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Fernando Bernstein,[a] Gregory A. DeCroix,[b] N. Bora Keskin[a]

[a] Fuqua School of Business, Duke University, Durham, North Carolina 27708; [b] Wisconsin School of Business, University of Wisconsin–Madison, Madison, Wisconsin 53706

Contact: fernando.bernstein@duke.edu, https://orcid.org/0000-0002-1013-6898 (FB); greg.decroix@wisc.edu, https://orcid.org/0000-0003-2638-6522 (GAD); bora.keskin@duke.edu, https://orcid.org/0000-0003-1828-0533 (NBK)

Abstract. *Problem definition*: This paper explores the impact of competition between platforms in the sharing economy. Examples include the cases of Uber and Lyft in the context of ride-sharing platforms. In particular, we consider competition between two platforms that offer a common service (e.g., rides) through a set of independent service providers (e.g., drivers) to a market of customers. Each platform sets a price that is charged to customers for obtaining the service provided by a driver. A portion of that price is paid to the driver who delivers the service. Both customers' and drivers' utilities are sensitive to the payment terms set by the platform and are also sensitive to congestion in the system (given by the relative number of customers and drivers in the market). We consider two possible settings. The first one, termed "single-homing," assumes that drivers work through a single platform. In the second setting, termed "multihoming" (or "multiapping," as it is known in practice), drivers deliver their service through both platforms. *Academic/practical relevance*: This is one of the first papers to study competition and multihoming in the presence of congestion effects typically observed in the sharing economy. We leverage the model to study some practical questions that have received significant press attention (and stirred some controversies) in the ride-sharing industry. The first involves the issue of surge pricing. The second involves the increasingly common practice of drivers choosing to operate on multiple platforms (multihoming). *Methodology*: We formulate our problem as a pricing game between two platforms and employ the concept of a Nash equilibrium to analyze equilibrium outcomes in various settings. *Results*: In both the single-homing and multihoming settings, we study the equilibrium prices that emerge from the competitive interaction between the platforms and explore the supply and demand outcomes that can arise at equilibrium. We build on these equilibrium results to study the impact of surge pricing in response to a surge in demand and to examine the incentives at play when drivers engage in multihoming. *Managerial implications*: We find that raising prices in response to a surge in demand makes drivers and customers better off than if platforms were constrained to charge the same prices that would arise under normal demand levels. We also compare drivers' and customers' performance when all drivers either single-home or multihome. We find that although individual drivers may have an incentive to multihome, all players are worse off when all drivers multihome. We conclude by proposing an incentive mechanism to discourage multihoming.

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1. Introduction

1.1. Motivation

Over the past decade, the growth of the sharing economy has had a substantial impact on several established industries, on the ways that people can earn income, and on the utilization of otherwise underused assets. As well-known examples, the taxi service and hotel industries have faced new competition from ride-sharing services such as Uber and Lyft and housing services such as Airbnb, respectively, and individuals with full-time jobs have begun earning extra money on the side by providing rides or renting out rooms. Though in many cases there is nothing new about the underlying services being provided, the structure of the system providing the service has some distinctive characteristics. First, rather than consisting of a firm with employees whose work schedules are directly controlled by the firm, the firm in a sharing-economy setting is typically a platform that facilitates matches between independent service providers and customers. Second, in many (though certainly not all) cases, the individuals

providing the service are doing so as a side activity rather than as their primary means of earning income. As a result, service providers are able to decide whether to participate in the market at any given point in time (based on the current pricing structure, the intensity of demand, and other factors), and platforms need to set high enough prices to incentivize a sufficient number of service providers to be active in the marketplace. On the other hand, prices cannot be set too high, because higher prices will tend to reduce customer demand for the service. Further complicating the problem faced by the platform is the fact that price is not the only factor affecting supply and demand—there is a feedback loop between supply and demand that also plays a role. At any given price, a service provider would be more likely to enter the market if demand is high, because that increases the likelihood of successfully completing a service and earning income (or reduces the time spent waiting to find a customer). Similarly, a customer would be more likely to use the service if supply is high, because that reduces the time spent waiting to receive service. This interaction complicates the platform's pricing problem, for example, by causing supply and demand to no longer be monotone in price. Choosing a good pricing policy is challenging enough if a platform is the only one of its kind operating in a given market, but it becomes even more challenging if multiple platforms are present, and they need to compete for customers and possibly share service providers.

This paper presents and analyzes a model capturing many of the key elements of such a setting. Two competing platforms operate in a single market for a particular service. Each platform sets a price for service through that platform—each customer must pay this full price to receive service, and each provider who completes a service earns a portion of this price (after paying a fixed commission and a fraction of the service price to the platform). A potential customer's decision about whether to seek service, and which platform to choose, is influenced by her underlying preference between the platforms, the prices offered by the two platforms, and the level of congestion in each platform—that is, the number of customers seeking service relative to the number of providers active in the market. A potential service provider's decision about whether to be active in the market is influenced by the provider's underlying cost to provide the service, the price for service set by the platform, and the level of congestion in the platform. A key feature of our model is the inclusion of these congestion effects, where the relative levels of supply and demand influence each other through a feedback loop. Such congestion effects are typically observed in ride-sharing services such as Uber and Lyft, and because these applications serve as our

primary motivation (and are familiar to most readers), we will hereafter use the terms "service provider" and "driver" interchangeably. To reflect realities that are emerging in some ride-sharing markets, we analyze two separate scenarios—one in which drivers can only operate on a single platform and one in which they operate on both platforms. We refer to the former scenario as *single-homing* and the latter as *multihoming*. (In the ride-sharing industry, the latter is also known as multiapping.)

Within this framework we examine a variety of research questions. On a fundamental level, we seek to understand how supply and demand in the market respond to changes in platform prices. From this understanding, we explore what types of supply and demand outcomes can arise at equilibrium prices and how various underlying cost and market parameters influence the relationships between supply and demand and how much of the customer market gets served (at equilibrium). We further leverage these insights to study some practical questions that have received significant press attention (and stirred some controversies) in the ride-sharing industry. The first involves the issue of surge pricing—the practice of increasing prices during times of high demand— which has been criticized by some as a form of price gouging (Lowrey 2014, David 2016). We investigate how equilibrium prices would change if the platforms are allowed to respond to a demand spike, and we also explore whether the parties in the market are better or worse off than if surge pricing were prohibited. The second involves the practice of drivers choosing to operate on both platforms—for example, drivers choosing to drive for both Uber and Lyft. On one hand, numerous ride-sharing platform-switching apps have been recently released (e.g., Mystro, Upshift, QuickSwitch), which allow drivers to operate for multiple platforms without the inconvenience of using multiple phones or having to manually switch apps. At the same time, there are reports of the platforms taking steps to prevent or minimize this practice (Fink 2014). By leveraging our analysis of both scenarios (drivers operating on one or both platforms), we compare the two scenarios to determine which leads to better outcomes for each of the parties in the market.

1.2. Main Contributions
For each of the scenarios considered, we derive expressions for how supply and demand on the platforms respond to any given pair of prices, and we establish conditions under which a price equilibrium is guaranteed to exist. At a given equilibrium price outcome, platforms may or may not compete directly for customers indifferent between the two platforms, and all drivers wishing to be active in the market may

or may not find a customer. We characterize regions of problem parameters such that each of these outcomes occurs based on which region the parameters fall into. We find that platforms actively compete for customers when overall demand intensity is lower or customers are less sensitive to congestion. On the other hand, all drivers are able to find customers when overall demand intensity is high or customers are less sensitive to congestion. We also find that platform competition leads to lower prices and lower congestion levels in equilibrium, compared with a setting with a monopolist platform. In addition, we find that, rather than being a form of exploitation, adjusting (i.e., raising) prices in response to a surge in demand actually makes both drivers and customers in the market better off than if platforms were constrained to hold prices fixed at the equilibrium that would arise at normal demand levels. Whereas higher prices in isolation would seem to harm customers, raising prices incentivizes additional drivers to enter the market, thus reducing the scarcity of supply, with the net effect being positive for customers. That increase in supply (relative to demand) is a negative for drivers, but from their perspective, the higher prices more than compensate, making them better off as well. By comparing the equilibrium outcomes when drivers multihome with the scenario where they single home, we find that all parties are weakly better off when drivers single-home. At first, this may seem to run counter to anecdotal experience—for example, it is becoming more and more common to see cars with both Uber and Lyft stickers in the window. However, this result reflects a prisoners' dilemma type of outcome. All else held constant, any individual driver would be better off choosing to multihome. However, if all (or a sufficient number of) drivers choose to do so, the platforms would respond with a new price equilibrium, and at this new equilibrium, prices and demand are weakly lower, and drivers and customers are weakly (and, in some parameter regions, strictly) worse off than when drivers single-home and prices are set accordingly. Finally, we propose an incentive mechanism that, if implemented by platforms, could help mitigate the practice of multihoming.

The rest of this paper is organized as follows. We review relevant streams of literature in Section 2 and then present the basic problem formulation in Section 3. In Section 4, we analyze the case of single-homing and examine the impact of surge pricing. In Section 5, we analyze the case of multihoming, and in Section 6, we compare market outcomes and participant preferences between single-homing and multihoming. Section 7 discusses some extensions and robustness checks of our results, and provides concluding

remarks. The proofs of all results are deferred to an online supplement.

2. Related Literature

This paper is related to two distinct streams of literature. The first consists of economics research studying pricing behavior of multiple platforms competing in two-sided markets, and the second is a stream of operations management papers that incorporate more detailed operational considerations such as congestion effects or the physical distribution of drivers throughout the network.

In the economics literature, Rysman (2009 p. 125) defines a two-sided market as "one in which (1) two sets of agents interact through an intermediary or platform, and (2) the decisions of each set of agents affects the outcomes of the other set of agents, typically through an externality." Examples of this body of work include Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), Hagiu (2006, 2009), Armstrong and Wright (2007), Ambrus and Argenziano (2009), and Choi (2010). A common focus of much of this literature is the structure of equilibrium prices, addressing issues such as asymmetry in pricing between the two sides of the market (e.g., one side being subsidized in order to attract many customers, which in turn attracts participants from the other side of the market), whether price discrimination is possible, and the impact of multihoming on market behavior (as well as when it will arise in equilibrium). Rysman (2009) provides a broad discussion of some of this literature and how the analysis of two-sided markets relates to various questions of economic and public policy interest. One key way that our model and analysis differ from this research is that we include features that reflect operational realities that arise in many sharing-economy settings. Specifically, we include a measure of market congestion—the relative balance between the number of drivers and the number of customers—in the utility functions of both drivers and customers. This introduces negative externalities within each side of the market (e.g., all else equal, a customer/driver is worse off when more customers/drivers are participating in the market) that are absent in the bulk of the economics literature. (An exception to this is the paper by Hagiu (2009), which assumes that competition exists on the supply side, resulting in net profit per customer being a decreasing function of the number of suppliers on the platform. This is somewhat different than the congestion we model, and Hagiu's model does not include negative externalities on the customer side.) Bai and Tang (2018) study whether multiple platforms can simultaneously be profitable in a two-sided market. The authors characterize conditions under which Bertrand-type equilibria may

arise, making competing platforms earn no profit. Nikzad (2018) investigates how labor market size and competition between two-sided platforms affect driver and customer welfare, but unlike our work, this study does not consider suppliers' sensitivity to congestion—that is, suppliers are assumed to receive a fixed wage, and their net profit per customer does not depend on the number of suppliers or customers on the platform. Siddiq and Taylor (2019) consider competition between two ride-sharing platforms where one of the platforms can generate some of its supply through the acquisition of autonomous vehicles. The authors study how intensity of competition in the labor market affects prices and the size of the autonomous vehicle fleet and how the cost of autonomous vehicles affects the competing platform's profits. Once again, this work differs from ours in that congestion effects are not incorporated into the supply or demand functions.

The treatment of operational issues such as market congestion is naturally more prevalent in the operations literature studying two-sided platforms. Several of these papers construct formal queueing models to capture these effects. Riquelme et al. (2015) develop a queueing network model capturing real-time dynamics of a market with Poisson customer and driver entries and exponential trip durations. Drivers decide whether to offer their service based on expected earnings (which are a function of the current price and expected time spent waiting to obtain a customer), and customers accept the service if a driver is available and the current price is acceptable. Using some large market limiting arguments, the authors derive equilibrium behavior under dynamic threshold pricing policies (prices rise if the number of available drivers drops below a threshold). They show that such dynamic pricing does not allow the platform to increase throughput or revenues, but it is more robust than fixed pricing to changes in problem parameters. Taylor (2018) uses an $M/M/k$ queueing model to capture customer waiting times and derives the equilibrium number of participating drivers and effective customer arrival rate (the potential arrival rate times the probability that a customer chooses to seek service) given fixed prices and wages, and for a given realization of customer valuations. The paper focuses on how agent independence (i.e., the platform cannot control the agents' actions) and congestion disutility affect optimal prices and wages. In the absence of uncertainty in customer valuation and driver opportunity cost, the author shows that concerns about congestion should be addressed by raising wages and lowering prices but that these insights can break down in specific ways in the presence of such uncertainties. Note that one property of the underlying congestion model is that there are economies of scale on the driver side—as the number of drivers (with a particular

opportunity cost) increases, the equilibrium demand allocated to each driver increases. This is in contrast to our model, where there are *negative* externalities within each side of the platform (e.g., a given driver is harmed when there are more drivers participating). Bai et al. (2019) present a similar modeling framework (an $M/M/k$ queueing model for customer waiting times, with similar customer and driver utility functions) but with continuous customer valuation and driver opportunity costs, as well as variable service volume demands by customers. Under a fixed price-to-wage ratio, the authors study how various parameters—demand rate, sensitivity to waiting, service rate, and size of the driver pool—affect the optimal wage and price decisions.

Other papers capture supply-demand balance issues in ways that do not involve detailed queueing models. Cachon et al. (2017) take a rationing approach. The authors assume that if supply exceeds demand, then demand is spread equally across all drivers, and if demand exceeds supply, then capacity is rationed by randomly rejecting some customers. (For this paper and others using a rationing approach, note that there is no explicit customer disutility from congestion effects—customers are either served or not based on available supply.) Drivers decide whether to participate based on their rational expectations of earnings, and customer demand is captured by downwardly sloping (in price) demand curves. The authors consider five different wage and price contract types that allow varying degrees of flexibility (in the face of different market conditions such as surges in demand). They find that contracts where the price/wage ratio is held constant perform quite well relative to contracts with full flexibility, providing support for their use in practice. They also explore surge pricing, finding that customers may be better off if both wages and prices can adjust to demand conditions but that customers are hurt by pricing flexibility if wages are fixed. Gurvich et al. (2019) also employ rationing to reject demand that exceeds available capacity, but stochastic demand is exogenous and is not influenced by platform decisions. The authors focus on choices the platform can make to control supply—that is, limiting the number of potential drivers allowed in the system and guaranteeing drivers a given wage rate per unit of time (which can also be implemented via a piece-rate wage per service). The authors derive newsvendor-type results that provide insights into the "cost" of having drivers free to choose whether to participate at a given point in time. They also find that drivers' wages are higher in high-demand periods, but despite this, a larger fraction of customers are rejected as a result of supply shortages. Hu and Zhou (2017) model supply and demand more abstractly using functions that are increasing in wage and decreasing in price,

respectively, but they allow for multiple different versions reflecting different states of the world (e.g., weather). The authors demonstrate that requiring higher wages for drivers (which increases supply) may actually benefit customers, because this might lead to lower prices to spur additional demand to match the new supply. Using similar arguments, they also note that surge pricing could benefit customers. The authors then analyze a fixed commission contract (and variations on it) and explore conditions under which that contract can be shown to be optimal or within certain bounds of optimality. Reflecting the "gig economy" aspect of ride-sharing platforms, Benjaafar et al. (2017) relax the distinction between drivers and customers, allowing for individuals to endogenously choose among several different transportation strategies where they might take on either (or neither) role. The authors analyze the impact of the existence of a ride-sharing platform on car ownership, transportation choices, and congestion, and they explore how ownership and usage costs affect these outcomes. In more recent work, Benjaafar et al. (2018) study how the congestion level in a two-sided platform affects labor welfare. The authors identify two regimes, one in which expanding the labor pool improves welfare and another in which the opposite result holds.

A final group of operations papers focuses more on geospatial issues and/or aspects of matching customers with drivers. Afèche et al. (2018) present a model with two locations and four routes (within each location and between locations in each direction). Queueing models capture customer waiting at each of the locations. The authors consider different degrees of platform control over customer admission and driver repositioning from one location to another. They find that it is sometimes optimal to deny admission to customers even in low-demand scenarios in order to incentivize drivers to reposition themselves. Bimpikis et al. (2019) also model a network of multiple locations and explore how pricing can be used to motivate driver participation and repositioning to meet demand that may not be evenly spread across locations. In the face of such unbalanced underlying demand, the authors find that it is optimal for the platform to set prices and compensation in a way that increases the geospatial supply-demand balance. Besbes et al. (2018) consider a similar type of setting and derive structural results for supply equilibria. The authors use their model to analyze the optimal response to a demand shock and find that it may be beneficial to adjust prices to reduce driver profitability around such a shock, thus pushing additional supply toward the shock. Guda and Subramanian (2017) study platform use of forecast sharing and surge pricing to influence driver positioning in a two-location, two-period model. The

authors show that drivers may not trust surge demand forecasts, so price changes can help increase credibility. Counterintuitively, the authors show that it may be most effective to increase price in the location with excess supply, thus choking off demand and giving drivers incentive to move away from that location. Feng et al. (2017) present a stylized model of a circular road and compare two methods of matching customers with drivers—one where the platform makes the matches and one where drivers pick up the first customer they pass. The authors find that the first method works well for high and low traffic intensity, whereas the second method works better for medium traffic. Özkan and Ward (2020) also take supply and demand to be exogenous, and they analyze the problem of matching drivers with customers with the goal of maximizing the number of customers served. The authors propose a policy that they show is optimal in a large market asymptotic regime.

Outside the context of platform research, Johari et al. (2010) analyze oligopoly competition in markets with congestion effects. Unlike our work and the aforementioned studies, the authors focus on a setting where market participants are assumed to set their own prices and the market does not involve a two-sided platform.

In addition to some of the modeling details and the specific research questions addressed, a key distinction between our paper and the preceding operations papers is that the latter focus on a single platform, whereas we study two-sided markets with two competing platforms. Thus our work can be seen as incorporating key elements of both streams of literature. We combine the multiplatform nature of the economics literature with some of the operational aspects (i.e., congestion effects) of the operations literature.

As the preceding discussions indicate, the issue of surge pricing has received a fair amount of attention in recent years. Some existing analytical studies—for example, Cachon et al. (2017) and Hu and Zhou (2017)—have identified scenarios where surge pricing can benefit customers. Our analysis adds evidence that surge pricing is beneficial and extends these results to settings with customer congestion effects and multiple competing platforms. Several empirical studies have also looked at this issue. Using Uber data gathered in New York and San Francisco, Chen et al. (2015) find that surge pricing seems to have a small effect on increasing supply and a larger effect on decreasing demand. The data used in Diakopoulos (2015) (Uber data from five locations in Washington, DC) suggest that drivers may not start driving in response to surge pricing but that they seem to move into the surge area from nearby locations. In an extensive study, Chen and Sheldon (2015) collected data on more than 25 million Uber trips in five different

cities over a span of 10 months and found that surge pricing resulted in drivers working longer shifts and completing more rides. Hall et al. (2015) identify specific examples involving Uber in New York where surge prices helped keep waiting times for customers relatively low (or the lack of surge pricing had the opposite effect), and they may have increased aggregate driver earnings. Note that these empirical results generally find that surge pricing helps better align supply and demand, by reducing the number of customers willing to pay for a ride while incentivizing drivers to increase supply. Although this is consistent with certain aspects of our model and the dynamics of our system, the empirical studies do not directly address questions of individual customer and driver utility. Also, with the exception of Hall et al. (2015), increases in demand and surge pricing occur together, which makes it difficult to identify the potential benefits of adding surge pricing on top of a given demand surge as we do.

3. Problem Formulation
Consider two platforms, indexed by $i \in \{0, 1\}$, such that each platform enables two types of agents—namely, *customers* and *drivers*—to interact with each other.

Before presenting the details of the model, it may be helpful to clarify the time scale that it is intended to capture. The decisions and actions in the model take place within a relatively short time frame—for example, on the order of minutes or hours. Events that occur on that time scale include (a) underlying market demand intensity arises (perhaps depending on weather, sporting events, or other factors), (b) platforms set the prices customers will pay for service, (c) customer demand is realized in light of those prices and the level of supply in the market, and (d) driver participation in the market arises in light of prices and the level of demand in the market. Decisions that are longer term in nature have been made prior to the short-term decisions we analyze, and they are assumed to be fixed for the purposes of our basic analysis. These include (a) setting the broader compensation structure between drivers and platforms (the commission rate and share of service price paid by drivers to the platforms) and (b) driver choice of which platform to operate through (or whether to operate through both).

3.1. Platform Demand
There is a population of customers with heterogeneous preferences for the two platforms. We encode the customers' taste for platforms as a "location" parameter $v_d \in [0, 1]$; higher values of v_d represent stronger affinity toward platform 1, whereas lower values of v_d correspond to stronger affinity toward platform 0. The customer population is composed of a

continuum of infinitesimal customers whose location parameters v_d are distributed uniformly over $[0, 1]$ with intensity $\lambda > 0$. A customer's utility from using platform i depends on (a) her location parameter, v_d; (b) the price that the customer pays for using that platform, p_i; and (c) the level of supply congestion in that platform, measured by a convex increasing function of the utilization rate in the system, $\rho_i = d_i / s_i$, where d_i is the mass of customers receiving service on platform i and s_i is the mass of drivers active on platform i. (These quantities will be explained in more detail below.) In particular, we focus on quadratic congestion functions.

The negative impact of congestion reflects the delay experienced by the customer before the start of service. This consists of two elements—the time between a service request and being matched with a driver and the time between matching and the beginning of service. In services such as ride-sharing platforms, matching is nearly instantaneous in most cases, whereas the time for a driver to reach the customer is nontrivial, so our model focuses on the second element. To estimate the impact of utilization on this delay, we simulated a geographical region with 100 drivers and anywhere from 1 to 100 customers (representing different utilization levels). Customers and drivers were (independently) distributed according to a bivariate normal distribution (mimicking a city center and outlying areas). Customers and drivers were matched in a sequential manner—customers were randomly ordered and assigned in this order to the driver nearest to them. The average customer delay was computed at each utilization level, and a quadratic model was used to capture delay as a function of utilization. The model fit the data very well, yielding an R^2 value of approximately 0.98. This led to the choice of an increasing quadratic function to capture the negative impact of congestion on customer utility.

A customer with location parameter v_d derives the following respective utilities from using platforms 0 and 1:

$$u_{d0} = 1 - v_d - p_0 - \gamma \rho_0^2, \tag{1}$$

$$u_{d1} = v_d - p_1 - \gamma \rho_1^2, \tag{2}$$

where γ denotes the customers' sensitivity to congestion in the system. On the other hand, if the customer uses neither platform, she derives a net utility of 0 from using an outside option. Denote by $\underline{v}_d$ the location parameter of the customer who is indifferent between using platform 0 and the outside option. Similarly, we let $\bar{v}_d$ be the location of the customer indifferent between using platform 1 and the outside option. Then, $\underline{v}_d = 1 - p_0 - \gamma \rho_0^2$ and $\bar{v}_d = p_1 + \gamma \rho_1^2$. (Note that we assume customers are able to use either platform, which is consistent, for example, with the

ease of setting up an account with multiple ride-sharing platforms simply by downloading apps and entering some basic personal information. In that sense, customers are always assumed to multihome. However, we assume that each customer will only choose to use one platform—the one that delivers higher utility as expressed in (1) and (2).)

Depending on the values of $\underline{v}_d$ and $\bar{v}_d$, there are two possible cases regarding how the platforms share the market. In the first case, when $\underline{v}_d \leq \bar{v}_d$, any given customer derives positive utility from at most one platform, so that the customer segments served by the platforms are separated from each other. We refer to this case as a *partially covered market*, hereafter abbreviated as case P. The respective demands for platforms 0 and 1 in case P are

$$d_0 = \lambda \underline{v}_d = \lambda\left(1 - p_0 - \gamma\rho_0^2\right), \tag{3}$$

$$d_1 = \lambda(1 - \bar{v}_d) = \lambda\left(1 - p_1 - \gamma\rho_1^2\right). \tag{4}$$

Note that if the platforms' prices exceed 1, then the preceding demand expressions would become negative. This observation implies that there is a natural upper bound $\bar{p} \leq 1$ on the platforms' prices such that the demand for the platforms are well defined if their prices are below $\bar{p}$.

Figure 1 illustrates the demand for both platforms in case P. As can be seen in this figure, the condition characterizing case P—namely, $\underline{v}_d \leq \bar{v}_d$—holds if and only if the demands as specified in (3) and (4) satisfy $d_0 + d_1 \leq \lambda$.

The second case is the complement of the first case, characterized by the condition $\underline{v}_d > \bar{v}_d$. In this case, every customer derives positive utility from at least one of the platforms, so between them, the platforms serve the entire customer population; that is, the platforms actively compete for customers. We refer to this case as a *fully covered market*, hereafter abbreviated as case F. To express the customer demand in this case, let $\tilde{v}_d$ denote the location of the customer who is indifferent between platforms 0 and 1. Then, $\tilde{v}_d$ satisfies $1 - \tilde{v}_d - p_0 - \gamma\rho_0^2 = \tilde{v}_d - p_1 - \gamma\rho_1^2$; thus,

$\tilde{v}_d = \frac{1}{2}[1 + p_1 - p_0 + \gamma(\rho_1^2 - \rho_0^2)]$. As a result, the respective demands for platforms 0 and 1 in case F are

$$d_0 = \lambda \tilde{v}_d = \frac{1}{2}\lambda\left[1 - p_0 + p_1 - \gamma\left(\rho_0^2 - \rho_1^2\right)\right], \tag{5}$$

$$d_1 = \lambda(1 - \tilde{v}_d) = \frac{1}{2}\lambda\left[1 - p_1 + p_0 - \gamma\left(\rho_1^2 - \rho_0^2\right)\right]. \tag{6}$$

Therefore, in case F, we have $d_0 + d_1 = \lambda$. Figure 2 illustrates the demand for both platforms in case F.

3.2. Platform Supply

In ride-sharing settings (and similarly, in other two-sided markets), the process for becoming a driver is a bit more involved than it is for becoming a customer. As a result, the drivers' platform choices occur on a longer time scale than the one modeled here, so we take those choices as given in our analysis. In the basic model presented here, and in the analysis in Section 4, we focus on the case of single-homing, where each driver has selected a single platform in which to operate. In Section 5, we extend our analysis to the case of multihoming, and in Section 6, we compare the outcomes between the two scenarios.

The supply for platform $i \in \{0, 1\}$ is generated by a population of drivers with heterogeneous operational costs, denoted by k_{si}, which represent the cost of exerting the effort to pursue a service transaction. The driver population for platform i consists of a continuum of infinitesimal drivers whose operational costs k_{si} are distributed uniformly over $[0, 1]$ with a density normalized to 1. In addition to incurring these effort costs, to provide service to a customer using platform i, a driver must pay a commission $c \geq 0$ and share a fraction $\alpha \in [0, 1]$ of the revenue with the platform. This contract scheme can be viewed as a two-part tariff, which is commonly used in two-sided platforms in practice. For example, ride-sharing platforms Uber, Lyft, and Didi typically employ such contracts (*Global Times* 2016, Antczak 2017, Lyft 2018, RideGuru 2018).

A driver's utility from serving in platform i is determined by (a) the driver's operational cost, k_{si}; (b) the drivers' share of the price that the customers

Figure 1. Demand in Case P (Partially Covered Market)
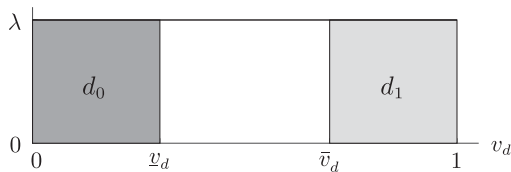
Notes. The dark shaded region labeled d_0 on the left shows the demand for platform 0, whereas the light shaded region labeled d_1 on the right shows the demand for platform 1. The unshaded region in the middle corresponds to the mass of customers who choose neither platform.

Figure 2. Demand in Case F (Fully Covered Market)
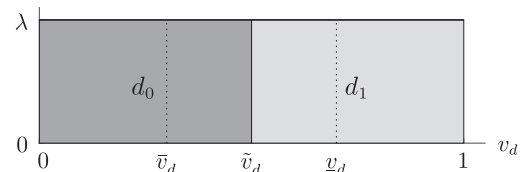
Note. The dark shaded region labeled d_0 on the left shows the demand for platform 0, and the light shaded region labeled d_1 on the right shows the demand for platform 1.

pay for using that platform, αp_i; (c) the commission charged by the platform for a service transaction, c; and (d) the level of congestion in the platform as captured by the utilization ρ_i. Because the latter equals the ratio of customers to drivers in the platform, it can be interpreted as the probability that a driver who chooses to operate will successfully receive net earnings of $\alpha p_i - c$. Therefore a driver with operational cost $k_{si} - c$ derives the following utility from being active in platform i:

$$u_{si} = -k_{si} + (\alpha p_i - c)\rho_i \quad \text{for } i \in \{0, 1\}. \tag{7}$$

If the driver chooses not to be active, then she derives a net utility of 0. Let $\bar{k}_{si}$ be the operational cost of the driver who is indifferent between being active in platform i and not. Then, $\bar{k}_{si} = (\alpha p_i - c)\rho_i$, and therefore, the supply for platform i is

$$s_i = \bar{k}_{si} = (\alpha p_i - c)\rho_i \quad \text{for } i \in \{0, 1\}. \tag{8}$$

There is a natural lower bound $\underline{p} > 0$ such that the supply for a platform is nonnegative if its price is above $\underline{p}$. We restrict attention to parameter values such that $\underline{p} < \overline{p}$, where $\overline{p}$ is the upper bound on prices that ensures that the demand for each platform is nonnegative (as explained in the paragraph following Equations (3) and (4)). Figure 3 illustrates the supply of platform $i \in \{0, 1\}$.

3.3. Pricing Game Formulation

The relations in (3), (5), and (8) imply a supply-demand balance for both platforms. For $i \in \{0, 1\}$, we let $D_i(p_i, p_j)$ and $S_i(p_i, p_j)$ denote the *balanced demand* and *balanced supply*, respectively, for platform i as functions of prices p_i and p_j, where $j \in \{0, 1\} \setminus \{i\}$. In subsequent sections, we provide derivations of these quantities.

At different stages in their development, platforms may focus on different objectives. Firms that are still establishing themselves may focus primarily on growth. This currently seems to be the case for ride-sharing platforms such as Uber and Lyft (see, e.g., *Knowledge@ Wharton* (2017), Sherman (2017), Wilhelm (2017), and Cook and Price (2018)). Once firms are more established they may focus on maximizing profit instead. We assume the profit maximization objective; that is,

Figure 3. Supply of a Platform

Notes. The shaded region labeled s_i on the left shows the supply of platform $i \in \{0, 1\}$. The unshaded region on the right shows the mass of drivers on platform i who choose not to be active.

platform $i \in \{0, 1\}$ selects price p_i to maximize $\Pi_i(p_i, p_j)$: $= [(1 - \alpha)p_i + c] \min\{D_i(p_i, p_j), S_i(p_i, p_j)\}$. When $\alpha = 1$, this objective is equivalent to maximizing the number of customers served.

Platforms choose prices in $[\underline{p}, \overline{p}]$ such that balanced demand and balanced supply are well defined and nonnegative. As will be shown later, there is a suitable choice of $[\underline{p}, \overline{p}]$ such that, for $i \in \{0, 1\}$ and all p_j, the mapping $p_i \mapsto \Pi_i(p_i, p_j)$ is concave, with its maximizer in the interior of $[\underline{p}, \overline{p}]$. The prices selected by the platforms must also satisfy the following supply constraint:

$$D_i(p_i, p_j) \leq S_i(p_i, p_j). \tag{9}$$

This constraint reflects the fact that, regardless of which objective a platform focuses on, the firm wants to avoid unsatisfied demand in order to establish and maintain customer acceptance of and satisfaction with the service. Note that under the supply constraint (9), we have $\min\{D_i(p_i, p_j), S_i(p_i, p_j)\} = D_i(p_i, p_j)$, and thus, $\Pi_i(p_i, p_j) = [(1 - \alpha)p_i + c]D_i(p_i, p_j)$ for $i \in \{0, 1\}$. We also note that as long as the demand congestion effect is not too small (i.e., $\gamma \geq \max\{[(\lambda - \alpha)(\alpha - c)]/(\alpha^2 + 3\alpha\lambda), 0\}$), the supply constraint is redundant in equilibrium—that is, prices that constitute an equilibrium when the supply constraint is present continue to be equilibrium prices in the absence of this constraint.

We say that a pair of prices $(p_0^*, p_1^*) \in [\underline{p}, \overline{p}] \times [\underline{p}, \overline{p}]$ is a Nash equilibrium if

$$\Pi_0(p_0^*, p_1^*) \geq \Pi_0(\tilde{p}_0, p_1^*)$$
$$\text{for all } \tilde{p}_0 \in [\underline{p}, \overline{p}] \text{ satisfying (9) for } i = 0,$$
$$\Pi_1(p_1^*, p_0^*) \geq \Pi_1(\tilde{p}_1, p_0^*)$$
$$\text{for all } \tilde{p}_1 \in [\underline{p}, \overline{p}] \text{ satisfying (9) for } i = 1.$$

As mentioned earlier, we focus on actions and events that are short term in nature. As a result, this formulation focuses on price equilibria and does not incorporate the commission structure in the decision variables (i.e., α and c are exogenous problem parameters), because that structure is a long-term decision that is typically not changed in response to short-term shocks. (See, e.g., Uber (2018) and RideGuru (2018) for discussions of this issue.) By focusing the model in this way, we can investigate how platforms react to short-term changes in market conditions; for example, we are interested in whether and how ride-sharing platforms should implement surge pricing when there is a positive demand shock. As will be explained in the following section, we will concentrate on symmetric equilibria to provide a crisp analysis of how platforms respond to shifts in problem parameters. This is why we assume that the commission structure (i.e., α and c) and the form of the supply functions (8) are the same for both platforms.
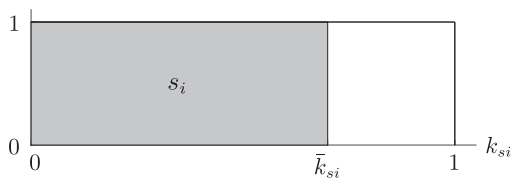
4. Analysis for Single-Homing Drivers
4.1. Characterizing Equilibria
This section studies price equilibria in the setting of single-homing drivers described in Section 3.

As alluded to earlier, the existence of price equilibria depends on well-defined balanced supply and demand functions. From Equations (3), (5), and (8), it is clear that supply and demand on a given platform interact with each other through the supply and demand congestion effects. As a first step in analyzing that interaction, we note that if drivers single-home, then the supply of a platform does not depend directly on the other platform's supply or demand. To be more precise, the supply of platform $i \in \{0, 1\}$ can be written as a function of the demand for the same platform: from (8), we have

$$s_i = \sqrt{(\alpha p_i - c)d_i} \quad \text{for } i \in \{0, 1\}. \tag{10}$$

An analogous statement can be made about demand in the case of a partially covered market, but not in the case of a fully covered market, and so the behavior of balanced demand and balanced supply differs depending on which of these two cases arises in response to the prices p_0 and p_1 charged by the platforms. We next derive the balanced demand and supply functions for the single-homing setting.

Lemma 1. *The balanced demand for platform i, as a function of prices p_i and p_j, is given by*

$$
\begin{aligned}
&D_i(p_i, p_j) \\
&= \begin{cases} d_i^{\mathrm{P}}(p_i) & \text{if } d_0^{\mathrm{P}}(p_0) + d_1^{\mathrm{P}}(p_1) \leq \lambda \quad [\text{Case P}], \\ d_i^{\mathrm{F}}(p_i, p_j) & \text{if } d_0^{\mathrm{P}}(p_0) + d_1^{\mathrm{P}}(p_1) > \lambda \quad [\text{Case F}], \end{cases}
\end{aligned}
\tag{11}
$$

where

$$d_i^{\mathrm{P}}(p_i) = \lambda \frac{(\alpha p_i - c)(1 - p_i)}{\alpha p_i - c + \lambda \gamma}, \tag{12}$$

$$
\begin{aligned}
&d_i^{\mathrm{F}}(p_i, p_j) \\
&= \lambda \frac{(\alpha p_i - c)[\lambda \gamma + (\alpha p_j - c)(1 - p_i + p_j)]}{2(\alpha p_i - c)(\alpha p_j - c) + \lambda \gamma [\alpha(p_i + p_j) - 2c]},
\end{aligned}
\tag{13}
$$

for $i, j \in \{0, 1\}$ with $i \neq j$. The corresponding balanced supply for platform i is given by

$$
\begin{aligned}
&S_i(p_i, p_j) \\
&= \begin{cases} s_i^{\mathrm{P}}(p_i) & \text{if } d_0^{\mathrm{P}}(p_0) + d_1^{\mathrm{P}}(p_1) \leq \lambda \quad [\text{Case P}], \\ s_i^{\mathrm{F}}(p_i, p_j) & \text{if } d_0^{\mathrm{P}}(p_0) + d_1^{\mathrm{P}}(p_1) > \lambda \quad [\text{Case F}], \end{cases}
\end{aligned}
\tag{14}
$$

where

$$s_i^{\mathrm{P}}(p_i) = (\alpha p_i - c)\sqrt{\frac{\lambda(1 - p_i)}{\alpha p_i - c + \lambda \gamma}}, \tag{15}$$

$$
\begin{aligned}
s_i^{\mathrm{F}}(p_i, p_j) &= (\alpha p_i - c) \\
&\times \sqrt{\frac{\lambda[\lambda \gamma + (\alpha p_j - c)(1 - p_i + p_j)]}{2(\alpha p_i - c)(\alpha p_j - c) + \lambda \gamma [\alpha(p_i + p_j) - 2c]}},
\end{aligned}
\tag{16}
$$

for $i, j \in \{0, 1\}$ with $i \neq j$.

In the expressions in (11)–(16), the superscript "P" denotes partial market coverage (case P), whereas the superscript "F" denotes full market coverage (case F). It follows from Lemma 1 that the condition characterizing case P (i.e., $\underline{v}_d \leq \bar{v}_d$) holds if $d_0^{\mathrm{P}}(p_0) + d_1^{\mathrm{P}}(p_1) \leq \lambda$, whereas the condition characterizing case F (i.e., $\underline{v}_d > \bar{v}_d$) holds if $d_0^{\mathrm{P}}(p_0) + d_1^{\mathrm{P}}(p_1) > \lambda$ (see Figures 1 and 2 for illustrations of these two conditions). Note that in case P, the platforms act similar to two local monopolies, setting the same prices as they would if the other platform did not exist. In case F, on the other hand, platforms actively compete through prices for customers who have positive utility for both platforms. We next show the existence of a price equilibrium between the platforms.

Theorem 1. *There exists a unique symmetric price Nash equilibrium for the platforms.*

To understand the equilibrium behavior for each set of input parameters, for the remainder of this section we assume that $\alpha = 1$ for analytical tractability. We revisit the case of general α in Section 7.

The next result derives conditions under which the equilibrium prices result in partial or full market coverage (i.e., local monopolistic behavior or active competition).

Proposition 1. *The following are the symmetric equilibrium prices for each set of parameters λ, γ, and c:*

$$
\begin{cases}
(a) \quad p^* = c + \sqrt{\lambda^2 \gamma^2 + \lambda \gamma (1 - c)} - \lambda \gamma \\
\qquad \text{for } \gamma \geq \frac{(\frac{1}{2} - c)^2}{2\lambda} \text{ and } \gamma \geq \frac{\lambda(1 - c)}{1 + 2\lambda}, \\
(b) \quad p^* = c + \frac{\lambda(1 - \gamma - c)}{1 + \lambda} \\
\qquad \text{for } \gamma \geq \frac{1}{2} - \frac{\lambda}{2} - c \text{ and } \gamma < \frac{\lambda(1 - c)}{1 + 2\lambda}, \\
(c) \quad p^* = c + \frac{\lambda}{2} \\
\qquad \text{for } \gamma < \frac{1}{2} - \frac{\lambda}{2} - c \text{ and } \gamma < \frac{\lambda}{2}, \\
(d) \quad p^* = c + \sqrt{\frac{\lambda \gamma}{2}} \\
\qquad \text{for } \gamma < \frac{(\frac{1}{2} - c)^2}{2\lambda} \text{ and } \gamma \geq \frac{\lambda}{2}.
\end{cases}
$$

In region (a), the platforms partially cover the market, and the equilibrium demand is strictly lower than equilibrium supply. In region (b), the platforms again partially cover the market, but equilibrium demand equals equilibrium

supply for each platform. In regions (c) and (d), the platforms fully cover the market, with equilibrium demand equal to equilibrium supply for each platform in region (c) and equilibrium demand strictly lower than equilibrium supply for each platform in region (d).

Figure 4 illustrates regions (a)–(d) in the (λ, γ) space, showing how some of the underlying parameters affect the relative balance between the number of customers who receive service and both the total market size and the number of participating drivers. As the overall market size (represented by λ) gets larger, it is more likely that platforms will not directly compete for marginal customers because some of the potential market will go unserved (i.e., we move toward regions (a) and (b) in Figure 4) and that all participating drivers will be occupied serving customers (i.e., we move toward regions (b) and (c) in Figure 4). As customers become more sensitive to congestion (i.e., γ increases), it is also more likely that some of the potential market will go unserved, but this is driven by reduced customer utility from getting service, and thus reduced customer participation, whereas the impact of λ stems from the growth in the potential market size outpacing the drivers' willingness to participate. Note that γ has the opposite effect of λ on the supply-demand balance—as γ increases, the decreased willingness of customers to participate moves the system toward having more drivers than customers.

The preceding visual representations of the equilibrium outcomes can also be useful for obtaining insights into the impact of competition between platforms in this market. In regions (a) and (b) in Figure 4, the platforms effectively operate as local monopolies because the customer segments they serve are distinct and separate. In these regions, neither platform is affected by the presence of a competitor. In regions (c) and (d), however, the platforms directly interact and compete for customers who have positive utilities for both platforms, yielding different outcomes than if only a single platform were operating in the market. The following result summarizes the impact on a platform's equilibrium outcomes of facing competition from a second platform, compared with a scenario where the platform has a true monopoly in the market. To compare utilities in this result, we refer to a customer who uses either of the platforms as a purchasing customer and to a driver who offers service in either of the platforms as an active driver.
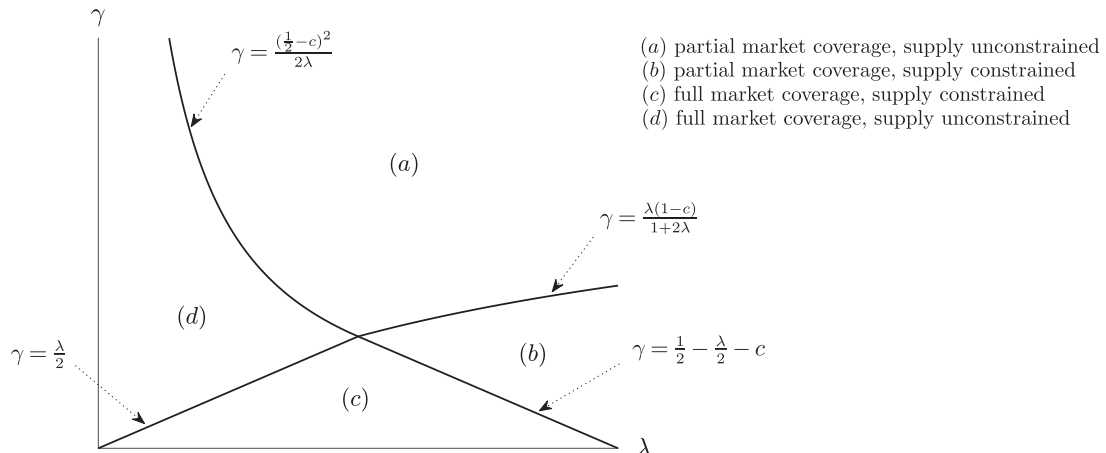
Proposition 2. *When a platform faces competition, the equilibrium values for platform price, supply, demand, utilization, and an active driver's utility are all (weakly) lower, whereas the equilibrium value for a purchasing customer's utility is (weakly) higher, relative to the case of a monopolist platform.*

As one might expect, the presence of competition drives equilibrium prices lower. Because the two platforms split the market, each platform's demand is lower (making a platform worse off than if it were a monopoly), and so it does not need to attract as many drivers. Proposition 2 also shows that competition results in lower congestion, benefiting customers. With lower prices and utilization, customers' utilities are higher but drivers are worse off in the presence of competition.

4.2. Analysis of Surge Pricing

We next build on our price equilibrium results to examine the impact of surge pricing on customers and drivers. More specifically, we study the impact of a surge in potential demand—as represented by an increase in the market size λ—on the equilibrium prices and other relevant quantities. Consistent with

Figure 4. Price Equilibrium Regions in the (λ, γ) Space When Drivers Are Single-Homing

(a) partial market coverage, supply unconstrained
(b) partial market coverage, supply constrained
(c) full market coverage, supply constrained
(d) full market coverage, supply unconstrained

$\gamma = \frac{(\frac{1}{2}-c)^2}{2\lambda}$

$\gamma = \frac{\lambda(1-c)}{1+2\lambda}$

$\gamma = \frac{1}{2} - \frac{\lambda}{2} - c$

$\gamma = \frac{\lambda}{2}$

Note. The cases (a), (b), (c), and (d) given in Theorem 1 are shown by regions (a), (b), (c), and (d), respectively, in the (λ, γ) space.

our focus on short time frames, the types of market size increase we consider are those of relatively short duration. Such market shifts can be the result of unusual weather events (e.g., rain, snow) or increased traffic related to public events (e.g., sport games, shows). In regions that are supply constrained (regions (b) and (c) in Figure 4), it is clearly necessary for the platforms to adjust prices to ensure sufficient supply to meet the higher demand levels resulting from an increased market size. For regions that are supply unconstrained, it is less clear whether allowing such an adjustment would benefit various participants in the market, and so we focus on these regions. It is first easy to verify that the equilibrium prices are increasing in λ, indicating that a surge in the market size is accompanied by surge pricing in equilibrium. Our following result establishes the impact of an increase in the market size on customer and driver utilities. To state this result, we let $p^*(\lambda)$ be the equilibrium price characterized in Theorem 1, with its dependence on the market size λ expressed explicitly. As before, let us refer to a customer who uses either of the platforms as a purchasing customer and to a driver who offers service in either of the platforms as an active driver. With a slight abuse of notation, we also let $u_{di}(p, \lambda)$ and $u_{si}(p, \lambda)$ denote the utilities of a purchasing customer and an active driver, respectively, for platform $i \in \{0, 1\}$, when both platforms charge the price p and the market size is λ.

Proposition 3. *Consider two levels of the market size, $\lambda < \hat{\lambda}$, such that the pairs (λ, γ) and $(\hat{\lambda}, \gamma)$ are in regions of unconstrained supply (i.e., regions (a) and (d) in Figure 4).*

i. If the platforms adjust their equilibrium prices in response to a market size surge from λ to $\hat{\lambda}$, then a purchasing customer's utility decreases and an active driver's utility increases. That is, for $i \in \{0, 1\}$,

$$u_{di}(p^*(\lambda), \lambda) > u_{di}(p^*(\hat{\lambda}), \hat{\lambda}) \quad and$$
$$u_{si}(p^*(\lambda), \lambda) < u_{si}(p^*(\hat{\lambda}), \hat{\lambda}).$$

ii. Given λ, there exists $\Lambda(\lambda, \gamma) \in (\lambda, \infty]$ such that for $\hat{\lambda} \in (\lambda, \Lambda(\lambda, \gamma)]$, purchasing customers and active drivers are better off if the platforms adjust their equilibrium prices in response to a surge in the market size than if the platforms keep their original equilibrium prices. That is, for $i \in \{0, 1\}$,

$$u_{di}(p^*(\lambda), \hat{\lambda}) < u_{di}(p^*(\hat{\lambda}), \hat{\lambda}) \quad and$$
$$u_{si}(p^*(\lambda), \hat{\lambda}) < u_{si}(p^*(\hat{\lambda}), \hat{\lambda}).$$

Both parts of Proposition 3 are illustrated in Figure 5. This figure displays two instances of the supply and demand curves for a particular platform (as a function of that platform's price)—the lower instance of each curve corresponds to the original market size λ, whereas the higher one corresponds to the larger market size $\hat{\lambda}$.

Figure 5. Surge Pricing When Drivers Are Single-Homing
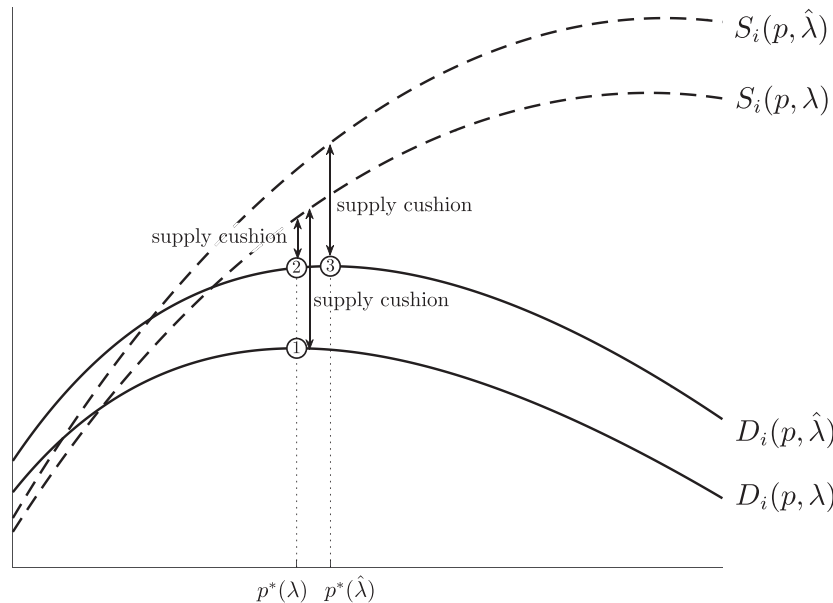
Notes. The lower and upper bold curves display the balanced demand functions when the market size is $\lambda = 0.6$ and $\hat{\lambda} = 0.8$, respectively. Similarly, the lower and upper dashed curves display the balanced supply functions when the market size is $\lambda = 0.6$ and $\hat{\lambda} = 0.8$, respectively. Vertical coordinates of points ①, ②, and ③ are $D_i(p^*(\lambda), \lambda)$, $D_i(p^*(\lambda), \hat{\lambda})$, and $D_i(p^*(\hat{\lambda}), \hat{\lambda})$, respectively, (and their horizontal coordinates are $p^*(\lambda)$, $p^*(\lambda)$, and $p^*(\hat{\lambda})$, respectively). The other problem parameters are $c = 0.2$ and $\gamma = 0.4$.

Part (i) of Proposition 3 reflects a comparison between point ①, which shows the original equilibrium prices at the original market size, and point ③, which shows the new (higher) equilibrium prices at the larger market size. The proposition states that an increase in the market size accompanied by surge pricing results in a higher utility for drivers and a lower utility for customers. Note that there is a larger "supply cushion" of drivers at point ① (the distance between that point and the lower supply curve) than at point ③ (the distance between that point and the higher supply curve), and this corresponds to a higher utilization ρ_i at point ③ than at point ①. This makes any original customers worse off at point ③, as does the higher price, and so their utility drops. Drivers experience the opposite—the higher utilization combined with the higher price causes their utilities to increase. Following the common belief on surge pricing (Lowrey 2014, David 2016), higher prices come at the expense of lower customer utility. However, this comparison is somewhat misleading. Given that an exogenous event caused demand to surge from its original level λ to a higher level $\hat{\lambda}$, the question is whether customers are better off when this demand surge is accompanied by a surge in prices, rather than by platforms keeping their original equilibrium prices that were associated with the original market size λ. Part (ii) of Proposition 3 explores that question.

A comparison of point ② and point ③ in Figure 5 helps illustrate part (ii). The move from point ② to point ③ reflects just each platform's reoptimization of price *given* the new larger market size $\hat{\lambda}$. Both customers and drivers experience a trade-off. As one might expect, and as opponents of surge pricing might argue, on its own, the increased price hurts customers but helps drivers. On the other hand, the supply cushion (the gap between the upper demand curve and the upper supply curve) increases as we move from point ② to point ③—that is, the utilization decreases, which is good for customers but bad for drivers. The net impact of these contrary effects is not obvious a priori, but Proposition 3 shows that both customers and drivers are better off at point ③—that is, they both benefit from allowing surge pricing. This is perhaps least easy to anticipate from the customer's perspective, and this further illustrates the importance of capturing the congestion effects on customer and driver utilities. Finally, whereas part (ii) of the proposition explicitly states that utilities increase for purchasing customers and active drivers, we note that this also implies that some additional customers and/or drivers may become active (because they now have positive utility), so all customers and drivers are at least weakly better off.

5. Analysis for Multihoming Drivers

This section extends our formulation and analysis to a setting in which drivers multihome (i.e., simultaneously participate in both platforms). Recall that we focus our analysis on events that happen within short time frames. Because the choice of which platform(s) to participate in is a longer-term decision by the drivers, we take this choice (including the choice of whether to single- or multihome) as exogenous. In this setting, the driver populations of the two platforms jointly serve the entire customer population that generates the demand for platforms. Accordingly, the utilizations of the platforms are based on aggregate supply and demand; that is,

$$\rho_0 = \rho_1 = \frac{d_0 + d_1}{s_0 + s_1}. \tag{17}$$

In the single-homing setting studied in preceding sections, a driver's utility from serving in platform $i \in \{0, 1\}$ (expressed as u_{si} in (7)) depends only on the price for using that platform, p_i. In the multihoming setting, drivers face a mixed population of customers across both platforms, implying that u_{si} depends on both prices. Supposing that the likelihood of serving a given platform's customer is proportional to the demand for that platform, we replace the price term p_i in (7) with $\frac{d_i}{d_0+d_1} p_i + \frac{d_j}{d_0+d_1} p_j$ to obtain the following expression for u_{si}:

$$u_{si} = -k_{si} + \left[\alpha \left(\frac{d_i}{d_0 + d_1} p_i + \frac{d_j}{d_0 + d_1} p_j \right) - c \right] \rho_i \tag{18}$$
$$\text{for } i, j \in \{0, 1\} \text{ with } i \neq j.$$

Based on this, the supply of platform i is given by

$$s_i = \left[\alpha \left(\frac{d_i}{d_0 + d_1} p_i + \frac{d_j}{d_0 + d_1} p_j \right) - c \right] \rho_i \tag{19}$$
$$\text{for } i, j \in \{0, 1\} \text{ with } i \neq j.$$

By (17) and (19), we can also express the supply of platforms as follows:

$$s_0 = s_1 = \sqrt{\frac{(\alpha p_0 - c)d_0 + (\alpha p_1 - c)d_1}{2}}, \tag{20}$$

which is analogous to (10) in the single-homing analysis. As in the preceding section, the analysis of price equilibria depends on well-defined balanced demand and supply functions. We next derive the balanced demand and supply functions for the multihoming setting.

Lemma 2. *There exists a compact set $\mathcal{P} \subseteq [\underline{p}, \overline{p}] \times [\underline{p}, \overline{p}]$ with $\mathcal{P} \supset \{(p_0, p_1) \in [\underline{p}, \overline{p}] \times [\underline{p}, \overline{p}] : p_0 = p_1\}$ such that for*

all price pairs $(p_0, p_1) \in \mathcal{P}$, *the balanced demand for platform i is well defined and given by*

$$
D_i(p_i, p_j)
$$
$$
= \begin{cases} d_i^{\mathrm{P}}(p_i, p_j) & \text{if } d_0^{\mathrm{P}}(p_0, p_1) + d_1^{\mathrm{P}}(p_1, p_0) \leq \lambda \\ & \qquad\qquad\qquad\qquad [\text{Case P}], \\ d_i^{\mathrm{F}}(p_i, p_j) & \text{if } d_0^{\mathrm{P}}(p_0, p_1) + d_1^{\mathrm{P}}(p_1, p_0) > \lambda \\ & \qquad\qquad\qquad\qquad [\text{Case F}], \end{cases}
$$
$$
(21)
$$

where

$$
d_i^{\mathrm{P}}(p_i, p_j) = \frac{\lambda}{2} [\xi_+(p_0, p_1) - p_i + p_j], \qquad (22)
$$

$$
d_i^{\mathrm{F}}(p_i, p_j) = \frac{\lambda}{2} (1 - p_i + p_j), \qquad (23)
$$

for $i, j \in \{0, 1\}$ *with* $i \neq j$, *and* $\xi_+(p_0, p_1) = \frac{-B(p_0,p_1) + \sqrt{B(p_0,p_1)^2 - 4A(p_0,p_1)C(p_0,p_1)}}{2A(p_0,p_1)}$ *with* $A(p_0, p_1) = 2[\lambda\gamma + \frac{\alpha(p_0+p_1)}{2} - c]$, $B(p_0, p_1) = 4[c - \frac{(\alpha+c)(p_0+p_1)}{2} + \alpha p_0 p_1]$, *and* $C(p_0, p_1) = \alpha(2 - p_0 - p_1)(p_0 - p_1)^2$. *Moreover, for all price pairs* $(p_0, p_1) \in \mathcal{P}$, $\xi_+(p_0, p_1)$ *is concave in* p_0 *and* p_1. *The corresponding balanced supply for platform i is*

$$
S_i(p_i, p_j)
$$
$$
= \begin{cases} s_i^{\mathrm{P}}(p_i, p_j) & \text{if } d_0^{\mathrm{P}}(p_0, p_1) + d_1^{\mathrm{P}}(p_1, p_0) \leq \lambda \\ & \qquad\qquad\qquad\qquad [\text{Case P}], \\ s_i^{\mathrm{F}}(p_i, p_j) & \text{if } d_0^{\mathrm{P}}(p_0, p_1) + d_1^{\mathrm{P}}(p_1, p_0) > \lambda \\ & \qquad\qquad\qquad\qquad [\text{Case F}], \end{cases}
$$
$$
(24)
$$

where

$$
s_i^{\mathrm{P}}(p_i, p_j)
$$
$$
= \frac{1}{2}\sqrt{\lambda\xi_+(p_0, p_1)[\alpha(p_0 + p_1) - 2c] - \lambda\alpha(p_0 - p_1)^2},
$$
$$
(25)
$$

$$
s_i^{\mathrm{F}}(p_i, p_j) = \frac{1}{2}\sqrt{\lambda[\alpha(p_0 + p_1) - 2c] - \lambda\alpha(p_0 - p_1)^2}, \quad (26)
$$

for $i, j \in \{0, 1\}$ *with* $i \neq j$.

Because of the interdependence of supply and demand quantities, the existence of a well-defined supply-demand balance entails additional conditions in the multihoming setting. We note that in the single-homing setting, both platforms have well-defined and concave payoff functions. To have a well-defined pricing game with concave payoffs in the multihoming setting, one needs to restrict the action space of the two platforms. For this purpose, we show in Lemma 2 that there exists a restricted subset of the price space—namely, $\mathcal{P}$—in which both platforms have well-defined and concave demand functions. As will be shown later, this also yields well-defined and concave payoff functions for

both platforms. Therefore, the multihoming setting can accommodate a well-defined pricing game with a restricted action space that contains all candidates for symmetric equilibria (i.e., the price pairs on the line $p_0 = p_1$).

It follows from Lemma 2 that the condition for case P (i.e., $\underline{v}_d \leq \bar{v}_d$) holds if $d_0^{\mathrm{P}}(p_0, p_1) + d_1^{\mathrm{P}}(p_1, p_0) \leq \lambda$, and the condition for case F (i.e., $\underline{v}_d > \bar{v}_d$) holds if $d_0^{\mathrm{P}}(p_0, p_1) + d_1^{\mathrm{P}}(p_1, p_0) > \lambda$ (see Figures 1 and 2 for illustrations of these two conditions). Given (21), platforms choose prices p_0 and p_1 from $\mathcal{P}$ in equilibrium, with platform i maximizing its profit $\Pi_i(p_i, p_j)$ while satisfying the following joint supply constraint:

$$
D_0(p_0, p_1) + D_1(p_1, p_0) \leq S_0(p_0, p_1) + S_1(p_1, p_0). \quad (27)
$$

As in single-homing, this constraint is redundant in equilibrium if $\gamma \geq \max\{[(\lambda - \alpha)(\alpha - c)]/(\alpha^2 + 3\alpha\lambda), 0\}$.

In the context of multihoming drivers, we say that a pair of prices $(p_0^*, p_1^*) \in \mathcal{P}$ is a Nash equilibrium if

$$
\Pi_0(p_0^*, p_1^*) \geq \Pi_0(\tilde{p}_0, p_1^*)
$$
$$
\text{for all } \tilde{p}_0 \text{ satisfying } (\tilde{p}_0, p_1^*) \in \mathcal{P} \text{ and } (27),
$$
$$
\Pi_1(p_1^*, p_0^*) \geq \Pi_1(\tilde{p}_1, p_0^*)
$$
$$
\text{for all } \tilde{p}_1 \text{ satisfying } (p_0^*, \tilde{p}_1) \in \mathcal{P} \text{ and } (27).
$$

In the next result, we show the existence of a price equilibrium between the platforms.

Theorem 2. *There is a set* Θ *of problem parameters such that if* $(\lambda, \gamma, \alpha, c) \in \Theta$, *then there exists a unique symmetric price Nash equilibrium for the platforms.*

As in the preceding section, with the goal of understanding the equilibrium behavior for each set of input parameters, we assume that $\alpha = 1$ going forward (and we revisit the case of general α in Section 7). The following result characterizes the set Θ in closed form and the equilibrium in the different cases of market coverage.

Proposition 4. *Let* $\Theta = \Theta_1 \cap \Theta_2$, *where* $\Theta_1 = \{(\lambda, \gamma, c) \in \mathbb{R}_+^3 : 2(1 - c)^2 \geq \lambda\gamma(1 - c) + \lambda^2\gamma^2\}$ *and* $\Theta_2 = \{(\lambda, \gamma, c) \in \mathbb{R}_+^3 : \gamma \leq \frac{1}{2} - \frac{\lambda}{2} - c \text{ or } \gamma \geq \frac{3\sqrt{3 - 2c + c^2} + c - 5}{2\lambda}\}$. *If* $(\lambda, \gamma, c) \in \Theta$ *then the following are the symmetric equilibrium prices for each set of parameters* λ, γ, *and* c:

$$
\begin{cases} (\tilde{a}) & p^* = c + \sqrt{\frac{\lambda^2\gamma^2 + \lambda\gamma(1-c)}{2}} - \lambda\gamma \\ & \text{for } \gamma \geq \frac{3\sqrt{3 - 2c + c^2} + c - 5}{2\lambda} \text{ and } \gamma \geq \frac{2\lambda(1-c)}{1 + 2\lambda - \lambda^2}, \\ (\tilde{b}) & p^* = c + \frac{\lambda(1 - \gamma - c)}{1 + \lambda} \\ & \text{for } \gamma \geq \frac{1}{2} - \frac{\lambda}{2} - c \text{ and } \gamma < \frac{2\lambda(1-c)}{1 + 2\lambda - \lambda^2}, \\ (\tilde{c}) & p^* = c + \frac{\lambda}{2} \\ & \text{for } \gamma < \frac{1}{2} - \frac{\lambda}{2} - c. \end{cases}
$$

In region ($\tilde{a}$), the platforms partially cover the market, and the equilibrium demand is strictly lower than equilibrium supply. In region ($\tilde{b}$), the platforms again partially cover the market, but equilibrium demand equals equilibrium supply. In region ($\tilde{c}$), the platforms fully cover the market, with equilibrium demand equal to equilibrium supply.

Figure 6 depicts regions ($\tilde{a}$)–($\tilde{c}$) in the (λ, γ) space. From this figure we see that the impacts the parameters have on driver utilization and whether the market is fully covered are qualitatively the same as for the case of single-homing. Specifically, as the market size λ increases, platform competition for customers is less intense, and all drivers are generally more likely to be occupied serving customers; as customer sensitivity to congestion γ increases, it is also more likely that the market will be partially covered but that there will be more drivers than customers in the market.

As stated in Theorem 2 and Proposition 5, there is a price equilibrium between the platforms as long as the problem parameters reside in the set Θ. If the problem parameters are outside Θ, then there is no price equilibrium because the balanced demand expression would either become negative or exceed the total market size, meaning that the balanced demand would be ill-defined in equilibrium. In particular, in the context of Proposition 5, if $(\lambda, \gamma, c) \notin \Theta_1$, then the balanced demand becomes negative, whereas if $(\lambda, \gamma, c) \notin \Theta_2$, the balanced demand exceeds the total market size.

6. Comparison of Single-Homing and Multihoming

In this section, we compare the equilibrium outcomes in the single-homing and multihoming settings. In particular, we study how price, demand, and utilities of customers and drivers change in equilibrium when drivers switch from single-homing (SH) to multihoming (MH). Consistent with our earlier focus on symmetric platforms and symmetric equilibria, in this section we generally suppress the platform index for notational simplicity.

Given problem parameters for which price equilibria exist as in Propositions 1 and 4, we let p^*_{SH} and p^*_{MH} be the equilibrium prices in the settings of single-homing and multihoming, respectively. To compare the performance of platforms in these two settings, we also let D^*_{SH} and D^*_{MH} be the equilibrium demand quantities under single-homing and multihoming, respectively. For a purchasing customer (i.e., a customer who uses either of the platforms), let $u^*_{d,\text{SH}}$ and $u^*_{d,\text{MH}}$ be the equilibrium utilities of said customer under single-homing and multihoming, respectively. Then,
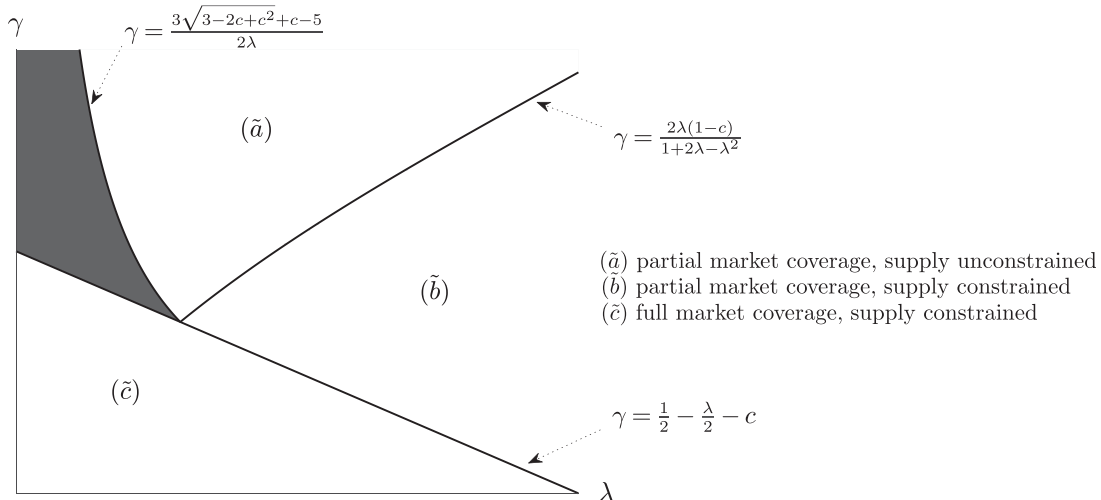
$$u^*_{d,\text{SH}} - u^*_{d,\text{MH}} = p^*_{\text{MH}} + \gamma \left(\rho^*_{\text{MH}} \right)^2 - p^*_{\text{SH}} - \gamma \left(\rho^*_{\text{SH}} \right)^2,$$

where ρ^*_{SH} and ρ^*_{MH} are the equilibrium utilizations under single-homing and multihoming, respectively. Similarly, for an active driver (i.e., a driver who offers service in either of the platforms), we let $u^*_{s,\text{SH}}$ and $u^*_{s,\text{MH}}$ be the equilibrium utility of this driver under single-homing and multihoming, respectively. That is,

$$u^*_{s,\text{SH}} - u^*_{s,\text{MH}} = \left(p^*_{\text{SH}} - c \right) \rho^*_{\text{SH}} - \left(p^*_{\text{MH}} - c \right) \rho^*_{\text{MH}}.$$

We note that the parameter regions characterized in Propositions 1 and 4 are distinct. Therefore, to compare the equilibrium outcomes in each possible case, we need to consider how the regions in Propositions 1 and 4 overlap in the (λ, γ) space. To that end, we define region (x-$\tilde{y}$) as the intersection of region (x) in the single-homing setting and region ($\tilde{y}$) in the multihoming setting, for $x \in \{a, b, c, d\}$ and $\tilde{y} \in \{\tilde{a}, \tilde{b}, \tilde{c}\}$ (see

Figure 6. Price Equilibrium Regions in the (λ, γ) Space When Drivers Are Multihoming

$\gamma = \dfrac{3\sqrt{3 - 2c + c^2} + c - 5}{2\lambda}$

$\gamma = \dfrac{2\lambda(1 - c)}{1 + 2\lambda - \lambda^2}$

($\tilde{a}$)

($\tilde{b}$)

($\tilde{c}$)

($\tilde{a}$) partial market coverage, supply unconstrained
($\tilde{b}$) partial market coverage, supply constrained
($\tilde{c}$) full market coverage, supply constrained

$\gamma = \dfrac{1}{2} - \dfrac{\lambda}{2} - c$

Notes. The cases ($\tilde{a}$), ($\tilde{b}$), and ($\tilde{c}$) given in Theorem 2 are shown by regions ($\tilde{a}$), ($\tilde{b}$), and ($\tilde{c}$), respectively, in the (λ, γ) space. In the dark shaded area at the top left, there is no equilibrium in the multihoming setting.

Propositions 1 and 4 and Figures 4 and 6 for the definitions of these regions). This means that if (λ, γ) is in region $(x - \tilde{y})$, then (λ, γ) is in Region (x) when drivers single-home and in region $(\tilde{y})$ when drivers multihome.

Based on this construction, the following result compares the equilibrium outcomes under single-homing and multihoming.

Proposition 5. *Let* $(\lambda, \gamma, c) \in \Theta = \Theta_1 \cap \Theta_2$, *where* $\Theta_1 = \{(\lambda, \gamma, c) \in \mathbb{R}^3_+ : 2(1 - c)^2 \geq \lambda\gamma(1 - c) + \lambda^2\gamma^2\}$ *and* $\Theta_2 = \{(\lambda, \gamma, c) \in \mathbb{R}^3_+ : \gamma \leq \frac{1}{2} - \frac{\lambda}{2} - c \text{ or } \gamma \geq \frac{3\sqrt{3 - 2c + c^2} + c - 5}{2\lambda}\}$. *We have* $p^*_{\text{SH}} \geq p^*_{\text{MH}}$, $D^*_{\text{SH}} \geq D^*_{\text{MH}}$, $u^*_{d,\text{SH}} \geq u^*_{d,\text{MH}}$, *and* $u^*_{s,\text{SH}} \geq u^*_{s,\text{MH}}$. *Moreover, if* (λ, γ) *is outside regions* $(b\text{-}\tilde{b})$ *and* $(c\text{-}\tilde{c})$, *then* $p^*_{\text{SH}} > p^*_{\text{MH}}$, $u^*_{d,\text{SH}} > u^*_{d,\text{MH}}$, *and* $u^*_{s,\text{SH}} > u^*_{s,\text{MH}}$, *whereas if* (λ, γ) *is outside regions* $(b\text{-}\tilde{b})$, $(c\text{-}\tilde{c})$, *and* $(d\text{-}\tilde{c})$, *then* $D^*_{\text{SH}} > D^*_{\text{MH}}$.

A simple verbal paraphrase of Proposition 5 is that under multihoming, equilibrium prices, demands, and the utilities of purchasing customers and active drivers are all lower than they are in the single-homing setting. Moreover, they are strictly lower under the conditions that are provided in the proposition. This also implies that the platforms are better off under single-homing. The regions referenced in this proposition are depicted in Figure 7.

Although one might intuitively anticipate the fact that the platforms prefer single-homing (i.e., preventing drivers from working for the competing platform), it may not be obvious that customers and drivers share that preference. This is because multihoming introduces externalities into the pricing decisions by the platforms. First, price increases by a platform are less effective at motivating driver participation under multihoming. In single-homing, the full impact of a platform's price influences drivers on that platform (as seen in (8)). In multihoming (see (19)), a platform's price has only a partial impact on motivating drivers from its own platform, but now it has an equal impact on motivating drivers from the other platform (who can also serve on the platform in question as a result of multihoming). However, a higher price causes that platform's demand to drop, altering the weights on the prices in (19) such that the overall motivating impact of a price increase is lower in multihoming. Second, if a platform uses higher prices to induce a multihoming driver to enter the market, that platform receives only part of the benefit of having that driver active—because a fraction of the driver's services will be through the competing platform. As a result, each platform gets less benefit from increasing prices under multihoming, and therefore equilibrium prices are lower than under single-homing. These lower prices cause drivers to earn less. Customers benefit from the lower prices but suffer from reduced driver participation—and it turns out that the latter

Figure 7. Changes in Price Equilibria When Drivers Switch from Single-Homing to Multihoming
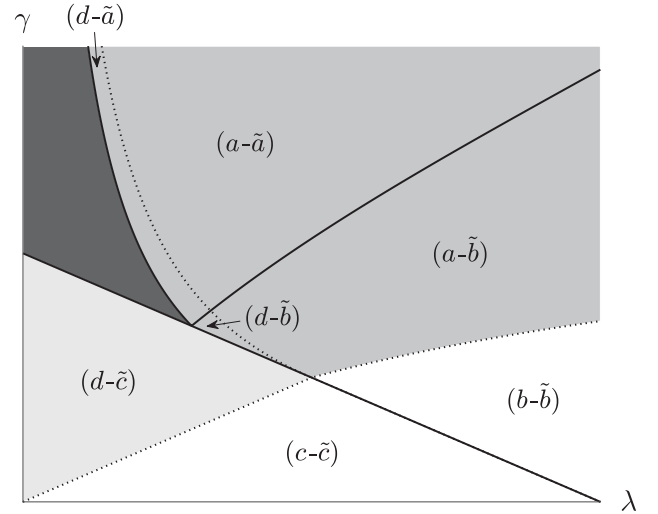
Notes. The dashed and bold curves show the boundaries of the regions in Figures 4 and 6, respectively. In the dark shaded area at the top left, there is no equilibrium in the multihoming setting. In the moderately shaded area at the top right, upon transitioning from single-homing to multihoming, price, demand, and utilities of customers and drivers all decrease in equilibrium. In the light shaded area in the bottom left, similar effects occur, except that equilibrium demand remains the same. In the unshaded area at the bottom right, equilibrium outcomes do not change.

dominates, making customers worse off. This in turn reduces demand in the market.

Given Proposition 5, one might wonder why multihoming is often observed in practice. As stated earlier, it is, for example, quite common to see a car with both Uber and Lyft stickers on it. The key observation here is that the equilibrium prices, supply, and demand under single-homing represent an equilibrium under the assumption that drivers *must* single-home (and similar for the multihoming case). If we consider a longer time frame in which drivers are able to choose which platform(s) to join, it turns out that they have an incentive to multihome.

To see this, consider a scenario where platform prices are fixed and each driver's single/multihoming status is fixed (though any configuration of single/multihoming across drivers of both platforms is allowed). To determine which customers get matched with which drivers, one can imagine a sample path with a random ordering of customers and a random ordering (ranking) of drivers. On this sample path, a customer is assigned to the first driver that can accommodate her (i.e., the first driver that either single-homes on that customer's platform or multihomes). For any sample path (random sequence of customer arrivals and random ranking of drivers), a failure of the focal driver to obtain a customer under multihoming must be the result of all customers being served by a

higher-ranked driver (because the focal driver could serve either type of customer). As a result, the focal driver would also fail to obtain a customer if that driver chose single-homing. Therefore the focal driver's probability of obtaining a customer (which is just the expected value of the number of customers obtained across all possible sample paths) must be at least weakly larger under multihoming than under single-homing. One can construct at least one positive-probability sample path such that the focal driver obtains a customer under multihoming but not under single-homing. As a result, the probability that the focal driver is matched with a customer is strictly higher under multihoming. Note that utilization ρ_i can be interpreted as the probability that a driver obtains a customer. Thus, given the drivers' utility expression (7), a higher probability of getting a customer in multihoming corresponds to higher driver utility, so the driver prefers to multihome. Because the preceding argument holds for any configuration of drivers, one can iteratively apply it to each driver in the market to conclude that all drivers ultimately have an incentive to choose multihoming and that if a driver makes that choice she will maintain that preference regardless of subsequent choices by other drivers. The end result is a situation where all drivers prefer to multihome.

To summarize, one can imagine the evolution of a market with competing platforms as follows. At first, when the platforms are relatively new, all drivers may affiliate with a single platform (single-home), and the platforms would select pricing policies under that assumption. Over time, the incentives described earlier may lead more and more drivers to choose to multihome, eventually leading to full multihoming across both platforms. At that point, the platforms would recognize this fact, resulting in a new multi-homing price equilibrium with the properties described in Proposition 5. (Note that the platforms could, in fact, adjust their prices along the way to respond to partial multihoming—this would not alter the driver incentives described previously.)

Because equilibrium prices and demands are lower, and customers and drivers are worse off under multihoming, platforms might wish to identify mechanisms to deter this behavior. For example, a platform may seek to structure driver compensation in a way that differentiates between single- and multihoming drivers to a sufficient extent that drivers would never choose to multihome when making their up-front platform choice. This might be achieved in practice through the use of some kind of loyalty or incentive program. For example, if a driver completes a certain number of rides τ within a fixed number of driving opportunities T, the commission could be reduced on all rides given by that driver during that time window—say, from $c + \delta$ to c.

The threshold τ could be set high enough that it would be difficult to reach if a driver is engaging in significant multihoming but relatively easy to reach if the driver primarily or exclusively single homes. For example, for a given utilization ρ_i, the expected number of successful rides given in T attempts would be $\rho_i T$, whereas that number would be about $\frac{1}{2}\rho_i T$ if the driver is splitting efforts between the two platforms. Setting τ such that $\frac{1}{2}\rho_i T < \tau < \rho_i T$ would allow single-homing drivers to pay the lower commission c, whereas multihoming drivers would pay $c + \delta$.

We seek to formalize this idea and establish how large δ would need to be for a single-homing driver to have no incentive to multihome. To that end, suppose the system starts with all drivers single-homing and prices at the resulting unique symmetric equilibrium (with all secondary quantities d_i, ρ_i, etc., at corresponding values). Also, suppose that the threshold τ and the opportunity window T are chosen in such a way that all single-homing drivers pay commission c and any driver choosing to multihome will pay $c + \delta$.

Proposition 6. *If drivers on platform i pay commission c if they single-home and $c + \delta$ if they multihome, then for any $\delta > (1 - \rho_i)(p_i - c)$, every platform i driver prefers to single-home.*

The lower bound on δ given in the proposition has an intuitive interpretation. Under single-homing, a driver earns an average of $\rho_i(p_i - c)$ per service attempt. At best, multihoming could increase the probability of earning the margin $p_i - c$ from ρ_i to 1, leading to a potential gain of $(1 - \rho_i)(p_i - c)$. As long as the additional commission incurred by pursuing that option is high enough to outweigh that gain, the driver will stick with single-homing. That bound also provides some insight into the types of settings where platforms might need to worry the most about multihoming and how big of a commission differential might be needed to deter it. In settings with high utilization ρ_i under single-homing, the bound on δ is quite small, becoming 0 when utilization is equal to 1. In such settings, multihoming is not as much of a concern and can be countered with a relatively modest incentive plan. (Note that this observation is consistent with Proposition 5, which states that when a platform is supply constrained—i.e., $\rho_i = 1$—the equilibrium outcomes are the same for both single- and multihoming.)

7. Conclusion

In this paper, we present a model of a two-platform sharing-economy system where each platform can only indirectly influence supply and demand through price. A key element of this model is the inclusion of two-sided congestion effects. On the demand side, a large number of customers seeking service relative to

the number of drivers offering service can result in longer customer delays before service begins, so that potential customers are less willing to participate in the market. On the supply side, a large number of drivers offering service relative to the number of customers seeking service reduces the likelihood that a driver will successfully be able to find a customer and earn income, which makes drivers less willing to be active. This creates an interaction effect between supply and demand that captures relatively subtle system behaviors and allows us to obtain deeper insights into system performance and questions of practical interest. For both single-homing and multihoming settings, we establish conditions under which a price equilibrium exists, and we identify regions in the parameter space under which different types of outcomes can arise. Examination of these regions provides insights regarding the impact that factors such as market size or customer sensitivity to delays have on outcomes such as the intensity of competition between the platforms or the level of congestion in the market. It also reveals insights regarding the impacts on prices, congestion, and participant utilities of having a second platform competing in the market.

We build on these results to make some comparisons that address managerial and policy questions that are currently being debated in sharing-economy circles. First, we examine the effect of surge pricing by platforms. We find that, despite resulting in higher prices for customers during periods of high demand, the net effect of that price shift is to make both customers and drivers better off. The additional supply induced by the higher price more than compensates for the negative price impact on customers, but it is not so large that it dominates the benefit drivers experience from the higher price. Next, we explore the differences—in terms of outcomes for customers and drivers—between single-homing and multihoming settings. We find that demand and prices are lower, and both customers and drivers are weakly (and sometimes strictly) better off under single-homing. On the surface, the conclusion regarding drivers seems to run counter to anecdotal observation, because it is common to see drivers working under both Uber and Lyft platforms. However, we identify a prisoners' dilemma type of phenomenon that can explain this apparent inconsistency. Starting with single-homing, any single driver is better off multihoming, and this preference continues for each driver in turn until all drivers shift to multihoming. Once platforms adjust prices to reflect this new reality, however, demand and prices are lower, and both customers and drivers are weakly worse off. We propose an incentive mechanism that, if implemented by platforms, could help deter the practice of multihoming, thus mitigating these negative effects. By

analyzing one form of such a mechanism, we find that less incentive is needed to deter multihoming in settings where single-homing results in high system utilization.

To test the robustness of our managerial insights (i.e., those discussed in Propositions 2, 3, and 5), we conducted an extensive numerical study where we dropped the supply constraint (9) or (27) and allowed for general α. We used the following parameter values: $c \in \{0.1, 0.2\}$, $\alpha \in \{0.8, 0.9, 1\}$, $\gamma \in \{0.1, 0.2, \ldots, 1\}$, and $\lambda \in \{0.1, 0.2, \ldots, 3\}$, for a total of 1,800 cases. These include parameter combinations under which the supply constraint is not guaranteed to be redundant (i.e., $\gamma < \max\{[(\lambda - \alpha)(\alpha - c)]/(\alpha^2 + 3\alpha\lambda), 0\}$). Almost all of the results continued to hold in more than 98% of the cases considered (and many of them in 100% of the cases). The only exceptions to this are the results that demand and customer utility are higher in single-homing than in multihoming; these results continued to hold in 94% of the cases. On the basis of this numerical study, our managerial insights do not appear to rely on the supply constraint in general, and they appear to be fairly robust to the value of α.

The analysis in this paper reflects two key features of our model: platform competition and market congestion. If competition between platforms were ignored, then it would not be possible to conduct a comparison of the single-homing and multihoming settings. Furthermore, modeling platform competition allows us to analyze (a) how platforms adjust prices and, in turn, congestion levels in response to competition and (b) how platforms respond to positive demand shocks in a competitive fashion. Therefore, explicitly modeling competition makes our work pertinent to some of the current challenges in the ridesharing industry.

Our model captures the impact of congestion on customers and drivers via disutility terms that depend on the level of utilization in a platform (the additive term $-\gamma \rho_i^2$ in (3.1) and the multiplicative term ρ_i in (7)). If we suppress the operational features of our model by removing the congestion effects (i.e., replacing the additive term $-\gamma \rho_i^2$ in (3.1) by 0 and the multiplicative term ρ_i in (7) by 1), then the resulting supply and demand curves would be linear, and the platforms' pricing problems would reduce to intersecting these supply and demand curves to determine the equilibrium price. Ignoring congestion effects would result in platforms no longer maintaining supply cushions (as illustrated in Figure 5), and it would not be possible to obtain our findings on the impact of surge pricing. Moreover, the equilibrium outcomes in the single-homing and multihoming settings would become the same, implying that all parties would be indifferent between these scenarios. Incorporating congestion effects allows us to explore a

richer set of questions that result from the more nuanced feedback between supply and demand.

In order to make more extensive analysis tractable and focus attention on the phenomena being studied, our model assumes that platforms are symmetric. To check the robustness of our equilibrium results, however, we have analyzed two types of platform asymmetry. The first modifies the customer location distribution shown in Figures 1 and 2 to have a nonzero mass at one end of the distribution, reflecting a group of customers with strong loyalty to one platform. Under that extension, for the case of single-homing, we are still able to (a) derive expressions for balanced demand and supply, (b) establish the existence of equilibrium prices, (c) (implicitly) characterize four regions with different outcomes similar to Figure 4, and (d) (implicitly) characterize the equilibrium prices in those regions. (For three of the regions, we are able to establish uniqueness of the equilibrium analytically, whereas in the fourth region, numerical trials always resulted in a unique equilibrium.) In addition, the platform with the loyal customer base charges higher prices but still enjoys higher demand in equilibrium. The second version of asymmetry allows the commission c to be different for each platform. Under that generalization, for single-homing, we can show that a unique equilibrium exists, and we can characterize analogues of the four regions in Figure 4, as well as equilibrium prices in each of these regions. One new outcome in this setting is that a new subregion may arise in which one platform's drivers are fully utilized while the other has some drivers idle. The analysis under either of these generalizations quickly becomes intractable, however, and further exploration of these cases is beyond the scope of this paper.

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