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Spillover Effect of Consumer Awareness on Third Parties' Selling Strategies and Retailers' Platform Openness

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Abstract. In this study, we explore why an online retailer would open its platform and why a third-party seller would join the platform when the third party carries products identical to those the retailer sells as well as products the retailer does not carry. When the third party joins the retailer's platform, more consumers become aware of the third party's existence and, thus, might become aware of the products it does not list on the retailer's platform (e.g., by searching the seller brand online), which is called the spillover effect. We develop a game-theoretic model to first examine the spillover effect on the third-party seller's product offering on the retailer's platform under an exogenous commission rate and then examine the effect on the retailer's platform-openness decision. We find that the third party's optimal selling strategies vary with its initial awareness level, the extent of the spillover effect, and the commission rate. Furthermore, we characterize how the initial awareness level and the extent of the spillover effect together determine the retailer's openness decision and equilibrium selling partnership. When the spillover effect is mild, the retailer opens its platform and the third-party seller sells its exclusive product on the platform. When the spillover effect is salient, if the retailer has a large awareness advantage over the third party, the retailer has no incentive to open; otherwise, the retailer opens its platform and the third party sells the identical product on the platform. Finally, we find that, compared with the case without a spillover effect, the spillover effect makes the retailer less likely to open its platform, but it makes the third party more likely to sell identical products on an open retailer platform; the spillover effect always (weakly) benefits the third party, but it does not necessarily hurt the retailer.

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Keywords: platform openness • coopetition • spillover effect • online retailing

1. Introduction

Global e-commerce sales have experienced unprecedented growth recently and have reached over $2 trillion annually. Along with this remarkable growth is the emergence of giant online retailers such as Amazon.com in the United States and JD.com in China. Traditionally, these retailers adopt merchant revenue models under which they buy products from suppliers and resell to consumers. Over the years, they have developed considerable consumer bases. For instance, Amazon has reached more than 195 million monthly unique visitors in the United States alone[1] and over 300 million active customer accounts worldwide,[2] making it one of the most valuable companies in the world. Its counterpart in China, JD.com, has also advanced rapidly with over 380 million active customer accounts.[3]

With the option of milking the dominance of their merchant revenue models, interestingly, these retailers open their platforms and allow third-party sellers to offer products there, thus inviting competition. Attracted by the promise of tapping into the huge user base, small merchants might use these platforms to reach customers who otherwise would not know of them. Third-party sellers report an average of 50% increase in sales when they join Amazon's platform (Reuters 2017). In turn, Amazon takes a commission for every sale (e.g., 6% for personal computers and 8% for cell phones). Intuitively, both the retailer and the third-party sellers benefit from the partnership if the third-party sellers sell products different from those offered by the retailer. However, one puzzling phenomenon is that these retailers allow third-party sellers to sell products identical to those offered by the retailers, and we often observe both a retailer and a third-party seller offering the same product on the retailer's platform.

Interestingly, many third-party sellers have their own websites and carry more products than the large retailer does in some specific category. When they join a retailer's platform, they list some of their products on the retailer's platform. Sometimes these third-party sellers may even choose to sell the same product as the retailer, rather than different products, on the retailer's platform. For example, cheersofa.com sells various sofas. In addition to selling on its own website, this firm lists some of its sofas on JD.com, and, interestingly, JD.com itself also carries some of this firm's sofas. Similarly, lovelyskin.com sells skincare products online directly on its own website. In addition, it lists some of its products on Amazon.com, some of which are also offered by Amazon itself. Conceivably, the sellers do so because their presence on a retailer's platform can increase traffic to their websites: When a third party's product is listed on a retailer's platform, exposing the seller to consumers, some consumers may also become aware of other products offered by the seller through various online tools (e.g., search engines). We call this cross-product awareness increase the *spillover effect* of consumer awareness. The spillover effect has been documented and examined in the context of advertising and consumer search (e.g., Chiou and Tucker 2012, Lewis and Nguyen 2015). Enabled by high-speed Internet connections and powerful search engines, once exposed to a seller, consumers can easily find and learn about the seller and its product offering. According to InternetLiveStats.com, daily searches on Google exceeded 3.5 billion in 2018.

This paper aims to answer the following questions. First, with an open platform and a given commission rate, how does spillover affect a third-party seller's incentive to join a retailer's platform and its product offering on that platform? Second, how does spillover affect a retailer's openness decision and the retailer's and the third party's profits? We develop a game-theoretic model to answer these questions. We first consider a game in which the platform is open and the commission rate is given. The third party chooses whether to join the platform. If it does so, the third party chooses to sell either a product also sold by the retailer, or an exclusive product not offered by the retailer, or both. We then consider a super game in which the retailer chooses whether to open its platform and, if so, the retailer sets the commission rate.

We find that the third party's best selling strategies vary with its initial awareness level, the extent of the spillover effect, and the commission rate. Specifically, for a low commission rate, when its initial awareness and the spillover effect are mild, the third party sells both products on the retailer's platform. When its initial awareness or the spillover effect is salient, the third party sells exclusive products. For an intermediate commission rate, the third party's best strategy is always to sell only exclusive products on the retailer's platform. For a high commission rate, the third party only sells identical products if the spillover effect is significant; otherwise, the third party offers no products through the retailer. Interestingly, even if the commission rate is extremely high (e.g., the retailer asks for the third party's entire revenue from its sales on the retailer's platform), when the spillover effect relative to the initial awareness level is significant, the third party optimally chooses to sell the identical product on the retailer's platform. This surprising result stems from the benefit of the increased demand for the exclusive product resulting from the spillover effect outweighing the cost of paying the identical-product revenue to the retailer.

Furthermore, we characterize how the retailer's optimal openness decision and the equilibrium selling partnership depend on the extent of the spillover effect and the retailer's awareness advantage. When the spillover effect is mild, the retailer opens its platform and the third party sells exclusive products on the platform. In this case, the retailer has no incentive to let the third party offer the identical product on its platform, because doing so increases the competition against the retailer in the identical product. As a result, the retailer sets a commission rate such that the third-party seller has incentive to offer the exclusive product but is disinclined to offer the identical product on the platform. When the spillover effect is salient, the third party is more inclined to sell the identical product on the platform than the exclusive product because of the benefit from the awareness increase for the exclusive product. If the retailer has a large awareness advantage over the third party, it has no incentive to open its platform because the retailer can benefit from the awareness advantage by excluding the third party from selling the identical product on its platform. If the awareness advantage is small, the retailer is better off earning the commission, anticipating that the third-party seller may sell the identical product on its platform. As a result, the retailer opens its platform, and the third party sells the identical product on the platform.

Finally, we find that, compared with the case without the spillover effect, the spillover effect makes the retailer less likely to open its platform, but it makes the third party more likely to sell identical products on an open retailer platform. Without the spillover effect, the retailer always welcomes the third party to sell the exclusive product on its platform, because this partnership increases the retailer's revenue without altering the competition for the retailer's own product offering. In the presence of the spillover effect, if the third party sells exclusive products on the retailer's platform, the retailer's awareness advantage decreases because of the spillover effect. As a result, the retailer may even induce the third party to sell identical products. In addition, we find that the spillover effect always (weakly) benefits the third party but does not necessarily hurt the retailer. The spillover effect in general benefits the third party because of the additional awareness it entails. Surprisingly, we find that the retailer may also benefit in some cases by strategically setting its platform policy. In particular, in the equilibrium that the retailer opens and the third party sells identical products on its platform, the retailer's profit increases in the extent of the spillover effect because the gain brought by the spillover effect to the third party increases in the extent of the spillover effect, and the retailer shares the gain via the commission.

The rest of this paper is organized as follows. In the next section, we discuss the related literature. In Section 3, we lay out the model. We then analyze a third-party seller's incentive to join a retailer's platform under an exogenous commission rate in Section 4. In Section 5, we examine the spillover effect of consumer awareness on a retailer's openness decision with an endogenous commission rate. Section 6 discusses a retailer's openness decision in the case with an exogenous commission rate. Section 7 concludes the paper.

2. Literature

Our study mainly relates to two streams of literature. The first related stream concerns the spillover effect in the context of advertising and consumer search. Existing work has studied demand spillover across different firms (e.g., Chiou and Tucker 2012, Lewis and Nguyen 2015, Sahni 2016, Shapiro 2018). For instance, Lewis and Nguyen (2015) find a positive spillover effect of display advertising from an advertiser to its competitors due to increasing the exposure of the advertised product category to customers. Based on field experiments on a restaurant-search website, Sahni (2016) documents a similar spillover effect and finds that the spillover effect decreases in the intensity of advertising. Li and Agarwal (2017) find positive spillover effects in both complementary and substitute markets. Using advertising data on Google which allows third-party sellers to use trademarks in their ads, Chiou and Tucker (2012) examine the spillover of advertising and show that, when sellers use trademarks in the ads, consumers are more likely to buy from the direct channel, because the use of trademarks in ads blurs the distinction between indirect and direct channels. In contrast, Fong et al. (2016) show that targeted promotions can effectively reduce consumers' search activities and lead to a negative spillover effect on cross-firm purchase activity.

Other work has studied the sales spillover across products of the same firm. For example, Erdem and Sun (2002) show that the spillover of advertising and sales promotion benefits the umbrella branding products because the marketing effort can increase consumer awareness and reduce uncertainty about the brand. Balachander and Ghose (2003) find that advertising of new-generation brands can even increase the sales of previous generations. In this article, we consider the spillover effect across products for the same firm. Different from the existing work, we consider the competition between a third-party seller and a giant retailer, in which the third-party seller might leverage the spillover effect by selling some of its products on the retailer's platform. We focus on how spillover affects the third-party seller's product offering on the retail platform and, furthermore, how it affects the retailer's platform openness and the resulting partnership with the third party.

The second related stream is about platform business models and platform openness in retailing (e.g., Hagiu and Wright 2015, Abhishek et al. 2016, Chen et al. 2016, Tan et al. 2016, Tan and Carrillo 2017). For example, Hagiu and Wright (2015) study how marketing-activity information affects an intermediary's business-model choice—wholesale, agency, or hybrid. Kwark et al. (2017) examine how third-party information influences the upstream competition and further influences the retailer's business-model choice—wholesale or agency. Abhishek et al. (2016) show that not only the competition between online retailers, but also the cross-channel effect on manufacturers impact the retailers' business-model choice. In contrast to these studies on platform business-model choice, we focus on platform openness.

Several studies have examined why online retailers would open their retailing platforms to third-party sellers (e.g., Jiang et al. 2011, Ryan et al. 2012, Mantin et al. 2014, Chen and Guo 2016, He et al. 2020). For example, Mantin et al. (2014) show that allowing others to sell on its platform gives the retailer an outside option: if it cannot negotiate with a supplier to a reasonable price, the retailer can let third-party sellers sell substitute goods. Chen and Guo (2016) take the perspective that new media reduces advertising costs and makes advertising a viable option even for small sellers, showing that the decrease in this advertising cost provides incentive for leading retailers to open their platforms, because, otherwise, small sellers can advertise with new media and improve their positions to compete with leading retailers anyway. Jiang et al. (2011) argue that, by letting small sellers sell on its platform, Amazon can learn the demand through small sellers' sales activities. In contrast to these studies, we study how spillover affects the third party's decision on which product(s) to offer on a retailer's platform and the retailer's openness decision. We find that, on the one hand, spillover generally makes the third party more likely to sell identical products on the retailer's platform (if open). On the other hand, spillover makes the retailer less likely to open its platform. Although the spillover effect always benefits the third party, it does not necessarily hurt the retailer, which implies a possible win-win outcome.

3. The Model

We consider two players, an online retailer R and a third-party seller S. The third-party seller sells two products, product 1 and product 2, and the retailer sells product 1 but not product 2. We may interpret this setting as that products 1 and 2 are in the same product category, and the third-party seller is a specialty seller in this product category, whereas the retailer is a general retailer that does not carry all products the specialty seller carries in this category. The general online retailers do not offer as much product variety as specialty sellers within specific product categories for different reasons, such as inventory and selling cost, special skills and expertise required, and exclusive deals arranged for specialty sellers.

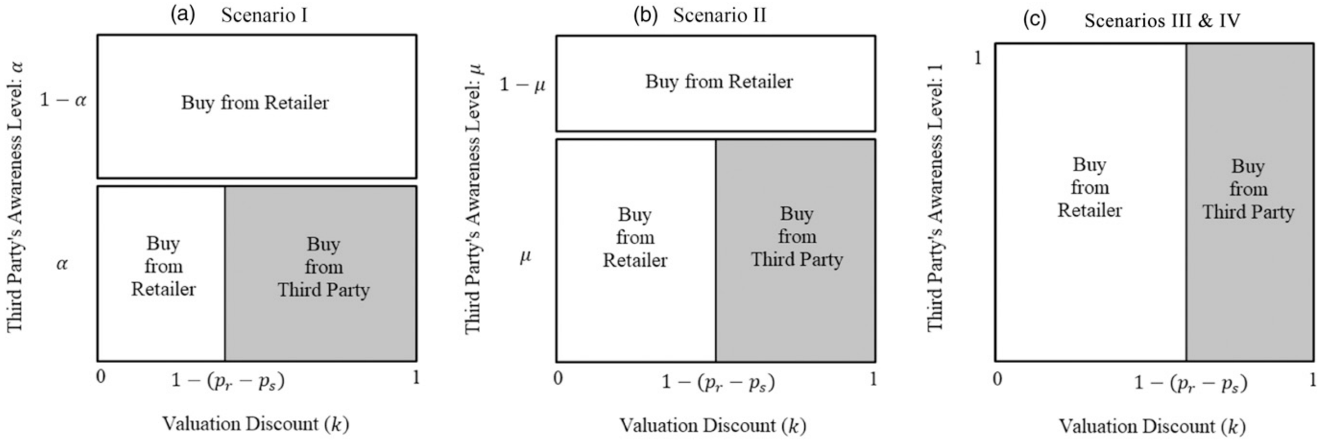
Compared with the third-party seller, the retailer has awareness advantage and valuation advantage. Awareness advantage means that more consumers are aware of the retailer than the third-party seller. We assume that all consumers are aware of the retailer, but only a proportion α of consumers are aware of the third-party seller, where $\alpha \in (0, 1)$. Valuation advantage means that, everything else being equal, consumers prefer to buy products from the retailer over the third party because of the retailer's established reputation of, for example, good customer service (e.g., in product shipping and return service). When consumers purchase product 1 from the retailer, the valuation they can derive is v. When consumers purchase product 1 from the third party, they can derive a discount valuation k, where k is uniformly distributed over [0, 1]. The retailer's valuation advantage over the third-party seller is essentially about vertical differentiation between their product offerings. Although our model of product differentiation is a variation of the most commonly used framework in which consumers have heterogeneous valuation of a high-quality product and the same discount to a low-quality product, the essential idea captured by those models is the same: Everything else being equal, consumers all prefer the high-quality product over the low-quality product.

The retailer may choose to open its platform and allow the third-party seller to sell product(s) on its platform by charging a commission with a commission rate θ, $\theta \in [0, 1]$. When the platform is open, if the third-party seller chooses to sell both products on the platform, we assume all consumers become aware of both of the third-party seller's offerings. If the third-party seller chooses to sell only one product on the platform, all consumers become aware of that product. In addition, some consumers also become aware of the third-party seller's other product, by, for example, searching for its brand online with search engines. In particular, we assume a proportion $\beta \in [0, 1]$ of consumers who otherwise do not know of the third-party seller's offering of the other product become aware of that offering. We call such a cross-product awareness increase the *spillover effect* of consumer awareness.

One unit of consumers is in the market, and each consumer has a unit demand for product 1. The utility that a consumer derives from a product is the valuation of the product net of its price. We normalize the valuation of buying product 1 from the retailer to 1, and denote the prices charged by the retailer and the third-party seller for product 1 as p_r and p_s, respectively. Thus, a consumer's utility of buying from the retailer is $1 - p_r$ and of buying from the third party is $k - p_s$. A consumer can only buy from a seller the consumer knows. For product 1, when a consumer is aware of both sellers, the consumer buys from the seller that gives the higher utility. When a consumer is only aware of the retailer, the consumer buys from it as long as the utility is nonnegative. We assume that the marginal cost of acquiring and selling product(s) is zero for both firms. Product 2 is independent of product 1. We denote as t the expected profit from product 2 for the third-party seller from each additional consumer who becomes aware of its product 2. For ease of exposition and to focus on more interesting cases, we assume t is not too small (e.g., $t > \frac{1}{4}$). Otherwise, the third-party seller would be more likely attracted to sell product 2 than product 1 on the retailer's platform, and we can show that the third party selling only product 1 on the retailer's platform might not arise as an equilibrium because of the low spillover benefit to product 2 and the competition against the retailer in product 1 market.

We first consider a game in which the retailer opens its platform and charges an exogenous commission rate θ. The timing of this game is as follows. In stage 1, the third-party seller chooses whether to join the retailer's platform. If it does so, the third-party seller may choose to sell product 1 only, product 2 only, or both products on the retailer's platform. In stage 2, the retailer and the third-party seller price their products, and consumers make their purchase decisions. We then consider a super game in which the retailer first decides whether to open its platform and, if so, sets the commission rate θ.

Figure 1. Product 1 Market Segmentation Under Different Scenarios



4. Third Party's Equilibrium Choice Under an Exogenous Commission Rate

In this section, we first examine the pricing competition subgames in stage 2 and present the equilibrium outcome. We then examine the equilibrium outcome of stage 1 and present the conditions under which the third party would sell no products, product 1, product 2, or products 1 and 2 on the retailer's platform, for a given commission rate θ.

4.1. Pricing Competition

Considering the third party's different choices in stage 1, we distinguish four scenarios: (I) the third party does not sell on the retailer's platform and sells its products separately (scenario I); (II) the third party sells product 2 on the retailer's platform (scenario II); (III) the third party sells product 1 on the retailer's platform (scenario III); and (IV) the third party sells both products on the retailer's platform (scenario IV).

Scenario I: Selling Separately. When the third party and the retailer sell their products separately, a portion α of consumers is aware of both sellers' offering of product 1, and a portion $(1-\alpha)$ of consumers is only aware of the retailer's offering of product 1. Note that $(1-\alpha)$ reflects the retailer's awareness advantage over the third party, which is also the retailer's exclusive demand. The consumers who are aware of both offerings will buy from the retailer as long as $1-p_r \geq k-p_s$. Therefore, among all consumers who are aware of both offerings, those with $k \leq 1-(p_r-p_s)$ buy from the retailer and the others buy from the third party. Figure 1(a) illustrates the market segmentation for product 1. We can formulate the profit functions for both firms as

$$\begin{aligned}\pi_r(p_r,p_s) &= p_r[\alpha(1-(p_r-p_s)) + (1-\alpha)],\\ \pi_s(p_s,p_r) &= \alpha p_s(p_r-p_s) + \alpha t,\end{aligned} \tag{1}$$

where $p_i \in [0,1]$, $i \in \{s,r\}$, and αt is the third party's profit from selling product 2.

Both firms maximize their profits by choosing optimal prices. Based on the best response functions, we can derive the equilibrium prices. Furthermore, by substituting the equilibrium prices into the profit functions in Equation (1), we can obtain the equilibrium profits. The following lemma summarizes the equilibrium outcome.

Lemma 1. *If the third party sells its products separately, not on the retailer's platform, the equilibrium prices are*

$$\begin{cases} p_r^* &= \min\{\frac{2}{3\alpha},1\},\\ p_s^* &= \frac{p_r^*}{2},\end{cases} \tag{2}$$

and the equilibrium profits are

$$\pi_r^* = \begin{cases}\frac{2-\alpha}{2} & \text{if } \alpha \leq \frac{2}{3},\\ \frac{4}{9\alpha} & \text{otherwise},\end{cases}$$
$$\pi_s^* = \begin{cases}\frac{\alpha}{4}+\alpha t, & \text{if } \alpha \leq \frac{2}{3},\\ \frac{1}{9\alpha}+\alpha t & \text{otherwise}.\end{cases} \tag{3}$$

Proof. All proofs are in the appendix unless indicated otherwise.

When the retailer has a significant awareness advantage (i.e., $(1-\alpha) \geq \frac{1}{3}$), $p_r^* = 1$ according to Equation (2). The retailer simply charges the monopoly price to fully exploit its exclusive demand and the residual demand left by the third party among the consumers who are aware of both sellers' offering of product 1. In contrast, when its awareness advantage is small (i.e., $(1-\alpha) < \frac{1}{3}$), the retailer competes with the third party aggressively and in equilibrium charges a price less than 1. Regardless of the equilibrium pricing strategies used, the retailer's equilibrium profit increases in its awareness advantage; that is, π_r^* increases in $(1-\alpha)$.

The third-party seller's profit consists of two parts. The first part (the first term in π_s^*) is the revenue from selling product 1, and the second part (the second term in π_s^*) is the revenue from selling product 2. Intuitively, the revenue from selling product 2 always monotonically increases in consumer awareness level (i.e., α), because a higher awareness level means a higher demand for product 2. In contrast, the revenue from selling product 1 is not monotonic, reaching a maximum at $\alpha = \frac{2}{3}$. When $\alpha \leq \frac{2}{3}$, the third party's profit from selling product 1 increases in α, and when $\alpha > \frac{2}{3}$, the profit decreases in α. The intuition for such a dichotomy is as follows. In the presence of a salient awareness advantage, the retailer is induced to simply charge the monopoly price and forgo competition. As a result, the third party can also charge a high price, and the demand and thus the profit increase in its awareness level. In contrast, when the third party's awareness level is comparable to the retailer's, the small gap triggers an intense competition between them and the retailer competes aggressively with the third party to benefit from its valuation advantage. In this case, a higher α leads to a more intense competition and results in a lower profit for the third party.

Scenario II: Selling Product 2 Through the Retailer. When the third party sells product 2 on the retailer's platform, it pays commission θ of each sale of product 2 to the retailer, which is the cost for the third party. The direct benefit of selling product 2 is the increased awareness: When the third party joins the retailer's platform, all consumers are aware of the third party's offering of product 2. Furthermore, an additional $\beta(1-\alpha)$ consumers become aware of product 1 because of the spillover effect, in addition to the *initial awareness* α. The *mixed awareness*, μ, includes the initial awareness and the spillover effect:

$$\mu = \alpha + \beta(1-\alpha). \tag{4}$$

Similar to scenario I, for product 1, the consumers who are aware of both offerings of product 1 buy from the retailer as long as $1 - p_r \geq k - p_s$. Thus, those with $k \leq 1 - (p_r - p_s)$ buy from the retailer and the rest buy from the third party. Figure 1(b) illustrates the market segmentation for product 1. We can formulate the profit functions for both firms as

$$\begin{aligned} \pi_r(p_r, p_s) &= p_r\big[\mu\big(1-(p_r-p_s)\big) + (1-\mu)\big] + \theta t, \\ \pi_s(p_s, p_r) &= \mu p_s\big(p_r - p_s\big) + (1-\theta)t. \end{aligned} \tag{5}$$

As in scenario I, both firms optimize their profits by determining the optimal prices. Based on the best response functions, we can derive the equilibrium prices. Furthermore, we can obtain the equilibrium profits by substituting the equilibrium prices into the profit functions. Lemma 2 summarizes the equilibrium outcome.

Lemma 2. *If the third party sells product* 2 *on the retailer's platform, the equilibrium prices are*

$$\begin{cases} p_r^* &= \min\left\{\frac{2}{3\mu}, 1\right\}, \\ p_s^* &= \frac{p_r^*}{2}, \end{cases} \tag{6}$$

and the equilibrium profits are

$$\begin{aligned} \pi_r^* &= \begin{cases} \frac{2-\mu}{2} + \theta t & \text{if } \mu \leq \frac{2}{3}, \\ \frac{4}{9\mu} + \theta t & \text{otherwise}, \end{cases} \\ \pi_s^* &= \begin{cases} \frac{\mu}{4} + (1-\theta)t, & \text{if } \mu \leq \frac{2}{3}, \\ \frac{1}{9\mu} + (1-\theta)t & \text{otherwise}. \end{cases} \end{aligned} \tag{7}$$

Note that the retailer does not compete with the third party in the market for product 2. The third party puts product 2 on the retailer's platform to increase its awareness, as well as to increase the awareness level for product 1 via the spillover effect. Similar to scenario I, when the retailer has significant awareness advantage for product 1 (i.e., $\mu \leq \frac{2}{3}$), the retailer simply charges a monopoly price (i.e., $p_r^* = 1$) to fully exploit its exclusive demand and the residual demand left by the third party. In contrast, when its awareness advantage is small (i.e., $\mu > \frac{2}{3}$), the retailer competes with the third party aggressively and in equilibrium charges a price less than 1. Regardless of the equilibrium pricing strategies used, the spillover effect decreases the retailer's awareness advantage and thus its profit.

Similar to scenario I, the relative awareness advantage, measured by $(1-\mu)$, alters the competition for product 1 between the two sellers. When the mixed awareness level is $\frac{2}{3}$, the third party obtains the maximum equilibrium profit: When the mixed awareness level is low, the third party's profit increases in μ; when it is high, the profit decreases. Because the mixed awareness is determined by the initial awareness and the spillover effect as defined in Equation (4), the spillover effect on the third party's profit is also not monotonic.

Scenario III: Selling Product 1 Through the Retailer. When the third party sells product 1 on the retailer's platform (and pays commission rate θ), all consumers become aware of this offering. Meanwhile, a proportion μ of consumers becomes aware of the third party's offering of product 2 via the spillover effect. Similar to the previous scenarios, consumers will buy product 1 from the retailer as long as $1 - p_r \geq k - p_s$, and thus, those with $k \leq 1 - (p_r - p_s)$ buy from the retailer and the rest buy from the third party. Figure 1(c) illustrates the market segmentation for product 1. We can formulate the profit functions for both firms as

$$\begin{aligned}\pi_r(p_r, p_s) &= p_r[1-(p_r-p_s)] + \theta p_s(p_r-p_s),\\ \pi_s(p_s, p_r) &= (1-\theta)p_s(p_r-p_s) + \mu t.\end{aligned} \tag{8}$$

Following the same approach as in scenarios I and II, we can derive the equilibrium outcome as summarized in Lemma 3.

Lemma 3. *If the third party sells product* 1 *on the retailer's platform, the equilibrium prices are*

$$\begin{cases} p_r^* &= \frac{2}{3-\theta},\\ p_s^* &= \frac{p_r^*}{2},\end{cases} \tag{9}$$

and the equilibrium profits are

$$\begin{cases} \pi_r^* &= \frac{4-\theta}{(3-\theta)^2},\\ \pi_s^* &= \frac{1-\theta}{(3-\theta)^2} + \mu t.\end{cases} \tag{10}$$

Two observations regarding the competition for product 1 are worth highlighting. First, the prices of product 1 increase in the commission rate θ. When the retailer charges a higher commission rate, a larger proportion of the third party's sales go to the retailer's revenue, and thus the competition between the third party and the retailer is softened, which results in a higher equilibrium product price. Second, we can verify that the retailer's profit increases and the third party's profit decreases in the commission rate θ. Intuitively, a higher commission rate directly benefits the retailer because a larger proportion of the third party's revenue goes to the retailer. In addition, a higher commission rate also softens the competition, which indirectly benefits the retailer. A higher commission rate hurts the third party because the third party has to transfer a larger proportion of its revenue to the retailer, and it is not compensated by the benefit of softened competition.

Note that, in this case, because the retailer does not compete in the product 2 market, the third party always benefits from the spillover effect (captured by β) via an increased mixed awareness and thus demand for product 2.

Scenario IV: Selling Both Products Through the Retailer. When the third party sells both products 1 and 2 on the retailer's platform, all consumers become aware of both offerings from the third party. Similar to the previous scenarios, we can formulate the profit functions for both firms as

$$\begin{aligned}\pi_r(p_r, p_s) &= p_r[1-(p_r-p_s)] + \theta p_s(p_r-p_s) + \theta t,\\ \pi_s(p_s, p_r) &= (1-\theta)p_s(p_r-p_s) + (1-\theta)t.\end{aligned} \tag{11}$$

Following the same approach as in the previous scenarios, we can derive the equilibrium outcome as summarized in Lemma 4.

Lemma 4. *If the third party sells both products* 1 *and* 2 *on the retailer's platform, the equilibrium prices are*

$$\begin{cases} p_r^* &= \frac{2}{3-\theta},\\ p_s^* &= \frac{p_r^*}{2},\end{cases} \tag{12}$$

and the equilibrium profits are

$$\begin{cases} \pi_r^* &= \frac{4-\theta}{(3-\theta)^2} + \theta t,\\ \pi_s^* &= \frac{1-\theta}{(3-\theta)^2} + (1-\theta)\,t.\end{cases} \tag{13}$$

Because the retailer does not compete with the third party in the product 2 market, having both products on the retailer's platform (in this scenario) or having product 1 only (in scenario III) does not alter the competition between the two sellers for product 1. As a result, the equilibrium prices in this scenario take the same forms as in scenario III (in Equation (9)).

In the same spirit, the profits from product 1 for both firms stay the same as in scenario III. In addition, the retailer makes commission θt by letting the third party sell product 2 on its platform, and the third party keeps the rest from selling product 2. Altogether, similar to scenario III, the retailer's profit increases, but the third party's profit decreases in the commission rate θ.

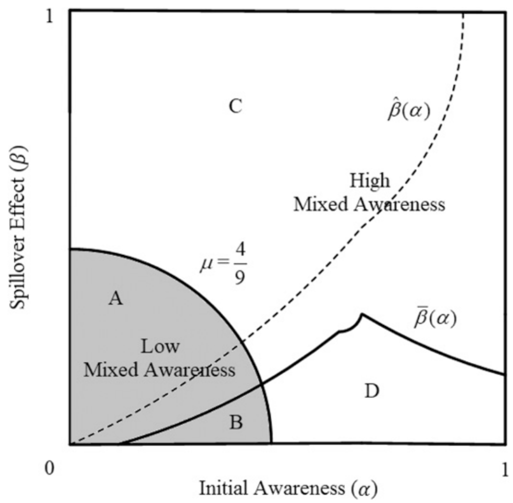
4.2. Third Party's Selling Choice

With the open platform and given commission rate θ, the third party has the above four selling choices. By comparing the equilibrium profit under scenario I (selling separately) with that under scenario II (selling product 2 through the retailer) and with that under scenario III (selling product 1 through the retailer), because of the monotonicity in the commission rate, we can derive the highest commission rates, θ_{s2}^* and θ_{s1}^*, under which the third party might still have incentive to sell product 2 and sell product 1 on the retailer's platform, respectively. These thresholds are functions of the initial awareness level, α, and the spillover effect, β. We note that θ_{s2}^* might be higher or lower than θ_{s1}^* in different cases. We define $\bar{\beta}(\alpha)$ as the boundary on which these two thresholds are equal to each other; that is,

$$\bar{\beta}(\alpha) \text{ is the solution to } \theta_{s2}^*(\alpha,\beta) = \theta_{s1}^*(\alpha,\beta). \tag{14}$$

Similarly, by comparing the equilibrium profits under scenarios II and III, we can derive the indifferent commission rate, θ_{21}^*, under which the third party is indifferent about selling product 1 only or selling product 2 only on the retailer's platform.

Figure 2. Third Party's Equilibrium Choice Under Different Awareness Structures ($t = 1$)



By comparing the equilibrium payoffs under these four selling choices, we can derive the conditions under which the third party chooses, in equilibrium, to sell no products, product 1 only, product 2 only, or both products 1 and 2 on the retailer's platform, as summarized in Proposition 1.

Proposition 1. *For a given commission rate θ, the third party sells on the retailer's platform only in the following cases:*

a. *When the mixed awareness is low (i.e., $\mu \le \frac{4}{9}$),*

a.1. *if the spillover effect is mild such that $\beta < \bar{\beta}(\alpha)$, when $\theta \le K(\frac{\mu}{4})$, the third party sells both products 1 and 2; when $\theta \in (K(\frac{\mu}{4}), \theta^*_{s2}]$, the third party sells product 2;*

a.2. *if $\beta \ge \bar{\beta}(\alpha)$, when $\theta \le K(\frac{\mu}{4})$, the third party sells both products 1 and 2; when $\theta \in (K(\frac{\mu}{4}), \theta^*_{21}]$, the third party sells product 2; when $\theta \in (\theta^*_{21}, \theta^*_{s1}]$, the third party sells product 1.*

b. *When the mixed awareness is high (i.e., $\mu > \frac{4}{9}$),*

b.1. *if $\beta < \bar{\beta}(\alpha)$, when $\theta \le \theta^*_{s2}$, the third party sells product 2;*

b.2. *if $\beta \ge \bar{\beta}(\alpha)$, when $\theta \le \theta^*_{21}$, the third party sells product 2; when $\theta \in (\theta^*_{21}, \theta^*_{s1}]$, the third party sells product 1, where*

$$K(x) = \frac{-1 + 6x + \sqrt{1 - 8x}}{2x}, \tag{15}$$

$\bar{\beta}(\alpha)$ is defined as in Equation (14), and θ^*_{s2}, θ^*_{s1}, and θ^*_{21} are defined as in Equations (20), (23), and (25) in the appendix.

Recall that the mixed awareness (μ) is a function of the initial awareness level (α) and the extent of the spillover effect (β), as defined in Equation (4). Proposition 1 prescribes the third party's optimal selling strategies along with α, β, and θ: how the equilibrium choice varies with its initial awareness level, the extent of the spillover effect, and the commission rate. To make it more concrete, we use the case with $t = 1$ to illustrate the optimal selling strategies, as depicted in Figure 2 (in which the two solid lines define four areas: A, B, C, and D) and Table 1. Specifically, when the expected mixed awareness is low, if the spillover effect relative to initial awareness is salient (area A in Figure 2), the third party sells products 1 and 2, product 2, product 1, and no products, as the commission rate varies from low to high; if the relative spillover effect is mild (area B in Figure 2), the third party sells products 1 and 2, product 2, and no products, as the commission rate varies from low to high. When the expected mixed awareness is high, if the relative spillover effect is salient (area C in Figure 2), the third party sells product 2, product 1, and no products, as the commission rate varies from low to high; if the relative spillover effect is mild (area D in Figure 2), the third party sells product 2 and no products, as the commission rate varies from low to high.

Table 1. Third Party's Equilibrium Choice Under Different Awareness Structures ($t = 1$)

	Commission rate θ		
Region	Products 1 and 2	Product 2	Product 1
A	$[0, K(\frac{\mu}{4})]$	$(K(\frac{\mu}{4}), \theta^*_{21}]$	$(\theta^*_{21}, \theta^*_{s1}]$
B	$[0, K(\frac{\mu}{4})]$	$(K(\frac{\mu}{4}), \theta^*_{s2}]$	Never
C	Never	$[0, \theta^*_{21}]$	$(\theta^*_{21}, \theta^*_{s1}]$
D	Never	$[0, \theta^*_{s2}]$	Never

A few observations are worth highlighting. First, the third party sells both products 1 and 2 on the retailer's platform only if both the commission rate and the expected mixed awareness are low. A low commission rate means the cost of doing so is low. A low mixed awareness level means the benefit of doing so is high, because selling on the retailer's platform increases the awareness from α or μ to 1. Otherwise, the third party would have chosen to sell no products on the retailer's platform if the commission rate is high or to sell one product on the platform and increase the awareness of the other product by leveraging the spillover effect if the commission rate is not too high but the mixed awareness is high.

Second, when the commission rate is low or intermediate, if it chooses only one product to sell on the retailer's platform, the third party chooses product 2 rather than product 1. Selling product 2 on the platform can increase the demand for product 2 to the greatest extent, thus benefiting the third party, because the retailer does not compete with the third party in the product 2 market. In contrast, selling product 1 on the platform may increase the competition between the two sellers in the product 1 market, while leaving the demand potential for product 2 out. When the commission rate is high, the retailer chooses to sell product 1 or not to sell at all.

Third, even when the commission rate is very high (e.g., $\theta = 1$ to the extreme), the third party may still have incentive to sell product 1 on the retailer's platform. In particular, when the spillover effect relative to the initial awareness level is significant, especially, when $\beta \geq \hat{\beta}(\alpha)$ in Figure 2, where

$$\hat{\beta}(\alpha) = \begin{cases} \frac{\alpha}{4(1-\alpha)t} & \text{if } \alpha \leq \frac{2}{3}, \\ \frac{1}{9\alpha(1-\alpha)t} & \text{otherwise,} \end{cases} \tag{16}$$

even if the retailer asks for all of the third party's sales on the retailer's platform (i.e., $\theta = 1$), the third party still optimally chooses to put product 1 on the retailer's platform. This surprising result is because, in this case, the spillover effect is more important than the initial awareness, and the benefit of increased demand for product 2 resulting from the spillover effect outweighs the cost of contributing the revenue from product 1 to the retailer.

Fourth, when $\beta < \bar{\beta}(\alpha)$ such that the spillover effect is mild (with no spillover effect, $\beta = 0$, as the extreme case), the third party has no incentive to only sell product 1 on the retailer's platform. In this case, when the commission rate is low, only selling product 1 is not as good as only selling product 2, because product 2, for which the retailer does not compete, can benefit from the increased awareness to the greatest extent, whereas selling product 1 on the retailer's platform triggers competition with the retailer. When the commission rate is high, the benefit from increased awareness of selling product 1 on the retailer's platform, together with a limited benefit from the spillover effect, cannot compensate for the cost of a high commission. Therefore, when the spillover effect is mild, selling just product 1 on the retailer's platform cannot arise as an equilibrium. Combining the above third and fourth observations, we can see that the spillover effect generally makes the third party more likely to sell product 1 on the retailer's platform.

5. Spillover Effects on Retailer's Openness Decision

In this section, we first investigate how the spillover effect of consumer awareness affects the retailer's decision of opening its platform to the third-party seller. We then examine the impact of the spillover effect on the retailer's and the third party's profits when the retailer strategically chooses whether to open its platform.

5.1. Retailer's Decision: Whether to Open Its Platform

In the super game, the retailer decides whether to open its platform and allow the third-party seller to sell on its platform, and if so, the retailer also decides the commission rate charged to the third party. Proposition 1 has prescribed how the third party reacts to the commission rate, depending on the initial awareness level and the extent of the spillover effect. Anticipating the third party's reaction, if deciding to open its platform, the retailer can deduce the optimal commission rate and thus the optimal profit. Comparing the optimal profits when opening its platform and when not opening, the retailer chooses whether to open its platform.

First, we show that the retailer has no incentive to induce the third party to sell both products on its platform, as summarized in the following lemma.

Lemma 5. *Selling both products 1 and 2 on the retailer's platform cannot arise as an equilibrium in the super game.*

Proposition 1 establishes that the third party has incentive to sell both products 1 and 2 on the retailer's platform only when both mixed awareness and the commission rate are low (i.e., $\mu \leq \frac{4}{9}$ and $\theta \leq K(\frac{\mu}{4})$). Essentially, the above lemma implies that, when the mixed awareness is low, the retailer will not open its platform and charge a low commission rate. The intuition is as follows. When the mixed awareness is low, the retailer can benefit from its significant awareness advantage by discouraging the third party from selling the identical product on its platform. A low commission rate might induce the third party to sell the identical product on the retailer's platform, thus significantly cutting the retailer's awareness advantage in the competition for that product. Instead, the retailer is better off by charging a higher commission rate, which induces the third party to sell only product 2 (by Proposition 1), resulting in more commission from product 2 and less-intense competition for product 1. Therefore, when the mixed awareness is low, the retailer has no incentive to open its platform and charge a low commission rate. Subsequently, the third party will not sell both products on the platform.

Next, we examine the conditions under which the retailer can benefit if the third party only sells product 2 or product 1 on its platform.

Lemma 6.

a. *If the third party only sells product 2 on the retailer's platform, the retailer can benefit if and only if $\theta \in (\theta^*_{r2}, 1]$, where θ^*_{r2} is defined as*

$$\theta^*_{r2} = \begin{cases} \frac{(1-\alpha)\beta}{2t} & \text{if } \mu \leq \frac{2}{3}, \\ \frac{1}{t}\left(1 - \frac{\alpha}{2} - \frac{4}{9\mu}\right) & \text{if } \mu > \frac{2}{3} \text{ and } \alpha \leq \frac{2}{3}, \\ \frac{1}{t}\left(\frac{4}{9\alpha} - \frac{4}{9\mu}\right) & \text{if } \alpha > \frac{2}{3}. \end{cases} \tag{17}$$

b. *If the third party only sells product* 1 *on the retailer's platform, the retailer can benefit if and only if* $\alpha \in (\frac{1}{2}, 1)$ *and* $\theta \in (\theta_{r1}^*, 1]$, *where* θ_{r1}^* *is defined as*

$$\theta_{r1}^* = \begin{cases} \frac{5-3\alpha-\sqrt{5-2\alpha}}{2-\alpha} & \text{if } \alpha \le \frac{2}{3}, \\ \frac{24-9\alpha-3\sqrt{9\alpha^2+16\alpha}}{8} & \text{if } \alpha > \frac{2}{3}. \end{cases} \tag{18}$$

When the third party sells product 2 on the retailer's platform, in the absence of the spillover effect (i.e., $\beta = 0$), the retailer benefits under any positive commission rate, because the retailer does not compete with the third party in the product 2 market and any commission from product 2 is pure profit for the retailer. However, in the presence of the spillover effect, having the third party sell product 2 on the retailer's platform reduces the retailer's awareness advantage in the competition for product 1. As a result, if and only if the commission rate is high enough (as prescribed in Lemma 6), the commission generated from product 2 for the retailer can compensate for its revenue loss in product 1 resulting from the reduced awareness advantage.

When the third party sells product 1 on the retailer's platform, all consumers become aware of its offering, which reduces the retailer's awareness advantage and hurts its profit from product 1. As a result, if and only if the commission rate is high enough (as prescribed in Lemma 6), the commission generated from product 1 might compensate its revenue loss in its direct profit from selling product 1. It is worth highlighting that this tradeoff between commission and direct profit loss can work for the retailer only when its awareness advantage is mild (i.e., $1 - \alpha < \frac{1}{2}$). When its awareness advantage is salient, even if the retailer receives all the product 1 revenue from the third party, the retailer is still better off by reaping its salient awareness advantage via excluding the third party from selling on its platform.

Next, we present the retailer's equilibrium decision on whether to open its platform and the third party's equilibrium selling strategies when the retailer's platform is open. For ease of exposition, we use the notation with subscript "2" (e.g., π_{r2}^*) to represent the equilibrium under "selling product 2 through the retailer" (scenario II) and the notation with subscript "1" (e.g., π_{r1}^*) to represent the equilibrium under "selling product 1 through the retailer" (scenario III).

Proposition 2. *In equilibrium, the retailer opens its platform in the following cases:*

a.1. *when the spillover effect is mild (i.e.,* $\beta < \bar{\beta}(\alpha)$*) and the retailer can benefit from inducing the third party to sell product* 2 *on its platform (i.e.,* $\theta_{r2}^* < \theta_{s2}^*$*);*

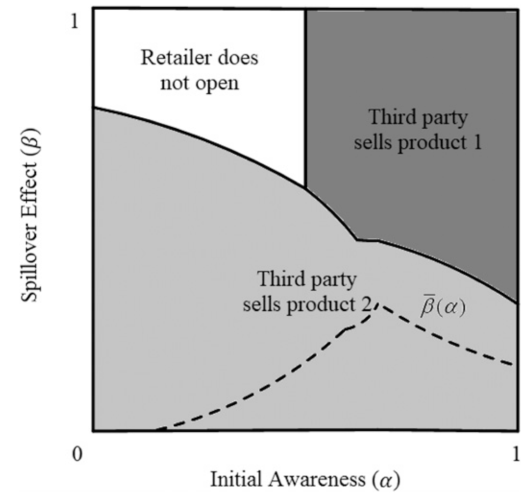
a.2. *when the spillover effect is salient (i.e.,* $\beta \ge \bar{\beta}(\alpha)$*), the third party's initial awareness is low (i.e.,* $\alpha \le \frac{1}{2}$*), and the retailer can benefit from inducing the third party to sell product* 2 *on its platform (i.e.,* $\theta_{r2}^* < \theta_{21}^*$*);*

a.3. *when the spillover effect is salient (i.e.,* $\beta \ge \bar{\beta}(\alpha)$*), the third party's initial awareness is high (i.e.,* $\alpha > \frac{1}{2}$*), and the retailer can benefit from inducing the third party to sell product* 2 *on its platform and benefit more than that from inducing the third party to sell product* 1 *(i.e.,* $\theta_{r2}^* < \theta_{21}^*$ *and* $\pi_{r2}^*(\theta_{21}^*) > \pi_{r1}^*(\min\{\theta_{s1}^*, 1\})$*);*

b. *when the spillover effect is salient (i.e.,* $\beta \ge \bar{\beta}(\alpha)$*), the third party's initial awareness is high (i.e.,* $\alpha > \frac{1}{2}$*), and the retailer can benefit from inducing the third party to sell product* 1 *on its platform and benefit more than that from inducing the third party to sell product* 2 *(i.e.,* $\theta_{r1}^* < \theta_{s1}^*$ *and* $\pi_{r2}^*(\theta_{21}^*) < \pi_{r1}^*(\min\{\theta_{s1}^*, 1\})$*).*

In cases (a.1)–(a.3), the third party sells product 2 *on the platform, and in case (b) the third party sells product* 1*. The thresholds* $\bar{\beta}(\alpha)$, θ_{s2}^*, θ_{s1}^*, θ_{21}^*, θ_{r2}^*, *and* θ_{r1}^* *are defined in Equations* (14), (20), (23), (25), (17), *and* (18)*, respectively, and the profit functions* π_{r2}^* *and* π_{r1}^* *are defined in Equations* (7) *and* (10).

Figure 3. Equilibrium Outcome in the Super Game ($t = 1$)



Proposition 2 shows that the retailer's equilibrium openness decision depends on the extent of the spillover effect and the third party's initial awareness level. To make it more concrete, we use the case with $t = 1$ to illustrate the equilibrium outcome, as depicted in Figure 3 (in which the solid lines define three possible outcomes). Specifically, when the spillover effect is mild (the gray area), the retailer opens its platform and the third party sells product 2 on the retailer's platform; when the spillover effect is salient but the initial awareness level of the third party is low (the white area), the retailer does not open its platform; when the spillover effect is salient and the initial awareness level of the third party is high (the dark area), the retailer opens its platform and the third party sells product 1 on the retailer's platform.

The intuition is as follows. First, when the spillover effect is mild (the gray area), the retailer induces the third party to sell product 2 on its platform because doing so can earn the commission from product 2 while keeping its relative awareness advantage to a large extent in the product 1 competition. In contrast, having the third party sell product 1 on the retailer's platform makes all consumers aware of the third party's offering of product 1, which is detrimental to the retailer's revenue from product 1. In this case, when the retailer has great awareness advantage, its revenue loss from product 1 is significant such that even a high commission cannot compensate for the loss. When the retailer's awareness advantage is mild, the third party has little incentive to pay a high commission for selling product 1 on the retailer's platform because of the limited benefit for product 1 and small spillover for product 2.

Second, when the spillover effect is salient, if the retailer opens its platform, regardless of which product the third party sells on the platform, the retailer always suffers from strong competition for product 1. Note that, even if the third party is induced to sell product 2 on its platform, the retailer is also hurt in the product 1 competition because of the salient spillover effect. In this case, whether to open depends on the extent of the retailer's awareness advantage. When the awareness advantage is salient (the white area), the best choice for the retailer is not to open its platform because, otherwise, the commission cannot be large enough to compensate for its revenue loss in the product 1 competition resulting from its lost awareness advantage. When the awareness advantage is mild (the dark area), the retailer induces the third party to sell product 1, because opening its platform only renders a small awareness advantage loss for the retailer regardless of which product the third party chooses to sell on the platform. In addition, according to Proposition 1, a low/high commission rate induces the third party to sell product 2/1, so the retailer is better off with the high commission rate. As a result, the retailer opens its platform and charges a high commission rate, inducing the third party to sell the identical product on the open platform.

5.2. Spillover Effect on the Retailer and the Third Party

In this section, we use the case without the spillover effect (i.e., $\beta = 0$) as the benchmark and analyze the spillover effect on the retailer's and the third party's equilibrium choices and profits. First, by letting $\beta = 0$ in Proposition 2, we draw the following conclusion.

Corollary 1. *Without the spillover effect (i.e., $\beta = 0$), in equilibrium the retailer opens its platform and the third party only sells product 2 on the retailer's platform.*

Combining Proposition 2 and Corollary 1, we observe two interesting results: (1) The spillover effect makes the retailer less likely to open its platform, and (2) the spillover effect can induce the third party to sell the identical product on the retailer's platform in equilibrium. Without the spillover effect, the commission always entices the retailer to allow the third party to sell the exclusive product on its platform. In contrast, in the presence of a strong spillover effect, when the third party's initial awareness level is low, the retailer chooses not to open its platform out of concern for the awareness-advantage loss. Otherwise, the third party sells product 2/1 under a low/high commission rate and either way increases the awareness level for its product 1, which explains result (1).

Result (2) can be seen in that, when the spillover effect is salient and the retailer's awareness advantage is mild (the dark area in Figure 3), in equilibrium the third party sells product 1 on the retailer's platform. The third party is attracted to this partnership because of the strong spillover effect for its product 2, and the retailer is attracted to this partnership by the high commission that does not seriously hurt its awareness advantage (because its awareness advantage is already mild).

Next, we show how the retailer's and third party's equilibrium profits change with the extent of the spillover effect. We also show the spillover effect on the retailer's and third party's profits by comparing the equilibrium profits with and without the spillover effect. According to Corollary 1, the equilibrium profit without the spillover effect can be derived by substituting the equilibrium commission rate into the profit functions in Lemma 2. The profits with the spillover effect can be derived for different cases according to Proposition 2.

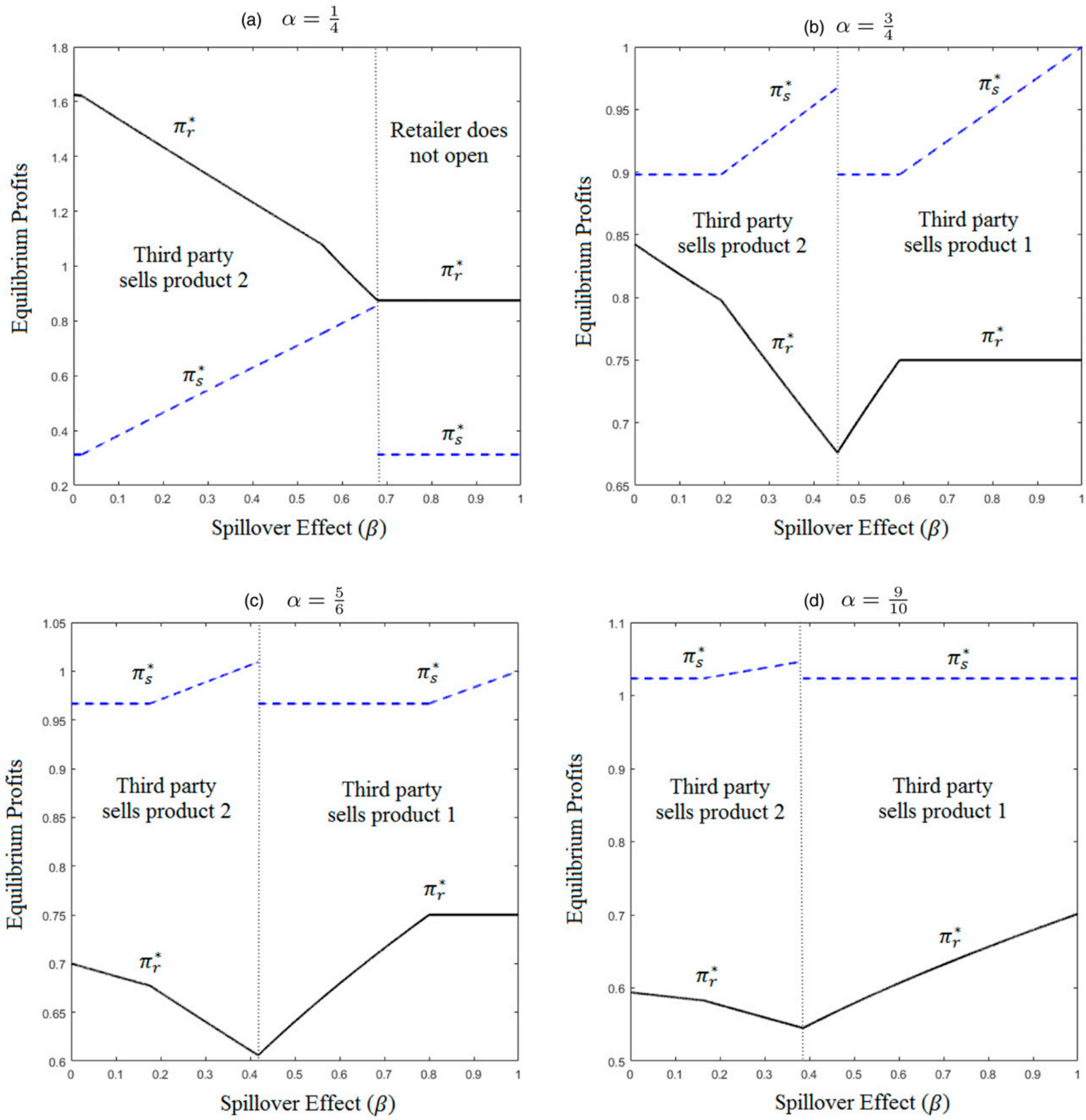
Proposition 3.

a. *Within the equilibrium in which the third party sells product 2 on the retailer's platform, the third party's profit (weakly) increases and the retailer's profit (weakly) decreases in the extent of the spillover effect. Within the equilibrium in which the third party sells product 1 on the retailer's platform, the third party's and the retailer's profits (weakly) increase in the extent of the spillover effect.*

b. *Compared with the case without the spillover effect (i.e., $\beta = 0$), the spillover effect always (weakly) benefits the third party, and it might benefit the retailer only when the retailer opens its platform and the third party sells product 1 on the platform.*

Figure 4 illustrates the comparative statics identified in Proposition 3(a)—the impact of the spillover effect on the retailer's and the third party's equilibrium profits—under different cases. As shown in the figure, the impact of the spillover effect is not monotonic. Such nonmonotonicity is primarily due

Figure 4. Spillover Effect on the Retailer's and Third Party's Profits ($t = 1$)



to the strategic interaction between the retailer and the third party—anticipating the third party's selling-strategy choice, the retailer strategically chooses whether to open its platform, and if so, the retailer optimally chooses the commission rate according to the extent of the spillover effect, which results in different partnerships in equilibrium. Consequently, we also observe discontinuities in the third party's equilibrium profit function. The discontinuities occur at the points where the equilibrium selling partnership shifts from one state to another (e.g., from the third party's selling product 2 to selling product 1 in Figure 4, (b), (c), and (d)).

In addition, we also observe kinks in the retailer's profit function within each type of equilibrium because of its strategic choices. For example, in the equilibrium in which the third party sells product 2 on the retailer's platform, the kinks occur at the points where the retailer starts strategically lowering the commission rate to induce the third party to sell product 2 instead of product 1 on its platform (e.g., when β reaches $\bar{\beta}(\alpha)$ in Figure 2). When the spillover effect relative to initial awareness is mild (i.e., $\beta < \bar{\beta}(\alpha)$), the retailer can charge a high commission rate without concern that the third party might sell product 1 on its platform.

When the relative spillover effect becomes salient (i.e., $\beta > \bar{\beta}(\alpha)$), the retailer must lower the commission rate to discourage the third party from selling the identical product on its platform. Corollary 2 summarizes the change in the equilibrium commission rate with the spillover effect.

Corollary 2. *Within the equilibrium in which the third party sells product* 2 *on the retailer's platform, the equilibrium commission rate increases in the extent of the spillover effect when* $\beta \le \bar{\beta}(\alpha)$ *and decreases when* $\beta > \bar{\beta}(\alpha)$*. Within the equilibrium in which the third party sells product* 1 *on the retailer's platform, the equilibrium commission rate (weakly) increases in the extent of the spillover effect.*

As demonstrated in Figure 4 and shown in Proposition 3(a), within the same equilibrium partnership in which the third party sells product 1 or 2 on the retailer's platform, the third party's profit (weakly) increases in the extent of the spillover effect. In other words, within the same equilibrium partnership, the spillover effect is, in general, beneficial to the third party. Specifically, in the equilibrium in which the third party sells the exclusive product on the retailer's platform, when the extent of the spillover effect increases beyond the kink point at $\bar{\beta}(\alpha)$, the retailer lowers the commission rate to induce the third party to sell the exclusive product instead of the identical product on its platform, benefiting the third party. When the third party sells the identical product on the retailer's platform, the awareness of the third party's exclusive product (not listed on the retailer's platform) increases in the extent of the spillover effect, benefiting the third party. However, across different equilibrium selling partnerships (e.g., from selling product 2 to selling product 1), the spillover effect can hurt the third party because of the retailer's strategic choice and dominant role. Despite the possible damage, Proposition 3(b) establishes that, compared with the case without the spillover effect, the spillover effect always benefits the third party regardless of its extent.

Surprisingly, we find that the retailer may also benefit from the spillover effect in some cases by strategically setting its platform policy. Proposition 3(a) first indicates that, within the equilibrium in which the third party sells product 2 on the retailer's platform, the retailer's profit decreases in the extent of the spillover effect (as illustrated in Figure 4). Intuitively, such a decrease is either because the increased awareness for the third party's product 1 resulting from spillover effect intensifies the competition with the retailer, or because the increased spillover effect limits the highest commission rate that the retailer can charge to prevent the third party from selling the identical product on its platform. Meanwhile, Proposition 3(a) also indicates that, in the equilibrium where the third party sells identical products on the retailer's platform, the retailer's profit increases in the extent of the spillover effect. This increase is because the gain brought by the spillover effect to the third party increases in the extent of the spillover effect and the retailer shares this gain via the commission. Therefore, Proposition 3(a) essentially states that the retailer's profit first decreases and then increases in the extent of the spillover effect. Proposition 3(b) establishes that it is possible that the increase can offset the decrease such that the retailer also benefits from the spillover effect compared with the case without it as shown in Figure 4, (c) and (d).

Furthermore, realizing that its profit depends on the spillover effect, the retailer might have incentive to manipulate the degree of the spillover effect. The manipulation can be implemented via its platform listing policy or design. For example, when selling on the retailer's platform, whether the third party is allowed to provide a link to its own web page affects the degree of the spillover effect. Next, we consider that to some extent the retailer can choose the spillover effect. For instance, we can consider that, given an organic spillover effect β_o, the retailer can enhance or dampen the spillover effect by an amount up to ε within the valid range $[0, 1]$; that is, the retailer can choose the degree of the spillover effect within $[\underline{\beta}, \overline{\beta}]$, where $\underline{\beta} = \max\{0, \beta_o - \varepsilon\}$ and $\overline{\beta} = \min\{1, \beta_o + \varepsilon\}$. Based on the monotonicity established in Proposition 3, we draw the following conclusion.

Corollary 3. *We assume that the retailer can manipulate the spillover effect and can choose an optimal* β *within* $[\underline{\beta}, \overline{\beta}]$.

a. *When the equilibrium without manipulation under any* $\beta \in [\underline{\beta}, \overline{\beta}]$ *is that the retailer opens its platform and the third party sells product* 1 *on the platform, the retailer optimally chooses the highest spillover effect* $\overline{\beta}$.

b. *When the equilibrium without manipulation under any* $\beta \in [\underline{\beta}, \overline{\beta}]$ *is either that the retailer does not open its platform or that the retailer opens its platform and the third party sells product* 2 *on the platform, the retailer optimally chooses the lowest spillover effect* $\underline{\beta}$.

c. *When the equilibrium without manipulation within* $[\underline{\beta}, \overline{\beta}]$ *is that the retailer opens its platform and the third party may sell product* 1 *or* 2 *on the platform, the retailer optimally chooses the lowest spillover effect if* $\pi^*_{r2}(\underline{\beta}) > \pi^*_{r1}(\overline{\beta})$*; otherwise, the retailer optimally chooses the highest spillover effect, where* π^*_{r2} *and* π^*_{r1} *are the* π^*_r *defined in Equations* (7) *and* (10), *respectively.*

In general, when it is possible to manipulate the spillover effect, the retailer's equilibrium profit can be (weakly) improved by optimally choosing the degree of the spillover effect. Furthermore, the manipulation can even alter the equilibrium partnership between the retailer and the third party. For example, the boundary between "retailer does not open" and "third party sells product 2" in Figure 3 may shift upward, because under the spillover effect level that is close to but above the current boundary, the retailer may optimally dampen the spillover effect and choose to open its platform to have "third party sells product 2," whereas the optimal choice without manipulation would be "retailer does not open." In this case, the possible spillover-effect manipulation makes the retailer more likely to open its platform.

6. Retailer's Choice Under Exogenous Commission Rates

In analyzing the retailer's openness decision in Section 5, we endogenize the commission rate and consider that the retailer can choose an optimal commission rate because the commission rate is an important decision variable when the retailer chooses to open its platform. However, in some cases, the commission rate might be determined externally. For example, a well-established commission rate may have already been in use, making the retailer's best interest to follow the practice. In this section, we first study the retailer's openness decision under exogenous commission rates. We then examine whether the retailer has incentive to continue to carry product 1 in this case.[4]

6.1. Retailer's Openness Decision

We first examine the retailer's openness decision under an exogenous commission rate. The timing of the game remains the same as in the main model except that the commission rate θ is an exogenous parameter.

Under an exogenous commission rate, first, the third party's selling choices are the same as prescribed in Proposition 1. Second, the conditions under which the retailer can benefit when the third party only sells product 2 or sells product 1 on its platform are the same as in Lemma 6. Different from that in Section 5 (prescribed by Lemma 5), we cannot rule out the possibility that the third party sells both products on the retailer's platform. Next, we derive the conditions under which the retailer can benefit when the third party sells both products on its platform.

Lemma 7. *If the third party sells both products 1 and 2 on the retailer's platform, the retailer can benefit if and only if $\theta \in (\theta^*_{r3}, 1]$, where θ^*_{r3} is the solution to $\pi^*_{r3} - \pi^*_{r0} = 0$, and π^*_{r0} and π^*_{r3} are the π^*_r defined in Equations (3) and (13), respectively.*

The next proposition discusses the conditions under which the retailer opens its platform.

Proposition 4. *For an exogenous commission rate θ, the retailer opens its platform and the third party sells on the retailer's platform only in the following cases:*

a. *When the mixed awareness is low (i.e., $\mu \le \frac{4}{9}$), if $\theta \le K(\frac{\mu}{4})$ and $\theta > \theta^*_{r3}$, the third party sells both products 1 and 2; otherwise,*

a.1. *when $\beta < \bar{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta^*_{s2}]$ and $\theta > \theta^*_{r2}$, the third party sells product 2;*

a.2. *when $\beta \ge \bar{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta^*_{21}]$ and $\theta > \theta^*_{r2}$, the third party sells product 2, and if $\theta \in (\theta^*_{21}, \theta^*_{s1}]$ and $\theta > \theta^*_{r1}$, the third party sells product 1.*

b. *When the mixed awareness is high (i.e., $\mu > \frac{4}{9}$),*

b.1. *when $\beta < \bar{\beta}(\alpha)$, if $\theta \le \theta^*_{s2}$ and $\theta > \theta^*_{r2}$, the third party sells product 2;*

b.2. *when $\beta \ge \bar{\beta}(\alpha)$, if $\theta \le \theta^*_{21}$ and $\theta > \theta^*_{r2}$, the third party sells product 2, and if $\theta \in (\theta^*_{21}, \theta^*_{s1}]$ and $\theta > \theta^*_{r1}$, the third party sells product 1, where $K(\cdot)$, $\bar{\beta}(\alpha)$, θ^*_{s1}, θ^*_{s2}, and θ^*_{21} are the same as in Proposition 1, and θ^*_{r1}, θ^*_{r2}, and θ^*_{r3} are as defined in Lemmas 6 and 7.*

When the commission rate is exogenous, the retailer has little influence on the third party's product offering if it opens its platform, and whether to open its platform is the only choice left to the retailer. If the retailer chooses to open its platform, Proposition 1 prescribes the third party's equilibrium choice. Comparing the retailer's profits resulting from the third party's equilibrium choice in the case of an open platform with that in the case of a closed platform, we conclude the retailer's openness decision as in Proposition 4. Intuitively, anticipating the third party's product offering choice in the presence of an open platform, the retailer chooses to open its platform if the commission rate is high enough to benefit it; otherwise, the retailer chooses not to open.

6.2. Whether Retailer Should Carry Product 1

As shown in Section 5, commission rates can be used by the retailer to influence the third party's product offering when it has the choice. Under exogenous commission rates, the retailer does not have this choice. Alternatively, the retailer might strategically choose whether to continue to offer product 1 to influence the third party's product offering. Next, we consider a game in which the retailer first decides whether to carry product 1 and then whether to open its platform to the third-party seller. Everything else remains the same as in the main model. We first examine the third party's product offering decision and the retailer's platform openness decision when the retailer does not carry product 1. We then compare the retailer's profits in the subgames where the retailer does and does not carry product 1 and conclude whether the retailer has incentive to continue to carry product 1.

When it does not sell product directly, on the one hand, the retailer always has incentive to open its platform for the commission. On the other hand, whether the third party has incentive to join the platform and, if so, what product(s) will be offered on the platform depend on the commission rate. The following lemma summarizes the findings.

Lemma 8. *When the retailer does not carry product 1, the retailer opens its platform, and the third party's product offering on the platform depends on the commission rate:*

a. *If $\theta \le 1 - \mu$, the third party sells both products.*

b. *If $\theta \in (1 - \mu, (1 - \alpha)(1 + 4\beta t)]$, the third party sells product 1.*

c. *If $\theta > (1 - \alpha)(1 + 4\beta t)$, the third party does not sell on the retailer's platform.*

When the retailer does not carry product 1 and acts as a pure platform, the third party does not compete with it directly. Therefore, when the commission rate is low, the third party sells both products on the retailer's platform to increase their awareness levels. When the commission rate is medium, the commission is nonnegligible, and the third party strategically sells only one product on the retailer's platform for the direct increase in the awareness for that product as well as the benefit to the other product from the spillover effect. When the commission rate is high, the benefit from the increase in product awareness level cannot compensate for the high commission, and thus, the third party does not sell on the retailer's platform.

By comparing the retailer's profits when it carries product 1 and when it does not, we conclude whether the retailer should carry product 1, as summarized in the following proposition.

Proposition 5. *Under any exogenous commission rate θ, $\theta \in [0, 1]$, the retailer always chooses to carry product 1.*

Note that, when it does not carry product 1, the retailer plays as a pure online platform. Because the third party is strategic, if the commission rate is high, the third party does not sell products on the retailer's platform (by Lemma 8), and the retailer earns no profit from the third party. If the commission rate is medium, the third party sells product 1 on the retailer's platform (by Lemma 8). In this case, the mediocre commission is not as good as what the retailer can make by selling product 1 itself, given its valuation advantage in the competition with the third party. Therefore, if the commission rate is medium or high, the retailer has incentive to carry product 1.

If the commission rate is low (i.e., $\theta \le 1 - \mu$), the third party sells both products on the retailer's platform when the retailer does not carry product 1 (by Lemma 8). If the commission rate is very low, the commission is so limited that the retailer is better off by selling product 1 itself and not opening its platform. If the commission rate is not too low, the retailer is still better off by carrying product 1, because it can make profit from selling product 1 and can also earn a commission by strategically opening its platform.

7. Conclusion

We examine the spillover effect of consumer awareness on a third-party seller's product offering on a retailer's platform and further on the retailer's platform-openness decision. We develop a game-theoretic model in which a third-party seller carries a product identical to a retailer's product as well as an exclusive product that the retailer does not carry. We first examine the third-party seller's product-offering strategy if the retailer opens its platform and allows the third party to sell on it. We find that the third party's optimal selling strategies vary with the extent of the spillover effect, its initial awareness level, and the commission rate. We further examine the retailer's incentive to open its platform and find that the spillover effect plays an important role in its openness decision and the resulting partnership. When the spillover effect is mild, the retailer opens its platform and the third-party seller sells the exclusive product on the platform. When the spillover effect is salient, if the retailer has a large awareness advantage over the third party, the retailer has no incentive to open; otherwise, the retailer opens its platform and the third party sells the identical product on the platform.

Our study has several managerial implications for online sellers that consider joining retail platforms. First, our study underscores the importance of the cross-product spillover effect of consumer awareness in determining online sellers' selling strategies on giant retail platforms. For instance, when the spillover effect is salient, online sellers might even be better off by sacrificing all of its revenue generated from a retailer's platform for the spillover-effect benefit. Our study thus calls online sellers' attention to the strategic role of the spillover effect. Furthermore, to benefit from the strategic selling choice, online sellers should use their accumulated data or conduct field experiments to collect data to estimate the spillover effect and take it into account in their product-offering decisions on retail platforms.

Second, our study offers a rule of thumb for online sellers—the optimal product offering varies with the initial awareness level of their products, the extent of the spillover effect, and the commission rates charged by retailers. The general rules are that online sellers should offer both identical and exclusive products on retail platforms when the initial awareness level is low, the spillover effect is mild, and the commission rate is low. Online sellers should only offer identical products when the spillover effect is significant and the commission rate is high. Otherwise, online sellers should only offer exclusive products.

Our study also provides managerial implications for the online retailers. First, our study calls retailers' attention to the role of the spillover effect in their platform-openness and platform-partnership decisions. Opening their platforms and letting third-party sellers sell products different from the products offered by the retailers sounds like a straightforward choice for retailers and can create a win-win outcome in the absence of the spillover effect—retailers make the commission and third parties reach the user base. However, as online search tools become more powerful and consumers become more experienced with online search, the spillover effect can be significant. With such change, our study suggests how retailers should adjust their platform-partnership strategies. In particular, in the presence of a strong spillover effect, when retailers' awareness advantage is mild, retailers should entice third parties to sell identical products on their platforms. In such a partnership, the adverse effect on retailers due to increased competition can be offset by charging a high commission, and third parties' commissions can be compensated by the spillover effect for their brands. Therefore, our results reveal that it is imperative for retailers to understand the underlying incentive, which calls for adjustment in their partnership strategies.

Second, our study demonstrates that the impact of the spillover effect on retailers varies under different partnerships, and it does not even necessarily hurt retailers. When third parties sell identical products on retailers' platforms, the spillover effect benefits third parties as well as the retailers. In this case, retailers should strive to enhance the spillover effect by, for example, highlighting seller names or providing links to third parties' own webpages. When third parties sell exclusive products on retailers' platforms, the spillover effect hurts retailers. In this case, retailers should discourage consumer search and, when appropriate, dampen the spillover effect by, for example, limiting the use of third parties' trademarks on their platforms (Chiou and Tucker 2012).

This paper has several limitations that suggest directions for future research. First, we consider a setting where a retailer carries the same product as the third-party seller and does not carry another product that the third-party seller carries. Further research can examine the retailer's product carrying choice in the first place. Second, we do not consider the competition among third-party sellers. For the products that a retailer does not carry itself, the retailer could allow multiple third-party sellers to sell and introduce competition. The retailer may be hurt by this competition in the short term if its platform service is commission based. Over time, such competition may benefit consumers and, thus, the retailer. Further research may extend our work by considering competition among third-party sellers.

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Appendix

A.1. Proof of Lemma 1

Proof. The third party's optimal price is characterized by the first-order condition on π_s in Equation (1):

$$\frac{d\pi_s}{dp_s} = \alpha(p_r - p_s) - \alpha p_s = 0.$$

Therefore, we conclude $p_s^*(p_r) = \frac{p_r}{2}$.

The retailer's optimal price is characterized by the first-order derivative of π_r in Equation (1):

$$\frac{d\pi_r}{dp_r} = (1-\alpha) + \alpha[1-(p_r - p_s)] - \alpha p_r = 1 - \frac{3\alpha}{2}p_r,$$

where the second equality comes from substituting $p_s^*(p_r)$. Note that, when $\alpha \leq \frac{2}{3}$, $\frac{d\pi_r}{dp_r} \geq 0$ for any $p_r \in [0,1]$, and thus $p_r^* = 1$. When $\alpha > \frac{2}{3}$, $p_r^* = \frac{2}{3\alpha}$ by letting $\frac{d\pi_r}{dp_r} = 0$. Altogether, the optimal price, p_r^*, can be written as in Equation (2).

Therefore, if $\alpha > \frac{2}{3}$, $p_r^* = \frac{2}{3\alpha}$ and $p_s^* = \frac{1}{3\alpha}$; otherwise, $p_r^* = 1$ and $p_s^* = \frac{1}{2}$. Substituting these optimal prices into Equation (1), we derive the equilibrium profits as in Equation (3). □

A.2 Proof of Lemma 2

Proof. The third party's optimal price is characterized by the first-order condition on π_s in Equation (5):

$$\frac{d\pi_s}{dp_s} = \mu(p_r - p_s) - \mu p_s = 0.$$

Therefore, we conclude $p_s^*(p_r) = \frac{p_r}{2}$.

The retailer's optimal price is characterized by the first-order derivative of π_r in Equation (5):

$$\frac{d\pi_r}{dp_r} = (1-\mu) + \mu[1-(p_r - p_s)] - \mu p_r = 1 - \frac{3\mu}{2}p_r,$$

where the second equality comes from substituting $p_s^*(p_r)$. Note that, when $\mu \leq \frac{2}{3}$, $\frac{d\pi_r}{dp_r} \geq 0$ for any $p_r \in [0,1]$, and thus $p_r^* = 1$. When $\mu > \frac{2}{3}$, $p_r^* = \frac{2}{3\mu}$ by letting $\frac{d\pi_r}{dp_r} = 0$. Altogether, the optimal price, p_r^*, can be written as in Equation (6).

Table A.1. Notation of Equilibrium Prices and Profits Under Different Scenarios

Scenario	R's price	S's price	R's profit	S's profit
I: Selling separately	p^*_{r0}	p^*_{s0}	π^*_{r0}	π^*_{s0}
II: S's product 2 on R	p^*_{r2}	p^*_{s2}	π^*_{r2}	π^*_{s2}
III: S's product 1 on R	p^*_{r1}	p^*_{s1}	π^*_{r1}	π^*_{s1}
IV: Both products on R	p^*_{r3}	p^*_{s3}	π^*_{r3}	π^*_{s3}

Therefore, if $\mu > \frac{2}{3}$, $p^*_r = \frac{2}{3\mu}$ and $p^*_s = \frac{1}{3\mu}$; otherwise, $p^*_r = 1$ and $p^*_s = \frac{1}{2}$. Substituting these optimal prices into Equation (5), we derive the equilibrium profits as in Equation (7). □

A.3. Proof of Lemma 3

Proof. The retailer's and the third party's optimal prices are characterized by the first-order conditions on π_r and π_s in Equation (8):

$$\frac{d\pi_r}{dp_r} = \left[1 - (p_r - p_s)\right] - p_r + \theta p_s = 0,$$
$$\frac{d\pi_s}{dp_s} = (1-\theta)(p_r - p_s) - (1-\theta)p_s = 0.$$

Solving the above system of equations, we can derive the equilibrium prices as in Equation (9). Substituting these optimal prices into Equations (8), we derive the equilibrium profits as in Equation (10). □

A.4. Proof of Lemma 4

Proof. The retailer's and the third party's optimal prices are characterized by the first-order conditions on π_r and π_s in Equation (11):

$$\frac{d\pi_r}{dp_r} = \left[1 - (p_r - p_s)\right] - p_r + \theta p_s = 0,$$
$$\frac{d\pi_s}{dp_s} = (1-\theta)(p_r - p_s) - (1-\theta)p_s = 0.$$

Solving the above system of equations, we can derive the equilibrium prices as in Equation (12). Substituting these optimal prices into Equations (11), we derive the equilibrium profits as in Equation (13). □

A.5. Notation

To ease the exposition, we use the notation in Table A.1 to represent the equilibrium outcomes under different scenarios for the rest of the appendix.

A.6. Proof of Propositiion 1

Proof. In the following steps, we first compare the third party's equilibrium payoffs under any two scenarios (in part A), and then derive its equilibrium choice (in part B).

Part A: Comparison of Equilibrium Payoffs Under Any Two Scenarios. Step 1: Comparing scenario II ("selling product 2 through the retailer") with scenario I ("selling separately"), we derive the equilibrium profit difference as

$$\pi^*_{s2} - \pi^*_{s0} = \begin{cases} \left(\frac{\mu}{4} - \frac{\alpha}{4}\right) - [\alpha - (1-\theta)]t & \text{if } \mu \le \frac{2}{3}, \\ \left(\frac{1}{9\mu} - \frac{\alpha}{4}\right) - [\alpha - (1-\theta)]t & \text{if } \mu > \frac{2}{3} \text{ and } \alpha \le \frac{2}{3}, \\ \left(\frac{1}{9\mu} - \frac{1}{9\alpha}\right) - [\alpha - (1-\theta)]t & \text{if } \alpha > \frac{2}{3}. \end{cases} \quad (19)$$

Because the above profit difference decreases in θ, we have $\pi^*_{s0} \le \pi^*_{s2}$ if and only if $\theta \le \theta^*_{s2}$, where θ^*_{s2} is the solution to $\pi^*_{s2} - \pi^*_{s0} = 0$ and can be derived as

$$\theta^*_{s2} = \begin{cases} 1 - \alpha + \frac{(1-\alpha)\beta}{4t} & \text{if } \mu \le \frac{2}{3}, \\ 1 - \alpha + \frac{1}{t}\left(\frac{1}{9\mu} - \frac{\alpha}{4}\right) & \text{if } \mu > \frac{2}{3} \text{ and } \alpha \le \frac{2}{3}, \\ 1 - \alpha + \frac{1}{t}\left(\frac{1}{9\mu} - \frac{1}{9\alpha}\right) & \text{if } \alpha > \frac{2}{3}. \end{cases} \quad (20)$$

Step 2: Comparing scenario III ("selling product 1 through the retailer") with scenario I, we derive the equilibrium profit difference as

$$\pi^*_{s1} - \pi^*_{s0} = \begin{cases} \frac{1-\theta}{(3-\theta)^2} - \left[\frac{\alpha}{4} - (\mu - \alpha)t\right] & \text{if } \alpha \le \frac{2}{3}, \\ \frac{1-\theta}{(3-\theta)^2} - \left[\frac{1}{9\alpha} - (\mu - \alpha)t\right] & \text{if } \alpha > \frac{2}{3}. \end{cases} \quad (21)$$

Note that $\frac{1-\theta}{(3-\theta)^2}$ decreases in θ, and it reaches its maximum $\frac{1}{9}$ when $\theta = 0$. If the term in brackets is greater than $\frac{1}{9}$ or, equivalently, if $\beta < \tilde{\beta}(\alpha)$, where

$$\tilde{\beta}(\alpha) = \begin{cases} \frac{9\alpha - 4}{36(1-\alpha)t} & \text{if } \alpha \le \frac{2}{3}, \\ \frac{1}{9\alpha t} & \text{if } \alpha > \frac{2}{3}, \end{cases} \quad (22)$$

$\pi^*_{s1} < \pi^*_{s0}$ for any $\theta \in [0,1]$; otherwise, because the above profit difference decreases in θ, we have $\pi^*_{s0} \le \pi^*_{s1}$ if and only if $\theta \le \theta^*_{s1}$, where θ^*_{s1} is the solution to $\pi^*_{s1} - \pi^*_{s0} = 0$. Altogether, we can derive

$$\theta^*_{s1} = \begin{cases} K\left(\frac{\alpha}{4} - (1-\alpha)\beta t\right) & \text{if } \alpha \le \frac{2}{3} \text{ and } \beta \ge \frac{9\alpha - 4}{36(1-\alpha)t}, \\ K\left(\frac{1}{9\alpha} - (1-\alpha)\beta t\right) & \text{if } \alpha > \frac{2}{3} \text{ and } \beta \ge \frac{1}{9\alpha t}, \\ \emptyset & \text{otherwise}, \end{cases} \quad (23)$$

where $K(x)$, as defined in Equation (15),[5] is a root of $\frac{1-\theta}{(3-\theta)^2} = x$ with respect to θ, and "$\emptyset$" is used to represent the cases in which $\pi^*_{s1} < \pi^*_{s0}$ for any $\theta \in [0,1]$.

Step 3: Comparing scenario II with scenario III, we derive the equilibrium profit difference as

$$\Delta_s^{2-1}(\theta) \equiv \pi^*_{s2} - \pi^*_{s1} = \begin{cases} \left(\frac{\mu}{4} - \frac{1-\theta}{(3-\theta)^2}\right) - [\mu - (1-\theta)]t & \text{if } \mu \le \frac{2}{3}, \\ \left(\frac{1}{9\mu} - \frac{1-\theta}{(3-\theta)^2}\right) - [\mu - (1-\theta)]t & \text{if } \mu > \frac{2}{3}. \end{cases} \quad (24)$$

We can verify that, when $t \ge \frac{1}{4}$, $\Delta_s^{2-1}(1) \le 0$ and $\Delta_s^{2-1}(0) > 0$. In addition,

$$\frac{\partial \Delta_s^{2-1}(\theta)}{\partial \theta} = \frac{1+\theta}{(3-\theta)^3} - t < 0$$

for $\theta \in [0,1)$. Therefore, $\pi^*_{s1} \le \pi^*_{s2}$ if and only if $\theta \le \theta^*_{21}$, where $\theta^*_{21} \in (0,1]$ is the solution to $\Delta_s^{2-1}(\theta^*_{21}) = 0$, or, equivalently,

$$\theta^*_{21} \text{ is the solution to } \frac{1-\theta}{(3-\theta)^2} + \mu t = \min\left\{\frac{\mu}{4}, \frac{1}{9\mu}\right\} + (1-\theta)t. \tag{25}$$

Step 4: Comparing scenario IV ("selling both products through the retailer") with scenario III, we derive the equilibrium profit difference as $\pi^*_{s3} - \pi^*_{s1} = (1-\theta)t - \mu t$. Because this profit difference decreases in θ, we have $\pi^*_{s1} \le \pi^*_{s3}$ if and only if $\theta \le \theta^*_{31}$, where θ^*_{31} is the solution to $\pi^*_{s3} - \pi^*_{s1} = 0$ and can be derived as $\theta^*_{31} \equiv 1 - \mu$.

Step 5: Comparing scenario IV with scenario II, we derive the equilibrium profit difference as

$$\pi^*_{s3} - \pi^*_{s2} = \begin{cases} \frac{1-\theta}{(3-\theta)^2} - \frac{\mu}{4} & \text{if } \mu \le \frac{2}{3}, \\ \frac{1-\theta}{(3-\theta)^2} - \frac{1}{9\mu} & \text{if } \mu > \frac{2}{3}. \end{cases}$$

We can verify that, when $\mu > \frac{4}{9}$, $\pi^*_{s3} - \pi^*_{s2} < 0$. Thus, when $\mu > \frac{4}{9}$, the choice of selling both products is dominated by selling product 2 only. When $\mu \le \frac{4}{9}$, because the above profit difference decreases in θ, $\pi^*_{s2} \le \pi^*_{s3}$ if and only if $\theta \le K(\frac{\mu}{4})$, where $K(\frac{\mu}{4})$ is the solution to $\pi^*_{s3} - \pi^*_{s2} = 0$ and $K(\cdot)$ is defined as in Equation (15).

Furthermore, when $\mu \le \frac{4}{9}$, according to Equation (24), we have

$$\begin{aligned} \pi^*_{s2}(\theta^*_{31}) - \pi^*_{s1}(\theta^*_{31}) &= \pi^*_{s2}(1-\mu) - \pi^*_{s1}(1-\mu) \\ &= \frac{\mu}{4} - \frac{\mu}{(2+\mu)^2} > 0, \end{aligned}$$

which implies that $\theta^*_{31} < \theta^*_{21}$. Meanwhile, we have

$$\begin{aligned} \theta^*_{31} - K\left(\frac{\mu}{4}\right) &= (1-\mu) - K\left(\frac{\mu}{4}\right) \\ &= 1 - \mu - \frac{-2 + 3\mu + 2\sqrt{1-2\mu}}{\mu} \\ &= \frac{\left(1-\sqrt{1-2\mu}\right)^2 - \mu^2}{\mu} > 0. \end{aligned}$$

Therefore, we conclude

$$K\left(\frac{\mu}{4}\right) < \theta^*_{31} < \theta^*_{21}. \tag{26}$$

Part B: Derivation of Equilibrium Choice. First, we notice that if $\beta < \tilde{\beta}(\alpha)$, $\pi^*_{s1} < \pi^*_{s0}$ and the choice of selling product 1 is dominated by selling separately and should not be considered; if $\beta \ge \tilde{\beta}(a)$, we have

$$\begin{aligned} \frac{\partial(\theta^*_{s1} - \theta^*_{s2})}{\partial\beta} &= \frac{\partial\theta^*_{s1}}{\partial\beta} - \frac{\partial\theta^*_{s2}}{\partial\beta} \\ &= \begin{cases} \left[\frac{(1-4x)-\sqrt{1-8x}}{2x^2\sqrt{1-8x}}(1-\alpha)t\right] - \left(\frac{1-\alpha}{4t}\right) > 0 & \text{if } \mu \le \frac{2}{3}, \\ \left[\frac{(1-4x)-\sqrt{1-8x}}{2x^2\sqrt{1-8x}}(1-\alpha)t\right] - \left(-\frac{1-\alpha}{9\mu^2 t}\right) > 0 & \text{if } \mu > \frac{2}{3}, \end{cases} \end{aligned} \tag{27}$$

where $x = \min\{\frac{\alpha}{4}, \frac{1}{9\alpha}\} - (1-\alpha)\beta t$. Because $\theta^*_{s1} - \theta^*_{s2}$ increases in β, $\theta^*_{s1} < \theta^*_{s2}$ if and only if $\beta < \bar{\beta}(\alpha)$, where $\bar{\beta}(\alpha)$ is the solution to $\theta^*_{s1} = \theta^*_{s2}$ (as defined in Equation (14)). In addition, we have $\tilde{\beta}(\alpha) < \bar{\beta}(\alpha)$ because, when $\beta = \tilde{\beta}(\alpha)$, $\theta^*_{s1} = 0 < \theta^*_{s2}$. Furthermore, we have

$$\begin{cases} \pi^*_{s2}(\theta^*_{s2}) = \pi^*_{s0} = \pi^*_{s1}(\theta^*_{s1}) > \pi^*_{s1}(\theta^*_{s2}) & \text{if } \beta < \bar{\beta}(\alpha), \\ \pi^*_{s2}(\theta^*_{s2}) = \pi^*_{s0} = \pi^*_{s1}(\theta^*_{s1}) \le \pi^*_{s1}(\theta^*_{s2}) & \text{if } \beta \ge \bar{\beta}(\alpha), \end{cases}$$

where the two equalities in each expression are due to the definitions of θ^*_{s2} and θ^*_{s1}, and the inequality is implied by the definition of $\bar{\beta}(\alpha)$. According to the threshold, θ^*_{21}, discussed in step 3, we therefore have

$$\begin{cases} \theta^*_{s1} < \theta^*_{s2} < \theta^*_{21} & \text{if } \beta < \bar{\beta}(\alpha), \\ \theta^*_{21} \le \theta^*_{s2} \le \theta^*_{s1} & \text{if } \beta \ge \bar{\beta}(\alpha). \end{cases} \tag{28}$$

We next analyze the third party's equilibrium selling choice.

a. The case with $\mu \le \frac{4}{9}$: If $\theta \le K(\frac{\mu}{4})$, $\pi^*_{s2} \le \pi^*_{s3}$ because of the analysis in step 5, $\pi^*_{s1} < \pi^*_{s3}$ because $K(\frac{\mu}{4}) < \theta^*_{31}$ by Equation (26), and $\pi^*_{s0} < \pi^*_{s2}$ because $K(\frac{\mu}{4}) < \theta^*_{31} = 1 - \mu < 1 - \alpha + \frac{(1-\alpha)\beta}{4t} = \theta^*_{s2}$. Therefore, in this case, the third party sells both products through the retailer.

If $\theta > K(\frac{\mu}{4})$, according to the analysis in step 5, the choice of selling both products is dominated by selling product 2 only and should not be considered.

a.1. When $\beta < \bar{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta^*_{s2}]$, $\pi^*_{s0} \le \pi^*_{s2}$ because of the definition of θ^*_{s2}; when $\beta < \tilde{\beta}(\alpha)$, $\pi^*_{s1} < \pi^*_{s0}$ because of the definition of $\tilde{\beta}(\alpha)$, and when $\beta \ge \tilde{\beta}(\alpha)$, $\pi^*_{s1} < \pi^*_{s2}$ because $\theta^*_{s2} < \theta^*_{21}$ by Equation (28). Therefore, the third party chooses to sell product 2 only. If $\theta > \theta^*_{s2}$, $\pi^*_{s2} < \pi^*_{s0}$ because of the definition of θ^*_{s2}, and $\pi^*_{s1} < \pi^*_{s0}$ either because of the definition of $\tilde{\beta}(\alpha)$ (when $\beta < \tilde{\beta}(\alpha)$) or because $\theta^*_{s1} < \theta^*_{s2}$ by Equation (28) (when $\beta \ge \tilde{\beta}(\alpha)$). Therefore, the third party chooses not to sell any product through the retailer.

a.2. When $\beta \ge \bar{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta^*_{21}]$, $\pi^*_{s1} \le \pi^*_{s2}$ because of the definition of θ^*_{21}, and $\pi^*_{s0} \le \pi^*_{s2}$ because $\theta^*_{21} \le \theta^*_{s2}$ by Equation (28). Therefore, the third party chooses to sell product 2 only. If $\theta \in (\theta^*_{21}, \theta^*_{s1}]$, $\pi^*_{s2} < \pi^*_{s1}$ because of the definition of θ^*_{21}, and $\pi^*_{s0} \le \pi^*_{s1}$ because $\theta^*_{21} \le \theta^*_{s1}$ by Equation (28). Therefore, the third party chooses to sell product 1 only. If $\theta > \theta^*_{s1}$, $\pi^*_{s1} < \pi^*_{s0}$ because of the definition of θ^*_{s1}, and $\pi^*_{s2} < \pi^*_{s0}$ because $\theta^*_{s2} \le \theta^*_{s1}$ by Equation (28). Therefore, the third party chooses not to sell any product through the retailer.

b. The case with $\mu > \frac{4}{9}$: According to the analysis in step 5, the choice of selling both products is dominated by selling product 2 only and should not be considered.

b.1. The same arguments in (a.1) explain the conditions under which the third party chooses to sell product 2 only or not any product through the retailer.

b.2. The same arguments in (a.2) explain the conditions under which the third party chooses to sell product 2, product 1, or not any product through the retailer. □

A.7. Proof of Lemma 5

Proof. According to Proposition 1, selling both products 1 and 2 can be the equilibrium choice for the third party only when $\mu \le \frac{4}{9}$ and $\theta \in [0, K(\frac{\mu}{4})]$. However, when $\mu \le \frac{4}{9}$, by Equation (26) and $\frac{\partial \pi^*_{r3}}{\partial \theta} > 0$, for any $\theta \in [0, K(\frac{\mu}{4})]$, we have

$$\pi^*_{r3}(\theta) \le \pi^*_{r3}\left(K\left(\frac{\mu}{4}\right)\right) < \pi^*_{r3}(\theta^*_{21}) < \pi^*_{r2}(\theta^*_{21}),$$

where the last inequality is because $\pi^*_{r3}(\theta) < \pi^*_{r2}(\theta)$ for $\theta \in [0,1]$ by Lemmas 2 and 4. Therefore, when $\mu \le \frac{4}{9}$, the retailer prefers to induce the third party to sell product 2 on its platform by setting $\theta \in (K(\frac{\mu}{4}), \theta^*_{21})$ rather than to induce the third party to sell both products. □

A.8. Proof of Lemma 6

Proof.

a. The retailer's equilibrium profit difference between scenario II ("selling product 2 through the retailer") and scenario I ("selling separately") is

$$\pi^*_{r2} - \pi^*_{r0} = \begin{cases} \theta t - \left(\frac{2-\alpha}{2} - \frac{2-\mu}{2}\right) & \text{if } \mu \le \frac{2}{3}, \\ \theta t - \left(\frac{2-\alpha}{2} - \frac{4}{9\mu}\right) & \text{if } \mu > \frac{2}{3} \text{ and } \alpha \le \frac{2}{3}, \\ \theta t - \left(\frac{4}{9\alpha} - \frac{4}{9\mu}\right) & \text{if } \alpha > \frac{2}{3}. \end{cases}$$

Because the above profit difference increases in θ, we have $\pi^*_{r0} \le \pi^*_{r2}$ if and only if $\theta^*_{r2} \le \theta$, where θ^*_{r2} is the solution to $\pi^*_{r2} - \pi^*_{r0} = 0$ and can be derived as in Equation (17).

b. The retailer's equilibrium profit difference between scenario III ("selling product 1 through the retailer") and scenario I is

$$\pi^*_{r1} - \pi^*_{r0} = \begin{cases} \frac{4-\theta}{(3-\theta)^2} - \frac{2-\alpha}{2} & \text{if } \alpha \le \frac{2}{3}, \\ \frac{4-\theta}{(3-\theta)^2} - \frac{4}{9\alpha} & \text{otherwise.} \end{cases}$$

The above profit difference increases in θ. Notice that, when $\alpha \le \frac{1}{2}$, $\pi^*_{r1} \le \pi^*_{r0}$ even with the highest commission rate 1, so the retailer has no incentive in this case to let the third party sell product 1 only. When $\alpha \in (\frac{1}{2}, 1]$, we have $\pi^*_{r0} \le \pi^*_{r1}$ if and only if $\theta^*_{r1} \le \theta$, where θ^*_{r1} is the solution to $\pi^*_{r1} - \pi^*_{r0} = 0$ and can be derived as in Equation (18). □

A.9. Proof of Proposition 2

Proof. By Lemma 5, S's selling both products on R's platform cannot arise as an equilibrium.

a.1. When $\beta < \bar{\beta}(\alpha)$: By Proposition 1, the third party has no incentive to sell product 1 on the retailer's platform. The retailer can induce the third party to sell product 2 on the retailer's platform if and only if $\theta^*_{r2} < \theta^*_{s2}$. In particular, when $\mu \le \frac{4}{9}$ ($\mu > \frac{4}{9}$), for any $\theta \in (\max\{K(\frac{\mu}{4}), \theta^*_{r2}\}, \theta^*_{s2})$ ($\theta \in (\theta^*_{r2}, \theta^*_{s2})$), the partnership of S selling product 2 through R can be established.

a.2. When $\beta \ge \bar{\beta}(\alpha)$ and $\alpha \le \frac{1}{2}$: By Lemma 6, the retailer has no incentive to let the third party sell product 1 on its platform. The retailer can induce the third party to sell product 2 on its platform if and only if $\theta^*_{r2} < \theta^*_{21}$. In particular, when $\mu \le \frac{4}{9}$ ($\mu > \frac{4}{9}$), for any $\theta \in (\max\{K(\frac{\mu}{4}), \theta^*_{r2}\}, \theta^*_{21})$ ($\theta \in (\theta^*_{r2}, \theta^*_{21})$), the partnership of S selling product 2 through R can be established. The upper bound θ^*_{21} ensures that S prefers to sell product 2 rather than product 1, and $\theta^*_{21} \le \theta^*_{s2}$ by Equation (28), which ensures that S chooses to sell product 2.

a.3 and b. When $\beta \ge \bar{\beta}(\alpha)$ and $\alpha > \frac{1}{2}$: First, we note that $\frac{\partial \pi^*_{r2}}{\partial \theta} > 0$ and $\frac{\partial \pi^*_{r1}}{\partial \theta} > 0$.

a.3. When $\beta \ge \bar{\beta}(\alpha)$ and $\alpha > \frac{1}{2}$, the retailer can induce the third party to sell product 2 on its platform if and only if $\theta^*_{r2} < \theta^*_{21}$ and $\pi^*_{r2}(\theta^*_{21}) > \pi^*_{r1}(\min\{\theta^*_{s1}, 1\})$. In particular, for any $\theta \in (\max\{\theta^*_{r2}, \theta^*\}, \theta^*_{21})$, where θ^* is the solution to $\pi^*_{r2}(\theta) = \pi^*_{r1}(\min\{\theta^*_{s1}, 1\})$, the partnership of S selling product 2 through R can be established. In this case, the third party has incentive to sell product 2 because $\pi^*_{s1}(\theta) < \pi^*_{s2}(\theta)$ by the definition of θ^*_{21} and $\pi^*_{s0} \le \pi^*_{s2}(\theta)$ due to $\theta^*_{21} \le \theta^*_{s2}$ by Equation (28), and the retailer has incentive because $\pi^*_{r0} < \pi^*_{r2}(\theta)$ by the definition of θ^*_{r2} and $\pi^*_{r1}(\min\{\theta^*_{s1}, 1\}) < \pi^*_{r2}(\theta)$ by the definition of θ^*.

The reason for "only if" is as follows: If $\theta^*_{r2} > \theta^*_{21}$, the two players' incentives cannot be aligned to have S sell product 2 through R; if $\theta^*_{r2} < \theta^*_{21}$ but $\pi^*_{r2}(\theta^*_{21}) < \pi^*_{r1}(\min\{\theta^*_{s1}, 1\})$, the retailer is better off by having S sell product 1.

b. When $\beta \ge \bar{\beta}(\alpha)$ and $\alpha > \frac{1}{2}$, the retailer can induce the third party to sell product 1 on its platform if and only if $\theta^*_{r1} < \theta^*_{s1}$ and $\pi^*_{r2}(\theta^*_{21}) < \pi^*_{r1}(\min\{\theta^*_{s1}, 1\})$. In particular, for any $\theta \in (\max\{\theta^*_{r1}, \theta^*_{21}, \theta^{**}\}, \min\{\theta^*_{s1}, 1\})$, where θ^{**} is the solution to $\pi^*_{r2}(\theta^*_{21}) = \pi^*_{r1}(\theta)$, the partnership of S selling product 1 through R can be established. In this case, the third party has incentive to sell product 1 because $\pi^*_{s2}(\theta) < \pi^*_{s1}(\theta)$ by the definition of θ^*_{21} and $\pi^*_{s0} < \pi^*_{s1}(\theta)$ because of the definition of θ^*_{s1}. Similarly, the retailer has incentive to have the third party sell product 1 because $\pi^*_{r0} < \pi^*_{r1}(\theta)$ by the definition of θ^*_{r1} and $\pi^*_{r2}(\theta^*_{21}) < \pi^*_{r1}(\theta)$ by the definition of θ^{**}.

The reason for "only if" is as follows: If $\theta^*_{r1} > \theta^*_{s1}$, the two players' incentives cannot be aligned to have S sell product 1 through R; if $\theta^*_{r1} < \theta^*_{s1}$ but $\pi^*_{r2}(\theta^*_{21}) > \pi^*_{r1}(\min\{\theta^*_{s1}, 1\})$, the retailer is better off by having S sell product 2. □

A.10. Proof of Corollary 1

Proof. When $\beta = 0$, $\beta < \bar{\beta}(\alpha)$. In addition, by substituting $\beta = 0$ into Equations (20) and (17), we have $\theta^*_{s2} = 1 - \alpha$ and $\theta^*_{r2} = 0$. Thus, $\theta^*_{r2} < \theta^*_{s2}$. By Proposition 2, this case belongs to category (a.1); thus, in equilibrium, the third party sells product 2 on the retailer's platform. □

A.11. Proof of Proposition 3

Proof.

a. Within the equilibrium in which the third party sells product 2 on the retailer's platform, by the proof of Proposition 2(a), the retailer's optimal commission rate is θ^*_{s2} when $\beta < \bar{\beta}(\alpha)$ and θ^*_{21} when $\beta \ge \bar{\beta}(\alpha)$.

When $\beta < \bar{\beta}(\alpha)$, we have $\frac{\partial \pi^*_{s2}(\theta^*_{s2})}{\partial \beta} = \frac{\partial \pi^*_{s0}}{\partial \beta} = 0$, and by Lemma 2,

$$\frac{\partial \pi^*_{r2}(\theta^*_{s2})}{\partial \beta} = \begin{cases} \frac{\partial\left(\frac{2-\mu}{2} + \theta^*_{s2} t\right)}{\partial \beta} = -\frac{1-\alpha}{2} + \frac{\partial \theta^*_{s2}}{\partial \beta} \cdot t \\ \qquad = -\frac{1-\alpha}{4} < 0 \quad \text{if } \mu \le \frac{2}{3}, \\ \frac{\partial\left(\frac{4}{9\mu} + \theta^*_{s2} t\right)}{\partial \beta} = -\frac{4(1-\alpha)}{9\mu^2} + \frac{\partial \theta^*_{s2}}{\partial \beta} \cdot t \\ \qquad = -\frac{5(1-\alpha)}{9\mu^2} < 0 \quad \text{otherwise,} \end{cases}$$

where the last equality in each case comes from substituting in $\frac{\partial \theta^*_{s2}}{\partial \beta}$ derived in Equation (27).

When $\beta \ge \bar{\beta}(\alpha)$, we have $\frac{\partial \pi^*_{s2}(\theta^*_{21})}{\partial \beta} = \frac{\partial \pi^*_{s2}}{\partial \theta^*_{21}} \cdot \frac{\partial \theta^*_{21}}{\partial \beta} > 0$, and by Lemma 2,

$$\frac{\partial \pi^*_{r2}(\theta^*_{21})}{\partial \beta} = \begin{cases} -\frac{1-\alpha}{2} + \frac{\partial \theta^*_{21}}{\partial \beta} \cdot t < 0 & \text{if } \mu \le \frac{2}{3}, \\ -\frac{4(1-\alpha)}{9\mu^2} + \frac{\partial \theta^*_{21}}{\partial \beta} \cdot t < 0 & \text{otherwise,} \end{cases} \tag{29}$$

where the inequality is because we can verify that $\frac{\partial \theta^*_{21}}{\partial \beta} < 0$ based on the definition of θ^*_{21}—the solution to $(1-\theta)t - \frac{1-\theta}{(3-\theta)^2} = \mu t - \min\{\frac{\mu}{4}, \frac{1}{9\mu}\}$. Altogether, we have $\frac{\partial \pi^*_{s2}}{\partial \beta} \ge 0$ and $\frac{\partial \pi^*_{r2}}{\partial \beta} \le 0$.

Within the equilibrium in which the third party sells product 1 on the retailer's platform, by the proof of Proposition 2(b), the optimal commission rate for the retailer is $\min\{\theta^*_{s1}, 1\}$. When $\beta > \hat{\beta}(\alpha)$, where $\hat{\beta}(\alpha)$ is defined as in Equation (16), $\min\{\theta^*_{s1}, 1\} = 1$. By Lemma 3, $\frac{\partial \pi^*_{s1}(1)}{\partial \beta} = (1-\alpha)t > 0$ and $\frac{\partial \pi^*_{r1}(1)}{\partial \beta} = 0$. When $\beta \le \hat{\beta}(\alpha)$, $\min\{\theta^*_{s1}, 1\} = \theta^*_{s1}$. Therefore, $\frac{\partial \pi^*_{s1}(\theta^*_{s1})}{\partial \beta} = \frac{\partial \pi^*_{s0}}{\partial \beta} = 0$, and by Lemma 3,

$$\frac{\partial \pi^*_{r1}\left(\min\{\theta^*_{s1}, 1\}\right)}{\partial \beta} = \frac{\partial \pi^*_{r1}\left(\theta^*_{s1}\right)}{\partial \beta} = \frac{5-\theta^*_{s1}}{\left(3-\theta^*_{s1}\right)^3} \cdot \frac{\partial \theta^*_{s1}}{\partial \beta} > 0, \quad (30)$$

where the last inequality comes from $\frac{\partial \theta^*_{s1}}{\partial \beta} = \frac{(1-4x)-\sqrt{1-8x}}{2x^2\sqrt{1-8x}}(1-\alpha)t > 0$ by Equation (27). Altogether, we have $\frac{\partial \pi^*_{s1}}{\partial \beta} \ge 0$ and $\frac{\partial \pi^*_{r1}}{\partial \beta} \ge 0$.

b. When $\beta = 0$, by Corollary 1, in equilibrium the third party sells product 2 on the retailer's platform and the retailer charges θ^*_{s2}. As a result, the third party is indifferent between joining and not joining the retailer's platform. In the presence of the spillover effect, when the third party is induced to join the retailer's platform, the third party must earn profit (weakly) higher than when not joining the retailer's platform. Therefore, the spillover effect always (weakly) benefits the third party.

For the retailer, when the third party sells product 1 on its platform, the spillover effect might benefit the retailer when the third party's initial awareness level and the extent of the spillover effect are high. For example, when $\alpha = \frac{5}{6}$, $t = 1$, and $\beta > \hat{\beta}(\alpha) = \frac{1}{9\alpha(1-\alpha)t} = \frac{4}{5}$, we have $\pi^*_{r1} = \frac{4-\theta}{(3-\theta)^2} = \frac{3}{4} > \pi^*_{r2} = \frac{4}{9\alpha} + (1-\alpha)t = \frac{7}{10}$. Therefore, the spillover effect might benefit the retailer. □

A.12. Proof of Corollary 2

Proof. The proof of Proposition 3 contains the proof of this corollary. □

A.13. Proof of Corollary 3

Proof.

a. By Proposition 3(a), the retailer's equilibrium profit increases in β. Thus, within $[\underline{\beta}, \overline{\beta}]$, the optimal β is $\overline{\beta}$.

b. Within $[\underline{\beta}, \overline{\beta}]$, if under any β the equilibrium without manipulation is that the retailer opens its platform and the third party sells product 2, by Proposition 3(a), the retailer's equilibrium profit decreases in β, and thus, the optimal β is $\underline{\beta}$. If under any β the equilibrium without manipulation is that the retailer does not open its platform, β does not affect the retailer's profit. If the retailer's equilibrium choice without manipulation is to open its platform under some β and not to open its platform under other β, by Proposition 3(a), the retailer opens its platform when β is low and does not open when β is high. Because of the monotonicity in its profit function, the retailer opens the platform, and the optimal β is $\underline{\beta}$. Altogether, the optimal β is $\underline{\beta}$.

c. If the retailer's equilibrium choice without manipulation is to open its platform and the third party sells product 2 under some β and sells product 1 under other β, by Propositions 2 and 3(a), the third party sells product 2 when β is low and sells product 1 when β is high. By Proposition 3(a), the retailer's equilibrium profit decreases in β when the third party sells product 2 and increases in β when the third party sells product 1. Therefore, the optimal β is $\underline{\beta}$ if $\pi^*_{r2}(\underline{\beta}) > \pi^*_{r1}(\overline{\beta})$; otherwise, the optimal β is $\overline{\beta}$. □

A.14. Proof of Lemma 7

Proof. The retailer's equilibrium profit difference between scenario IV ("selling both products through the retailer") and scenario I ("selling separately") is

$$\Delta_r^{3-0}(\theta) = \pi^*_{r3} - \pi^*_{r0} = \begin{cases} \frac{4-\theta}{(3-\theta)^2} + \theta t - \frac{2-\alpha}{2} & \text{if } \alpha \le \frac{2}{3}, \\ \frac{4-\theta}{(3-\theta)^2} + \theta t - \frac{4}{9\alpha} & \text{otherwise.} \end{cases}$$

Because $\frac{4-\theta}{(3-\theta)^2}$ increases in θ, $\Delta_r^{3-0}(\theta)$ increases in θ. We can verify that $\Delta_r^3(0) \le 0$ and $\Delta_r^3(1) > 0$. Therefore, there exists a unique $\theta^*_{r3} \in [0, 1)$ that makes $\pi^*_{r3} - \pi^*_{r0} = 0$, and $\pi^*_{r0} \le \pi^*_{r3}$ if and only if $\theta^*_{r3} \le \theta$. □

A.15. Proof of Proposition 4

Proof.

a. When $\mu \le \frac{4}{9}$, if $\theta \le K(\frac{\mu}{4})$, by Proposition 1, the third party sells both products 1 and 2 on the retailer's platform if the retailer opens its platform; if $\theta > \theta^*_{r3}$, by Lemma 7, the retailer has incentive to open its platform.

a.1. When $\beta < \overline{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta^*_{s2}]$, by Proposition 1, the third party sells product 2 on the retailer's platform if the retailer opens its platform; if $\theta > \theta^*_{r2}$, by Lemma 6, the retailer has incentive to open its platform.

a.2. When $\beta \ge \overline{\beta}(\alpha)$, if $\theta \in (K(\frac{\mu}{4}), \theta^*_{21}]$ and $\theta > \theta^*_{r2}$, by Proposition 1, the third party sells product 2 on the retailer's platform if the retailer opens its platform, and by Lemma 6, the retailer has incentive to open its platform; if $\theta \in (\theta^*_{21}, \theta^*_{s1}]$ and $\theta > \theta^*_{r1}$, by Proposition 1, the third party sells product 1 on the retailer's platform if the retailer opens its platform, and by Lemma 6, the retailer has incentive to open its platform.

b. When $\mu > \frac{4}{9}$:

b.1. When $\beta < \overline{\beta}(\alpha)$, if $\theta \le \theta^*_{s2}$ and $\theta > \theta^*_{r2}$, by Proposition 1, the third party sells product 2 on the retailer's platform if the retailer opens its platform, and by Lemma 6, the retailer has incentive to open its platform.

b.2. When $\beta \ge \overline{\beta}(\alpha)$, if $\theta \le \theta^*_{21}$ and $\theta > \theta^*_{r2}$, by Proposition 1, the third party sells product 2 on the retailer's platform if the retailer opens its platform, and by Lemma 6, the retailer has incentive to open its platform; if $\theta \in (\theta^*_{21}, \theta^*_{s1}]$ and $\theta > \theta^*_{r1}$, by Proposition 1, the third party sells product 1 on the retailer's platform if the retailer opens its platform, and by Lemma 6, the retailer has incentive to open its platform. □

A.16. Proof of Lemma 8

Proof. We first derive the retailer's and third party's equilibrium profits when the retailer does not carry product 1. Noting that the expected profits per user of product 1 and product 2 for the third party are $\frac{1}{4}$ and t, respectively, we can derive the equilibrium profits under different scenarios as summarized in Table A.2.

Table A.2. Equilibrium Profits when Retailer Does Not Carry Product 1

Scenario	Retailer's profit	Third party's profit
Scenario I: Selling separately	0	$\frac{\alpha}{4}+\alpha t$
Scenario II: S's product 2 on R	θt	$\frac{\mu}{4}+(1-\theta)t$
Scenario III: S's product 1 on R	$\frac{\theta}{4}$	$\frac{1-\theta}{4}+\mu t$
Scenario IV: Both products on R	$\frac{\theta}{4}+\theta t$	$\frac{1-\theta}{4}+(1-\theta)t$

We follow the same notation in Table A.1 and add superscript "0" to distinguish the case when the retailer does not carry product 1. By simple algebra, we have

$$\pi^{0*}_{s1}-\pi^{0*}_{s0}=\frac{(1-\theta-\alpha)}{4}+(1-\alpha)\beta t,$$
$$\pi^{0*}_{s2}-\pi^{0*}_{s1}=(1-\theta-\mu)\left(t-\frac{1}{4}\right),$$
$$\pi^{0*}_{s3}-\pi^{0*}_{s1}=(1-\theta-\mu)t,$$
$$\pi^{0*}_{s3}-\pi^{0*}_{s2}=\frac{1-\theta-\mu}{4}.$$

a. If $\theta\le 1-\mu$, then $\pi^{0*}_{s3}-\pi^{0*}_{s1}\ge 0$, $\pi^{0*}_{s3}-\pi^{0*}_{s2}\ge 0$, and $\pi^{0*}_{s1}-\pi^{0*}_{s0}\ge 0$ (because $\mu\ge\alpha$). Therefore, the third party sells both products through the retailer.

b. If $\theta\in(1-\mu,(1-\alpha)(1+4\beta t)]$, then $\pi^{0*}_{s3}-\pi^{0*}_{s1}<0$, $\pi^{0*}_{s2}-\pi^{0*}_{s1}<0$, and $\pi^{0*}_{s1}-\pi^{0*}_{s0}\ge 0$. Therefore, the third party sells product 1 through the retailer.

c. If $\theta>(1-\alpha)(1+4\beta t)$, then $\pi^{0*}_{s1}-\pi^{0*}_{s0}\le 0$, $\pi^{0*}_{s2}-\pi^{0*}_{s1}<0$, and $\pi^{0*}_{s3}-\pi^{0*}_{s1}<0$. Therefore, the third party does not sell through the retailer. □

A.17. Proof of Proposition 5

Proof. When $\theta>1-\mu$, if the retailer does not carry product 1, by Lemma 8, the retailer opens its platform and the third party sells product 1 or no products through the retailer. Because $\max\{\pi^{0*}_{r0},\pi^{0*}_{r1}\}=\frac{\theta}{4}\le\frac{1}{4}<\pi^{*}_{r0}$, where the last inequality is by Equation (3), it is optimal for the retailer to carry product 1.

Next, we consider $\theta\le 1-\mu$. In this case, if the retailer does not carry product 1, the retailer opens its platform and the third party sells both products via the retailer.

a. The cases with $\mu\le\frac{4}{9}$: When $\theta\le K(\frac{\mu}{4})$, if the retailer carries product 1, by Proposition 4, the retailer opens its platform and the third party sells both products through the retailer if $\theta>\theta^*_{r3}$. We have

$$\pi^{*}_{r3}-\pi^{0*}_{r3}=\frac{4-\theta}{(3-\theta)^2}+\theta t-\left(\frac{\theta}{4}+\theta t\right)=\frac{4-\theta}{(3-\theta)^2}-\frac{\theta}{4}>0.$$

If $\theta\le\theta^*_{r3}$ (which implies that $\pi^*_{r0}\ge\pi^*_{r3}$), by Proposition 4, the retailer does not open its platform. We have $\pi^*_{r0}-\pi^{0*}_{r3}\ge\pi^*_{r3}-\pi^{0*}_{r3}>0$. Therefore, in this case, it is optimal for the retailer to carry product 1.

When $\theta>K(\frac{\mu}{4})$, we can verify that $1-\mu<\theta^*_{s2}$ when $\beta<\bar\beta(\alpha)$ and $1-\mu<\theta^*_{21}$ when $\beta\ge\bar\beta(\alpha)$. When the retailer carries product 1, by Proposition 4, if $\theta\in(K(\frac{\mu}{4}),1-\mu]$ and $\theta>\theta^*_{r2}$, the retailer opens its platform and the third party sells product 2 through the retailer. We have

$$\pi^*_{r2}-\pi^{0*}_{r3}=\begin{cases}\frac{2-\mu}{2}+\theta t-\left(\frac{\theta}{4}+\theta t\right)>0 & \text{if } \mu\le\frac{2}{3},\\ \frac{4}{9\mu}+\theta t-\left(\frac{\theta}{4}+\theta t\right)>0 & \text{otherwise.}\end{cases}$$

If $\theta\le\theta^*_{r2}$ (which implies that $\pi^*_{r0}\ge\pi^*_{r2}$), the retailer does not open its platform. We have $\pi^*_{r0}-\pi^{0*}_{r3}\ge\pi^*_{r2}-\pi^{0*}_{r3}>0$. Therefore, in this case, the retailer optimally chooses to carry product 1.

b. The cases with $\mu>\frac{4}{9}$: We can also verify that $1-\mu<\theta^*_{s2}$ when $\beta<\bar\beta(\alpha)$ and $1-\mu<\theta^*_{21}$ when $\beta\ge\bar\beta(\alpha)$. When the retailer carries product 1, by Proposition 4, if $\theta>\theta^*_{r2}$, the retailer opens its platform and the third party sells product 2 through the retailer. If $\theta\le\theta^*_{r2}$, the retailer does not open its platform. Similar to the analysis in part (a), we conclude that it is optimal for the retailer to carry product 1. □

Endnotes

[1] See https://sell.amazon.com/zh_CN/beginners-guide.html?lang=zh_CN (accessed August 8, 2020).

[2] See https://sell.amazon.com/?ref_=footer_soa_sellp2_t1&ld=AZFSSOA_SELLP2_T1 (accessed August 8, 2020).

[3] See http://corporate.jd.com/.

[4] We thank an anonymous reviewer for suggesting this direction.

[5] We define $K(0)\equiv\lim_{x\to 0}K(x)=\lim_{x\to 0}\frac{-1+6x+\sqrt{1-8x}}{2x}=1$.

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