Innovation and Price Competition in a Two-Sided Market

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ABSTRACT: We examine a platform's optimal two-sided pricing strategy while considering seller-side innovation decisions and price competition. We model the innovation race among sellers in both finite and infinite horizons. In the finite case, we analytically show that the platform's optimal seller-side access fee fully extracts the sellers' surplus, and that the optimal buyer-side access fee mitigates price competition among sellers. The platform's optimal strategy may be to charge or subsidize buyers depending on the degree of variation in the buyers' willingness to pay for quality; this optimal strategy induces full participation on both sides. Furthermore, a wider quality gap among sellers' products lowers the optimal buyer-side fee but leads to a higher optimal seller-side fee. In the infinite innovation race, we perform computations to find the stationary Markov equilibrium of sellers' innovation rate. Our results show

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that when all sellers innovate, there exists a parameterization under which a higher seller-side access fee stimulates innovation.

KEY WORDS AND PHRASES: innovation, price competition, two-sided markets.

IN RECENT YEARS, ELECTRONIC MARKETS HAVE SUCCESSFULLY ADOPTED two-sided business models. Mobile commerce is experiencing unprecedented growth. Apple sold 47.5 million iPhones in 2010, due in part to an ever-expanding Apple's App Store [20]. Google's Android Market experienced a sixfold increase in the number of applications since the end of 2009 [8] and currently accounts for 25.5 percent of worldwide smartphone sales [17]. As a result of rapidly expanding mobile platforms, Internet traffic in the mobile industry has increased four to six times since 2008 [1]. These growth statistics suggest that mobile platforms are creating vital application markets for both users and application developers.[1]

Existing studies of two-sided platforms are based primarily on cross-side network externalities [5, 6, 7, 27]. They analyze the platform's pricing and governance issues, given that users on each side of the market benefit from the number of users on the other side and that demand is the driving force for user participation. In a buyer–seller platform, interactions between the two sides exhibit rich characteristics beyond pure network externalities. Even though network size may be the platform's strategic focus before the number of platform users reaches a critical mass, other factors, such as innovation and price competition in the platform market, are increasingly important for platform growth. Furthermore, increasing network size may have complex effects on the innovation incentives in the platform market [10]. In this paper, we endogenize the cross-side externality by explicitly modeling seller-side price competition; we also incorporate sellers' innovation decisions and analyze the platform's optimal two-sided pricing strategy.

On a platform, sellers' pricing decisions and buyers' purchase decisions ultimately determine both sides' benefits from the platform. Apple's App Store offers a wide variety of applications. Within each application category, developers (sellers) compete in quality, creativity, and price. Application users (buyers) have access to application ratings, rankings, and often free trials to assess quality before making purchase decisions. Our study focuses on any one category of products by examining the price competition in a vertically differentiated platform market. The equilibrium outcome yields the sellers' revenues and the buyers' values in the platform market.

To the best of our knowledge, past studies on two-sided platforms typically take product development as given. However, platform growth is fundamentally driven by continuous innovation that supplies products to the platform market. An intrinsic characteristic of information technology (IT) platforms is that products go through finite cycles in the market. Sellers compete aggressively for a chance to serve a sizable market and innovate intensively to sustain or recapture market share [4]. The sellers' innovation intensity then determines the rate of market turnover. For instance, Google's

Android Market became active much later than Apple's App Store, but it has been growing at a much faster pace as a result of application innovations.

By setting access fees on both sides of the market, the platform owner plays a direct role in jointly regulating sellers' innovation incentives and buyers' participation and purchase decisions. A proprietary software platform (e.g., Windows) typically charges the buyer a licensing fee. When the platform owner integrates the platform hardware and software, it imposes a cost on the buyer for purchasing the hardware with the software preinstalled (e.g., iPad). A typical proprietary platform owner, such as Apple, also imposes a fee and certain hardware requirements on the seller side for using the software development kit necessary to develop and test new apps.[2] Based on this up-front cost and other research and development (R&D) expenses, the seller evaluates potential payoffs of innovation and considers the chance of success when facing competition from the other sellers.

In this paper, we address questions that arise from introducing seller-side price competition and innovation into the platform's decisions in setting two-sided access fees: How do two-sided access fees affect price competition in the platform market? What is the platform's optimal strategy in setting two-sided access fees? And when sellers engage in continuous innovation, how do the platform access fees affect innovation incentives?

The equilibrium characterization of price competition in the platform market reveals several new insights into the effect of platform access fees on buyers' participation decisions. We find that increasing the buyer-side access fee up to a certain threshold does not exclude any buyers. Given a buyer-side fee below this threshold, price competition among sellers drives down the prices in the platform market, which induces all buyers to join the platform in equilibrium. For the buyer-side access fee above the threshold, sellers have an incentive to subsidize buyers, which may still lead to buyers' full participation in equilibrium.

Based on the finite-horizon setting of the sellers' innovation problem, we find that the platform sets the optimal seller-side access fee to fully extract the sellers' surplus and sets the optimal buyer-side access fee to moderate price competition among sellers and to disincentivize seller subsidization of the buyers' fee. Interestingly, subsidizing buyers may be optimal for the platform if the buyers' willingness to pay for quality is sufficiently dispersed; otherwise, the platform charges a positive optimal buyer-side fee. In both cases, the platform's optimal pricing strategy induces full participation by both sides of the platform. Furthermore, as the quality gap in the platform market widens, the optimal seller-side access fee increases because of higher revenues in the platform market, whereas the optimal buyer-side access fee decreases, leading to buyers' full participation.

Our results in the infinite case indicate that when all sellers innovate, a parameterization exists under which a higher seller-side access fee stimulates innovation. This is because as innovation failure becomes more costly, sellers balance their expected payoffs by innovating more intensively for a greater chance of success. However, when the seller-side access fee exceeds a certain threshold, sellers do not perceive the platform market to be profitable and do not innovate. We find that the buyer-side access fee has a different effect. A higher buyer-side access fee reduces buyers' willingness to pay and induces the sellers to subsidize buyers in equilibrium. Therefore, increasing the buyer-side fee lowers the sellers' equilibrium innovation rate.

The rest of the paper is organized as follows. Next, we review the related literature. The third section provides an overview of the model. We then formulate seller-side price competition and characterize the equilibrium of the platform market in the fourth section. In the fifth section, we analyze the platform's optimal pricing strategy given sellers' innovation problem in a finite horizon and then illustrate the effects of the platform's fees when sellers engage in an infinite dynamic innovation race. Finally, we conclude in the last section.

Literature Review

A STREAM OF RESEARCH STUDIES TWO-SIDED PRICING in various types of platform markets. Rochet and Tirole [22] were the first to analyze system competition in the context of a payment card association. They examine the set of rules governing the transactions in the payment cards industry, such as cooperative determination of the *interchange fee,* the *honor-all-cards rule,* and the *no-surcharge rule* [22]. They further study (e.g., [23]) competing platforms' two-sided price allocation problems under different governance structures (for profit and not for profit) while taking into account other factors, such as user multihoming. Caillaud and Jullien [11, 12] examine the multihoming factor in platform pricing, but their emphasis is on platforms with indirect network externality such as those in matchmaking. They introduce a more complex fee structure. Platforms can impose two types of fees, the ex ante registration fee and the ex post transaction fee; the latter may allow competing platforms to differentiate given the possibility of multihoming [12].

Armstrong [5] focuses on two-sided markets that have cross-side network externality and explores platform pricing strategies based on such externality, on different types of fees, and on multihoming. He finds that the side of the platform that provides substantial benefit (or exerts substantial network externality) to the other side enjoys a price subsidized by the platform; however, a per transaction fee structure mitigates such price cuts in a competitive setting [5]. Moreover, when one side of the market multihomes (the other side singlehomes), the platform generates high profits from the multihoming side but subsidizes the singlehoming side [5]. Armstrong and Wright further study a competitive setting where only one side perceives differentiation among competing platforms [6].

In the context of online intermediaries, Yoo et al. [27] study a two-sided pricing problem of a business-to-business (B2B) market owner. Their findings on the effect of network externality on the optimal prices are consistent with those suggested by Armstrong [5]; Yoo et al. also reveal additional insights into the roles of negative network effects and switching costs in platform pricing. Galbreth et al. [16] endogenize participation of the two sides of a platform market and find the equilibrium to predict the growth of the market. Bakos and Katsamakas [7] examine the platform's design decisions, in which network externality is endogenized under independent, owned-by-one-side, and spin-off ownerships. They find that the platform's optimal design decision often involves an asymmetric design and is independent of the platform ownership structure [7]. Most of these frameworks use a static model (with the exception of Galbreth et al. [16], which focuses on market growth rather than pricing) and assume a certain utility form that is often symmetric for the two sides of the market. Differing from this traditional framework, Economides and Katsamakas [15] model two-sided pricing of an operating system platform and incorporate price competition among the applications. Their focus is to compare industries of a proprietary platform (e.g., Windows) and an open source platform (e.g., Linux) [15]. They find that users' strong preference for application variety may lead to higher total profits in a proprietary industry compared to those in an industry based on an open source platform [15]. They also show that the proprietary platform industry may dominate the open source platform industry in market share [15].

Departing from purely static analysis in which two sides of the market make decisions simultaneously, Hagiu [19] explores the platform's price commitment strategy when one side arrives first. He finds that the platform's commitment may not lead to optimal pricing. The reason is that without committing to charging the late-arrival side a low price, the platform could still get the early arrival side on board, which would allow the platform to charge the late-arrival side a higher price [19].

We contribute to the literature on two-sided markets by explicitly modeling transactions between two sides and characterizing the market equilibrium. Moreover, we consider sellers' innovation problem prior to their market participation. Discussion of innovation in the context of two-sided markets is currently limited, as most research continues in the direction of pure network externality. An empirical study by Boudreau [10] is one of the few recent studies to examine the effects of innovation in the platform market. By analyzing a data set on the applications for handheld computers and PDAs (personal digital assistants), Boudreau identifies a negative effect of increasing developer network size on innovation incentives in a two-sided platform [10]. We incorporate dynamic innovation by sellers into the interactions between the platform and the two sides of the market and discuss the effect of the platform's fees on sellers' innovation incentives.

There is a rich literature in economics on innovation incentives, policies, and economic growth. The innovation race carried by one side of the platform described in our work rests on the set of theories and assumptions fundamental to this stream of literature. Grossman and Helpman [18] investigate the interaction between innovation and imitation in a general equilibrium framework and identify the steady-state equilibrium and its response to various policies. Aghion et al. [2] identify a complementary relationship between antitrust and patent policies, and find that (1) product market competition leads to growth by incentivizing firms to innovate to escape competition, and (2) some neck-and-neck competition resulting from imitation promotes growth. Segal and Whinston [24] focus on a number of antitrust policies and conclude that policies that protect new market entrants raise the innovation rate. Our work differentiated from these studies by incorporating the interactions between two sides of the market and by discussing innovation in the context of the platform's pricing decisions. We adopt model setup of price competition among vertically differentiated sellers from Shaked and Sutton [25] and introduce additional factors related to two-sided prices. Furthermore, we discuss our findings in the context of the platform's pricing decisions.

The Model

Consider a setting where the platform sets lump-sum access fees on two sides of the market: f for the seller side and p_b for the buyer side. Suppose the platform generates revenues from these fees collected from both sides.[3] Observing such fees, both sides make their decisions.

Sellers make decisions in two stages. They first choose whether to innovate and the contingent innovation rate, which determines the probability of successful invention and market entry. Sellers also choose the equilibrium prices in competition with the other market incumbents contingent on market entry. To distinguish between the innovating sellers and those that successfully enter the platform market and serve buyers, we use the term *incumbent* for the latter type of seller. We first derive the platform's optimal two-sided fees in the finite case, where sellers make the decision to enter the innovation race only once (i.e., there is no reentry). We then present the dynamic case, in which sellers engage in continuous innovation in the infinite horizon, and we illustrate numerically the effect of the platform fees on sellers' equilibrium innovation rate. For both formulations, we assume that the innovation is superior to the existing goods in the market; thus, a seller is positioned as the top-quality incumbent upon market entry but may be demoted to a lower-quality position as the innovation race among the other sellers continues. Figure 1 shows an overview of the interactions between the platform and participants from two sides of the market.

In the platform market, competing incumbents are vertically differentiated and set prices based on buyers' preferences. In each period, buyers make decisions on whether to join the platform, and contingently they make the purchase decision after observing the buyer-side access fee, p_b, set by the platform and the prices set by the incumbents. Buyers incur p_b only if they choose to purchase from any incumbent; otherwise, they resort to an outside option and do not join the platform. The equilibrium characterization of the platform market explicitly captures the transactions between the two sides while taking into account quality differentiation, price competition, and buyer heterogeneity.

Using backward induction, we first analyze the equilibrium of the platform market while taking p_b and f as given. Based on the equilibrium characterization, we then find the platform's optimal pricing strategies given sellers' innovation problem in the finite horizon. Finally, we incorporate dynamic features into sellers' innovation decisions and present the numerical findings.

The Platform Market

Consider a market of products that differ in quality. Denote by $k = 1, \ldots, n$ the index for product quality, where a higher k represents a higher quality. Let buyers be

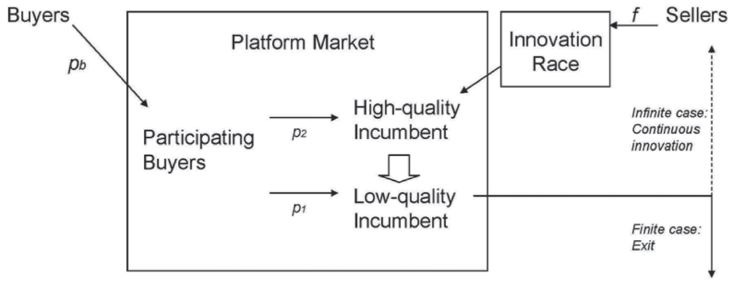

Figure 1. An Overview of the Two-Sided Platform

heterogeneous in their willingness to pay for quality in the platform market. Denote a buyer's marginal willingness to pay by the random variable z, which follows the uniform distribution: $z \sim U[\underline{z}, \bar{z}]$. Following the approach in Shaked and Sutton [25], we start with a general setting of n qualities and then impose a constraint on the bounds of buyers' willingness to pay. This condition will lead to a two-product market and allows for analytical tractability in deriving the equilibrium.

Observing the buyer-side access fee p_b, buyers maximize the following utility function:

$$U\left(z, k, p_b\right) = u_k \cdot z - p_b, \tag{1}$$

where, u_k is the value offered by a product of quality k, such that $u_0 < u_1 < \ldots < u_n$ (i.e., higher quality provides higher value). A buyer's utility from consuming a quality k good is the value of the good weighted by his or her willingness to pay for quality. One may interpret u_0 as the value of an outside option, to which buyers resort if they choose not to transact on the platform market. For notational simplicity, we normalize u_0 to 0, while our main findings still hold for other values of u_0.

A buyer incurs the buyer-side access fee p_b only if he or she chooses to transact on the platform, in which case the buyer also pays the price of the product purchased. Let z_k be the indifference level, such that the buyer with the willingness to pay, z_k, is indifferent between products k and $k - 1$ at their respective prices. Thus,

$$U\left(z_k - p_k, k, p_b\right) = U\left(z_k - p_{k-1}, k-1, p_b\right). \tag{2}$$

Based on Equation (1), buyers with willingness to pay above z_k would prefer the quality k good at p_k over the quality $k - 1$ good at p_{k-1}. In other words, buyers with taste $z > z_k$ have the preference order $(k, p_k) \quad (k - 1, p_{k-1})$. Assume $p_k \geq 0$ for all k. Furthermore, because buyers do not pay p_b when resorting to the outside option, $U(z, 0, p_b) = u_0 z = 0$.

From Equation (2), we can then derive the following indifference levels:

$$z_1 = p_1 + \frac{p_b}{u_1}, \tag{3}$$

$$z_k = \frac{-u_{k-1} p_{k-1}}{u_k - u_{k-1}} + \frac{u_k p_k}{u_k - u_{k-1}}. \tag{4}$$

We focus on sellers' innovation costs (e.g., f and $\varepsilon c(\phi)$ in the model of seller innovation and assume zero marginal production cost for those that become incumbents. Let incumbents compete in price. Their revenue functions are

$$R_1\left(p_1,\ldots,p_n\right)=p_1\left(z_2-\underline{z}\right),\ \text{if } z_1\le\underline{z} \tag{5}$$

$$R_1\left(p_1,\ldots,p_n\right)=p_1\left(z_2-\underline{z}\right),\ \text{if } z_1\ge\underline{z} \tag{6}$$

$$R_k\left(p_1,\ldots,p_n\right)=p_k\left(z_{k+1}-z_k\right),\ \text{for } 1<k<n \tag{7}$$

$$R_n\left(p_1,\ldots,p_n\right)=p_n\left(\overline{z}-z_n\right). \tag{8}$$

When p_1 is sufficiently high, buyers with lower willingness to pay may not purchase even the lowest-quality good and, in turn, do not join the platform, in which case the market is not covered and the indifference level z_1 lies above the lower bound of buyers' willingness to pay. Equations (5) and (6) correspond to the cases where the lowest-quality incumbent does and does not cover the market, respectively.

Lemma 1: Let $\overline{z} < 4\underline{z}$, then in any Nash equilibrium, at most two incumbents (of quality n and $n-1$) obtain positive market shares.

Proof: See the Appendix.

When the range of buyers' willingness to pay satisfies the condition indicated in Lemma 1, the incumbents of the top two qualities engage in an intense competition resulting in equilibrium prices that are sufficiently low, such that all buyers choose to purchase from one of the two products on the platform. Adopted from Shaked and Sutton [25], this assumption allows for a tractable equilibrium characterization. In the other cases, three or more incumbents may serve the market, and the equilibrium would require complex conditions without adding significant insights.

Two-Quality Market Equilibrium

Based on Lemma 1, the following analysis rests on a two-quality market with high- and low-quality incumbents. Given the buyer-side access fee, p_b, set by the platform, incumbents and buyers make their pricing and purchase decisions, respectively.

Applying Equations (3) and (4), we have

$$p_1=z_1\frac{p_b}{u_1} \tag{9}$$

$$p_2=\frac{1}{u_2}\left[\left(u_2-u_1\right)z_2+u_1z_1-p_b\right]. \tag{10}$$

Equations (9) and (10) express the high- and low-quality incumbents' prices in z_1 and z_2, which are the indifference levels of buyers' willingness to pay, and they allow us to characterize the equilibrium market segmentation. The existence of equilibrium requires an additional condition that specifies the relationship between the bounds of buyers' willingness to pay:

Table 1. Boundary Conditions for Equilibrium Regions I, II, and III

Region	Range of buyer-side access fee
Region I	$p_b \leq \frac{1}{3}\left[(2u_2+u_1)\underline{z}-(u_2-u_1)\overline{z}\right]$
Region II	$\frac{1}{3}\left[(2u_2+u_1)\underline{z}-(u_2-u_1)\overline{z}\right] \leq p_b \leq \frac{u_1}{2u_2+u_1}\left[(4u_2-u_1)\underline{z}-(u_2-u_1)\overline{z}\right]$
Region III	$\frac{u_1}{2u_2+u_1}\left[(4u_2-u_1)\underline{z}-(u_2-u_1)\overline{z}\right] \leq p_b \leq \frac{u_1}{2}\overline{z}$

Table 2. Equilibrium Characterization for the Two-Quality Market

Region	Equilibrium prices	Equilibrium revenues
I	$p_1^* = \frac{u_2-u_1}{3u_1}(\overline{z}-2\underline{z})$	$R_1^* = \frac{u_2-u_1}{u_1}\left(\frac{\overline{z}-2\underline{z}}{3}\right)^2$
	$p_2^* = \frac{u_2-u_1}{3u_2}(2\overline{z}-\underline{z})$	$R_2^* = \frac{u_2-u_1}{u_2}\left(\frac{2\overline{z}-\underline{z}}{3}\right)^2$
II	$p_1^* = \frac{1}{u_1}\left[u_1\underline{z}-p_b\right]$	$R_1^* = \frac{p_1^*}{2}\left[\overline{z}-2\underline{z}-\frac{1}{u_2-u_1}\left[u_1\underline{z}-p_b\right]\right]$
	$p_2^* = \frac{1}{2u_2}\left[(u_2-u_1)\overline{z}+u_1\underline{z}-p_b\right]$	$R_2^* = \frac{u_2-u_1}{4u_2}\left[\overline{z}+\frac{1}{u_2-u_1}\left[u_1\underline{z}-p_b\right]\right]^2$
III	$p_1^* = \frac{u_2-u_1}{4u_2-u_1}\left[\overline{z}-\frac{2p_b}{u_1}\right]$	$R_1^* = \frac{u_2(u_2-u_1)}{(4u_2-u_1)^2}\left[\overline{z}-\frac{2p_b}{u_1}\right]^2$
	$p_2^* = \frac{u_2-u_1}{4u_2-u_1}\left[2\overline{z}-\frac{p_b}{u_2}\right]$	$R_2^* = \frac{u_2-u_1}{u_2(4u_2-u_1)^2}\left[2u_2\overline{z}-p_b\right]^2$

Proposition 1: If $2\underline{z} < \overline{z} < 4\underline{z}$, then a unique equilibrium exists. Moreover, the equilibrium lies in one of the three regions based on the conditions specified in Table 1. The equilibrium characterization is shown in Table 2.

Proof: See the Appendix.

The buyer-side access fee set by the platform results in one of three equilibrium regions (shown in Table 1) and affects the equilibrium pricing strategies of the competing incumbents. The buyer-side access fee and the incumbents' equilibrium prices then jointly determine the buyers' decisions of platform participation. In Region I, the buyer-side access fee is sufficiently low so that the platform has a high potential appeal, even to the buyers with a low willingness to pay for quality. As a result, the competition drives down incumbents' prices such that, in equilibrium, buyers with the lowest willingness to pay strictly prefer to join the platform and purchase from the low-quality incumbent. In Region II, the buyer-side access fee is above such a threshold. As buyers are faced with a higher barrier to join the platform, the force of market competition is mitigated—in equilibrium, the low-quality incumbent sets the price such that buyers with the lowest willingness to pay are indifferent between joining the platform and resorting to the outside option. As the buyer-side access fee further increases and leads to the Region III equilibrium, competing incumbents strategize to exclude buyers with a low willingness to pay. Thus, the market is not covered.

Figure 2 indicates three equilibrium regions based on the relationship between the low-quality level (u_1) and the quality gap ($u_2 - u_1$), given the buyer-side access fee p_b. When the quality gap is narrow, the equilibrium falls in Region I for higher values of u_1. In this case, price competition among incumbents is fierce because of a low differentiation and buyers' strong inclination to purchase. However, at a lower u_1, the low-quality good offers lower utility to buyers and dampens their purchasing incentives; thus, the equilibrium switches to Region II (and then to Region III for even lower values of u_1), as incumbents compete less aggressively on price. With a wider quality gap, the equilibrium falls in Region II for higher values of u_1, where price competition is mitigated. For lower values of u_1, buyers are less inclined to purchase; therefore, we may have a Region III equilibrium, in which the market is not covered.

Lemma 2: When $p_b \geq 1/3[(2u_2 + u_1)\underline{z} - (u_2 - u_1)\overline{z}]$ *(Region II or Region III), both incumbents' equilibrium prices and revenues decrease in the buyer-side access fee.*

Proof: See the Appendix.

This result implies that incumbents may have an incentive to subsidize buyers when the buyer-side access fee imposed by the platform is sufficiently high. Furthermore, such subsidizing by both incumbents—which occurs in equilibrium—leads to lower incumbent revenues as the buyer-side access fee increases. Our findings reveal two new insights into the externality of the buyer-side access fee under the forces of price competition.

The first key insight is that the buyer-side access fee may exert a negative cross-side externality on the seller-side equilibrium revenues without having any direct effect on the buyer-side network size. In particular, all buyers join the platform in Region II as a result of incumbents' strategy to subsidize buyers in equilibrium. Within the bounds of this region, the platform can increase the buyer-side access fee without trading off any demand.

The second notable finding is that this negative cross-side externality is absent for lower values of the buyer-side fee. Illustrated in the Region I equilibrium, price competition among incumbents dominates any effect from the buyer-side access fee; thus, incumbents' equilibrium revenues in this region are independent of the buyer-side access fee, and the market is covered. Therefore, the platform can set the buyer-side access fee up to a certain level without trading off any demand or taxing the incumbents

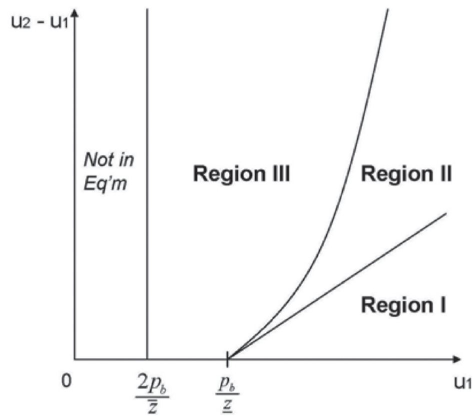

Figure 2. The Equilibrium Regions in $(u_1, u_2 - u_1)$-Space

indirectly, and it is optimal to price the buyer-side access fee at least at this level. (The next section presents the analysis of the platform's optimal pricing strategy.)

In Region III, competing incumbents subsidize buyers nevertheless, as the buyer-side access fee increases. However, in this region, this subsidizing is not sufficient to bring all buyers on board. Therefore, in this case, increasing the buyer-side access fee reduces the buyer-side network size as well as incumbents' revenues.

Seller Innovation and Platform Pricing

In this section, we set up the sellers' innovation decision problem and discuss the finite and infinite cases separately. In the finite case, sellers face a two-period horizon when choosing the innovation rate. In this setup, we analytically derive the platform's optimal buyer-side and seller-side access fees. In the infinite case, we incorporate dynamic features into the innovation race where sellers' innovation decisions also take into account the prospects of both exiting the market (because of continuing innovation by the other sellers) and reentering the innovation race. We present the numerical results showing the effect of the platform's two-sided fees on the sellers' equilibrium innovation rate. We first introduce the notations that are common to both cases.

Let there be M homogeneous sellers that compete in the R&D race. Their variable R&D cost is described by $\varepsilon c(\phi_i)$, where ε is a constant parameter. Following the assumption in Segal and Whinston [24], suppose the cost function is convex in the form $c(\phi_i) = \phi_i^2$ so that innovation efforts become increasingly costly. And let $\phi_i \in [0, 1]$ denote seller i's innovation rate as well as its probability of producing a successful innovation. Multiple sellers may succeed in developing a new product. However, suppose one out of all the sellers that successfully innovate is rewarded with market entry at random, which follows the assumption in the literature that the success of innovation that leads to market dominance over existing products often depends on serendipitous factors [10, 14, 18, 24].

Given homogeneity among sellers, we are interested in a symmetric equilibrium, where all sellers choose the identical innovation rate. Let $\pi(\phi)$ denote the probability that *at least* one seller successfully develops a new product when M sellers innovate at the rate of $\phi \in [0,1]$:

$$\pi(\phi) = 1 - (1-\phi)^M . \tag{11}$$

For any one seller, the probability of market entry conditional on successful innovation is denoted by $r_M(\phi)$, where $\phi \in [0,1]$ is the innovation effort of $M - 1$ competing sellers:

$$r_M(\phi) = \sum_{k=0}^{M-1} \left(\frac{1}{k+1}\right)\binom{M-1}{k} \phi^k (1-\phi)^{M-1-k} = \frac{1-(1-\phi)^M}{\phi \cdot M}. \tag{12}$$

This seller's probability of market entry, $\lambda(\phi', \phi)$, is then the product of the probability of successful innovation, ϕ', and the conditional probability, $r_M(\phi)$: $\lambda(\phi', \phi) = \phi' r_M(\phi)$.

Finite Innovation Horizon

Consider a two-period model where M sellers make innovation decisions in each period. If a seller enters the market, in period 1, it will become the high-quality incumbent and generate the equilibrium revenue, R_2^*, which is derived in Table 2. In period 2, this seller (which has become the high-quality incumbent) will be demoted to the low-quality position and receive the revenue, R_1^*, as long as one successful innovation is produced in this period (i.e., at least one other seller enters the market and becomes the new high-quality incumbent); we assume that in the case of no successful innovation in period 2, the high-quality incumbent remains in its dominant position. When a seller does not enter the market, it obtains zero revenue. However, a seller incurs the seller-side access fee f if it decides to innovate, regardless of the outcome of innovation. When a seller chooses not to innovate, it receives nothing. Suppose a seller innovates when it is indifferent between innovating and not innovating. Let the discount factor $\beta = 1$. Each seller solves the following set of problems, given the other sellers' innovation rate ϕ:

$$\max\left\{0, \max_{\phi' \in [0,1]} \lambda(\phi', \phi) V_2(\phi) - \varepsilon c(\phi') - f\right\} \tag{13}$$

$$V_2(\phi) = R_2^* + \pi(\phi) V_1(\phi) + (1 - \pi(\phi)) V_2(\phi) \tag{14}$$

$$V_1(\phi) = R_1^* + (1 - \pi(\phi)) V_1(\phi). \tag{15}$$

$V_2(\phi)$ and $V_1(\phi)$ are the value functions for the high-quality and low-quality incumbents, respectively. The expected values of the incumbents also depend on $\pi(\phi)$, the probability of at least one seller's successfully developing a new product,[4] in which case market entry is guaranteed. By rearranging Equations (14) and (15), $V_1(\phi) = R_1^*/\pi(\phi)$ and $V_2(\phi) = (R_2^* + R_1^*)/\pi(\phi)$. Thus, the seller's problem can be reduced to

$$\max\left\{0, \max_{\phi'\in[0,1]} \frac{\phi'\left(R_2^* + R_1^*\right)}{M\phi} - \varepsilon c\left(\phi'\right) - f\right\}. \tag{16}$$

Proposition 2: If $f \leq (R_2^* + R_1^*)/2M$, the sellers' equilibrium innovation rate is $f^* = ((R_2^* + R_1^*)/2\varepsilon M)^{1/2}$; otherwise, no seller will participate in the innovation race.

Proof: See the Appendix.

Based on Proposition 2, the number of sellers joining the platform by paying the access fee and engaging in innovation can be denoted by the following function of f and p_b:

$$\Omega\left(f, p_b\right) = \begin{cases} M & \text{if } f \leq \dfrac{R_2^* + R_1^*}{2M} \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

Similarly, denote by $\Psi(f, p_b)$ the fraction of buyers who participate in the platform market:

$$\Psi\left(f, p_b\right) = \begin{cases} 1 & \text{if } p_b \leq \dfrac{u_1}{2u_2 + u_1}\left[\left(4u_2 - u_1\right)\underline{z} - \left(u_2 - u_1\right)\overline{z}\right] \\ \dfrac{\overline{z} - z_1^*}{\overline{z} - \underline{z}} & \text{otherwise.} \end{cases} \tag{18}$$

Recall that

$$p_b \leq \frac{u_1}{2u_2 + u_1}\left[\left(4u_2 - u_1\right)\underline{z} - \left(u_2 - u_1\right)\overline{z}\right]$$

implies that the market equilibrium is in Region I or Region II, where the market is covered. In Region III, the market is not covered, and

$$z_1^* = \frac{1}{4u_2 - u_1}\left[\left(u_2 - u_1\right)\overline{z} + \frac{2u_2 + u_1}{u_1} p_b\right]$$

(see the proof for Proposition 1 in the Appendix) is the willingness to pay of the buyer who is indifferent between not participating and consuming the low-quality product on the platform in equilibrium. By normalizing the size of the buyer market to 1, the profit-maximizing platform chooses f and p_b to solve

$$\max_{(f, p_b)\in R^2} \Pi\left(f, p_b\right) = f\Omega\left(f, p_b\right) + p_b \Psi\left(f, p_b\right). \tag{19}$$

In a finite-horizon problem, the seller-side access fee affects the sellers' participation decision but not their equilibrium innovation rate. Therefore, the platform clearly would set f at the maximum value $(R_2^* + R_1^*)/2M$ to ensure participation while fully extracting the sellers' surplus. We can then express the platform's revenue in terms of p_b only:

$$\Pi\left(p_b\right)=\begin{cases} p_b+\dfrac{R_2^*+R_1^*}{2} & \text{if } p_b \le \dfrac{u_1}{2u_2+u_1}\left[\left(4u_2-u_1\right)\underline{z}-\left(u_2-u_1\right)\overline{z}\right] \\ \dfrac{\overline{z}-z_1^*}{\overline{z}-\underline{z}}p_b+\dfrac{R_2^*+R_1^*}{2} & \text{otherwise.} \end{cases} \quad (20)$$

Lemma 3: The platform's revenue function is increasing in p_b in Region I. If $\overline{z}-\underline{z} \ge (u_2(4u_2-u_1))/(u_2-u_1)$, the platform's revenue function is decreasing in p_b in both Region II and Region III.

Proof: See the Appendix.

In Region I, the buyer-side access fee exerts no externality on the competition in the platform market; thus, the platform faces no trade-off from increasing the buyer-side fee. However, in Region II, given that some buyers have sufficiently high willingness to pay for quality, the effect that increasing the buyer-side access fee has on cutting the incumbents' revenues and in turn on reducing the platform's revenues outweighs the platform's direct gains from the buyers' fees. Furthermore, given a sufficiently high degree of variation in buyers' willingness to pay, the platform suffers more losses than gains by increasing the buyer-side fee in Region III, where increasing the buyer-side fee reduces not only the incumbents' equilibrium revenues but also the buyer-side demand.

Proposition 3: If $\overline{z}-\underline{z} \ge (u_2(4u_2-u_1))/(u_2-u_1)$, the platform sets the optimal buyer-side access fee at $p_b^* = 1/3[(2u_2+u_1)\underline{z}-(u_2-u_1)\overline{z}]$ and the optimal seller-side access fee at

$$f^*=\frac{\left(u_2-u_1\right)}{18M}\left[\frac{\left(\overline{z}-2\underline{z}\right)^2}{u_1}+\frac{\left(2\overline{z}-\underline{z}\right)^2}{u_2}\right].$$

Full participation by both sides of the market is achieved.

Proof: See the Appendix.

The condition $\overline{z}-\underline{z} \ge (u_2(4u_2-u_1))/(u_2-u_1)$ ensures concavity in the platform's revenue function and leads to an interior solution of the optimal buyer-side access fee.[5] Given a symmetric equilibrium of innovation decisions among homogeneous sellers, it is clear that the platform would capture all sellers to ensure transactions in the market. Based on the platform's optimal pricing, price competition among the incumbents is moderated. At the low-quality incumbent's equilibrium price, the buyer with the lowest willingness to pay is indifferent between participating on the platform and choosing the outside option. In other words, the platform's optimal pricing strategy leads to the highest possible incumbent prices while covering the buyer market in equilibrium. Moreover, the platform's optimal buyer-side fee disincentivizes seller subsidization of the buyer-side fee.

Proposition 4: If $\underline{z} \ge ((u_2-u_1)\overline{z})/(2u_2+u_1)$ (which holds given $\overline{z} \ge (u_2(4u_2-u_1)(2u_2+u_1))/((u_2-u_1)(u_2+2u_1))$), the platform's optimal buyer-side access fee *is positive. If* $\underline{z} < ((u_2 - u_1)\overline{z})/(2u_2 + u_1)$ *(which holds for* $5u_1 < 2u_2$*), the platform subsidizes the buyers by setting a negative optimal buyer-side fee.*

Proof: See the Appendix.

The platform's optimal strategy may be to charge or subsidize buyers depending on their willingness to pay. The relationship between the upper and lower bounds of the buyers' willingness to pay in Proposition 4 indicates the degree of variation in their preferences for quality. The more dispersed these preferences are, the higher the incumbents' equilibrium revenues; thus, when the variation of these preferences is sufficiently high (i.e., $\underline{z} < ((u_2 - u_1)\overline{z})/(2u_2 + u_1)$), the platform has an incentive to subsidize buyers to induce their full participation and in turn extract higher revenues from the seller-side of the market. Moreover, given such dispersion, incumbents are less inclined to serve buyers that have lower willingness to pay. Therefore, platform subsidization of buyer-side fees is necessary to cover the market and to achieve the maximum platform revenue. Following the same intuition, given a lower variation of buyers' willingness to pay for quality, the competition among incumbents is sufficient to result in a market-covered equilibrium; thus, the platform's optimal strategy is to set a positive buyer-side access fee.

We now consider the effect of quality gap on the platform's optimal pricing strategy by studying the comparative statics of the optimal fees with respect to buyers' valuation of the high-quality good, u_2. Increasing u_2 while fixing buyers' valuation for the low-quality good, u_1, effectively increases quality differentiation.

Proposition 5: The platform's optimal buyer-side access fee is decreasing in the product quality gap, whereas the optimal seller-side access fee is increasing in the product quality gap.

Proof: See the Appendix.

The product quality gap in the platform market in part determines the platform's optimal allocation of entry costs between the two sides of the market. First, intuitively, a wider quality gap mitigates competition and raises the incumbents' equilibrium revenues. The platform would then extract a higher surplus from the sellers by increasing the seller-side access fee. However, without also adjusting the buyer-side access fee downward to the optimal level, the platform would induce a market equilibrium where the incumbents strategically subsidize the buyers for the access fee. Thus, to fully appropriate the gains from a wider quality gap, the platform needs to lower the buyer-side access fee to eliminate the suboptimal competitive tension among incumbents.

Extension: Infinite Innovation Race

In this section, we numerically illustrate the effect of the platform's fees when sellers can continue innovation after exiting the market in an attempt to regain incumbent market positions.[6]

With the discount factor $\beta \in (0, 1)$, the sellers choose their innovation rate ϕ' based on the following value functions of a dynamic programming problem:

$$V^0(\phi) = \max\left\{0, -f + V^E(\phi)\right\} \tag{21}$$

$$V^E(\phi) = \max_{\phi' \in [0,1]} \left\{\lambda(\phi', \phi) V_2^I + \left(1 - \lambda(\phi', \phi)\right)\beta V^0(\phi) - \varepsilon c(\phi')\right\} \tag{22}$$

$$V_2^I(\phi) = R_2^* + \beta\pi(\phi) V_1^I(\phi) + \beta\left(1 - \pi(\phi)\right) V_2^I(\phi) \tag{23}$$

$$V_1^I(\phi) = R_1^* + \beta\pi(\phi) V^0(\phi) + \beta\left(1 - \pi(\phi)\right) V_1^I(\phi). \tag{24}$$

$V^0(\phi)$ is the value function of sellers at the start of the game: if they choose to innovate, the expected value is $-f + V^E(\phi)$; when this value is negative, they would choose not to innovate and would obtain value 0. $V^E(\phi)$ is the value function of sellers participating in the R&D race; they choose the innovation effort, ϕ' (given the effort of others, ϕ), to maximize their expected value, while taking into account the probability of market entry and the variable innovation cost. $V_2^I(\phi)$ and $V_1^I(\phi)$ are the value functions of the high- and low-quality incumbents, respectively. A higher market entry probability may reduce the expected value of incumbency because incumbents shift down in quality (and the lowest-quality incumbent exits) when entry occurs.

We are interested in the stationary Markov equilibrium [21] among sellers, consisting of a set of strategies: given other sellers' strategies, (1) each seller's best response of whether to enter the innovation race, (2) the innovation rate ϕ that maximizes its expected value, and (3) its equilibrium price as an incumbent given the other incumbent's pricing strategies (presented in Table 2). The equilibrium innovation rate is solved computationally. Sellers' innovation decisions are dynamic as they care about intertemporal trade-offs; however, their prices as incumbents in the market are determined under a static equilibrium in each period in response to the strategy of the other incumbent in the current market. The implicit assumptions here are twofold: (1) incumbents do not engage in collusive contracts (particularly not with unknown future entrants) to obtain higher revenues (i.e., they do not set a higher price in the initial period at the expense of lower revenues in the discounted future periods), and (2) sellers perceive finite periods of market incumbency so that their pricing decisions in each period are equivalent to those in a static equilibrium; therefore, the equilibrium prices are independent intertemporally.

Lemma 4: Let $X = [0, 1]$, and denote by $B(X)$ a space of bounded functions $g: X \to R$. The value function V^0, defined in Equations (21) through (24), is unique in $B(X)$.

Proof: See the Appendix.

Because we are interested in the stationary symmetric equilibria, sellers' choice sets include two types. Sellers either never enter the innovation race or always enter the innovation race after exiting the market. We write the dynamic problem of the latter case as follows:

$$V^A(\phi) = \max_{\phi' \in [0,1]} \left\{-f + \lambda(\phi', \phi) V_2^I + \left(1 - \lambda(\phi', \phi)\right)\beta V^A(\phi) - \varepsilon c(\phi')\right\} \tag{25}$$

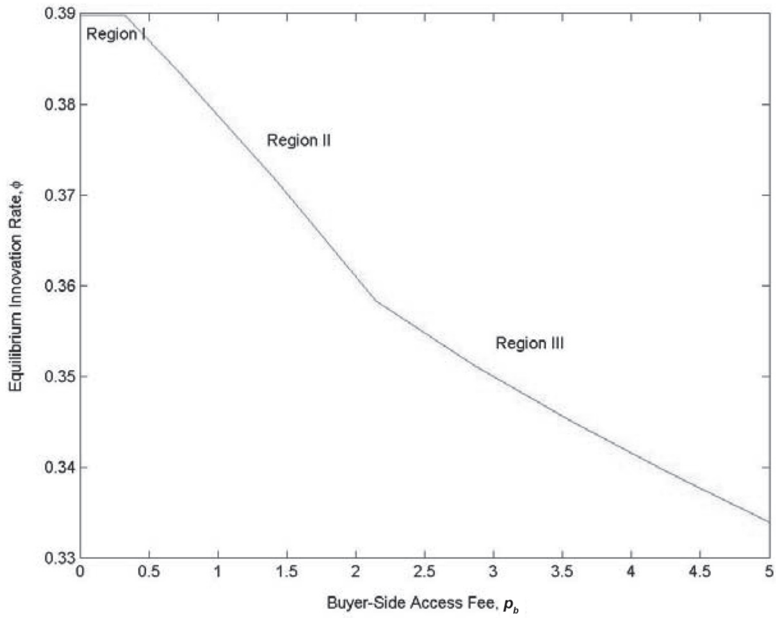

Figure 3. Equilibrium Innovation Rate and p_b

$$V_2^I(\phi) = R_2^* + \beta\pi(\phi)V_1^I(\phi) + \beta(1-\pi(\phi))V_2^I(\phi) \tag{26}$$

$$V_1^I(\phi) = R_1^* + \beta\pi(\phi)V^A(\phi) + \beta(1-\pi(\phi))V_1^I(\phi), \tag{27}$$

where $V^A(\phi)$ represents the value of a seller that always enters the innovation race when the other sellers choose the innovation rate ϕ. When $\phi' = \phi$, all sellers choose the same innovation rate. If $V^A(\phi) \geq 0$, the optimal innovation rate $\phi^* = \phi$. Otherwise, no seller enters the innovation race. The innovation rate is determined by the following equation:

$$\phi = \arg\max_{\phi' \in [0,1]} \left\{-f + \lambda(\phi',\phi)V_2^I + (1-\lambda(\phi',\phi))\beta V^A(\phi) - \varepsilon c(\phi')\right\}. \tag{28}$$

Because of the complexity of the dynamic problem, we use computational methods to find the numerical solution ϕ for the problem described by Equation (28). (See the Computation and Parameterization section in the Appendix for this computation approach and the parameterization.) After computing the value of $V^A(\phi)$, we find whether sellers enter the innovation race, given f and p_b.

We first fix the seller-side access fee, f, and vary the buyer-side access fee, p_b, to understand the manner in which the buyer-side access fee affects the sellers' innovation decisions.

As shown in Figure 3, the sellers' equilibrium innovation rate decreases in the buyer-side access fee, p_b, as the market equilibrium shifts from Region I to Region II and then to Region III. Increases in the buyer-side access fee may diminish the sellers'

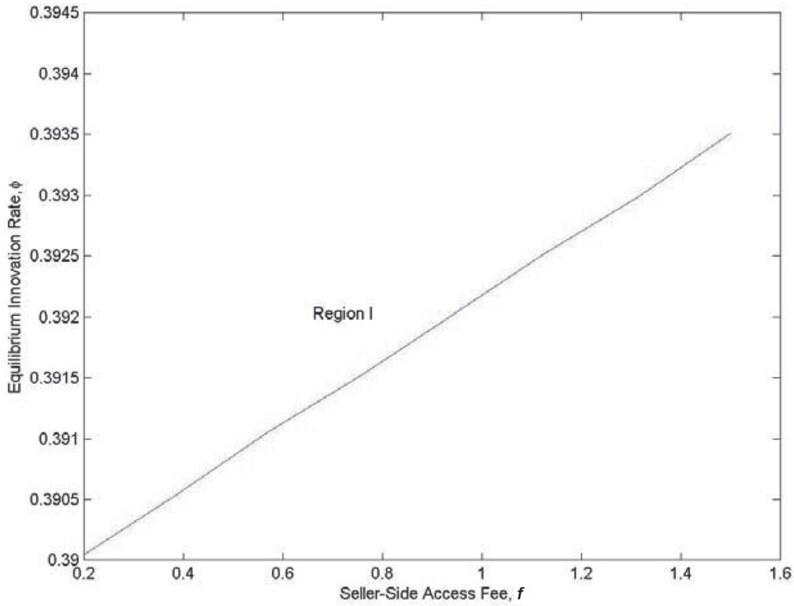

Figure 4. Equilibrium Innovation Rate and *f*

equilibrium revenues in case of market entry (Lemma 2). The sellers' innovation incentive is driven by the expected payoff of innovation, which is the sum of the values in different states weighted by the probability of shifting from one state to the next (e.g., the probability of market entry, and that of being demoted from the high-quality incumbent to the low-quality incumbent). As p_b increases, if the market equilibrium falls in Region I, the market equilibrium revenues are not affected (Lemma 2), and neither is the sellers' expected innovation payoff; thus, the equilibrium innovation rate remains constant. In Regions II and III, the incumbents of both qualities have decreasing revenues in p_b; therefore, the equilibrium innovation rate declines as p_b increases. Notice that this result is consistent with the finite case, where $f^* = ((R_2^* + R_1^*)/2\varepsilon M)^{1/2}$ and $\partial f^*/\partial p_b$ is clearly negative.

We now take p_b as given and vary the seller-side access fee, f. When f exceeds the sellers' expected payoff of innovation, no seller chooses to enter the innovation race. Thus, we focus on the symmetric equilibrium, where all sellers innovate, and we examine the effect of the seller-side access fee on the equilibrium innovation rate.

We find that the seller-side access fee induces higher innovation efforts (Figure 4),[7] as sellers are motivated to innovate more intensively for a higher probability of innovation success and market entry. This contrasts with the finite-horizon case, where the seller-side fee affects only the sellers' decision of participation, but it has no effect on their equilibrium innovation rate. In the finite case, the seller-side fee incurs only once if a seller chooses to innovate, and the innovation rate does not interact with this up-front lump-sum fee. The key difference in the infinite case is that ex ante sellers consider future innovation after their market exit; thus, this fee must be paid at each future reentry into the innovation race. As f increases, the failure to innovate becomes more costly, and the expected payoff is diminished. Therefore, in equilibrium, sellers balance the expected values of different states by innovating more intensively to achieve higher probabilities of generating revenues and to take into account more costly innovation failure. Similarly, a lower f would then make sellers "lazier." Our result shows that competition for market entry leads to an overall increase in the symmetric equilibrium innovation rate.

Conclusion

Motivated by recent developments in mobile platform markets, we analyze a platform's optimal two-sided pricing strategy while taking into consideration seller-side price competition and innovation. In the finite-horizon innovation race, our findings indicate that the platform sets the optimal seller-side access fee at the maximum to fully extract the sellers' surplus and may charge or subsidize buyers, depending on the degree of variation of buyers' willingness to pay for quality. The optimal buyer-side access fee moderates the competition among sellers and disincentivizes their subsidizing of the buyer-side fee. The platform's optimal two-sided pricing strategy and the interactions between sellers and buyers enable full participation by both sides of the market in equilibrium. When sellers engage in innovation in the infinite horizon, our parameterization shows that increasing the buyer-side fee may discourage innovation and that when all sellers innovate, a higher seller-side fee may stimulate innovation in equilibrium.

The presence of seller price competition allows the platform to serve buyers with all levels of willingness to pay even at a positive optimal price. For instance, marketed as a superior platform device, the iPhone has successfully penetrated a wide range of user market segments, in part because of attractive apps that are priced to appeal even to users with a low willingness to pay. At the same time, an excessive buyer-side access fee may lead to a suboptimal competitive intensity, where the seller-side indirectly bears the cost to ensure the buyers' participation. Although raising the price for the device may not significantly compromise the buyer network size, it can lead to significantly lower app prices that reduce the sellers' surplus and thus the platform's revenues. Thus, while setting a positive price for the mobile device, the platform should also support profitable transactions in the app market. The iPad, arguably the most appealing tablet, is not priced substantially higher than its lower-quality substitutes. Our findings suggest that this strategy provides leverage for the app sellers to avoid an intense price competition and to achieve higher profits. Furthermore, as the high-end apps become increasingly superior, the platform's optimal strategy could be to further reduce the buyers' cost and to raise the seller-side fee.

To the best of our knowledge, most past studies on two-sided platforms do not explicitly model seller-side price competition (with the exception of Economides and Katsamakas's [15] work, which focuses on comparing industry structures based on proprietary and open source platforms). Past studies often rest on the assumption that two sides possess highly symmetric characteristics and primarily exert network externality.

Introduction of price competition unveils results that contrast with previous findings, which show that the side of the platform that provides substantial benefits to the other side is rewarded with a lower price subsidized by the platform [5]. By characterizing the market equilibrium, we find that the platform may subsidize the buyer side given a high variation of buyers' preferences. In this case, platform subsidies help to bring on board the buyer segment that would otherwise be excluded in equilibrium. In the other cases, the platform sets a positive fee on both sides. Our findings also contrast with the insight in Economides and Katsamakas [15], showing that the platform may subsidize the application providers (analogous to sellers in our work) when users (i.e., buyers) have a strong preference for application variety. While Economides and Katsamakas use a linear demand function to capture the degrees of complementarity and substitution among applications, we model a vertically differentiated market of sellers and show that a wider quality gap affects the platform's optimal allocation of fees: The buyer side experiences a reduction in the access fee, whereas the fee for the seller side increases.

Furthermore, studies on the role of innovation in a two-sided market are limited, with the exception of Boudreau's empirical study on the effect of network size on software developers' innovation incentives [10]. Our work incorporates sellers' innovation decisions in both finite and infinite horizons and shows that the platform access fees on both sides affect sellers' innovation incentives. Increasing the buyer-side fee can have a negative effect on innovation. But increasing the seller-side fee may have a favorable effect on innovation as sellers strive for a greater chance of market entry to offset the cost of innovation failure.

It remains a challenge to model a competitive framework in which multiple platforms set prices on two sides simultaneously. Furthermore, as a future research direction, it may be interesting to consider innovating sellers that have heterogeneous characteristics. The asymmetric innovation decisions could lead to insights that expand on the current findings.

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NOTES

1. Among the many examples of two-sided technology platforms are operating systems platforms, such as Linux and Windows, and gaming hardware platforms, such as PlayStation and the Xbox.
2. See the iOS Developer Program, http://developer.apple.com/iphone/program/.
3. The other type of fee commonly considered in two-sided markets is transaction based, which the platform collects from each successful seller–buyer transaction. Our qualitative results still hold if the platform imposes both types of fees. Thus, we focus on the fixed-fee structure in our analysis for tractability.
4. Notice that in this stage, the seller is an incumbent in the market and does not participate in the R&D race.

5. In the absence of this condition, a different condition is needed to justify a possible corner solution, in which case the platform may set the optimal buyer-side access fee at its maximum. The proof of optimality would then require further comparison between this corner value and the one established in Proposition 3. For tractability and a crisp insight, we focus on the interior solution in this problem.

6. This setup is analogous to the potential monopolist's innovation process studied in Segal and Whinston [24].

7. The figure shows only one equilibrium region because varying f while holding p_b fixed does not change the equilibrium region. The numerical experiments under the p_b values of all regions show the same result.

8. See a discussion of the Bellman's equation in Bertsekas [9, p. 8].

References

1. AdMob mobile metrics: Our insights on the mobile ecosystem. May 2010 Mobile Metrics Report. AdMob, June 30, 2010 (available at http://metrics.admob.com).

2. Aghion, P.; Harris, C.; Howitt, P.; and Vickers, J. Competition, imitation, and growth with step-by-step innovation. *Review of Economic Studies, 68,* 3 (2001), 467–492.

3. Aghion, P.; Bloom, N.; Blundell, R.; Griffith, R.; and Howitt, P. Competition and innovation: An inverted-U relationship. *Quarterly Journal of Economics, 120,* 2 (2005), 701–728.

4. Andrew, J.; Sirkin, H.; and Butman, J. *Payback: Reaping the Rewards of Innovation.* Boston: Harvard Business School Press, 2006.

5. Armstrong, M. Competition in two-sided markets. *RAND Journal of Economics, 37,* 3 (2006), 668–691.

6. Armstrong, M., and Wright, J. Two-sided markets, competitive bottlenecks, and exclusive contracts. *Economic Theory, 32,* 2 (2007), 353–380.

7. Bakos, Y., and Katsamakas, E. Design and ownership of two-sided networks: Implications for Internet platforms. *Journal of Management Information Systems, 25,* 2 (Fall 2008), 171–202.

8. Ben-Aaron, D. Android beats Apple App Store growth, Distimo says. *BusinessWeek,* January 12, 2011 (available at www.businessweek.com/news/2011-01-12/android-beats-apple-app-store-s-growth-distimo-says.html).

9. Bertsekas, D. *Dynamic Programming and Optimal Control: Volume II*. Belmont, MA: Athena Scientific, 1995.

10. Boudreau, K. Let a thousand flowers bloom? Growing an applications software platform and the rate and direction of innovation. *Organization Science* (2011), forthcoming.

11. Caillaud, B., and Jullien, B. Competing cybermediaries. *European Economic Review, 45,* 4–6 (2001), 797–808.

12. Caillaud, B., and Jullien, B. Chicken and egg: Competing matchmakers. *RAND Journal of Economics, 34,* 2 (2003), 309–328.

13. Choi, Y.C.; Hummels, D.; and Xiang, C. Explaining import quality: The role of the income distribution. *Journal of International Economics, 77,* 2 (2009), 265–275.

14. Christensen, C.M. *The Innovator's Dilemma.* Boston: Harvard Business School Press, 1997.

15. Economides, N., and Katsamakas, E. Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry. *Management Science, 52,* 7 (2006), 1057–1071.

16. Galbreth, M.R.; March, S.T.; Scudder, G.D.; and Shor, M. A game-theoretic model of e-marketplace participation growth. *Journal of Management Information Systems, 22,* 1 (Summer 2005), 295–319.

17. Gartner says worldwide mobile phone sales grew 35 percent in third quarter 2010; Smartphone sales increased 96 percent. Press Release, Gartner, Stamford, CT, November 2010 (available at www.gartner.com/it/page.jsp?id=1466313/).

18. Grossman, G.M., and Helpman, E. Quality ladders in the theory of growth. *Review of Economic Studies, 58,* 1 (1991), 43–61.

19. Hagiu, A. Pricing and commitment by two-sided platforms. *RAND Journal of Economics, 37,* 3 (2006), 720–737.
20. Lia, S. iPhone market share analysis for 2010/2011. *infoSync World,* January 28, 2011 (available at www.infosyncworld.com/reviews/cell-phones/iphone-market-share-analysis-2010-2011/11744.html).
21. Mailath, G.J., and Samuelson, L. *Repeated Games and Reputations: Long-Run Relationships.* New York: Oxford University Press, 2006.
22. Rochet, J., and Tirole, J. Cooperation among competitors: Some economics of payment card associations. *RAND Journal of Economics, 33,* 4 (2002), 549–570.
23. Rochet, J., and Tirole, J. Platform competition in two-sided markets. *Journal of the European Economic Association, 1,* 4 (2003), 990–1029.
24. Segal, I., and Whinston, M. Antitrust in innovative industries. *American Economic Review, 97,* 5 (2007), 1703–1730.
25. Shaked, A., and Sutton, J. Relaxing price competition through product differentiation. *Review of Economic Studies, 49,* 1 (1982), 3–13.
26. Stokey, N., and Lucas, R.E. *Recursive Methods in Economic Dynamics.* Cambridge: Harvard University Press, 1989.
27. Yoo, B.; Choudhary, V.; and Mukhopadhyay, T. A model of neutral B2B intermediaries. *Journal of Management Information Systems, 19,* 3 (Winter 2002–3), 43–68.

Appendix

Proof of Lemma 1

Let us define $C_k \equiv u_k/(u_k - u_{k-1})$ (note that $C_k > 1$). Equations (3) and (4) can be rewritten as

$$z_1 = p_1 + \frac{p_b}{u_1} \tag{A1}$$

$$z_k = p_{k-1}\left(1 - C_k\right) + p_k C_k. \tag{A2}$$

The first-order conditions of incumbents' revenues in Equations (5) through (8) are as follows:

For $k = 1$,

$$z_2 - \underline{z} - p_1\left(C_2 - 1\right) = 0, \text{ if } z_1 \le \underline{z} \tag{A3}$$

$$z_2 - z_1 - p_1\left[\left(C_2 - 1\right) + 1\right] = 0, \text{ if } z_1 \ge \underline{z}. \tag{A4}$$

For $k = 2,\ldots,n - 1$,

$$z_{k+1} - z_k - p_k\left[\left(C_{k+1} - 1\right) + C_k\right] = 0. \tag{A5}$$

For $k = n$,

$$\overline{z} - z_n - p_n C_n = 0. \tag{A6}$$

Assume there are more than two incumbents in equilibrium; we can rewrite the first-order conditions using Equation (A2) as

$$z_{k+1} - 2z_k - p_k\left(C_{k+1} - 1\right) - p_{k-1}\left(C_k - 1\right) = 0 \tag{A7}$$

$$\overline{z} - 2z_n - p_{n-1}\left(C_n - 1\right) = 0. \tag{A8}$$

We can then get $\overline{z} > 2z_n$ and $z_{k+1} > 2z_k$, which yield $2z_n > 4z_{n-1}$, and then $\overline{z} > 4z_n - 1$. By the assumption $\overline{z} < 4\underline{z} - 3p_b$, we get $4\underline{z} > \overline{z}$. Therefore, $z_{n-1} < \underline{z}$, implying that the incumbents of the two qualities cover the market.

Proof of Proposition 1

Based on Equations (9) and (10), rewriting the first-order conditions of the low-quality incumbent yields the following:

$$z_2 = \underline{z} + \frac{1}{u_2 - u_1}\left(u_1 z_1 - p_b\right), \text{ if } z_1 \leq \underline{z} \tag{A9}$$

$$z_2 = \frac{2u_2 - u_1}{u_2 - u_1} z_1 - \frac{u_2 p_b}{u_1\left(u_2 - u_1\right)}, \text{ if } z_1 \geq \underline{z}. \tag{A10}$$

And for the high-quality incumbent, we get

$$\overline{z} - 2z_2 = \frac{1}{u_2 - u_1}\left(u_1 z_1 - p_b\right). \tag{A11}$$

Figure A1 is a plot of the low-quality incumbent's first-order conditions (Equations (A9) and (A10)) showing increasing values of z_2 as functions of z_1 in three regions. In the high-quality incumbent's first-order condition, Equation (A11), z_2 is a decreasing function of z_1. Thus, the point where two incumbents' first-order conditions intersect is the equilibrium of interest. In an equilibrium where two incumbents share the market, the point of intersection of their first-order conditions necessarily lies in one of the three regions. In particular, z_2, the indifference level for the two products, must exceed the lower bound $\underline{z}$; otherwise, the high-quality incumbent obtains the entire market. Shown in Figure A1, on the lower bound of Region I, $z_2 = \underline{z}$ and $z_1 = p_b/u_1$; firm 2's first-order condition must cross from above, requiring $z_2 > \underline{z}$, for the existence of equilibrium.

Both firms' revenue functions are concave in each firm's own price holding the competitor's price constant; thus, the second-order conditions are satisfied. Given $\overline{z} > 2\underline{z}$, firm 2's first-order condition (Equation (A11)) lies above $\underline{z}$ at $z_1 = p_b/u_1$ in Figure A1. Because z_2 is decreasing in z_1 in Equation (A11), it necessarily intersects with firm 1's first-order condition in one of the three regions, thus resulting in a unique equilibrium.

At $z_1 = \underline{z}$, if

$$z_2 \leq \frac{u_2}{u_2 - u_1}\underline{z} - \frac{p_b}{u_2 - u_1}$$

for firm 2 (Equation (A11)), then the equilibrium occurs in Region I; thus, the condition

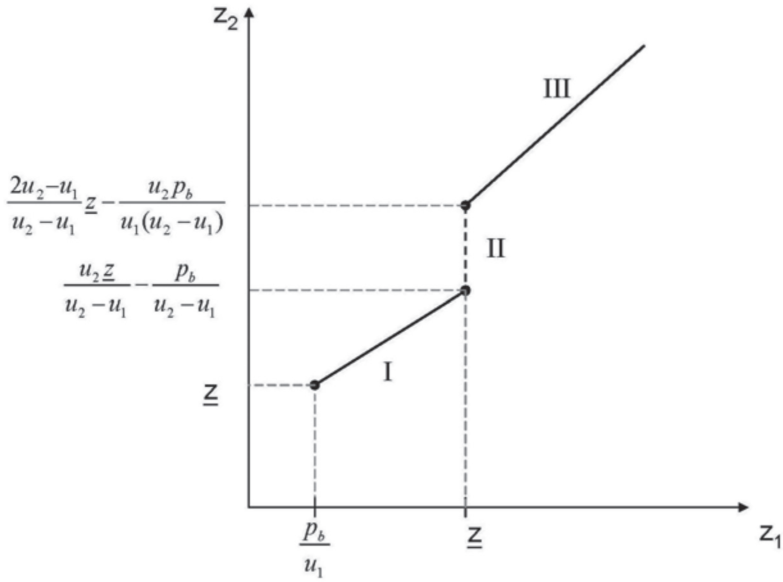

Figure A1. The Low-Quality Incumbent's First-Order Conditions

$$p_b \le \frac{1}{3}\left[\left(2u_2+u_1\right)\underline{z}-\left(u_2-u_1\right)\overline{z}\right]$$

for Region I equilibrium is obtained. Similarly, at $z_1 = \underline{z}$, if

$$\frac{u_2}{u_2-u_1}\underline{z}-\frac{p_b}{u_2-u_1} \le z_2 \le \frac{2u_2-u_1}{u_2-u_1}\underline{z}-\frac{u_2 p_b}{u_1\left(u_2-u_1\right)}$$

for firm 2, then a Region II equilibrium follows;

$$z_2 \ge \frac{2u_2-u_1}{u_2-u_1}\underline{z}-\frac{u_2 p_b}{u_1\left(u_2-u_1\right)}$$

meanwhile, leads to a Region III equilibrium. The corresponding conditions of p_b can be derived as shown in Table 1.

For Region II to be feasible, we need to show that the lower bound of p_b is indeed less than the upper bound for this region; in other words, we need to show that

$$\frac{1}{3}\left[\left(2u_2+u_1\right)\underline{z}-\left(u_2-u_1\right)\overline{z}\right] < \frac{u_1}{2u_2+u_1}\left[\left(4u_2-u_1\right)\underline{z}-\left(u_2-u_1\right)\overline{z}\right].$$

The lower bound can be rearranged as follows:

$$\frac{1}{3}\left[\left(2u_2-2u_2+u_1+2u_1\right)\underline{z}+2\left(u_2-u_1\right)\underline{z}-\left(u_2-u_1\right)\overline{z}\right]$$

$$=u_1\underline{z}+\frac{1}{3}\left(u_2-u_1\right)\left(2\underline{z}-\overline{z}\right).$$

And the upper bound can be rearranged in a similar way:

$$\frac{u_1}{2u_2+u_1}\left[\left(4u_2-2u_2-u_1+2u_1\right)\underline{z}+2\left(u_2-u_1\right)\underline{z}-\left(u_2-u_1\right)\overline{z}\right]$$

$$=u_1\underline{z}+\frac{u_1}{2u_2+u_1}\left(u_2-u_1\right)\left(2\underline{z}-\overline{z}\right).$$

Clearly, $[u_1/(2u_2+u_1)] < 1/3$ because $2u_2+u_1 > 3u_1$. And given that $2\underline{z}-\overline{z}<0$, the lower bound is less than the upper bound.

For Region I, from the first-order conditions (Equations (A9) and (A11)), we can derive that

$$z_1^*=\frac{1}{u_1}\left[\frac{u_2-u_1}{3}\left(\overline{z}-2\underline{z}\right)+p_b\right]$$

and

$$z_2^*=\left[\overline{z}+\underline{z}\right]/3.$$

The equilibrium prices can be derived by substituting these expressions into Equations (9) and (10). Then, based on the revenue functions, Equations (5) to (8), the corresponding equilibrium revenues can be derived by substituting in z_1^*, z_2^*, and the equilibrium prices.

For Region II, $z_1^*=\underline{z}$, and thus,

$$z_2^*=\left[\overline{z}-\frac{u_1}{u_2-u_1}\underline{z}+\frac{p_b}{u_2-u_1}\right]/2$$

based on the first-order condition Equation (A11). The rest of the equilibrium expressions can be derived in a similar manner as in Region I.

For Region III, using Equations (A10) and (A11), it follows that

$$z_1^*=\frac{1}{4u_2-u_1}\left[\left(u_2-u_1\right)\overline{z}+\frac{2u_2+u_1}{u_1}p_b\right]$$

and

$$z_2^*=\frac{1}{4u_2-u_1}\left[\left(2u_2-u_1\right)\overline{z}+p_b\right].$$

And similarly, by substituting these expressions into the corresponding equations, the rest of the equilibrium results follow.

By the assumption $2\underline{z}<\overline{z}<4\underline{z}$, the equilibrium prices of Region I are clearly positive. In Region II, positive pricing implies that $p_b \le u_1\underline{z}$. We can show that at $\overline{z}=2\underline{z}$, the upper bound meets $u_1\underline{z}$. Because the upper bound is a decreasing function of $\overline{z}$ and $\overline{z}>2\underline{z}$, for all values of p_b in this region, the equilibrium prices will be positive. In Region III, to ensure positive equilibrium prices, p_b must satisfy $p_b \le (u_1/2)\overline{z}$.

We also need to show that Region III is feasible—that is, that the conditions

$$p_b \le \frac{u_1}{2}\overline{z}$$

and

$$p_b \geq \frac{u_1}{2u_2 + u_1}\left[\left(4u_2 - u_1\right)\underline{z} - \left(u_2 - u_1\right)\overline{z}\right]$$

(see Table 1) can be satisfied simultaneously. Because $2\underline{z} < \overline{z}$, if the lower bound at $\underline{z} = \overline{z}/2$ is less than or equal to the upper bound, then this region will be feasible. We can easily show that at $\underline{z} = \overline{z}/2$, the lower bound yields exactly $(u_1/2)\overline{z}$.

Thus, all three equilibrium regions are feasible, and all equilibrium prices and revenues are positive.

Proof of Lemma 2

Region II:

$$\frac{\partial p_1^*}{\partial p_b} = -1/u_1 < 0 \text{ and } \frac{\partial p_2^*}{\partial p_b} = -1/\left(2u_2\right) < 0.$$

$$\frac{\partial R_1^*}{\partial p_b} = \frac{1}{2u_1}\left[-\overline{z} + 2\underline{z} + \frac{2}{u_2 - u_1}\left[u_1\underline{z} - p_b\right]\right].$$

Substituting in the lower bound

$$p_b = \frac{1}{3}\left[\left(2u_2 + u_1\right)\underline{z} - \left(u_2 - u_1\right)\overline{z}\right],$$

$$\begin{aligned} &-\overline{z} + 2\underline{z} + \frac{2}{u_2 - u_1}\left[u_1\underline{z} - p_b\right] \\ &= -\frac{1}{3}\overline{z} + 2\underline{z} - \frac{4\left(u_2 - u_1\right)}{3\left(u_2 - u_1\right)}\underline{z} \\ &= \frac{1}{3}\left(2\underline{z} - \overline{z}\right) < 0. \end{aligned}$$

Because $\partial R_1^*/\partial p_b$ is decreasing in p_b, $\partial R_1^*/\partial p_b < 0$ for all values of p_b in Region II:

$$\frac{\partial R_2^*}{\partial p_b} = \frac{-1}{2u_2}\left[\overline{z} + \frac{1}{u_2 - u_1}\left[u_1\underline{z} - p_b\right]\right] < 0$$

(note that $p_b \leq u\underline{z}$).

Region III:

$$\frac{\partial p_1^*}{\partial p_b} = \frac{-2\left(u_2 - u_1\right)}{u_1\left(4u_2 - u_1\right)} < 0 \text{ and } \frac{\partial p_2^*}{\partial p_b} = \frac{-\left(u_2 - u_1\right)}{u_2\left(4u_2 - u_1\right)} < 0.$$

Given

$$p_b \leq \frac{u_1}{2}\overline{z}$$

$$\frac{\partial R_1^*}{\partial p_b} = \frac{-4u_2\left(u_2 - u_1\right)}{u_1\left(4u_2 - u_1\right)^2}\left[\overline{z} - \frac{2p_b}{u_1}\right] < 0$$

and

$$\frac{\partial R_2^*}{\partial p_b} = \frac{-2\left(u_2 - u_1\right)}{u_2\left(4u_2 - u_1\right)^2}\left[2u_2\overline{z} - p_b\right] < 0.$$

Proof of Proposition 2

Suppose a seller decides to innovate. It solves

$$\max_{\phi'} \frac{\phi'\left(R_2^* + R_1^*\right)}{M\phi} - \varepsilon\phi'^2 - f.$$

The first-order condition yields $(R_2^* + R_1^*)/M\phi = 2\varepsilon\phi'$. Given homogeneous sellers, we are interested in a symmetric equilibrium, where $\phi' = \phi$. Therefore, we have $\phi^* = ((R_2^* + R_1^*)/2\varepsilon M)^{1/2}$. By substituting ϕ^* into the seller's value function, we get $((R_2^* + R_1^*)/2M) - f$, which is positive when $f \leq (R_2^* + R_1^*)/2M$. Thus, all sellers will innovate when the seller-side access fee is below $(R_2^* + R_1^*)/2M$.

Proof of Lemma 3

We find the first-order derivative of the platform's revenue function with respect to p_b in all three regions.

Region I

Based on Proposition 2, R_1^* and R_2^* are constant in p_b in Region I. Therefore, the first-order condition for the platform's revenue function in this region is simply $\Pi'(p_b) = 1$. The revenue is increasing in p_b in Region I.

Region II

The first-order derivative of the platform's revenue function is

$$\frac{\partial \Pi}{\partial p_b} = 1 + \frac{1}{2}\frac{\partial R_2^*}{\partial p_b} + \frac{1}{2}\frac{\partial R_1^*}{\partial p_b},$$

which is

$$\begin{aligned} &1 + \frac{1}{4u_1}\left(2\underline{z} - \overline{z}\right) + \frac{1}{u_2 - u_1}\left(\frac{1}{2u_1} - \frac{1}{4u_2}\right)\left[u_1\underline{z} - p_b\right] - \frac{\overline{z}}{4u_2} \\ &= 4u_1 + \left(2\underline{z} - \overline{z}\right) + \frac{1}{u_2 - u_1}\left(2 - \frac{u_1}{u_2}\right)\left[u_1\underline{z} - p_b\right] - \frac{u_1\overline{z}}{u_2}. \end{aligned}$$

Because this expression is decreasing in p_b, it is sufficient to show that it is negative at the lower bound of p_b for this region, where

$$p_b = \frac{1}{3}\left[\left(2u_2 + u_1\right)\underline{z} - \left(u_2 - u_1\right)\overline{z}\right].$$

By substituting in this p_b value, we get

$$\begin{aligned} &4u_1 + \left(2\underline{z} - \overline{z}\right) + \frac{1}{3\left(u_2 - u_1\right)}\left(2 - \frac{u_1}{u_2}\right)\left[-2\left(u_2 - u_1\right)\underline{z} + \left(u_2 - u_1\right)\overline{z}\right] - \frac{u_1\overline{z}}{u_2} \\ &= 4u_1 + \left(2\underline{z} - \overline{z}\right) - \left(\frac{2}{3} - \frac{u_1}{3u_2}\right)\left(2\underline{z} - \overline{z}\right) - \frac{u_1\overline{z}}{u_2} \\ &= 4u_1 + \frac{1}{3}\left(2\underline{z} - \overline{z}\right) + \frac{u_1}{3u_2}\left(2\underline{z} - \overline{z}\right) - \frac{u_1\overline{z}}{u_2}. \end{aligned}$$

Because $2\underline{z} < \overline{z}$, this expression is negative, given $\overline{z} \geq 4u_2$. The revenue is decreasing in p_b in Region II.

Region III

The platform's revenue function for Region III is

$$\Pi\left(p_b\right) = \frac{p_b}{\left(4u_2 - u_1\right)\left(\overline{z} - \underline{z}\right)}\left[3u_2\overline{z} - \frac{2u_2 + u_1}{u_1}p_b\right] + \frac{1}{2}\left(R_2^* + R_1^*\right). \tag{A12}$$

The first-order derivative of the platform's revenue function with respect to p_b is

$$\begin{aligned} &\frac{1}{\left(4u_2 - u_1\right)\left(\overline{z} - \underline{z}\right)}\left[3u_2\overline{z} - \frac{2\left(2u_2 + u_1\right)}{u_1}p_b\right] \\ &- \frac{u_2 - u_1}{u_2\left(4u_2 - u_1\right)^2}\left[2u_2\overline{z} - p_b\right] - \frac{2u_2\left(u_2 - u_1\right)}{u_1\left(4u_2 - u_1\right)^2}\left[\overline{z} - \frac{2p_b}{u_1}\right]. \end{aligned}$$

We can increase the last term slightly by replacing u_1 with u_2 (to simplify analysis) and show that the resulting expression yields a negative value:

$$\frac{1}{\left(4u_2 - u_1\right)\left(\overline{z} - \underline{z}\right)}\left[3u_2\overline{z} - 2p_b - \frac{4u_2}{u_1}p_b\right] - \frac{u_2 - u_1}{u_2\left(4u_2 - u_1\right)^2}\left[4u_2\overline{z} - p_b - \frac{4u_2}{u_1}p_b\right].$$

Clearly,

$$3u_2\overline{z} - 2p_b - \frac{4u_2}{u_1}p_b < 4u_2\overline{z} - p_b - \frac{4u_2}{u_1}p_b.$$

Therefore, for

$$\overline{z} - \underline{z} \geq \frac{u_2\left(4u_2 - u_1\right)}{u_2 - u_1},$$

the expression is negative, and the platform's revenue is decreasing in p_b in Region III.

Because $\overline{z} \leq 4\underline{z}$, $\overline{z} - \underline{z} \leq (3/4)\overline{z}$. The condition

$$\overline{z} - \underline{z} \geq \frac{u_2(4u_2 - u_1)}{u_2 - u_1}$$

implies that

$$\overline{z} \geq \frac{4u_2(4u_2 - u_1)}{3(u_2 - u_1)} > 4u_2.$$

Therefore, given the condition,

$$\overline{z} - \underline{z} \geq \frac{u_2(4u_2 - u_1)}{u_2 - u_1},$$

the platform's revenue is decreasing in both Regions II and III.

Proof of Proposition 3

Lemma 3 establishes that the platform's revenues are increasing in p_b in Region I, and decreasing in Regions II and III. Therefore, the optimal p_b lies in the point where Region I overlaps Region II,

$$p_b^* = \frac{1}{3}\left[(2u_2 + u_1)\underline{z} - (u_2 - u_1)\overline{z}\right].$$

By Proposition 2, the platform sets f at the maximum conditional on sellers' entry into the innovation race. Therefore,

$$f = \frac{(u_2 - u_1)}{18M}\left[\frac{(\overline{z} - 2\underline{z})^2}{u_1} + \frac{(2\overline{z} - \underline{z})^2}{u_2}\right].$$

Proof of Proposition 4

Clearly,

$$p_b^* = \frac{1}{3}\left[(2u_2 + u_1)\underline{z} - (u_2 - u_1)\overline{z}\right] \geq 0,$$

if and only if

$$\underline{z} \geq \frac{(u_2 - u_1)\overline{z}}{2u_2 + u_1}.$$

We can show that this condition for $\underline{z}$ can be satisfied in this problem. Because $2\underline{z} < \overline{z}$, we must satisfy

$$\frac{\bar{z}}{2} > \frac{(u_2 - u_1)\bar{z}}{2u_2 + u_1},$$

which holds, given

$$\frac{1}{2} > \frac{(u_2 - u_1)}{2u_2 + u_1}.$$

For

$$\bar{z} \geq \frac{u_2(4u_2 - u_1)(2u_2 + u_1)}{(u_2 - u_1)(u_2 + 2u_1)},$$

the condition

$$\bar{z} - \underline{z} \geq \frac{u_2(4u_2 - u_1)}{u_2 - u_1}$$

holds as well. Therefore, if

$$\underline{z} \geq \frac{(u_2 - u_1)\bar{z}}{2u_2 + u_1},$$

p_b^* is positive.

Similarly, $p_b^* < 0$, if and only if

$$\underline{z} < \frac{(u_2 - u_1)\bar{z}}{2u_2 + u_1}.$$

Because $\bar{z} < 4\underline{z}$, we need to show that

$$\frac{\bar{z}}{4} < \frac{(u_2 - u_1)\bar{z}}{2u_2 + u_1},$$

which holds for $5u_1 < 2u_2$. Thus, in this case, p_b^* is negative.

Proof of Proposition 5

To find the effect of the quality gap on the platform's optimal fees, we study the comparative statics of p_b^* and f^* with respect to u_2, the buyers' valuation for the high-quality good, while keeping u_1 fixed.

It is straightforward to derive the following:

$$\frac{\partial p_b^*}{\partial u_2} = \frac{1}{3}(2\underline{z} - \bar{z}) < 0$$

and

$$\frac{\partial f^*}{\partial u_2} = \frac{(\bar{z} - 2\underline{z})}{18Mu_1} + \frac{(2\bar{z} - \underline{z})^2}{18Mu_2}\frac{u_1}{u_2} > 0.$$

Therefore, it immediately follows that p_b^* decreases as the quality gap widens, whereas f^* increases as the quality gap widens.

Proof of Lemma 4

We first substitute Equations (23) and (24) into Equation (22). The problem of interest can be written as follows:

$$V^0(\phi) = \max\left\{0, -f + V^E(\phi)\right\}$$

$$V^E(\phi) = \max_{\phi' \in [0,1]} \left\{ \frac{\lambda(\phi',\phi)}{1-\beta(1-\pi(\phi))} R_2 + \frac{\lambda(\phi',\phi)\beta\pi(\phi)}{\left(1-\beta(1-\pi(\phi))\right)} R_1 - \varepsilon c(\phi') + \tilde{\beta}(\phi',\phi) V^0(\phi) \right\},$$

where

$$\tilde{\beta}(\phi',\phi) \equiv \left(\frac{\beta\pi(\phi)}{1-\beta(1-\pi(\phi))} \right)^2 \lambda(\phi',\phi) + \left(1-\lambda(\phi',\phi)\right)\beta.$$

Let

$$\tilde{R}(\phi',\phi) = -f + \frac{\lambda(\phi',\phi)}{1-\beta(1-\pi(\phi))} R_2 + \frac{\lambda(\phi',\phi)\beta\pi(\phi)}{\left(1-\beta(1-\pi(\phi))\right)^2} R_1 - \varepsilon c(\phi').$$

After rearranging the dynamic programming problem, we have the following Bellman's equation:

$$V^0(\phi) = \max\left\{0, \max_{\phi' \in [0,1]} \left\{ \tilde{R}(\phi',\phi) + \tilde{\beta}(\phi',\phi) V^0(\phi) \right\} \right\}. \quad \text{(A13)}$$

Define the operator $T: B(X) \to B(X)$ as follows:

$$Tv(\phi) = \max\left\{0, \max_{\phi' \in [0,1]} \left\{ \tilde{R}(\phi',\phi) + \tilde{\beta}(\phi',\phi) v(\phi) \right\} \right\}.$$

Bellman's equation can be written as

$$v = Tv.$$

Thus, the optimal value function v is a fixed point of the mapping T.[8] Next, we need to show that the mapping T satisfies the requirement of theorems 3.2 and 3.3 in Stokey and Lucas [26]. So the mapping T has a unique fixed point. We first check the requirement of theorem 3.3 (Blackwell's sufficient conditions for a contraction) in Stokey and Lucas [26].

1. (Monotonicity) $\forall g, h \in B(X)$, $g(x) \le h(x)$ for all $x \in X$. We want to show $(Tg)(x) \le (Th)(x)$ for all $x \in X$. Let $\phi_1 = \arg\max_{\phi' \in [0,1]}\{\tilde{R}(\phi', x) + \tilde{\beta}(\phi', x) g(x)\}$ and $\phi_2 = \arg\max_{\phi' \in [0,1]}\{\tilde{R}(\phi', x) + \tilde{\beta}(\phi', x) h(x)\}$, $x \in X$. Clearly,

$$(Tg)(x) = \max\left\{0, \tilde{R}(\phi_1, x) + \tilde{\beta}(\phi_1, x)g(x)\right\} \le \max\left\{0, \tilde{R}(\phi_1, x) + \tilde{\beta}(\phi_1, x)h(x)\right\}$$
$$\le \max\left\{0, \tilde{R}(\phi_2, x) + \tilde{\beta}(\phi_2, x)h(x)\right\} = (Th)(x).$$

2. (Discounting) $\forall x \in X$, $\forall g \in B(X)$, $a \ge 0$, we want to show that there exists some $\iota \in (0,1)$ such that $T(g + a)(x) \le (Tg)(x) + \iota a$. Parameter a is a constant. Here, $(g + a)(x)$ denotes the function $(g + a)(x) = g(x) + a$. $\forall x \in X$, as $((\beta\pi(x))/(1 - \beta(1 - \pi(x))))^2 \in [0, \beta^2]$, $\lambda(\phi', x) \in [0.1]$, $\beta \in (0,1)$, thus $\tilde{\beta}(\phi', x) \in [0, \beta]$.

$$T(g+a)(x) = \max\left\{0, \max_{\phi' \in [0,1]}\left\{\tilde{R}(\phi', x) + \tilde{\beta}(\phi', x)(g(x) + a)\right\}\right\} \tag{A14}$$

$$\le \max\left\{0, \max_{\phi' \in [0,1]}\left\{\tilde{R}(\phi', x) + \beta(\phi', x)g(x) + \tilde{\beta}(\phi', x)a\right\}\right\} \tag{A15}$$

$$\le (Tg)(x) + \tilde{\beta}(\phi', x)a \tag{A16}$$

$$\le (Tg)(x) + \beta a. \tag{A17}$$

Therefore, the mapping defined above is a contraction mapping with modulus β. By applying theorem 3.2 (contraction mapping theorem) in Stokey and Lucas [26], we know that the mapping T has a unique fixed point. Therefore, Bellman's equation has a unique solution.

Computation and Parameterization

We solve the dynamic programming problem in Equations (21) through (24) using the value function iteration method and derive the policy function $\phi' = v(\phi)$, which is a seller's innovation rate given other sellers' innovation rate. We then evaluate the distance between the derived innovation rate $v(\phi)$ and the original guess and update the ϕ to find the solutions of these equations. We use the "fsolve" function in Matlab to solve this system of equations. We also try a large set of initial guesses to determine whether multiple equilibria might exist. Our computation results are robust under different initial guesses.

The discount rate $\beta = 0.95$ implies that the annual interest rate is approximately 5 percent. For u_2, u_1, and u_0, we consider a functional form for u_k similar to that in Choi et al. [13]: $u_k = e^{ak}$, where a is an exogenous parameter, such that a higher a gives a greater difference in quality, and ak reflects the elasticity of utility with regard to quality. Parameter a in the utility function is set at 0.5. For the uniform distribution of buyers' willingness to pay, $U[\underline{z}, \overline{z}]$, the upper bound $\overline{z}$ is set at 22, and the lower bound $\underline{z}$ is 6. As for the functional form of innovation cost $c(\cdot)$, we follow Aghion et al.'s [3] model and use quadratic form, $c(\varepsilon) = \varepsilon\phi^2$, where $\varepsilon = 22$. We also let the number of potential entrants, M, be 10 in each period, for a reasonable computation load.