges both groups the same price. Armstrong shows that PD is profitable if and only if,

$$(t_1 - t_2)^2 > (\alpha_1 - \alpha_2)^2. \quad (16)$$

Our condition (15) for a profitable PD is qualitatively different from Armstrong's condition (16). In our case the levels matter, whereas in Armstrong's case the differences matter more. If the transportation parameters are the same across groups ($t_1 = t_2$), as it is the case in our model, then PD is never profitable in Armstrong's model, whereas it may be in our model.

Finally, as it is well-known (e.g., Thisse and Vives, 1988) PD, in one-sided markets when preferences are uniformly distributed and platforms are symmetric, always leads to a prisoners' dilemma. The profits under perfect PD are $t/4$, whereas under a uniform pricing rule they are $t/2$ (with marginal cost c equal to zero). In contrast, in two-sided markets, when (15) is satisfied perfect PD yields higher profits than uniform prices.

3.4 POLICY IMPLICATIONS

Our result has important theoretical and policy implications because it demonstrates that PD is more likely to be anticompetitive in two-sided markets than it is in one-sided markets. More fundamentally, it suggests that two-sided markets can be very different from one-sided markets (see Economides and Tåg, 2007, where a similar conclusion, regarding the difference between one-sided and two-sided markets, is reached).

Our analysis can also apply to intermediate goods markets where PD is more likely to raise antitrust concerns than in final goods markets. Indeed, in the United States PD is illegal in intermediate goods markets under the Robinson-Patman act. Each platform in our model can be viewed as a Business-to-Business (B2B) web site which matches input suppliers with producers (e.g., Caillaud and Jullien, 2003). The Internet facilitates the collection and application of information about the users' preferences and characteristics, see FTC (2000). An interesting question which arises then is whether platforms should be restricted to charge uniform prices.

More specifically, when the market is an intermediate goods markets, then our result implies that PD will lead to higher input prices if and only if platforms have detailed information about the preferences of the participants and the marginal cost is low relative to network externalities. To arrive at this result, we can assume that each firm is seeking to buy only one unit of the input and each input seller sells only one unit. The platforms facilitate the matching process between the two sides. Let's assume that platforms have very good (perfect) information about the agents. If platforms are allowed to customize their prices then firms end up paying higher prices for the right to trade a unit of the input. Now if we assume that the prices the participants pay to join a platform do not affect the bargaining process between an input supplier and a firm that will ensue once a matching takes place, then a higher price charged by a platform will lead to a higher overall price a firm will have to pay in order to acquire its input. If firms can pass part of this extra cost on to consumers, then PD is anti-competitive.

However, the reverse is true if platforms do not possess very detailed information about the participants (as in the uniform price case). In this case the cost of acquiring the input is reduced due to PD.

3.5 PRICES ARE PRIVATE

So far we have assumed that each agent observes all prices before he chooses which platform to join. Here we assume that prices are private, in the sense that each agent only observes his own price. Given the cross-group externalities, beliefs are important in this case. What is an agent's belief about the offers made to other agents if he receives an out-of-equilibrium offer? If beliefs are passive (e.g., McAfee and Schwartz, 1994) and price offers are secret, then equilibrium prices do not depend on the cross-group externalities. In particular, the equilibrium prices are,

$$p_{\ell A} = t(1 - 2x) + c \quad \text{and} \quad p_{\ell B} = c, \quad \text{for } x \leq \frac{1}{2} \text{ and} \\ p_{\ell A} = c \quad \text{and} \quad p_{\ell B} = t(2x - 1) + c, \quad \text{for } x \geq \frac{1}{2}.$$

To see this, suppose that a platform raises unilaterally its prices to a group of agents in its territory. Each agent, however, continues to believe that market shares will not change and given that agents are indifferent, in equilibrium, between the two platforms they will all switch to the rival platform. Hence, such a deviation is unprofitable. Price cuts would also be unprofitable because a reduction in price to an agent (or a group of agents) will not lead to higher market share. Thus, when prices are private equilibrium discriminatory prices do not depend on the externality parameter, as in the case of public prices and low marginal cost, that is, case (ii) of Proposition 2. Given that no new insights are derived under the assumption of private prices, in the rest of the paper we go back to assuming that prices are public.

4. AGENTS ARE ALLOWED TO MULTIHOME

We would like to investigate the robustness of the comparison between uniform and discriminatory pricing to an extension to the benchmark model. We allow agents to multihome. Prices are public. Our result does not change qualitatively. One difference is that equilibrium discriminatory prices, when agents multihome, depend positively (on average) on the cross-group externality. In order to cut down on the number of different cases that we will have to examine, we assume that the marginal cost c is zero, which is the analogue of case (ii) in Proposition 2. We maintain the assumption that the distribution is uniform and we set $\alpha_1 = \alpha_2 = \alpha$. The indirect utility of an agent from group ℓ who is located at point $x \in [0, 1]$ is given by,²⁰

$$U_{\ell} = \begin{cases} V + \alpha n_{mA}^e - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha n_{mB}^e - t(1 - x) - p_{\ell B}(x), & \text{if he joins platform } B \\ V + \theta + \alpha - t - p_{\ell A}(x) - p_{\ell B}(x) & \text{if he joins both platforms.} \end{cases} \quad (17)$$

20. This utility specification has also been used in Kim and Serfes (2006) in a one-sided framework.

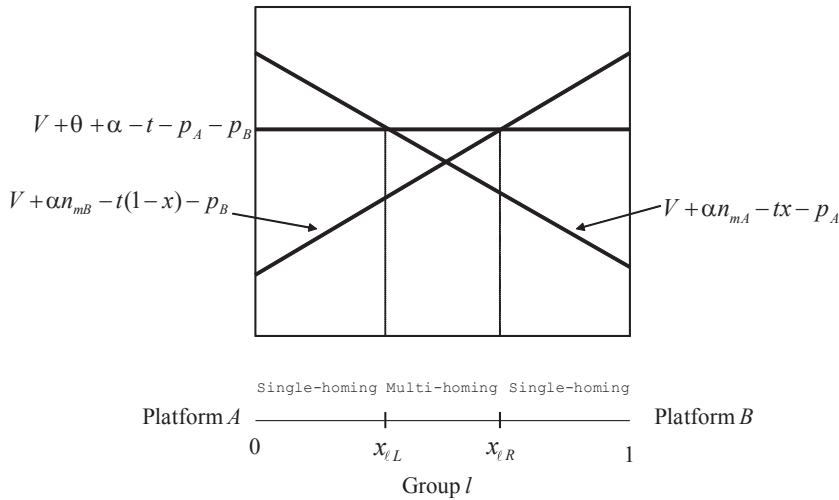


FIGURE 1. INDIRECT UTILITIES: PARTIAL MULTIHOMING

The incremental maximum willingness to pay of an agent from group ℓ who chooses to multihome by joining platform k is given by $\theta + \alpha(1 - n_{mk}^e)$. The first effect (*product variety effect*) is captured by the parameter θ , where $\theta \in [0, V]$, and the second effect (*indirect externality effect*) is given by the term $\alpha(1 - n_{mk}^e)$. For example, the utility of an agent who chooses to read a second newspaper increases because he gets to see more classified advertisements (indirect externality effect), but also because the second newspaper covers different issues than the first one (product variety effect). Or, a second credit card allows the holder to have transactions with more merchants (indirect externality effect), but also increases his credit limit (product variety effect). More generally, agent utility can increase, when he joins a second platform, independent of the indirect externality effect, because platforms are differentiated and agents value “variety.”

The disutility of the agent who chooses to multihome also increases and this is captured by the parameter t ($t = tx + t(1 - x)$). We assume that the total transportation cost is additive. Agents choose the option that gives them the highest indirect utility. We maintain the assumption that $t > \alpha$.

In general, there are three possible type of equilibria: (i) single-homing, (ii) partial multihoming, and (iii) complete multihoming. In the first equilibrium, no agent multihomes, in the second one a fraction of the agents multihomes and in the third one all agents multihome. Due to symmetry the outcomes are the same across the two groups of agents. We will focus on the second type of equilibrium. Figure 1 is consistent with the partial multihoming equilibrium and depicts the indirect utilities when prices within each group are uniform.

There are two marginal agents in group ℓ , where $\ell = 1, 2$, located at $x_{\ell L}$ and $x_{\ell R}$, respectively (the subscript L stands for left and the subscript R stands for right). $x_{\ell L}$ is indifferent between joining platform A only and joining both platforms, whereas $x_{\ell R}$ is indifferent between joining both platforms and joining B only. Agents in $[0, x_{\ell L}]$ and in $[x_{\ell R}, 1]$ single-home and in $[x_{\ell L}, x_{\ell R}]$ they multihome.

4.1 UNIFORM PRICES (UP)

We assume that prices are constant within each group of agents. Let $p_{\ell A}$ and $p_{\ell B}$ denote platform A and B 's price in group $\ell = 1, 2$, respectively. The marginal agents in group $\ell = 1, 2$ are defined by,

$$x_{\ell L}^{UP}: V + \alpha n_{mA}^e - tx - p_{\ell A} = V + \theta + \alpha - t - p_{\ell A} - p_{\ell B},$$

$$x_{\ell R}^{UP}: V + \alpha n_{mB}^e - t(1 - x) - p_{\ell B} = V + \theta + \alpha - t - p_{\ell A} - p_{\ell B},$$

with $n_{mA}^e = x_{mR}^{UP}$, and $n_{mB}^e = 1 - x_{mL}^{UP}$, $m = 1, 2$. From these equations, we can obtain the marginal agents in group ℓ as follows,

$$x_{\ell L}^{UP} = \frac{(-t^2 + \alpha p_{mA} - \alpha\theta + t\alpha - tp_{\ell B} + t\theta)}{(\alpha - t)(t + \alpha)} \quad \text{and}$$

$$x_{\ell R}^{UP} = \frac{(\alpha\theta - t\alpha - t\theta + tp_{\ell A} + \alpha^2 - \alpha p_{mB})}{(\alpha - t)(t + \alpha)}.$$

Then, the platforms' problems are,

$$\max_{\{p_{1A}, p_{2A}\}} \pi_A = p_{1A} x_{1R}^{UP} + p_{2A} x_{2R}^{UP},$$

$$\max_{\{p_{1B}, p_{2B}\}} \pi_B = p_{1B}(1 - x_{1L}^{UP}) + p_{2B}(1 - x_{2L}^{UP}).$$

Solving the first order conditions, we can obtain the candidate equilibrium prices and profits as,

$$p_{1A}^* = p_{1B}^* = p_{2A}^* = p_{2B}^* = \frac{(t - \alpha)(\alpha + \theta)}{2t - \alpha},$$

$$\pi_A = \pi_B = \frac{2t(t - \alpha)(\alpha + \theta)^2}{(2t - \alpha)^2(\alpha + t)}. \quad (18)$$

In this candidate equilibrium,

$$0 < x_{\ell L}^{UP} \equiv \frac{2t^2 - \alpha^2 - t\theta}{(t + \alpha)(2t - \alpha)} < x_{\ell R}^{UP} \equiv \frac{t(\alpha + \theta)}{(t + \alpha)(2t - \alpha)} < 1,$$

if $\theta \in [\theta_1 \equiv \frac{6t(t-\alpha)}{(5t-\alpha)}, \theta_2 \equiv \frac{2t^2-\alpha^2}{t}]$.²¹ That is, only the agents strictly in the middle multihome (see Figure 1), and we have partial multihoming.

What is worth observing is that the equilibrium profits (18) are decreasing in α when α exceeds a given threshold (its specific value is omitted) and approach zero as α tends to t . For low values of α equilibrium profits can be increasing in the externality

21. The restrictions on the parameter θ can be understood as follows: i) $\theta < \theta_2$ guarantees that $x_{\ell L}^{UP} > 0$ and $x_{\ell R}^{UP} < 1$ and ii) $\theta > \frac{2t^2-\alpha^2-\alpha t}{2t}$ guarantees that $x_{\ell L}^{UP} < x_{\ell R}^{UP}$. Moreover, $\theta \geq \theta_1 \geq \frac{2t^2-\alpha^2-\alpha t}{2t}$, ensures that no global unilateral deviation is profitable. Profit functions are strictly concave when partial multihoming is assumed, but a platform can deviate in a way that partial multihoming vanishes. We want our solutions to the first order conditions to be immune from such a deviation. So, if θ falls in the above interval, then the candidate equilibrium becomes an equilibrium.

parameter. There are two opposing effects present as the indirect externality α increases. First, as in the single-homing case, incentives for unilateral price cuts increase. Second, agents are willing to pay more to join a second platform, which gives platforms incentives to raise their prices. This second effect arises because of the multihoming assumption.²² When α is high, the first effect is more dominant, whereas for low α the second effect may be more dominant.

4.2 PERFECT PRICE DISCRIMINATION

Now, we assume that platforms can identify the exact location of each agent. We first need to identify the locations of the marginal agents. The agent who is located at $x_{\ell L}^{PD}$ is indifferent between joining platform A only and joining both platforms. This implies that this agent obtains zero utility from joining B , when platform B is charging zero price. Then, $x_{\ell L}^{PD}$ is defined by,

$$x_{\ell L}^{PD} : V + \alpha n_{mA}^e - tx - p_{\ell A} = V + \theta + \alpha - t - (p_{\ell A} + 0) \Rightarrow \theta + \alpha(1 - n_{mA}^e) = t(1 - x).$$

Similarly, $x_{\ell R}^{PD}$ is defined by,

$$x_{\ell R}^{PD} : V + \alpha n_{mB}^e - t(1 - x) - p_{\ell B} = V + \theta + \alpha - t - (0 + p_{\ell B}) \Rightarrow \theta + \alpha(1 - n_{mB}^e) = tx.$$

Also, $n_{mA}^e = x_{mR}^{PD}$ and $n_{mB}^e = 1 - x_{mL}^{PD}$, $m = 1, 2$. From these equations, we can obtain the locations of the marginal agents, $\ell = 1, 2$,

$$x_{\ell L}^{PD} = \frac{t - \theta}{t + \alpha} \quad \text{and} \quad x_{\ell R}^{PD} = \frac{\theta + \alpha}{t + \alpha}.$$

Then, $n_{mA} = n_{mB} = \frac{\theta + \alpha}{t + \alpha}$. When $x \in (x_{\ell L}^{PD}, x_{\ell R}^{PD})$, both platforms set prices so that all agents are indifferent between joining both platforms or that platform alone,

$$V + \alpha n_{mA}^e - tx - p_{\ell A}(x) = V + \alpha n_{mB}^e - t(1 - x) - p_{\ell B}(x) = V + \theta + \alpha - t - p_{\ell A}(x) - p_{\ell B}(x).$$

From this equation, we can solve for $p_{\ell A}(x)$ and $p_{\ell B}(x)$. When $\theta \in [\frac{t-\alpha}{2}, t]$ we have $0 \leq x_{\ell L}^{PD} \leq x_{\ell R}^{PD} \leq 1$, and partial multihoming is an equilibrium. The equilibrium prices and profits are given by,

$$p_{\ell A}^*(x) = \begin{cases} \frac{t(\theta + \alpha - x(t + \alpha))}{(t + \alpha)}, & \text{for } x \in \left[x_{\ell L}^{PD} \equiv \frac{t - \theta}{t + \alpha}, x_{\ell R}^{PD} \equiv \frac{\theta + \alpha}{t + \alpha} \right] \\ t(1 - 2x), & \text{for } x \leq x_{\ell L}^{PD} \equiv \frac{t - \theta}{t + \alpha} \\ 0, & \text{for } x \geq x_{\ell R}^{PD} \equiv \frac{\theta + \alpha}{t + \alpha} \end{cases} \quad (19)$$

22. Multihoming changes the platforms' incentives to unilaterally change prices and therefore it generates new insights. Choi (2006), for example, shows that, when multihoming is allowed in two-sided markets, tying can be welfare-enhancing because it induces more consumers to multihome.

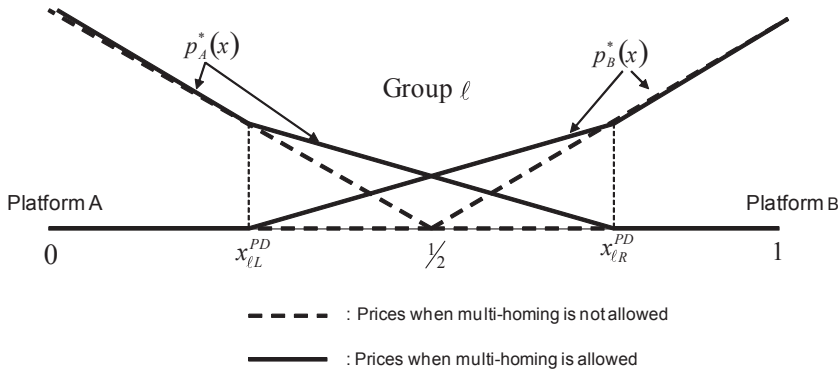


FIGURE 2. PRICE COMPARISON UNDER PERFECT DISCRIMINATION BETWEEN WHEN MULTIHOMING IS ALLOWED AND WHEN IT IS NOT

and

$$p_{\ell B}^*(x) = \begin{cases} \frac{t(\theta - t + x(t + \alpha))}{(t + \alpha)}, & \text{for } x \in \left[x_{\ell L}^{PD} \equiv \frac{t - \theta}{t + \alpha}, x_{\ell R}^{PD} \equiv \frac{\theta + \alpha}{t + \alpha} \right] \\ t(2x - 1), & \text{for } x \geq x_{\ell R}^{PD} \equiv \frac{\theta + \alpha}{t + \alpha} \\ 0, & \text{for } x \leq x_{\ell L}^{PD} \equiv \frac{t - \theta}{t + \alpha}. \end{cases}$$

$$\pi_A = \pi_B = \frac{t(t^2 - 2t\theta + 2\theta^2 + 2\alpha\theta + \alpha^2)}{(t + \alpha)^2}. \quad (20)$$

The solid lines in Figure 2 depict the equilibrium prices as given by (19). (The dashed lines on the same figure depict the equilibrium prices under perfect discrimination when multihoming is not allowed, as given by (9)). The price functions under multihoming exhibit two kinks, one at $x_{\ell L}^{PD}$ and the other at $x_{\ell R}^{PD}$. The agents that multihome are located in the interval $[x_{\ell L}^{PD}, x_{\ell R}^{PD}]$. The agents in $[0, x_{\ell L}^{PD}]$ join platform A exclusively and the agents in $[x_{\ell R}^{PD}, 1]$ join platform B exclusively. The differences between when multihoming is not allowed and when it is are the following: (i) when multihoming is allowed platforms make more sales (i.e., $x_{\ell L}^{PD} < \frac{1}{2}$ and $x_{\ell R}^{PD} > \frac{1}{2}$) and (ii) equilibrium prices are (weakly) higher when agents are allowed to multihome. In particular, the prices are the same between the two regimes for the agents who join one platform exclusively, but higher when multihoming is allowed for the agents who join both platforms. This is because under multihoming each agent is indifferent between joining one platform exclusively and joining both platforms, which softens price competition.²³

It can be readily verified that equilibrium prices (for the agents who multihome) and profits increase with α . When some agents multihome equilibrium prices are affected by the cross-group externalities. The reason is that equilibrium prices keep the agents who multihome indifferent between joining one and two platforms. In other words,

23. This can be better seen by observing that in the multihoming region the prices are falling slower (slope is equal to $-t$), as we move in the middle of the intervals, than when multihoming is not allowed (slope is equal to $-2t$). In the latter case a unilateral price cut induces agents to switch platforms (*business stealing*), whereas in the former case a similar price cut results in more agents joining both platforms (*demand creation*).

platforms in equilibrium extract all the incremental surplus from the agents who choose to multihome. Hence, the externalities are priced in the discriminatory equilibrium prices. (Recall that when multihoming is not allowed, perfect discriminatory prices are free of the externality parameters, when $c < \min\{\alpha_1, \alpha_2\}$ (case (ii) of Proposition 2), which holds here because we have assumed that $c = 0$). As the indirect externality increases the incremental benefit from joining a second platform also increases. This allows platforms to sustain higher equilibrium discriminatory prices as a function of α . On the other hand, as expected, the prices for the agents who single-home are not affected by α .

4.3 COMPARING UNIFORM PRICING AND PERFECT PD

4.3.1 PRICE AND PROFIT COMPARISON

In this comparison, for brevity, we focus implicitly on the parameter range that is common between the two parameter ranges for which (18) and (20) constitute an equilibrium. By comparing (18) with (20), it can be shown that PD is always more profitable.

This sharp prediction is very likely to be model specific. However, we believe that the effects we have identified at the end of each of the previous two subsections are likely to hold in more general models. These effects yield the following predictions as the indirect externality α increases: (i) uniform prices decrease (after the externality exceeds a given threshold) and (ii) discriminatory prices increase. Hence, PD should yield higher profits than uniform pricing at least when these externalities are strong enough. This result echoes the prediction from our benchmark model where multihoming is not allowed.

4.3.2 SOCIAL WELFARE

Due to multihoming, aggregate demand is elastic, so social welfare comparisons are meaningful. The equilibrium under perfect PD is efficient. This can be seen as follows. Given symmetry and the fact that each agent who single-homes joins the closest platform, what matters for efficiency is total output, that is, the number of agents who multihome. First, note that, in the partial multihoming case, each platform is a local monopoly, because platforms do not compete head-on for agents. Second, each platform extracts each agent's entire incremental surplus from multihoming (i.e., from joining a second platform). Therefore, the private benefit is aligned with the social benefit, which implies that social surplus is maximized under perfect PD. If the equilibrium under uniform prices differs, then we can conclude that the uniform price equilibrium is inefficient. This is indeed the case. Comparing $x_{\ell R}^{UP}$ and $x_{\ell R}^{PD}$, we can easily find that,

$$x_{\ell R}^{UP} < x_{\ell R}^{PD}, \quad (21)$$

because $t > \alpha$.

By symmetry, we can show that,

$$x_{\ell L}^{UP} < x_{\ell L}^{PD}. \quad (22)$$

This implies that more agents multihome under perfect PD than under uniform pricing. We can conclude by stating that perfect price discrimination is efficient, whereas uniform prices result in an inefficient equilibrium (less output than the first-best).

5. CONCLUSION

We examine the issue of PD in two-sided markets. We assume that there are two symmetric horizontally differentiated platforms and two groups of agents. Agents from both groups must join a platform for successful trades to take place. Platforms possess information about the agents' brand preferences which can be used to customize prices. We derive new results regarding the equilibrium discriminatory prices. When indirect externality is weak (relative to marginal cost), contrary to predictions from one-sided models, equilibrium prices are not distribution-free. Moreover, they do depend on both group externalities, as opposed to uniform prices in two-sided models which only depend on the other-group externality.

Then, we compare the profitability of PD with uniform pricing in a two-sided market. Our main result indicates that when the marginal cost is low relative to externalities perfect PD yields higher profits relative to those under uniform prices. This result is in sharp contrast with the prisoners' dilemma prediction in oligopolistic one-sided PD models.

Our results have new and clear managerial implications, regarding pricing strategies in two-sided markets. Moreover, in a two-sided market, firms may have stronger incentives to collect consumer information which allows them to price discriminate. This should happen when the marginal cost is low relative to cross-group network externalities.

APPENDIX

Proof of Proposition 1. The location of the marginal agent from group ℓ , who is indifferent between A and B , is given by,

$$V + \alpha_\ell n_{mA}^e - tx - p_{\ell A} = V + \alpha_\ell n_{mB}^e - t(1 - x) - p_{\ell B} \Rightarrow$$

$$x_\ell = \frac{\alpha_\ell (n_{mA}^e - n_{mB}^e) - p_{\ell A} + p_{\ell B} + t}{2t}, \quad (\text{A1})$$

where $n_{mA}^e = F_m(x_\ell^e)$ and $n_{mB}^e = 1 - F_m(x_\ell^e)$. Therefore, the implicit functions for the market shares are given by,

$$x_1 = \frac{\alpha_1 [2F_2(x_2^e) - 1] - (p_{1A} - p_{1B}) + t}{2t} \quad \text{and} \quad x_2 = \frac{\alpha_2 [2F_1(x_1^e) - 1] - (p_{2A} - p_{2B}) + t}{2t}.$$

As expectations are rational we must have $x_\ell = x_\ell^e$, or $n_{mk}^e = n_{mk}$. By invoking the implicit function theorem we can derive the effect of prices on the market shares,

$$\frac{\partial x_1}{\partial p_{1A}} = \frac{\partial x_2}{\partial p_{2A}} = -\frac{t}{2[t^2 - \alpha_1 \alpha_2 f_1(x_1) f_2(x_2)]}, \quad \frac{\partial x_1}{\partial p_{2A}} = -\frac{\alpha_1 f_2(x_2)}{2[t^2 - \alpha_1 \alpha_2 f_1(x_1) f_2(x_2)]} \quad (\text{A2})$$

$$\text{and} \quad \frac{\partial x_2}{\partial p_{1A}} = -\frac{\alpha_2 f_1(x_1)}{2[t^2 - \alpha_1 \alpha_2 f_1(x_1) f_2(x_2)]}.$$

For the Jacobian of the system of the implicit functions to have a nonzero determinant it must be that $t^2 - \alpha_1 \alpha_2 f_1(x_1) f_2(x_2) \neq 0$, for all x_1 and x_2 . We further assume that $t^2 - \alpha_1 \alpha_2 f_1(x_1) f_2(x_2) > 0$, for all x_1 and x_2 .

The platforms' profit functions are given by,

$$\pi_A = (p_{1A} - c)n_{1A} + (p_{2A} - c)n_{2A} = (p_{1A} - c)F_1(x_1) + (p_{2A} - c)F_2(x_2) \text{ and}$$

$$\pi_B = (p_{1B} - c)n_{1B} + (p_{2B} - c)n_{2B} = (p_{1B} - c)[1 - F_1(x_1)] + (p_{2B} - c)[1 - F_2(x_2)].$$

The first order conditions of platform A are given by,

$$\frac{\partial \pi_A}{\partial p_{1A}} = F_1(x_1) + (p_{1A} - c)f_1(x_1)\frac{\partial x_1}{\partial p_{1A}} + (p_{2A} - c)f_2(x_2)\frac{\partial x_2}{\partial p_{1A}} = 0,$$

$$\frac{\partial \pi_A}{\partial p_{2A}} = F_2(x_2) + (p_{2A} - c)f_2(x_2)\frac{\partial x_2}{\partial p_{2A}} + (p_{1A} - c)f_1(x_1)\frac{\partial x_1}{\partial p_{2A}} = 0.$$

Each first order condition has three terms. Suppose platform A lowers its price to group ℓ agents. The first two terms in each first order condition capture the reduction in inframarginal rents and the increase in marginal agents, respectively. As more agents from group ℓ join platform k , platform k becomes more attractive to the members of group m . The third term represents the additional revenue from the increase in the number of agents from group m that join platform k .

We look for a symmetric equilibrium where platforms charge the same prices to each group. We assume that regularity conditions hold so that a symmetric sharing equilibrium exists.²⁴ Using (A2), the symmetric solution to the system of first order conditions is given by,

$$p_{1A}^* = p_{1B}^* = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)} + c \quad \text{and} \quad p_{2A}^* = p_{2B}^* = \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)} + c.$$

The equilibrium profits are,

$$\pi_A = \pi_B = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{2f_1\left(\frac{1}{2}\right)} + \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{2f_2\left(\frac{1}{2}\right)}.$$

□

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24. We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform, then the profit functions are strictly concave provided that $2t > (\alpha_1 + \alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

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