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Drivers, Riders, and Service Providers: The Impact of the Sharing Economy on Mobility

Saif Benjaafar,^a Harald Bernhard,^b Costas Courcoubetis,^b Michail Kanakakis,^b Spyridon Papafragkos^b

^a University of Minnesota, Minneapolis, Minnesota 55455; ^b Singapore University of Technology and Design, Singapore 487372

Contact: saif@umn.edu,  <https://orcid.org/0000-0002-1949-0105> (SB); bernhard.harald@gmail.com (HB); costas@sutd.edu.sg,  <https://orcid.org/0000-0001-9568-0640> (CC); michail_kanakakis@sutd.edu.sg,  <https://orcid.org/0000-0002-6155-0802> (MK); spyridon_papafragkos@sutd.edu.sg,  <https://orcid.org/0000-0002-1393-4414> (SP)

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Abstract. It is widely believed that ride sharing, the practice of sharing a car such that more than one person travels in the car during a journey, has the potential to significantly reduce traffic by filling up cars more efficiently. We introduce a model in which **individuals may share rides for a certain fee**, paid by the rider(s) to the driver through a ride-sharing platform. Collective decision making is modeled as an anonymous nonatomic game with a finite set of strategies and payoff functions among individuals who are heterogeneous in their income. We examine how ride sharing is organized and how traffic and ownership are affected if a platform, which chooses the seat rental price to maximize either revenue or welfare, is introduced to a population. We find that the ratio of ownership to usage costs determines how ride sharing is organized. If this **ratio is low, ride sharing is offered as a peer-to-peer (P2P) service**, and if this ratio is high, ride sharing is offered as a business-to-customer (B2C) service. In the P2P case, rides are initiated by drivers only when the drivers need to fulfill their own transportation requirements. In the B2C case, cars are driven all the time by full-time drivers taking rides even if these are not motivated by their private needs. We show that, although the introduction of ride sharing may reduce car ownership, it can lead to an increase in traffic. We also show that traffic and ownership may increase as the ownership cost increases and that a revenue-maximizing platform might prefer a situation in which cars are driven with only a few seats occupied, causing high traffic. We contrast these results with those obtained for a social welfare-maximizing platform.

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Keywords: ride sharing • sharing economy • transportation • equilibrium analysis • nonatomic games • social welfare

1. Introduction

Ride sharing denotes the practice of sharing a privately owned car such that more than one person travels in a car during a journey. Sharing rides was traditionally restricted to family members and close friends or long-distance journeys scheduled well before the intended time of departure. Only the emergence of mobile computing technologies and GPS location services in combination with electronic payments and online reputation systems provided for the technological cornerstones to make on-demand, short-distance ride sharing among strangers viable. Typically, individuals enter their trip details on an online platform that facilitates the matching of riders with cars and drivers. To be successful, such platforms must attract both supply and demand for rides.

Ride sharing includes settings in which drivers offer rides to others while undertaking a journey themselves. It also includes settings in which the

driver derives no utility from the journey but instead offers rides solely to generate income. Examples of the former include Bla Bla Car in Europe, Waze Carpool and Ryde in Southeast Asia. Examples of the latter include UberX in the United States, Didi in China, and Grab in Southeast Asia.

In dense urban areas, ride-sharing platforms offer an alternative mode of transportation to established modes, such as the use of public transport or one's own private car. As such, ride sharing may significantly impact traffic, car ownership, and the population's welfare in terms of mobility. For example, Santi et al. (2014) and Alonso-Mora et al. (2017) argue that ride sharing has the potential to significantly reduce congestion. Although it is clear that ride-sharing platforms have the potential to significantly increase the number of passengers per car, it is less clear whether, without strict control over demand and supply for rides, increasing car occupancy

(the number of occupied seats per car per trip) would indeed lead to less traffic and less car ownership. For example, it is conceivable that the potential income from offering rides to others could encourage low-income individuals to pick up driving as an alternative form of employment, potentially resulting in a net increase in the number of cars.¹ Moreover, if shared rides are priced sufficiently lower than private car costs, demand for such rides could increase and potentially lead to higher traffic. Hence, although ride sharing is heralded by some as a more sustainable and environmentally friendly form of urban mobility, the economic opportunities provided by a sharing platform might result in different outcomes by either increasing the supply of seats (through new car ownership) or the demand for seats (by shifting demand away from other forms of transport). This paper aims to illuminate these issues.

We introduce a model in which a population of individuals, heterogeneous in the income they earn when not driving (and, in an extension, in their utility for private transportation), interacts with a ride-sharing platform. Individuals may act on the supply-and-demand side of the platform. They may supply seats to the platform by renting out empty seats in their cars on a casual or full-time basis or use the platform to find a ride. Individuals are assumed rational and optimize their long-term average payoffs, deciding on whether and how to interact with the platform and whether to own a personal car. Collective decision making is modeled as an anonymous nonatomic game. We study possible equilibria of this game and how these depend on system parameters, including the seat rental price, usage and ownership costs for cars, and the distribution of income and utility in the population. We do not focus on the engineering challenges associated with operating a ride-sharing platform, but study the incentive structure and potential negative externalities brought about by the self-interested behavior of the individuals.

Our model distinguishes between casual ride-sharing services (e.g., carpooling) in which the driver of the car initiates the journey to fulfill the driver's own need for transportation—we call this peer-to-peer (P2P) ride sharing—and professional ride-sharing services with which the driver's motivation to drive is the seat rental price paid by the riders. We denote the latter as business-to-customer (B2C) ride sharing. The model allows both casual and professional ride-sharing services to arise as possible outcomes in equilibrium. Using the model, we seek to investigate the following important questions. Under what conditions is a particular type of ride supply (P2P or B2C) likely to arise, and is it possible for both to coexist? Is one type more desirable than the other (e.g., in terms of reducing car ownership or traffic)? If so, are there levers at the disposal of a regulator to induce the more desirable outcomes? More generally,

does the introduction of ride sharing necessarily improve outcomes in terms of car ownership and traffic?

To find answers to these questions, we study a platform that optimizes either revenue or social welfare and chooses the seat rental fee accordingly. Some of our findings include the following:

- Only two types of equilibria are expected to prevail with the introduction of ride sharing.

- A P2P equilibrium, which separates society into two classes: individuals who own cars but use them casually to primarily fulfill their transportation needs and share rides whenever they drive to make additional income and individuals who prefer not to own a car and use the service of the platform for transportation.

- A B2C equilibrium, which separates society into two classes: a class of service providers who own cars and forgo their income from employment they generate when they are not driving to offer rides to the platform on a full-time basis and a class of individuals with high income who exclusively use the platform to satisfy their transportation needs.

The ratio of ownership to usage costs determines whether ride sharing is organized as a P2P or B2C service.

- Whether the introduction of ride sharing as the dominant form of private transportation increases or decreases car ownership or traffic depends crucially on the ownership and usage costs of cars. In particular, when the cost of ownership is sufficiently high, it is possible for the introduction of ride sharing to reduce ownership but to increase traffic.

- Ownership and traffic are affected differently by increases in the costs of ownership and usage with ownership being nonmonotonic in both. Whether ownership or traffic increases or decreases with the cost of ownership and cost usage depends on whether the prevailing equilibrium is P2P, B2C, or a mix of both. In the B2C regime, we observe the paradoxical situation in which increasing ownership costs can increase both traffic and ownership.

- In general, the platform's and society's interests may not be aligned. Our analysis suggests that the interests are much less aligned when B2C equilibria appear. We identify parameter combinations such that, under a revenue-maximizing platform, there is a large number of professional drivers who constantly drive their cars, thus causing high traffic, but transport very few riders. In contrast, under a welfare-maximizing platform, for the right rental price, every car is always fully occupied, and hence, traffic is no higher than is absolutely necessary to transport all the riders for the socially optimal level of demand. This is consistent with the popular belief that sharing can indeed reduce ownership and congestion in society.

- In various extensions, we analyze the cases of the platform providing subsidies (positive or negative) to

service providers, the presence of a congestion cost that is increasing in the amount of traffic generated by the platform, and casual drivers experiencing greater matching friction than service providers (professional drivers). In most cases, we show that our main results continue to hold. In the others, the results are modified in obvious ways.

Results in the paper inform the debate about whether ride sharing may or may not decrease ownership and traffic and under what conditions. They also point to potential levers a regulator may be able to deploy to induce desired outcomes with regard to ownership and usage (e.g., affecting the cost of ownership and usage via taxes, subsidies, or other forms of financial transfer). More significantly, the results point to the importance of tailoring such regulation to the prevailing equilibrium regime.

The rest of this paper is organized as follows. In Section 2, we review related literature. In Section 3, we describe our model. In Section 4, we characterize the equilibrium. In Section 5, we consider the problem of a revenue-maximizing platform. In Section 6, we consider the impact of a revenue-maximizing platform on traffic and ownership. In Section 7, we consider the problem from the perspective of a social planner. In Section 9, we analyze various extensions. In Section 10, we offer concluding comments. Proofs and supplementary material can be found in the online appendix.

2. Related Literature

There is a growing literature in operations management and related disciplines that deals with shared mobility (mobility enabled by business models that eschew private ownership of vehicles in favor of on-demand access to shared vehicles). Some of this literature considers settings in which a third party (e.g., a car rental company) owns a fleet of vehicles and makes these vehicles available for short-term rentals; see, for example, Bellos et al. (2017), He et al. (2017), Kabra et al. (2018), Shu et al. (2013), and Benjaafar et al. (2019a, 2021). A stream within this literature considers the design and operations of such a system (e.g., size of the fleet, location of pickup and drop-off points, and the rebalancing of inventory across locations over time); see Kabra et al. (2018), Benjaafar et al. (2019a, 2021), and He et al. (2017) for discussion and related literature. Another stream examines the economics of such systems and their implications on consumers, service providers, and society. In particular, Bellos et al. (2017) examine the economic incentives for a car manufacturer to also operate a car-sharing system. They show that a manufacturer may have an incentive to increase the fuel efficiency of shared cars although offering shared cars may not necessarily lead to an overall reduction

in environmental impact. Agrawal and Bellos (2017) examine the broader implications of business models, of which car sharing can be viewed as an example, built around selling the functionality of a product instead of the product itself.

A second body of literature considers car sharing that is enabled by peer-to-peer car rentals. Under such a system, car owners have the option of putting their cars up for rent when they are not using them. Fraiberger et al. (2015) develop a dynamic model of peer-to-peer car sharing and use it to study, among others, the impact of peer-to-peer car sharing on ownership and vehicle usage. For parameter values calibrated using data from a car-sharing service, they observe that peer-to-peer car sharing leads to a reduction in new and used car ownership, an increase in the fraction of the population who do not own, and an increase in the usage intensity per vehicle. Jiang and Tian (2016) study a similar model but focus on the decision of the car manufacturer. In particular, they study how the manufacturer should choose product quality and price in anticipation of sharing by consumers. Benjaafar et al. (2019b) study a model of peer-to-peer product sharing and examine the impact of the cost of ownership. Their model precludes the simultaneous use of the product by the owner and the renter, an essential feature of our model. Also, in contrast to our setting, (i) the owner of the product is not involved in providing service to the renter, and (ii) the usage cost is not a factor of the model and the results depend entirely on the cost of ownership. Moreover, the strategies considered by individuals are much simpler and consist of being either an owner or a renter. Abhishek et al. (2020) also consider a model of peer-to-peer product sharing and examine the role of consumer usage heterogeneity.

A third body of literature deals with the peer-to-peer provisioning of transportation services by which a platform matches drivers willing to offer rides with passengers. The platform decides on wages and prices, which, in turn, determine driver supply and passenger demand. Examples from this literature include Benjaafar et al. (2021), Cachon et al. (2017), Taylor (2018), Bai et al. (2019), Cohen and Zhang (2017), Bimpikis et al. (2019), and Chen and Hu (2018), among many others.² An important focus of this literature is the development of price and wage mechanisms that ensure effective matching of supply and demand for rides. This literature typically assumes that the drivers and passengers are drawn from separate populations whose sizes are exogenously specified. In this paper, we consider a setting in which the population of drivers is endogenously determined with drivers and passengers drawn from the same overall population.

The literature on ride sharing (the practice of pooling multiple passengers undertaking the same trip at the

same time that we consider in this paper) is relatively limited. Much of the existing literature on this topic focuses on the optimization of the logistics of ride-sharing systems; see Agatz et al. (2012) and Furuhata et al. (2013) for reviews. Santi et al. (2014) and Alonso-Mora et al. (2017) develop optimization algorithms to pool trip requests and match these requests with available vehicles. They apply their algorithms to a large data set of taxi trips from New York City. They show that pooling passengers can significantly reduce the number of taxis needed. Their analysis treats trip demand as being exogenous and not affected by the extent to which trips are pooled. They also treat the supply of seats and demand for rides as being independent (i.e., drivers and riders are not drawn from the same population).

There is relatively limited empirical research that documents the impact of car sharing in general, and ride sharing in particular, on ownership and traffic or congestion. Li et al. (2016) investigate how Uber affects traffic in urban areas of the United States, where it operates, and find evidence that the entry of Uber leads to a decrease in traffic congestion. In contrast, Barrios et al. (2019) find that the entry of Uber and Lyft in U.S. cities is associated with an increase in fatal car accidents, vehicle miles traveled, traffic delays, and new car registrations. Clark et al. (2014) present results from a survey of British users of a peer-to-peer car-sharing service. They find that peer-to-peer car sharing has led to a net increase in the number of miles driven by car renters. Ballús-Armet et al. (2014) find that approximately 25% of surveyed car owners would be willing to share their personal vehicles through peer-to-peer car sharing with liability and trust concerns being the primary deterrents. Nijland et al. (2015) (and, also, Martin and Shaheen 2011) study car sharing that involves a third-party service provider and find that car sharing would lead to a net decrease in traffic. On the other hand, a study by KPMG (Silberg et al. 2015) projects a significant increase in miles driven by cars and attributes this to increased usage of on-demand transportation services. In general, there does not appear to be a consensus yet on the impact of car and ride sharing on traffic and ownership.

Our tool for modeling collective decision making relies on the theory of anonymous games with a continuum of agents. The study of such games usually follows the formulations of either Schmeidler (1973) or Mas-Colell (1984) with our formulation being closest to the former. In both models, the existence of pure strategy equilibria is guaranteed for general assumptions, including for finite strategy spaces such as ours. Such games are an approximation to a game with a finite but large number of players as is the case for ride-sharing platforms, which can only function at scale. Considering a very large number of players “help[s]

overcome modeling difficulties” (Kalai 2004, p. 1632). In particular, the equilibrium predictions become robust to modifications to the rules of the game as, for example, the order of play or allowed revisions of strategy choices. Furthermore, any equilibrium in our continuum-of-agents game translates into an ε -Nash equilibrium of a finite game with a large enough number of players (e.g., Carmona and Podczeck 2009). See Khan and Sun (2002) for an introduction to games with many players.

3. The Model

3.1. Agents' Description

We consider a population consisting of individuals who, besides being employed and obtaining income, need also to perform activities that require transportation (for example, going to work or shopping at some other location). Each such activity results in a request for a *ride* by that individual. Our model is the simplest possible in terms of topology and assumes that, for all individuals, transportation takes place between a single source–destination pair and that all rides are of the same type, that is, take the same average time or require the same average distance of travel. Time is measured in time units (e.g., days), and we assume that all individuals must satisfy one transportation activity per unit of time.

A ride can be obtained through different means, which influence an individual's payoff. Namely, an individual may fulfill these transportation needs using one of three options:

- Use public transport: an individual may choose to use public transport, which we assume is always available. This corresponds to the outside option for all individuals in the population.³
- Drive a car: an individual may use the individual's own car to undertake the journey and perform a *trip*. When performing a trip, the individual may offer to take others along with the platform providing the individual with potential riders who are willing to rent seats in the individual's car. The fraction of seats offered that is filled by riders is $\bar{p} \in [0, 1]$, where \bar{p} is determined endogenously. Trips can be initiated either because of self-fulfilling a need for a ride, to generate income, or for both reasons. Driving a car requires individuals to choose the long-term option of owning a car.
- Ride share: an individual may seek a ride in someone else's car with the platform providing the individual with potential willing drivers. This is only possible for a fraction $p \in [0, 1]$ of ride requests in which p is determined endogenously. For a fraction $1 - p$ of requests, the individual must resort to the outside option.

We require that every individual satisfies the need for a ride. If the individual owns a car, the individual uses the car. Otherwise, the individual requests a seat

from the platform and uses the outside option if the request is not successful. Individuals derive utility $\rho > 0$ if their request for a ride is fulfilled using a car (either the individual's own car or a car driven by someone else) and are homogeneous in their utility ρ (we consider the case in which ρ is heterogeneous in Section 8). Without loss of generality, we assume that the utility an individual derives from using public transport is zero.

Besides fulfilling the need for transportation, individuals may pursue work to increase their payoff. In our model, they may choose between two long-term options:

- Work for a wage at rate $\nu > 0$ per unit time. This may, for example, correspond to full-time employment in some company.
- Forgo the wage ν and offer seats to the platform to attract fare-paying riders although the trip is of no personal value to the driver. This corresponds to becoming a full-time professional driver for the platform.

Hence, an individual may choose a guaranteed wage (rate) ν by keeping a previous job, which is heterogeneous in the population, or rely fully on a platform-dependent income that is affected by the population's collective behavior. We assume a unit mass of such nonatomic individuals whose wages are distributed according to a distribution M on the set of individual types $X = (0, \infty)$. Specifically, we assume that the density of M with respect to ν is given by $m_\nu(\nu) = \frac{1}{(1+\nu)^2}$, which implies $M(\{1\} \times [\nu, \infty)) = \frac{1}{1+\nu}$, a form of a generalized Pareto distribution. Pareto distributions have been widely used in economics to model distribution of income. This assumption also allows us to characterize various quantities of interest in closed form.

3.2. The Platform

We assume there exists a platform operated by a third party that matches supply and demand for seats. It also chooses the market price r for seats. Every individual may use the platform free of subscription charge.

There are two ways for an individual to interact with the platform. The individual may either (i) request a seat on the platform to fulfill the need for a ride or, (ii) whenever the individual is driving the individual's own car and performing a trip, put the remaining seats up for rent on the platform for the duration of the trip. The former contributes to seat demand and the latter to seat supply with which the precise definition of the resource "seat" refers to the capability of using a physical seat in a car over the duration of a ride.

Every car is assumed capable of carrying, in addition to the driver, up to a maximum of $k > 0$ riders

per trip. Whenever the platform manages to find a rider for an empty seat, an amount r corresponding to the seat rental price specified by the platform gets transferred from the rider to the driver of the car.

The capability of the platform to successfully match riders with seats depends on the current levels of supply and demand. Let α denote the number of cars starting new trips per unit of time (i.e., the rate of offered trips defined in our unit of time). Then, $k\alpha$ correspond to the rate at which cars offer seats (as defined) to the platform. Let β denote the rate of seat requests, that is, the number of individuals requiring a ride and using the platform. To account for imbalances in seat supply and demand, we define matching functions p and \bar{p} dependent on α and β , where p indicates the fraction of seat requests that the platform is able to match and \bar{p} the fraction of seats sold (of those put up for sale) per trip per car. We require $p, \bar{p}: [0, \infty)^2 \rightarrow [0, 1]$. In addition, we require $p\beta \leq k\alpha$ (number of seats sold no more than number of seats available) and $\bar{p}k\alpha \leq \beta$ (number of seats occupied no more than number of seats requested). That is, the number of seats sold through the platform is bounded by $k\alpha \wedge \beta$, where $a \wedge b := \min\{a, b\}$. For simplicity we assume the upper bound is achieved and model the matching with the two elementary functions.

- Fraction of demand (seat requests) that is served:

$$p(\alpha, \beta) := \frac{k\alpha}{\beta} \wedge 1, \text{ and}$$

- Fraction of supply (seats available) that is matched with demand:

$$\bar{p}(\alpha, \beta) := \frac{\beta}{k\alpha} \wedge 1.$$

3.3. Strategies

Given the different decisions an individual may make as described in Section 3.1, we consider four different strategies that reflect the decision choices. Namely, we allow individuals to choose among the four strategies in $\Sigma = \{A, D, U, S\}$.

- $[A]$, the abstinent: an individual who chooses this strategy generates income ν during each unit of time and does not own a car, and whenever the individual needs a ride, the individual chooses public transport (deriving zero utility from the ride).

- $[D]$, the driver: an individual who chooses this strategy generates income ν during each unit of time, owns a car, and drives the individual's own car once per unit of time to satisfy the individual's own need for a ride, deriving utility ρ per unit of time. Empty seats in the individual's car are offered on the platform, generating a supply of k seats per unit of time. Any seat rental fee paid to the driver reduces the individual's trip costs and may create net profits.

- [S], the service provider: an individual who chooses this strategy owns a car but does not earn a wage. The individual's income is generated by renting seats on the platform. During each time unit, the individual performs $\lambda > 1$ trips, each offering k seats, that is, supplies $k\lambda$ seats per unit of time. Because the individual owns a car, we assume that the individual can always find a way to fulfill the individual's personal need for transportation while driving, offering seats to others (i.e., at no additional trip) and, hence, deriving utility ρ per unit of time.

- [U], the user: an individual who chooses this strategy generates income v during each unit of time and does not own a car, and whenever the individual needs to take a ride, the individual tries to obtain a seat through the sharing platform, contributing a unit of seat demand. If that request is successful, the individual derives utility ρ per unit of time. If that request is unsuccessful, the individual takes the outside option and derives zero utility from the ride.

Although there are three modes of transportation (public transport, driving, and ride sharing) and a decision between two choices on how to generate professional income during each period, other feasible strategies are strictly dominated by these once we account for associated costs. For example, the strategy of owning a car (which incurs a positive cost) but always using public transport is strictly dominated by strategy *A* in terms of payoff. Similarly, a cost-sensitive individual who drives the individual's own car always puts the additional seats up for rent on the platform as we do not model any inconvenience associated with sharing a ride. *A*'s strategy, allowing part time work for the platform, is also dominated by either strategy *D* or strategy *S* as a rational individual makes an either/or decision depending on the higher rate of income: income from either the platform or work. A positive seat rental price r induces this behavior.

We denote by μ_σ the fraction of individuals playing strategy σ and by $\mu = (\mu_A, \mu_D, \mu_S, \mu_U)$ the corresponding vector with its components summing to one. In Section 3.2, we specify the matching functions p and \bar{p} as functions of supply and demand for seats. Having defined the behavior of each strategy, along with specifying an aggregate strategy distribution μ , allows us to compute supply and demand rates and, thus, the matching functions p and \bar{p} as functions of μ . To do so, we evaluate how each strategy contributes to seat supply and demand. Drivers offer one trip per unit time, whereas service providers offer λ trips per unit time, and users generate seat demand at a unit rate. This yields the following system of equations:

$$\begin{aligned}\alpha(\mu) &= \mu_D + \lambda\mu_S, \\ \beta(\mu) &= \mu_U,\end{aligned}$$

and

$$\begin{aligned}p(\alpha(\mu), \beta(\mu)) &= \frac{k\alpha(\mu)}{\beta(\mu)} \wedge 1, \\ \bar{p}(\alpha(\mu), \beta(\mu)) &= \frac{\beta(\mu)}{k\alpha(\mu)} \wedge 1.\end{aligned}$$

Substituting the expressions for $\alpha(\mu), \beta(\mu)$ yields the matching functions

$$p(\mu) = \frac{k\lambda\mu_S + k\mu_D}{\mu_U} \wedge 1, \quad (1)$$

and

$$\bar{p}(\mu) = \frac{\mu_U}{k\lambda\mu_S + k\mu_D} \wedge 1. \quad (2)$$

In case $\mu_U = 0$ and μ_S or $\mu_D > 0$, we use $p(\mu) = 1$, and similarly, in case $\mu_D = \mu_S = 0$ and $\mu_U > 0$, we use $\bar{p}(\mu) = 1$. When there is no supply and demand (i.e., no sharing economy), only strategies *A* and *D* are available, and $\bar{p} = 0$.

3.4. Payoffs

Each individual performs one trip per unit time. If it is performed using private transportation, the individual obtains utility ρ per unit time. Otherwise, zero utility is obtained. Additionally, all individuals not employing strategy *S* earn a wage at rate v per unit time. Individuals who use the platform to find a seat pay r per ride to the driver, who is offering the seat as compensation for the service. Those who offer rides pay a cost c per trip, which includes any variable costs, such as fuel costs, wear and tear to the car, and road tolls, among others. Additionally, in case supply exceeds demand, we assume the rental fees paid by the riders are shared uniformly among all seat suppliers (i.e., all seat suppliers obtain the same average revenue per seat in the long run). Finally, everyone who owns a car (i.e., individuals who choose strategies *D* or *S*) pays a cost ω per unit time to cover the cost of ownership. Each strategy's payoff only depends on an individual's type $v \in (0, \infty)$ and the aggregate distribution of strategies played, μ .

In the following, we specify the payoff functions (per unit time) for each strategy.

- [A]: Abstinent individuals earn income v per unit time and have no additional costs. Hence, the corresponding payoff π_A for an individual with type v is given by

$$\pi_A(v, \mu) = v.$$

- [D]: Drivers always use their own car when satisfying their need for a ride. Hence, they obtain utility ρ

per unit time for the trip, pay usage cost c , and earn $k\bar{p}(\mu)r$ by selling excess seats on the platform. In addition, they pay the car ownership cost ω per unit time. They earn a wage at rate v . Hence, the corresponding payoff is given by

$$\pi_D(v, \mu) = \rho + k\bar{p}(\mu)r - c - \omega + v.$$

- [S]: A service provider realizes utility ρ per unit time for a personal trip. All other trips are of no personal utility to the provider. The provider pays ω for car ownership and undertakes trips at rate $\lambda > 1$ with a value of $k\bar{p}(\mu)r - c$ per trip. Thus, the payoff of a service provider is given by

$$\pi_S(v, \mu) = \rho - \omega + \lambda(k\bar{p}(\mu)r - c).$$

- [U]: Users earn wage v , do not own a car, and get payoff $\rho - r$ whenever they successfully secure a ride. Thus, the payoff of a user is given by

$$\pi_U(v, \mu) = v + p(\mu)(\rho - r).$$

Together with the expressions for $p(\mu)$ and $\bar{p}(\mu)$ in (1) and (2), the payoff functions are fully characterized. We use the notation $G := ((X, M), \Sigma, \pi)$ to denote the game defined by the triplet of the agents (X, M) , strategies Σ , and payoffs π . Without loss of generality, we scale ρ such that $\rho = 1$.

We conclude this section by introducing the concept of the “traditional economy.” In this case, $\pi_D(v, \mu) = \rho - c - \omega + v$. This corresponds to the game in which individuals can choose strategies among $\{A, D\}$; that is, they can fulfill their transportation needs by either using public transport or driving their own car. This is because, in the absence of a platform, there can be no users or service providers. We assume that $\omega + c < \rho$ because, otherwise, there may be no drivers in the traditional economy. In an extension in which we allow for utility from transport to be heterogeneous in the population, we relax this assumption (see Section 8).

3.5. Summary of Key Model Assumptions

After introducing our model, it is useful to summarize and discuss its key assumptions.

We assume a monopolist platform that can freely set prices in the absence of competition. Some more specific assumptions of our model are that (i) the fraction of each trip revenue kept as a profit by the platform is negligible, (ii) cars cost the same for professional and nonprofessional drivers, (iii) there are no negative externality effects in the payoffs associated with traffic, and (iv) we do not distinguish between the features of the seats offered by professionals and nonprofessionals.

The assumption of a monopoly is done for two reasons. The first is that understanding the monopoly

case is a first nontrivial step toward the study of more complex market structures. The second is that most of the existing markets are not highly competitive in car-sharing services. In several markets in which competition existed, a merger occurred between the two dominant players (e.g., Uber and Didi in China, Uber and Grab in Singapore, Uber and Careem in the Middle East). There are good reasons for that, including positive externalities of the platform size in terms of the number of users and drivers and economies of scope because these platforms can easily add more services to their portfolio. Hence, analyzing the case of monopoly is both illuminating and practical.

We turn now to the rest of the assumptions. Assumptions (i)–(ii) are relaxed in Section 9.1 in which we allow for the platform to keep a fraction $1 - \theta$ of the revenue from each trip. We also allow for a subsidy ϕ to the cost of ownership for professional drivers who join the platform. This may correspond to economies of scale (cheaper cars) or monetary transfers (positive or negative) to the service providers. Assumption (iii) is addressed in Section 9.3 in which we reduce the utility ρ of individuals traveling in private cars by a congestion cost term that is convex and increasing in the amount of system traffic. Assumption (iv) is addressed in Section 9.2 in which we consider a setting with additional “friction” in matching the supply of seats by nonprofessionals to demand. This captures the practical situation in which paths of trips offered by carpooling drivers are not flexible and may not match to rides demanded by users. It can also address the case in which a fraction of users refuses to use seats in cars driven by nonprofessionals or when, occasionally, nonprofessional drivers prefer to drive alone in their car. Finally, it can lead to a model of “market design.” A strategic platform may choose to restrict the supply of seats by nonprofessionals using different criteria in matching supply with demand.

4. Equilibrium Analysis

In this section, we study equilibria of the game G .⁴ In Section 4.1, we introduce our notion of equilibrium and equivalence of such equilibria. In Section 4.2, we provide the main characterization for equilibria in the game G and a description of how ride sharing is organized based on the particular equilibrium.

4.1. Preliminaries

We require that, in equilibrium, every individual of type $v \in X$ plays a strategy $\sigma(v)$, which represents the individual’s (weakly) best choice among the strategies defined in Section 3.3 given the aggregate behavior of everyone else. This motivates our equilibrium definition to be a partition $\{P_\sigma\}_{\sigma \in \Sigma}$ of X into measurable sets with a one-to-one correspondence between the elements of the partition and our set of

strategies. Let $\mu_\sigma = M(P_\sigma)$ be the total mass of agents choosing strategy σ , and $\mu(P)$ be the distribution of the strategies according to P , that is, the vector $\mu = \{\mu_\sigma\}_{\sigma \in \Sigma}$ for the given partition P .

Definition 4.1. An “equilibrium” is a partition $P = \{P_\sigma\}_{\sigma \in \Sigma}$ of X such that, for all individuals contained in a partition set P_{σ_0} , the (weakly) best strategy choice is σ_0 :

$$\forall v \in P_{\sigma_0} : \pi_{\sigma_0}(v, \mu(P)) \geq \pi_\sigma(v, \mu(P)) \quad \forall \sigma \in \Sigma.$$

Two equilibria P^1 and P^2 are called *revenue-equivalent* if they induce the same strategy distributions $\mu(P^1) = \mu(P^2)$, and the payoff for each individual is identical under both partitions.

In many cases of system parameters, the resulting equilibria may not be unique, but we can show that they belong to a unique equivalence class. Because the payoffs are affine functions of the types of individuals (for fixed values of p and \bar{p}), it turns out that such an equivalence class of equilibria is a partition P of X into convex sets in which, now, (i) each strategy σ appearing in equilibrium is associated with a unique element of P , but (ii) any element in P can be associated with multiple strategies $J \subset \Sigma$, denoted by P_J , with the property that each individual in P_J weakly prefers strategies in J from other strategies but is indifferent between strategies in J . An equivalence class containing such a set $P_J, J \subset \Sigma$ can generate arbitrarily many equilibria by arbitrarily assigning the strategies in J to agents in P_J while keeping constant the total mass μ_σ of agents playing each strategy $\sigma \in J$.

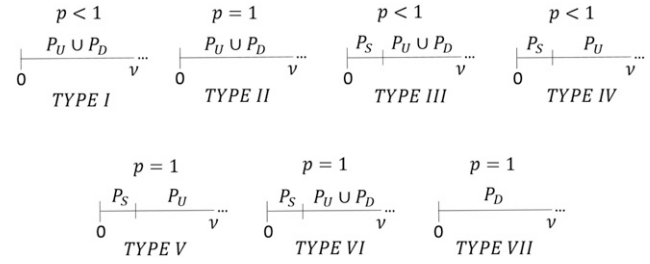
We prove that, if an equilibrium exists, then it corresponds to a unique such equivalent class modulo how to choose strategies on the boundaries of the partition sets, that is, on a set of measure zero with respect to M . This equivalent class is fully defined by specifying, for each multistrategy set P_J of the partition, the corresponding measures of strategies $\mu_\sigma, \sigma \in J$.

Lemma 4.1. Given any equilibrium partition P^0 , we can construct a unique equivalence class represented by a convex partition P of X that characterizes all the partitions of X that are revenue-equivalent equilibria with P^0 .

4.2. Characterization of Equilibria in the Sharing Economy

We first define seven types of partitions, which we show to be associated with all the possible equilibria. Figure 1 illustrates these partitions graphically, in which we use notation $P_U \cup P_D$ to refer to the multistrategy set $P_{\{U,D\}}$. The type of a partition is determined by the value of p it induces (note that $p < 1$ if and only if demand is strictly greater than supply) and the geometry of the partition sets. Because $X = (0, \infty)$, a partition set corresponds to an interval over the positive

Figure 1. Illustrations of the Seven Partition Types That Correspond to Possible Equilibria



Note. Types III and VI correspond to equivalence classes of equilibria (i.e., multiple equilibria that are revenue equivalent), whereas the rest correspond to single equilibria.

real line, which specifies a range of wage values for individuals. Each equilibrium type represents a different way the sharing economy materializes.

A formal definition of these equilibrium types is in Definition A.1 in the online appendix and provides exact algebraic expressions for all seven partition types. The following is a summary description:

- Equilibria of types I and II consist of individuals choosing among strategies D and U (i.e., individuals are either drivers or users). In equilibrium of type I, supply is less than demand, and in equilibrium of type II, supply is greater than or equal to demand.
- In equilibria of types III and VI, individuals are drivers, users, or service providers. In particular, there exists a threshold on income v such that individuals whose income is below this threshold choose to be service providers, and others are either users or drivers. In equilibrium of type III, supply is less than demand, and in equilibrium of type VI supply is greater than or equal to demand.
- In equilibria of types IV and V, individuals are either users or service providers. In particular, similarly to the previous case, there exists a threshold on income v such that individuals whose income is below this threshold choose to be service providers, and others are users. In equilibrium of type IV, supply is less than demand, and in equilibrium of type V, supply is greater than or equal to demand.
- An equilibrium of type VII has only drivers and no users and can be thought as a nonsharing/traditional economy equilibrium because there is no demand for seats.

Theorem 4.1. For each combination of parameter values for $c, r, k, \omega > 0$ and $\lambda \geq 2$, there exists a unique equilibrium modulo equivalence in the game $G = ((X, M), \Sigma, \pi)$. In each case, the equilibrium is of type I–VI.

In the online appendix, we provide a more elaborate version of Theorem 4.1 in which we fully characterize the type of equilibria for each combination of parameter values, which is needed for the rest of the proofs of the paper.

5. The Platform's Problem

We have so far treated seat rental price as being exogenously specified. This price is typically a decision the platform makes. In this section, we consider a platform that decides on the rental price to maximize revenue. Analyzing a revenue-maximizing platform is motivated by the practical case of a platform that deducts a service charge proportional to the rental price for every transaction on the platform. That is, each rider pays the price r to the platform, which keeps $(1 - \theta)r$ as a service fee and pays the driver θr for some exogenously fixed θ . Thus, the platform's profits are proportional to the revenue generated. For ease of exposition, we assume that $1 - \theta$ is negligible, and hence, r is fully transferred from seat renters to seat providers. The results of the analysis remain similar if we do not make this simplification; see Section 9.1.

Let $R(r)$ denote the revenue generated per unit time for a given seat rental price r (i.e., the total amount of seat rental prices paid through the platform per unit of time). Note that the seat rental price is only paid by individuals who choose strategy U and only in case the platform is able to match a seat request. Thus, $R(r)$ is given by $R(r) = pr\mu_U$, where p and μ_U (the mass of individuals choosing strategy U) denote the respective quantities in equilibrium for the given seat rental price r . By Theorem 4.1, $R(r)$ is well defined for all values of r . Then, the platform's problem in finding a revenue-maximizing price r_R can be stated as follows:

$$r_R \in \arg \max_{r \in [0, \infty)} R(r). \quad (3)$$

To obtain closed-form expressions for the optimal price, we restrict ourselves to $k = 1$, as this is commonly observed in practice for most existing popular ride-hailing platform services and leads to a simple form of the optimal price. Most of the results hold for arbitrary values of k , see the online appendix for details. The analysis of the game leads to the following proposition that characterizes the equilibrium under optimal pricing.

Proposition 5.1. *The platform revenue is maximized at a unique price $r_R < \omega + c$. Moreover,*

- *If $\omega \leq c$, then the resulting equilibrium is of type II, an equilibrium involving users and drivers, with supply equal to demand.*
- *If $c < \omega < \hat{\omega}$, where $\hat{\omega} = c + \frac{2}{\lambda(\lambda-1)}$, the resulting equilibrium is of type VI, an equilibrium involving users, drivers, and service providers, with supply equal to demand.*
- *If $\omega \geq \hat{\omega}$, then the resulting equilibrium is of type V, an equilibrium involving users and service providers, with supply exceeding demand.*

Proposition 5.1 shows that, under revenue maximization, equilibrium outcomes reduce to only three

possibilities: types II, V, and VI. Which outcomes arises depend on the relative magnitude of the ownership cost (ω) to the usage cost (c). In particular, if the ownership cost is low relative to the usage cost, ride sharing is P2P based (type II equilibrium involves only users and drivers but no service providers). If the ownership cost is in an intermediate range, both P2P and B2C ride-sharing regimes coexist (under type VI equilibrium, drivers and professional service providers coexist). If the ownership cost is high, only B2C ride sharing takes place (type V equilibrium involves users and service providers but no drivers). We also observe that, because professional drivers spend considerably more time on the road compared with nonprofessional drivers, for practical values of $\lambda > 1$, we get $c \approx \hat{\omega}$, and hence, equilibria under a revenue maximizing platform are either of type II or V.

In the case of type II and VI equilibria, supply equals demand. In the case of type V equilibrium, the revenue-maximizing platform finds it desirable to choose a price for which the supply of seats is higher than demand, resulting in partially empty cars. The reason is that, in type V equilibria, revenue is increasing in price,⁵ leading to higher supply and lower demand because users do not benefit more from the higher supply (recall that $p = 1$). This price increase stops when it becomes profitable for the drivers to appear and supply seats. Because drivers do not pay for self-transportation, this leads to a decrease in revenue for the platform.

5.1. Synopsis

Observation 5.1. The magnitude of the cost of ownership relative to the cost of usage determines how demand and supply are organized under a revenue-maximizing ride-sharing platform. When the cost of cars is (relatively) low, many individuals end up owning and using a car for fulfilling their own transportation needs. This creates a high supply of seats from carpooling and does not incentivize supply by professionals. Demand is fully served by a P2P service regime. In contrast, when cars are (relatively) expensive to own, car ownership is justified only by generating sufficient revenue from driving, which requires car owners to use their assets professionally. In this case, demand is fully served by a B2C service regime in which supply strictly exceeds demand. The intermediate case of car ownership costs leads to a market in which carpooling and professional driving coexist.

6. Car Ownership and Traffic in the Sharing Economy

In this section, we define two important metrics—car ownership and traffic—and examine how these

metrics are affected by the costs of ownership and usage when ride sharing is available. We also answer the important question as to what extent does ride sharing lead to less car ownership and traffic (relative to a setting in which ride sharing is not available).

We use the term “traffic” to refer to the number of cars on the road at any given time. In Section 3.2, we defined α as the rate of new trips by cars, each trip supplying seats to the platform. Then, if $T < 1$ is the (typical) duration of a trip, we expect αT cars on the road at any given time. For a distribution μ of strategies, we have $\alpha T = (\lambda\mu_S + \mu_D)T$.

We can now define

- **Traffic:**⁶ The number of cars on the road, scaled by the trip time T :

$$\Gamma(\mu) = \lambda\mu_S + \mu_D. \quad (4)$$

- **Ownership:** The total amount of cars owned by the population:

$$\Omega(\mu) = \mu_D + \mu_S. \quad (5)$$

The reader should note that actual road congestion is related to Γ . Higher values of Γ lead to more congestion in the actual transportation network. However, translating our results into actual congestion metrics goes beyond this current work because it depends on the specifics of the particular transportation network. In Section 9.3, we consider a setting in which congestion affects the utility individuals derive from car transportation.

Our model allows us to estimate other statistics of interest, such as average *car utilization*, that is, the fraction of time a typical car is used. The utilization of a specific car depends on the strategy of its owner. Specifically, drivers and service providers use their cars for T and λT fractions of time, respectively.

Note that all our performance metrics are invariant under equivalent equilibria. Theorem 4.1, thus, implies that we may define our performance metrics Γ and Ω as functions of the underlying game parameters instead of the equilibrium partitions.

Finally, note that we assume that all λ trips undertaken by a service provider, whether full (with a passenger) or empty (without a passenger) contribute to congestion. This is consistent with the reality of service providers continuing to “cruise” even without a passenger. For example, a recent study by New York City (2019) finds that “30% of Manhattan Core traffic is for-hire vehicle (FHV) services, most of which work with apps like Uber and Lyft, and that these vehicles are cruising empty 41% of the time.” As a result, the city has recently enacted regulation requiring that “large app-based companies bring their Manhattan Core cruising time—when drivers are working but not earning money—down to 31% during

the peak hours. . . .As a result of these rules, FHV-related traffic congestion below 96th Street during rush hours is projected to decrease by more than 20%” (see also Taxi and Limousine Commission 2019).

6.1. Impact of Ownership and Usage Costs

The following proposition characterizes how ownership and traffic are affected by the cost of ownership and usage.

Proposition 6.1. *The following hold:*

- If $\omega \leq c$, both traffic and ownership are unaffected by ω and c .*
- If $c < \omega < \hat{\omega}$, traffic is increasing in ω and decreasing in c , and ownership is decreasing in ω and increasing in c .*
- If $\omega \geq \hat{\omega}$, both traffic and ownership are increasing in ω and decreasing in c .*

Proposition 6.1 shows that ownership and traffic are affected differently by increases in the costs of ownership and usage with ownership being nonmonotonic in both. In particular, in the P2P regime (part (a) in which the equilibrium is of type II), neither ownership nor traffic are sensitive to changes in ownership or usage costs. This is because, under this regime, there are only drivers and users. A revenue-maximizing platform chooses a price that induces an equal fraction of users and drivers in the economy and, hence, keeps all performance parameters constant (an equal number of drivers and users maximizes revenue because it maximizes the number of rentals).

The results in part (b), in which the equilibrium is of type VI, have a simple explanation in terms of substitution between drivers and service providers offering service (remember that a type VI equilibrium is one in which both D and S coexist). First, it is easy to show that increasing any of the costs negatively impacts all agents offering service, and as a result, the platform must raise its price. An increase in ω penalizes S and D equally, but the resulting increase in price by the platform increases the profit per ride for everyone, which benefits S more compared with D (because the profit per ride of S is multiplied by $\lambda > 1$). Hence, we have substitution of agents from D to S . This decreases the number of cars needed to provide the same amount of transportation, and it increases the number of cars on the road because S cars drive more. Using similar arguments, an increase in c deters more S than D because S spend more time on the road. In this case, we see a substitution from S to D reversing the previous results.

The results in part (c), in which the equilibrium is of type V (corresponding to a B2C regime), can be explained as follows. Cars in this case are very expensive. When ownership costs are much higher compared with usage costs, service at the optimal price is provided solely by S . Any increase in

ownership costs increases platform prices in a way to *overcompensate* service providers. As a result, more service providers are present (but with emptier cars), which increases both ownership and traffic.⁷ An increase in c has the opposite effect, resulting in a decrease in both ownership and traffic.

These results point to potential levers a regulator may be able to deploy to induce desired outcomes with regard to ownership and usage. This may involve affecting the cost of ownership and usage via taxes, subsidies, or other forms of financial transfer tied to owning or using a car. The results point to the importance of tailoring such regulation to the prevailing equilibrium regime. For example, although increasing the cost of owning a car through, say, a tax on car purchases would reduce car ownership in a mixed P2P/B2C regime, such an increase would backfire in a B2C regime. The results also highlight how affecting ownership and usage costs could lead to different outcomes. For example, taxing usage in a B2C regime would reduce ownership and traffic, in contrast to taxing ownership, which would increase both.

A regulator could also potentially use the levers of ownership and usage costs to induce an equilibrium that is of either type P2P or B2C. Note that the choice between the two types of equilibria involves non-trivial trade-offs. A P2P equilibrium involves higher ownership but lower traffic than a B2C equilibrium (see Lemma A.16 in the online appendix for a proof). A regulator may prefer one over the other, depending on the magnitude of the negative externalities associated with ownership and usage.

6.1.1. Synopsis.

Observation 6.1. Ownership and traffic are affected differently by increases in the costs of ownership and usage with ownership being nonmonotonic in both. In particular, we observe the paradoxical situation that, when cars are expensive to own, higher ownership costs are associated with higher traffic and higher ownership (more cars on the road driving less full). In the case of carpooling, which occurs when cars are inexpensive to own, traffic and ownership are insensitive to small variations of the costs. In this case, supply equals demand.

6.2. The Sharing vs. the Traditional Economy

In this section, we examine how the introduction of ride sharing into an economy affects ownership and traffic. Given our assumption about common valuation for car usage and the assumption that $\omega + c < 1$, this translates into an economy in which everyone is a driver,⁸ and hence, our model captures more accurately a situation in which there is a high amount of ownership in the traditional economy. Let $\Delta_\pi(v)$ denote the

difference in payoff with and without ride sharing for an individual with income level v .

Proposition 6.2. *The following hold:*

- *The introduction of ride sharing always reduces ownership but can increase traffic. In particular, traffic increases with the introduction of ride sharing if and only if $\omega > \frac{2\lambda-1+c\lambda(\lambda-1)^2}{(\lambda-1)^3} \geq \hat{\omega}$.*
- *The benefit derived from ride sharing is the same among all individuals in the population when $\omega \leq c$ (the P2P regime). That is, in this case, $\Delta_\pi(v)$ is invariant to v . Lower income individuals benefit more when $\omega > c$ (the B2C regime). Specifically, among the individuals who choose to be service providers, $\Delta_\pi(v)$ is decreasing in v . The benefit is the same among all individuals who choose to be users.*

This proposition suggests that, when cars are abundant in the traditional economy (as implied by the assumption $\omega + c < 1$), the sharing economy reduces car ownership. But, perhaps contrary to intuition, traffic does not always decrease. When cars are expensive to own, under a revenue-maximizing platform, traffic increases even though fewer cars are owned. The reason is that driving cars is provided by professionals being all the time on the road with cars inefficiently utilized, contributing to an increase in the total traffic compared with the case without sharing.⁹

6.2.1. Synopsis.

Observation 6.2. When cars are abundant in the traditional economy, the sharing economy reduces car ownership. But traffic may not decrease because cars may be driven by professionals spending more time on the road and not being fully utilized.

7. The Social Planner's Problem

In this section, we consider a platform whose objective is to maximize social welfare; social welfare is defined as the cumulative payoff of all the individuals in the population given any partition P . Platforms that are social welfare-driven may be those that are operated by a government entity or a not-for-profit organization. Studying a social welfare-maximizing platform also provides us with a benchmark against which to compare equilibrium outcomes under a revenue-maximizing platform.

We call an assignment a function $q : X \rightarrow \Delta^3$ from agents to the probability simplex of mixed strategies and denote as \mathcal{C} the set of all possible assignments by a social planner. The aggregate distribution of strategies played is given by $\mu_\sigma(q) := \int_X q_\sigma(v)M(dv)$, $\sigma \in \Sigma$. Formally, we define social welfare as the function $W : \mathcal{C} \rightarrow \mathbb{R}$

$$W(q) := \sum_{\sigma \in \Sigma} \int_X \pi_\sigma(v, \mu(q)) q_\sigma(v) dM(v).$$

We abuse notation and use $W(P)$ to refer to the welfare generated by a partition of X into strategy assignments. The welfare optimum corresponds to $W^* := \sup_{q \in C} W(q)$.

Proposition 7.1. *For any distribution of individual types M and parameter combination $\omega, c, k > 0$, and $\lambda > 1$, there exists a unique seat rental price r^* such that the corresponding equilibrium partition P^* achieves the welfare maximum W^* . In this partition, supply equals demand; that is $p = \bar{p} = 1$. Furthermore,*

- *If $\omega \leq c$, the equilibrium is of type II,*
- *If $c < \omega < \hat{\omega}$, the equilibrium is of type VI,*
- *If $\omega \geq \hat{\omega}$, the equilibrium is of type V, and*
- *$r^* = r_R$ if $\omega < \hat{\omega}$, and $r^* < r_R$ otherwise.*

The proposition¹⁰ shows that the social welfare maximum (given ω, c, k, λ) indeed exists and constitutes an equilibrium of game G . A social planner is, thus, able to induce the maximum welfare simply by using the seat rental price as a coordinating mechanism. The socially optimum price is either the same (for small-to-medium values of ω) or strictly smaller than the revenue maximizing price. Proposition 7.1 shows that, again, equilibrium under social welfare maximization is of type II, V, or VI and depends on the cost of ownership in the same way equilibrium does in the case of revenue maximization.

A policy implication of these results is that, when the cost of ownership is relatively low, a regulator need not intervene in the operation of a platform if it is revenue maximizing (because the price that maximizes revenue also maximizes social welfare). A regulator may want to intervene when the cost of ownership is high by using policy instruments that would nudge the revenue-maximizing platform to choose a lower price (e.g., by placing a price ceiling).

The following proposition provides results that parallel those in Proposition 6.1 regarding ownership and traffic

Proposition 7.2. *The following hold under social welfare maximization:*

- *If $\omega \leq c$ or $\omega \geq \hat{\omega}$, then both traffic and ownership are unaffected by the costs of ownership and usage, ω and c .*
- *If $c < \omega < \hat{\omega}$, then traffic is increasing in ω and decreasing in c while ownership is decreasing in ω and increasing in c .*

These results are consistent with the characterization of the equilibria in Proposition 7.1. When $\omega \leq c$ or $\omega \geq \hat{\omega}$, because supply is provided by the same type of individuals and equals demand, there is a fixed fraction of society that provides transportation, and this is not sensitive to the change of parameters. When $c < \omega < \hat{\omega}$, supply is shared by drivers and service providers. Increasing ω justifies more the professional use of assets and, hence, induces substitution from drivers to service providers increasing traffic but decreasing

ownership (fewer cars are needed to provide the same number of seats). Increasing c has the opposite effect by penalizing service providers that perform more trips per unit of time.

Proposition 7.3. *Under a social welfare-maximizing platform, both ownership and traffic are always lower than those in the traditional economy.*

This is in contrast to the result in Proposition 6.2, in which, under a revenue-maximizing platform, traffic is higher in the sharing economy for high enough values of ω . The popular belief that a sharing economy reduces traffic and ownership can be indeed valid but only when the platform uses socially optimal prices that induce supply to equal demand.

Finally, to assess the extent to which social welfare is reduced from its maximum when the platform is revenue maximizing, we consider the difference $\Delta_{SW} = SW(r^*) - SW(r_R)$, where $SW(r^*)$ is the social welfare when the platform uses the social welfare-maximizing price and $SW(r_R)$ is the social welfare when the platform uses the revenue-maximizing price.

Proposition 7.4. *If $\omega \leq \hat{\omega}$, then $\Delta_{SW} = 0$; otherwise, Δ_{SW} is increasing in ω .*

Proposition 7.4 shows that, under revenue maximization, the more expensive cars are to own, the less efficiently they tend to be used (because supply in excess of demand increases; see Proposition 6.1), hence, the loss in social welfare. Using numerical analysis, we observe that Δ_{SW} is decreasing in c . Hence, the reverse is true concerning usage cost c : the more costly cars are to use, the more efficiently they tend to be used (as supply in excess of demand shrinks).

7.1. Synopsis

Observation 7.1. In general, the platform's and society's interests may not be aligned. It is possible, under a revenue-maximizing platform, that there is a large number of professional drivers who constantly drive their cars, thus causing high traffic, but transport very few riders. In contrast, under a welfare-maximizing platform, every car is always fully occupied, and hence, traffic is no higher than absolutely necessary to transport all the riders for the socially optimal level of demand. This is consistent with the popular belief that sharing can indeed reduce ownership and congestion in society. A regulator may be able to steer a revenue-maximizing platform toward a more socially desirable equilibrium by constraining the seat rental price the platform charges.

8. The Case of Heterogeneous Utilities

In this section, we consider the more general setting in which individuals are heterogeneous in both their

utility for car transport and income. In particular, we assume the density $m(\rho, \nu) = \mathbb{1}_{[[0,1]^2]}(\rho, \nu)$, that is, M follows the uniform distribution on the unit square, and we allow for arbitrary k . Note that we no longer need to impose $\omega + c < 1$. Also, because both income and utility derived from car transportation are heterogeneous and vary from zero to one, we can no longer eliminate the abstinent strategy (strategy A) from the equilibrium.

Although it is possible to characterize analytically the equilibrium for an exogenously specified price (and a result similar to Theorem 4.1 can be obtained), doing so when the platform chooses the price that maximizes revenue is difficult. Therefore, in this section, we rely on numerical analysis.

8.1. Characterizing Equilibrium Outcomes

To get the full picture for the different types of equilibria that arise when the platform is revenue-maximizing, for different combinations of problem parameters, we carried out a comprehensive numerical study in which we considered all parameter combinations $\omega, c \in (0, 1)$ working with a discretized grid of 200×200 points. This analysis was replicated for different values of $k \in \{1, 2, 3, 4\}$ and $\lambda \in \{2, 3, \dots, 8\}$. Representative results are shown in Figure 2 for $k = 2$ and $\lambda = 6$, and the corresponding equilibria partitions are shown in Figure 3.

As illustrated in Figure 2, the results are consistent with the findings of Proposition 5.1. In particular, we observe that, except for a thin region of cost parameter values in which the P2P and B2C regimes coexist, the

Figure 2. Equilibrium Type Outcomes as a Function of Ownership Costs ω and Usage Costs c When Customers Are Heterogeneous in Their Wage ν and Utility ρ

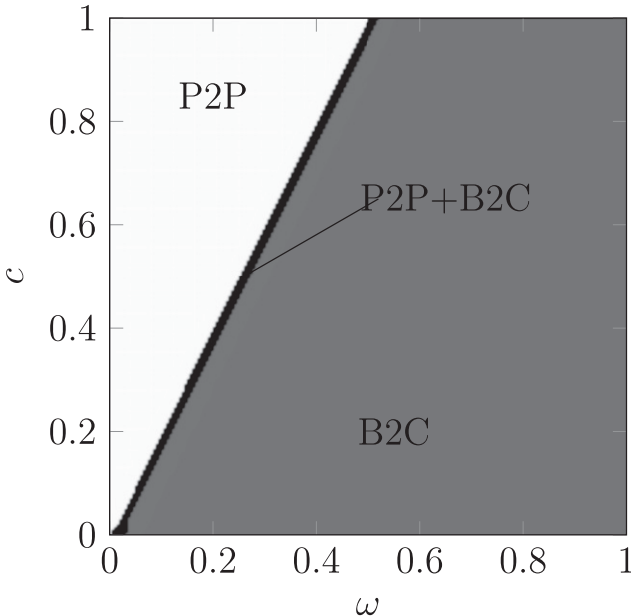
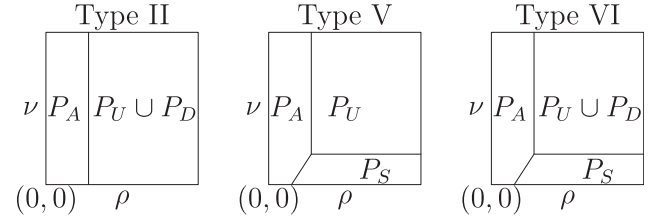


Figure 3. The Possible Equilibria Partitions of the Unit Square in the ρ - ν Axes for Individuals with Nonhomogeneous ρ When the Platform Is Revenue Maximizing



Note. Types II, V, and VI correspond to P2P, B2C, and mixed P2P-B2C regimes, respectively.

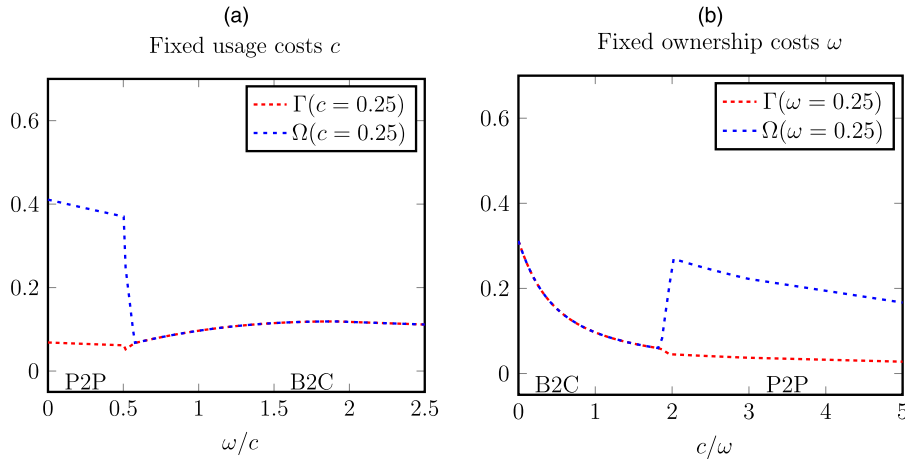
resulting equilibria are either P2P (when $\omega < c/k$) or B2C (when $\omega > \hat{\omega}$) as predicted analytically for the model with common utility. Also, as predicted analytically for the model with common utility, the region in which P2P and B2C coexist occurs when $c/k < \omega < \hat{\omega}$. Note that, in each regime, strategy A is present (i.e., individuals with sufficiently low utility from car transport choose to abstain); see Figure 3.

8.2. Ownership and Traffic

In Figure 4, we illustrate how ownership level and traffic in the sharing economy vary with the ownership cost ω and usage cost c . Note that, in the x -axis of plot (a), the two points at which regimes change correspond to c/k and $\hat{\omega}$, respectively, as suggested by Proposition 6.1. This is reversed in plot (b). In contrast to the results in Proposition 6.1, ownership and traffic are no longer invariant to changes in the cost of ownership and usage in the P2P regime. Both are now decreasing. This is because, in the P2P regime, the abstinent strategy (strategy A), is now present.¹¹ An increase in either ω or c leads to an increase in the fraction of the population that chooses strategy A, resulting in a decrease in both ownership and traffic. In the B2C regime, the effect of the ownership cost is more nuanced. The initial increase in ω leads to an increase in ownership for reasons similar to those discussed in Section 6. However, further increases lead to a decrease in ownership as abstaining becomes more preferable for a growing fraction of the population. The effect of the usage cost is consistent with that described in Proposition 6.1.

Note the following:

- In the B2C regime, ownership and traffic coincide because we plot the normalized traffic given by $\Gamma(\mu) = \mu_S + \mu_D/\lambda$. Because $\mu_D = 0$, $\Gamma(\mu) = \mu_S = \Omega(\mu)$.
- There is a drastic drop of ownership when the equilibrium regime transitions from P2P to B2C. This is because the same supply of seats is provided by fewer cars, each used more intensely (a similar effect is observed for the original with common utility for car transport).

Figure 4. (Color online) Ownership and Traffic When Users Are Heterogeneous in Their Utility ρ 

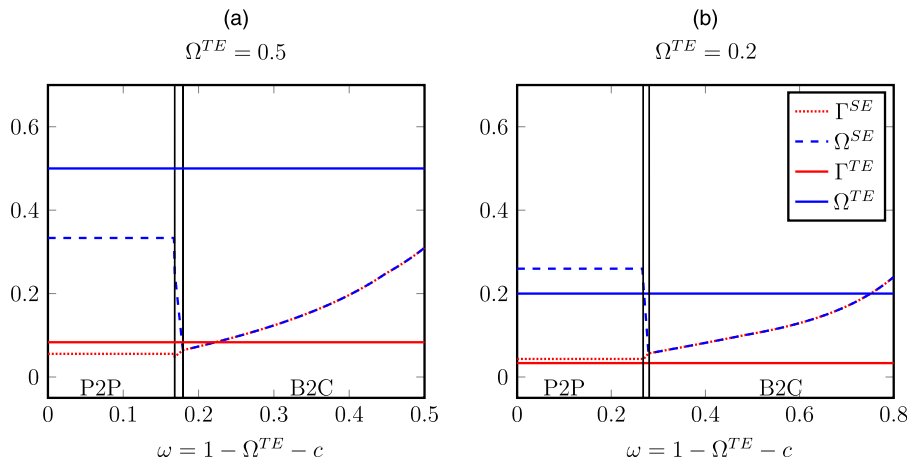
Notes. In both plots, we assume $k = 2$ and $\lambda = 6$. For plot (a), we vary the ratio ω/c by fixing usage cost c and varying ownership cost ω . We do the reverse in plot (b). The solid vertical lines in the plots denote the boundaries between equilibria of types II, V, and VI. The points at which these lines intersect the x -axis define the regions suggested by Proposition 6.1.

8.3. The Traditional vs. the Sharing Economy

In Figure 5, we compare ownership and traffic in the traditional and sharing economies. Note that a setting in which the utility ρ is heterogeneous among individuals allows for partial car ownership in the traditional economy, in which individuals with sufficiently low utility forego ownership (recall that, when utility is common, there is either full or zero ownership in the traditional economy). The results shown are obtained by considering all values of $\omega, c \in (0, 1)$ that yield the same constant level of ownership (and, hence, also of traffic) in the traditional economy. That is, we vary $\omega, c \in (0, 1)$ while keeping their sum constant. Because the payoffs of drivers in the traditional economy depends on $\omega + c$, such a change does not affect their strategy, and consequently, ownership

and traffic remain constant. This is not the case in the sharing economy in which ownership and traffic are affected, per the previous section, by ownership and usage costs. Figure 5, (a) and (b), corresponds to different levels of ownership in the traditional economy.

Figure 5(a) corresponds to the case of high car ownership in the traditional economy. As in Figure 4, the solid vertical lines in the plots denote the boundaries between equilibria of types II, V, and VI as suggested by Proposition 6.1. We observe that the results of Proposition 6.2 (which assumes high car ownership in the traditional economy) hold. Under a sharing economy, ownership is always reduced, but for high ownership costs, traffic is increased relative to the traditional economy.

Figure 5. (Color online) Ownership and Traffic in the Sharing Economy (Ω^{SE}, Γ^{SE}) for Various Ownership and Usage Costs While Keeping Ownership and Traffic in the Traditional Economy (Ω^{TE}, Γ^{TE}) Fixed

Notes. Ownership in the traditional economy, $\Omega^{TE} = 1 - \omega - c$ is kept constant. The solid lines correspond to ownership and traffic in the traditional economy. Note that traffic and ownership coincide in the B2C regime because each car is driven all the time. The results shown are for $k = 4$.

Figure 5(b) corresponds to the case of low car ownership in the traditional economy. In this case, the sharing economy always generates more traffic. Ownership is reduced only when service providers become the only seat suppliers, and fewer cars are needed to serve demand. This occurs for medium ownership costs. When these costs increase, as suggested by Proposition 6.1, ownership increases and may eventually become higher than in the traditional economy as the high values of ω in Figure 5(b) suggest.

9. Extensions

In this section, we consider several extensions of the original model (discussed in Sections 3–7). In particular, we consider settings in which (1) the platform charges a commission and may offer a subsidy to service providers, (2) there are differences in the efficiency with which drivers and service providers are matched with riders, and (3) users experience a disutility that is increasing in the volume of traffic. In each case, we examine the extent to which our main results (namely, Propositions 5.1, 6.1, and 6.2(a)) continue to hold and the degree to which the specifications of the features associated with each extension affect outcomes. For each of the extensions, we consider, unless stated otherwise, the applicable assumptions are those of the original model.

9.1. Subsidies and Commissions

In this section, we consider a setting in which the platform keeps a fraction $1 - \theta$ of the revenue and each service provider obtains a subsidy ϕ (positive or negative). A positive subsidy may correspond to the platform contributing toward the purchase or lease of a car, and a negative subsidy may correspond to an access fee for using the platform.¹²

In the presence of commissions and subsidies, the payoffs of participants are modified as follows: users pay r per seat, service providers and drivers receive θr per seat, and service providers receive a payment ϕ (positive or negative) per unit time. That is,

$$\pi_U = v + p(1 - r), \pi_D = v + k\theta\bar{p}r + 1 - \omega - c, \text{ and} \\ \pi_S = \lambda(k\theta\bar{p}r - c) + 1 - \omega + \phi.$$

The case of $\theta = 1$ and $\phi = 0$ corresponds to our original model. The case of $\theta > 0$ and $\phi = 0$ corresponds to the typical setting in which platforms earn income by charging a commission on each successful transaction but neither offers a subsidy or charges an access fee.

For general values of ϕ , we distinguish two cases of practical relevance: (a) $\phi \geq 0$ comes at no cost to the platform and is exogenously specified. This models possible discounts in the purchase price of vehicles for service providers enrolling full time with the platform that are negotiated by the platform with the manufacturers on

behalf of the service providers. (b) ϕ is a monetary transfer from the platform to the service providers and is endogenously determined, in which case it affects directly the profit of the platform. In the first case, the platform's profit can be stated as

$$\Pi_{\theta,\phi}(r) = (1 - \theta)rp(r)\mu_U(r),$$

and in the second case,

$$\Pi_{\theta}(r, \phi) = (1 - \theta)rp(r, \phi)\mu_U(r, \phi) - \phi\mu_S(r, \phi).$$

In both cases, we assume that the commission rate θ is exogenously determined. This reflects the reality that commission rates vary within a narrow band in practice and are not typically adjusted based on demand and supply of seats (e.g., many on-demand services use a uniform commission rate across the markets/regions in which they operate; see Benjaafar et al. 2019 for further discussion).

9.1.1. The Case in Which $\phi > 0$ Is at No Cost to the Platform. In the online appendix, we show that, for fixed θ and ϕ , results similar to those obtained in Theorem 4.1 and Propositions 5.1, 6.1, and 6.2(a) continue to hold (see Theorem A.25 and Propositions A.29–A.31 in the online appendix). In particular, we show that, when the platform chooses price optimally, the resulting equilibrium, except for a narrow region, is P2P when the cost of ownership is sufficiently low and is B2C otherwise. We also obtain results similar to those obtained for the original model regarding the sensitivity of ownership and traffic to ownership and usage costs and regarding comparisons between the traditional and sharing economies. These results confirm the robustness of our main observations for systems with commission and/or subsidy. In the online appendix, we also show that, even though the subsidy to the service providers is at no cost to the platform, a higher subsidy is not always preferable to the platform. In particular, in Proposition A.35, we show that platform profit in the B2C regime is non-monotonic in $\phi > 0$, first increasing and then decreasing. This is because, in the B2C regime, in which the population is divided between users and service providers, a higher value of ϕ increases the pool of service providers by decreasing the pool of users. Having either too few service providers or too many is detrimental to revenue because some balance between service providers and users is needed to maximize revenue.

9.1.2. The Case in Which ϕ Is a Monetary Transfer Between the Platform and the Service Providers. In this case, ϕ can be either positive or negative and affects directly the profit of the platform. Unfortunately, this case is analytically intractable as it involves optimization on the part of the platform over two dimensions,

r and ϕ (although the first-order conditions for optimality lead to closed-form expressions for the optimal values of r and $\phi(r)$, it is difficult to check analytically in all cases the corresponding second order conditions; see Online Appendix A.5 for details). However, based on extensive numerical experiments, we observe the following to hold, and this validates the findings of Proposition 5.1.

- If $\omega \leq c$, then $\phi = 0$, resulting in an equilibrium of type II (P2P regime).

- If $\omega \geq c$, then there exists a threshold θ_0 such that $\phi < 0$ if and only if $\theta_0 < \theta \leq 1$, where θ_0 is decreasing in ω . Moreover, for any $0 < \theta \leq 1$,

- If $c < \omega < \tilde{\omega}$, the equilibrium is of type VI (B2C–P2P regime).

- If $\omega \geq \tilde{\omega}$, the equilibrium is of type V (B2C regime).

These results can be explained as follows. When $\omega < c$, there is very little incentive for individuals to become service providers, and the required subsidy in order to attract service providers is too high to be profitable. When $\omega > c$ and θ is large, there are strong incentives for individuals to be service providers: (i) cars are expensive to be used only occasionally, and (ii) the commission charged by the platform is low. In this case, the platform gains by charging service providers a participation fee $\phi < 0$ because it will not significantly affect supply. But when θ is low, that is, the platform keeps a large share of the revenue, service providers must be further incentivized by the platform, and hence, the platform prefers $\phi > 0$. For very large values of ω , θ_0 is negative, implying always $\phi < 0$. When the equilibrium regime is B2C and the platform charges a participation fee, it operates not unlike a *traditional taxi* service. The financial transfer between the service providers and the platform can be reinterpreted as the platform assuming the ownership of cars and charging a rental fee to service providers. This taxi service-like business model arises when the cost of ownership is sufficiently high and the commission rate charged by the platform is sufficiently low.

Sensitivity analysis with respect to ownership and usage costs leads to similar conclusions as in Proposition 6.1 except that there exists $\tilde{\omega}$ such that, for $\tilde{\omega} < \omega \leq \tilde{\omega}$, ownership and traffic are not affected by the ownership and usage costs. This is because, for this range of ω , supply equals demand. That is, the possibility for the platform of choosing not only price but also subsidy level allows it to induce a B2C equilibrium with better utilization of the cars when $\tilde{\omega} < \omega \leq \tilde{\omega}$. Finally, our numerical analysis suggests that the results of Proposition 6.2(a) hold.

9.2 Matching Friction

In our original model, all seats, regardless of whether they are supplied by drivers or service providers, have an equal chance of being matched with users. A

sensible extension is to assume that there is greater friction in the matching of seats supplied by drivers compared with those supplied by service providers, reflecting the reality that drivers tend to follow specific routes that may be incompatible with the routes demanded by users at the time the driver makes the trip. In turn, this decreases the probability that a seat supplied by a driver would be successfully matched. Such a constraint is less pertinent for service providers, who are typically more willing to *chase* fares. The matching friction faced by drivers could also arise from decisions made by the platform (e.g., giving preferences to service providers when matching seats with demand).

In the presence of matching friction (in addition to the friction that arises because of the mismatch between supply and demand), the payoff of drivers is modified as follows:

$$\pi_D = v + k\zeta\bar{p}r + 1 - \omega - c,$$

where $0 < \zeta \leq 1$ is a friction parameter that modulates the probability that a seat provided by a driver would be successfully matched with lower values of ζ corresponding to higher matching friction. Given a distribution μ of strategies in the population, this implies that

$$p = \frac{k(\lambda\mu_S + \zeta\mu_D)}{\mu_U} \wedge 1 \quad \text{and} \quad \bar{p} = \frac{\mu_U}{k(\lambda\mu_S + \zeta\mu_D)} \wedge 1.$$

In the online appendix (see Theorem A.36 and Propositions A.37–A.39), we state the equivalent results of Theorem 4.1 and Propositions 5.1, 6.1, and 6.2(a). Hence, our results for the original model are robust to the matching friction feature.

A question that naturally arises is to what extent does matching friction on the driver side affect the platform. All else remaining the same, lower values of ζ (higher matching friction for drivers), leads to fewer seats being offered by drivers, incentivizing more individuals to become service providers. Under what conditions is this beneficial to the platform? Clearly, if supply exceeds demand, decreasing ζ increases platform revenue. This is because reducing the revenue of drivers converts a fraction of them to users, increasing demand and, hence, platform revenue. Is, hence, a smaller ζ always better for the platform? Should the platform choose $\zeta = 0$? The answer depends on whether the platform, by discouraging drivers, can still guarantee an adequate supply of seats by service providers. To attract service providers, prices must be high enough. But such prices are not possible when both ω and c are small because raising prices above $\omega + c$ would drive demand to zero. Small ω and c lead to P2P or mixed P2P–B2C regimes with supply provided mainly by drivers. Hence, reducing ζ would

reduce the main source of supply, reducing platform revenue. In this case, the platform may prefer $0 < \zeta \leq 1$, allowing for both drivers and service providers to be seat suppliers. The following proposition provides a sufficient condition under which the platform would prefer $\zeta = 0$ (i.e., a setting in which seats are exclusively supplied by service providers).

Proposition 9.1. *A revenue-maximizing platform would prefer $\zeta = 0$ if $\lambda\omega + c \geq 1/\lambda$.*

The condition $\lambda\omega + c > 1/\lambda$ is sufficient for the revenue-maximizing equilibrium to be of type V (B2C) when $\zeta = 0$. In this case, the platform does not gain by increasing ζ and attracting drivers. We can show that social welfare is always increasing in ζ , implying that the interests of the platform and society can be at odds when it comes to reducing matching friction.

We expect similar results to hold for any extension of the model that considers forms of inconvenience associated with seats offered by nonprofessionals. This may include the case of users experiencing a disutility when riding with a nonprofessional (e.g., because of concerns about safety, lower quality, or personal discomfort). Our results imply that eliminating such inconvenience may not necessarily be beneficial to the platform.

9.3. Congestion Cost

In this section, we extend our original model by accounting for the negative externality generated by traffic. In particular, we incorporate an additional cost, associated with traffic, in the payoff of all individuals who rely on cars to fulfill their transportation needs. Specifically, we consider a *congestion cost*, $g(\Gamma)$,

that is convex increasing in traffic $\Gamma = \lambda\mu_S + \mu_D$ and of the form $g(\Gamma) = \frac{a\Gamma}{1-a\Gamma}$, where $a > 0$ is a parameter that captures the sensitivity of individuals to traffic. In the presence of a congestion cost, the payoffs associated with various strategies (other than the abstain strategy whose payoff $\pi_A = v$ remains the same) are modified as follows:

$$\begin{aligned}\pi_U &= v + p(1 - r - g(\Gamma)), \\ \pi_D &= v + k\bar{p}r + 1 - \omega - c - g(\Gamma), \text{ and} \\ \pi_S &= \lambda(k\bar{p}r - c) + 1 - \omega - g(\Gamma).\end{aligned}$$

Note that the congestion cost, by reducing the payoffs of strategies U, D, S , makes strategy A (use public transport) more desirable, and it is now possible for this strategy to appear in equilibrium.

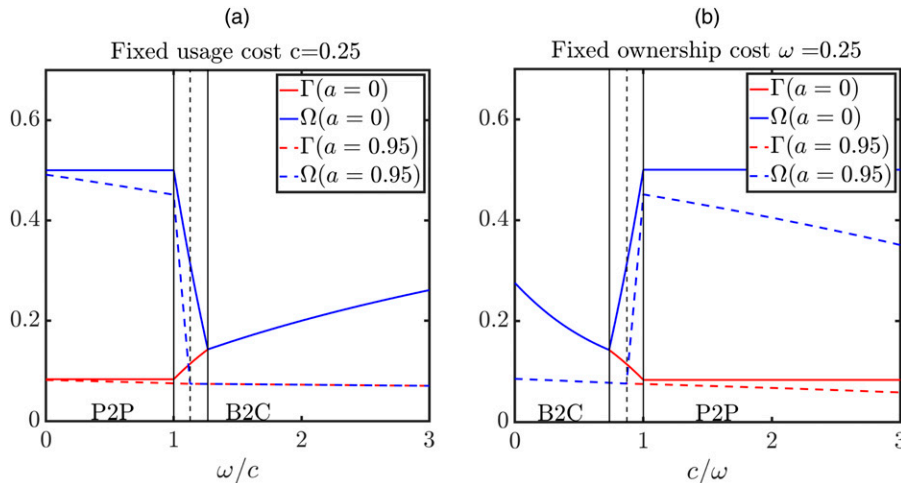
The following proposition shows that, when a is sufficiently small, results obtained for the original model continue to hold because strategy A never appears in equilibrium.

Proposition 9.2. *If $a \leq a_0 = \frac{1-\omega-c}{\lambda(2-\omega-c)}$, identical results to those in the original model with $a = 0$, that is, Theorem 4.1 and Propositions 5.1, 6.1, and 6.2(a) hold.*

When $a > a_0$, the analysis becomes intractable because it is no longer possible to eliminate strategy A as one that would arise in equilibrium (the difficulty is compounded by the now-complex form of the payoff functions). However, based on extensive numerical experiments, we observe the following to hold; see also Figures 6 and 7.

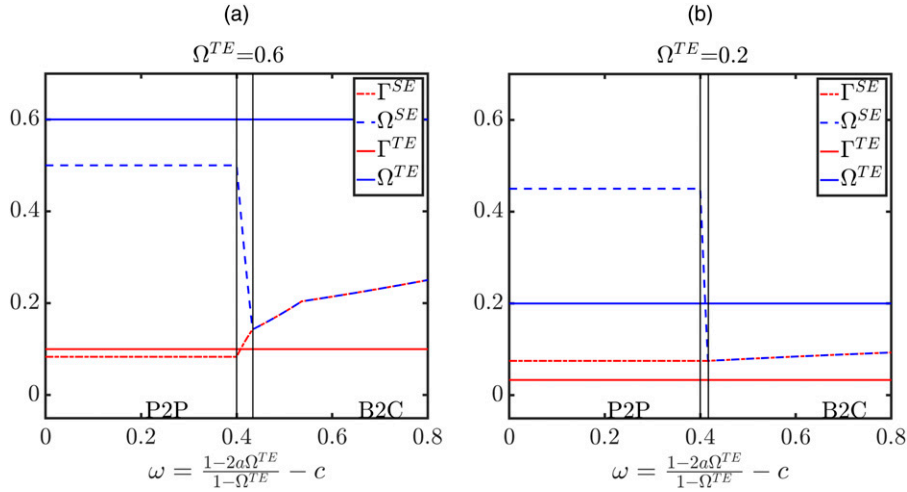
- There exists $a_1 > a_0$ such that, when $a_0 < a \leq a_1$, strategy A never arises in equilibrium (when the platform chooses the revenue-maximizing price). Results equivalent

Figure 6. (Color online) We Plot the Ownership and Traffic Under High Congestion Cost $a = 0.95$ (Dotted Lines) and Compare This with the Case of No Congestion Cost $a = 0$ (Solid Lines)



Notes. In plot (a), we observe that, as ω increases, under high congestion cost, both ownership and traffic decrease in both the P2P and B2C regimes contrary to the case of $a = 0$. Similarly, in plot (b), as c increases, ownership and traffic decrease in the P2P regime. In all cases, ownership and traffic decrease when the congestion cost is high.

Figure 7. (Color online) Traffic (Scaled) and Ownership in the Sharing Economy Compared with the Traditional Economy When Keeping the Sum of the Costs ω and c Constant When the Cost of Congestion Is Low (Plot (a), $a = 0.27$) or High (Plot (b), $a = 0.83$)



Note. Both traffic and ownership can be higher or lower under a sharing economy.

to Propositions 5.1 and 6.1 hold. However, Proposition 6.2(a) may not hold. Ownership may be increased in the sharing economy compared with the traditional economy because drivers and service providers can mitigate the congestion cost with the income derived from offering rides.

- When $a > a_1$, strategy A can be part of the equilibrium. In this case, we observe the same equilibrium structure as predicted by Proposition 5.1 with the possible addition of strategy A .¹³ However, the impact of the ownership and usage costs on ownership and traffic can be different. In particular, it is now possible for an increase in the ownership cost to lead to lower ownership in both the P2P and B2C regimes. This is because, in both regimes, an increase in the cost of ownership can lead more individuals to become abstinent, which, in turn, can result in less ownership and traffic. Similarly, an increase in the cost of usage can result in less ownership and usage in the P2P regime because of an increase in the fraction of individuals who choose to be abstinent. Proposition 6.2(a) may not hold for similar reasons as those described in the previous bullet.

10. Concluding Comments

In this paper, we introduce a model to study ride sharing in a population of individuals who are heterogeneous in the utility they derive from private transportation and income. We examine how ride sharing is organized and how it affects traffic and ownership. Among our findings, we show that the ratio of ownership to usage costs largely determines how ride sharing is organized. If this ratio is low, ride sharing is offered as a P2P service, and if this ratio is

high, ride sharing is offered as a B2C service. Moreover, we show that the introduction of ride sharing may not necessarily lead to a decrease in either ownership or traffic and that the impact of ownership and usage costs on ownership and traffic may depend on the prevailing regime (P2P or B2C). We compare outcomes obtained by a profit-seeking platform to those of a social welfare-maximizing platform, highlighting that a profit-seeking platform may find it preferable to induce more car ownership than what is socially optimal and for cars to be used less efficiently. We discuss levers, such as taxes on ownership/usage and caps on prices, that a regulator may employ to induce desirable outcomes.

Many extensions of our model are possible, including adding a spatial element to how ride sharing takes place. For example, one may consider a setting in which demand and supply are organized along a set of many origin–destination pairs (routes). In such a setting, each “driver” serves demand for a specific route (corresponding to the driver’s own origin and destination) but “service providers” form a common pool that serves all routes. In equilibrium, we expect service providers (if they exist) to be distributed so that they incur the same profit per route, that is, induce the same \bar{p} for all routes. We expect the same type of analysis to carry through and the ratio of ownership to usage costs to continue to determine the type of equilibria along with outcomes regarding ownership and traffic.

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helpful comments and suggestions. A short preliminary version of the paper (Benjaafar et al. 2017) was presented at NetEcon '17.

Endnotes

¹This is supported by empirical evidence as reported in Gong et al. (2017); see also recent reports by Bhattarai (2017) and Schmitt (2019). The empirical evidence on congestion is mixed. Li et al. (2016) find evidence that Uber's entry leads to a significant decrease in congestion. However, there is also evidence to the contrary (see San Francisco County Transportation Authority 2018 and Agrawal 2018), leading several municipalities, such as New York City to curb the growth of ride hailing.

²See Benjaafar and Hu (2020) for a comprehensive review.

³We use the term "public transport" loosely to refer to, among others, the use of a bus, subway, or taxi to fulfill transport needs.

⁴In this section, we consider the case in which the seat rental price is exogenously specified. In Section 5, we consider the case in which the platform chooses price to maximize revenue. In that case, game G can be viewed as the subgame in a leader-follower game in which the platform is the leader who first decides on the price (knowing how agents behave). Then agents respond by choosing among the various agent strategies.

⁵The revenue is maximized at the highest possible price resulting in a type V equilibrium, just before the equilibrium becomes of type VI .

⁶A common metric for measuring the consumption of car travel is total vehicles miles traveled (VMT) per unit of time. Our definition for traffic Γ is proportional to the VMT. To compute the VMT, we just need to multiply the total rate of trips $\mu_S \lambda + \mu_D$ by the average length of a trip.

⁷The actual overcompensation argument is more subtle. It involves the properties of the equilibrium at the revenue-maximizing price. We prove that, if $\omega > \hat{\omega}$, the revenue-maximizing price is the highest price that keeps strategy D being dominated, hence preventing any positive mass of drivers from being present in the equilibrium: if drivers appear, the revenue of the platform drops sharply because drivers are nonpaying individuals and reduce revenue resulting from conversion of paying to nonpaying customers. An increase in ω de-incentivizes drivers more than service providers from being present because a service provider can amortize that cost over λ trips per unit time. This allows the price to be raised much more than before, resulting in even more service providers supplying seats. The reverse happens if c increases, and this has a more severe effect on service providers because they do more trips.

⁸Our model of homogeneous utility from transportation cannot address intermediate ranges of car ownership in the traditional economy. A fuller picture of the impact of ride sharing on ownership and traffic is provided in Section 8, in which we relax the assumption of common utility for car transport. In that case we show, for example, that it is possible not only for traffic to increase with the introduction of sharing, but also for ownership.

⁹In Section 7, we show that this is never the case when the platform chooses the social welfare-maximizing price, which results in full car utilization. In this case, ownership and traffic are always reduced.

¹⁰Propositions 7.1, 7.2, and 7.3, as we show in the online appendix, hold with modification for $k \geq 1$.

¹¹Note that we assume abstaining has a negligible impact on traffic (i.e., abstaining individuals use the subway or other public transportation that is anyway available). In settings in which abstaining can affect traffic, for example, when more buses or taxis are needed to accommodate these individuals, a term $\gamma \mu_A$ could be added to the expression of traffic in (4), in which $\gamma \in (0, 1)$ corresponds to a traffic intensity parameter associated with abstaining.

¹²Subsidies can also be used to model a setting, similar to that of a traditional taxi, in which the platform owns the vehicles and charges the service providers a usage fee. In this case, $\phi = \omega - \gamma$, where $\gamma \geq 0$ is the usage fee.

¹³With the exception that, when $\omega > \hat{\omega}$ and a is very large, we may get a B2C equilibrium in which $p < 1$.

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