Centralized versus Decentralized Pricing Controls for Dynamic Matching Platforms

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An important operational decision for online service platforms is how much control they have over pricing decisions. Motivated by recent regulations and increased public scrutiny on platform control, we study the effects of decentralized pricing using a fluid model of dynamic two-sided matching. In the centralized setting, the platform dictates the market price by optimizing a certain objective function. In contrast, under full decentralization, each supplier picks a price, adapted to their private cost, to maximize expected earnings. In turn, based on how much they value the service, customers can accept or reject the proposed matches. Our main contribution is to uncover the structure of stationary market equilibria under various degrees of centralization and analyze the resulting social welfare. We show that there exists a unique equilibrium in centralized and decentralized platforms. In the general case, where the platform can restrict the *price menu*, we characterize the equilibria through a tractable piecewise polynomial system. We establish that, within "impatient" markets (i.e., agents depart after a single match attempt), centralized platforms with plausible objective functions achieve higher social welfare than the decentralized equilibria. By contrast, the decentralized equilibrium converges to the *first-best* outcome as market participants' patience level uniformly increases. We demonstrate the robustness of these findings through a numerical study and identify semi-centralized pricing rules that retain some flexibility but recover near-optimal outcomes in most regimes. Overall, our work suggests that decentralized pricing creates friction in the matching process, except if supply and demand arrivals are balanced or if the agents are patient.

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1. Introduction

Online service platforms have gained widespread popularity by connecting customers with suppliers in real-time. To achieve efficient matches, platforms often exercise tight control over market operations using dispatch and pricing systems (Lobel 2021). For example, ride-hailing platforms use various pricing incentives to suppress demand and reduce pick-up times under high utilization (Castillo et al. 2017), reposition workers to undersupplied areas (Banerjee et al. 2022), or mitigate rejections of match requests (Garg and Nazerzadeh 2022). These algorithmic controls have significantly reduced costs and enhanced market efficiency in mobility and delivery services; see, Han et al. (2022). Nonetheless, by acting as "central planners", platforms risk undermining the flexibility endorsed by the gig economy. Considerable attention has been paid lately to the classification of gig workers as independent contractors rather than employees; freedom in decisions that directly influence earnings, such as prices, is sometimes presented as a requirement (Paul 2017).

In the ride-hailing sector, several emerging platforms, such as Empower, InDrive, and Bolt, propose alternative operating models. Drivers can flexibly set prices or adjust the input parameters of rider fare calculations (e.g., base distance rates). Major platforms have also tested more "decentralized" price-setting mechanisms, prompted by new regulations. For instance, in 2019, the state of California passed a law (Assembly Bill 5) that generalized strict criteria for classifying workers as independent contractors, among which the need to demonstrate that workers are free from the control and direction of hiring platforms. In this context, Uber partially relinquished control of prices by introducing a "fare multiplier" on the app, that each driver could set in 10% increments (Said 2020). With the subsequent exemption of ridesharing firms from the AB5 requirements, Uber rolled back that product feature, reporting an increase in rider cancellations and degradation of the matching process (Sandler 2021).

At first glance, decentralized pricing has both costs and benefits. On the one hand, service platforms set a spot price (e.g., surge multiplier) to counteract demand-supply imbalances and facilitate the matching process. When supply is scarce relative to demand, a higher price deters service requests from low-valuation customers and attracts additional suppliers with higher marginal costs. Additionally, since customers and suppliers can decline or cancel matching requests, an adequate price is important to reduce friction in the search process, leading to either successful matches or abandonments. One could expect that decentralized pricing—understood as *selfish pricing decisions made by suppliers*—might not internalize such matching-related externalities in equilibrium. On the other hand, however, suppliers have private information about their preferences, which is often hard to elicit (Filippas et al. 2022), or which platforms cannot use to price-discriminate. Specifically, in the context of mobility platforms, drivers may incur different fixed costs (e.g., vehicle rental or leasing) and time-based costs (e.g., depending on access to other paid activities). A decentralized price system can leverage such dispersed private information: the equilibrium prices can reflect supplier heterogeneity, potentially expanding the pool of suppliers or increasing their participation in the market.

This paper studies the above tradeoff between *operational efficiency* and *supplier information*. We formulate a fluid model of dynamic two-sided matching markets and quantify the effects on social welfare of different degrees of centralization in price controls.

Our results. We formulate a fluid model of two-sided matching, facilitated by a platform, between customers and suppliers endowed with private valuations and costs, respectively. We express the platform's degree of centralization in pricing through the notion of *price menu*, an interval of prices from which suppliers can choose. Centralized platforms offer a single price that maximizes a certain objective function, whereas decentralized ones do not impose any constraint on sellers' prices. Customers and suppliers are involved in multiple match attempts; in stationary equilibrium, their strategies maximize expected long-term payoffs with respect to the system state. Our main technical contribution is to uncover the structure of stationary market equilibria under various levels of centralization and market characteristics and to analyze the resultant social welfare. We prove the existence of a unique equilibrium in centralized and decentralized platforms (**Theorem 1**). For arbitrary price menus, we establish that any equilibrium is determined by three variables, which must be the roots of a fully-determined piecewise polynomial system.

Building on this analysis, we find that, within "impatient" markets (i.e., agents depart after one match attempt), decentralization is costly to social welfare compared with centralized pricing controls (**Propositions 1 - 2**); under our model assumption, the worst-case loss is a factor of 2 or 4 relative to revenue-maximal or welfare-maximal prices, respectively. By contrast, within "patient" markets (i.e., agents are involved in many match attempts), the decentralized equilibrium approaches the *first-best* outcome as agents' departure rates converge to zero (**Theorem 2**). Numerical studies show that, other than the patience level, the degree of supply-demand balance and the specific objective used by a centralized platform to set prices critically affect the comparison between centralized and decentralized pricing (**Section 6**). Although a welfare-maximal price control consistently generates higher welfare than decentralization, we identify a *semi-centralized* pricing rule, where suppliers retain some flexibility in pricing, but outcomes close to the welfare maximum are achieved in most regimes. Using simulation-based experiments, we verify that our findings are robust to different model assumptions (e.g., distributions of valuations and costs).

2. Related Literature

There has been considerable research recently on matching in dynamic two-sided markets to balance market thickness and waiting-times (Anderson et al. 2017, Tsitsiklis and Xu 2017, Akbarpour et al. 2020, Özkan and Ward 2020, Aouad and Sarıtaç 2022, Hu and Zhou 2022). However, such papers focus on centralized matching policies in stochastic networks, but often do not incorporate pricing decisions or agents' incentives. In parallel, the operations management literature has examined operational interventions to reduce search frictions and market congestion (Arnosti et al. 2021, Halaburda et al. 2018, Papanastasiou et al. 2018, Allon et al. 2012). Our modeling approach is very similar to that of Kanoria and Saban (2021), who analyze matching in stationary equilibrium over a continuum of long-lived agents, making rational decisions to propose or accept/reject match requests. However, the paper questions how to constrain the information and initiative in matching to facilitate the search process; in this context, pricing is stated by the authors as an open direction for future work. The design of optimal matching processes with strategic agents was also analyzed in several other works (Baccara et al. 2019, Immorlica et al. 2022), absent pricing incentives. Technically, these papers often consider a finite set of agent types, whereas our pricing model needs to capture heterogeneity as a continuum of valuations and costs.

Connecting matching efficiency with pricing, Shi (2022, 2023) examines how to efficiently clear a two-sided market with endogenous price formation, respectively in terms of communication complexity and social welfare. Chen and Hu (2020) discover fixed base prices with waiting cost adjustments to be asymptotically optimal in a dynamic market with strategic agents but no horizontal utility differentiation; however, the effect of the degree of centralization is not explored. A few recent papers more explicitly consider the issue of comparing centralized and decentralized pricing controls. Tackling a similar research question, Pang et al. (2022) considers a networked Cournot competition model between firms, which is not specifically tailored to matching markets. Atasu et al. (2022) examines whether price delegation can encourage better local learning of demand from a contract design perspective. In the sharing economy setting, Filippas et al. (2022) give experimental evidence that suppliers retaining partial control over prices can be more effective than fully centralized pricing controls. Closer to our setting, Cachon et al. (2022) analyze decentralized and centralized pricing in service platforms. Their work focuses on optimal contract design for the platform's payment structure in decentralized markets. By contrast, our model depicts the interplay between decentralized pricing and the matching process, with an aim to capture search frictions in dynamic markets.

Finally, our work is naturally connected to long-standing literature in economics on labor markets; e.g., Diamond (1982a,b), Mortensen (1982), Pissarides (1984, 1985), Hosios (1990), Acemoglu (2001), Shimer and Smith (2001). Although our model and results have technical similarities, the focus on pricing controls for service platforms leads to a different set-up (e.g., open-loop model).

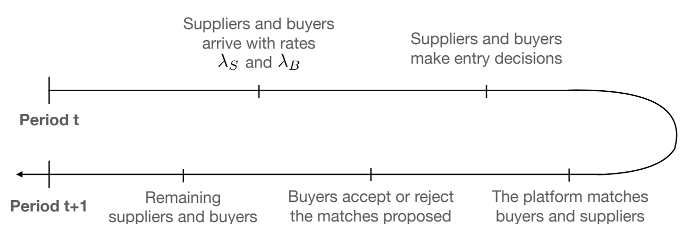


Figure 1 **Timing of events.**

3. Model

We study a two-sided matching market mediated by a central platform over an infinite discrete-time horizon. We refer to the two sides of the market as "supply" and "demand", and the agents associated with each side as "suppliers" and "customers". They make strategic decisions to maximize their expected long-term payoffs based on their private costs and valuations, respectively. We consider different types of platforms – centralized, semi-centralized, or decentralized – depending on how flexible suppliers are in setting prices. At the beginning of each time period, customers and suppliers arrive and decide whether to enter the market. Among those who enter, the platform tentatively matches customers with suppliers. After observing the supplier's price in the requested match, each customer chooses whether to accept their current offer or to stay in the market until the next period. Finally, a fraction of the agents leave the system at the end of the time period. Although we consider a fluid system, we assume that each infinitesimal agent solves a stochastic decision process.[1] Figure 1 illustrates the timeline of events in each time period, which includes agent arrivals, entry decisions, matching process, and departures. We discuss the main limitations of modeling assumptions in Appendix B.

3.1. Preliminaries

Platform type: Degrees of centralization. We model the extent to which the platform is centralized by a price menu $[p_{\min}, p_{\max}]$ offered to the suppliers in the market. Here, $p_{\min} \in \mathbb{R}^+$ is a lower bound on the price that can be selected by any supplier, and $p_{\max} \in \mathbb{R}^+ \cup \{+\infty\}$ is an upper bound. We say that a platform is *centralized* if suppliers do not have any flexibility, meaning that $p_{\min} = p_{\max}$. In other words, the menu of prices is picked in the set $\{[p, p] : p \in \mathbb{R}^+\}$. By contrast, the *decentralized* setting is unconstrained from the perspective of suppliers, meaning that $p_{\min} = 0$ and $p_{\max} = +\infty$. The corresponding collection of price menu is $[0, +\infty]$. Finally, we refer to any intermediary setting as *semi-centralized*, which subsumes all price menus in $\{[y_1, y_2] : y_1 \in \mathbb{R}^+, y_2 \in [y_1, \infty]\}$. Note that the platform type is exogenous and the price menu is fixed over time. That is, the platform chooses its operating model before agents strategically interact in the market.

Agent types and strategies. We model a continuum of vertically differentiated customers and suppliers. Specifically, each customer on the demand side has a private type v that represents their valuation of the service, and each supplier has a type private c that represents the cost of providing the service to the customer; the platform does not observe agents' types. We normalize the maximum customer valuation to 1 and the minimum cost to 0. Note we do not require valuations to be non-negative because the final utility of a match will account for idiosyncratic random shocks, meaning that having a customer with a negative valuation may still obtain a positive utility from matching with certain suppliers. For similar reasons, suppliers' costs can exceed 1.

Going forward, each agent type is associated with a strategy. That is, the strategy space is formed by deterministic mappings from costs and valuations to actions in each time period. On the supply side, suppliers choose whether or not to enter the market, and if so, they choose a price within the menu $[p_{\min}, p_{\max}]$. On the demand side, customers choose whether or not to enter the market and accept any given match request. We formally describe these strategies in the next section.

[1] The Exact Law of Large Numbers was not formally established for our model formulation; e.g., results in Remarks 1 and 2 of Duffie and Sun (2012) for (infinitely sized) complete separable metric type spaces require a full matching process in each time period and no random type mutation. As seen later, that is violated by our setting, where (i) the matching is partial; (ii) a random post-matching mutation occurs (random success of a match and departure).

Table 1 Notation

Market parameters:	
$[p_{\min}, p_{\max}]$	Menu of prices allowed by the platform
λ_C, λ_S	Density of arrivals for customers and suppliers, respectively
μ_C, μ_S	Fraction of departing customers and suppliers in each period, respectively
ϵ	Horizontal differentiation parameter
State variables:	
$N_C^t(v)$	Density of customers with type v in the system in period t
$N_S^t(c)$	Density of suppliers with type c in the system in period t
$M^t(v,c)$	Match density between type-v customers and type-c suppliers in period t
Strategies:	
T_C^t	Set of types of customers that enter the market in time period t
T_S^t	Set of types of suppliers that enter the market in time period t
$\tau(v)$	The threshold picked by customers of type v
$p(c)$	The price picked by suppliers of type c

3.2. Dynamical system

In this section, we introduce the sequence of events that occur in one time period $t \in \mathbb{N}$. Here, the agents' strategies throughout the horizon are fixed; our equilibrium concept is presented in Section 3.3. As a direct consequence of the model set-up, the distribution of agent types is non-atomic in every time period.[2] Hence, we denote by $N_S^t(\cdot)$ and $N_C^t(\cdot)$ the density of supplier and customer types at the beginning of period t, respectively.

Exogenous arrivals and entry decisions. At the beginning of the new time period, new customers arrive in the system with a uniform density λ_C of valuations $v \in (-\infty, 1]$, and similarly, new suppliers arrive with a uniform density λ_S over costs $c \in [0, +\infty)$. (Recall from Section 3.1 that we do not ex-ante eliminate negative valuations and costs larger than 1 because the final utility has a random component.) The uniformity assumption is a strong imposition, which we require for our analysis. However, we relax this assumption in our numerical study of Section 6.

After arrival, each customer and supplier decides whether or not to enter the market. We denote by T_C^t and T_S^t the set of customer and supplier types that enter the market in period t, respectively. As mentioned in Section 3.3, in equilibrium, these decisions are endogenous and guided by the long-term surplus or profit expected from the matching process. We define the truncated distributions of customers $\bar{N}_C^t(\cdot)$ and suppliers $\bar{N}_S^t(\cdot)$ that enter the market after arrivals, i.e., $\bar{N}_C^t(v) = N_C^t(v) + \lambda_C$ for all $v \in T_C^t$, otherwise $\bar{N}_C^t(v) = 0$, and $\bar{N}_S^t(c) = N_S^t(c) + \lambda_S$ for all $c \in T_S^t$, otherwise $\bar{N}_S^t(c) = 0$. That is, the densities $\bar{N}_S^t(c)$ and

[2] This property does not hold for the distribution of agents' strategies in equilibrium, which may not be a diffuse measure.

$\bar{N}_C^t(v)$ are the same as $N_S^t(c)$ and $N_C^t(v)$ for types that enter the market plus the new arrivals while they are equal to 0 for those who do not enter the market.

Matching process. Given the set of customers and suppliers who enter the market, the platform proposes to match them uniformly at random while maximizing the total mass of matches.[3] The resultant *batch-matching* in time period t is given by the density function

$$M^t(v,c) = \min\left\{\frac{\bar{N}_C^t(v)\cdot\bar{N}_S^t(c)}{\|\bar{N}_S^t\|}, \frac{\bar{N}_C^t(v)\cdot\bar{N}_S^t(c)}{\|\bar{N}_C^t\|}\right\},$$

where $\|\bar{N}_S^t\|$ and $\|\bar{N}_C^t\|$ denote the total masses of suppliers and customers that enter the market in time period t, respectively. To interpret this matching function, each customer of type $v \in T_C^t$ is offered a match with a supplier of type $c \in T_S^t$ with probability $\alpha_t \cdot \bar{N}_S^t(c)/\|\bar{N}_S^t\|$, and remains unmatched with probability $1-\alpha_t$, where $\alpha_t = \min\{1, \|\bar{N}_S^t\|/\|\bar{N}_C^t\|\}$. Hence, all agents on the short side (customers if $\alpha_t = 1$ and suppliers if $\alpha_t < 1$) of the market are offered a potential match partner, and all agents on the long side have an equal chance of being matched. Similar stylized assumptions (i.e., uniformly random matching) are used in previous literature to represent a search process or a periodic matching proposed by the platform; see, e.g., Lo et al. (2021), Kanoria and Saban (2021).[4]

Matching outcomes and departures. Given any match proposed by the platform between a customer of type v and a supplier of type c, the customer observes the random utility from the proposed match, which takes the quasi-linear form $v - p^t(c) + \boldsymbol{\eta}$, where $p^t(c)$ is the price chosen by the supplier's strategy in period t and $\boldsymbol{\eta}$ is an independent random shock, revealed after the match request (we reserve boldface font to designate random experiments). From a modeling perspective, $\boldsymbol{\eta}$ can be interpreted as horizontal differentiation in the market, representing idiosyncratic customer preferences for different suppliers. Throughout our analysis, we will assume that $\boldsymbol{\eta} \sim U_\epsilon$ is uniform over $[-\epsilon, \epsilon]$ for some fixed parameter $\epsilon > 0$. In a commoditized service platform, one should think of ϵ as small relative to other market characteristics, indicating that the vertical component of the utility

[3] A matching is formally defined as a density function $M : T_C^t \times T_S^t \to \mathbb{R}^+$ that satisfies the packing constraints $\int_{T_C^t} M(v,c)dv \le N_S^t(c)$ for all $c \in T_S^t$ and $\int_{T_S^t} M(v,c)dc \le N_C^t(v)$ for all $v \in T_C^t$.

[4] The platform does not observe customers' valuations and suppliers' costs, and thus, it cannot utilize this information in matching decisions. However, our model allows the platform to leverage price information by choosing $[p_{\min}, p_{\max}]$, which effectively constrains the entry of customers and suppliers before the matching.

differentiation (governed by valuations and costs) outweighs the horizontal component. Nevertheless, we will consider arbitrary $\epsilon > 0$ values throughout the paper.

The customer observes the matching utility and decides whether or not to accept the corresponding match based on their strategy. We denote by $\mathbb{I}^t(v, v - p^t(c) + \eta) \in \{0,1\}$ the corresponding outcome, which we assume to depend on the customer type and the utility of the match. At the end of period t, the departures from the platform occur in two ways. First, the matched customers and suppliers immediately leave the market. Second, the remaining agents may depart due to impatience. To capture this phenomenon (which guarantees the stability of the system), we assume that a fraction $\mu_C \in (0,1]$ of the remaining customers leave the market uniformly over all valuations v. Similarly, a fraction $\mu_S \in (0,1]$ of the remaining suppliers leave the market.

System evolution. Summarizing the above events, the agents' strategies in period t are described by the types of agents who enter the market T_C^t, T_S^t on each side of the market, the suppliers' prices $p^t(\cdot)$, and the customers' acceptance decisions $\mathbb{I}^t(\cdot,\cdot)$. Consequently, the distribution of agents in the next time period $N^{t+1} = (N_C^{t+1}, N_S^{t+1})$ is given by the following system of equations:

$$N_C^{t+1}(v) = (1-\mu_C) \cdot \left(N_C^t(v) + \lambda_C - \int_0^{+\infty} M^t(v,c) \cdot \mathbb{E}_{\boldsymbol{\eta}} \left[\mathbb{I}^t(v, v - p^t(c) + \boldsymbol{\eta}) \right] dc \right), \tag{1}$$

where $\boldsymbol{\eta} \sim U_\epsilon$ stands for an independent draw from the uniform distribution over the interval $[-\epsilon, \epsilon]$. Equation (1) captures the arrivals of new agents (with density λ_C), the outcomes of the matching process $M^t(\cdot)$ with suppliers' prices $p^t(\cdot)$ and customers' acceptance rule $\mathbb{I}^t(\cdot)$, and the uniform departures at rate μ_C. Similarly, the supplier side of the market evolves as follows:

$$N_S^{t+1}(c) = (1-\mu_S) \cdot \left(N_S^t(c) + \lambda_S - \int_{-\infty}^{1} M^t(v,c) \cdot \mathbb{E}_{\boldsymbol{\eta}} \left[\mathbb{I}^t(v, v - p^t(c) + \boldsymbol{\eta}) \right] dv \right). \tag{2}$$

Throughout the paper, we analyze this system in steady-state. Specifically, we define the notion of steady-state as the combination of a stationary measure $N^\infty = (N_C^\infty, N_S^\infty)$ and time-invariant strategies $(T_S^\infty, T_C^\infty, p^\infty(\cdot), \mathbb{I}^\infty(\cdot))$ that induce a fixed point with respect to equations (1)-(2). Letting M^∞ denote the matching over the type distribution N^∞ with entry decisions T_S^∞, T_C^∞, the fixed-point property amounts to the equations:

$$\lambda_C = \frac{\mu_C}{1-\mu_C} \cdot N_C^\infty(v) + \int_0^{+\infty} M^\infty(v,c) \cdot \mathbb{E}_{\boldsymbol{\eta}} \left[\mathbb{I}^\infty(v, v - p^\infty(c) + \boldsymbol{\eta}) \right] dc, \quad \text{for all } v \in T_C, \tag{3}$$

$$\lambda_S = \frac{\mu_S}{1-\mu_S} \cdot N_S^\infty(c) + \int_{-\infty}^{1} M^\infty(v,c) \cdot \mathbb{E}_{\boldsymbol{\eta}}\left[\mathbb{I}^\infty(v, v - p^\infty(c) + \boldsymbol{\eta})\right] dv, \qquad \text{for all } c \in T_S. \quad (4)$$

Put simply, these equations ensure that if the initial distribution is N^∞ and the agents adopt the strategies $(T_S^\infty, T_C^\infty, p^\infty(\cdot), \mathbb{I}^\infty(\cdot))$, then the system state does not vary over time. We will now describe our solution concept, explaining how such stationary outcome may arise from the strategic game between customers and suppliers.

3.3. Equilibrium and platform objective

We draw upon the notion of stationary equilibrium, introduced by Hopenhayn (1992), which was also in related literature for large-market regimes (Kanoria and Saban 2021, Immorlica et al. 2022). In such equilibria, the system is in steady state, and the agent's strategies are the best responses with respect to the aggregate system state. The information required is that the agents know the aggregate distribution of types and strategies in the steady-state market. In what follows, we formulate the best-response optimization problem solved by customers and suppliers in equilibrium, and then formally introduce our solution concept. Since policies are time-invariant, we drop the superscript and use the simplified notation $(T_S, T_C, p(\cdot), \mathbb{I}(\cdot), N)$ for agents' strategies.

Customers' best-response strategies. Each infinitesimal customer with valuation v chooses whether or not to enter the market and accept match requests based on their utility. The objective of the customer's strategy is to maximize the long-term expected surplus, i.e., customers may reject a match request if it involves a sufficiently high price or low $\boldsymbol{\eta}$. Agents enter the market if and only if they can obtain a positive expected utility from entering.[5] It follows that, at equilibrium, $T_C = (\underline{v}, 1]$ where $\underline{v} = \inf\{p(c) - \epsilon : c \geq 0, \bar{N}_S(c) > 0\}$. Indeed, any customer with a valuation larger than $\underline{v}$ can benefit from entering the market and accepting the first random match with positive utility. Reciprocally, customers with valuations smaller than or equal to $\underline{v}$ do not find it worthwhile to enter because all subsequent matches have nonpositive utilities with probability 1.

Given that $v \in T_C$, the customer's best response can be formulated as a dynamic program. Since the actions of each infinitesimal customer do not affect the steady-state, it follows that the reward-to-go is a constant $\tau(v)$ that only depends on v. Hence, the Bellman equation for the best-response strategy takes a simple threshold form. The customer accepts the current

[5] For simplicity, we posit that positive utility is necessary for entry as a rationality condition. More formally, this setting can be viewed as having an entry cost to the market, in the limit where this cost tends to zero.

match if and only if the current utility is larger than or equal to $\tau(v)$. This observation implies that the strategy space can be reduced without loss of generality to type-dependent thresholds. Formalizing the customer's optimization problem, let $\phi_C(v,x,T_C,T_S,\tau,p,N) = \min\left\{1, \|\bar{N}_S\|/\|\bar{N}_C\|\right\} \cdot \Pr_{\boldsymbol{c},\boldsymbol{\eta}}[v - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq x]$ represent the fraction of customers of valuation v and threshold x with successful matches in every steady-state period, where $\boldsymbol{c} \sim \bar{N}_S$ stands for the cost of a randomly drawn infinitesimal supplier from the distribution $\bar{N}_S$ of entering agents on the supply side. Here, the first term is the fraction of customers who are matched; if the total mass of customers exceeds that of suppliers, this fraction is smaller than 1. With this notation, the customer chooses a threshold $x \in \mathbb{R}$ that maximizes the expected long-term surplus $\Phi_C(v,x,T_C,T_S,\tau,p,N)$, calculated as follows:

$$\Phi_C\left(v,x,T_C,T_S,\tau,p,N\right) = \frac{\min\left\{1, \frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\} \cdot \mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}[\mathbb{I}(v - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq x) \cdot (v - p(\boldsymbol{c}) + \boldsymbol{\eta})]}{\phi_C(v,x,T_C,T_S,\tau,p,N) \cdot (1-\mu_C) + \mu_C}. \quad (5)$$

Hence, each customer of type v picks $\tau^*(v,T_C,T_S,\tau,p,N) \in \arg\max_{x\in[0,1+\epsilon]} \Phi_C(v,x,T_C,T_S,\tau,p,N)$, where we restrict the search to the interval $[0,1+\epsilon]$ as the match probability (thus, the expected surplus) is otherwise zero.

Suppliers' best-response strategies. Each infinitesimal supplier with cost c chooses a price and whether or not to enter the market. The objective is to maximize the expected long-term profit generated by this strategy. Suppliers enter the market if and only if they obtain a positive expected profit from matching, i.e., at equilibrium $T_S = [0, \bar{c})$, where $\bar{c} = \sup\{v - \tau(v) + \epsilon : v \leq 1,\ \bar{N}_C(v) > 0\}$.

Next, we formalize the supplier's profit-maximization problem. Given a price $y \in [p_{\min}, p_{\max}]$, let $\phi_S(y,T_C,T_S,\tau,p,N) = \min\left\{1, \|\bar{N}_C\|/\|\bar{N}_S\|\right\} \cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}\left[\boldsymbol{v} - y + \boldsymbol{\eta} \geq \tau(\boldsymbol{v})\right]$ represent the fraction of suppliers of price y successfully matched per period, where $\boldsymbol{v} \sim \bar{N}_C$ stands for the valuation of a randomly drawn infinitesimal customer from the distribution $\bar{N}_C$ of entering agents on the customer side. Then, the supplier chooses a price $y \in \mathbb{R}$ that maximizes the expected long-term profit $\Phi_S(c,y,T_C,T_S,\tau,p,N)$, calculated as follows:

$$\Phi_S(c,y,T_C,T_S,\tau,p,N) = \frac{(p-c)\cdot\phi_S(y,T_C,T_S,\tau,p,N)}{\phi_S(y,T_C,T_S,\tau,p,N)(1-\mu_S)+\mu_S}. \quad (6)$$

Hence, each supplier picks $p^*(c,T_C,T_S,\tau,p,N) \in \arg\max_{y\in[p_{\min},p_{\max}]} \Phi_S(c,y,T_C,T_S,\tau,p,N)$, where the price search is restricted to the menu $[p_{\min}, p_{\max}]$ as per the platform type. Consequently, we formally define the notion of stationary equilibrium.

Definition 1 *(Stationary Equilibrium) We say that* (T_C, T_S, τ, p, N) *is a stationary equilibrium if the fixed-point equations* (3)-(4) *and entry conditions are satisfied, and the suppliers' and customers' strategies are in best-response with respect to* N*, i.e.,* $\forall v \in T_C$*,* $\tau(v) \in \tau^*(v, T_C, T_S, \tau, p, N)$ *and for all* $c \in T_S$*,* $p(c) \in p^*(c, T_C, T_S, \tau, p, N)$*.*

In equilibrium, the entry conditions are straightforward and directly proceed from the strategies $\tau(\cdot), p(\cdot)$ and the distribution N. Hence, in the remainder of the paper, we simplify our notation for best-response strategies $\tau^*(\cdot), p^*(\cdot)$, match probabilities $\phi_C(\cdot), \phi_S(\cdot)$, and expected surplus/profit $\Phi_C(\cdot), \Phi_S(\cdot)$ by dropping the arguments T_S, T_C.

Pricing controls and the first-best. Within our framework, a semi-centralized platform can control market outcomes by choosing the menu $[p_{\min}, p_{\max}]$ of prices offered to suppliers. A pre-requisite for this intervention is that the platform must be able to anticipate the stationary equilibria, which in general might not exist or be unique. Hence, much of our analysis will focus on analyzing the equilibrium structure. If the platform has access to a mapping from observable market characteristics and price menus to stationary equilibria, then we can formulate a platform optimization problem. We will take the perspective that the platform chooses a price menu to maximize a certain ex-ante objective function. Deferring the formal definition of the platform objectives to Section 4, we will consider two standard objectives – gross revenue and social welfare. These operational metrics are often used by practitioners to optimize pricing and matching decisions. In ridesharing, platforms charge a fixed commission rate, implying that gross revenue is a proxy for the platform's revenue. Platforms may perceive social welfare as a proxy (or "surrogate") for long-term growth; it is often utilized as an outcome measure (Yan et al. 2020).

Additionally, we formulate a notion of *first-best*, which provides us with an upper bound on social welfare. Here, we assume that the platform observes the private agent types (i.e, valuations and costs), but it does not know ex-ante the realizations of idiosyncratic shocks. Based on this information, it chooses suppliers' prices, customers' thresholds, and the matching. The corresponding first-best optimization problem is formulated as a continuous linear program in Appendix C.

4. Warm-up Case: Impatient Markets

We start with analyzing *impatient markets*, which amounts to $\mu_S = \mu_C = 1$. In this market setting, there are no multiple match attempts, and agents leave immediately after one batch-matching. The resulting one-shot game can be analyzed in closed form, as shown in Proposition 1 below; the proof is deferred to Appendix 1.

Proposition 1 (Equilibrium Structure) *For impatient markets (i.e., $\mu_S = \mu_C = 1$), there exists a unique equilibrium for any centralized or decentralized platform types. In the centralized equilibrium with price z, suppliers with costs below z and customers with valuations above $z-\epsilon$ enter the market, i.e., $T_S = [0, z)$ and $T_C = (z-\epsilon, 1]$. In the decentralized equilibrium, suppliers with costs below $1+\epsilon$ and customers with valuations above $\max\{1/2-\epsilon, (1-2\epsilon)/3\}$ enter the market, i.e., $T_S = [0, 1+\epsilon)$ and $T_C = (\max\{1/2-\epsilon, (1-2\epsilon)/3\}, 1]$. Customers have a uniform threshold $\tau(v) = 0$ for all $v \in T_C$ in both platform types. In the decentralized equilibrium, suppliers choose the following prices:*

$$p(c) = \begin{cases} \frac{1+c}{2}, & \text{if } \ 0 \leq c < \max\{0, 1-2\epsilon\}, \\ \frac{1+2c+\epsilon}{3}, & \text{if } \max\{0, 1-2\epsilon\} \leq c < 1+\epsilon. \end{cases}$$

Next, we compare the total social welfare attained by centralized and decentralized platform types. As mentioned in Section 3.3, we consider two plausible objective functions for centralized platforms: the per-period social welfare $\min\{\|\bar{N}_C\|, \|\bar{N}_S\|\} \cdot \mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v} - \boldsymbol{c} + \boldsymbol{\eta}) \cdot \mathbb{I}(\boldsymbol{v} - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq \tau(\boldsymbol{v}))]$, and the platform's gross revenue $\min\{\|\bar{N}_C\|, \|\bar{N}_S\|\} \cdot \mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[p(\boldsymbol{c}) \cdot \mathbb{I}(\boldsymbol{v} - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq 0)]$. Given an instance $(I, \epsilon) = (\lambda_C, \lambda_S, \mu_C, \mu_S, \epsilon)$, we denote by $W^{\text{dec}}_{I,\epsilon}$ the social welfare in the decentralized equilibrium and by $W^{\text{cen}}_{I,\epsilon}(z)$ the social welfare in the centralized equilibrium with price $z = p_{\min} = p_{\max}$. Similarly, we denote by $R^{\text{cen}}_{I,\epsilon}(z)$ the platform's gross revenue in the centralized equilibrium with price z. Based on these objectives, we consider centralized platforms choosing either a welfare-maximal price $p^{*W}_{I,\epsilon} \in \arg\max_{z\in[0,1+\epsilon]} W^{\text{cen}}_{I,\epsilon}(z)$, or a revenue-maximal price $p^{*R}_{I,\epsilon} \in \arg\max_{z\in[0,1+\epsilon]} R^{\text{cen}}_{I,\epsilon}(z)$.

In the next proposition, we identify and compare the social welfare corresponding to different platform types in the limit $\epsilon \to 0$. The proof appears in Appendix D.2.

Proposition 2 (Welfare Comparisons) *For impatient markets (i.e., $\mu_S = \mu_C = 1$), as ϵ tends to 0, the welfare-maximal and the revenue-maximal prices converge to $p^{*W}_{I,\epsilon} = \lambda_C/(\lambda_C + \lambda_S)$ and $p^{*R}_{I,\epsilon} = \max\{1/2, p^{*W}_{I,\epsilon}\}$, respectively. Customers' net surplus and suppliers'* *profits converge to the closed-form expressions provided in Table 2, Appendix A. The welfare ratio of the decentralized platform relative to a welfare-maximizing centralized platform is tightly bounded as follows:*

$$\inf_I \lim_{\epsilon\to 0^+} \frac{W_{I,\epsilon}^{\text{dec}}}{W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*W})} = \frac{1}{4} \quad \text{and} \quad \sup_I \lim_{\epsilon\to 0^+} \frac{W_{I,\epsilon}^{\text{dec}}}{W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*W})} = \frac{3}{4}.$$

Similarly, the welfare ratio relative to a revenue-maximizing platform is bounded as follows:

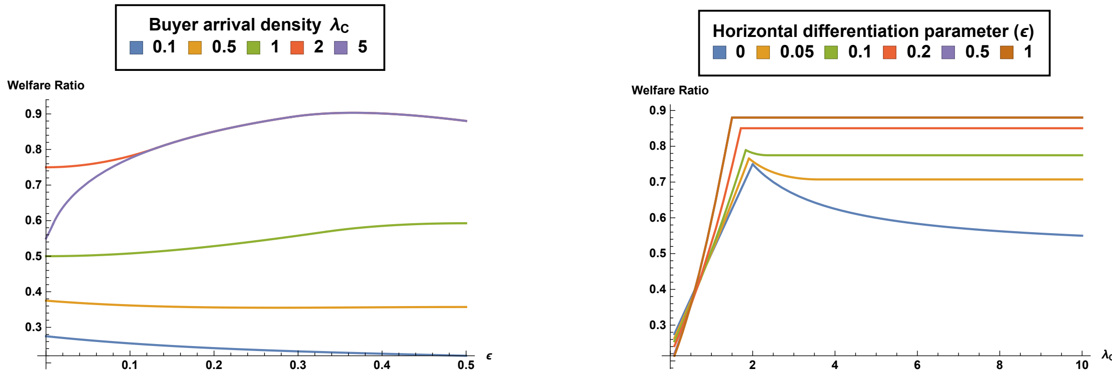
$$\inf_I \lim_{\epsilon\to 0^+} \frac{W_{I,\epsilon}^{\text{dec}}}{W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*R})} = \frac{1}{2} \quad \text{and} \quad \sup_I \lim_{\epsilon\to 0^+} \frac{W_{I,\epsilon}^{\text{dec}}}{W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*R})} = \frac{3}{4}.$$

Proposition 2 implies that centralization leads to higher social welfare for impatient markets as the horizontal differentiation parameter ϵ tends to 0. Even the maximization of gross revenue as a platform objective achieves higher social welfare than decentralization, although the worst-case welfare ratio is improved from 1/4 to 1/2. Put simply, decentralization harms social welfare, compared to centralized platforms with plausible objectives.

We extend our analysis to $\epsilon > 0$ in Appendix D.3. In this setting, the expressions for customers' surplus and suppliers' profit are more complex. We plot the effect of horizontal differentiation on the welfare ratio (as defined in Proposition 2) in Figure 2; also see Figure 5 in Appendix A for a 3D plot. The main findings of Proposition 2 continue to hold, as the centralized platform generates higher social welfare than the decentralized platform for all market characteristics. The effect of the horizontal differentiation parameter ϵ is non-monotone. For large ϵ, the welfare ratio can be as low as 0.2, breaching the lower bound in Proposition 2. In general, we observe that decentralization is more competitive in a relatively balanced market, and the efficiency loss widens as the demand-supply imbalances increase. To explain this, as shown in Proposition 1, the decentralized prices do not internalize the demand-supply imbalances as they do not depend on λ_C, λ_S. Accordingly, the welfare ratio $W_{I,\epsilon}^{\text{dec}}/W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*W})$ peaks at $\lambda_C/\lambda_S = (1+\epsilon)/\min\{1/2+2\epsilon, (2+5\epsilon)/3\}$ where the masses of suppliers and customers are equal at equilibrium.[6]

Now, how do these findings generalize to patient markets? When $\mu_S, \mu_C < 1$, the market becomes dynamic: agents are involved in multiple match attempts before they depart. In particular, customers are no longer myopic (i.e., they do not accept all matches that generate positive utility) and optimize their acceptance thresholds based on their expectations

[6] The peak is not at $\lambda_C/\lambda_S = 1$ because suppliers earn positive profits and set prices higher than their costs; thus, a fraction of low-valuation customers do not enter the market.



(a) Welfare ratio against ϵ (b) Welfare ratio against λ_C

Figure 2 **The welfare ratio between the decentralized and centralized settings, $W^{\text{dec}}_{I,\epsilon}/W^{\text{cen}}_{I,\epsilon}(p^{*W}_{I,\epsilon})$, for the impatient market case ($\mu_S = \mu_C = 1$). Instances generated by varying ϵ and λ_C while $\lambda_S = 1$.**

of future surplus. It is unclear how such dynamic and strategic behaviors affect centralized and decentralizing pricing. Intuitively, smaller values of μ_S, μ_C reduce the "cost" of unsuccessful matches, because customers and suppliers are more likely to be rematched in the future. However, this effect is internalized by the customers' and suppliers' strategies, potentially leading to market congestion. Additionally, the type mix might exhibit more low-valuation customers and high-cost suppliers, which are harder to match.

5. General Case: Equilibrium Analysis

In this section, we analyze the market equilibria in a general setting for different platform types, i.e., $\mu_C, \mu_S \in (0,1]$, $\epsilon > 0$, and $p_{\min}, p_{\max} \in \mathbb{R}^+ \cup \{+\infty\}$.

5.1. Main results

Our first set of results uncovers the structure of the equilibria and characterizes them as the solutions of a system of piecewise polynomial equations. To this end, we introduce the functional change of variable $\omega(v) = v - \tau(v)$; we call this quantity the *willingness-to-pay* of customers with valuation v noting that $\omega(v)$ is the maximum acceptable price in the absence of horizontal differentiation. As seen subsequently, the equilibrium properties can be expressed more easily in terms of $\omega(\cdot)$ rather than $\tau(\cdot)$. Given a stationary equilibrium (τ, p, N), we introduce the following notation for the upper and lower bounds on the distribution of willingness-to-pay:

$$\underline{\omega} = \inf\{v - \tau(v) : \bar{N}_C(v) > 0\} \quad \text{and} \quad \overline{\omega} = \sup\{v - \tau(v) : \bar{N}_C(v) > 0\},$$

where, for simplicity, our notation omits the dependence of $\underline{\omega}$, $\overline{\omega}$ on (τ, p, N). Note that $\underline{\omega}$, $\overline{\omega}$ are well-defined if a positive mass of customers enters the market. In the remainder of our analysis, we overlook trivial equilibria in which no agent participates in the market. Finally, we denote by $\delta_S(p_{\min})$ the point mass of suppliers who choose price $p_{\min}$ exactly.[7]

Theorem 1 **(Equilibrium structure)**. *For the centralized and decentralized platform types, there exists a unique stationary equilibrium. In general, for the semi-centralized platform type, every stationary equilibrium (τ, p, N) is uniquely determined by the variables $(\underline{\omega}, \overline{\omega}, \delta_S(p_{\min}))$, which must be the roots of a system formed by three piecewise-polynomial equations of degree 3, whose coefficients only depend on the instance parameters $\lambda_C, \lambda_S, \mu_C, \mu_S, \epsilon, p_{\min}$, and $p_{\max}$.*

Underlying this theorem is a fine-grained characterization of the equilibrium structure. We show in Section 5.2 that, in equilibrium, the willingness-to-pay and prices are well-behaved functions of the valuations and costs, respectively. In particular, the density of customers' willingness-to-pay in equilibrium is uniform with rate λ_C/μ_C. The distribution of suppliers' prices is slightly more complex, with a piecewise uniform density and potential point masses at the boundaries $\underline{p}, \overline{p}$. The platform's intervention through the price menu $[p_{\min}, p_{\max}]$ may restrict the price spread in equilibrium.

As a result, in the proof of Theorem 1, we reduce the determination of equilibria to three market variables $(\underline{\omega}, \overline{\omega}, \delta_S(p_{\min}))$. Our proof, presented in Section 5.3, is constructive: the polynomial system is explicit and it can be easily solved for any given instance. In particular, this result gives us a tool to evaluate and compare different platform types $[p_{\min}, p_{\max}]$ in terms of social welfare and revenue in Section 6. Note, however, that Theorem 1 does not guarantee the existence and uniqueness of equilibria in general (semi-centralized case) because we cannot theoretically argue that our polynomial system always admits a unique solution. Nonetheless, by solving this system numerically, we verify the existence and uniqueness of the equilibrium on a large array of instances, which suggests that this property might be generally true.[8]

Based on the equilibrium characterization, we determine the efficiency of decentralized platform types within patient markets. Recall from Section 4 that, for impatient markets, decentralization is costly to social welfare, in the sense that centralized platforms with

[7] If the suppliers' price distribution is non-atomic, then $\delta_S(p_{\min}) = 0$, but this may not be the case in equilibrium.

[8] Specifically, we vary $\lambda_S/\lambda_B \in \{0.01, 0.02, 0.1, 0.2, 0.5, 0.75, 1, 1.25, 1.5, 2, 5, 10, 50, 100\}$, $\epsilon \in \{0, 0.1, 0.5\}$, $\mu_C = \mu_S \in \{0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$ and vary $p_{\min}$ and $p_{\max}$ in 0.05 increments such that $p_{\min} \leq p_{\max}$.

plausible objective functions generate higher welfare. By contrast, we establish that as customers' and suppliers' patience increases, i.e., μ_C, μ_S tend to 0, the decentralized platform converges to the first-best outcome.

THEOREM 2 **(Welfare ratio)**. *As μ_C, μ_S tend to 0 subject to $\mu_C/\mu_S = \Theta(1)$, the welfare ratio between the decentralized platform and the first-best outcome converges to 1. Specifically, suppliers' prices uniformly converge to $(1+\epsilon)\cdot\lambda_C/(\lambda_C+\lambda_S)$ and the price spread $\overline{p} - \underline{p}$ decreases to zero.*

At first sight, it was unclear how lower departure rates would affect market outcomes. As mentioned in Section 4, one could postulate a vicious circle caused by strategic behaviors and market congestion. Contrary to this hypothesis, Theorem 2 shows that market efficiency is restored by uniformly increasing the patience level of the market participants. In other words, if customers and suppliers are willing to engage in a lengthy search process or to consider multiple match requests, then full decentralization attains efficient market prices in equilibrium. This result implies that decentralization might be more plausible when there is no time pressure to match the market participants on both sides. For example, in the transportation sector, this platform type might be more adequate for pre-scheduled rides than for on-demand marketplaces.

One can draw a parallel between Theorem 2 and Coasian insights for durable good markets with uncertain valuations (Coase 1972, Fudenberg et al. 1985). The asymptotic regime on departure rates $\mu_C, \mu_S \to 0$ can be viewed as per-period discount rates approaching 1 for future expected utilities. The Coase conjecture states that in equilibrium, by sequential rationality, the price exerted by a monopoly seller converges to the competitive price as the discount rate goes to 1– the seller's market power dissipates, and the profit is cut to zero. In a sense, Theorem 2 shows a robust Coase conjecture for two-sided matching markets with competing buyers and sellers. One difference, however, is that the counterpart notion of competitive price is a market clearing price, where most sellers continue to exert a price markup and earn positive profits in the asymptotic regime.

5.2. Equilibrium strategies

Here, we analyze the structure of market equilibria, starting with the customer side.

Proposition 3 *For every stationary equilibrium (τ, p, N), customers' thresholds $\tau(v)$ and willingness-to-pay $v - \tau(v)$ are monotonically increasing in v for all $v \in T_C$. Moreover, the* *distribution of customers' willingness-to-pay has the uniform density* $f_C(w) = \lambda_C/\mu_C$ *over* $w \in [\underline{\omega}, \overline{\omega}]$.

The proof of this proposition is presented in Appendix E.1. We derive the first-order condition for the customers' best-response problem, applying the envelope theorem; Proposition 3 follows by combining this condition with Bellman's equation and the inverse sampling property. Note that the result only requires τ to be a best-response to p and N.

To interpret Proposition 3, recall that customers with valuation v decide on whether to accept the current match requests offered to them by comparing their net surplus $v - p(\boldsymbol{c}) + \boldsymbol{\eta}$ to their threshold $\tau(v)$. Therefore, an optimal threshold strikes a balance between the likelihood of being matched ultimately and the net surplus achieved by successful matches. The monotonicity properties of Proposition 3 show that the customer-side strategies exhibit a vertical differentiation in equilibrium. Customers with higher valuations can afford a higher willingness-to-pay, preserving a competitive edge in the matching process. Simultaneously, they demand a higher net surplus from successful matches, as implied by the monotonicity of their thresholds. More importantly, Proposition 3 shows that the density of the customers' willingness-to-pay values is uniform and equal to λ_C/μ_C, which increases inversely with μ_C as customers become more patient. In other words, the distribution of willingness-to-pay is a scaled version of the arrival distribution of valuations. One may wonder if this property is a coincidence or if it reflects the distribution of customers' private valuations upon arrival. In the next corollary, we show that this property is indeed more general.

Corollary 1. *Suppose that customers arrive with density* $f(v)$ *over valuations* $v \in (-\epsilon, 1]$. *In equilibrium, the distribution of customers' willingness-to-pay has the density* $(1/\mu_C) \cdot f(\omega^{-1}(w))$ *for all* $w \in [\underline{\omega}, \overline{\omega}]$.

Next, we characterize the distribution of suppliers' prices in any stationary equilibrium; the proof is presented in Appendix E.2.

Proposition 4 *For every stationary equilibrium* (τ, p, N), *suppliers' prices* $p(c)$ *are monotonically increasing in* c, *and their profits* $p(c) - c$ *per successful match are decreasing in* c *over the interval* $\{c \in T_S : p(c) \in (p_{\min}, p_{\max})\}$. *The support of the distribution of suppliers' prices is the interval* $[\underline{p}, \overline{p}]$, *where* $\underline{p} = \underline{\omega} + \epsilon$ *and* $\overline{p} = \min\{p_{\max}, \overline{\omega} + \epsilon\}$. *This distribution* *has a density function $f_S(y)$ over $y \in (\underline{p}, \overline{p})$ and potential point masses $\delta_S(\underline{p}), \delta_S(\overline{p})$ at the boundaries. Moreover, $\delta_S(\underline{p}) > 0$ implies $\underline{p} = p_{\min}$, $\delta_S(\overline{p}) > 0$ implies $\overline{p} = p_{\max}$, and $f_S(y)$ is piecewise-uniform over $y \in (\underline{p}, \overline{p})$:*

$$f_S(y) = \begin{cases} 2 \cdot \frac{\lambda_S}{\mu_S}, & \text{if } \ \underline{p} < y < \min\{p_{\max}, \overline{\omega} - \epsilon\}, \\ \frac{3}{2} \cdot \frac{\lambda_S}{\mu_S}, & \text{if } \ \max\{\underline{p}, \min\{p_{\max}, \overline{\omega} - \epsilon\}\} \le y < \overline{p}. \end{cases}$$

This proposition shows similarities and differences between the customer and supplier sides of the market. Similarly to customers, the suppliers' strategies are vertically differentiated: lower-cost suppliers enjoy both higher profits and larger match rates, retaining a competitive edge in the matching process. However, one important difference is that the platform can operationally control the level of heterogeneity via the price menu $[p_{\min}, p_{\max}]$: prices may cluster at the boundaries of the interval. The distributional analysis of $p(\boldsymbol{c})$ is more intricate than the customer-side counterpart due to a boundary effect for large prices; it uses the first-order optimality condition for sellers' prices and the uniform density of customers' willingness-to-pay in Proposition 3. Therefore, our analysis may not generalize under different distributional assumptions (contrary to Corollary 1).

5.3. Proof outline of Theorem 1

In this section, we present the crucial piecewise polynomial system used to characterize the stationary equilibria and outline the proof ideas used to derive it. Since there are lengthy technical details, we defer the proofs of all lemmas to Appendix F.

Piecewise polynomial system. Given a stationary equilibrium (τ, p, N), we define $p^{\text{foc}}(c)$ as the price value that satisfies the first-order optimality condition of a cost-c supplier (see equation (35) in Appendix E.2). Let $c_0 = \inf\{c \in T_S : p(c) = p^{\text{foc}}(c)\}$, where $c_0 = -\infty$ if the set is empty.[9] Interestingly, the next lemma shows that, when $c_0 = -\infty$, all suppliers' prices cluster at $p_{\max}$.

Lemma 1. *For any stationary equilibrium such that $c_0 = -\infty$, i.e., $p(c) \neq p^{\text{foc}}(c)$ for all $c \in T_S$, all suppliers choose the price $p_{\max}$, i.e., $p(c) = p_{\max}$ for all $c \in T_S$.*

In this special case, the decentralized equilibrium is identical to the centralized setting, and our polynomial system takes a simplified form, which is separately presented in Appendix F.2. Hence, in the remainder of this section, we focus on the interesting case

[9] A supplier's profit-maximizing price may not satisfy $p^{\text{foc}}(c) \in [p_{\min}, p_{\max}]$ in equilibrium, meaning that $p(c) \neq p^{\text{foc}}(c)$.

$c_0 \neq -\infty$. Using the change of variables $x = \overline{\omega} - \underline{\omega}$, $y = p_{\max} - \underline{\omega} - \epsilon$, and $z = \delta_S(p_{\min})$, we show that the equilibrium is uniquely determined by (x, y, z), which must be a root of the system $P_1(x,y,z) = 0$, $P_2(x,y,z) = 0$, and $P_3(y,z) = 0$, where

$$P_1(x,y,z) = \left(\frac{1}{\mu_C} - 1\right) \cdot \left(\begin{cases} \min\left\{ \frac{(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{\mu_S z + \lambda_S(x+\min\{x,y\}-\epsilon)}, \frac{\mu_C(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{x\lambda_C \mu_S} \right\}, & \text{if } \; x > 2\epsilon \\ \min\left\{ \frac{x^2(\lambda_S x + 2\mu_S z)}{4\epsilon(2\mu_S z + \lambda_S(x+2\min\{x,y\}))}, \frac{x\mu_C(\lambda_S x + 2\mu_S z)}{8\epsilon\mu_S\lambda_C} \right\}, & \text{if } \; x \leq 2\epsilon \end{cases} \right)$$

$$+ x - y + p_{\max} - \epsilon - 1,$$

$$P_2(x,y,z) = \left(\frac{1}{\mu_S} - 1\right) \cdot \left(\begin{cases} \min\left\{ \frac{(x-\epsilon)(\mu_S z + \lambda_S(x-\epsilon))}{x}, \frac{\lambda_C \mu_S (x-\epsilon)(\mu_S z + \lambda_S(x-\epsilon))}{\mu_C(\mu_S z + \lambda_S(x+\min\{x,y\}-\epsilon))} \right\}, & \text{if } \; x > 2\epsilon \\ \min\left\{ \frac{x(2\mu_S z + \lambda_S x)}{8\epsilon}, \frac{x^2\lambda_C\mu_S(2\mu_S z + \lambda_S x)}{4\mu_C\epsilon(2\mu_S z + \lambda_S(x+2\min\{x,y\}))} \right\}, & \text{if } \; x \leq 2\epsilon \end{cases} \right)$$

$$+ \left(\begin{cases} \lambda_S(x-\epsilon), & \text{if } \; x > 2\epsilon \\ \frac{\lambda_S x}{2}, & \text{if } \; x \leq 2\epsilon \end{cases} \right) + \mu_S z - \lambda_S(p_{\max} - y),$$

$$P_3(y,z) = (y - p_{\max} + p_{\min}) \cdot z \; . \tag{7}$$

Uniqueness of the (x,y,z)-parametrization. The customer-side strategies are expressed by Proposition 3 in terms of the variables $\underline{\omega}$ and $\overline{\omega}$. Specifically, Proposition 3 shows that the density of willingness-to-pay is λ_C/μ_C for any willingness-to-pay value $\omega \in [\underline{\omega}, \overline{\omega}]$, and the point-mass of customers is 0 for any such willingness-to-pay value. On the other hand, the supplier side is fully characterized by Proposition 4 in terms of the variables $\underline{\omega}$, $\overline{\omega}$, $\delta_S(p_{\min})$, $\delta_S(p_{\max})$. Namely, Proposition 4 identifies the density of suppliers' prices within the open interval $(\underline{p}, \overline{p})$ where $\underline{p} = \max\{p_{\min}, \underline{\omega} + \epsilon\}$ and $\overline{p} = \min\{\overline{\omega} + \epsilon, p_{\max}\}$. Hence, it suffices to show that $\delta_S(p_{\max})$ can be expressed in terms of market parameters and $\overline{\omega}$.

LEMMA 2. *Suppose that $p_{\min} < p_{\max}$ and $c_0 \neq -\infty$. Then, the mass of suppliers who pick price $p_{\max}$ is*

$$\delta_S(p_{\max}) = \begin{cases} \frac{\lambda_S}{\mu_S} \cdot (\overline{\omega} - p_{\max}), & \text{if } \; \underline{\omega} + \epsilon < p_{\max} \leq \overline{\omega} - \epsilon, \\ \frac{\lambda_S}{2\mu_S} \cdot (\overline{\omega} - p_{\max} + \epsilon), & \text{if } \; \overline{\omega} - \epsilon < p_{\max} < \overline{\omega} + \epsilon, \\ 0, & \text{if } \; \overline{\omega} + \epsilon \leq p_{\max}. \end{cases}$$

Thus, knowing $\underline{\omega}$, $\overline{\omega}$, and $\delta_S(p_{\min})$ is sufficient to characterize agents' strategies uniquely.

Derivation of $P_3(y,z)=0$. Recall that $p_{\min} \le p(0) = \underline{\omega} + \epsilon$ by the customer-side entry conditions. If $p_{\min} < \underline{\omega} + \epsilon = p(0)$, we infer that the mass of suppliers with price $p_{\min}$ is 0, i.e., $\delta_S(p_{\min}) = 0$. Hence, we conclude that the following equation always holds $(p_{\min} - \underline{\omega} - \epsilon) \cdot \delta_S(p_{\min}) = 0$, which yields the relationship $P_3(y,z) = (y - p_{\max} + p_{\min}) \cdot z = 0$.

Derivation of $P_1(x,y,z)=0$. Observe that the average match rate m_C of a uniformly randomly chosen customer is

$$m_C = \min\left\{1, \frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\} \cdot \Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p(\boldsymbol{c})\right]. \tag{8}$$

By combining equation (8) with Proposition 3, we infer the following structural equation that expresses the total mass of customers in steady-state:

$$\frac{\lambda_C(\overline{\omega} - \underline{\omega})}{\mu_C} = \frac{\lambda_C(1-\underline{\omega})}{(1-\mu_C)m_C + \mu_C}. \tag{9}$$

The left-hand side proceeds from the density λ_C/μ_C of customers' willingness-to-pay in equilibrium. To explain the right-hand side, at the beginning of each period, a mass $(1-\underline{\omega})\lambda_C$ of customers enters the market, of which a fraction $(1-\mu_C)(1-m_C)$ survives to the following period. Indeed, customers enter with a uniform density λ_C over the valuations in $T_C = (\underline{\omega}, 1]$, where $\underline{\omega} = \omega(\underline{v}) = \underline{v} - \tau(\underline{v}) = \underline{v}$.

Recall that $x = \overline{\omega} - \underline{\omega}$, $y = p_{\max} - \underline{\omega} - \epsilon$, and $z = \delta_S(p_{\min})$. Hence, to derive the desired equation, it suffices to express m_C in terms of (x,y,z) and substitute in equation (9). In the next lemma, we express two crucial quantities in the definition of m_C (see equation (8)) as a function of x,y, and z. The proof requires lengthy technical details, which are presented in Appendix F.4.

LEMMA 3. *Suppose that* $p_{\min} < p_{\max}$ *and* $c_0 \neq -\infty$. *The average match success rate is*

$$\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p(\boldsymbol{c})\right] = \begin{cases} \frac{(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{x(\mu_S z + \lambda_S(x+\min\{x,y\}-\epsilon))}, & \text{if } x > 2\epsilon, \\ \frac{\lambda_S x^2 + 2x\mu_S z}{4\epsilon(2\mu_S z + \lambda_S(x+2\min\{x,y\}))}, & \text{if } x \le 2\epsilon. \end{cases}$$

Moreover, the total mass of suppliers is $\|\bar{N}_S\| = z + (\lambda_S/\mu_S)(x + \min\{x,y\} - \epsilon)$ *if* $x > 2\epsilon$, *and* $\|\bar{N}_S\| = z + (\lambda_S/2\mu_S)(x + 2\min\{x,y\})$ *otherwise.*

With Lemma 3, $\|\bar{N}_C\|$ is the only remaining term in equation (8) that is not expressed in terms of the variables x,y, and z. It follows from Proposition 3 that $\|\bar{N}_C\| = (\lambda_C/\mu_C)\cdot(\overline{\omega} - \underline{\omega}) = (\lambda_C/\mu_C)\cdot x$. Combining this observation with Lemma 3, we obtain:

$$m_C = \begin{cases} \min\left\{\frac{(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{x(\mu_S z+\lambda_S(x+\min\{x,y\}-\epsilon))}, \frac{\mu_C(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{x^2\lambda_C\mu_S}\right\}, & \text{if } x > 2\epsilon, \\ \min\left\{\frac{\lambda_S x^2 + 2x\mu_S z}{4\epsilon(2\mu_S z+\lambda_S(x+2\min\{x,y\}))}, \frac{\mu_C(\lambda_S x + 2\mu_S z)}{8\epsilon\mu_S\lambda_C}\right\}, & \text{if } x \le 2\epsilon. \end{cases} \tag{10}$$

Finally, by substituting in equation (9) the expression for m_C from (10), we obtain:

$$
\begin{aligned}
0 &= (\overline{\omega}-\underline{\omega})\left(\frac{1-\mu_C}{\mu_C}\cdot m_C+1\right)-(1-\underline{\omega}) = x\cdot\left(\frac{1}{\mu_C}-1\right)\cdot m_C + x - y + p_{\max} - \epsilon - 1\\
&= \left(\frac{1}{\mu_C}-1\right)\cdot\left(\begin{cases}\min\left\{\frac{(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{\mu_S z+\lambda_S(x+\min\{x,y\}-\epsilon)}, \frac{\mu_C(x-\epsilon)(\lambda_S(x-\epsilon)+\mu_S z)}{x\lambda_C\mu_S}\right\}, & \text{if } x>2\epsilon\\ \min\left\{\frac{x^2(\lambda_S x+2\mu_S z)}{4\epsilon(2\mu_S z+\lambda_S(x+2\min\{x,y\}))}, \frac{x\mu_C(\lambda_S x+2\mu_S z)}{8\epsilon\mu_S\lambda_C}\right\}, & \text{if } x\le 2\epsilon\end{cases}\right)\\
&\quad + x - y + p_{\max} - \epsilon - 1\\
&= P_1(x,y,z).
\end{aligned}
$$

Derivation of $P_2(x,y,z)=0$. Here, we derive an equation for the lowest price $\underline{p}$ in the market. Specifically, we distinguish between two cases, $\delta_S(p_{\min})>0$ and $\delta_S(p_{\min})=0$. In the former case, we have $\underline{p}=p_{\min}$, and we express $\delta_S(p_{\min})$ as a function of $\phi_S(\underline{p},\tau,N)$ and other market parameters.

LEMMA 4. *Suppose that $c_0\neq-\infty$ and $\underline{p}=p_{\min}$. Then, the point mass of suppliers with price $p_{\min}$ is characterized by the following equation:*

$$\delta_S(p_{\min}) = \frac{p_{\min}\cdot\lambda_S}{(1-\mu_S)\phi_S(p_{\min},\tau,N)+\mu_S} + \frac{\lambda_S}{\mu_S}\cdot\frac{\phi_S(p_{\min},\tau,N)}{\phi_S'(p_{\min},\tau,N)}.$$

In the opposite case, i.e., $\delta_S(p_{\min})=0$, it follows that $c_0=0$. Therefore, we invoke the first-order optimality condition at c_0 (see (35) in the proof of Proposition 4 for a formal derivation)

$$p(c_0)-c_0 = \frac{-\phi_S(p(c_0),\tau,N)}{\phi_S'(p(c_0),\tau,N)}\cdot\frac{(1-\mu_S)\phi_S(p(c_0),\tau,N)+\mu_S}{\mu_S}. \tag{11}$$

By noting that $p(c_0)=p_{\min}$ when $\delta_S(p_{\min})>0$, the key observation is that equation (11) and Lemma 4 can be rearranged into a single, unconditional relationship:

$$\delta_S(p_{\min}) = \frac{\underline{p}\cdot\lambda_S}{(1-\mu_S)\phi_S(\underline{p},\tau,N)+\mu_S} + \frac{\lambda_S}{\mu_S}\cdot\frac{\phi_S(\underline{p},\tau,N)}{\phi_S'(\underline{p},\tau,N)}. \tag{12}$$

Now, given that $z=\delta_S(p_{\min})$ and $\underline{p}=\underline{\omega}+\epsilon=p_{\max}-y-\epsilon$, it suffices to express $\phi_S(\underline{p},\tau,N)$ as a function of x,y, and z. Using this approach, equation (12) yields our final piecewise polynomial equation $P_2(x,y,z)=0$. Technical details are left to Appendix F.6.

Existence and uniqueness properties. In the special cases of centralized and decentralized platform types, we show that there exists a unique equilibrium. Our proof exploits the piecewise polynomial system in simplified form. In the decentralized case, we note that $z = 0$ (meaning that the variable z and the equation $P_3(y,z) = 0$ can be eliminated) and the polynomials P_1, P_2 can be further simplified by noting that $y < x$. Using such simplified expressions, we show a strict monotonicity property of y relative to x over the system's roots, which then implies that there exists a unique equilibrium. This reasoning is developed in Appendix F.7. For the centralized case, we invoke the simplified system for centralized prices, and derive an analogous monotonicity argument in Appendix F.8.

5.4. Proof of Theorem 2

We fix all instance parameters, except $(\mu_C, \mu_S) = (\mu, \alpha\mu)$, where $\alpha > 0$ is a constant and $\mu \in (0,1)$ varies. For simplicity, here, we assume that μ_C/μ_S is constant, but our proof argument easily extends to $\mu_C/\mu_S = \Theta(1)$. Now, let (τ_μ, p_μ, N_μ) be arbitrarily chosen equilibria for each $\mu = \mu_C \in (0,1)$. We define (x_μ, y_μ, z_μ) (when there is no ambiguity, we drop the reference to μ, i.e., (x,y,z)) as the corresponding root in the piecewise polynomial system of Section 5.3.[10] Our proof proceeds in three steps. First, we show that $\underline{p}_\mu, \overline{p}_\mu$ converge to $((1+\epsilon)\lambda_C)/(\lambda_C + \lambda_S)$ in the limit $\mu \to 0$. Then, we derive an upper bound U^* on the first-best social welfare for all $\mu \in (0,1)$. Finally, we show that, in the limit $\mu \to 0$, the social welfare in the decentralized setting converges to U^*.

Step 1: Price spread and convergence. In the decentralized setting, we take $p_{\max}$ arbitrarily large, meaning that $\min\{x_\mu, y_\mu\} = x_\mu$. Moreover, we clearly have $p_{\min} = 0$, and thus, $z_\mu = \delta_{S,\mu}(p_{\min}) = 0$. Using these simplifications, the equation $P_1(x_\mu, y_\mu, z_\mu) = 0$ amounts to

$$\left(\begin{cases} \min\left\{\frac{(x_\mu-\epsilon)^2}{(2x_\mu-\epsilon)}, \frac{\alpha\lambda_S(x_\mu-\epsilon)^2}{x_\mu\lambda_C}\right\}, & \text{if } x_\mu > 2\epsilon \\ \min\left\{\frac{x_\mu^2}{12\epsilon}, \frac{\alpha\lambda_S x_\mu^2}{8\epsilon\lambda_C}\right\}, & \text{if } x_\mu \le 2\epsilon \end{cases}\right) = \left(\frac{\mu}{1-\mu}\right) \cdot (1 - x_\mu + y_\mu - p_{\max} + \epsilon). \quad (13)$$

Recalling that we define $x_\mu = \overline{\omega}_\mu - \underline{\omega}_\mu \in (0, 1+\epsilon)$ and $y_\mu = p_{\max} - \underline{\omega}_\mu - \epsilon \in (0, 1+\epsilon)$, it follows that $x_\mu - y_\mu + p_{\max} - \epsilon - 1$ is uniformly bounded over $\mu \in (0,1)$. As a result, we infer from equation (13) that x_μ reduces as μ decreases and $x_\mu \to 0$ as $\mu \to 0$; note that it cannot be the case that $x_\mu > 2\epsilon$ for sufficiently small $\mu \in (0,1)$ as the right-hand side of equation (13) converges to zero. In this regime, equation (13) implies

$$\frac{1}{\mu} \cdot \min\left\{\frac{x_\mu^2}{12\epsilon}, \frac{\alpha\lambda_S x_\mu^2}{8\epsilon\lambda_C}\right\} = -x_\mu + y_\mu - p_{\max} + \epsilon + 1 = 1 - \overline{\omega}_\mu. \quad (14)$$

[10] Recall that $x = \overline{\omega} - \underline{\omega}$, $y = p_{\max} - \underline{\omega} - \epsilon$, and $z = \delta_S(p_{\min})$ in the notation of Section 5.3.

Now, using the above-mentioned simplifications, $z=0$ and $\min\{x_\mu, y_\mu\} = x_\mu$, and noting that $x_\mu \le 2\epsilon$ for sufficiently small $\mu \in (0,1)$, our second piecewise polynomial equation $P_2(x_\mu, y_\mu, z_\mu) = 0$ can be rearranged as:

$$\frac{1}{\mu_S} \cdot \min\left\{\frac{\lambda_S x_\mu^2}{8\epsilon}, \frac{x_\mu^2 \lambda_C}{12\alpha\epsilon}\right\} = \frac{\lambda_S}{2}\left(2(p_{\max} - y) - x\right) = \frac{\lambda_S}{2}\left((\overline{\omega}_\mu - \underline{\omega}_\mu) + 2(\underline{\omega}_\mu + \epsilon)\right). \tag{15}$$

By dividing both sides of equation (14) by those of (15), and using the fact that $x_\mu = \overline{\omega}_\mu - \underline{\omega}_\mu$ converges to 0 in the limit $\mu \to 0$, we infer that $(1 - \underline{\omega}_\mu)/(\underline{\omega}_\mu + \epsilon) \to \lambda_S/\lambda_C$. Hence, by rearranging, $\underline{p}_\mu = \underline{\omega}_\mu + \epsilon$ and $\overline{p}_\mu = \underline{p}_\mu + x_\mu$ both converge to $(1+\epsilon)\lambda_C/(\lambda_C + \lambda_S)$ as μ tends to 0.

Step 2: Upper bound on first-best. Define $U^* = \lambda_C \int_{p^*-\epsilon}^1 (v+\epsilon)dv - \lambda_S \int_0^{p^*} cdc$ where $p^* = (1+\epsilon)\cdot \lambda_C/(\lambda_C + \lambda_S)$; intuitively, this is the social welfare if one were able to match customers with valuations $[p^* - \epsilon, 1]$ with suppliers of valuation $[0, p^*]$ in each period, while cherry-picking the maximum possible realization ϵ of the random shock η. The next claim shows that U^* is an upper bound on the *first-best* welfare, which we denote by W_μ^{fb} (see Appendix C for a formal definition).

CLAIM 1. $U^* \ge W_\mu^{\text{fb}}$.

Step 3: Matching lower bound. The final piece is to prove that the welfare W_μ^{dec} attained by the decentralized platform converges to U^* in the limit $\mu \to 0$. Based on step 1, fix $\delta > 0$ and choose μ sufficiently small such that $p^* - \delta \le \underline{p} \le \overline{p} \le p^* + \delta$. The social welfare is lower bounded by:

$$\begin{aligned} W_\mu^{\text{dec}} &\ge \lambda_C \int_{p^*-\epsilon+\delta}^1 \Phi_C\left(v, \tau_\mu(v), p_\mu, N_\mu\right) dv + \lambda_S \int_0^{p^*-\delta} (p_\mu(c) - c)dc \\ &\ge \lambda_C \int_{p^*-\epsilon}^1 \Phi_C\left(v, \tau_\mu(v), p_\mu, N_\mu\right) dv + \lambda_S \int_0^{p^*} (p^* - c)dc - (\lambda_C + \lambda_S)\delta(1+\epsilon). \end{aligned} \tag{16}$$

To justify the first inequality, observe that $(p^* - \epsilon + \delta, 1] \subseteq T_{B,\mu}$ by the entry conditions on the customer side since the lowest price is at most $p^* + \delta$. Customers of valuation v receive a net surplus of $\Phi_C(v, \tau_\mu(v), p_\mu, N_\mu)$ in expectation. Similarly, we have $[0, p^* - \delta) \subseteq T_{S,\mu}$ by the entry conditions on the supplier side as the highest price is at least $\overline{p}_\mu \ge p^* - \delta$. Suppliers of type c earn an expected profit of at most $p_\mu(c) - c$. The second inequality follows from $\Phi_C(v, \tau_\mu(v), p_\mu, N_\mu) \le (1+\epsilon)$ for all v as well as $p_\mu(c) \le p^* + \delta$ and $p^* \le (1+\epsilon)$. Below, we use the shorthand $\kappa = \min\left\{1, \frac{3\lambda_S \mu_C}{2\mu_S \lambda_C}\right\} \cdot \frac{1}{2\epsilon}$.

Claim 2. *For every* $v \in (p^* - \epsilon, 1]$ *and* $\mu \in (0, \kappa\delta^2)$*, we have* $\Phi_C(v, \tau_\mu(v), p_\mu, N_\mu) \geq v + \epsilon - p^* - 3(1+\epsilon)\delta$*.*

Plugging Claim 2 into inequality (16), for sufficiently small μ, we obtain

$$\begin{aligned} W_\mu^{\text{dec}} &\geq \lambda_C \int_{p^*-\epsilon}^{1} (v + \epsilon - p^* - 3(1+\epsilon)\delta) dv - \lambda_S \int_0^{p^*} (p^* - c) dc - (\lambda_C + \lambda_S)\delta(1+\epsilon) \\ &= \lambda_C \int_{p^*-\epsilon}^{1} (v+\epsilon) dv - \lambda_S \int_0^{p^*} c dc - 4(\lambda_C + \lambda_S)\delta(1+\epsilon) = U^* - 4(\lambda_C + \lambda_S)\delta(1+\epsilon). \end{aligned}$$

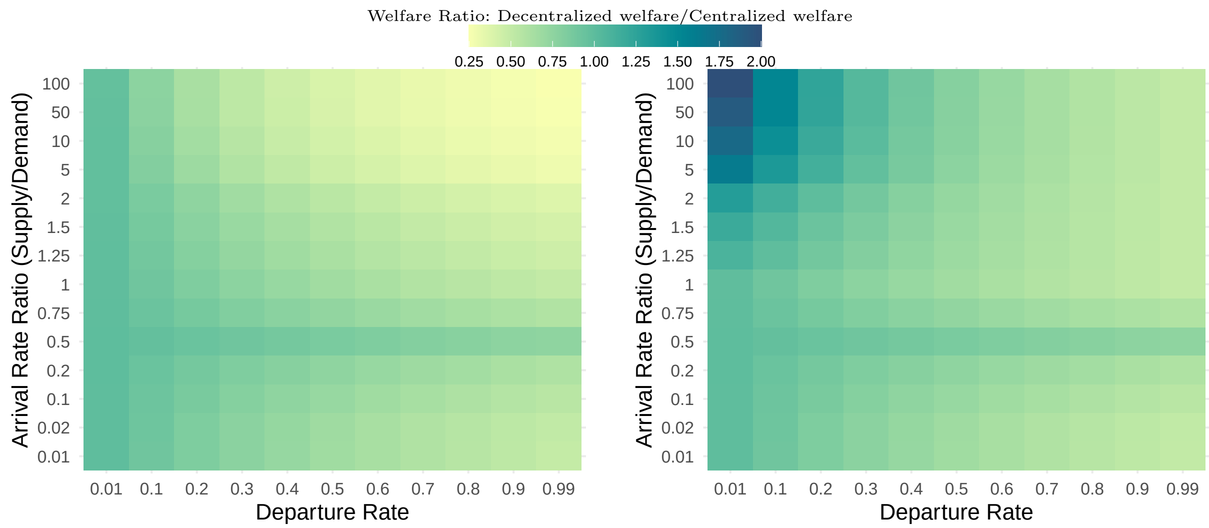
As δ can be taken arbitrarily small, this completes the proof of Theorem 2.

6. Numerical Study

Using the characterization of Theorem 1, we study the market outcomes of centralized and decentralized platform types numerically. We relax some of the assumptions required in our theoretical analyses (i.e., the limiting cases $\mu_C = \mu_S = 1$ and $\mu_C, \mu_S \to 0$), evaluate semi-centralized price menus, and test the robustness of our findings to other distributional assumptions.

Welfare comparison for general μ_S, μ_C*.* First, we compute the welfare ratios $W^{\text{dec}}/W^{\text{cen}}(p)$ for the welfare-maximal price $p = p^{*W}$ and revenue-maximal price $p = p^{*R}$. (Here, we omit the subscripts indicating the dependence on instance parameters (I, ϵ).) We vary the supply/demand arrivals λ_S/λ_C and departure rates $\mu_S = \mu_C$ to capture different levels of supply availability and willingness-to-wait. Figure 3 shows that the welfare-maximal centralized platform achieves higher social welfare than the decentralized setting under all market characteristics, with larger gaps for imbalanced and impatient markets. Centralization can increase total welfare by up to a factor of 4 when demand is scarce and a factor of 2 when supply is scarce. Corroborating Theorem 2, the decentralized platform converges to the first-best outcome for small departure rates. Contrary to the insight of Proposition 2 for the special case $\mu_C = \mu_S = 1$, the comparison between centralization and decentralization should generally be moderated by the platform's objective. Figure 3 shows that when the platform maximizes revenue, decentralization leads to superior welfare (with a welfare ratio of 2) in demand-constrained, patient markets.

Operational controls via semi-centralization. Additionally, we consider simple semi-centralized price menus of the form $[0, p_{\max}]$: the platform imposes a maximum allowed price. More specifically, we choose $p_{\max} = p^{\text{bal}}$ to be the price point at which the stationary



(a) Decentralized equilibrium vs. welfare-maximal pricing

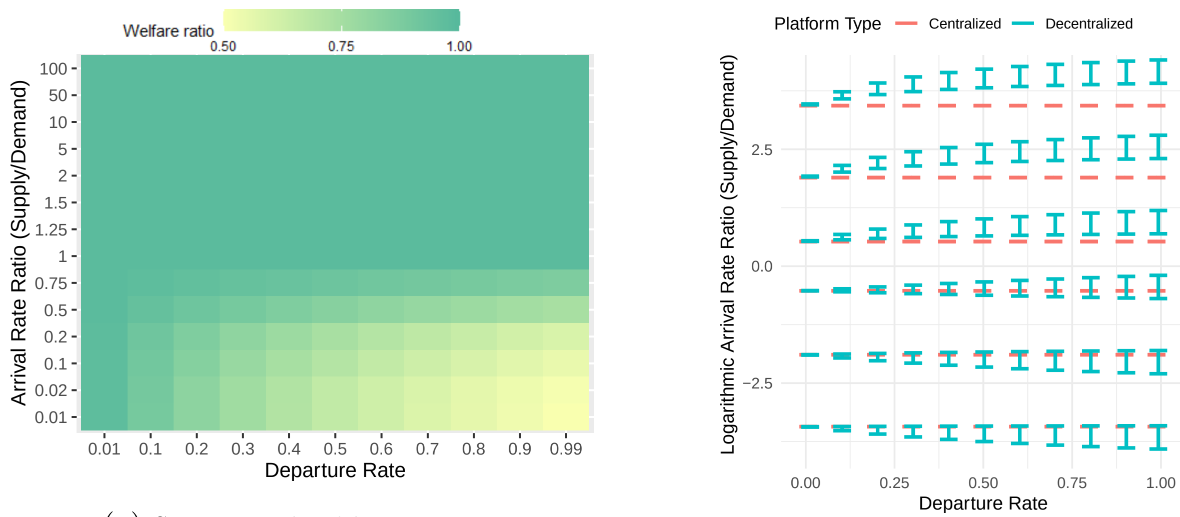
(b) Decentralized equilibrium vs. revenue-maximal pricing

Figure 3 **Welfare ratio between the decentralized and centralized settings; on the left, $W^{\text{dec}}/W^{\text{cen}}(p^{*W})$, and on the right, $W^{\text{dec}}/W^{\text{cen}}(p^{*R})$. Instances are generated by varying λ_S/λ_C and $\mu_C = \mu_S$ in the limit $\epsilon \to 0$.**

equilibrium balances demand and supply, i.e., $\|\bar{N}_S\| = \|\bar{N}_C\|$, if there exists such a price, otherwise $p^{\text{bal}} = 1 + \epsilon$ takes the largest possible value for small λ_S values. This choice of $p_{\max}$ is motivated by simplicity (i.e., the platform does not need to estimate the market parameters, but only react to $\|\bar{N}_S\|, \|\bar{N}_C\|$ in equilibrium).

Comparing Figure 4a to Figure 3, we observe that semi-centralized pricing increases welfare compared to the decentralized pricing (up to a 4-factor), and recovers the first-best for demand-constrained, patient markets (here, we denote by W^{semi} the social welfare from $[0, p^{\text{bal}}]$). This setting can be viewed as a compromise between price flexibility and platform control. A natural follow-up question is whether a carefully chosen price menu $[p_{\min}, p_{\max}]$ may outperform the optimal centralized price p^{*W}. Remarkably, this is not the case across the instances we tested; see Figure 6, Appendix A. Moreover, we numerically see that $p_{\max} = p^{\text{bal}}$ maximizes welfare out of the price menus of the form $[0, p_{\max}]$; see Figure 7, Appendix A.

Market price structure. Next, we examine the pricing strategies within each platform type and propose mechanisms to explain the observed welfare gaps. As is evident in Figure 4b, decentralized platforms exhibit ranges of prices in equilibrium that are adapted to suppliers' heterogeneous costs. In line with Theorem 2, this price spread vanishes as $\mu_C = \mu_S \to 0$, with convergence to the welfare-maximal centralized pricing. Comparing the top



(a) Semi-centralized heuristic vs. welfare-maximal pricing

(b) Decentralized vs. welfare-maximal

Figure 4 On the left, we plot the welfare ratio $W^{\text{semi}}(p^{\text{bal}})/W^{\text{cen}}(p^{*W})$ between the semi-centralized and centralized settings. On the right, the range of prices in decentralized settings and welfare-maximal centralized price p^{*W}. Instances are generated similarly to Figure 3.

and bottom rows in Figure 4b, it can be observed that the decentralized prices are not as effective as p^{*W} in balancing demand and supply levels: in demand-constrained markets, prices are too high relative to p^{*W}, whereas in supply-constrained markets, prices are too low relative to p^{*W}. This observation is consistent with Proposition 1, showing that the equilibrium prices do not depend on λ_C/λ_S. However, a benefit of price decentralization is higher participation from the supply side (as high-cost suppliers find it worthwhile to enter the market). Hence, one may wonder if decentralization benefits the supplier side of the market. In Figures 8-9, Appendix A, we report a finer analysis of decentralized equilibria in terms of customer surplus and supplier profit relative to the Pareto-efficiency frontier attained by centralized prices. Decentralized market outcomes are generally not Pareto-efficient, except in patient markets, meaning that suppliers' profits are not systematically improved.

Robustness tests. We conduct several batches of experiments in which we relax our model assumptions; the plots are presented in Appendix A. We generate instances where customers' arrival valuations and suppliers' arrival costs are Beta-distributed, including left-skewed, right-skewed, U-shaped, and unimodal distributions. In this setting, it is unknown whether equilibria exist and how to compute them; we implement a simulation-based heuristic, which is described in Appendix H. Consistently with our previous results, Figures 10 and 11 show that the welfare-maximizing platform outperforms the decentralized platform in general, while the revenue-maximizing platform performs worse than demand-constrained, patient markets. Further, we observe in Figure 12 that our results are mostly unaffected by the horizontal differentiation parameter ϵ.

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Appendix A: Supplementary Tables and Figures

Table 2 **Market outcomes for different platform types with $\mu_C = \mu_S = 1$ and $\epsilon \to 0$.**

	Suppliers' Profit	Customers' Surplus
Welfare-Maximizing Platform	$\frac{\lambda_S \lambda_C^2}{2(\lambda_S+\lambda_C)^2}$	$\frac{\lambda_S^2 \lambda_C}{2(\lambda_S+\lambda_C)^2}$
Revenue-Maximizing Platform	$\min\left\{\frac{\lambda_C}{8}, \frac{\lambda_S \lambda_C^2}{2(\lambda_S+\lambda_C)^2}\right\}$	$\min\left\{\frac{\lambda_C}{8}, \frac{\lambda_S^2 \lambda_C}{2(\lambda_S+\lambda_C)^2}\right\}$
Decentralized Platform	$\min\left\{\frac{\lambda_C}{12}, \frac{\lambda_S}{6}\right\}$	$\min\left\{\frac{\lambda_C}{24}, \frac{\lambda_S}{12}\right\}$

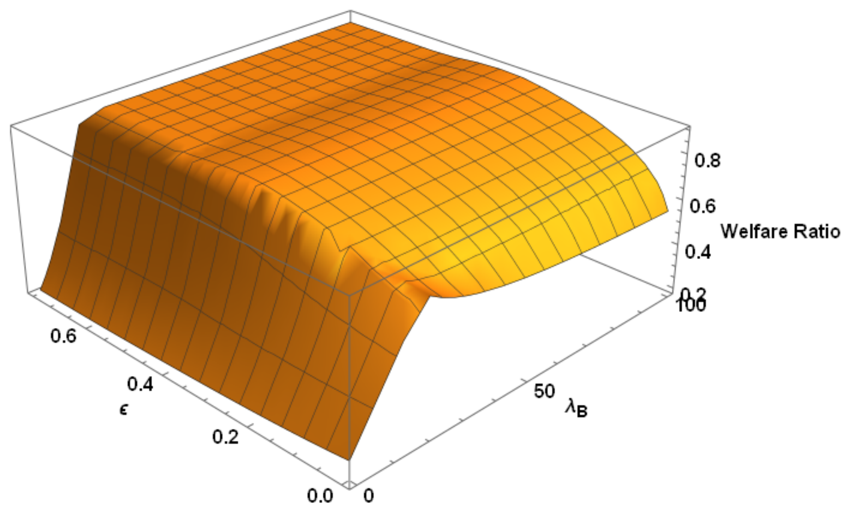


Figure 5 **Welfare ratio, $W^{\text{dec}}_{I,\epsilon}/W^{\text{cen}}_{I,\epsilon}(p^{*W}_{I,\epsilon})$, as a function of ϵ and λ_C, for $\lambda_S = 10$.**

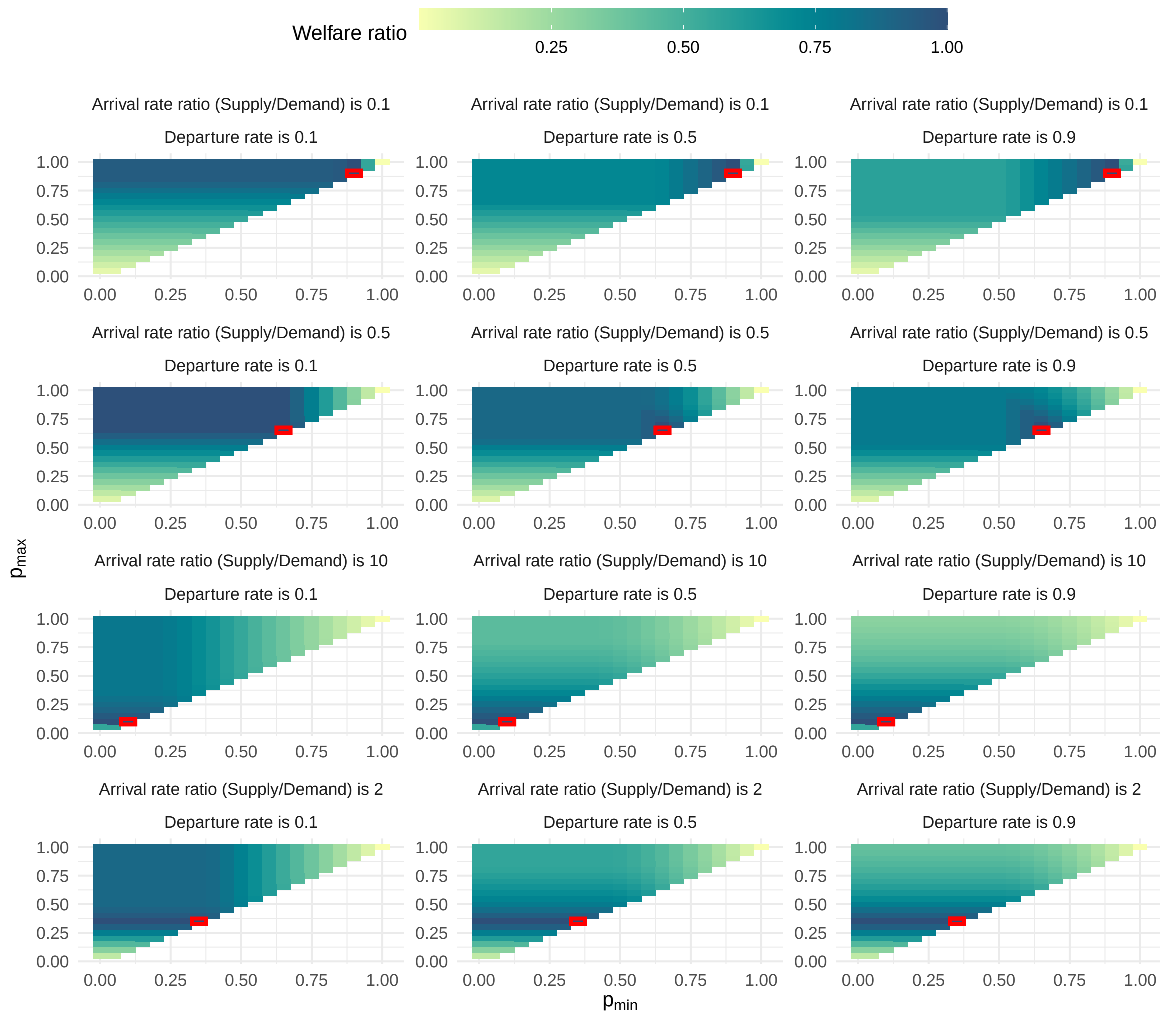


Figure 6 **Welfare ratio of semi-centralized platforms with various price menus $[p_{\min}, p_{\max}]$ to the welfare achieved by the best semi-centralized platform, where $p_{\min}$ and $p_{\max}$ are varied at 0.05 increments. Instances are generated similarly to Figure 3.**

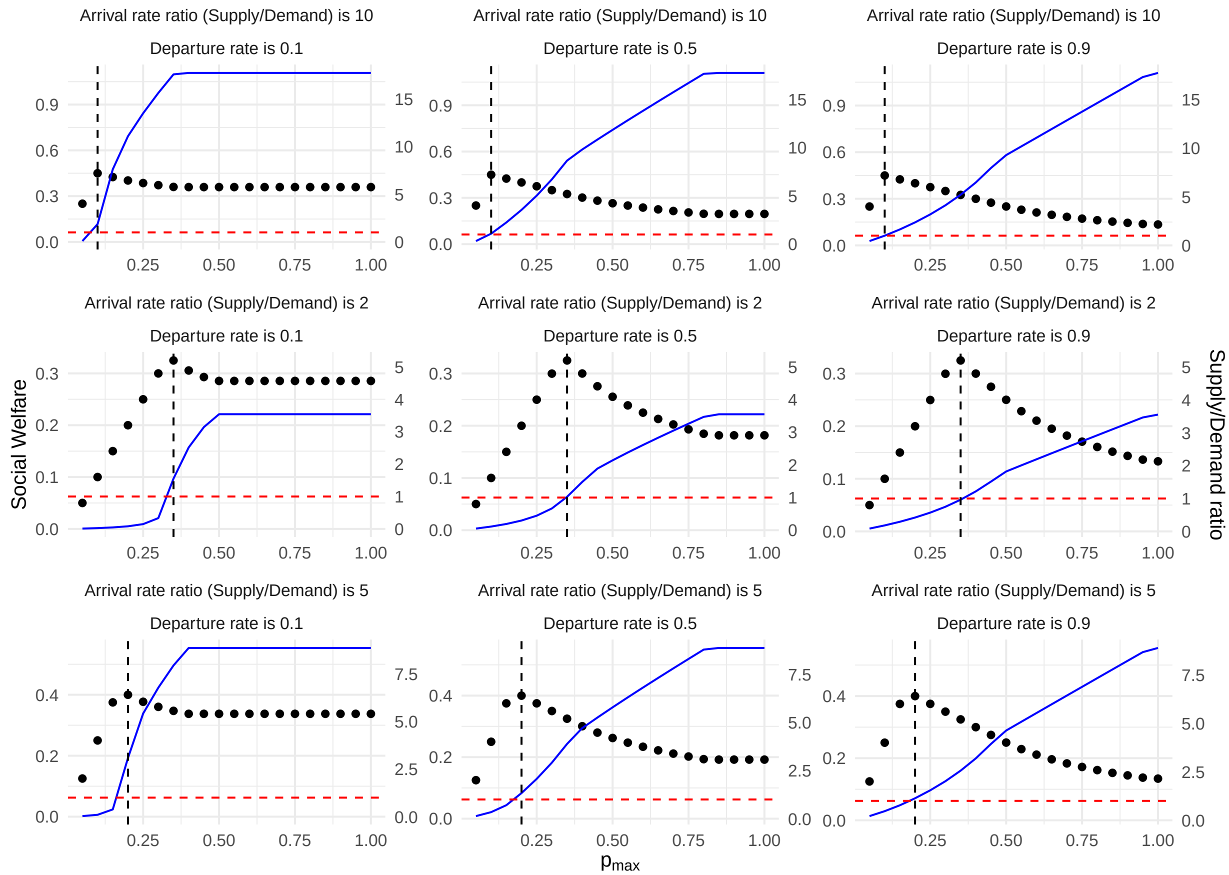
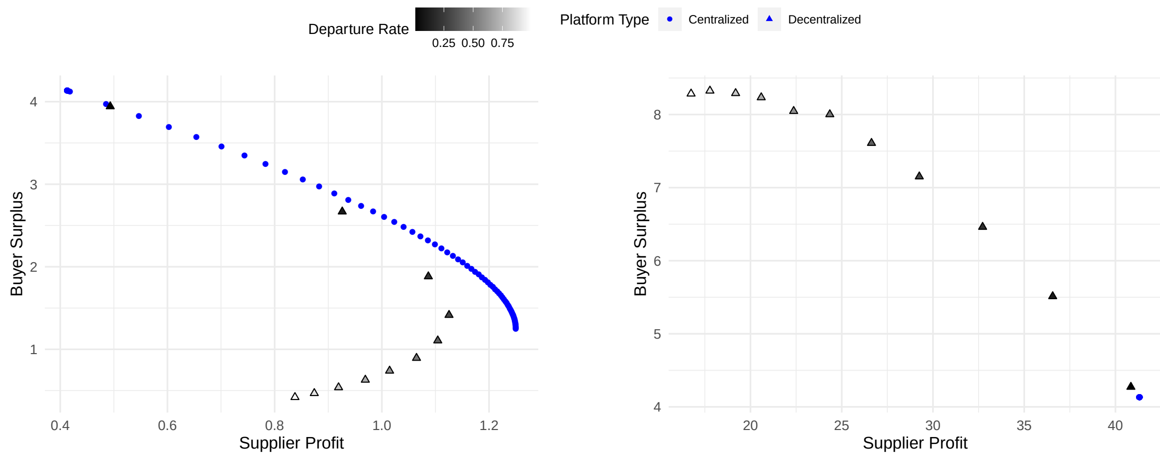


Figure 7 **Effect of $p_{\max}$ values on social welfare and the corresponding supply-balance ratio values, keeping $p_{\min} = 0$. The black dots correspond to welfare values with the corresponding axis on the left. The blue curves correspond to supply/demand ratio $\|\bar{N}_S\|/\|\bar{N}_C\|$ with the corresponding axis on the right. The vertical black dashed lines indicate the welfare-maximal $p_{\max}$ value, and the horizontal red dashed lines indicate where $\|\bar{N}_S\|/\|\bar{N}_C\| = 1$. Instances are generated similarly to Figure 3.**



(a) Demand-constrained setting (b) Supply-constrained setting

Figure 8 Customer surplus and supplier profit in decentralized equilibria and Pareto-efficiency frontier of centralized prices. On the left, we consider a demand-constrained setting ($\lambda_C = 10$ and $\lambda_S = 100$). On the right, we consider a supply-constrained setting ($\lambda_C = 1000$ and $\lambda_S = 100$). The departure rates used are $\mu_C = \mu_S \in \{0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$.

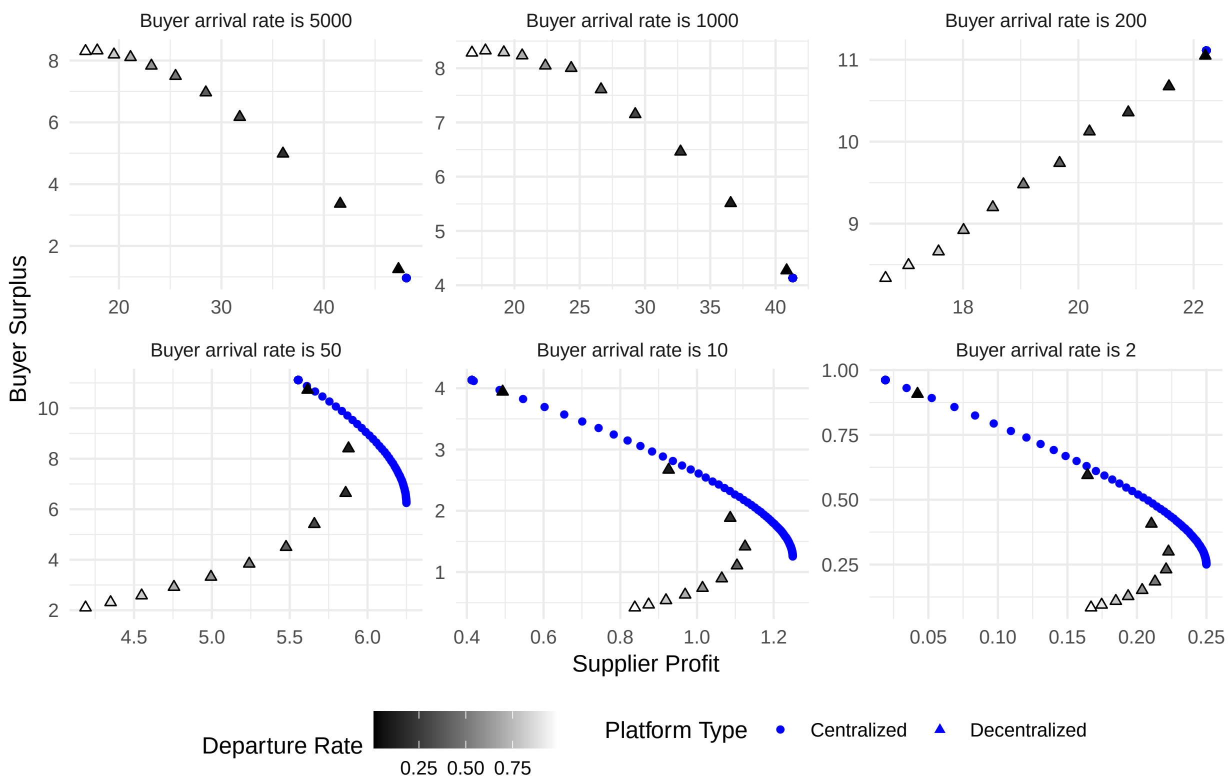


Figure 9 Customer surplus and supplier profit in decentralized equilibria and Pareto-efficiency frontier of centralized prices. Instances are generated by varying λ_S. We fix $\lambda_C = 100$.

Figure 10 Welfare ratio, $W^{\text{dec}}/W^{\text{cen}}(p^{*R})$, between the decentralized and centralized revenue-maximizing platform settings for Beta-distributed valuations and costs. We vary $\mu_S = \mu_C$ and λ_S/λ_C.

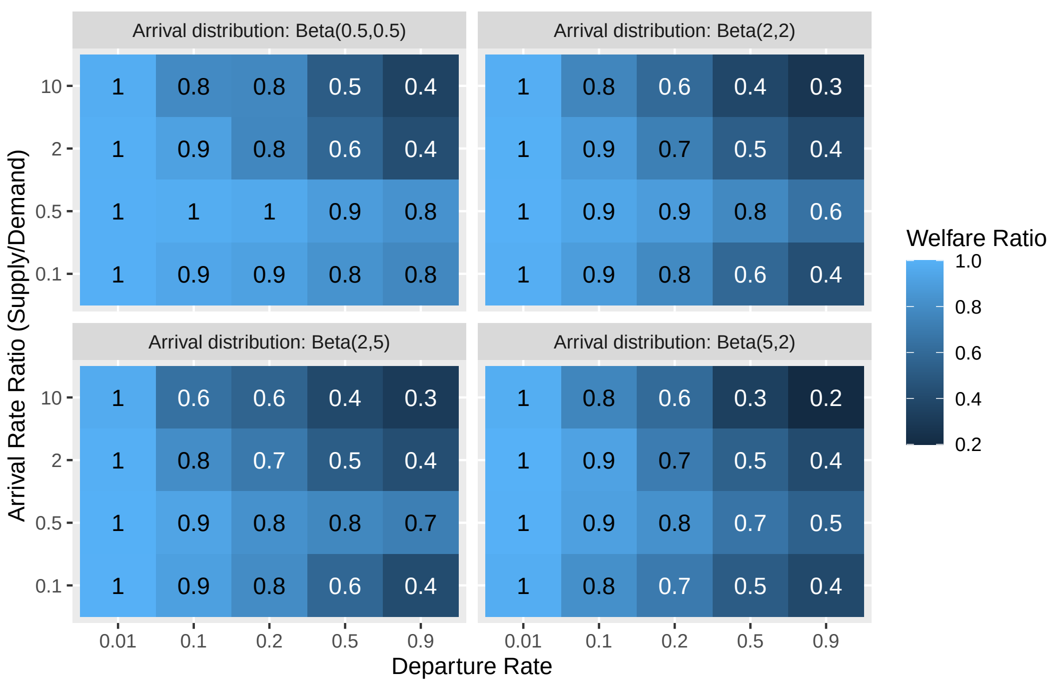


Figure 11 Welfare ratio, $W^{\text{dec}}/W^{\text{cen}}(p^{*W})$, between the decentralized and the centralized welfare-maximizing platform settings for Beta-distributed valuations and costs. We vary $\mu_S = \mu_C$ and λ_S/λ_C.

Figure 12 **Welfare ratio, $W^{\text{dec}}/W^{\text{cen}}(p^{*W})$, between the decentralized and centralized welfare-maximizing platform settings. Instances are generated by varying λ_C, $\mu_C = \mu_S$, and ϵ, with $\lambda_S = 100$.**

Appendix B: Discussion of model assumptions

Our model is stylized vis-a-vis real-world applications in two important ways. Matching markets often have a complex network structure, describing the matching compatibilities and preferences on both sides of the market. By contrast, our model only features horizontal and vertical differentiation. Since strategic behaviors in dynamic matching settings are difficult to analyze, our simplified network structure enables us to develop theoretical insights. Moreover, in spatial service markets such as ride-hailing, our model can represent a local approximation of the market in a small region (e.g., an airport). Although we restrict attention to uniform distributions for suppliers' costs and customers' valuations, we consider other distributional assumptions in the simulation-based study. As shown in Section 6, our findings extend beyond the restricted theoretical setup.

The other key assumption is to represent the dynamic matching market as an open-loop system. Similarly to related literature on dynamic matching, we do not explicitly track suppliers' long-term expected earnings and utilization levels. Instead, we represent an opportunity cost of waiting and an outside option through the suppliers' departures. This design choice is motivated by the objective of analyzing search frictions in the matching process through the acceptance/rejection of potential match offers. However, the analysis of closed-loop models is a relevant direction for future research. Should suppliers' decisions be explicitly guided by long-term expected earnings, our open-loop model would still be valid under some assumptions. Indeed, we view the price menu in our model as expressed in "normalized unit". In the examples of Section 1, decentralized pricing is implemented by exposing a fare multiplier or a variable cost parameter for calculating fares. However, the platform still determines the spatiotemporal variation of prices. In this context, carefully designed platform pricing incentives can make drivers indifferent across trip types (Garg and Nazerzadeh 2022, Ma et al. 2022).

Appendix C: Platform's First-best Optimization Problem

In this section, we study the platform's first-best problem. Namely, we allow the platform to dictate agents' strategies and utilize any matching strategy after observing suppliers' costs and customers' valuations. The following Continuous Linear Program (CLP) picks a matching strategy and implicitly picks the suppliers' prices and customers' thresholds such that the total social welfare is maximized.

$$
\begin{aligned}
&\sup_{x,M,N_S^{e,\infty},N_C^{e,\infty}} \int_0^{1+\epsilon}\int_{-\epsilon}^{1}\int_{-\epsilon}^{\epsilon} \underbrace{x(v,c,\eta)}_{\text{density of matches}} \cdot \underbrace{(v-c+\eta)}_{\text{match utility}} \cdot d\eta dv dc \\
&\text{subject to} \\
&x(v,c,\eta) \le M(v,c)\cdot f(\eta), && \forall c\in[0,1+\epsilon], \forall v\in[-\epsilon,1], \forall \eta\in[-\epsilon,\epsilon], \\
&\bar{N}_C(v) = \lambda_C + \left(\bar{N}_C(v) - \int_0^1\int_{-\epsilon}^{\epsilon} x(v,c,\eta)d\eta dc\right)\cdot(1-\mu_C), && \forall v\in[-\epsilon,1], \\
&\bar{N}_S(c) = \lambda_S + \left(\bar{N}_S(c) - \int_0^1\int_{-\epsilon}^{\epsilon} x(v,c,\eta)d\eta dv\right)\cdot(1-\mu_S), && \forall c\in[0,1+\epsilon], \\
&\int_0^{1+\epsilon} M(v,c)dc \le \bar{N}_C(v), && \forall v\in[-\epsilon,1], \\
&\int_{-\epsilon}^{1} M(v,c)dv \le \bar{N}_S(c), && \forall c\in[0,1+\epsilon], \\
&x(v,c,\eta)\ge 0, && \forall c\in[0,1+\epsilon], \forall v\in[-\epsilon,1], \forall \eta\in[-\epsilon,\epsilon], \\
&M(v,c)\ge 0, && \forall c\in[0,1+\epsilon], \forall v\in[-\epsilon,1].
\end{aligned}
\tag{CLP}
$$

In (CLP), $M(v,c)$ represents the density of customers with valuations v that the platform matches with suppliers of cost c. $x(v,c,\eta)$ represents the density of matches realized between customers of valuations v and suppliers of costs c where the randomness in the total match utility is realized at the level η. $f(\eta)$ represents the density corresponding to the random variable representing the randomness in the total match utility at the point η. Hence, $f(\eta) = 1/2\epsilon$ for any $\eta \in [-\epsilon,\epsilon]$ since $\boldsymbol{\eta}$ is uniformly distributed between $-\epsilon$ and ϵ. Nonetheless, we keep the general notation $f(\eta)$ to demonstrate that the first-best formulation holds irrespective of the distribution of the random variable $\boldsymbol{\eta}$. The first constraint makes sure that the density of successful matches realized at the η-level is at most as much as the density of matches allowed by the platform's matching strategy. The second and third constraints are flow balance constraints that make sure that the market is in a steady state. The fourth and fifth constraints ensure that the total density of matches for each time period and each type respect the flow balance constraints. The remaining constraints ensure that the mass of agents and matches is non-negative.

Deterministic matching utility. Next, we investigate the case where the utility from successfully matching a customer of type v and a supplier of type c is deterministic and is equal to $v-c$. In this case, we can represent the platform's first-best optimization problem as the following continuous linear program:

Proposition 5 *The platform's first-best optimization problem is equivalent to the following continuous linear program when $\epsilon = 0$:*

$$\begin{aligned} & sup_{\boldsymbol{M}} \quad \int_0^1 \int_0^1 M(v,c)\cdot(v-c)dvdc \\ & subject\ to \\ & \lambda_S \geq \int_0^1 M(v,c)dv, && \forall c \in [0, 1+\epsilon], \\ & \lambda_C \geq \int_0^1 M(v,c)dc, && \forall v \in [-\epsilon, 1], \\ & M(v,c) \geq 0, && \forall c \in [0,1+\epsilon], \forall v \in [-\epsilon, 1]. \end{aligned} \tag{CLP-D}$$

Proof. Due to the fact that all matches picked by the platform will be successful, in every time period, there do not remain any agents who can be matched in subsequent time periods. Therefore, in every time period, among the incoming λ_C density of customers of each type $v \in [-\epsilon, 1]$ and the incoming λ_S density of suppliers of each type $c \in [0, 1+\epsilon]$, the platform decides on the density of matches $M(v,c)$ for all v, c. Out of matching customers of type v with suppliers of type c, the social welfare of $(v-c)\cdot M(v,c)$ is gained. The platform picks the density of matches between any customer and supplier types v and c in order to maximize the total social welfare $\int_0^1 \int_0^1 (v-c)\cdot M(v,c)dvdc$. ∎

Proposition 5 immediately implies that assortative matching, where the platform matches customers of the highest valuations with suppliers of the lowest costs is first-best. Hence, Proposition 5 immediately implies that the centralized platform that imposes a single price achieves the first best.

Corollary 2. *The centralized platform achieves first-best if $\epsilon = 0$.*

Appendix D: Proofs from Section 4

Throughout the proofs in this section, we let the best response prices $p^*(c) = p^*(c, \tau, p, N)$ for the sake of simplicity.

D.1. Proof of Proposition 1

Under the decentralized platform when $\mu_C = \mu_S = 1$, by (6), suppliers with cost c pick price y to maximize their expected profits

$$\Phi_S(c, y, \tau, p, N) = (y-c)\cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - y + \boldsymbol{\eta} \geq \tau(\boldsymbol{v})] \cdot \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}. \tag{17}$$

Next, we express $\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - y + \boldsymbol{\eta} \geq \tau(\boldsymbol{v})]$ as a function of y. One can show that $p^*(c)$ is increasing in c (this is shown to be true for the general case $\mu_C, \mu_S \in (0,1]$ in Proposition 4). Therefore, since $\tau(v) = 0$ in the impatient case where the reward-to-go is 0, $\boldsymbol{v} - \tau(\boldsymbol{v}) = \boldsymbol{v}$ is uniformly distributed between $p^*(0) - \epsilon$ and 1 while $\boldsymbol{\eta}$ is uniformly distributed between $-\epsilon$ and ϵ. Importantly, since the mass of suppliers with any cost $c \in [0, 1+\epsilon]$ is 0, no infinitesimal agent's strategy affects other agents' strategies. Hence, without loss of generality, we can take $p^*(0)$ as constant. Therefore, one can show for $p^*(0) \leq y \leq 1+\epsilon$ that

$$\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - y + \boldsymbol{\eta} \geq \tau(\boldsymbol{v})] = \begin{cases} \frac{1-y}{1-p^*(0)+\epsilon}, & \text{if } \min\{p^*(0), 1-\epsilon\} \leq y < 1-\epsilon, \\ \frac{(1+\epsilon-y)^2}{4\epsilon(1-p^*(0)+\epsilon)}, & \text{if } \max\{p^*(0), 1-\epsilon\} \leq y \leq 1+\epsilon, \\ 0, & \text{if } 1+\epsilon \leq y. \end{cases}$$

Subsequently, we find y that maximizes equation (17) by considering two cases separately. For $y \leq 1-\epsilon$, by simple algebra, we can show that equation (17) is maximized at $p^*(c) = (1+c)/2$ since such $p^*(c)$ is the global maximizer of $(p^*(c)-c)(1-p^*(c))$. For the other case where $\max\{p^*(0), 1-\epsilon\} \leq y \leq 1+\epsilon$, we need to maximize the slightly more complicated polynomial $(y-c)(1+\epsilon-y)^2$. Taking the derivative of this expression with respect to y yields the following polynomial after rearranging the terms

$$\frac{\partial \Phi_S}{\partial y} = \frac{\min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}}{1-p^*(0)+\epsilon}\left[(1+\epsilon-y)^2+(y-c)\cdot(2y-2\epsilon-2)\right].$$

Therefore, the first-order optimality condition becomes

$$\begin{aligned}
&(1+\epsilon-y)^2+(y-c)\cdot(2y-2\epsilon-2) \\
=&(1+\epsilon-y)\cdot(1+\epsilon-y-2(y-c)) \\
=&(1+\epsilon-y)\cdot(1+\epsilon-3y+2c)=0.
\end{aligned}$$

We have two potential solutions in this case: one is where $1+\epsilon-y=0$, and the other is where $1+\epsilon-3y+2c=0$. We know that $y=1+\epsilon$ makes the profit 0. One can also check that $\partial\Phi_S/\partial y>0$ for $\max\{p^*(0), 1-\epsilon\} \leq y < (1+2c+\epsilon)/3$ and $\partial\Phi_S/\partial y<0$ for $(1+2c+\epsilon)/3 \leq y < 1+\epsilon$. Therefore, the suppliers pick the other solution, where $y=(1+2c+\epsilon)/3$. Hence, $p^*(c)=(1+2c+\epsilon)/3$ for $\max\{p^*(0), 1-\epsilon\} \leq p^*(c) \leq 1+\epsilon$. Since $p^*(c)=1-\epsilon$ is attained when $c=1-2\epsilon$, we have the following characterization of optimal prices for suppliers:

$$p^*(c)=\begin{cases} \frac{1+c}{2}, & 0 \leq c < \max\{0, 1-2\epsilon\}, \\ \frac{1+2c+\epsilon}{3}, & \max\{0, 1-2\epsilon\} \leq c \leq 1+\epsilon. \end{cases}$$

D.2. Proof of Proposition 2

First, we look at the market outcomes under the altruistic platform.

Welfare-maximizing platform. We calculate the average match utility and the number of total matches to find the total welfare of the system. Suppose that the welfare-maximizing platform picks the price y. Then, customers with valuations $v>y$, and suppliers of type $c<y$ enter the market. Hence, a uniformly randomly chosen match has the expected utility $U=(1+y)/2-y/2=1/2$. This is due to the fact that all matches will be successful since $v>c$ for any customers and suppliers that enter the market and that on average, a supplier with cost $c=y/2$ and a customer with $v=(1+y)/2$ is picked by the platform. On the other hand, the mass of matches is $N(y)=\min\{\lambda_S\cdot y, \lambda_C\cdot(1-y)\}$. Therefore, in order to maximize the total welfare $N(y)/2$, the welfare-maximizing platform chooses the price $y=y^*$ that maximizes the mass of matches $N(y)$. Note that $N(y)=\min\{\lambda_S\cdot y, \lambda_C\cdot(1-y)\}$ is maximized at the point where $\lambda_S\cdot y^*=\lambda_C\cdot(1-y^*)$, which implies that $y^*=\lambda_C/(\lambda_S+\lambda_C)$. Hence, the total welfare under the welfare-maximizing platform is

$$\frac{N(y^*)}{2}=\frac{1}{2}\cdot\frac{\lambda_S\lambda_C}{\lambda_S+\lambda_C}.$$

Next, we calculate how welfare is distributed among suppliers and customers. The customers' total net surplus is calculated as

$$\frac{1-y^*}{2}\cdot N(y^*)=\frac{\lambda_S\lambda_C}{\lambda_S+\lambda_C}\cdot\frac{\lambda_S}{\lambda_S+\lambda_C}\cdot\frac{1}{2}=\lambda_S\cdot\frac{\lambda_S\lambda_C}{2(\lambda_S+\lambda_C)^2}$$

and the suppliers' total profit is calculated as

$$\frac{y^*}{2}\cdot N(y^*)=\frac{\lambda_S\lambda_C}{\lambda_S+\lambda_C}\cdot\frac{\lambda_C}{\lambda_S+\lambda_C}\cdot\frac{1}{2}=\lambda_C\cdot\frac{\lambda_S\lambda_C}{2(\lambda_S+\lambda_C)^2}.$$

Revenue-maximizing platform. In this case, the platform maximizes the quantity $N(y)\cdot y=\min\{\lambda_S\cdot y^2,\lambda_C\cdot(1-y)y\}$. First, we observe that $\lambda_S\cdot y^2$ and $\lambda_C\cdot(1-y)y$ meet at $y=\lambda_C/(\lambda_S+\lambda_C)$. Hence, for $y<\lambda_C/(\lambda_S+\lambda_C)$, we have that $\min\{\lambda_S\cdot y^2,\lambda_C\cdot(1-y)y\}=\lambda_S\cdot y^2$ and for $y\geq\lambda_C/(\lambda_S+\lambda_C)$, we have that $\min\{\lambda_S\cdot y^2,\lambda_C\cdot(1-y)y\}=\lambda_C\cdot(1-y)y$. This implies that since $\lambda_C\cdot(1-y)y$ takes its maximum value at $y=1/2$, the optimal price is $y^*=1/2$ when $\lambda_S>\lambda_C$ and the optimal price is $y^*=\lambda_C/(\lambda_S+\lambda_C)$ as in the case of welfare-maximizing platform when $\lambda_S<\lambda_C$. Consequently, the total welfare under revenue-maximizing platform is

$$\frac{1}{2}\cdot N(y^*)=\begin{cases}\frac{\lambda_C}{4} & \text{if } \lambda_S>\lambda_C\\ \frac{1}{2}\cdot\frac{\lambda_S\lambda_C}{\lambda_S+\lambda_C} & \text{otherwise}\end{cases}=\min\left\{\frac{\lambda_C}{4},\frac{\lambda_C\lambda_S}{2(\lambda_S+\lambda_C)}\right\}.$$

Next, we calculate how welfare is distributed among suppliers and customers. Note that for the case where $\lambda_S<\lambda_C$, the total welfare, as well as the distribution of the total welfare among the suppliers and the customers, are the same as in the welfare-maximizing platform case. On the other hand, when $\lambda_S>\lambda_C$, since the optimal price is $y^*=1/2$, the total welfare is distributed equally between suppliers and customers. Hence, the suppliers' total profit and the customers' total net surplus is $\lambda_C/8$.

D.3. Derivation of the closed form expressions for $\epsilon>0$ under impatient market

Decentralized platform. By Proposition 1, we know that in impatient markets ($\mu_S=\mu_C=1$), suppliers with cost $c\in[0,\max\{0,1-2\epsilon\})$ choose prices $p^*(c)=(1+c)/2$ while suppliers with cost $c\in[\max\{0,1-2\epsilon\},1+\epsilon]$ pick prices $p^*(c)=(1+2c+\epsilon)/3$. The customers pick the threshold $\tau(v)=0$ for all $v\in[-\epsilon,1]$. Via this information, we will calculate the average welfare from a random match in the market:

$$\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta}\geq 0)]. \tag{18}$$

Note that when $\epsilon\leq 1/2$, $\boldsymbol{v}$ is uniformly distributed between $1/2-\epsilon$ and 1, and when $\epsilon>1/2$, $\boldsymbol{v}$ is uniformly distributed between $(1+\epsilon)/3-\epsilon$ and 1. This is due to (i) the entry conditions that require that the minimum valuation in the market is $p^*(0)-\epsilon$ and (ii) Proposition 1, which implies that $p^*(0)=1/2$ when $\epsilon\leq 1/2$, and $p^*(0)=(1+\epsilon)/3$ when $\epsilon>1/2$. With this in mind, we analyze the following two cases: $\epsilon\leq 1/2$ and $\epsilon>1/2$.

Case 1 ($\epsilon\leq 1/2$). We first find the probability density function associated with the random variable $\boldsymbol{v}+\boldsymbol{\eta}$:

$$f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)=\begin{cases}\frac{2x+4\epsilon-1}{2\epsilon(1+2\epsilon)}, & \text{if } 1/2-2\epsilon<x\leq 1/2,\\ \frac{2}{1+2\epsilon}, & \text{if } 1/2<x\leq 1-\epsilon,\\ \frac{1+\epsilon-x}{\epsilon(1+2\epsilon)}, & \text{if } 1-\epsilon<x\leq 1+\epsilon,\\ 0, & \text{otherwise.}\end{cases}$$

Next, we calculate the average welfare $\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta}\geq 0)]$:

$$\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta})]=\int_0^{1+\epsilon}\int_{\max\{p^*(c),1/2-2\epsilon\}}^{1+\epsilon}(x-y)f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)f_{\boldsymbol{c}}(y)dxdy,$$

where $f_{\boldsymbol{c}}(y)=1/(1+\epsilon)$ is the probability density function corresponding to the random variable $\boldsymbol{c}$. Since $p^*(c)>1/2-2\epsilon$ for all $c\in[0,1+\epsilon]$ by Proposition 1, we have that

$$\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta})]=$$

$$
\begin{aligned}
&\frac{1}{1+\epsilon}\left[\int_0^{1-2\epsilon}\int_{(1+y)/2}^{1+\epsilon}(x-y)f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)dxdy+\int_{1-2\epsilon}^{1+\epsilon}\int_{(1+2y+\epsilon)/3}^{1+\epsilon}(x-y)f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)dxdy\right]\\
&=\frac{1}{1+\epsilon}\left[\int_0^{1-2\epsilon}\frac{9+4\epsilon^2-18y+9y^2}{3(1+2\epsilon)}+\int_{1-2\epsilon}^{1+\epsilon}\frac{10(1+\epsilon-y)^3}{81\epsilon(1+2\epsilon)}\right]\\
&=\frac{3+4\epsilon^2-2\epsilon^3}{12(1+3\epsilon+2\epsilon^2)}.
\end{aligned}
$$

Case 2 ($\epsilon > 1/2$). In this case, the probability density function of the random variable $\boldsymbol{v}+\boldsymbol{\eta}$ is the following:

$$
f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)=\begin{cases}\frac{3(1-x+\epsilon)}{4\epsilon(1+\epsilon)}, & \text{if } (1+\epsilon)/3\le x<(1+\epsilon),\\ \frac{1}{2\epsilon}, & \text{if } \min\{1-\epsilon,1/3\}\le x<(1+\epsilon)/3,\\ \frac{-1+5\epsilon+3x}{4\epsilon(1+\epsilon)}, & \text{if } (1-5\epsilon)/3\le x<\min\{1-\epsilon,1/3\}.\end{cases}
$$

Next, we calculate the average welfare $\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta}\ge 0)]$:

$$
\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta}\ge 0)]=\int_0^{1+\epsilon}\int_{\max\{p^*(c),(1+\epsilon)/3-2\epsilon\}}^{1+\epsilon}(x-y)f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)f_{\boldsymbol{c}}(y)dxdy,
$$

where $f_{\boldsymbol{c}}(y)=1/(1+\epsilon)$ denotes the probability density function corresponding to the random variable $\boldsymbol{c}$. Since $p^*(c)>(1+\epsilon)/3-2\epsilon$ for all $c\in[0,1+\epsilon]$ by Proposition 1, we have that

$$
\begin{aligned}
\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}-p^*(\boldsymbol{c})+\boldsymbol{\eta}\ge 0)]&=\frac{1}{1+\epsilon}\left[\int_0^{1+\epsilon}\int_{(1+2y+\epsilon)/3}^{1+\epsilon}(x-y)f_{\boldsymbol{v}+\boldsymbol{\eta}}(x)dxdy\right]\\
&=\frac{1}{1+\epsilon}\left[\int_0^{1+\epsilon}\frac{5(1+\epsilon-y)^3}{54\epsilon(1+\epsilon)}\right]\\
&=\frac{5(1+\epsilon)^2}{216\epsilon}.
\end{aligned}
$$

Next, we calculate the total mass of match attempts. For $\epsilon<1/2$, since $p^*(0)=1/2$, customers having valuations between $1/2-\epsilon$ and 1 enter the market. This makes the total mass of customers that enter the market $\lambda_C\cdot(1/2+\epsilon)$. For $\epsilon\ge 1/2$, since $p^*(0)=(1+\epsilon)/3$, customers having valuations between $(1-2\epsilon)/3$ and 1 enter the market. This makes the total mass of customers that enter the market $2\lambda_C(1+\epsilon)/3$. On the other hand, all suppliers enter the market since $p^*(c)>c$ for all $c\in[0,1+\epsilon]$, which makes the total mass of suppliers that enter the market $(1+\epsilon)\lambda_S$. Hence, for $\epsilon\le 1/2$, the total mass of match attempts is

$$
\min\{(1+\epsilon)\lambda_S,(1/2+\epsilon)\lambda_C\},
$$

while for $\epsilon>1/2$, it is

$$
\min\{(1+\epsilon)\lambda_S,2\lambda_C(1+\epsilon)/3\}.
$$

Hence, the total welfare that the decentralized platform achieves is equal to the following

$$
\begin{cases}\frac{5(1+\epsilon)^2\min\{(1+\epsilon)\lambda_S,2\lambda_C(1+\epsilon)/3\}}{216\epsilon}, & \text{if } \epsilon\le 1/2,\\ \frac{(3+4\epsilon^2-2\epsilon^3)\min\{(1+\epsilon)\lambda_S,(1/2+\epsilon)\lambda_C\}}{12(2\epsilon^2+3\epsilon+1)}, & \text{if } \epsilon>1/2.\end{cases}
$$

Next, we derive the total welfare for the centralized platform.

Centralized platform. We let the single price picked by the centralized platform be y. First, given this price, we will calculate the social welfare with respect to y, after which we will optimize for y. For the market setting where agents are impatient ($\mu_C = \mu_S = 1$) and the single price picked by the centralized platform is y, we know that customers with valuations $v \in (y-\epsilon, 1]$, and suppliers with costs $c \in [0, y)$ enter the market. Hence, the random variable corresponding to valuations $\boldsymbol{v}$ is uniformly distributed in the interval $[y-\epsilon, 1]$, and the random variable $\boldsymbol{c}$ corresponds to costs that are uniformly distributed in the interval $[0, y]$. Hence, one can show that the probability of a match attempt being successful is

$$\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v}+\boldsymbol{\eta} \geq y] = \begin{cases} \frac{1-y+\epsilon}{4\epsilon}, & \text{if } \epsilon \geq 1-y, \\ \frac{1-y}{1+\epsilon-y}, & \text{if } \epsilon < 1-y. \end{cases}$$

We also have that given a match attempt is successful, the welfare from the match is

$$\mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta} \mid \boldsymbol{v}+\boldsymbol{\eta} \geq y] = \begin{cases} \frac{y+2\epsilon+2}{6}, & \text{if } \epsilon \geq 1-y, \\ \frac{3+\epsilon^2-3y}{6(1-y)}, & \text{if } \epsilon < 1-y. \end{cases}$$

Hence, the average welfare from a match attempt is

$$\begin{aligned} \mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[(\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta})\mathbb{I}(\boldsymbol{v}+\boldsymbol{\eta} \geq y)] &= \mathbb{E}_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v}-\boldsymbol{c}+\boldsymbol{\eta} \mid \boldsymbol{v}+\boldsymbol{\eta} \geq y]\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v}+\boldsymbol{\eta} \geq y] \\ &= \begin{cases} \frac{(1+\epsilon-y)(y+2\epsilon+2)}{24\epsilon}, & \text{if } \epsilon \geq 1-y, \\ \frac{3+\epsilon^2-3y}{6(1+\epsilon-y)}, & \text{if } \epsilon < 1-y. \end{cases} \end{aligned}$$

The total mass of matches is equal to the minimum of the total masses of customers and suppliers $\min\{\lambda_C(1-y+\epsilon), \lambda_S y\}$. Hence, the social welfare that the centralized platform with price y achieves is equal to

$$W^{\text{cen}}_{I,\epsilon}(y) = \begin{cases} \frac{(1+\epsilon-y)(y+2\epsilon+2)\min\{\lambda_C(1-y+\epsilon),\lambda_S y\}}{24\epsilon}, & \text{if } \epsilon \geq 1-y, \\ \frac{(3+\epsilon^2-3y)\min\{\lambda_C(1-y+\epsilon),\lambda_S y\}}{6(1+\epsilon-y)}, & \text{if } \epsilon < 1-y. \end{cases}$$

Next, we find the welfare-maximal price

$$p^* = \underset{y \in [0,1+\epsilon]}{\arg\max}\, W^{\text{cen}}_{I,\epsilon}(y).$$

First, we express $W^{\text{cen}}_{I,\epsilon}(y)$ in another form where $W^{\text{cen}}_{I,\epsilon}(y) = \min\{g_1(y), g_2(y)\}$. In order to achieve this, we let $g_1(y)$ equal to

$$g_1(y) = \begin{cases} \frac{\lambda_C(1+\epsilon-y)^2(y+2\epsilon+2)}{24\epsilon}, & \text{if } \epsilon \geq 1-y, \\ \frac{\lambda_C(3+\epsilon^2-3y)}{6}, & \text{if } \epsilon < 1-y. \end{cases} \tag{19}$$

and $g_2(y)$ equal to

$$g_2(y) = \begin{cases} \frac{\lambda_S y(1+\epsilon-y)(y+2\epsilon+2)}{24\epsilon}, & \text{if } \epsilon \geq 1-y, \\ \frac{\lambda_S y(3+\epsilon^2-3y)}{6(1+\epsilon-y)}, & \text{if } \epsilon < 1-y. \end{cases} \tag{20}$$

In the following lemmas, we show various properties of the functions of $g_1(y)$ and $g_2(y)$ that are going to be instrumental in our proof.

LEMMA 5. *We have the following properties about the function $g_1(\cdot)$:*

- *$g_1(y)$ is convex and monotonically decreasing in $y \in [0, 1+\epsilon]$.*
- *$g_1(0) > 0$.*
- *$g_1(1+\epsilon) = 0$.*

Proof. We first note that $g_1(0) > 0$ since

$$g_1(0) = \begin{cases} \frac{\lambda_C(1+\epsilon)^3}{12\epsilon}, & \text{if } \epsilon \geq 1, \\ \frac{\lambda_C(3+\epsilon^2)}{6}, & \text{if } \epsilon < 1. \end{cases}$$

Next, we also have that $g_1'(y) < 0$ for all $y \in [0, 1+\epsilon]$ since

$$g_1'(y) = \begin{cases} \frac{\lambda_C(y^2-(1+\epsilon)^2)}{8\epsilon}, & \text{if } \epsilon > 1-y, \\ -\lambda_C/2, & \text{if } \epsilon \leq 1-y. \end{cases}$$

Moreover, $g_1(y)$ is convex for $y \in [0, 1+\epsilon]$ since

$$g_1''(y) = \begin{cases} \frac{\lambda_C y}{4\epsilon}, & \text{if } \epsilon > 1-y, \\ 0, & \text{if } \epsilon < 1-y. \end{cases}$$

is non-negative and $g_1'(y)$ is non-decreasing in the neighborhood of $1-\epsilon$. Finally, we note that $g_1(1+\epsilon) = 0$ since the term $(1+\epsilon-y)^2$ of (19) in the numerator is 0 for $y = 1+\epsilon$. ∎

LEMMA 6. *We have the following properties about the function $g_2(\cdot)$:*

- *$g_2(y)$ is strictly concave in $y \in [0, 1+\epsilon]$.*
- *$g_2(0) = 0$.*
- *$g_2(1+\epsilon) = 0$.*

Proof. First, we note that $g_2(0) = 0$ trivially by definition. Next, we show that $g_2(y)$ is concave for all $y \in [0, 1+\epsilon]$:

$$g_2''(y) = \begin{cases} \frac{-\lambda_S(1+\epsilon+3y)}{12\epsilon}, & \text{if } \epsilon > 1-y, \\ \frac{\lambda_S(-3+\epsilon)\epsilon(1+\epsilon)}{3(1+\epsilon-y)^3}, & \text{if } \epsilon < 1-y. \end{cases}$$

By this expression, it is clear that $g_2''(y) < 0$ for the case where $\epsilon > 1-y$ since the numerator is negative and the denominator is positive by non-negativity of y, ϵ, and λ_S. We can also readily show that $g_2(y)$ is concave when $\epsilon < 1-y$ due to y and ϵ being non-negative. Since $g_2'(y)$ is decreasing in the neighborhood of $1-\epsilon$, we conclude $g_2(y)$ is strictly convex in the domain $[0, 1+\epsilon]$. Finally, we have that $g_2(1+\epsilon) = 0$ since the term $(1+\epsilon-y)$ of (20) in the numerator is zero for $y = 1+\epsilon$. ∎

Building on the previous two lemmas, Lemmas 5 and 6, we show in the following Lemma that $g_1(y)$ and $g_2(y)$ has exactly one meeting point in $[0, 1+\epsilon]$. Moreover, this result leads to an important property of the maximizer of $W_{I,\epsilon}^{\text{cen}}(y) = \min\{g_1(y), g_2(y)\}$, which will be key in allowing us to express the welfare-maximal price in closed form.

LEMMA 7. *The set $\{y \in (0, 1+\epsilon) : g_1(y) = g_2(y)\}$ is a singleton that only includes the point $y_0 = \frac{\lambda_C(1+\epsilon)}{\lambda_C+\lambda_S}$. Then, we have the following equality that describes social welfare under the centralized platform:*

$$W_{I,\epsilon}^{cen}(y) = \min\{g_1(y), g_2(y)\} = \begin{cases} g_2(y), & \text{if } y < y_0, \\ g_1(y), & \text{if } y \geq y_0. \end{cases}$$

Proof. First, we note that one can easily verify that for $y = \frac{\lambda_C(1+\epsilon)}{\lambda_C+\lambda_S}$, we have the equality $g_1(y) = g_2(y)$.

By Lemma 6, we know that $g_2(y)$ is concave for all $y \in [0, 1+\epsilon]$ where it hits zero at $g_2(0)$ and $g_2(1+\epsilon)$. We also have by Lemma 5 that $g_1(y)$ is convex and decreasing for all $y \in [0, 1+\epsilon]$. These imply that $g_1(y)$ and $g_2(y)$ meet exactly at one point within $(0, 1+\epsilon)$.[11]

[11] The reader might find it helpful to visually inspect the graphs of the functions $g_1(y)$ and $g_2(y)$ with the properties of these functions given in Lemmas 5 and 6.

An alternative way to reach this conclusion is to observe that the function $h(y) = g_1(y) - g_2(y)$ is strictly convex for $y \in [0, 1+\epsilon]$ and $h(0) > 0$, $h(1+\epsilon) = 0$ by Lemmas 5 and 6. Let $y_0 \in (0, 1+\epsilon)$ where $h(y_0) = 0$. Hence, by the definition of strict convexity, for all $y \in (y_0, 1+\epsilon)$, we have that $h(y) < 0 = \theta \cdot h(y_0) + (1-\theta) \cdot h(1+\epsilon)$ for any $\theta \in (0,1)$. Therefore, there does not exist $y \in (y_0, 1+\epsilon)$ such that $h(y) = 0$. For any $y_1 \in (0, p_0)$, it is also true that $h(y_1) \neq 0$ since y_0 is a convex combination of y_1 and $1+\epsilon$: $0 = h(p_0) < \theta \cdot h(y_1) + (1-\theta) \cdot h(1+\epsilon) = \theta \cdot h(y_1)$ for some $\theta \in (0,1)$, proving that $h(y_1) > 0$. Hence, there exists a unique $y \in (0, 1+\epsilon)$ such that $g_1(y) = g_2(y)$. ∎

We start expressing the welfare-maximal price in closed form. By Lemma 7, we can express the welfare-maximal price of the centralized platform as explained in the corollary below:

COROLLARY 3. *The welfare-maximal price of the centralized platform can be expressed as follows:*

$$p^* = \underset{y \in [0, 1+\epsilon]}{\arg\max}\, W_{I,\epsilon}^{\text{cen}}(y) = \begin{cases} \lambda_C(1+\epsilon)/(\lambda_C + \lambda_S), & \text{if } \arg\max_y g_2(y) \geq y_0, \\ \arg\max_y g_2(y), & \text{if } \arg\max_y g_2(y) < y_0. \end{cases}$$

In order to make use of this corollary, we find $\arg\max_y g_2(y)$ in the next lemma.

LEMMA 8. *Let $g_2(y)$ is as defined in* (20). *Then*

$$\underset{y}{\arg\max}\, g_2(y) = \begin{cases} 1+\epsilon - \sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)}, & \text{if } \epsilon < 2\sqrt{7} - 5, \\ \frac{(1+\epsilon)(\sqrt{7}-1)}{3}, & \text{if } \epsilon \geq 2\sqrt{7} - 5. \end{cases}$$

Proof. Since $g_2(y)$ is a piecewise function, we analyze it in two cases:

Case 1 : $\epsilon < 1 - y$. In this case, we maximize $\log(g_2(y))$ out of convenience instead of $g_2(y)$ without loss of generality since $\log(\cdot)$ is a strictly monotone function. First, we find the first-order optimality condition for $\log(g_2(y))$.

$$\begin{aligned} \frac{d(\log(g_2(y)))}{dy} &= \frac{1}{y} - \frac{3}{3+\epsilon^2 - 3y} + \frac{1}{1+\epsilon - y} \\ &= \frac{1+\epsilon}{y(1+\epsilon-y)} - \frac{3}{3+\epsilon^2-3y} = 0, \end{aligned}$$

which implies that

$$3y(1+\epsilon - y) = (3+\epsilon^2 - 3y)(1+\epsilon),$$

and that

$$-3y^2 + 6y(1+\epsilon) - (3+\epsilon^2)(1+\epsilon) = 0.$$

We find the two roots of this second-degree polynomial

$$y_{1,2} = (1+\epsilon) \pm \sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)}.$$

Since $y < 1-\epsilon$ in this case, the only potential local optimum is the point

$$y_1 = (1+\epsilon) - \sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)},$$

which should be smaller than $1-\epsilon$ for it to be a valid local maximum. Hence, we need to find the set of ϵ values that makes y_1 smaller than $1-\epsilon$ in order to find for what ϵ values y_1 is a valid local maximum. We have

$$y_1 = (1+\epsilon) - \sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)} < (1-\epsilon),$$

if and only if

$$2\epsilon < \sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)} = \sqrt{(1+\epsilon)\epsilon(1-\epsilon/3)},$$

which is equivalent to the following inequality when the term inside the square-root is non-negative, which necessitates that $0 \leq \epsilon \leq 3$:

$$\epsilon^2 + 10\epsilon - 3 = (\epsilon+5)^2 - 28 < 0,$$

This equivalency implies that $\epsilon < 2\sqrt{7} - 5 < 3$, deeming the condition that ϵ must be less than 3 redundant.

Case 2 : $\epsilon \geq 1-y$. In this case, the first-order optimality condition is

$$(1+\epsilon-2y)(y+2+2\epsilon) + y(1+\epsilon-y) = 0,$$

which is true if and only if

$$-3y^2 - 2y(1+\epsilon) + (1+\epsilon)^2 = 0.$$

We find the two roots of this polynomial:

$$y_{1,2} = -\frac{1+\epsilon}{3} \pm (1+\epsilon)\frac{\sqrt{7}}{3}.$$

We can eliminate one of the roots since it is negative, which leaves us with the local optimum

$$y_2 = (1+\epsilon)\frac{\sqrt{7}-1}{3}.$$

Next, we find the set of values for which $y_2 \geq 1-\epsilon$ in order to find when y_2 is a valid local optimum for g_2. One can verify by simple algebra that $(1+\epsilon)\frac{\sqrt{7}-1}{3} \geq 1-\epsilon$ if and only if $\epsilon \geq 2\sqrt{7}-5$. ■

Hence, we know the closed-form expressions for all the potential welfare-maximal prices that are given by Lemma 7 and that there is a unique welfare-maximal price for any given set of market parameters. The next step in the proof is to find the conditions that determine when each of these potential welfare-maximal prices is valid. Put another way, we need to find the set of market parameters for which $y_0 = \lambda_C(1+\epsilon)/(\lambda_C+\lambda_S) < \arg\max_y g_2(y)$ in order to use Lemma 7. Solving this problem, the next lemma characterizes the welfare-maximal price.

LEMMA 9. *The welfare-maximal price can be characterized as follows:*

$$\arg\max_y W^c_{I,\epsilon}(y) = \begin{cases} \frac{(1+\epsilon)(\sqrt{7}-1)}{3}, & \epsilon > 2\sqrt{7}-5 \text{ and } \frac{\lambda_S}{\lambda_C} > \frac{4-\sqrt{7}}{\sqrt{7}-1}, \\ \frac{(1+\epsilon)\lambda_C}{\lambda_S+\lambda_C}, & \epsilon > 2\sqrt{7}-5 \text{ and } \frac{\lambda_S}{\lambda_C} \leq \frac{4-\sqrt{7}}{\sqrt{7}-1}, \\ \frac{(1+\epsilon)\lambda_C}{\lambda_S+\lambda_C}, & 0 \leq \epsilon \leq \min\left\{2\sqrt{7}-5, \frac{3}{2}\frac{\lambda_C}{\lambda_S+\lambda_C} - \sqrt{\frac{9}{4}\frac{\lambda_C^2}{(\lambda_S+\lambda_C)^2} - 3\frac{\lambda_S}{\lambda_S+\lambda_C}}\right\} \\ & \text{or } \frac{\lambda_C}{\lambda_C+\lambda_S} < 2/3, \\ 1+\epsilon - \sqrt{\frac{\epsilon(1+\epsilon)(3-\epsilon)}{3}}, & \frac{3}{2}\frac{\lambda_C}{\lambda_S+\lambda_C} - \sqrt{\frac{9}{4}\frac{\lambda_C^2}{(\lambda_S+\lambda_C)^2} - 3\frac{\lambda_S}{\lambda_S+\lambda_C}} \leq \epsilon \leq 2\sqrt{7}-5 \\ & \text{and } \frac{\lambda_C}{\lambda_C+\lambda_S} \geq 2/3. \end{cases}$$

Proof. By Lemma 8, $\arg\max_y g_2(y)$ is piecewise with respect to ϵ. There are two cases that define these pieces: $\epsilon < 2\sqrt{7}-5$ and $\epsilon \geq 2\sqrt{7}-5$. We will be using Lemma 7 to determine the set of values of ϵ, λ_S, and λ_C for which the welfare-maximal price is $y_0 = \lambda_C(1+\epsilon)/(\lambda_S+\lambda_C)$ or $\arg\max_y g_2(y)$, where $g_2(y)$ is defined in (20).

Case 1 ($\epsilon \geq 2\sqrt{7}-5$). When $\epsilon \geq 2\sqrt{7}-5$, one can verify that

$$\arg\max_y g_2(y) = \frac{(1+\epsilon)(\sqrt{7}-1)}{3} < \frac{\lambda_C(1+\epsilon)}{\lambda_S+\lambda_C} = y_0$$

whenever

$$\frac{\lambda_S}{\lambda_C} > \frac{4-\sqrt{7}}{\sqrt{7}-1}.$$

Hence, by Lemma 7, provided that $\epsilon < 2\sqrt{7}-5$, the welfare-maximal price is $\arg\max_y g_2(y) = \frac{(1+\epsilon)(\sqrt{7}-1)}{3}$ when $\lambda_S/\lambda_C > (4-\sqrt{7})/(\sqrt{7}-1)$, and it is equal to $\lambda_C(1+\epsilon)/(\lambda_S+\lambda_C)$ otherwise.

Case 2 ($\epsilon < 2\sqrt{7}-5$). When $\epsilon < 2\sqrt{7}-5$, in order to find the condition for which welfare-maximal price is equal to $\arg\max_y g_2(y)$ by Corollary 3, we need to find the set of ϵ values for which

$$p_0 = \frac{\lambda_C(1+\epsilon)}{\lambda_C+\lambda_S} > (1+\epsilon) - \sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)} = \arg\max_y g_2(y). \tag{21}$$

First, we note that the term inside the square-root can be factorized,

$$\sqrt{(1+\epsilon)^2 - \frac{1}{3}(1+\epsilon)(3+\epsilon^2)} = \sqrt{\frac{\epsilon(1+\epsilon)(3-\epsilon)}{3}}\ .$$

Hence, the term inside the square-root is positive since we are analyzing the case of $\epsilon < 2\sqrt{7}-5 < 3$. Next, we let $K = \lambda_C/(\lambda_S+\lambda_C)$ for convenience since it will be a recurring term. Then, isolating the term with the square-root in (21), squaring both sides, then dividing both sides by $(1+\epsilon)$, and rearranging, we get

$$\epsilon^2 - 3\epsilon K + 3(1-K) < 0.$$

By completing the square, this is true if and only if

$$\left(\epsilon - \frac{3K}{2}\right)^2 - \frac{9K^2}{4} + 3(1-K) < 0.$$

Since $-\frac{9K^2}{4} + 3(1-K)$ can be factorized nicely, this inequality is equivalent to

$$\left(\epsilon - \frac{3K}{2}\right)^2 < \frac{3}{4}(3K-2)(K+2),$$

which clearly does not hold for $0 \leq K < 2/3$ since the right-hand side becomes negative and left-hand side is a perfect square. Hence, we have for $1 > K \geq 2/3$ that

$$\frac{3K}{2} - \sqrt{\frac{3}{4}(3K-2)(K+2)} < \epsilon < \frac{3K}{2} + \sqrt{\frac{3}{4}(3K-2)(K+2)}. \tag{22}$$

Next, we show that the upper bound for ϵ in this expression, $\frac{3K}{2} + \sqrt{\frac{3}{4}(3K-2)(K+2)}$, is redundant in the sense that it is always larger than $2\sqrt{7}-5$. The expression $\frac{3K}{2} + \sqrt{\frac{3}{4}(3K-2)(K+2)}$ is increasing in K for $K \in [2/3, 1)$. Hence, it takes its minimum value when $K = 2/3$, which makes the expression equal to 1. Since

$2\sqrt{7}-5=\sqrt{28}-\sqrt{25}<1=\sqrt{36}-\sqrt{25}$, the upper bound on ϵ in (22) is redundant and can be replaced by $2\sqrt{7}-5$. Hence, by Lemma 7, we conclude that

$$\arg\max_y W_{I,\epsilon}^{\text{cen}}(y) = \begin{cases} \frac{(1+\epsilon)\lambda_C}{\lambda_S+\lambda_C}, & 0 \le \epsilon \le \min\left\{2\sqrt{7}-5, \frac{3}{2}\frac{\lambda_C}{\lambda_S+\lambda_C} - \sqrt{\frac{9}{4}\frac{\lambda_C^2}{(\lambda_S+\lambda_C)^2} - 3\frac{\lambda_S}{\lambda_S+\lambda_C}}\right\} \\ & \text{or } \frac{\lambda_C}{\lambda_C+\lambda_S} < 2/3, \\ 1+\epsilon-\sqrt{\frac{\epsilon(1+\epsilon)(3-\epsilon)}{3}}, & \frac{3}{2}\frac{\lambda_C}{\lambda_S+\lambda_C} - \sqrt{\frac{9}{4}\frac{\lambda_C^2}{(\lambda_S+\lambda_C)^2} - 3\frac{\lambda_S}{\lambda_S+\lambda_C}} \le \epsilon \le 2\sqrt{7}-5 \\ & \text{and } \frac{\lambda_C}{\lambda_C+\lambda_S} \ge 2/3. \end{cases}$$

when $\epsilon < 2\sqrt{7}-5$. ■

Finally, we use our characterization of the welfare-maximal price in Lemma 9 to find a closed-form expression for the maximum welfare of the centralized platform. In order to do this, we just plug in the welfare-maximal prices described in Lemma 9 and use Lemma 7 to determine whether $W_{I,\epsilon}^{\text{cen}}(y)$ is equal to $g_1(y)$ or $g_2(y)$.

Note that whenever $y^* = \arg\max_y W_{I,\epsilon}^c(y) = \lambda_C(1+\epsilon)/(\lambda_C+\lambda_S)$ in Lemma 9, it holds that $W_{I,\epsilon}^{\text{cen}}(y^*) = g_1(y^*) = g_2(y^*)$ by Lemma 7. On the other hand, whenever $y^* = \arg\max_y W_{I,\epsilon}^{\text{cen}}(y) \neq \lambda_C(1+\epsilon)/(\lambda_C+\lambda_S)$, then it holds that $W_{I,\epsilon}^{\text{cen}}(y^*) = g_2(y^*)$ by the Corollary 3. Hence, in order to find the total welfare, we use $y^* = \arg\max_y W_{I,\epsilon}^{\text{cen}}(y)$ as defined in Lemma 9, and derive $g_2(y^*)$ using the definition of $g_2(\cdot)$ in (20). Using the procedure expressed in this paragraph and rearranging the arising expressions appropriately, one can verify that the maximum social welfare of the centralized platform can be expressed as follows:

$$\max_y W_{I,\epsilon}^{\text{cen}}(y) = \begin{cases} \frac{\lambda_S(7\sqrt{7}-10)(1+\epsilon)^3}{324\epsilon}, & \epsilon > 2\sqrt{7}-5 \text{ and } \frac{\lambda_S}{\lambda_C} > \frac{4-\sqrt{7}}{\sqrt{7}-1}, \\ \frac{\lambda_C\lambda_S^2(1+\epsilon)^3(3\lambda_C+2\lambda_S)}{24\epsilon(\lambda_C+\lambda_S)^3}, & \epsilon > 2\sqrt{7}-5 \text{ and } \frac{\lambda_S}{\lambda_C} \le \frac{4-\sqrt{7}}{\sqrt{7}-1}, \\ \frac{\lambda_C}{6}\left(3+\epsilon^2-\frac{3(1+\epsilon)\lambda_C}{\lambda_C+\lambda_S}\right), & 0 \le \epsilon \le \min\left\{2\sqrt{7}-5, \frac{3}{2}\frac{\lambda_C}{\lambda_S+\lambda_C} - \sqrt{\frac{9}{4}\frac{\lambda_C^2}{(\lambda_S+\lambda_C)^2} - 3\frac{\lambda_S}{\lambda_S+\lambda_C}}\right\} \\ & \text{or } \frac{\lambda_S}{\lambda_C} > 1/2, \\ \frac{\lambda_S}{6}\left(6\epsilon+3-\epsilon^2-2\sqrt{3(3-\epsilon)\epsilon(1+\epsilon)}\right), & \frac{3}{2}\frac{\lambda_C}{\lambda_S+\lambda_C} - \sqrt{\frac{9}{4}\frac{\lambda_C^2}{(\lambda_S+\lambda_C)^2} - 3\frac{\lambda_S}{\lambda_S+\lambda_C}} \le \epsilon \le 2\sqrt{7}-5 \\ & \text{and } \frac{\lambda_S}{\lambda_C} \le 1/2. \end{cases}$$

Decentralized platform. Under the decentralized platform, with a slight abuse of notation, we let suppliers with cost c pick prices $p^*(c)$ to maximize their expected profits

$$\Phi_S(c, p^*(c), \tau, p, N) = (p^*(c)-c)\cdot\frac{1-p^*(c)}{1-p(0)}\cdot\min\left\{1, (1-p(0))\cdot\frac{\lambda_C}{\lambda_S}\right\}.$$

The first term in the multiplication refers to the net profit by the suppliers with cost c. The second term represents the fraction of customers that enter the market and that are willing to accept the price $p^*(c)$. Finally, the third term refers to the probability that any supplier is picked for a match.

Since the mass of suppliers with any cost c is 0, the fact that they are picking the optimal prices does not affect the large market. Hence, without loss of generality, we can take $p(0)$ as constant. Given this, the suppliers with cost c maximize the quantity $(y-c)\cdot(1-y)$ over y, which can be shown to be maximized at $y = p^*(c) = (1+c)/2$ by simple algebra.

Next, given $p^*(c) = (1+c)/2$, we calculate the supplier's total profit and the customers' total net surplus. The total suppliers' profit is calculated as

$$\lambda_S \int_0^1 \Phi_S(c, p^*(c), \tau, p, N) dc = \lambda_S \int_0^1 \frac{(1-c)^2}{2} \cdot \min\left\{1, \frac{\lambda_C}{2\lambda_S}\right\} dc$$
$$= \min\left\{\frac{\lambda_S}{6}, \frac{\lambda_C}{12}\right\}.$$

In order to calculate the customers' total net surplus, we first observe that $p^*(0) = 1/2$, which implies that only the customers with types $v > 1/2$ enter the market whereas all suppliers enter the market since $\tau(v) = 0$ for all $v \in [-\epsilon, 1]$. This is simply due to the fact that in a static market, the continuation value associated with the corresponding optimal stopping problem is always 0 for customers with any type $v \in [-\epsilon, 1]$. Hence, the mass of matches is $\min\{\lambda_C/2, \lambda_S\}$. Next, we calculate the average net surplus of a customer from a match. We observe that both the suppliers' prices and the customers' valuations for those who enter the market are uniformly distributed between 1/2 and 1. Hence, the average net surplus of a customer from a match is $\mathbb{E}[(U_1 - U_2)^+] = 1/12$, where U_1 and U_2 are uniform random variables distributed between 1/2 and 1. Multiplying the average net surplus of a match with the mass of matches $\min(\lambda_C/2, \lambda_S)$, we calculate the customers' total net surplus as $\min\{\lambda_C/24, \lambda_S/12\}$.

Hence, the customers' total net surplus is $\min\{\lambda_C/24, \lambda_S/12\}$ while the suppliers' total profit are $\min\{\lambda_C/12, \lambda_S/6\}$, which makes the total welfare $\min\{\lambda_C/8, \lambda_S/4\}$ under the decentralized platform.

Bounds on welfare ratios. Deriving the bounds on the welfare ratios for the revenue-maximal price and the welfare-maximal price are straightforward. We have the following inequalities:

$$\lim_{\epsilon\to 0^+} \frac{W_{I,\epsilon}^{\text{dec}}}{W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*W})} = \frac{\min\left(\frac{\lambda_C}{8}, \frac{\lambda_S}{4}\right)}{\frac{\lambda_S\lambda_C}{2(\lambda_S+\lambda_C)}} = \left(\begin{cases} \frac{1}{4}\left[1 + \frac{\lambda_C}{\lambda_S}\right] & \text{if } \frac{\lambda_C}{\lambda_S} \le 2 \\ \frac{1}{2}\left[1 + \frac{\lambda_S}{\lambda_C}\right] & \text{if } \frac{\lambda_C}{\lambda_S} > 2 \end{cases}\right) \in \left[\frac{1}{4}, \frac{3}{4}\right],$$

$$\lim_{\epsilon\to 0^+} \frac{W_{I,\epsilon}^{\text{dec}}}{W_{I,\epsilon}^{\text{cen}}(p_{I,\epsilon}^{*R})} = \frac{\min\left(\frac{\lambda_C}{8}, \frac{\lambda_S}{4}\right)}{\min\left(\frac{\lambda_C}{4}, \frac{\lambda_S\lambda_C}{2(\lambda_S+\lambda_C)}\right)} = \left(\begin{cases} \frac{1}{2} & \text{if } \frac{\lambda_C}{\lambda_S} < 1 \\ \frac{1}{4}\left[1 + \frac{\lambda_C}{\lambda_S}\right] & \text{if } 1 < \frac{\lambda_C}{\lambda_S} \le 2 \\ \frac{1}{2}\left[1 + \frac{\lambda_S}{\lambda_C}\right] & \text{if } \frac{\lambda_C}{\lambda_S} > 2 \end{cases}\right) \in \left[\frac{1}{2}, \frac{3}{4}\right].$$

Appendix E: Proofs from Section 5.2

E.1. Proof of Proposition 3

First, we note that in all notation we use throughout this proof, we omit the reference to the mappings the agents use for ease of exposition. Namely, the dependence on the price mapping $p(\cdot)$, the threshold mapping $\tau(\cdot)$, and N are omitted.

We divide this proof into two parts to improve the readability: The monotonicity of customers' strategies and distributional properties.

The monotonicity of customers' strategies. First, we remind the reader of the expected surplus utility of a customer with the threshold strategy x and type v:

$$\Phi_C(v, x)$$

$$
\begin{aligned}
&= \underbrace{\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}\left[\mathbb{I}((v-p(\boldsymbol{c})+\boldsymbol{\eta})\geq x)\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})\right]}_{\text{Expected customer surplus given the customer is picked for a match}} \cdot \underbrace{\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}}_{\text{Probability of being picked}} \\
&\quad \cdot \sum_{t=0}^{\infty} \underbrace{\left(1-\phi_C(v,x)\right)^t\left(1-\mu_C\right)^t}_{\substack{\text{Probability of no departure} \\ \text{t time periods into the future}}} \\
&= \frac{\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}\cdot\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}[\mathbb{I}(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x)\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})]}{\phi_C(v,x)\cdot(1-\mu_C)+\mu_C},
\end{aligned}
$$

We let $g_v(\cdot)$ be the probability density function for the random variable equal to the utility surplus $(v-p(\boldsymbol{c})+\boldsymbol{\eta})$ of a customer with a fixed type $v\in[-\epsilon,1]$ where the randomness is over $\boldsymbol{c}\sim\bar{N}_S$ and $\boldsymbol{\eta}\sim$ Uniform$[-\epsilon,\epsilon]$. Then, we define the expected utility surplus conditional on the customer being picked for a match for a customer with type $v\in[-\epsilon,1]$ and threshold x:

$$E(v,x) := \mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}[\mathbb{I}(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x)\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})] = \int_x^{\infty} g_v(y)\cdot y\cdot dy.$$

Before expressing the first-order optimality condition with respect to the customer threshold x, in order to ease the exposition, we first determine the partial derivative of $E(v,x)$ with respect to x:

$$\frac{\partial E(v,x)}{\partial x} = -g_v(x)\cdot x. \tag{23}$$

Similarly, we determine the partial derivative of $\phi_C(v,x)$ with respect to x as follows:

$$\frac{\partial \phi_C(v,x)}{\partial x} = -\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}\cdot g_v(x). \tag{24}$$

Given equations (23) and (24), we use the simple fact that for any two differentiable functions f and g, $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$ whenever $g(x)\neq 0$, in order to obtain the following first-order optimality condition:

$$
\begin{aligned}
&\frac{\partial \Phi_C(v,x)}{\partial x} = \\
&\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}\cdot\Bigg[\frac{\overbrace{-g_v(x)\cdot x}^{\text{Derivative of the numerator by (23)}}\cdot\overbrace{\left(\phi_C(v,x)\cdot(1-\mu_C)+\mu_C\right)}^{\text{Denominator of }\Phi_C(v,x)}}{\left(\phi_C(v,x)\cdot(1-\mu_C)+\mu_C\right)^2} + \\
&\frac{\overbrace{\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}\left[\mathbb{I}(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x)\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})\right]}^{\text{Numerator of }\Phi_C(v,x)}\cdot\overbrace{\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}\cdot g_v(x)(1-\mu_C)}^{\text{Derivative of the denominator by (24)}}}{\left(\phi_C(v,x)\cdot(1-\mu_C)+\mu_C\right)^2}\Bigg] \\
&= 0.
\end{aligned}
$$

It can be shown that the optimal threshold $x=\tau^*(v)$ obtained by setting the first-order optimality condition to zero can also be obtained by using Bellman's equation. Specifically, $\tau^*(v)$ is equal to the expected utility surplus of a type v customer who remains in the system at the current time period. Put another way, the optimal threshold on the net utility surplus makes the customer indifferent between staying in the market and accepting the proposed match. Thus, by the fact that the reward-to-go is equal to $(1-\mu_C)\cdot\Phi_C(v,\tau^*(v))$ by definition[12], we infer that:

$$\begin{aligned}\frac{\tau^*(v)}{1-\mu_C} &= \Phi_C(v,\tau^*(v)) \\ &= \frac{\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}\cdot\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}\left[\mathbb{I}(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq\tau^*(v))\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})\right]}{\phi_C\left(v,\tau^*(v)\right)\cdot(1-\mu_C)+\mu_C},\end{aligned} \tag{25}$$

which implies that

$$\frac{1}{1-\mu_C}\cdot(\tau^*)'(v)=\frac{\partial\Phi_C(v,\tau^*(v))}{\partial v}+(\tau^*)'(v)\cdot\frac{\partial\Phi_C(v,\tau^*(v))}{\partial\tau}=\frac{\partial\Phi_C(v,\tau^*(v))}{\partial v} \tag{26}$$

because $\frac{\partial\Phi_C(v,\tau^*(v))}{\partial\tau}=0$ by the first-order optimality condition. We note that equation (26) also follows from the envelope theorem for an optimization problem with one decision variable, one parameter, and no constraints. We proceed with calculating the partial derivative of Φ_C with respect to v. In order to compute this partial derivative, we derive the partial derivative of the expected utility surplus from a match, $E(v,x)$, with respect to v using the definition of partial derivative:

$$\begin{aligned}\frac{\partial E(v,x)}{\partial v} &= \lim_{h\to0}\frac{E(v+h,x)-E(v,x)}{h} \\ &= \lim_{h\to0}\left(\frac{\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}\left[\mathbb{I}\left(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x-h\right)\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})\right]}{h}\right. \\ &\quad\left.+\frac{\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}\left[\mathbb{I}\left(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x-h\right)\cdot h\right]-E(v,x)}{h}\right) \\ &= \lim_{h\to0}\frac{\int_{x-h}^{\infty}g_v(y)\cdot y\cdot dy+h\cdot\Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x\right]-\int_x^{\infty}g_v(y)y\cdot dy}{h} \\ &= \Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq x\right]+g_v(x)\cdot x.\end{aligned} \tag{27}$$

We calculate the partial derivative of $\Phi_C(v,\tau^*(v))$ with respect to v by the definition of $\Phi_C(v,\tau^*(v))$:

$$\begin{aligned}&\frac{\partial\Phi_C(v,\tau^*(v))}{\partial v} \\ &= \min\left\{1,\frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\left[\frac{\partial E(v,\tau^*(v))}{\partial v}\cdot\sum_{t=0}^{\infty}\left(1-\phi_C(v,\tau^*(v))\right)^t(1-\mu_C)^t\right. \\ &\quad\left.+E(v,\tau^*(v))\cdot\frac{\partial\left(\sum_{t=0}^{\infty}(1-\phi_C(v,\tau^*(v)))^t(1-\mu_C)^t\right)}{\partial v}\right] \\ &= \min\left\{1,\frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\left[\underbrace{\frac{\Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq\tau^*(v)\right]+g_v(\tau^*(v))\cdot\tau^*(v)}{\phi_C\left(v,\tau^*(v)\right)\cdot(1-\mu_C)+\mu_C}}_{\text{By (27)}}\right. \\ &\quad\left.-\mathbb{E}_{\boldsymbol{c},\boldsymbol{\eta}}\left[\mathbb{I}(v-p(\boldsymbol{c})+\boldsymbol{\eta}\geq\tau^*(v))\cdot(v-p(\boldsymbol{c})+\boldsymbol{\eta})\right]\cdot\underbrace{\frac{g_v(\tau^*(v))\cdot(1-\mu_C)\cdot\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}}{\left(\phi_C\left(v,\tau^*(v)\right)\cdot(1-\mu_C)+\mu_C\right)^2}}_{\text{By }\frac{\partial\phi_C}{\partial v}=g(v,\tau^*(v))\cdot\min\left\{1,\|\bar{N}_S\|/\|\bar{N}_C\|\right\}}\right]\end{aligned}$$

[12] Without loss of generality, we set the reward-to-go to $(1-\mu_C)\cdot\Phi_C(v,\tau^*(v))$ when there are multiple best-response thresholds with equal reward. This choice does not affect any of the customers' net surpluses, suppliers' profits, or the platform's gross revenue.

$$
\begin{aligned}
&= \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \left[\frac{\Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq \tau^*(v)\right] + g_v(\tau^*(v)) \cdot \tau^*(v)}{\phi_C\left(v, \tau^*(v)\right) \cdot (1-\mu_C) + \mu_C}\right. \\
&\quad - \left. \underbrace{\Phi_C(v, \tau^*(v)) \cdot (1-\mu_C)}_{\text{Replace by } \tau^*(v) \text{ via (25) in the next row}} \cdot \frac{g_v(\tau^*(v))}{\phi_C\left(v, \tau^*(v)\right) \cdot (1-\mu_C) + \mu_C}\right] \\
&= \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \left[\frac{\Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq \tau^*(v)\right] + g_v(\tau^*(v)) \cdot \tau^*(v)}{\phi_C\left(v, \tau^*(v)\right) \cdot (1-\mu_C) + \mu_C}\right. \\
&\quad - \left. \frac{g_v(\tau^*(v)) \cdot \tau^*(v)}{\phi_C\left(v, \tau^*(v)\right) \cdot (1-\mu_C) + \mu_C}\right] \\
&= \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{\Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v - p(\boldsymbol{c}) + \boldsymbol{\eta} \geq \tau^*(v)\right]}{\phi_C\left(v, \tau^*(v)\right) \cdot (1-\mu_C) + \mu_C} \\
&= \frac{\phi_C(v, \tau^*(v))}{\phi_C(v, \tau^*(v)) \cdot (1-\mu_C) + \mu_C}. \qquad (28)
\end{aligned}
$$

Now, by combining equation (28) with (26), we obtain the derivative of the best-response threshold $\tau^*(v)$ with respect to the type v, which we show to be positive:

$$
(\tau^*)'(v) = (1-\mu_C) \cdot \frac{\partial \Phi_C(v, \tau^*(v))}{\partial v} = \frac{\phi_C(v, \tau^*(v)) \cdot (1-\mu_C)}{\phi_C(v, \tau^*(v)) \cdot (1-\mu_C) + \mu_C} > 0, \qquad \forall v \in T_C, \qquad (29)
$$

which implies that the willingness-to-pay $\omega^*(v) = v - \tau^*(v)$ is monotonically increasing:

$$
(\omega^*)'(v) = 1 - (\tau^*)'(v) = \frac{\mu_C}{\phi_C(v, \tau^*(v)) \cdot (1-\mu_C) + \mu_C} > 0, \qquad \forall v \in T_C. \qquad (30)
$$

The results in equations (29) and (30) establish the first part of Proposition 3.

Distributional properties. We proceed with establishing the second part of Proposition 3. Let $\tilde{\lambda}_C(w)$ denote the arrival density of customers with the willingness-to-pay w. Note that at the end of each time period, $(1-\mu_C) \cdot (1 - \phi_C\left(\omega^{-1}(w), \tau\left(\omega^{-1}(w)\right)\right))$ fraction of customers who enter the market with willingness-to-pay w stays in the market for the next time period. Hence, the density of customers with willingness-to-pay $w \in [-\epsilon, 1]$ such that $\omega^{-1}(w) \in T_C$ in each steady-state period after entry is

$$
\begin{aligned}
f_C(w) &= \tilde{\lambda}_C(w) \cdot \sum_{t=0}^{\infty} (1-\mu_C)^t \left(1 - \phi_C\left(\omega^{-1}(w), \tau\left(\omega^{-1}(w)\right)\right)\right)^t \\
&= \tilde{\lambda}_C(w) \cdot \frac{1}{(1-\mu_C) \cdot \phi_C\left(\omega^{-1}(w), \tau\left(\omega^{-1}(w)\right)\right) + \mu_C}. \qquad (31)
\end{aligned}
$$

Before we proceed with the proof, we present the following textbook lemma that is instrumental not only for the proof of Proposition 3 but also for the proof of Proposition 4.

LEMMA 10. *(Billingsley 1986) Suppose $\omega : S \to T \subseteq \mathbb{R}$ is monotonically increasing and differentiable, and X is a continuous random variable on S with the associated probability density function $f_X(x)$. Then, the probability density function associated with $\omega(X)$, denoted by $f_\omega(x)$, is given by*

$$
f_\omega(x) = \frac{f_X(\omega^{-1}(x))}{\omega'\left(\omega^{-1}(x)\right)}, \quad \forall x \in T.
$$

Proof. We let $F_\omega(x)$ and $F_X(x)$ denote the cumulative distribution functions associated with the random variables $\omega(X)$ and X, respectively. Then, we have the following equality:

$$
F_\omega(x) = \Pr_X\left[\omega(X) \leq x\right] = \Pr_X\left[X \leq \omega^{-1}(x)\right] = F_X(\omega^{-1}(x)).
$$

Therefore, the probability density function associated with $\omega(X)$ is

$$\frac{dF_\omega}{dx} = f_\omega(x) = F'_X(\omega^{-1}(x)) \cdot (\omega^{-1})'(x) = f_X(\omega^{-1}(x)) \cdot \frac{1}{\omega'(\omega^{-1}(x))}$$

for all $x \in T$. ■

Now, we focus on expressing $\tilde{\lambda}_C(w)$. We denote the total mass of customers that enter the market $\|\tilde{\lambda}_C\|$ and let $f^\lambda_\omega(w) = \tilde{\lambda}_C(w)/\|\tilde{\lambda}_C\|$ be the probability density function associated with arrival densities of agents with willingness-to-pay w. Note that agents of any type $v \in T_C$ have the arrival density λ_C, i.e., $\|\tilde{\lambda}_C\| = (1-\underline{\omega})\lambda_C$ (The arrival distribution of valuations is uniform between $\underline{v} = \underline{\omega}$ and 1.). Hence, we have the following[13] by Lemma 10 and equation (30):

$$\begin{aligned} f^\lambda_\omega(w) &= \underbrace{\frac{1}{(1-\underline{\omega})}}_{\substack{\text{pdf associated with}\\ \text{arrival density}\\ \text{(numerator in Lemma 10)}}} \cdot \underbrace{\frac{1}{(\omega^*)'((\omega^*)^{-1}(w))}}_{\substack{\text{Use equation (30) to}\\ \text{replace in the next row}}} \qquad (32)\\ &= \frac{1}{(1-\underline{\omega}) \cdot \left(1 - \frac{\phi_C((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w)))\cdot(1-\mu_C)}{\phi_C((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w)))\cdot(1-\mu_C)+\mu_C}\right)} \\ &= \frac{\phi_C((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w)))\cdot(1-\mu_C)+\mu_C}{(1-\underline{\omega})\cdot\mu_C}, \end{aligned}$$

for all $w \in [-\epsilon, 1]$ such that $(\omega^*)^{-1}(w) \in T_C$. Plugging $\tilde{\lambda}_C(w) = f^\lambda_\omega(w) \cdot \|\tilde{\lambda}_C\|$ into (31), we obtain the density of customers with willingness-to-pay $w \in [-\epsilon, 1]$ such that $(\omega^*)^{-1}(w) \in T_C$ as follows[14]:

$$\begin{aligned} f_C(w) =& \frac{1}{(1-\mu_C)\cdot\phi_C\big(\omega^{-1}(w), \tau(\omega^{-1}(w))\big)+\mu_C} \cdot \tilde{\lambda}_C(w) \\ =& \frac{1}{(1-\mu_C)\cdot\phi_C\big((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w))\big)+\mu_C} \\ & \cdot \frac{\phi_C((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w)))\cdot(1-\mu_C)+\mu_C}{(1-\underline{\omega})\cdot\mu_C} \cdot \|\tilde{\lambda}_C\| \\ =& \frac{1}{(1-\mu_C)\cdot\phi_C\big((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w))\big)+\mu_C} \\ & \cdot \frac{\phi_C((\omega^*)^{-1}(w), \tau^*((\omega^*)^{-1}(w)))\cdot(1-\mu_C)+\mu_C}{(1-\underline{\omega})\cdot\mu_C} \cdot (1-\underline{\omega})\cdot\lambda_C \\ =& \frac{\lambda_C}{\mu_C}. \qquad (33) \end{aligned}$$

Hence, we have the following desired result:

$$f_C(w) = \begin{cases} \frac{\lambda_C}{\mu_C}, & \text{if } \underline{\omega} \le w \le \overline{\omega}, \\ 0, & \text{otherwise.} \end{cases}$$

E.2. Proof of Proposition 4

First, we note that in the notation we use throughout this proof, we suppress the dependence on the price $p(\cdot)$ and N for the sake of simplicity. We proceed with this proof in two subsections: The monotonicity of suppliers' prices and the distributional properties of the suppliers' prices.

[13] The probability that arriving customer has a valuation $v \in T_C = [\underline{\omega}, 1]$, which appears as the first term, is $1/(1-\underline{\omega})$ due to the uniformity of arrivals.

[14] We note that Corollary 1 can be readily proven by replacing the first expression in (32) with the probability density function associated with the arrival density $f(v)$.

The monotonicity of the suppliers' prices. We let $P(y,\tau)$ be the fraction of suppliers with price y that are successfully matched before leaving the market at steady-state when customers use the mapping τ to decide on their thresholds:

$$\begin{aligned} P(y,\tau) &= \phi_S(y,\tau)\sum_{t=0}^{\infty}(1-\phi_S(y,\tau))^t(1-\mu_S)^t \\ &= \frac{\phi_S(y,\tau)}{\phi_S(y,\tau)(1-\mu_S)+\mu_S}. \end{aligned} \tag{34}$$

We remind the reader that $\phi_S(y,\tau)=\min\left\{1,\|\bar{N}_C\|/\|\bar{N}_S\|\right\}\cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}\left[\boldsymbol{v}-\tau(\boldsymbol{v})+\boldsymbol{\eta}\geq y\right]$ denotes the fraction of suppliers with price y when customers use the mapping τ to decide on their threshold strategies. Hence, suppliers with type c pick price y to maximize their expected profits

$$\Phi_S(c,y,\tau)=(y-c)\cdot P(y,\tau),$$

subject to the constraint that $y\in[p_{\min},p_{\max}]$. We let

$$P'(y,\tau)=\frac{\partial P(y,\tau)}{\partial y}$$

denote the partial derivative of $P(y,\tau)$ with respect to y. Therefore, we have the following first-order optimality condition that describes the best-response price of $p^*(c,\tau)$:

$$p^*(c,\tau)-c=\frac{-P(p^*(c,\tau),\tau)}{P'(p^*(c,\tau),\tau)}, \tag{35}$$

for any $c\in[0,1+\epsilon]$ such that $p^*(c,\tau)\in(p_{\min},p_{\max})$.[15] Note that

$$P'(y,\tau)=\frac{\mu_S\cdot\phi_S'(y,\tau)}{\left((1-\mu_S)\phi_S(y,\tau)+\mu_S\right)^2}, \tag{36}$$

for any $y\in(p_{\min},p_{\max})$ where we let $\phi_S'(y,\tau)=\partial\phi_S(y,\tau)/\partial y$ denote the partial derivative of $\phi_S(y,\tau)$ with respect to y and let $\phi_S''(y,\tau)=\partial^2\phi_S(y,\tau)/\partial y^2$ denote the second partial derivative. Then, plugging equation (36) in equation (35), we obtain the following equality that describes the maximum net profit of suppliers with cost c at steady-state:

$$p^*(c,\tau)-c=\frac{-\phi_S(p^*(c,\tau),\tau)}{\phi_S'(p^*(c,\tau),\tau)}\cdot\frac{(1-\mu_S)\phi_S(p^*(c,\tau),\tau)+\mu_S}{\mu_S}. \tag{37}$$

Before continuing with the proof, we show that the suppliers' expected profit $\Phi_S(c,y,\tau)$ is strictly log-concave in y for all $c\in T_S$. This result, which is presented next in Lemma 11, shows that there exists a unique maximizer y for the suppliers' profit function $\Phi_S(c,y,\tau)$ for any $c\in T_S$.

LEMMA 11. *The profit function for suppliers with cost c, $\Phi_S(c,y,\tau)=(y-c)\cdot P(y,\tau)$, is unimodal in y for all $c\in T_S$.*

[15] As will be clearer later on, for the price $p(c)\in\{p_{\min},p_{\max}\}$, the first-order optimality condition may not hold. In other words, the price that satisfies the first-order optimality condition may not lie in $[p_{\min},p_{\max}]$.

Proof. We will show the profit function for suppliers with cost c, $\Phi_S(c,y,\tau)=(y-c)\cdot P(y,\tau)$ is strictly log-concave, which will imply that it is also unimodal. Accordingly, we first note that

$$\log(\Phi_S(c,y,\tau))=\log(y-c)+\log(P(y,\tau)).$$

By equation (36), taking the derivative of both sides reveals the following relationship:

$$\begin{aligned}\frac{d\log(\Phi_S)}{dy}&=\frac{1}{y-c}+\frac{P'(y,\tau)}{P(y,\tau)}\\&=\frac{1}{y-c}+\frac{\phi_S'(y,\tau)}{\phi_S(y,\tau)}\cdot\frac{\mu_S}{(1-\mu_S)\phi_S(y,\tau)+\mu_S}.\end{aligned}$$

Taking the derivative of both sides, we obtain the following equality:

$$\begin{aligned}\frac{d^2\log(\Phi_S)}{dy^2}=&-\frac{1}{(y-c)^2}+\frac{\phi_S''(y,\tau)\phi_S(y,\tau)-(\phi_S'(y,\tau))^2}{(\phi_S(y,\tau))^2}\cdot\frac{\mu_S}{(1-\mu_S)\phi_S(y,\tau)+\mu_S}\\&-\frac{\phi_S'(y,\tau)}{\phi_S(y,\tau)}\cdot\frac{\mu_S\cdot\phi_S'(y,\tau)}{((1-\mu_S)\phi_S(y,\tau)+\mu_S))^2}.\end{aligned}\tag{38}$$

Making use of the next lemma, we will conclude that the right-hand side of equation (38) is negative.

LEMMA 12. *For any stationary equilibrium, we have the following equality:*

$$\frac{\phi_S''(y,\tau)\cdot\phi_S(y,\tau)}{(\phi_S'(y,\tau))^2}=\begin{cases}\frac{1}{2}, & \max\{\underline{\omega}+\epsilon,\overline{\omega}-\epsilon\}<y<\overline{\omega}+\epsilon,\\ 0, & \text{otherwise.}\end{cases}$$

The proof is deferred to Appendix E.3.

We conclude by Lemma 12 that $\phi_S''(y,\tau)\phi_S(y,\tau)-(\phi_S'(y,\tau))^2\leq 0$. We also know that the third term on the right-hand side is positive since it is a squared term multiplied with the positive quantity $\mu_S/\phi_S(y,\tau)$. Hence, we conclude that the right-hand side of equation (38) is negative, showing the strict log-concavity of the profit function $\Phi_S(c,y,\tau)=(y-c)\cdot P(y,\tau)$ for all $c\in T_S$. ■

Now that the uniqueness of $p^*(c,\tau)$ is established by the unimodality property of the suppliers' profit function of Lemma 11, we are interested in finding $(p^*)'(c,\tau)=\partial(p^*(c,\tau))/\partial c$. To this end, in order to ease the exposition, we let $\tilde{\phi}_S(y,\tau)=-\phi_S(y,\tau)/\phi_S'(y,\tau)$ which corresponds to the first term on the right-hand side of equation (37). First, we find the partial derivative of $\tilde{\phi}_S(p^*(c),\tau)$ with respect to c:

$$\begin{aligned}\frac{\partial\tilde{\phi}_S(p^*(c,\tau),\tau)}{\partial c}&=\frac{-(\phi_S'(p^*(c,\tau),\tau))^2\cdot(p^*)'(c,\tau)+\phi_S(p^*(c,\tau),\tau)\cdot\phi_S''(p^*(c,\tau),\tau)\cdot(p^*)'(c,\tau)}{(\phi_S'(p^*(c,\tau),\tau))^2}\\&=(p^*)'(c,\tau)\cdot\left[\frac{\phi_S(p^*(c,\tau),\tau)\cdot\phi_S''(p^*(c,\tau),\tau)}{(\phi_S'(p^*(c,\tau),\tau))^2}-1\right].\end{aligned}$$

Then, taking the partial derivative of both sides in (37) with respect to c, we obtain

$$\begin{aligned}(p^*)'(c,\tau)-1=&(p^*)'(c,\tau)\cdot\left[\frac{\phi_S(p^*(c,\tau),\tau)\cdot\phi_S''(p^*(c,\tau),\tau)}{(\phi_S'(p^*(c,\tau),\tau))^2}-1\right]\cdot\left[\frac{1-\mu_S}{\mu_S}\cdot\phi_S(p^*(c,\tau),\tau)+1\right]\\&-(p^*)'(c,\tau)\cdot\frac{\phi_S(p^*(c,\tau),\tau)}{\phi_S'(p^*(c,\tau),\tau)}\cdot\frac{1-\mu_S}{\mu_S}\cdot\phi_S'(p^*(c,\tau),\tau),\end{aligned}$$

which implies that

$$(p^*)'(c,\tau)=\left[\left(\frac{1-\mu_S}{\mu_S}\cdot\phi_S(p^*(c,\tau),\tau)+1\right)\cdot\left(2-\frac{\phi_S(p^*(c,\tau),\tau)\cdot\phi_S''(p^*(c,\tau),\tau)}{(\phi_S'(p^*(c,\tau),\tau))^2}\right)\right]^{-1}\tag{39}$$

for any $c \in T_S$ in any stationary equilibrium. By Lemma 12, we know that the expression

$$\left(2 - \frac{\phi_S(p^*(c,\tau),\tau) \cdot \phi_S''(p^*(c,\tau),\tau)}{(\phi_S'(p^*(c,\tau),\tau))^2}\right)$$

is equal to 3/2 or 2 for $p^*(c,\tau) \in (p_{\min}, p_{\max})$ in any stationary equilibrium. Hence, since $(1-\mu_S) \cdot \phi_S(p^*(c,\tau),\tau)/\mu_S + 1 > 1$, we have that $0 < (p^*)'(c,\tau) < 2/3$. Consequently, $p^*(c,\tau)$ is increasing and $p^*(c,\tau) - c$ is decreasing in c for all $c \in [0, 1+\epsilon]$ such that $p^*(c,\tau) \in (p_{\min}, p_{\max})$. With this, we have completed the proof of the first part of Proposition 4.

Distributional properties. We proceed with establishing the second part of Proposition 4. We let $\tilde{\lambda}_S(y,\tau)$ be the arrival density of suppliers with price y for any $y \in (p_{\min}, p_{\max})$ at steady-state where the customers use the mapping τ to pick their threshold strategies and let $\|\tilde{\lambda}_S\|$ denote the total mass of suppliers that enter the market. Consequently, we let $f_p(y,\tau) = \tilde{\lambda}_S(y,\tau)/\|\tilde{\lambda}_S\|$ be the probability density function associated with arrival densities of agents with price y in any stationary equilibrium. Noting that the arrival distribution of costs is uniform with density λ_S, we obtain the following for any $c \in T_S$ by Lemma 10 and equation (39),

$$\begin{aligned} f_p(p^*(c,\tau),\tau) &= \frac{\frac{\lambda_S}{\|\tilde{\lambda}_S\|}}{(p^*)'(c,\tau)} \\ &= \frac{\lambda_S}{\|\tilde{\lambda}_S\|} \cdot \left(\frac{1-\mu_S}{\mu_S} \cdot \phi_S(p^*(c,\tau),\tau) + 1\right) \cdot \left(2 - \frac{\phi_S(p^*(c,\tau),\tau) \cdot \phi_S''(p^*(c,\tau),\tau)}{(\phi_S'(p^*(c,\tau),\tau))^2}\right). \end{aligned} \tag{40}$$

Note that at the end of each time period, $(1-\mu_S) \cdot (1-\phi_S(y,\tau))$ fraction of suppliers with price y stays in the market for the next time period. Hence, using the equality $\tilde{\lambda}_S(y,\tau) = \|\tilde{\lambda}_S\| \cdot f_p(y,\tau)$, the equilibrium density of suppliers with price y in the market is

$$\begin{aligned} \tilde{\lambda}_S(y,\tau) \cdot \sum_{t=0}^{\infty} (1-\mu_S)^t \big(1-\phi_S(y,\tau)\big)^t &= \frac{\tilde{\lambda}_S(y,\tau)}{(1-\mu_S)\phi_S(y,\tau) + \mu_S} \\ &= \frac{\lambda_S}{\mu_S} \cdot \left(2 - \frac{\phi_S(y,\tau) \cdot \phi_S''(y,\tau)}{(\phi_S'(y,\tau))^2}\right) . \\ &= \begin{cases} 2 \cdot \frac{\lambda_S}{\mu_S} & \underline{p} < y < \min\{p_{\max}, \overline{\omega} - \epsilon\} \\ \frac{3}{2} \cdot \frac{\lambda_S}{\mu_S} & \max\{\underline{p}, \min\{p_{\max}, \overline{\omega} - \epsilon\}\} \le y < \overline{p}. \end{cases} \end{aligned}$$

This concludes the proof for the piecewise-uniform property of Proposition 4.

Possibility of positive point masses of suppliers with prices $p_{\min}$ *or* $p_{\max}$. The unimodality of the suppliers' profit function $\Phi_S(c,y,\tau)$ for all $c \in T_S$, which is established in Lemma 11, implies that $\Phi_S(c,y,\tau)$ is increasing for $y < p^*(c,\tau)$, and decreasing for $y > p^*(c,\tau)$, for all $c \in T_S$. Hence, operating under the constraint $p(c) \in [p_{\min}, p_{\max}]$, all suppliers with $p^*(c,\tau) \le p_{\min}$ choose the price $p(c) = p_{\min}$ whereas all suppliers with $p^*(c,\tau) \ge p_{\max}$ choose the price $p(c) = p_{\max}$. This leads to the possibility of having positive masses of suppliers with prices $p_{\min}$ or $p_{\max}$.

E.3. Proof of Lemma 12

By Proposition 3, we know that $f_C(\cdot)$ is uniformly distributed between $\underline{\omega}$ and $\overline{\omega}$. We let $f_\pi(\cdot)$ be the probability density function for the random variable associated with $\omega(\boldsymbol{v}) + \boldsymbol{\eta}$ where the randomness is over $\boldsymbol{v} \sim \bar{N}_C$ and $\boldsymbol{\eta} \sim \text{Uniform}(-\epsilon, \epsilon)$. Note that since the suppliers with the lowest price who enter the market have the price $\underline{\omega} + \epsilon$ by the entry conditions described in Section 3, we only need to consider the case where the price $y \ge \underline{\omega} + \epsilon$. We first analyze the case where $\epsilon \le \frac{\overline{\omega} - \underline{\omega}}{2}$:

Case 1 : $\epsilon \leq (\bar{\omega} - \underline{\omega})/2$. Note that by Proposition 3, we know that willingness-to-pay is uniformly distributed between $\underline{\omega}$ and $\overline{\omega}$. Then, we can determine the probability density function $f_\pi(\cdot)$:

$$f_\pi(z) = \begin{cases} \frac{z-(\underline{\omega}-\epsilon)}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \underline{\omega}-\epsilon \leq z < \underline{\omega}+\epsilon, \\ \frac{1}{\overline{\omega}-\underline{\omega}}, & \text{if } \underline{\omega}+\epsilon \leq z < \overline{\omega}-\epsilon, \\ \frac{\overline{\omega}+\epsilon-z}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \overline{\omega}-\epsilon \leq z < \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases}$$

Using this, we find $\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq y]$:

$$\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq y] = \begin{cases} \frac{\overline{\omega}-y}{\overline{\omega}-\underline{\omega}}, & \text{if } \underline{\omega}+\epsilon \leq y < \overline{\omega}-\epsilon, \\ \frac{(\overline{\omega}+\epsilon-y)^2}{4\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \overline{\omega}-\epsilon \leq y < \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{41}$$

Hence, noting that $\phi_S(y,\tau,N) = \min\{1, \|\bar{N}_C\|/\|\bar{N}_S\|\} \cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq y]$, we can calculate the first and second derivatives of $\phi_S(y,\tau,N)$ with respect to y:

$$\phi'_S(y,\tau,N) = \begin{cases} -\min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{1}{\overline{\omega}-\underline{\omega}}, & \text{if } \underline{\omega}+\epsilon \leq y < \overline{\omega}-\epsilon, \\ -\min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{\overline{\omega}+\epsilon-y}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \overline{\omega}-\epsilon \leq y < \overline{\omega}+\epsilon, \\ 0, & \text{otherwise,} \end{cases} \tag{42}$$

$$\phi''_S(y,\tau,N) = \begin{cases} 0, & \text{if } \underline{\omega}+\epsilon \leq y < \overline{\omega}-\epsilon, \\ \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{1}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \overline{\omega}-\epsilon \leq y < \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

which implies that our quantity of interest is either 0 or 1/2:

$$\frac{\phi_S(y,\tau,N) \cdot \phi''_S(y,\tau,N)}{(\phi'_S(y,\tau,N))^2} = \begin{cases} \frac{1}{2}, & \text{if } \overline{\omega}-\epsilon \leq y < \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases}$$

Next, we look at the case where $\epsilon > \frac{\overline{\omega}-\underline{\omega}}{2}$.

Case 2 : $\epsilon > (\overline{\omega} - \underline{\omega})/2$. Similarly, to the Case 1, we calculate $f_\pi(\cdot)$:

$$f_\pi(z) = \begin{cases} \frac{z-(\underline{\omega}-\epsilon)}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \underline{\omega}-\epsilon \leq z < \overline{\omega}-\epsilon, \\ \frac{1}{2\epsilon}, & \text{if } \overline{\omega}-\epsilon \leq z < \underline{\omega}+\epsilon, \\ \frac{\overline{\omega}+\epsilon-z}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \underline{\omega}+\epsilon \leq z < \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases}$$

Using this, we find $\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq y]$:

$$\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq y] = \begin{cases} \frac{(\overline{\omega}+\epsilon-y)^2}{4\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \underline{\omega}+\epsilon \leq y \leq \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{44}$$

This equation yields the following that describes $\phi'_S(y,\tau,N)$ and $\phi''_S(y,\tau,N)$:

$$\phi'_S(y,\tau,N) = \begin{cases} -\min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{\overline{\omega}+\epsilon-y}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \underline{\omega}+\epsilon \leq y \leq \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{45}$$

$$\phi''_S(y,\tau,N) = \begin{cases} \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{1}{2\epsilon(\overline{\omega}-\underline{\omega})}, & \text{if } \underline{\omega}+\epsilon \leq y \leq \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{46}$$

which implies that our quantity of interest, $\frac{\phi_S(y,\tau,N) \cdot \phi''_S(y,\tau,N)}{(\phi'_S(y,\tau,N))^2}$ is 1/2:

$$\frac{\phi_S(y,\tau,N) \cdot \phi''_S(y,\tau,N)}{(\phi'_S(y,\tau,N))^2} = \begin{cases} \frac{1}{2}, & \text{if } \underline{\omega}+\epsilon \leq y \leq \overline{\omega}+\epsilon, \\ 0, & \text{otherwise.} \end{cases}$$

Appendix F: Proofs from Section 5.3

F.1. Proof of Lemma 1

We demonstrate that the following statement holds:

$$p_{\max} < p^{\text{foc}}(0) \quad \text{or} \quad p_{\min} > p^{\text{foc}}(\overline{p}). \tag{47}$$

Suppose ad absurdum that $p_{\max} \geq p^{\text{foc}}(0)$ and $p_{\min} \leq p^{\text{foc}}(\overline{p})$, and define $\tilde{c}$ such that $p^{\text{foc}}(\tilde{c}) = \max\{p_{\min}, p^{\text{foc}}(0)\}$. Note that $\tilde{c}$ exists as we have shown in Proposition 4 that $p(c)$ is non-decreasing, continuous over $c \in [0,\overline{p}]$ and $p^{\text{foc}}(0) \leq \max\{p_{\min}, p^{\text{foc}}(0)\} \leq p^{\text{foc}}(\overline{p})$. From $p_{\max} \geq p^{\text{foc}}(0)$, we infer that $p_{\min} \leq p^{\text{foc}}(\tilde{c}) = \max\{p_{\min}, p^{\text{foc}}(0)\} \leq p_{\max}$. As a result, $p(\tilde{c}) = p^{\text{foc}}(\tilde{c})$ which contradicts that $p(c) \neq p^{\text{foc}}(c)$ for all $c \in T_S = [0, \overline{p}]$.

Due to the monotonicity of $p^{\text{foc}}(\cdot)$ and the requirement that $p^{\text{foc}}(p_{\min}) - p_{\min} \geq 0$, the second inequality in statement (47), $p_{\min} > p^{\text{foc}}(\overline{p}) \geq p^{\text{foc}}(p_{\min})$, cannot be true. Therefore, the first inequality $p_{\max} < p^{\text{foc}}(0)$ holds, which implies that $p_{\max} < p^{\text{foc}}(c)$ for all $c \in T_S$ by the monotonicity of $p^{\text{foc}}(c)$ in $c \in T_S$. We conclude that all prices cluster at $p_{\max}$, i.e., $p(c) = p_{\max}$ for all $c \in T_S$, noting that the profit of a supplier with cost c is increasing in the price p for $p < p^{\text{foc}}(c)$ by the unimodality of the profit function, established in Lemma 11.

F.2. Centralized case

Using the shorthand $z = \|\bar{N}_S\|$, we will show that in this case, the following piecewise polynomial system comprised of Q_1 and Q_2 holds:

$$\begin{aligned}
Q_1(\overline{\omega}, z) &= \left(\frac{1}{\mu_S} - 1\right) \cdot \mu_S z \cdot \min\left\{1, \frac{\lambda_C(\overline{\omega} + \epsilon - p_{\max})}{\mu_C z}\right\} \cdot \left(\begin{cases} \frac{\overline{\omega} - p_{\max}}{\overline{\omega} + \epsilon - p_{\max}} & \text{if } \; p_{\max} < \overline{\omega} - \epsilon \\ \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon} & \text{if } \; p_{\max} \geq \overline{\omega} - \epsilon \end{cases} \right) \\
&\quad + \mu_S z - \lambda_S p_{\max} = 0 \ , \\
Q_2(\overline{\omega}, z) &= \left(\frac{1}{\mu_C} - 1\right) \cdot (\overline{\omega} + \epsilon - p_{\max}) \\
&\quad \cdot \min\left\{1, \frac{\mu_C z}{\lambda_C(\overline{\omega} + \epsilon - p_{\max})}\right\} \cdot \left(\begin{cases} \frac{\overline{\omega} - p_{\max}}{\overline{\omega} + \epsilon - p_{\max}} & \text{if } \; p_{\max} < \overline{\omega} - \epsilon \\ \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon} & \text{if } \; p_{\max} \geq \overline{\omega} - \epsilon \end{cases} \right) \\
&\quad + \overline{\omega} - 1 = 0 \ .
\end{aligned} \tag{48}$$

Uniqueness of the $(\overline{\omega}, z)$-parametrization. With Lemma 1, we argue that the equilibrium is uniquely determined by the mass of suppliers $\|\bar{N}_S\|$ and $\overline{\omega}$. By Lemma 1, we know that $p(c) = p_{\max}$ for all $c \in T_S$. Since all prices are equal to $p_{\max}$, it suffices to determine the mass of suppliers $\|\bar{N}_S\|$ to describe the equilibrium strategies on the supplier side. As for the customer side, we know that $p(0) = \underline{\omega} + \epsilon$ due to the entry condition, which implies that $\underline{\omega} = p_{\max} - \epsilon$. Proposition 3 states that the customers' willingness-to-pay has density λ_C/μ_C for any $\omega \in [\underline{\omega}, \overline{\omega}] = [p_{\max} - \epsilon, \overline{\omega}]$. Therefore, the distribution of strategies on the customer side is fully determined by the knowledge of $\overline{\omega}$.

Derivation of $Q_1(\overline{\omega}, z)$. To derive the first piecewise polynomial expression, we leverage equation (9), which determines the mass of customers in the system. Although this equation was derived for the case $c_0 \neq -\infty$, it continues to hold when $c_0 = -\infty$. In this context, we have seen that prices cluster at $p_{\max}$ by Lemma 1, and thus, the next claim shows that $Q_1(\overline{\omega}, z) = 0$. The proof follows from equation (9); we defer it to the end of this section.

CLAIM 3. *For any stationary equilibrium such that $p(c) = p_{\max}$ for all $c \in T_S$, we have:*

$$\left(\frac{1}{\mu_C} - 1\right) \cdot (\overline{\omega} + \epsilon - p_{\max}) \cdot \min\left\{1, \frac{\mu_C z}{\lambda_C(\overline{\omega} + \epsilon - p_{\max})}\right\} \cdot \left(\begin{cases} \frac{\overline{\omega} - p_{\max}}{\overline{\omega} + \epsilon - p_{\max}} & \text{if } \; p_{\max} < \overline{\omega} - \epsilon \\ \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon} & \text{if } \; p_{\max} \geq \overline{\omega} - \epsilon \end{cases} \right)$$
$$+ \overline{\omega} - 1 = 0.$$

Derivation of $Q_2(\overline{\omega}, z)$. By leveraging the fact that all prices cluster at $p_{\max}$, we characterize the total mass of suppliers $\|\bar{N}_S\|$. The following claim is proved at the end of this section.

CLAIM 4. *For any stationary equilibrium such that $p(c) = p_{\max} \in [0, 1+\epsilon]$ for all $c \in T_S$, the total mass of suppliers $\|\bar{N}_S\|$ can be expressed as follows:*

$$\|\bar{N}_S\| = \frac{\lambda_S \cdot p_{\max}}{(1 - \mu_S)\phi_S(p_{\max}, \tau, N) + \mu_S}.$$

Following Claim 4, it remains to express $\phi_S(p_{\max}, \tau, N)$ as a function of $(\overline{\omega}, z)$. Using the fact that $\|\bar{N}_C\| = (\overline{\omega} - \underline{\omega}) \cdot \lambda_C / \mu_C$ by Proposition 3, and the entry condition $\underline{\omega} = p_{\max} - \epsilon$ as all prices cluster at $p_{\max}$, we invoke equations (41) and (44) in Appendix E.3 to derive the following:

$$\begin{aligned}
\phi_S(p_{\max}, \tau, N) &= \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \Pr_{\boldsymbol{v}, \boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\max}\right] \\
&= \begin{cases} \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{\overline{\omega} - p_{\max}}{\overline{\omega} - p_{\max} + \epsilon}, & \text{if} \quad \underline{\omega} + \epsilon \leq p_{\max} < \overline{\omega} - \epsilon, \\ \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon}, & \text{if} \quad \overline{\omega} - \epsilon \leq p_{\max} < \overline{\omega} + \epsilon, \\ 0, & \text{otherwise,} \end{cases} \\
&= \begin{cases} \min\left\{1, \frac{\lambda_C(\overline{\omega} - p_{\max} + \epsilon)}{\mu_C \cdot \|\bar{N}_S\|}\right\} \cdot \frac{\overline{\omega} - p_{\max}}{\overline{\omega} - p_{\max} + \epsilon}, & \text{if} \quad \underline{\omega} + \epsilon \leq p_{\max} < \overline{\omega} - \epsilon, \\ \min\left\{1, \frac{\lambda_C(\overline{\omega} - p_{\max} + \epsilon)}{\mu_C \cdot \|\bar{N}_S\|}\right\} \cdot \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon}, & \text{if} \quad \overline{\omega} - \epsilon \leq p_{\max} < \overline{\omega} + \epsilon, \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}$$

By plugging this equation into Claim 4, while noting that the third case above is ruled out since $\underline{\omega} + \epsilon = \underline{p} = p_{\max}$, we obtain the desired piecewise polynomial equation:

$$\begin{aligned}
0 &= (1 - \mu_S) \cdot z \cdot \phi_S(p_{\max}, \tau, N) + \mu_S \cdot z - \lambda_S \cdot p_{\max} \\
&= (1 - \mu_S) \cdot z \cdot \min\left\{1, \frac{\lambda_C(\overline{\omega} + \epsilon - p_{\max})}{\mu_C z}\right\} \cdot \left(\begin{cases} \frac{\overline{\omega} - p_{\max}}{\overline{\omega} + \epsilon - p_{\max}} & \text{if } \; p_{\max} < \overline{\omega} - \epsilon \\ \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon} & \text{if } \; p_{\max} \geq \overline{\omega} - \epsilon \end{cases} \right) \\
&\quad + \mu_S z - \lambda_S p_{\max} \\
&= Q_2(\overline{\omega}, z) \; .
\end{aligned}$$

Proof of Claim 3. By Proposition 3, we now that $\|\bar{N}_C\| = (\overline{\omega} - \underline{\omega}) \cdot \lambda_C/\mu_C$. we begin by deriving $\phi_S(p_{\max}, \tau, N) = \min\left\{1, \|\bar{N}_C\|/\|\bar{N}_S\|\right\} \cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p_{\max}]$ by equations (41) and (44), and using the equality $\underline{\omega} = p_{\max} - \epsilon$, which is due to all prices clustering at $p_{\max}$:

$$\begin{aligned}
&\min\left\{1, \frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\} \cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p_{\max}], \\
&= \begin{cases} \min\left\{1, \frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\} \cdot \frac{\overline{\omega} - p_{\max}}{\overline{\omega} - p_{\max} + \epsilon} & \text{if} \quad \underline{\omega} + \epsilon \le p_{\max} < \overline{\omega} - \epsilon, \\ \min\left\{1, \frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\} \cdot \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon}, & \text{if} \quad \overline{\omega} - \epsilon \le p_{\max} < \overline{\omega} + \epsilon, \\ 0, & \text{otherwise,} \end{cases} \\
&= \begin{cases} \min\left\{1, \frac{\mu_C \cdot \|\bar{N}_S\|}{\lambda_C(\overline{\omega} - p_{\max} + \epsilon)}\right\} \cdot \frac{\overline{\omega} - p_{\max}}{\overline{\omega} - p_{\max} + \epsilon}, & \text{if} \quad \underline{\omega} + \epsilon \le p_{\max} < \overline{\omega} - \epsilon, \\ \min\left\{1, \frac{\mu_C \cdot \|\bar{N}_S\|}{\lambda_C(\overline{\omega} - p_{\max} + \epsilon)}\right\} \cdot \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon}, & \text{if} \quad \overline{\omega} - \epsilon \le p_{\max} < \overline{\omega} + \epsilon, \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}$$

Substituting this into equation (8), we obtain the desired equality:

$$\begin{aligned}
&(\overline{\omega} - \underline{\omega}) \cdot \frac{(1 - \mu_C)m_C + \mu_C}{\mu_C} - (1 - \underline{\omega}) \\
&= (\overline{\omega} - p_{\max} + \epsilon) \cdot \frac{(1 - \mu_C)m_C + \mu_C}{\mu_C} - (1 - p_{\max} + \epsilon) \\
&= (\overline{\omega} - p_{\max} + \epsilon) \cdot \left(\frac{(1 - \mu_C)m_C}{\mu_C} + 1\right) - (1 - p_{\max} + \epsilon) \\
&= \left(\frac{1 - \mu_C}{\mu_C}\right) \cdot (\overline{\omega} + \epsilon - p_{\max}) \cdot \min\left\{1, \frac{\mu_C z}{\lambda_C(\overline{\omega} + \epsilon - p_{\max})}\right\} \cdot \left(\begin{cases} \frac{\overline{\omega} - p_{\max}}{\overline{\omega} + \epsilon - p_{\max}} & \text{if} \;\; p_{\max} < \overline{\omega} - \epsilon, \\ \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon} & \text{if} \;\; p_{\max} \ge \overline{\omega} - \epsilon \end{cases}\right) \\
&\quad + \overline{\omega} - 1 = 0.
\end{aligned}$$

This completes the proof. ■

Proof of Claim 4. Since all prices are equal to $p_{\max}$, the probability of a successful match for any supplier can be written as follows:

$$\phi_S(p_{\max}, \tau, N) = \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p_{\max})].$$

Therefore, since the total mass of suppliers that enter the market per period is equal to $\lambda_S \cdot p_{\max}$ (as $T_S = [0, p_{\max}]$), the total mass of suppliers in equilibrium can be written as follows:

$$\|\bar{N}_S\| = \lambda_S \cdot p_{\max} \cdot \sum_{t=0}^{\infty} ((1 - \mu_S)(1 - \phi_S(p_{\max}, \tau, N)))^t = \frac{\lambda_S \cdot p_{\max}}{(1 - \mu_S)\phi_S(p_{\max}, \tau, N) + \mu_S}.$$

This equation holds due to the influx of $\lambda_S \cdot p_{\max}$ suppliers offering price $p_{\max}$ at the start of every time period. Among those who enter, the fraction that persists into the next time period is given by $(1 - \phi_S(p_{\max}, \tau, N))(1 - \mu_S)$. ■

F.3. Proof of Lemma 2

We define the infimum cost of suppliers who pick price $p_{\max}$ as $c_1 = \inf\{c \in T_S : p(c) = p_{\max}\}$, where we assume that the infimum of an empty set is taken as ∞. Note that if $p_{\max} > \overline{p}$, i.e., $p_{\max} > \overline{\omega} + \epsilon$, the total mass of suppliers with price $p_{\max}$ is $\delta_S(p_{\max}) = 0$ simply because there exists no supplier with price $p_{\max}$

that enter the market. Hence, it suffices to prove the lemma for $p_{\max} \leq \overline{p}$, which implies that $c_1 < \infty$ and that there exists $c \in T_S$ such that $p(c) = p_{\max}$. With this, we write down the total mass of the suppliers with price $p_{\max}$ as

$$\delta_S(p_{\max}) = \int_{c_1}^{p_{\max}} \bar{N}_S(c)dc = (p_{\max} - c_1) \cdot \frac{\lambda_S}{(1-\mu_S)\phi_S(p_{\max},\tau,N) + \mu_S}. \tag{49}$$

To explain the equation (49), we first note that at the beginning of every time period $\lambda_S \cdot (p_{\max} - c_1)$ mass of suppliers with price $p_{\max}$ enter the market. Out of those who enter, a fraction survives into the following time period, which causes the total mass of suppliers at the stationary equilibrium to be all those accumulated throughout the horizon. Here, the fraction that survives into the following time period is given by $(1 - \phi_S(p_{\max}, \tau, N))(1 - \mu_S)$ that is equal to the denominator on the right-hand side of the equation, where we find it worthwhile to remark that all such suppliers accumulate at the same rate since they have the same match probability due to them having the same price $p_{\max}$.

We proceed by showing that for any stationary equilibrium such that there exist suppliers with $c \in T_S$ who pick the price that satisfies their first-order optimality conditions, i.e, $p(c) = p^{\text{foc}}(c)$, there also exists $c \in T_S$ such that $p(c) = p^{\text{foc}}(c) = p_{\max}$.

LEMMA 13. *For any stationary equilibrium such that there exists $c \in T_S$ satisfying the equality $p(c) = p^{\text{foc}}(c)$, there also exists $c \in T_S$ such that $p(c) = p^{\text{foc}}(c) = p_{\max}$.*

Proof. By the entry conditions, suppliers that enter the market have positive profits and we are investigating the case $p_{\max} \leq \bar{p}$. Therefore, since $p_{\max} \in T_S = [0, p_{\max})$, we have that $p^{\text{foc}}(p_{\max}) > p_{\max}$. We also know from the statement in the lemma that there exists $c \in T_S$ satisfying the equality $p(c) = p^{\text{foc}}(c) \in [p_{\min}, p_{\max}]$. By the last two statements and the continuity of $p^{\text{foc}}(c)$ for $c \in T_S$, which is a direct consequence of Proposition 4, we conclude using the intermediate value theorem that there exists $c \in T_S$ such that $p(c) = p^{\text{foc}}(c) = p_{\max}$. ■

By Lemma 13, we know that $p(c_1) = p^{\text{foc}}(c_1)$. Thus, we utilize the first-order optimality condition given by equation (37) to obtain the following equality:

$$p_{\max} - c_1 = -\frac{\phi_S(p_{\max},\tau,N)}{\phi'_S(p_{\max},\tau,N)} \cdot \left(\frac{(1-\mu_S)\phi_S(p_{\max},\tau,N) + \mu_S}{\mu_S}\right).$$

Plugging this equation into (49), we obtain the following simplified expression:

$$\delta_S(p_{\max}) = -\frac{\lambda_S}{\mu_S} \cdot \frac{\phi_S(p_{\max},\tau,N)}{\phi'_S(p_{\max},\tau,N)}.$$

The expression $\frac{\phi_S(p_{\max},\tau,N)}{\phi'_S(p_{\max},\tau,N)}$ takes a different form depending on whether or not $\epsilon > (\overline{\omega} - \underline{\omega})/2$. By combining equations (44) and (45), we first examine the case where $\epsilon > (\overline{\omega} - \underline{\omega})/2$ to find the total mass of suppliers with price $p_{\max}$ as

$$\delta_S(p_{\max}) = -\frac{\lambda_S}{\mu_S} \cdot \frac{\phi_S(p_{\max},\tau,N)}{\phi'_S(p_{\max},\tau,N)} = \frac{\lambda_S}{2\mu_S} \cdot (\overline{\omega} - p_{\max} + \epsilon).$$

When $\epsilon < (\overline{\omega} - \underline{\omega})/2$, the expression $\frac{\phi_S(p_{\max},\tau,N)}{\phi'_S(p_{\max},\tau,N)}$ has a jump discontinuity at $p_{\max} = \overline{\omega} - \epsilon$, preceding from the discontinuity in the density of suppliers in equilibrium, $f_S(y)$, given by Proposition 4. We combine the equations (41) and (42) to find the total mass of suppliers with price $p_{\max}$:

$$\delta_S(p_{\max}) = -\frac{\lambda_S}{\mu_S} \cdot \frac{\phi_S(p_{\max},\tau,N)}{\phi'_S(p_{\max},\tau,N)} = \begin{cases} \frac{\lambda_S}{\mu_S} \cdot (\overline{\omega} - p_{\max}), & \text{if } \ \underline{\omega} + \epsilon < p_{\max} < \overline{\omega} - \epsilon, \\ \frac{\lambda_S}{2\mu_S} \cdot (\overline{\omega} - p_{\max} + \epsilon), & \text{if } \ \overline{\omega} - \epsilon \le p_{\max} < \overline{\omega} + \epsilon, \\ 0, & \text{if } \ \overline{\omega} + \epsilon \le p_{\max}. \end{cases}$$

F.4. Proof of Lemma 3

We decompose the probability of a successful match over the random variables $\boldsymbol{v}, \boldsymbol{c}, \boldsymbol{\eta}$ using the law of total probability as follows:

$$\begin{aligned} &\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p(\boldsymbol{c})\right] \\ &= \Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p_{\min}\right] \Pr_{\boldsymbol{c}}\left[p(\boldsymbol{c}) = p_{\min}\right] \\ &\quad + \Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \Pr_{\boldsymbol{c}}\left[p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \\ &\quad + \Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p_{\max}\right] \Pr_{\boldsymbol{c}}\left[p(\boldsymbol{c}) = p_{\max}\right]. \end{aligned} \tag{50}$$

To prove the desired claim, we express as a function of the variables x, y, z the conditional probabilities in equation (50) and the probabilities of sampling extremal prices, i.e., $\Pr_{\boldsymbol{c}}[p(\boldsymbol{c}) = p_{\min}]$ and $\Pr_{\boldsymbol{c}}[p(\boldsymbol{c}) = p_{\max}]$, as well as the residual probability $\Pr_{\boldsymbol{c}}[p_{\min} < p(\boldsymbol{c}) < p_{\max}]$.

To this end, we observe that the agents being chosen uniformly randomly for a match, $\Pr_{\boldsymbol{c}}[p(\boldsymbol{c}) = p_{\max}]$ is equal to the ratio of the mass of suppliers $\delta_S(p_{\max})$ with price $p_{\max}$, which is given by Lemma 2, to the total mass of suppliers $\|\bar{N}_S\|$. By a similar argument, we have $\Pr_{\boldsymbol{c}}[p(\boldsymbol{c}) = p_{\min}] = \delta_S(p_{\min})/\|\bar{N}_S\|$. Finally, the residual probability $\Pr_{\boldsymbol{c}}[p_{\min} < p(\boldsymbol{c}) < p_{\max}]$ is the ratio between the total mass of suppliers choosing a price $p(\boldsymbol{c}) \in (p_{\min}, p_{\max})$ to $\|\bar{N}_S\|$. The former quantity is obtained by integrating the densities of Proposition 4 over the interval of prices $(p_{\min}, p_{\max})$. To simplify the notation, let $\underline{p} = \underline{\omega} + \epsilon$, $B = \min\{p_{\max}, \overline{\omega} - \epsilon\}$ and $\overline{p} = \min\{p_{\max}, \overline{\omega} + \epsilon\}$. Recall that $\overline{p}, \underline{p}$ are the (endogenous) boundaries of the price interval. Then, the desired mass of suppliers with price $p(\boldsymbol{c}) \in (p_{\min}, p_{\max})$ is given by:

$$\begin{aligned} &\frac{\lambda_S}{\mu_S}\left(2(B - \underline{p}) + \frac{3}{2}(\overline{p} - B)\right) \\ &= \frac{\lambda_S}{2\mu_S}(3\overline{p} + B - 4\underline{p}) \\ &= \begin{cases} \frac{\lambda_S}{2\mu_S} \cdot (3\min\{x,y\} + x - 2\epsilon), & \text{if } \ 0 < x - 2\epsilon < y, \\ \frac{3\lambda_S}{2\mu_S} \cdot \min\{x,y\}, & \text{if } \ x - 2\epsilon \le 0 < y, \\ \frac{2\lambda_S}{\mu_S} y, & \text{if } \ y \le x - 2\epsilon. \end{cases} \end{aligned} \tag{51}$$

Consequently, we express $\delta_S(p_{\max})$ and $\|\bar{N}_S\|$ as a function of x, y, z. By Lemma 2, the point mass $\delta_S(p_{\max})$ can be written with respect to x and y:

$$\delta_S(p_{\max}) = \begin{cases} \frac{\lambda_S}{2\mu_S} \cdot (x - \min\{x,y\}), & \text{if } \ x - 2\epsilon < y, \\ \frac{\lambda_S}{\mu_S} \cdot (x - y - \epsilon), & \text{if } \ y \le x - 2\epsilon. \end{cases} \tag{52}$$

We proceed by calculating $\|\bar{N}_S\|$. Combining equations (51) and (52), we obtain the following expression for $\|\bar{N}_S\|$:

$$\|\bar{N}_S\| = \begin{cases} z + \frac{\lambda_S}{\mu_S} \cdot (x + \min\{x,y\} - \epsilon), & \text{if } \ 2\epsilon < x, \\ z + \frac{\lambda_S}{2\mu_S} \cdot (x + 2\min\{x,y\}), & \text{if } \ 2\epsilon \ge x, \end{cases}$$

$$= \begin{cases} \frac{\mu_S z + \lambda_S (x + \min\{x,y\} - \epsilon)}{\mu_S}, & \text{if } 2\epsilon < x, \\ \frac{2\mu_S z + \lambda_S (x + 2\min\{x,y\})}{2\mu_S}, & \text{if } 2\epsilon \geq x. \end{cases} \tag{53}$$

Hence, it remains to compute the conditional probabilities appearing in equation (50): $\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\min}]$, $\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\max}]$, and $\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}]$. Since $p_{\min} \leq \underline{\omega} + \epsilon$, we know by equations (41) and (44) that

$$\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\min}] = \begin{cases} \frac{x-\epsilon}{x}, & \text{if } p_{\min} = \underline{\omega} + \epsilon \text{ and } \epsilon < x/2, \\ \frac{x}{4\epsilon}, & \text{if } p_{\min} = \underline{\omega} + \epsilon \text{ and } \epsilon \geq x/2, \\ 1, & \text{if } p_{\min} < \underline{\omega} + \epsilon. \end{cases}$$

and that

$$\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\max}] = \begin{cases} \frac{x-y-\epsilon}{x}, & \text{if } y < x - 2\epsilon, \\ \frac{(x-y)^2}{4\epsilon x}, & \text{if } x - 2\epsilon \leq y < x, \\ 0, & \text{if } x \leq y. \end{cases}$$

Combining these expressions with equations (52) and (53), we obtain:

$$\begin{aligned} &\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\max}] \cdot \Pr_{\boldsymbol{c}}[p(\boldsymbol{c}) = p_{\max}] \\ &= \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\max}] \cdot \frac{\delta_S(p_{\max})}{\|\bar{N}_S\|} \\ &= \begin{cases} \frac{\lambda_S (x-y-\epsilon)^2}{x(\mu_S z + \lambda_S (x+y-\epsilon))}, & \text{if } y < x - 2\epsilon, \\ \frac{\lambda_S (x - \min\{x,y\})^3}{8\epsilon x(\mu_S z + \lambda_S (x + \min\{x,y\} - \epsilon))}, & \text{if } 0 < x - 2\epsilon \leq y, \\ \frac{\lambda_S (x - \min\{x,y\})^3}{4\epsilon x(2\mu_S z + \lambda_S (x + 2\min\{x,y\}))}, & \text{if } x - 2\epsilon \leq 0 < y. \end{cases} \end{aligned} \tag{54}$$

Similarly, using equation (53) and the fact that $z = \delta_S(p_{\min}) = 0$ if and only if $p_{\min} < \underline{\omega} + \epsilon$, we obtain the folowing:

$$\begin{aligned} &\Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\min}] \cdot \Pr_{\boldsymbol{c}}[p(\boldsymbol{c}) = p_{\min}] \\ &= \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p_{\min}] \cdot \frac{\delta_S(p_{\min})}{\|\bar{N}_S\|} \\ &= \begin{cases} \frac{(x-\epsilon)\mu_S z}{x(\mu_S z + \lambda_S (x + \min\{x,y\} - \epsilon))}, & \text{if } p_{\min} = \underline{\omega} + \epsilon \text{ and } \epsilon < x/2, \\ \frac{x \mu_S z}{2\epsilon(2\mu_S z + \lambda_S (x + 2\min\{x,y\}))}, & \text{if } p_{\min} = \underline{\omega} + \epsilon \text{ and } \epsilon \geq x/2, \\ 0, & \text{if } p_{\min} < \underline{\omega} + \epsilon, \end{cases} \\ &= \begin{cases} \frac{(x-\epsilon)\mu_S z}{x(\mu_S z + \lambda_S (x + \min\{x,y\} - \epsilon))}, & \text{if } \epsilon < x/2, \\ \frac{x \mu_S z}{2\epsilon(2\mu_S z + \lambda_S (x + 2\min\{x,y\}))}, & \text{if } \epsilon \geq x/2. \end{cases} \end{aligned} \tag{55}$$

To fully express the right-hand side of (50) as a function of the variables x, y, z, it remains to compute the conditional success probabilities of matching $\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}]$, which is accomplished by the next lemma.

LEMMA 14. *The success probability of a match conditional to $p(\boldsymbol{c}) \in (p_{\min}, p_{\max})$ is*

$$\begin{aligned} &\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}] = \\ &\begin{cases} \frac{(x - \min\{x,y\})^3 - 8\epsilon(x-\epsilon)^2}{4\epsilon x(2\epsilon - x - 3\min\{x,y\})}, & \text{if } 0 < x - 2\epsilon < y, \\ \frac{2x - y - 2\epsilon}{2x}, & \text{if } 0 < y \leq x - 2\epsilon, \\ \frac{3x^2 - 3x\min\{x,y\} + \min\{x,y\}^2}{12x\epsilon}, & \text{if } x - 2\epsilon < 0 \leq y. \end{cases} \end{aligned}$$

The proof is deferred to the end of this subsection. We are now ready to conclude by examining each case in Lemma 14 separately.

Case 1: $0 < x - 2\epsilon < y$. By combining equations (51) and (53) with Lemma 14, we derive

$$\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \Pr_{\boldsymbol{c}}\left[p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \\
&= \frac{(x - \min\{x,y\})^3 - 8\epsilon(x-\epsilon)^2}{4\epsilon x(2\epsilon - x - 3\min\{x,y\})} \cdot \frac{\lambda_S}{2\mu_S}(3\min\{x,y\} + x - 2\epsilon) \cdot \frac{\mu_S}{\mu_S z + \lambda_S(x + \min\{x,y\} - \epsilon)} \\
&= \frac{\lambda_S(8\epsilon(x-\epsilon)^2 - (x - \min\{x,y\})^3)}{8\epsilon x(\mu_S z + \lambda_S(x + \min\{x,y\} - \epsilon))}.
\end{aligned}$$

Now, we plug this equation along with (54) and (55) into (50) to obtain the success probability of a random match:

$$\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c})\right] \\
&= \frac{\lambda_S(8\epsilon(x-\epsilon)^2 - (x - \min\{x,y\})^3 + (x - \min\{x,y\})^3)}{8\epsilon x(\mu_S z + \lambda_S(x + \min\{x,y\} - \epsilon))} + \frac{(x-\epsilon)\mu_S z}{x(\mu_S z + \lambda_S(x + \min\{x,y\} - \epsilon))} \\
&= \frac{(x-\epsilon)(\lambda_S(x-\epsilon) + \mu_S z)}{x(\mu_S z + \lambda_S(x + \min\{x,y\} - \epsilon))}.
\end{aligned}$$

Case 2: $0 \leq y \leq x - 2\epsilon$. Similarly, by combining equations (51) and (53) with Lemma 14:

$$\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \cdot \Pr_{\boldsymbol{c}}\left[p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \\
&= \frac{2x - y - 2\epsilon}{2x} \cdot \frac{2\lambda_S}{\mu_S} y \cdot \frac{\mu_S}{\mu_S z + \lambda_S(x + y - \epsilon)} \\
&= \frac{\lambda_S(2x - y - 2\epsilon)y}{x(\mu_S z + \lambda_S(x + y - \epsilon))}.
\end{aligned}$$

Plugging this equation along with (54) and (55) into (50), we have:

$$\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c})\right] \\
&= \frac{\lambda_S(x - y - \epsilon)^2 + \lambda_S(2xy - y^2 - 2\epsilon y)}{x(\mu_S z + \lambda_S(x + y - \epsilon))} + \frac{(x-\epsilon)\mu_S z}{x(\mu_S z + \lambda_S(x + y - \epsilon))} \\
&= \frac{\lambda_S(x^2 + y^2 + \epsilon^2 - 2xy - 2\epsilon x + 2\epsilon y + 2xy - y^2 - 2\epsilon y) + (x-\epsilon)\mu_S z}{x(\lambda_S(x + y - \epsilon) + \mu_S z)} \\
&= \frac{\lambda_S(x^2 + \epsilon^2 - 2\epsilon x) + (x-\epsilon)\mu_S z}{x(\lambda_S(x + y - \epsilon) + \mu_S z)} \\
&= \frac{\lambda_S(x-\epsilon)^2 + (x-\epsilon)\mu_S z}{x(\lambda_S(x + y - \epsilon) + \mu_S z)} \\
&= \frac{(x-\epsilon)(\lambda_S(x-\epsilon) + \mu_S z)}{x(\lambda_S(x + y - \epsilon) + \mu_S z)}.
\end{aligned}$$

Case 3: : $x - 2\epsilon < 0 \leq y$. Finally, by combining equations (51) and (53) with Lemma 14, we obtain:

$$\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \Pr_{\boldsymbol{c}}\left[p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \\
&= \frac{3x^2 - 3x\min\{x,y\} + \min\{x,y\}^2}{12x\epsilon} \cdot \frac{3\lambda_S}{2\mu_S}\min\{x,y\} \cdot \frac{2\mu_S}{2\mu_S z + \lambda_S(x + 2\min\{x,y\})} \\
&= \frac{3\lambda_S(3x^2 - 3x\min\{x,y\} + \min\{x,y\}^2) \cdot \min\{x,y\}}{12x\epsilon(\mu_S z + \lambda_S(x + 2\min\{x,y\}))}.
\end{aligned}$$

Plugging this equation along with (54) and (55) into (50), we have:

$$\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c})\right]$$

$$
\begin{aligned}
&= \frac{3\lambda_S(3x^2 - 3x\min\{x,y\} + \min\{x,y\}^2)\cdot\min\{x,y\}}{12x\epsilon(2\mu_S z + \lambda_S(x + 2\min\{x,y\}))} + \frac{\lambda_S(x-\min\{x,y\})^3}{4\epsilon x(2\mu_S z + \lambda_S(x+2\min\{x,y\}))} \\
&\quad + \frac{x\mu_S z}{2\epsilon(2\mu_S z + \lambda_S(x + 2\min\{x,y\}))} \\
&= \frac{3\lambda_S x^3}{12x\epsilon(2\mu_S z + \lambda_S(x+2\min\{x,y\}))} + \frac{x\mu_S z}{2\epsilon(2\mu_S z + \lambda_S(x+2\min\{x,y\}))} \\
&= \frac{\lambda_S x^2 + 2x\mu_S z}{4\epsilon(2\mu_S z + \lambda_S(x + 2\min\{x,y\}))}.
\end{aligned}
$$

Proof of Lemma 14. Throughout the proof, we repeatedly refer to the following equation:

$$
\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \\
&= \frac{1}{2\epsilon}\cdot\frac{1}{\overline{\omega} - \underline{\omega}}\cdot\int_{-\epsilon}^{\epsilon}\int_{\underline{\omega}}^{\overline{\omega}} \Pr_{\boldsymbol{c}}\left[\omega + \eta \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] d\omega d\eta.
\end{aligned} \tag{56}
$$

Equation (56) directly follows from the law of total probability, using the fact that $\boldsymbol{v} - \tau(\boldsymbol{v})$ is uniformly distributed over $[\underline{\omega}, \overline{\omega}]$ by Proposition 3, and $\boldsymbol{\eta}$ is uniformly distributed over $[-\epsilon, \epsilon]$. Moreover, $\boldsymbol{v}$, $\boldsymbol{c}$, and $\boldsymbol{\eta}$ are mutually independent. To calculate the integral on the right-hand side of (56), we consider multiple cases depending on whether $\epsilon < (\overline{\omega} - \underline{\omega})/2$ and on where the value of $\overline{p}$ resides relative to $\underline{\omega}$ and $p_{\max}$.

Case 1: $\epsilon < (\bar{\omega} - \underline{\omega})/2$. It follows from Proposition 4 that

$$
\Pr_{\boldsymbol{c}}\left[u \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] = \begin{cases} 0, & \text{if } \ u < \underline{p}, \\ \frac{4(u - \underline{p})}{-4\underline{p} + B + 3\overline{p}}, & \text{if } \ \underline{p} \leq u < B, \\ \frac{-4\underline{p} + B + 3u}{-4\underline{p} + B + 3\overline{p}}, & \text{if } \ B \leq u < \overline{p}, \\ 1, & \text{otherwise.} \end{cases}
$$

Hence, we start calculating the integral in (56):

$$
\begin{aligned}
&\int_{-\epsilon}^{\epsilon}\int_{\underline{\omega}}^{\overline{\omega}} \Pr_{\boldsymbol{c}}\left[\omega + \eta \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] d\omega d\eta \\
&= \int_{-\epsilon}^{\epsilon}\left(\int_{\min\{p_{\max}-\eta,\overline{\omega}\}}^{\overline{\omega}} 1\cdot d\omega + \int_{\min\{B-\eta,\overline{\omega}\}}^{\min\{p_{\max}-\eta,\overline{\omega}\}} \frac{-4\underline{p} + B + 3(\omega+\eta)}{-4\underline{p} + B + 3\overline{p}} d\omega\right. \\
&\quad \left. + \int_{\min\{\underline{p}-\eta,\overline{\omega}\}}^{\min\{B-\eta,\overline{\omega}\}} \frac{4(\omega + \eta - \underline{p})}{-4\underline{p} + B + 3\overline{p}} d\omega\right) d\eta \\
&= \int_{-\epsilon}^{\epsilon}\left(\max\{\overline{\omega} - p_{\max} + \eta, 0\} + \int_{B-\eta}^{\min\{p_{\max}-\eta,\overline{\omega}\}} \frac{-4\underline{p} + B + 3(\omega+\eta)}{-4\underline{p} + B + 3\overline{p}} d\omega\right. \\
&\quad \left. + \int_{\underline{p}-\eta}^{B-\eta} \frac{4(\omega + \eta - \underline{p})}{-4\underline{p} + B + 3\overline{p}} d\omega\right) d\eta \\
&= \int_{-\epsilon}^{\epsilon}\left(\max\{\overline{\omega} - p_{\max} + \eta, 0\}\right. \\
&\quad + \frac{(-8\underline{p} + 2B)(\min\{p_{\max} - \eta, \overline{\omega}\} - B + \eta)}{-8\underline{p} + 2B + 6\overline{p}} + \frac{3((\min\{p_{\max}, \overline{\omega} + \eta\})^2 - B^2)}{-8\underline{p} + 2B + 6\overline{p}} \\
&\quad \left. + \frac{2(\underline{p} - B)^2}{-4\underline{p} + B + 3\overline{p}}\right) d\eta \\
&= \int_{-\epsilon}^{\epsilon}\Bigg(\max\{\overline{\omega} - p_{\max} + \eta, 0\}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{(-8\underline{p}+2B)(\min\{p_{\max},\overline{\omega}+\eta\}-B)}{-8\underline{p}+2B+6\overline{p}}+\frac{3(\min\{p_{\max},\overline{\omega}+\eta\}+B)(\min\{p_{\max},\overline{\omega}+\eta\}-B)}{-8\underline{p}+2B+6\overline{p}}\\
&+\left.\frac{2(\underline{p}-B)^2}{-4\underline{p}+B+3\overline{p}}\right)d\eta\\
=&\int_{-\epsilon}^{\epsilon}\Bigg(\max\{\overline{\omega}-p_{\max}+\eta,0\}\\
&+\frac{(-8\underline{p}+2B+3\min\{p_{\max},\overline{\omega}+\eta\}+3B)(\min\{p_{\max},\overline{\omega}+\eta\}-B)}{-8\underline{p}+2B+6\overline{p}}+\frac{2(\underline{p}-B)^2}{-4\underline{p}+B+3\overline{p}}\Bigg)d\eta\\
=&\int_{-\epsilon}^{\epsilon}\Bigg(\max\{\overline{\omega}-p_{\max}+\eta,0\}\\
&+\frac{(-8\underline{p}+5B+3\min\{p_{\max},\overline{\omega}+\eta\})\cdot(\min\{p_{\max},\overline{\omega}+\eta\}-B)}{-8\underline{p}+2B+6\overline{p}}\Bigg)d\eta+\frac{4\epsilon(\underline{p}-B)^2}{-4\underline{p}+B+3\overline{p}}\\
=&\int_{\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\}}^{\epsilon}\left(\overline{\omega}-p_{\max}+\eta+\frac{(-8\underline{p}+5B+3p_{\max})(p_{\max}-B)}{-8\underline{p}+2B+6\overline{p}}\right)d\eta\\
&+\int_{-\epsilon}^{\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\}}\frac{(-8\underline{p}+5B+3\overline{\omega}+3\eta)\cdot(\overline{\omega}+\eta-B)}{-8\underline{p}+2B+6\overline{p}}d\eta+\frac{4\epsilon(\underline{p}-B)^2}{-4\underline{p}+B+3\overline{p}}\\
=&\int_{\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\}}^{\epsilon}(\overline{\omega}-p_{\max}+\eta)d\eta\\
&+\left((\epsilon-\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\})\cdot\frac{(-8\underline{p}+5B+3p_{\max})(p_{\max}-B)}{-8\underline{p}+2B+6\overline{p}}\right)\\
&+\int_{-\epsilon}^{\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\}}\frac{(-8\underline{p}+5B+3\overline{\omega}+3\eta)\cdot(\overline{\omega}+\eta-B)}{-8\underline{p}+2B+6\overline{p}}d\eta+\frac{4\epsilon(\underline{p}-B)^2}{-4\underline{p}+B+3\overline{p}}. \qquad (57)
\end{aligned}
$$

To ease the exposition, we examine the expression on the right-hand side of (57) in three subcases.

Case 1.1: $\epsilon<(\overline{\omega}-\underline{\omega})/2$ *and* $p_{\max}>\overline{\omega}+\epsilon$*.* In this case, we have $\overline{p}=\overline{\omega}+\epsilon$, $B=\overline{\omega}-\epsilon$ and $\underline{p}=\underline{\omega}+\epsilon$ and $\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\}=\epsilon$. Thus, the expression in (57) is equal to:

$$
\begin{aligned}
&\int_{-\epsilon}^{\max\{\min\{p_{\max}-\overline{\omega},\epsilon\},-\epsilon\}}\frac{(-8\underline{p}+5B+3\overline{\omega}+3\eta)(\overline{\omega}+\eta-B)}{-8\underline{p}+2B+6\overline{p}}d\eta+\frac{4\epsilon(\underline{p}-B)^2}{-4\underline{p}+B+3\overline{p}}\\
&=\int_{-\epsilon}^{\epsilon}\frac{(8(\overline{\omega}-\underline{\omega})-13\epsilon+3\eta)\cdot(\epsilon+\eta)}{4(2(\overline{\omega}-\underline{\omega})-\epsilon)}d\eta+\frac{4\epsilon(\overline{\omega}-\underline{\omega}-2\epsilon)^2}{2(2(\overline{\omega}-\underline{\omega})-\epsilon)}\\
&=\frac{8\epsilon^2(2(\overline{\omega}-\underline{\omega})-3\epsilon)}{4(2(\overline{\omega}-\underline{\omega})-\epsilon)}+\frac{4\epsilon(\overline{\omega}-\underline{\omega}-2\epsilon)^2}{2(2(\overline{\omega}-\underline{\omega})-\epsilon)}\\
&=\frac{8\epsilon^2(2(\overline{\omega}-\underline{\omega})-3\epsilon)+8\epsilon((\overline{\omega}-\underline{\omega})^2-4\epsilon(\overline{\omega}-\underline{\omega})+4\epsilon^2)}{4(2(\overline{\omega}-\underline{\omega})-\epsilon)}\\
&=\frac{2\epsilon(\overline{\omega}-\underline{\omega}-\epsilon)^2}{2(\overline{\omega}-\underline{\omega})-\epsilon}.
\end{aligned}
$$

Hence, by using the change of variables $x=\overline{\omega}-\underline{\omega}$ and $y=p_{\max}-\underline{\omega}-\epsilon$ and by equation (56) we have the desired equality:

$$
\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v}-\tau(\boldsymbol{v})+\boldsymbol{\eta}\geq p(\boldsymbol{c})\mid p_{\min}<p(\boldsymbol{c})<p_{\max}]=\frac{(x-\epsilon)^2}{x(2x-\epsilon)}.
$$

Case 1.2: $\epsilon < (\overline{\omega} - \underline{\omega})/2$ *and* $\overline{\omega} - \epsilon \leq p_{\max} \leq \overline{\omega} + \epsilon$. In this case, we have $\overline{p} = p_{\max}$, $B = \overline{\omega} - \epsilon$, and $\underline{p} = \underline{\omega} + \epsilon$. Therefore, we obtain $\max\{\min\{p_{\max} - \overline{\omega}, \epsilon\}, -\epsilon\} = p_{\max} - \overline{\omega}$. Using the change of variables $x = \overline{\omega} - \underline{\omega}$ and $y = p_{\max} - \underline{\omega} - \epsilon$, one can simplify the expression in (57) as follows:

$$\begin{aligned}
&= \int_{p_{\max}-\overline{\omega}}^{\epsilon} (\overline{\omega} - p_{\max} + \eta) d\eta \\
&\quad + \left((\epsilon - p_{\max} + \overline{\omega}) \frac{(5(\overline{\omega} - \underline{\omega}) + 3(p_{\max} - \underline{\omega} - \epsilon) - 10\epsilon) \cdot (p_{\max} - \overline{\omega} + \epsilon)}{6y + 2x - 4\epsilon} \right) \\
&\quad + \int_{-\epsilon}^{p_{\max}-\overline{\omega}} \frac{(8(\overline{\omega} - \underline{\omega}) - 13\epsilon + 3\eta) \cdot (\epsilon + \eta)}{6y + 2x - 4\epsilon} d\eta + \frac{4\epsilon(\overline{\omega} - \underline{\omega} - 2\epsilon)^2}{3y + x - 2\epsilon} \\
&= \int_{y-x+\epsilon}^{\epsilon} (\overline{\omega} - p_{\max} + \eta) d\eta \\
&\quad + \left((x - y) \frac{(5x + 3y - 10\epsilon) \cdot (y - x + 2\epsilon)}{6y + 2x - 4\epsilon} \right) \\
&\quad + \int_{-\epsilon}^{y-x+\epsilon} \frac{(8x - 13\epsilon + 3\eta) \cdot (\epsilon + \eta)}{6y + 2x - 4\epsilon} d\eta + \frac{4\epsilon(x - 2\epsilon)^2}{3y + x - 2\epsilon} \\
&= \frac{(x - y)^2}{2} \\
&\quad + \left((x - y) \cdot \frac{(5x + 3y - 10\epsilon)(y - x + 2\epsilon)}{6y + 2x - 4\epsilon} \right) \\
&\quad + \frac{(3x + y - 6\epsilon)(x - y - 2\epsilon)^2}{6y + 2x - 4\epsilon} + \frac{4\epsilon(x - 2\epsilon)^2}{3y + x - 2\epsilon} \\
&= \frac{(x - y)^2}{2} \\
&\quad + \frac{(x - y - 2\epsilon)(6\epsilon^2 - x^2 + y^2 - \epsilon(x + 3y))}{3y + x - 2\epsilon} + \frac{4\epsilon(x - 2\epsilon)^2}{3y + x - 2\epsilon} \\
&= \frac{(2\epsilon - x - 3y)(x - y)^2 - 2(x - y - 2\epsilon)(6\epsilon^2 - x^2 + y^2 - \epsilon(x + 3y)) - 8\epsilon(x - 2\epsilon)^2}{2(2\epsilon - x - 3y)} \\
&= \frac{(x - y - 2x - 2y + 2\epsilon)(x - y)^2 - 8\epsilon((x - \epsilon) - \epsilon)^2}{2(2\epsilon - x - 3y)} \\
&\quad - \frac{2(x - y - 2\epsilon)(6\epsilon^2 - x^2 + y^2 - \epsilon(x + 3y))}{2(2\epsilon - x - 3y)} \\
&= \frac{(x - y)^3 - 8\epsilon(x - \epsilon)^2 - 2(x - y)^2(x + y - \epsilon) + 8\epsilon^2(2x - 3\epsilon)}{2(2\epsilon - x - 3y)} \\
&\quad - \frac{2(x - y - 2\epsilon)(6\epsilon^2 - x^2 + y^2 - \epsilon(x + 3y))}{2(2\epsilon - x - 3y)} \\
&= \frac{(x - y)^3 - 8\epsilon(x - \epsilon)^2 - 2(x - y)^2(x + y - \epsilon) + 8\epsilon^2(2x - 3\epsilon)}{2(2\epsilon - x - 3y)} \\
&\quad - \frac{2(-x^3 + x^2 y + x y^2 - y^3 + x^2\epsilon - 2xy\epsilon + y^2\epsilon + 8x\epsilon^2 - 12\epsilon^3)}{2(2\epsilon - x - 3y)} \\
&= \frac{(x - y)^3 - 8\epsilon(x - \epsilon)^2 - 2(x - y)^2(x + y - \epsilon) + 16x\epsilon^2 - 24\epsilon^3}{2(2\epsilon - x - 3y)} \\
&\quad - \frac{-2x^3 - 2x^2 y - 2xy^2 + 2y^3 - 2x^2\epsilon + 4xy\epsilon - 2y^2\epsilon - 16x\epsilon^2 + 24\epsilon^3}{2(2\epsilon - x - 3y)} \\
&= \frac{(x - y)^3 - 8\epsilon(x - \epsilon)^2 - 2(x^2 - 2xy + y^2)(x + y - \epsilon)}{2(2\epsilon - x - 3y)}
\end{aligned}$$

$$
\begin{aligned}
&\quad - \frac{-2x^3 - 2x^2y - 2xy^2 + 2y^3 - 2x^2\epsilon + 4xy\epsilon - 2y^2\epsilon}{2(2\epsilon - x - 3y)} \\
&= \frac{(x-y)^3 - 8\epsilon(x-\epsilon)^2}{2(2\epsilon - x - 3y)} \\
&\quad + \frac{-2(x^3 + x^2y + xy^2 - y^3 + x^2\epsilon - 2xy\epsilon + y^2\epsilon)}{2(2\epsilon - x - 3y)} \\
&\quad - \frac{-2x^3 - 2x^2y - 2xy^2 + 2y^3 - 2x^2\epsilon + 4xy\epsilon - 2y^2\epsilon}{2(2\epsilon - x - 3y)} \\
&= \frac{(x-y)^3 - 8\epsilon(x-\epsilon)^2}{2(2\epsilon - x - 3y)}.
\end{aligned}
$$

Therefore, by equation (56), we have the desired equality:

$$
\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] = \frac{(x-y)^3 - 8\epsilon(x-\epsilon)^2}{4\epsilon x(2\epsilon - x - 3y)}.
$$

Case 1.3: $\epsilon < (\overline{\omega} - \underline{\omega})/2$ *and* $p_{\max} \leq \overline{\omega} - \epsilon$. In this case, we have $\overline{p} = p_{\max}$, $B = p_{\max}$ and $\underline{p} = \underline{\omega} + \epsilon$. In addition, we have $\max\{\min\{p_{\max} - \overline{\omega}, \epsilon\}, -\epsilon\} = -\epsilon$. Using the change of variables $x = \overline{\omega} - \underline{\omega}$ and $y = p_{\max} - \underline{\omega} - \epsilon$, one can simplify the expression in (57) as follows:

$$
\begin{aligned}
&\int_{-\epsilon}^{\epsilon} (\overline{\omega} - p_{\max} + \eta) d\eta + \frac{4\epsilon(\underline{p} - B)^2}{-4\underline{p} + B + 3\overline{p}} \\
&= 2\epsilon(\overline{\omega} - p_{\max}) - \frac{4\epsilon(\underline{\omega} + \epsilon - p_{\max})^2}{4(\underline{\omega} + \epsilon - p_{\max})} \\
&= 2\epsilon(\overline{\omega} - p_{\max}) - \epsilon(\underline{\omega} + \epsilon - p_{\max}) \\
&= \epsilon(2\overline{\omega} - p_{\max} - \underline{\omega} - \epsilon) \\
&= \epsilon(2x - y - 2\epsilon).
\end{aligned}
$$

Hence, by equation (56), we have the desired equality:

$$
\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) \mid p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] = \frac{2x - y - 2\epsilon}{2x}.
$$

Case 2: $\epsilon \geq (\bar{\omega} - \underline{\omega})/2$. In this case, it follows from Proposition 4 that

$$
\Pr_{\boldsymbol{c}}\left[u \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] = \begin{cases} 0, & u < \underline{p}, \\ \frac{u - \underline{p}}{\overline{p} - \underline{p}}, & \underline{p} \leq u < \overline{p}, \\ 1, & \text{otherwise.} \end{cases}.
$$

Moreover, the distribution of the random variable $\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta}$ has the following probability density function:

$$
f_{\omega+\eta}(x) = \begin{cases} \frac{x - \underline{\omega} + \epsilon}{2\epsilon(\overline{\omega} - \underline{\omega})}, & \text{if } \underline{\omega} - \epsilon < x \leq \overline{\omega} - \epsilon, \\ \frac{1}{2\epsilon}, & \text{if } \overline{\omega} - \epsilon < x \leq \underline{\omega} + \epsilon, \\ \frac{\overline{\omega} + \epsilon - x}{2\epsilon(\overline{\omega} - \underline{\omega})}, & \text{if } \underline{\omega} + \epsilon < x \leq \overline{\omega} + \epsilon, \\ 0, & \text{otherwise.} \end{cases}
$$

Based on these observations, we calculate the integral in (56) as follows:

$$
\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] = \\
&= \int_{\underline{\omega}+\epsilon}^{\overline{\omega}+\epsilon} \Pr_{\boldsymbol{c}}\left[x \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \cdot f_{\omega+\eta}(x) dx \\
&= \int_{\underline{\omega}+\epsilon}^{\overline{p}} \Pr_{\boldsymbol{c}}\left[x \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \cdot f_{\omega+\eta}(x) dx
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{\overline{p}}^{\overline{\omega}+\epsilon} \Pr_{\boldsymbol{c}}\left[x \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \cdot f_{\omega+\eta}(x) dx \\
&= \int_{\underline{\omega}+\epsilon}^{\overline{p}} \frac{x-\underline{\omega}-\epsilon}{\overline{p}-\underline{\omega}-\epsilon} \cdot \frac{\overline{\omega}+\epsilon-x}{2\epsilon(\overline{\omega}-\underline{\omega})} dx + \int_{\overline{p}}^{\overline{\omega}+\epsilon} 1 \cdot \frac{\overline{\omega}+\epsilon-x}{2\epsilon(\overline{\omega}-\underline{\omega})} dx \\
&= \frac{(\overline{p}-\underline{\omega}-\epsilon)(3\overline{\omega}-\underline{\omega}-2\overline{p}+2\epsilon)}{12\epsilon(\overline{\omega}-\underline{\omega})} + \frac{(\overline{\omega}+\epsilon-\overline{p})^2}{4\epsilon(\overline{\omega}-\underline{\omega})}.
\end{aligned}
$$

By applying the change of variables $x = \overline{\omega} - \underline{\omega}$ and $y = p_{\max} - \underline{\omega} - \epsilon$, and noting that $\min\{x, y\} = \overline{p} - \underline{\omega} - \epsilon$, we obtain the desired equality:

$$
\begin{aligned}
&\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \geq p(\boldsymbol{c}) | p_{\min} < p(\boldsymbol{c}) < p_{\max}\right] \\
&= \frac{(\overline{p}-\underline{\omega}-\epsilon)(3\overline{\omega}-3\underline{\omega}-2(\overline{p}-\underline{\omega}-\epsilon))}{12\epsilon(\overline{\omega}-\underline{\omega})} + \frac{(\overline{\omega}-\underline{\omega}-(\overline{p}-\underline{\omega}-\epsilon))^2}{4\epsilon(\overline{\omega}-\underline{\omega})} \\
&= \frac{\min\{x,y\}(3x - 2\min\{x,y\})}{12\epsilon x} + \frac{(x-\min\{x,y\})^2}{4\epsilon x} \\
&= \frac{3x^2 - 3x\min\{x,y\} + \min\{x,y\}^2}{12\epsilon x}.
\end{aligned}
$$

This completes the proof. ■

F.5. Proof of Lemma 4

Since $p_{\min} = \underline{\omega} + \epsilon$ from the statement of the lemma and $p(0) = \underline{\omega} + \epsilon$, we conclude that there exists $c \in T_S$ such that $p(c) = p_{\min}$. With this, we write down the total mass of suppliers with price $p_{\min}$ as

$$
\delta_S(p_{\min}) = \int_0^{c_0} \bar{N}_S(c) dc = c_0 \cdot \frac{\lambda_S}{(1-\mu_S)\phi_S(p_{\min}, \tau, N) + \mu_S}. \tag{58}
$$

This stems from the influx of $\lambda_S \cdot c_0$ mass of suppliers offering price $p_{\min}$ at the beginning of every time period. Out of those who enter, a fraction survives into the following time period, which causes the total mass of suppliers at the steady-state to be all those accumulated throughout the horizon. Here, the fraction that survives into the following time period is given by $(1 - \phi_S(p_{\min}, \tau, N))(1 - \mu_S)$.

We proceed by showing that for any stationary equilibrium such that there exist suppliers with $c \in T_S$ who pick the price that satisfies their first-order optimality conditions, i.e, $p(c) = p^{\text{foc}}(c)$, there also exists $c \in T_S$ such that $p(c) = p^{\text{foc}}(c) = p_{\min}$.

Lemma 15. *For any stationary equilibrium such that there exists $c \in T_S$ satisfying the equality $p(c) = p^{\text{foc}}(c)$, there also exists $c \in T_S$ such that $p(c) = p^{\text{foc}}(c) = p_{\min}$.*

Proof. We take any $c \in T_S$ such that $p(c) = p_{\min}$. We begin by showing that for such c, we have that $p^{\text{foc}}(c) \leq p_{\min}$. To prove this by contradiction, suppose that $p^{\text{foc}}(c) > p_{\min}$. Since the profit of suppliers is decreasing in their price $p(c)$ for $p(c) > p^{\text{foc}}(c)$, this leads to a contradiction. Hence, we conclude that there exists $c \in T_S$ such that $p^{\text{foc}}(c) \leq p_{\min}$. We also know from the statement in the lemma that there exists $c \in T_S$ satisfying the equality $p(c) = p^{\text{foc}}(c) \in [p_{\min}, p_{\max}]$. By the last two statements and the continuity of $p^{\text{foc}}(c)$ for $c \in T_S$, which is a direct consequence of Proposition 4, we conclude using the intermediate value theorem that there exists $c \in T_S$ such that $p(c) = p^{\text{foc}}(c) = p_{\min}$. ■

By Lemma 15, we know that the suppliers with cost c_0 choose the price $p_{\min} = p^{\text{foc}}(c_0)$ that satisfies their first-order optimality condition given by (37). Therefore, we have the following equality by equation (37):

$$\underline{\omega} + \epsilon - c_0 = -\frac{\phi_S(\underline{\omega}+\epsilon,\tau,N)}{\phi_S'(\underline{\omega}+\epsilon,\tau,N)} \cdot \left(\frac{(1-\mu_S)\phi_S(\underline{\omega}+\epsilon,\tau,N)+\mu_S}{\mu_S}\right).$$

By substituting c_0 of this expression in the expression (58), we obtain the following equality:

$$\delta_S(p_{\min}) = \frac{(\underline{\omega}+\epsilon)\cdot\lambda_S}{(1-\mu_S)\phi_S(\underline{\omega}+\epsilon,\tau,N)+\mu_S} + \frac{\lambda_S}{\mu_S}\cdot\frac{\phi_S(\underline{\omega}+\epsilon,\tau,N)}{\phi_S'(\underline{\omega}+\epsilon,\tau,N)}.$$

F.6. Derivation of $P_2(x,y,z)=0$

We express equation (12) in terms of the variables x, y, and z. To this end, we note that by combining equations (42) and (45), we obtain the following:

$$\phi_S'(\underline{p},\tau) = \begin{cases} -\min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\frac{1}{x}, & \text{if } \ x > 2\epsilon, \\ -\min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\frac{1}{2\epsilon}, & \text{if } \ x \le 2\epsilon. \end{cases} \tag{59}$$

By equations (41) and (44), we obtain the following:

$$\phi_S(\underline{p},\tau) = \begin{cases} \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\frac{x-\epsilon}{x}, & \text{if } \ x > 2\epsilon, \\ \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\frac{x}{4\epsilon}, & \text{if } \ x \le 2\epsilon. \end{cases} \tag{60}$$

Hence, by combining equations (59) and (60), we obtain

$$\frac{\phi_S(\underline{p},\tau)}{\phi_S'(\underline{p},\tau)} = -\begin{cases} x-\epsilon, & \text{if } \ x > 2\epsilon, \\ \frac{x}{2}, & \text{if } \ x < 2\epsilon. \end{cases} \tag{61}$$

Finally, to express equation (12) in terms of the variables x, y, and z, we use equations (59), (60), and (61). In doing this, we remark that $\|\bar{N}_C\| = x\cdot(\lambda_C/\mu_C)$ as a direct consequence of Proposition 3. Finally, we use the characterization of $\|\bar{N}_S\|$ obtained in Lemma 3. We divide the exposition into two cases.

Case 1: $x > 2\epsilon$. In this case, equation (12) yields the following relationship:

$$\begin{aligned}
0 &= \left(\delta_S(p_{\min}) - \frac{\lambda_S}{\mu_S}\cdot\frac{\phi_S(\underline{p},\tau)}{\phi_S'(\underline{p},\tau)}\right)\left((1-\mu_S)\phi_S(\underline{p},\tau)+\mu_S\right) - \underline{p}\cdot\lambda_S \\
&= \phi_S(\underline{p},\tau)\left(\delta_S(p_{\min})(1-\mu_S) - \frac{\lambda_S(1-\mu_S)}{\mu_S}\cdot\frac{\phi_S(\underline{p},\tau)}{\phi_S'(\underline{p},\tau)}\right) + \mu_S\delta_S(p_{\min}) \\
&\quad - \lambda_S\left(\frac{\phi_S(\underline{p},\tau)}{\phi_S'(\underline{p},\tau)} + \underline{p}\right) \\
&= \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\}\cdot\frac{x-\epsilon}{x}\cdot\left(z(1-\mu_S) + \frac{\lambda_S(1-\mu_S)}{\mu_S}\cdot(x-\epsilon)\right) \\
&\quad + \mu_S z - \lambda_S(\epsilon - x + p_{\max} - y) \\
&= \min\left\{1, \frac{\lambda_C\mu_S x}{\mu_C(\mu_S z + \lambda_S(x+\min\{x,y\}-\epsilon))}\right\}\cdot\frac{(1-\mu_S)(x-\epsilon)(\mu_S z + \lambda_S(x-\epsilon))}{x\mu_S} \\
&\quad + \lambda_S(x-\epsilon) + \mu_S z - \lambda_S(p_{\max}-y) \\
&= \left(\frac{1}{\mu_S}-1\right)\cdot\min\left\{\frac{(x-\epsilon)(\mu_S z+\lambda_S(x-\epsilon))}{x}, \frac{\lambda_C\mu_S(x-\epsilon)(\mu_S z+\lambda_S(x-\epsilon))}{\mu_C(\mu_S z+\lambda_S(x+\min\{x,y\}-\epsilon))}\right\} \\
&\quad + \lambda_S(x-\epsilon) + \mu_S z - \lambda_S(p_{\max}-y) \\
&= P_2(x,y,z).
\end{aligned}$$

Case 2: $x \leq 2\epsilon$. In this case, equation (12) yields the following expression:

$$
\begin{aligned}
0 &= \left(\delta_S(p_{\min}) - \frac{\lambda_S}{\mu_S} \cdot \frac{\phi_S(\underline{p},\tau)}{\phi'_S(\underline{p},\tau)}\right)\left((1-\mu_S)\phi_S(\underline{p},\tau) + \mu_S\right) - \underline{p}\cdot\lambda_S \\
&= \phi_S(\underline{p},\tau)\left(\delta_S(p_{\min})(1-\mu_S) - \frac{\lambda_S(1-\mu_S)}{\mu_S} \cdot \frac{\phi_S(\underline{p},\tau)}{\phi'_S(\underline{p},\tau)}\right) + \mu_S\delta_S(p_{\min}) \\
&\quad - \lambda_S\left(\frac{\phi_S(\underline{p},\tau)}{\phi'_S(\underline{p},\tau)} + \underline{p}\right) \\
&= \min\left\{1, \frac{\|\bar{N}_C\|}{\|\bar{N}_S\|}\right\} \cdot \frac{x}{4\epsilon} \cdot \left(z(1-\mu_S) + \frac{\lambda_S(1-\mu_S)}{\mu_S} \cdot \frac{x}{2}\right) + \mu_S z - \lambda_S\left(-\frac{x}{2} + p_{\max} - y\right) \\
&= \min\left\{1, \frac{2\lambda_C\mu_S x}{\mu_C(2\mu_S z + \lambda_S(x + 2\min\{x,y\}))}\right\} \cdot \frac{(1-\mu_S)x(2\mu_S z + \lambda_S x)}{8\mu_S\epsilon} + \frac{\lambda_S x}{2} \\
&\quad + \mu_S z - \lambda_S(p_{\max} - y) \\
&= \left(\frac{1}{\mu_S} - 1\right) \cdot \min\left\{\frac{x(2\mu_S z + \lambda_S x)}{8\epsilon}, \frac{2x^2\lambda_C\mu_S(2\mu_S z + \lambda_S x)}{8\mu_C\epsilon(2\mu_S z + \lambda_S(x + 2\min\{x,y\}))}\right\} + \frac{\lambda_S x}{2} \\
&\quad + \mu_S z - \lambda_S(p_{\max} - y) \\
&= P_2(x,y,z).
\end{aligned}
$$

F.7. Existence and uniqueness of equilibrium: Decentralized setting

In Proposition 4, the distribution of suppliers' prices, $f_S(\cdot)$, is given as a function of $\underline{\omega}$ and $\overline{\omega}$. Thus, to show the uniqueness of the stationary equilibrium, it is sufficient to show the uniqueness of $\underline{\omega}, \overline{\omega}$ that satisfies the requirements of Definition 1. To this end, we first argue that the value of $\overline{\omega} - \underline{\omega}$ is identical across all stationary equilibria by comparing any two stationary equilibria $S_1 = (\tau_1, p_1, N_1)$ and $S_2 = (\tau_2, p_2, N_2)$; the proof appears at the end of this section.

LEMMA 16. *For any two stationary equilibria S_1 and S_2 such that $p_{\min} = 0$ and $p_{\max} = \infty$, the equality $\overline{\omega}(S_1) - \underline{\omega}(S_1) = \overline{\omega}(S_2) - \underline{\omega}(S_2)$ must hold.*

Next, we show that the value of $\underline{\omega}$ is uniquely determined by $\overline{\omega} - \underline{\omega}$ in every stationary equilibrium. Since the value of $\overline{\omega} - \underline{\omega}$ is constant across all stationary equilibria by Lemma 16, the latter claim implies that $\underline{\omega}$ is uniform across all stationary equilibria. As an immediate consequence, the value of $\overline{\omega}$ is also uniform across all stationary equilibria.

Now, to show that $\underline{\omega}$ is indeed uniquely determined by $\overline{\omega} - \underline{\omega}$ for every stationary equilibrium, we invoke the equation $P_2(x,y,z) = 0$. By the fact that $x < y$ and $z = 0$ for the decentralized platform (where $p_{\min} = 0$ and $p_{\max} = \infty$), we have the following relationship:

$$
\begin{aligned}
\underline{\omega} + \epsilon &= p_{\max} - y \qquad (62) \\
&= \left(\frac{1}{\mu_S} - 1\right) \cdot \left(\begin{cases} \min\left\{\frac{(x-\epsilon)^2}{x}, \frac{\lambda_C\mu_S(x-\epsilon)^2}{\mu_C\lambda_S(2x-\epsilon))}\right\} & \text{if } x > 2\epsilon \\ \min\left\{\frac{x^2}{8\epsilon}, \frac{x^2\lambda_C\mu_S}{12\mu_C\lambda_S\epsilon}\right\} & \text{if } x \leq 2\epsilon \end{cases}\right) + \left(\begin{cases} x - \epsilon & \text{if } x > 2\epsilon \\ \frac{x}{2} & \text{if } x \leq 2\epsilon \end{cases}\right).
\end{aligned}
$$

This equation expresses $\underline{\omega}$ as a function $x = \overline{\omega} - \underline{\omega}$, which implies that $\underline{\omega}$ and $\overline{\omega}$ are uniform across all stationary equilibria. Now, by Propositions 3 and 4, we know that any stationary equilibrium such that $p_{\min} = 0$ and $p_{\max} = \infty$ is fully characterized by the parameters $\underline{\omega}$ and $\overline{\omega}$. Therefore, we conclude that there can be at most one stationary equilibrium. It remains to show that a stationary equilibrium indeed exists, which is the point of the next lemma.

Lemma 17. *There exists a stationary equilibrium in the decentralized platform type.*

Proof of Lemma 16. We begin by showing the following monotonicity property of m_C, defined in (8) as the average match rate of a customer chosen uniformly randomly, which is instrumental in our proof.

Claim 5. *Given any two stationary equilibria S_1 and S_2 such that $p_{\min}=0$ and $p_{\max}=\infty$, $\overline{\omega}(S_1)-\underline{\omega}(S_1)>\overline{\omega}(S_2)-\underline{\omega}(S_2)$ if and only if $m_C(S_1)>m_C(S_2)$.*

Proof. We first use Lemma 3 to express m_C in terms of $x=\overline{\omega}-\underline{\omega}$. By the fact that $x<y$ and $z=0$ in the decentralized platform type (where $p_{\min}=0$ and $p_{\max}=\infty$), Lemma 3 yields the following relationship:

$$\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v}-\tau(\boldsymbol{v})+\boldsymbol{\eta}\geq p(\boldsymbol{c})]= \\ \begin{cases} \frac{(x-\epsilon)^2}{x(2x-\epsilon)}, & \text{if } x>2\epsilon, \\ \frac{x}{12\epsilon}, & \text{if } x\leq 2\epsilon. \end{cases}$$

This expression is clearly increasing in x since $\frac{(x-\epsilon)^2}{x(2x-\epsilon)}=\frac{1}{2}\left(1-\frac{\epsilon}{x}\right)\left(1-\frac{\epsilon}{2x-\epsilon}\right)$. Next, we show that $\min\left\{1,\|\bar{N}_S\|/\|\bar{N}_C\|\right\}$ is non-decreasing in x, which will complete the proof that $m_C=\Pr_{\boldsymbol{v},\boldsymbol{c},\boldsymbol{\eta}}[\boldsymbol{v}-\tau(\boldsymbol{v})+\boldsymbol{\eta}\geq p(\boldsymbol{c})]\cdot\min\left\{1,\|\bar{N}_S\|/\|\bar{N}_C\|\right\}$ is increasing x. To this end, we note that $\|\bar{N}_C\|=x\cdot\lambda_C/\mu_C$, which is a direct consequence of Proposition 3. By Lemma 3, we also obtain $\|\bar{N}_S\|$ in terms of x. Accordingly, since $x<y$ and $z=0$, Lemma 3 yields the following relationship:

$$\|\bar{N}_S\|=\begin{cases} \frac{\lambda_S(2x-\epsilon)}{\mu_S}, & \text{if } x>2\epsilon, \\ \frac{3\lambda_S x}{2\mu_S}, & \text{if } x\leq 2\epsilon. \end{cases}$$

Therefore, we express $\min\left\{1,\|\bar{N}_S\|/\|\bar{N}_C\|\right\}$ with respect to x:

$$\min\left\{1,\frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\}=\begin{cases} \min\left\{1,\frac{\lambda_S\mu_C}{\mu_S\lambda_C}\left(2-\frac{\epsilon}{x}\right)\right\}, & \text{if } \quad x>2\epsilon, \\ \min\left\{1,\frac{3\lambda_S\mu_C}{2\mu_S\lambda_C}\right\}, & \text{if } \quad x\leq 2\epsilon. \end{cases}$$

which is non-decreasing in x.

■

Next, we show that the type of customer in the market with the lowest valuation, $\underline{\omega}$, is an increasing function of $\overline{\omega}-\underline{\omega}$.

Claim 6. *Given any two stationary equilibria S_1 and S_2 such that $p_{\min}=0$ and $p_{\max}=\infty$, $\overline{\omega}(S_1)-\underline{\omega}(S_1)>\overline{\omega}(S_2)-\underline{\omega}(S_2)$ holds if and only $\underline{\omega}(S_1)>\underline{\omega}(S_2)$.*

Proof. We begin by using the equation $P_2(x,y,z)=0$ to express $\underline{\omega}=p_{\max}-y-\epsilon$ with respect to x. By the fact that $x<y$ and $z=0$ for the decentralized platform (where $p_{\min}=0$ and $p_{\max}=\infty$), the equation $P_2(x,y,z)=0$ yields the following relationship:

$$\begin{aligned} &\underline{\omega}+\epsilon=p_{\max}-y \\ &=\left(\frac{1}{\mu_S}-1\right)\cdot\left(\begin{cases} \min\left\{\frac{(x-\epsilon)^2}{x},\frac{\lambda_C\mu_S(x-\epsilon)^2}{\mu_C\lambda_S(2x-\epsilon))}\right\} & \text{if } x>2\epsilon \\ \min\left\{\frac{x^2}{8\epsilon},\frac{x^2\lambda_C\mu_S}{12\mu_C\lambda_S\epsilon}\right\} & \text{if } x\leq 2\epsilon \end{cases}\right)+\left(\begin{cases} x-\epsilon & \text{if } x>2\epsilon \\ \frac{x}{2} & \text{if } x\leq 2\epsilon \end{cases}\right). \end{aligned} \tag{63}$$

One can readily observe that this expression is increasing in x. This concludes the proof. ■

To conclude, we remind the reader that equation (9), which is restated subsequently, holds for any stationary equilibrium:

$$\frac{\lambda_C(\overline{\omega}-\underline{\omega})}{\mu_C}=\frac{\lambda_C(1-\underline{\omega})}{(1-\mu_C)m_C+\mu_C}\ .$$

On the one hand, Claims 5 and 6 reveal that the right-hand side expression is decreasing in $x=\overline{\omega}-\underline{\omega}$ over the set of stationary equilibria. On the other hand, it is straightforward that the left-hand side of this equation is increasing in $x=\overline{\omega}-\underline{\omega}$. Furthermore, at $x=0$, the left-hand side is equal to 0 while the right-hand side is positive. Therefore, we conclude that there exists a unique value $x\in\mathbb{R}^+$ value that satisfies equation (9) across all stationary equilibria. ■

Proof of Lemma 17. Initially, we show the existence of values $\underline{\omega},\overline{\omega}\in[-\epsilon,1]$ that satisfy the piecewise polynomial system. We first note that $P_2(x,y,z)=0$ implies that y can be expressed with respect to x for the decentralized platform (where $p_{\min}=0$ and $p_{\max}=\infty$) since $x<y$ and $z=0$ under the decentralized platform. By eliminating y using this relationship, we can express P_1 with respect to x only (since $z=0$), and focus only on the equation $P_1(x)=0$ to solve the piecewise polynomial system. We already know that there exists a unique $x=\overline{\omega}-\underline{\omega}\in\mathbb{R}^+$ value that satisfies equation (9) as shown in the last paragraph of the proof of Lemma 16. Since equation (9) is equivalent to the equation $P_1(x)=0$, we conclude that there exists a unique $x\in\mathbb{R}^+$ value that satisfies $P_1(x)=0$. Furthermore, as demonstrated by the equation $P_2(x)=0$, the value of $\underline{\omega}$ is uniquely identified by x. This is specified in equation (62). Therefore, we conclude that there exists a unique pair of $\underline{\omega},\overline{\omega}$ that fits the piecewise polynomial system specified by $P_1(x)=0$ and $P_2(x)=0$. We finally note that it can be seen from equation (62) that the inequality $\underline{\omega}>-\epsilon$ is satisfied and the next inequality, which is a direct consequence of equation (9), shows that $\underline{\omega},\overline{\omega}$ values that satisfy the piecewise polynomial system are such that $\underline{\omega}<\overline{\omega}<1$:

$$\underline{\omega}<\overline{\omega}=\frac{\mu_C(1-\underline{\omega})}{\mu_C+m_C(1-\mu_C)}+\underline{\omega}<1\ .$$

Recalling that $z=0$ for the decentralized platform, we conclude that $P_3(x,y,z)=0$ is also satisfied and there exists a unique pair of $\underline{\omega},\overline{\omega}\in[-\epsilon,1]$ that satisfy the piecewise polynomial system given by (7).

Next, we show that the distribution of strategies characterized by Propositions 3 and 4 with the unique pair of $\underline{\omega},\overline{\omega}$ of the piecewise polynomial system fulfills the conditions of stationary equilibrium. The stationary equilibrium requires that each supplier employs a best-response strategy while fixing the strategies in the rest of the market. To this end, take $\underline{\omega},\overline{\omega}$ as the unique pair that satisfies the piecewise polynomial system. Then, fix the distribution of customers' willingness-to-pay as described in Proposition 3. Note that the equation $P_1(x)=0$ in the piecewise polynomial system ensures that the first-order condition for the suppliers' profit function with cost $c=0$ is satisfied with $p^*(0)=\underline{\omega}+\epsilon$. By employing the proof from Proposition 4, we can demonstrate that with $\underline{\omega},\overline{\omega}$ fulfilling the requirements of the piecewise polynomial system, we can derive $p^*(c)$ for all values of c within T_S and that the resultant price distribution is aligned with Proposition 4. Knowing the initial point $p^*(0)=\underline{\omega}+\epsilon$ and $\frac{dp^*}{dc}(c)$ for the best-response prices will suffice to recover the whole distribution of best-response prices by the fundamental theorem of calculus. To show this, note that by equation (40) and the fact that $f_p(y,\tau) = \tilde{\lambda}_S(y,\tau)/\|\tilde{\lambda}_S\|$, we know that the arrival density of suppliers with price y, $\tilde{\lambda}_S(y,\tau)$, is equal to the following in any stationary equilibrium:

$$\tilde{\lambda}_S(y,\tau) = \lambda_S \cdot \left(\frac{1-\mu_S}{\mu_S} \cdot \phi_S(y,\tau,N) + 1\right) \cdot \left(2 - \frac{\phi_S(y,\tau,N)\phi_S''(y,\tau,N)}{(\phi_S'(y,\tau,N))^2}\right)$$

for all $y \in (\underline{p}, \bar{p})$. Using this equation and Lemma 10, we see that equation (39) that determines the derivative of any best response price $\frac{dp^*}{dc}(c)$, is met. Note that we also know that if the highest willingness-to-pay in the market is $\overline{\omega}$, the highest price in the market is $\overline{\omega} + \epsilon$. Thus, using $\underline{\omega}, \overline{\omega}$ obtained from solving our piecewise polynomial system, we recover the best-response prices $p^*(c)$ for all $c \in T_S$. Each resultant value of $p^*(c)$ for all $c \in T_S$ constitutes a best-response to the prices given by Proposition 4 and the willingness-to-pay distribution specified by Proposition 3.

Finally, we implement a similar approach for the demand side of the market. We take the unique pair of $\underline{\omega}, \overline{\omega}$ that satisfies the piecewise polynomial system and fix the distribution of suppliers' prices to $f_S(\cdot)$ as described in Proposition 4. We show that the distribution of customers' willingness-to-pay described in Proposition 3 with $\underline{\omega}, \overline{\omega}$ obtained from the piecewise polynomial system ensures that each customer type chooses a best response. Note that since $P_1(x,y,z) = 0$ ensures that $p^*(0) = \underline{\omega} + \epsilon$, we know that $\underline{\omega}$ of the piecewise polynomial system is indeed the lowest willingness-to-pay in the market. Using the proof of Proposition 3, by equation (33), we have that

$$\tilde{\lambda}_C(w,p) = \frac{\lambda_C}{\mu_C} \cdot \left[(1-\mu_C)\phi_C(\omega^{-1}(w), \tau(\omega^{-1}(w))) + \mu_C\right] .$$

Hence, by Lemma 10 and the fact that the arrival density of customers is λ_C, we obtain the equation (30) that describes that derivative of the willingness-to-pay, $\omega'(v) = 1 - \tau'(v)$. We know the derivative of the function $\omega(v)$ for all $v \in (\underline{v}, 1]$ and the value of the function at $\underline{\omega} = \underline{v}$. Therefore, we also recover $\omega(v)$ for all $v \in (\underline{v}, 1]$ by the fundamental theorem of calculus. We conclude that the recovered $\omega(v)$ values constitute best responses to the willingness-to-pay distribution characterized by Proposition 3 and the suppliers' price distribution described by Proposition 4. ∎

F.8. Existence and uniqueness of equilibrium: Centralized setting

Uniqueness. As shown in Section 5.3, any stationary equilibrium such that $p(c) = p_{\max} = p_{\min}$ for all $c \in T_S$ can be represented by the variable $\overline{\omega}$ and the total mass of suppliers $z = \|\bar{N}_S\|$ since the lowest willingness-to pay in the market is determined by $p_{\max}$, i.e., $\underline{\omega} = p_{\min} - \epsilon = p_{\max} - \epsilon$. In order to prove that z and $\overline{\omega}$ remain constant across all stationary equilibria such that $p(c) = p_{\max} \in [0, 1+\epsilon]$ for $c \in T_S$, we proceed by establishing two claims. The first claim shows that for any two stationary equilibria S_1 and S_2, the inequality $\overline{\omega}(S_1) > \overline{\omega}(S_2)$ holds if and only if $z(S_1) < z(S_2)$. In contrast, the second claim shows that the inequality $\overline{\omega}(S_1) > \overline{\omega}(S_2)$ holds if and only if $z(S_1) > z(S_2)$. By combining these two claims, we deduce that the equalities $\overline{\omega}(S_1) = \overline{\omega}(S_2)$ and $z(S_1) = z(S_2)$ must hold. We begin by showing that the first claim holds.

Claim 7. *For any two stationary equilibria S_1 and S_2 such that $p(c) = p_{\max} = p_{\min}$ for all $c \in T_S$, the inequality $\overline{\omega}(S_1) > \overline{\omega}(S_2)$ is true if and only if $z(S_1) < z(S_2)$.*

Proof. By the fact that $Q_2(\overline{\omega}, z) = 0$ in (48), which we restate subsequently, one can readily conclude that the desired property that z is decreasing in $\overline{\omega}$ must hold:

$$0 = (1-\mu_S) \cdot \left(\begin{cases} \min\left\{z, \frac{\lambda_C(\overline{\omega}-p_{\max}+\epsilon)}{\mu_C}\right\} \cdot \frac{\overline{\omega}-p_{\max}}{\overline{\omega}-p_{\max}+\epsilon} & \text{if} \quad \underline{\omega}+\epsilon \le p_{\max} \le \overline{\omega}-\epsilon, \\ \min\left\{z, \frac{\lambda_C(\overline{\omega}-p_{\max}+\epsilon)}{\mu_C}\right\} \cdot \frac{\overline{\omega}+\epsilon-p_{\max}}{4\epsilon} & \text{if} \quad \overline{\omega}-\epsilon \le p_{\max} \le \overline{\omega}+\epsilon, \\ 0 & \text{otherwise.} \end{cases} \right) + \mu_S z - \lambda_S p_{\max}. \quad (64)$$

This completes the proof. ■

Next, we show that our second claim outlined at the beginning of this subsection also holds.

Claim 8. *For any two stationary equilibria S_1 and S_2 such that $p(c) = p_{\max}$ for all $c \in T_S$, the inequality $\overline{\omega}(S_1) > \overline{\omega}(S_2)$ holds if and only if $z(S_1) > z(S_2)$.*

Proof. We begin by invoking Claim 4, which implies that the following equality holds:

$$z = \|\bar{N}_S\| = \frac{\lambda_S \cdot p_{\max}}{(1-\mu_S)\phi_S(p_{\max}, \tau, N) + \mu_S} \ . \quad (65)$$

Rearranging this equation, we obtain the following equality:

$$\phi_S(p_{\max}, \tau, N) = \left(\frac{\lambda_S p_{\max}}{z} - \mu_S\right) \cdot \frac{1}{1-\mu_S}. \quad (66)$$

Subsequently, by means of equation (9), we derive the following equivalence:

$$m_C(p_{\max}, \tau, N) = \left(\frac{\lambda_C(1-\underline{\omega})}{\|\bar{N}_C\|} - \mu_C\right) \cdot \frac{1}{1-\mu_C}. \quad (67)$$

Dividing (66) by (67) yields the following relationship:

$$\frac{\phi_S(p_{\max}, \tau, N)}{m_C(p_{\max}, \tau, N)} = \frac{\|\bar{N}_C\|}{z} \cdot \frac{\lambda_S p_{\max} - \mu_S z}{\lambda_C(1-\underline{\omega}) - \mu_C\|\bar{N}_C\|} \cdot \frac{1-\mu_C}{1-\mu_S}. \quad (68)$$

Given that $z = \|\bar{N}_S\|$, if we substitute the definitions of $\phi_S(p_{\max}, \tau, N) = \min\{1, \|\bar{N}_C\|/\|\bar{N}_S\|\} \cdot \Pr_{\boldsymbol{v},\boldsymbol{\eta}}[\boldsymbol{v} - \tau(\boldsymbol{v}) + \boldsymbol{\eta} \ge p_{\max}]$ and the average match rate $m_C(p_{\max}, \tau, N)$ for a uniformly randomly selected customer given by equation (8), we can infer that the left side of equation (68) is equal to

$$\frac{\phi_S(p_{\max}, \tau, N)}{m_C(p_{\max}, \tau, N)} = \frac{\min\{1, \|\bar{N}_C\|/\|\bar{N}_S\|\}}{\min\{1, \|\bar{N}_S\|/\|\bar{N}_C\|\}} = \frac{\|\bar{N}_C\|}{z} \ .$$

Now, by noting that $\|\bar{N}_C\| = (\overline{\omega} - p_{\max} + \epsilon) \cdot \frac{\lambda_C}{\mu_C}$, which is a direct consequence of Proposition 3, we obtain the following equality:

$$\frac{\|\bar{N}_C\|}{z} = \frac{\|\bar{N}_C\|}{z} \cdot \frac{\lambda_S p_{\max} - \mu_S z}{\lambda_C(1-\underline{\omega}) - \lambda_C(\overline{\omega} - p_{\max} + \epsilon)} \cdot \frac{1-\mu_C}{1-\mu_S}.$$

Rearranging this equality reveals the following relationship, which shows the desired property that z is increasing in $\overline{\omega}$ since $\underline{\omega} = p_{\min} - \epsilon = p_{\max} - \epsilon$:

$$(1-\mu_C) \cdot (\mu_S z - \lambda_S p_{\max}) = (1-\mu_S) \cdot (\lambda_C(\overline{\omega} - p_{\max} + \epsilon) - \lambda_C(1-\underline{\omega})) \ . \quad (69)$$

This completes the proof. ■

Existence. We first show that the zero of the piecewise polynomial system for the centralized system given in (48) is such that $\overline{\omega} \in (\underline{\omega}, 1)$[16]. Subsequently, with the identified $\overline{\omega}$, we show the existence of equilibrium. That is, assuming that the willingness-to-pay distribution aligns with Proposition 3, i.e., if the density of buyers with any willingness-to-pay within the range of $\underline{\omega} = p_{\max} - \epsilon$ and $\overline{\omega}$ equals λ_C/μ_C, then the market is at stationary equilibrium. First, we note that equation (69) can be rewritten as follows:

$$\mu_S \cdot z = \lambda_S(\overline{\omega} - 1) \cdot \frac{1-\mu_S}{1-\mu_C} + \lambda_S p_{\max} \ .$$

Plugging this into $Q_2(\overline{\omega}, z) = 0$, stated in (64), yields following relationship:

$$0 = (1-\mu_S) \cdot \left(\begin{cases} \min\left\{z, \frac{\lambda_C(\overline{\omega} - p_{\max} + \epsilon)}{\mu_C}\right\} \cdot \frac{\overline{\omega} - p_{\max}}{\overline{\omega} - p_{\max} + \epsilon} & \text{if} \quad \underline{\omega} + \epsilon \le p_{\max} \le \overline{\omega} - \epsilon, \\ \min\left\{z, \frac{\lambda_C(\overline{\omega} - p_{\max} + \epsilon)}{\mu_C}\right\} \cdot \frac{\overline{\omega} + \epsilon - p_{\max}}{4\epsilon} & \text{if} \quad \overline{\omega} - \epsilon \le p_{\max} \le \overline{\omega} + \epsilon, \\ 0 & \text{otherwise} \end{cases} \right)$$
$$+ \frac{\lambda_S(1-\mu_S)}{1-\mu_C}(\overline{\omega} - 1) \ .$$

This equation reveals that $\overline{\omega}$ that solves Q_2 lies in $(\underline{\omega}, 1)$ since the right-hand side is continuous in $\overline{\omega}$; setting $\overline{\omega} = 1$ makes the right-hand side positive while setting $\overline{\omega} = \underline{\omega} = p_{\max} - \epsilon$ makes the right-hand side negative.

Now, $\overline{\omega}$ and $z = \|\bar{N}_S\|$ are fixed by the piecewise polynomial system of $Q_1(\overline{\omega}, z) = 0$ and $Q_2(\overline{\omega}, z) = 0$. One can show by following the steps of the existence proof for the decentralized setting in Appendix F.7 that we can recover willingness-to-pay values $\omega(v)$ for all $v \in T_C$ of customers and that each recovered $\omega(v)$ for all $v \in T_C$ constitutes a best response to the market comprised of a single price $p(c) = p_{\max}$ for all $c \in T_S$ and the customers' willingness-to-pay distribution given by Proposition 3.

Appendix G: Proofs from Section 5.4

G.1. Proof of Claim 1

For every $v \in (-\epsilon, 1]$, we denote by $M_C^*(v) \in [0, \lambda_C]$ the per-period average density of customers with valuation v that are successfully matched in the first-best outcome. Similarly, we denote by $M_S^*(c) \in [0, \lambda_S]$ an analogous quantity for suppliers of cost $c \in [0, 1+\epsilon)$. Define $c^+ = \frac{1}{\lambda_S}\|M_S^*\|$ and $v^- = \frac{1}{\lambda_C}\|M_C^*\|$. Since the random shocks are uniformly distributed between $-\epsilon$ and ϵ, we have

$$\begin{aligned} W_\mu^{\text{fb}} &\le \epsilon\lambda_C \cdot \|M_C^*\| + \lambda_C \cdot \int_{-\epsilon}^{1} M_C^*(v)vdv - \lambda_S \cdot \int_0^{1+\epsilon} M_S^*(c)cdc \\ &\le \epsilon\lambda_C \cdot \|M_C^*\| + \lambda_C \cdot \int_{v^-}^{1} vdv - \lambda_S \cdot \int_0^{c+} cdc, \end{aligned} \tag{70}$$

where the first inequality proceeds from the decomposition of social welfare into the aggregate contributions of random shocks, customer valuations, and supplier costs. The second inequality follows by noting that we integrate over the mass $\lambda_S \cdot \int_0^{c+} 1dc = \|M_S^*\|$ of suppliers with lowest costs, meaning that $\lambda_S \cdot \int_0^{c+} cdc \le \lambda_S \cdot \int_0^{1+\epsilon} M_S^*(c)cdc$. Similarly, we integrate over the mass $\lambda_C \cdot \int_{v^-}^{1} 1dc = \|M_C^*\|$ of customers with highest valuations, implying that $\lambda_C \cdot \int_{v^-}^{1} vdv \ge \lambda_C \cdot \int_{-\epsilon}^{1} M_C^*(v)vdv$. Now, we distinguish between two cases.

[16] Given $\overline{\omega}$, the value of $z = \|\bar{N}_S\|$ is determined by equation (69). Moreover, one can show that equation (69) is a consequence of the piecewise polynomial system of $Q_1(\overline{\omega}, z) = 0$ and $Q_2(\overline{\omega}, z) = 0$.

Case 1. Suppose that $v^- \geq p^* - \epsilon$. Then, we have

$$\begin{aligned} c^+ &= \frac{1}{\lambda_S}\|M_S^*\| = \frac{1}{\lambda_S}\|M_C^*\| = \frac{\lambda_C}{\lambda_S}(1-v^-) \\ &\leq \frac{\lambda_C}{\lambda_S}(1-p^*+\epsilon) = \frac{(1+\epsilon)\lambda_C}{\lambda_C+\lambda_S} = p^*. \end{aligned} \tag{71}$$

Now, we can decompose U^* as follows:

$$\begin{aligned} U^* &= \lambda_C \int_{p^*-\epsilon}^{1} (v+\epsilon)dv - \lambda_S \int_0^{p^*} cdc \\ &= \epsilon\lambda_C \cdot \|M_C^*\| + \lambda_C \cdot \int_{v^-}^{1} vdv - \lambda_S \cdot \int_0^{c+} cdc + \lambda_C \int_{p^*-\epsilon}^{v^-} (\epsilon+v)dv - \lambda_S \int_{c^+}^{p^*} cdc \\ &\geq \epsilon\lambda_C \cdot \|M_C^*\| + \lambda_C \cdot \int_{v^-}^{1} vdv - \lambda_S \cdot \int_0^{c+} cdc \\ &\geq W_\mu^{\mathrm{fb}}. \end{aligned}$$

To justify the first inequality, we use the fact that $c^+ \leq p^* = p^* - \epsilon + \epsilon$ from inequality (71) and $\lambda_C(v^- - p^* + \epsilon) = \lambda_S(p^{*`} - c^+)$, noting that $\lambda_S p^* = \lambda_C(1 - p^* + \epsilon)$ by the definition of p^* and $\lambda_C(1-v^-) = \|M_C^*\| = \|M_S^*\| = \lambda_S c^+$ from the matching property. Together, these properties yield $\lambda_C \int_{p^*-\epsilon}^{v^-}(\epsilon+v)dv - \lambda_S \int_{c^+}^{p^*} cdc \geq 0$ by a straightforward change of variable. The last inequality proceeds from (70).

Case 2. Conversely, suppose that $v^- < p^* - \epsilon$. In this case, we observe that $c^+ > p^*$ by the same transformations as inequalities (71). Consequently,

$$\begin{aligned} W_\mu^{\mathrm{fb}} &\leq \lambda_C \cdot \int_{v^-}^{1} (v+\epsilon)dv - \lambda_S \cdot \int_0^{c+} cdc \\ &= \lambda_C \cdot \int_{p^*-\epsilon}^{1} (v+\epsilon)dv - \lambda_S \cdot \int_0^{p^*} cdc + \int_{v^-}^{p^*-\epsilon} (v+\epsilon)dv - \lambda_S \cdot \int_{p^*}^{c^+} cdc \\ &\leq \lambda_C \cdot \int_{p^*-\epsilon}^{1} (v+\epsilon)dv - \lambda_S \cdot \int_0^{p^*} cdc \\ &= U^*, \end{aligned}$$

where the first inequality follows from (70). In the second inequality, we note that $v^- + \epsilon < p^*$ and $\lambda_C(v^- - p^* + \epsilon) = \lambda_S(p^* - c^+)$ using precisely the same arguments as in Case 1. Together, these inequalities yield $\int_{v^-}^{p^*-\epsilon}(v+\epsilon)dv - \lambda_S \cdot \int_{p^*}^{c^+} cdc \leq 0$.

G.2. Proof of Claim 2

Choose $\tau = v + \epsilon - p^* - 2\delta$. In each period, the probability of being matched is

$$\begin{aligned} \phi_C(v,\tau,p_\mu,N_\mu) &= \min\left\{1, \frac{\|\bar{N}_S\|}{\|\bar{N}_C\|}\right\} \cdot \Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v - p_\mu(\boldsymbol{c}) + \boldsymbol{\eta} \geq \tau\right] \\ &\geq \min\left\{1, \frac{3\lambda_S\mu_C}{2\mu_S\lambda_C}\right\} \cdot \Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[v - p^* - \delta + \boldsymbol{\eta} \geq v + \epsilon - p^* - 2\delta\right] \\ &= \min\left\{1, \frac{3\lambda_S\mu_C}{2\mu_S\lambda_C}\right\} \cdot \Pr_{\boldsymbol{c},\boldsymbol{\eta}}\left[\boldsymbol{\eta} \geq \epsilon - \delta\right] \\ &= \min\left\{1, \frac{3\lambda_S\mu_C}{2\mu_S\lambda_C}\right\} \cdot \frac{\delta}{2\epsilon} \\ &= \kappa \cdot \delta, \end{aligned} \tag{72}$$

where the second inequality follows from Propositions 3 and 4 and the fact that $\delta_{S,\mu}(p_{\min}) = \delta_{S,\mu}(p_{\max}) = 0$ in the decentralized setting. Consequently, we have

$$\begin{aligned}
\Phi_C\left(v, \tau_\mu(v), p_\mu, N_\mu\right) &\geq \Phi_C\left(v, \tau, p_\mu, N_\mu\right) \\
&\geq (v + \epsilon - p^* - 2\delta) \cdot \frac{\phi_C(v, \tau, p_\mu, N_\mu)}{(1-\mu)\phi_C(v, \tau, p_\mu, N_\mu) + \mu} \\
&\geq (v + \epsilon - p^* - 2\delta) \cdot \frac{\kappa \cdot \delta}{\kappa \cdot \delta + \mu} \\
&\geq (v + \epsilon - p^* - 2\delta) \cdot \frac{\kappa \cdot \delta}{\kappa \cdot (\delta + \delta^2)} \\
&\geq v + \epsilon - p^* - 3(1+\epsilon)\delta,
\end{aligned}$$

where the second inequality holds because each successful match procures a net surplus of at least $\theta = v + \epsilon - p^* - 2\delta$. The third inequality follows from (72). The fourth inequality proceeds from the fact that $\mu \in (0, \kappa\delta^2)$. The last inequality holds since $v + \epsilon \leq 1 + \epsilon$.

Appendix H: Simulation details

We use an iterative algorithm that iterates over two levels to compute the stationary equilibrium characterized in Section 3. The low-level iteration performed by our algorithm is akin to policy-iteration, which iterates over a set of discrete types. We denote the discretization level of our numerical experiments as $\Delta \in [0,1]$, for which we specify the set of types as $\mathcal{T} = \{0, \Delta, 2\Delta, \ldots, \lfloor \frac{1}{\Delta} \rfloor \Delta\}$ for both sides of the market. The low-level iterative procedure is executed I many times to ensure convergence, where I is a parameter we specify subsequently. Given an arbitrary state of the market, the algorithm computes the best-response of the agents of each type in $\mathcal{T}$ in each low-level iteration. As we refine our discretization $\Delta \to 0$ using a large number of discrete types, we approach the mean-field equilibrium setting. In the mean-field equilibrium setting, changing the strategy of any atomless type does not affect the overall market state. For this reason, in the limit $\Delta \to 0$, the fixed points of our algorithm correspond to stationary equilibria of the dynamical system. Even though the condition $\Delta \to 0$ is not satisfied in our numerical experiments, for appropriate values of Δ and I, our algorithm converges to a unique equilibrium. In particular, we use the values $\Delta = 0.01$ and $I = 10$ in our simulation study.

ALGORITHM 1: Iterative algorithm to compute best-response strategies

Result: $p_i,\ \forall i \in \{1, 2, \ldots, \lfloor \frac{1}{\Delta} \rfloor\}$ and $\tau_i,\ \forall i \in \{1, 2, \ldots, \lfloor \frac{1}{\Delta} \rfloor\}$

Initialization 1. I: Number of high-level iterations, Δ: Discretization level ;

Initialization 2. $p_i,\ \forall i \in \{1, 2, \ldots, \lfloor \frac{1}{\Delta} \rfloor\}$ and $\tau_i,\ \forall i \in \{1, 2, \ldots, \lfloor \frac{1}{\Delta} \rfloor\}$;

```
for i in 1:I do
    for j in 1:⌊1/Δ⌋ do
        Compute best-response strategy for the supplier of type j given other strategies:
        p_j ← Compute_j^S(p_{-j}, τ);
    end
    for j in 1:⌊1/Δ⌋ do
        Compute best-response strategy for the customer of type j given other strategies:
        τ_j ← Compute_j^B(p, τ_{-j});
    end
end
```

Computing best-response strategies. For every type in $\mathcal{T}$, keeping the strategies of all other types fixed, we determine the best response of the corresponding agents with respect to the steady-state of the market. Specifically, customers of type $v \in \mathcal{T}$ pick a threshold τ that maximizes their expected utility surplus $\Phi_C(v,\tau,p,N)$ according to equation (5), while suppliers of type $c \in \mathcal{T}$ pick a price p that maximizes their expected profit $\Phi_S(c,\tau,p,N)$ according to equation (6) that is described in Section 3. Note that we do not have a closed-form expression for these objective functions. Hence, finding the best response for all agent types is the main computational bottleneck in our experiments. Nevertheless, by the equilibrium characterizations of the customers' willingness-to-pay and suppliers' prices given in Proposition 3 and Proposition 4, customers' expected utility $\Phi_C(v,\tau,p,N)$ as well as the suppliers' expected to profit $\Phi_S(c,\tau,p,N)$ are both unimodal at the stationary equilibrium. These results motivate us to operate under the assumption that these functions are unimodal (an assumption that is verified in all the experimental settings we tested). Consequently, based on the unimodality property, we compute the best-response strategies by employing efficient one-dimensional search algorithms, rather than searching exhaustively over all possible strategies. More specifically, we implement the golden-section search method to determine the best-response strategies.

Simulation of steady-state. Next, we describe our simulation procedure that computes the objective function corresponding to any strategy and agent type. The simulation takes as input a fixed set of strategies $\{\{\tau^*(v)\}_{v\in\mathcal{T}},\{p^*(c)\}_{c\in\mathcal{T}}\}$ and calculates the mass of agents amongst all agent types in $\mathcal{T}$ who enter the market at steady-state. The simulation starts with an empty market; i.e, $N_S^0(c) = N_S^0(v) = 0$ for all $v, c \in \mathcal{T}$. At the beginning of each time period $t \geq 1$, a mass of λ_S and λ_C of suppliers and customers of each type arrive at the system. The entry decisions are made in accordance with the entry conditions described in Section 3. Next, we compute the outcomes of the matching process, where the fraction of agents who are matched and leave the market are as described in Section 3. While we do not formally establish that the dynamical system converges to a unique steady-state for all market parameters, this convergence property is observed in our numerical study in all market settings we experiment with. To check whether the dynamical system converges, we assume that the steady-state is reached at time period t if the following criterion is satisfied:

$$\sum_{c\in\mathcal{T}} |\bar{N}_S^t(c) - \bar{N}_S^{t-1}(c)| + \sum_{v\in\mathcal{T}} |\bar{N}_C^t(v) - \bar{N}_C^{t-1}(v)| < 10^{-5}.$$