

Newsvendor Pricing Problem in a Two-Sided Market

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We study the pricing problem of a “platform” intermediary to jointly determine the selling price of the platforms (hardware) sold to consumers and the royalty charged to content developers for content (software), when the demands for content and for platforms are interdependent. Our model elucidates the impact of supply chain replenishment costs and demand uncertainty on the strategic issues of platform pricing in a two-sided market.

Key words: two-sided market; newsvendor problem; pricing strategy

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1. Introduction

In the classical newsvendor pricing model (cf. Whiting 1955), the challenge is to determine jointly the price and the ordering quantity of a product when demand for the product is a (random) function of the price charged (cf. Petruzzini and Dada [1999] and the references therein). Unfortunately, for products like the Xbox 360, the demand–price relationship may be so complex that it cannot be captured by conjoint data collected at the consumer end alone (cf. Raz and Porteus 2006). Each consumer needs an Xbox 360 console to play the console games developed by the content developers. Microsoft plays more the role of a “platform” intermediary. In this market, there is a strong *cross-side* network effect, that is, the value of the product on one side of the platform intermediary is correlated to the number of users on the other side. This property is also known as an *indirect* network externality. Such network externalities play an important role in the pricing strategies of intermediaries in many two-sided markets, and provide the theoretical basis to justify the pricing strategy adopted by numerous two-sided markets—where one side of the market is often treated as a profit center, while the other is treated as a loss leader, or at best financially neutral.

Interest in understanding two-sided markets is relatively new (cf. Eisenmann et al. 2006, Parker and van Alstyne 2005, Rochet and Tirole 2003, 2006). Their

recent popularity is mainly the result of the need to explain the workings of the software market and related industries. One of the most counter-intuitive observations in a two-sided network market is that profits can still accrue to the intermediary even if one side of the market is heavily subsidized by the intermediary (cf. Parker and van Alstyne 2005). Other works on two-sided markets include Argentesi and Filistrucchi (2007), Armstrong (2006), Caillaud and Jullien (2003), Parker and van Alstyne (2005), and Rochet and Tirole (2003, 2006).

In this paper, we examine a class of pricing and inventory problems faced by such platform intermediaries in a two-sided market. We propose a model to integrate the effects of indirect network externalities into the newsvendor pricing decisions. Recent results in the two-sided network literature synthesizes the spillover effect with optimal pricing decisions (cf. Parker and van Alstyne 2005) and shows that the optimal pricing decision necessitates subsidizing one side of the market so that profits can be accrued at the other side. Interestingly, despite adding supply chain operational costs in our model, such peculiar pricing behavior in the optimal solution may persist, though we discover another strategic option: the intermediary could also charge a surplus to both sides of the market to compensate for supply chain costs, despite the positive indirect network externalities in the markets. This appears to be the preferred strategy in the high-

end fashion magazine industry, where both readers and advertisers are charged a surplus despite the positive network externality of readership on the advertising market. The key contribution of this paper is the complete characterization of the different regimes of strategic pricing options, demonstrating clearly the importance of supply chain operational concerns on the strategic pricing decisions of a firm operating in a two-sided market.

2. Demand Models in Two-Sided Market

The key decision variables in our model are as follows. p_c denotes the price charged to consumers for an Xbox console. p_j denotes the price (royalty) charged to content providers for each unit of software sold. Let $q_c(p_c, p_j, \mathbf{Y})$ and $q_j(p_c, p_j, \mathbf{Y})$ denote the demand for consoles and the total units of software sold in the market, respectively. Our demand model assumes that

$$q_c(p_c, p_j, \mathbf{Y}) = D_c(p_c, \mathbf{Y}) + e_{jc} \times D_j(p_j), \quad (1)$$

$$q_j(p_c, p_j, \mathbf{Y}) = D_j(p_j) + e_{cj} \times D_c(p_c, \mathbf{Y}). \quad (2)$$

Here, D_c denotes the (inherent) demand from consumers for Xbox consoles without taking into account the cross-network effect. It depends on p_c and a random parameter \mathbf{Y} , which has a positive probability density function $f_Y(y)$ and cumulative density function $F_Y(y)$, $y \in \mathbb{R}$. The parameter \mathbf{Y} denotes the market elements that affect console demand but that cannot be attributed to the pricing decision. We assume $E_Y(\mathbf{Y}) = 0$. The shape of D_c and the distribution Y , can be obtained from standard conjoint experiment. Similarly, D_j denotes the expected (inherent) demand for software products without taking into account the cross-network effects. This is a function of p_j , because the royalties charged to the content providers affect their willingness to develop new software titles, and thus indirectly affect the number of software titles developed for the platform which subsequently affect the level of D_j .

We augment the demand models with the additional terms of $e_{jc}D_j(p_j)$ and $e_{cj}D_c(p_c, \mathbf{Y})$ in (1) and (2), respectively, to capture the indirect network effects in the two-sided market. This demand model ([1] and [2]) is motivated by the work in Parker and van Alstyne (2005)

$$q_c(p_c, p_j, \mathbf{Y}) = (1 - e_{cj}e_{jc})D_c(p_c, \mathbf{Y}) + e_{jc}q_j(p_c, p_j, \mathbf{Y}), \quad (3)$$

$$q_j(p_c, p_j, \mathbf{Y}) = (1 - e_{cj}e_{jc})D_j(p_j) + e_{cj}q_c(p_c, p_j, \mathbf{Y}). \quad (4)$$

Let us consider (3). Differentiating q_c w.r.t. p_j using (3), we obtain $\frac{\partial q_c}{\partial p_j} = e_{jc} \frac{\partial q_j}{\partial p_j}$, while differentiating q_j w.r.t. p_c using (4), we obtain $\frac{\partial q_j}{\partial p_c} = e_{cj} \frac{\partial q_c}{\partial p_c}$. The parameters e_{jc} and e_{cj} have the following natural interpretations: e_{jc} denotes the content-to-console *internetwork* externality term, which measures how much purchases on

the content side affect sales of the console market, while e_{cj} denotes the console-to-content *internetwork* externality term, which measures how much purchases on the console side affect the purchases of copies of game titles in the content market.

Let $\bar{D}_c(p_c) = E_Y[D_c(p_c, \mathbf{Y})]$, $\bar{q}_c(p_c, p_j) = E_Y[q_c(p_c, p_j, \mathbf{Y})]$, $\bar{q}_j(p_c, p_j) = E_Y[q_j(p_c, p_j, \mathbf{Y})]$. We define

- $\frac{\partial \bar{q}_c}{\partial p_j}$ = spillover effect from the content side to the console side;
- $\frac{\partial \bar{q}_j}{\partial p_c}$ = spillover effect from the console side to the content side; and
- $r = \frac{\frac{\partial \bar{q}_j}{\partial p_c}}{\frac{\partial \bar{q}_c}{\partial p_j}}$ = ratio of spillover effects.

The parameter r plays a key role in the two-sided network literature, as the optimal pricing behavior (which side to subsidize) can be connected to this single parameter in the existing theory (cf. Parker and van Alstyne 2005). We further assume that $D_c(p_c, \mathbf{Y}) = \bar{D}_c(p_c) + H_c(p_c)\mathbf{Y}$, where either $H_c(p_c) > 0$ for all p_c , or $H_c(p_c) = 0$ for all p_c . The latter reduces to the classic two-sided market problem, because there is no demand uncertainty. Henceforth, we assume the more interesting case, where $H_c(p_c) > 0$ for all p_c . Note that as $H_c(p_c)$ need not be monotonic in p_c , the above demand model allows us to handle the situation where the variability of demand could be small for either low or high values of p_c . This appears to be the norm, rather than the exception, in most settings.

To cope with demand uncertainty, let x denote the total number of the commodity (e.g., Xbox consoles) that will be produced at the beginning of the selling season. The decision for x is affected by the following parameters: w —wholesale price per unit of commodity; s —salvage value per unit of commodity; and p —penalty cost per unit of commodity shortage. Note that $w > s$. The total profit that can be attained is given by

$$\begin{aligned} \pi(x, p_c, p_j, \mathbf{Y}) = & p_c \min\{q_c(p_c, p_j, \mathbf{Y}), x\} \\ & + p_j q_j(p_c, p_j, \mathbf{Y}) - wx - p(q_c(p_c, p_j, \mathbf{Y}) - x)^+ \\ & + s(x - q_c(p_c, p_j, \mathbf{Y}))^+. \end{aligned} \quad (5)$$

We would like to choose x , p_c , and p_j to maximize the expected total supply chain profit $E[\pi(x, p_c, p_j, \mathbf{Y})]$. More importantly, we would like to answer the following question: How do we tell which side of the market to subsidize? It turns out that the ratio of spillover effects r is insufficient to characterize the strategic pricing decision. In many cases, we need to understand the impact of supply chain mismatch cost to answer this question. The magnitude of lost sales and the total overall market demand for the console can influence the strategic pricing decision—the optimal strategy may call for subsidization on the console market, even if the spillover effect of the

content market into the console market is larger, i.e., $r < 1$. The reason for this is intuitive: the spillover effects by subsidizing the console market can be fully captured in the content market, whereas the spillover effect from the content to the console market cannot be fully captured due to demand uncertainty.

In the rest of the paper, for ease of exposition, we assume the following: (A) $e_c e_{jc} < 1$, omitting the degenerate case when $e_c e_{jc} = 1$. Also, we assume that $e_{cj} \geq 0$, which means that sales of Xbox consoles always have a nonnegative internetwork effect on sales of titles; (B) Any optimal solution (x^*, p_c^*, p_j^*) to (6), where $\pi(x, p_c, p_j, \mathbf{Y})$ is given by (5), is such that $x^* > 0$, $\bar{q}_c^* > 0$, and $\bar{q}_j^* > 0$. Assumption A is an assumption often used in the literature on two-sided network effects, as in Parker and van Alstyne (2005). Assumption B is a technical assumption introduced for ease of exposition, and removes the need to digress to the discussion of degenerate cases.

3. Additive Demand Model

To determine jointly the optimal supply of Xbox consoles and the optimal selling prices in different markets, we consider the following maximization problem:

$$\max E_Y(\pi(x, p_c, p_j, \mathbf{Y})) \quad \text{subject to } x \geq 0. \quad (6)$$

We consider additive (inherent) console demand, whereby $H_c(p_c) \equiv 1$. Its extension, when $H_c(p_c)$ may not be a constant, will be discussed in section 3.4.

3.1. Preliminaries

Let us rewrite the maximization problem (6) in an equivalent form. The latter will be used instead of (6). Let (x^*, p_c^*, p_j^*) be an optimal solution to the maximization problem (6). Now, there exists $y_0^* \in \mathbb{R}$ such that $\bar{D}_c(p_c^*) + e_{jc} D_j(p_j^*) + y_0^* = x^*$. Note that y_0^* is the difference between actual expected console demand and the supply of Xbox consoles at optimality. Introducing this difference into (6), we can rewrite (6) as

$$\max E_Y(\pi(x, p_c, p_j, \mathbf{Y})) \quad \text{subject to } x \geq 0, \quad (7)$$

$$\bar{q}_c(p_c, p_j) + y_0 = x,$$

where $\bar{q}_c(p_c, p_j) := E_Y(q_c(p_c, p_j, \mathbf{Y})) = \bar{D}_c(p_c) + e_{jc} D_j(p_j)$. Equation (7) is equivalent to

$$\max E_Y(\pi(\bar{q}_c(p_c, p_j) + y_0, p_c, p_j, \mathbf{Y})) \quad \text{subject to} \quad (8)$$

$$\bar{q}_c(p_c, p_j) + y_0 \geq 0.$$

Henceforth, we use (8) in our investigation. Let us now give an explicit form for the objective function in (8) in Proposition 2. First, let $\bar{q}_j(p_c, p_j) := E_Y(q_j(p_c, p_j, \mathbf{Y}))$.

REMARK 1. Note that both \bar{q}_c and \bar{q}_j depends on p_c and p_j . From now onwards, we suppress this explicit dependence for ease of exposition. We also use \bar{q}_c^* and \bar{q}_j^* to denote $\bar{q}_c(p_c^*, p_j^*)$ and $\bar{q}_j(p_c^*, p_j^*)$, respectively.

PROPOSITION 1.

$$E_Y(\pi(\bar{q}_c + y_0, p_c, p_j, \mathbf{Y})) = (p_c - w)\bar{q}_c + p_j \bar{q}_j$$

$$+ \underbrace{(p_c + p - w)y_0 + (p_c + p - s) \int_{-\infty}^{y_0} (y - y_0) f_Y(y) dy}_{\text{supply chain mismatch cost}}. \quad (9)$$

From the explicit expression for $E_Y(\pi(\bar{q}_c + y_0, p_c, p_j, \mathbf{Y}))$ in Proposition 1, we see that the objective function in (8) comprises of production cost, revenue gained from the console and content markets, and also supply chain mismatch cost.

In Proposition 2, we provide a system of equations that (p_c^*, p_j^*, y_0^*) —an optimal solution to (8)—needs to satisfy. Based on this system of equations, we are able to derive a set of criteria that decide which market to subsidize or surcharge under optimality.

Note that we have $\bar{q}_c^* + y_0^* > 0$, by Assumption B, where we assume that $x^* > 0$. From the KKT conditions, the following holds:

PROPOSITION 2. Let (p_c^*, p_j^*, y_0^*) be an optimal solution to (8). It then satisfies

$$p_c^* = \frac{w - s \int_{-\infty}^{y_0^*} f_Y(y) dy}{\int_{y_0^*}^{\infty} f_Y(y) dy} - p, \quad (10)$$

$$\bar{q}_c^* = (w - p_c^*) \left(\frac{\partial \bar{q}_c}{\partial p_c} \right)^* - p_j^* \left(\frac{\partial \bar{q}_j}{\partial p_c} \right)^* + \int_{y_0^*}^{\infty} (y - y_0^*) f_Y(y) dy, \quad (11)$$

$$\bar{q}_j^* = (w - p_c^*) \left(\frac{\partial \bar{q}_c}{\partial p_j} \right)^* - p_j^* \left(\frac{\partial \bar{q}_j}{\partial p_j} \right)^*. \quad (12)$$

Note that (10) is equivalent to

$$\text{Prob}(q_c(p_c^*, p_j^*, \mathbf{Y}) \leq x^*) = \frac{p_c^* + p - w}{p_c^* + p - s}. \quad (13)$$

This is merely the optimality condition for the newsvendor problem with p_c^* and p_j^* fixed. Equations (11) and (12) are standard conditions relating demand with price at optimality. In Equation (11), the last term arises to account for lost sales due to demand uncertainty. When $\mathbf{Y} = 0$ a.s., then (11) and (12) reduce to the optimality conditions for the two-sided market problem studied in Parker and van Alstyne (2005).

We now introduce an important assumption in this paper: (C) When $e_{cj} = e_{jc} = 0$, (p_c^0, p_j^0, y_0^0) is the optimal solution to (8) if and only if it satisfies the KKT conditions for (8). Also, it is the only optimal solution to (8). This assumption is a technical assumption introduced to remove the need to digress to the issues of uniqueness of the optimal pricing strategy. While it rules out some

special cases, it nevertheless holds for the majority of interesting problem instances.

Let us define $p_c(y_0) := \frac{w-s \int_{-\infty}^{y_0} f_Y(y) dy}{\int_{y_0}^{\infty} f_Y(y) dy} - p$. Using $p_c(y_0)$,

we can then write (10) in a more compact form as $p_c^* = p_c(y_0^*)$.

PROPOSITION 3. $p_c(y_0)$ is an increasing function of y_0 .

Let $\bar{q}_c := \bar{q}_c - G_c(y_0)$, where $G_c(y_0) := \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy$. $G_c(y_0)$ can be thought of as expected lost sales at inventory level y_0 , when demand is Y . \bar{q}_c can be thought of as the effective consumer demand captured in the console market, after taking into account the expected lost sales, denoted by $G_c(y_0)$. If there is no console demand uncertainty, e.g., $Y = 0$ a.s., then $G_c(y_0)$ is identical to 0. In case $Y \neq 0$ a.s., $G_c(y_0) > 0$.

The conditions (10)–(12) can be rewritten in the following manner:

PROPOSITION 4. Let (p_c^*, p_j^*, y_0^*) be an optimal solution to (8). It satisfies

$$p_c^* = p_c(y_0^*), \quad (14)$$

$$p_c^* = w + \frac{e_{cj} \bar{D}'_c(p_c^*) \bar{q}_j^* - D'_j(p_j^*) \bar{q}_c^*}{(1 - e_{cj} e_{jc}) \bar{D}'_c(p_c^*) D'_j(p_j^*)}, \quad (15)$$

$$p_j^* = \frac{\bar{D}'_c(p_c^*) \bar{q}_j^* - e_{jc} D'_j(p_j^*) \bar{q}_c^*}{(e_{cj} e_{jc} - 1) \bar{D}'_c(p_c^*) D'_j(p_j^*)}. \quad (16)$$

3.2. Results

To determine whether to subsidize or surcharge the markets, we use the situation when there are no inter-network effects, that is, when $e_{cj} = e_{jc} = 0$, as the basis for comparison.

Suppose (p_c^0, p_j^0, y_0^0) is the optimal solution to (8) when $e_{cj} = e_{jc} = 0$. We have the following characterization of (p_c^0, p_j^0, y_0^0) :

PROPOSITION 5. Under Assumption C, $p_c = w - \frac{\bar{D}_c(p_c) - G_c(y_0)}{\bar{D}'_c(p_c)}$, and $p_c = p_c^0, y_0 = y_0^0 \Leftrightarrow p_c = p_c(y_0)$; $p_j = p_j^0 \Leftrightarrow p_j = -\frac{D_j(p_j)}{D'_j(p_j)}$.

To highlight that we are considering the ratio of spillover effects at optimality, let r^* be the ratio of spillover effects, r , evaluated at (p_c^*, p_j^*) . That is

$$r^* = \frac{e_{cj} \bar{D}'_c(p_c^*)}{e_{jc} D'_j(p_j^*)}.$$

As mentioned earlier, the ratio of spillover effects alone is not sufficient to characterize the strategic pricing decision. We now introduce another two key parameters r_0^* and r_1^* that affect optimal pricing

behavior. Their relationships with r^* affect how the optimal prices p_c^*, p_j^* behave w.r.t. p_c^0, p_j^0 —the prices when there are no cross-network effects.

DEFINITION 1.

$$r_0^* := 1 \text{ and } r_1^* := 1 - \frac{G_c^*}{\bar{q}_c^*}. \quad (17)$$

Let us call r_0^* the console pricing threshold, and r_1^* the content provider pricing threshold. Since G_c^* measures the expected lost sales in the additive demand case, r_1^* can be viewed as the proportion of console market demand captured relative to the size of the total market demand for console, at the optimal pricing strategy. Similarly, in view of the spillover effect, r_0^* can be viewed as the proportion of content market demand captured relative to the size of the total market demand for content, at the optimal pricing strategy. By comparing r^* with $r_0^*(r_1^*)$, we can determine the pricing strategy of the platform, that is, whether to surcharge or subsidize the console (content provider). In other words, measuring the console and content provider pricing thresholds, and comparing them with the ratio of spillover effects, provides a means to decide the pricing strategy of the platform.

THEOREM 1. Suppose Assumption C holds.

- If $e_{jc} > 0$, then $r^* \leq r_1^* \Leftrightarrow p_j^* \leq p_j^0$. Equality holds on the left-hand side if and only if equality holds on the right-hand side.
- If $e_{jc} < 0$, then $r^* \leq r_1^* \Leftrightarrow p_j^* \geq p_j^0$. Equality holds on the left-hand side if and only if equality holds on the right-hand side.

We examine next the relationship between r^* and r_0^* (the console pricing threshold) and how it affects p_c^* and p_j^0 . For ease of exposition, we first describe our main result for the case when the console demand function is linear in p_c .

THEOREM 2. Suppose Assumption C holds. Furthermore, suppose the (inherent) demand of the console is linear in its price, that is, $\bar{D}_c(p_c) = A_c - b_c p_c$ for some $A_c, b_c > 0$.

- If $e_{jc} > 0$, then $r^* \leq r_0^* \Leftrightarrow p_c^* \geq p_c^0$. Equality holds on the left-hand side if and only if equality holds on the right-hand side.
- If $e_{jc} < 0$, then $r^* \leq r_0^* \Leftrightarrow p_c^* \leq p_c^0$. Equality holds on the left-hand side if and only if equality holds on the right-hand side.

Again, as in Theorem 1, we see from Theorem 2 that the console pricing threshold, r_0^* , serves as a threshold, such that depending on whether r^* is less than or

greater than r_0^* , the console market is surcharged or subsidized accordingly.

In the above proposition and theorems, Assumption C plays an important role in the proof to show that our criteria—which is comparing r^* with r_0^* , r_1^* —can be used to decide the subsidizing/surcharging strategy for different markets. r_0^* and r_1^* are thresholds that reflect operational considerations—due to the presence of G_c^* —and through them, we see the effect of these considerations in deciding optimal pricing strategy.

3.3. Summary

Theorems 1 and 2 provide generalizations of similar results in Parker and van Alstyne (2005). Our results indicate that if r^* lies between r_1^* and r_0^* , then it is possible to charge both sides more than the base selling prices in the case when $e_{jc} > 0$. This situation of charging both sides more at the same time cannot be reflected in the model of Parker and van Alstyne (2005). This situation arises when $r_1^* < r^* < r_0^*$, that is, when the spillover effect from the console side to the content side is small enough compared with that from the content side to the console side for the console market to be surcharged, but not small enough for the content market to be subsidized. Table 1 provides a summary of results obtained thus far. As shown in Table 1, the subsidy direction depends on whether e_{jc} is positive or negative. The subsidy direction reverses when e_{jc} changes sign.

REMARK 2. In the degenerate case when $e_{jc} = 0$ and $e_{cj} > 0$, we always have $p_j^* > p_j^0$, while $p_c^* < p_c^0$.

Although we can use r_0^* and r_1^* , and compare them with r^* to determine the console and content provider pricing strategy through Theorems 1 and 2, a limitation is that we cannot determine the exact subsidy or surcharge made.

Table 1 Subsidy Schemes for Different Internetwork Externality Terms for Additive Demand

	$e_{jc} < 0, e_{cj} \geq 0$	$e_{jc} < 0, e_{cj} \leq 0$
p_c	$r^* < r_0^* \Leftrightarrow p_c^* < p_c^0$	$r^* < r_0^* \Leftrightarrow p_c^* < p_c^0$
p_j	$r^* < r_1^* \Leftrightarrow p_j^* < p_j^0$	$r^* < r_1^* \Leftrightarrow p_j^* < p_j^0$

3.4. Extension

In this subsection, we consider the case when $H_c(p_c)$ is not a constant. In this case, G_c in section 3.1 has to incorporate the change in console demand variance as p_c changes, and is given by

$$G_c(p_c, y_0) := H_c(p_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy + H'_c(p_c) \left((w - p_c - p) y_0 + (s - p_c - p) \int_{-\infty}^{y_0} (y - y_0) f_Y(y) dy \right).$$

In this general case, which includes multiplicative demand, Theorem 1 still holds. It turns out that the console pricing strategy for this general case is the same as for the additive, linear demand case (Theorem 2). Unfortunately, in this general demand setting, we do not know of any nice physical meaning on the parameter r_0^* , nor do we have the physical interpretation of $G_c(\cdot)$ vis-a-vis the supply chain operation.

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