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One-side value-added service investment and pricing strategies for a two-sided platform

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To widen user participation and increase profit, two-sided platforms may invest on value-added services (VASs) for users. Due to the cross-market network externality, the investment for one side would affect the utility of users on two sides, thus affect the demand and profit of the platform. In this paper, we investigate one-side VAS investment and pricing strategies for a two-sided platform. It is revealed that it is optimal to invest at the maximum level for any marginal investing cost below a certain threshold, and to decrease the investment when the marginal investing cost increases above the threshold. We find that compared with the case of no investment, the invested user side will always be priced higher, while the uninvested user side may either be priced higher or lower, depending on the relative magnitude of mutual cross-market network externalities. If both sides are priced higher after the investment, the price increment for uninvested user side could be larger than that for invested user side.

Keywords: networks; optimisation; two-sided market; value-added service investment

1. Introduction

Two-sided platforms connect two distinct groups of users and create surplus for them by realising interactions or transactions between the two groups (Schiff 2007). Examples of two-sided platforms are numerous. To name a few, shopping malls and newspapers are traditional typical two-sided platforms, enabling merchants and advertising agencies to sell products to buyers and exhibit advertisements to readers; Microsoft's Windows for PC, Apple's IOS and Google's Android for cell phones are operating system platforms, from which end users could download and use applications from third-party developers; electronic commerce websites such as Amazon, eBay and Alibaba prosper the online trading between buyers and sellers; Sony's PlayStation and Microsoft's Xbox are game consoles on which players could enjoy numerous of games provided by developers.

In order to create more surplus for users and widen their participation, value-added service (VAS) has been developed by many two-sided platforms (Kuo, Wu, and Deng 2009; Zhang et al. 2015). The VAS can be divided into three categories. The first category of VAS is beneficial to individuals. For example, people who use the cloud storage of Windows OneDrive manage to share documents on different electronic devices; customers who use the short message alerting service can get information of completed transactions instantly. The second category of VAS directly benefits (nearly) all users of two sides. For instance, the electronic paying system developed by Google's Apps Marketplace benefits both application developers and users directly as it brings more convenience during transactions; online store decorating service of electronic commerce platforms (e.g. Tmall) realises higher store attractiveness for merchants and better user experience for consumers. Differently, the third category of VAS directly benefits users on merely one side. For example, the electronic commerce platforms (e.g. eBay, Alibaba) provide market information and data mining services, according to which more effective marketing measures can be taken by merchants to promote transactions. These services does not contribute direct benefit to consumers. Besides, games such as 'Halo' and 'Wii Sports' provided by video game consoles (e.g. Microsoft's Xbox 360, Nintendo) directly benefit game players rather than game developers. In these two examples, for the consumers and game developers, they get indirect benefit when there are more merchants and game players accessing the platform, respectively. Moreover, the buyer side users can obtain an intrinsic usage benefit without the seller side users. In particular, game players can still enjoy movies or music without game developers on game consoles, but without game players, it is meaningless for the developers to access the platform.

In this paper, we focus on the last category of VAS which is developed for one side exclusively. We are doing so mainly because VAS is usually developed for one-side users and its impact on strategies for both sides could be

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explored. Two-sided platforms exhibit cross-market network externality (Katz and Shapiro 1985; Liebowitz and Margolis 1994), which leads to correlation and mutual influence of the two sides. Therefore, the strategy for one side has a great influence on the strategy for the other (Evans 2003; Rochet and Tirole 2003). From the first category of service discussed above, the influence cannot be learned because it is purchased by only a proportion of consumers, platforms maintain the strategy for those who have not chosen the service. Besides, we don't allow for the second category of VAS as it enhances the utility of users on two sides straightforward, leading to complexity in studying the effect of one-side investment on the other side.

One-side VAS enhances users' utility and thus higher users' demand on the corresponding side will be generated. Resulting from the cross-market network externality, the demand increment of invested user side improves the utility of users on the other side, of which a demand increment and different operational strategies are led to. In addition, platform's pricing has a close relationship with users' utility as well, the pricing changes if users' demand increases. Therefore, how to make the optimal investment strategy when developing VAS for one side, and what impacts will it have on the optimal pricing are issues quite worth studying. The issues are significant in the sense that, without expending too much to develop VAS for all users, one-side investment would pay off in realising wider user participation and higher profit. To explore one-side VAS investment strategy and its impact, we formulate models in two scenarios where the platform invests for each side. In the analysis, we not only consider the two sides to be differentiated concerning the intrinsic usage benefit but also model users on each side to be heterogeneous from certain perspectives.

The remainder of this paper is organised as follows. In Section 2, we review the related literature. In Section 3, we formulate two scenarios to analyse the investment and pricing strategies where VAS is developed for sellers and buyers, respectively. Also, equilibrium results and optimal pricing properties are presented. In Sections 4, we make comparisons between settings with and without investment and draw some conclusions. In Section 5, we make a discussion to present our implications to the practical problems. Finally, we summarise the paper and give some suggestions on topics for future research in Section 6. All proofs are relegated to the Appendix 1.

2. Literature review

The growing economic literature on two-sided platforms unveils that two-sided markets perform special characteristics differentiated from the traditional ones (Anderson, Parker, and Tan 2013), and therefore leading to distinct operation strategies.

Lots of the existing studies about two-sided platform put emphasis on pricing allowing for the cross-market network externality. For instance, to explore key principles that drive pricing in two-sided markets, Schiff (2007) illustrates basic pricing issues in two-sided markets using some simple linear models based on Rochet and Tirole (2006) and Armstrong (2006). Rochet and Tirole (2003) build a model of platform competition with two-sided markets, they unveil the determinants of pricing and end user surplus in different governance structures. Armstrong (2006) presents models of monopoly and competing two-sided markets, and gives the determinants of equilibrium prices including the cross-side network effects, the fee structure and whether the agents are single-homing or multi-homing. Economides and Katsamakas (2006) develop a framework to characterise the optimal two-sided pricing strategy based on a proprietary platform and an open source platform. Chandra and Collard-Wexler (2009) present an economic model of the newspaper market. They show that a monopolist may choose to raise prices on one side and lower them on the other side.

The research above reveals that for two-sided platforms exhibiting cross-market network externality, a novel pricing strategy that subsidising could be employed in achieving wider user participation and higher profit. However, relative little research is done to explore other controls beyond pricing. Taking information level as an operation method, Hagiu and Hałaburda (2014) study the effect of different information levels on two-sided platform profit under monopoly and competition. They show that the monopoly platform prefers informed users while the platform facing competition has the opposite preference. The study indicates that information could be taken as an operation method as well as pricing. Nonetheless, strategies concerning the non-price controls are usually studied in the setting that the platforms' service remains unimproved. Few studies about the platform's optimal strategy are done considering developing additional service. From this perspective, our paper addresses operation strategies allowing for one-side VAS investment in the presence of cross-market network externality. In this context, the optimal pricing has to be decided simultaneously with the investment strategy, the platform has to make a trade-off between attracting more users vs. spending less cost.

Closely related to our work, Hagiu and Spulber (2013) analyse the strategic use of first-party content, which is invested for buyers and can be substituted or compensated in relation to sellers' participation. We study a similar investing problem, but sellers in their research are considered homogeneous, all the sellers join the platform if the profit is non-negative or none joins otherwise. Thus, the price for sellers is not taken as the decision variable in the profit maximisation problem. In comparison, sellers in our setting are considered heterogeneous that a proportion of sellers choose to access the platform. Additionally, we also study the scenario where the platform invests for sellers. In each scenario, the prices for both sides together with the amount of investment are modelled as decision variables. In addition to Hagiu and Spulber (2013), Anderson, Parker and Tan (2013) build a strategic model to investigate the trade-off between investing in high platform performance vs. reducing investment to facilitate the third-party content development. Higher platform performance requires higher cost for sellers to access. But in their paper, both the profit from one transaction and the per-transaction fee are taken exogenous, while in our analysis, we assume that both sides charge membership fees, and we model them both as decision variables.

Our paper makes a combination of exploring operation method beyond pricing and developing additional service by exploring one-side VAS investment and pricing strategies. We not only model users on the two sides to be differentiated concerning the intrinsic usage benefit but also consider the users on the same side to be heterogeneous. Besides, the prices for both sides are taken as decision variables. These considerations together enable this work to differentiate itself from the existing research.

3. Models

We model a monopolistic two-sided platform connecting buyers and sellers (e.g. China UnionPay). Following Armstrong (2006), users pay a membership fee p_i on side i ($i = b, s$) to access the platform. There are M_i ($i = b, s$) users on side i and we normalise the potential maximum market size to 1, so that $M_i \le 1$. Assume that a user enjoys utility α_i ($i = b, s$, $\alpha_i \in (0, 1)$) per user from the other side (Hagiu and Hałaburda 2013). For the ease of analysis, we don't allow for the direct network externality in the same side here (Armstrong 2006). Thus, the gross utility associated with a buyer and a seller can be formulated as:

$$u_b = u_k + \alpha_b n_s - p_b \tag{1}$$

$$u_s = \alpha_s n_b - p_s - f_m \tag{2}$$

where n_i ($i = b, s$) is users' demand on side i ($i = b, s$). Note that α_i can be interpreted as the marginal cross-market network externality (Rasch and Wenzel 2013), $\alpha_i n_j$ ($i, j = b, s$, $i \neq j$) corresponds to the cross-market utility. To describe the heterogeneity of users, u_k characterises the intrinsic usage benefit of each buyer from using the platform's fundamental service without sells' participation (Rochet and Tirole 2006); f_m characterises each seller's cost of supplying product or service to buyers (Anderson, Parker, and Tan 2013). Following many existing studies (Anderson and Coate 2005; Armstrong 2006; Armstrong and Wright 2007; Anderson, Parker, and Tan 2013), we assume that both u_k and f_m are uniformly distributed over the interval [0, 1].

In the following part, we formulate two scenarios where the platform develops VAS for sellers and buyers in Sections 3.1 and 3.2, respectively. The optimal investment and pricing strategies are derived and some other analyses are done.

3.1 *The platform invests for sellers*

When the platform invests for sellers, Equation (2) can be reformulated as:

$$u_s = \alpha_s n_b + \beta_s x - p_s - f_m \tag{3}$$

where x measures the amount of VAS (Hagiu and Spulber 2013). We normalise the maximum amount of VAS to 1 so that $x \in [0, 1]$. Here we define all the VAS that can be developed based on the platforms' essential functionality as the meaning of $x = 1$. We don't allow for the situation where platforms extend their business to other regions since users involved in new businesses perform different characteristics. In practice, when an electronic commerce platform develops all the possible marketing service for merchants (e.g. advertising, business promotion, market survey, etc.), the amount of VAS can be viewed as 1. For each seller, $\beta_s \in (0, 1)$ measures the utility from an additional unit of VAS, and $\beta_s x$ corresponds to the total utility from the investment.

Users joins the platform when their gross utility is non-negative, i.e. $u_b \ge 0$, $u_s \ge 0$ in Equations (1) and (3). Denote by $\bar{u}_k$ the intrinsic usage benefit and $\bar{f}_m$ the cost which make a user indifferent between joining and not joining the platform, a buyer whose intrinsic usage benefit lies in the interval $[\bar{u}_k, 1]$ and a seller whose cost belongs to the region $[0, \bar{f}_m]$ will access the platform. With the participation rates of $1 - \bar{u}_k$ and $\bar{f}_m$, users' demand can be given as:

$$n_b = M_b(1 + \alpha_b n_s - p_b) \tag{4}$$

$$n_s = M_s(\alpha_s n_b + \beta_s x - p_s) \tag{5}$$

Suppose that it incurs cost $\phi x^2/2$ in providing the amount of VAS x to sellers (Hagiu and Spulber 2013), by choosing p_i ($i = b, s$) and x, the platform maximises its' profit from the following problem:

$$\max \prod(p_b, p_s, x) = p_b n_b + p_s n_s - \phi x^2/2 \tag{6}$$

By maximising the profit in (6) with (4) and (5), we can derive the optimal investment and pricing strategies summarised in the following theorem.

Theorem 1.

Let $I = 4 - (\alpha_b + \alpha_s)^2 M_b M_s$, $\bar{\phi} = [(\alpha_b + \alpha_s)M_b + 2\beta_s]\beta_s M_s/I$, $H = \phi I - 2\beta_s^2 M_s$, the optimal VAS investment and pricing strategies can be given as:

(1) If $\phi \le \bar{\phi}$, then $x^* = 1$, $p_s^* = \{(\alpha_s - \alpha_b)M_b + \beta_s[2 - \alpha_b(\alpha_b + \alpha_s)M_b M_s]\}/I$ and $p_b^* = [2 - \alpha_s(\alpha_b + \alpha_s)M_b M_s + (\alpha_b - \alpha_s)M_s\beta_s]/I$;

(2) If $\phi > \bar{\phi}$, then $x^* = (\alpha_b + \alpha_s)\beta_s M_s M_b/H$, $p_s^* = [\phi(\alpha_s - \alpha_b) + \alpha_b\beta_s^2 M_s]M_b/H$ and $p_b^* = \{\phi[2 - \alpha_s(\alpha_b + \alpha_s)M_s M_b] - \beta_s^2 M_s\}/H$.

The theorem demonstrates a one threshold policy for VAS investment. It is optimal to provide the maximum amount of VAS for a marginal investing cost below the threshold $\bar{\phi}$ and the optimal pricing is irrelevant to the cost parameter. Otherwise, the amount of VAS decreases for a larger ϕ, and ϕ becomes a determinant of the optimal pricing.

Form the optimal pricing in Theorem 1, we characterise when to charge and subsidise users. The results are summarised in the proposition below.

Proposition 1.

Let $E = 2 - \alpha_s(\alpha_b + \alpha_s)M_b M_s$, $F = 2 - \alpha_b(\alpha_b + \alpha_s)M_b M_s$, then we have the pricing strategies for users shown in Table 1:

Table 1 shows both positive and negative prices on both sides under different conditions. Specifically, in Rows 2 and 3, Column 4, $p_s^* > 0$ if $\alpha_s > \alpha_b$; in Rows 4 and 5, Column 5, $p_b^* > 0$ if $\alpha_s < \alpha_b$. To interpret, recall that Theorem 1 indicates that the investment is maximum when the marginal cost is small, i.e. $\phi \le \bar{\phi}$. Then if $\alpha_s > \alpha_b$, considerable utility can be derived by sellers, including the utility from the maximal amount of VAS and the cross-market utility devoted by buyers. Otherwise, if $\alpha_s < \alpha_b$, buyers obtain considerable utility indirectly from VAS through the cross-market network externality. Charging users to whom the platform contributes plentiful utility increases profit.

Meanwhile, in Rows 2 and 3, when $\alpha_s > \alpha_b$, the pricing will be that $p_b^* < 0$ $(p_b^* \ge 0)$ if $\beta_s > E/(\alpha_s - \alpha_b)M_s$ $(\beta_s \le E/(\alpha_s - \alpha_b)M_s)$. The maximal investment carried out when $\phi \le \bar{\phi}$ benefits seller considerably if the marginal investing benefit is relatively large, i.e. $\beta_s > E/(\alpha_s - \alpha_b)M_s$. Therefore, facilitating the investment to benefit more sellers can be more profitable. To attract more sellers, subsidising buyers pays off as it improves buyers' utility and demand, which in turn further improves sellers' utility and demand through cross-market network externality. Otherwise, $(\beta_s \le E/(\alpha_s - \alpha_b)M_s)$, the total investment benefit is relatively small, it won't be as profitable as the former situation to subsidise buyers in attracting more sellers. Since buyers obtain considerable utility indirectly from VAS, they can be charged to gain more profit.

Table 1. The conditions on which to charge and subsidise users.

Marginal investing cost	Cross-market network externality	Marginal investing benefit (cost)	Pricing for sellers	Pricing for buyers
$\phi \le \bar{\phi}$	$\alpha_s > \alpha_b$	$\beta_s \le E/(\alpha_s - \alpha_b)M_s$	$p_s^* > 0$	$p_b^* \ge 0$
		$\beta_s > E/(\alpha_s - \alpha_b)M_s$		$p_b^* < 0$
	$\alpha_s < \alpha_b$	$\beta_s < (\alpha_b - \alpha_s)M_b/F$	$p_s^* < 0$	$p_b^* > 0$
		$\beta_s \ge (\alpha_b - \alpha_s)M_b/F$	$p_s^* \ge 0$	
	$\alpha_s = \alpha_b$		$p_s^* > 0$	$p_b^* > 0$
$\phi > \bar{\phi}$	$\alpha_s \ge \alpha_b$	$\bar{\phi} < \phi < \beta_s^2 M_s/E$	$p_s^* > 0$	$p_b^* < 0$
		$\phi \ge \beta_s^2 M_s/E$		$p_b^* \ge 0$
	$\alpha_s < \alpha_b$	$\bar{\phi} < \phi < \beta_s^2 M_s/E$	$p_s^* > 0$	$p_b^* < 0$
		$\beta_s^2 M_s/E \le \phi < \alpha_b\beta_s^2 M_s/(\alpha_b - \alpha_s)$	$p_s^* > 0$	$p_b^* \ge 0$
		$\phi \ge \alpha_b\beta_s^2 M_s/(\alpha_b - \alpha_s)$	$p_s^* \le 0$	$p_b^* > 0$

Rows 4 and 5 show that when $\alpha_s < \alpha_b$, the pricing will be that $p_s^* \geq 0$ $\left(p_s^* < 0\right)$ when $\beta_s \geq (\alpha_b - \alpha_s)M_b/F$ $(\beta_s < (\alpha_b - \alpha_s)M_b/F)$. On the seller side, when the marginal investing benefit is relatively large, i.e. $\beta_s \geq (\alpha_b - \alpha_s)M_b/F$, the maximal investment benefits sellers significantly, for which the platform charges sellers. Otherwise, $(\beta_s < (\alpha_b - \alpha_s)M_b/F)$, subsidising sellers improves their utility and demand, which in turn realises a wider participation of buyers through cross-market network externality. Though the profit from sellers is negative, higher profit could be ensured from the buyer side.

If $\alpha_s = \alpha_b$, Row 6 in Table 1 shows positive prices for both sides. On one hand, $\alpha_s = \alpha_b$ suggests equal cross-market network externalities of the two sides. Thus, unlike the case that $\alpha_s > \alpha_b$ (or $\alpha_s < \alpha_b$), users' demand on one side cannot be enlarged effectively through subsidising the other side. On the other hand, sellers' gross utility gets improved from VAS, then the demand increment of sellers devotes cross-market utility for buyers. Consequently, VAS contributes extra utility for both sides, for which the platform could set positive prices to make more profit.

If the investment cost is large, i.e. $\phi > \bar{\phi}$, the platform also has both positive and negative prices for buyers when $\alpha_s > \alpha_b$, as is shown in Rows 7 and 8. When the cost increases to a region that $\bar{\phi} < \phi < \beta_s^2 M_s/E$, the amount of investment is still relatively large. That is, realising a wider participation of sellers to benefit from the investment can still be more profitable. Thus, buyers are subsidised to attract more sellers. However, if the cost increases to a very high level, i.e. $\phi \geq \beta_s^2 M_s/E$, the investment decreases a lot, benefiting sellers slightly. In this situation, charging buyers compensates some of the profit loss caused by the high cost.

The last three rows show that when $\phi > \bar{\phi}$, both positive and negative prices can be set on each side if $\alpha_s < \alpha_b$. For a relatively small cost, i.e. $\bar{\phi} < \phi < \beta_s^2 M_s/E$, the level of investment will be relatively high. As we discussed above, buyers can be subsidised to realise a wilder participation of sellers to benefit from the investment. With a further cost increase that $\beta_s^2 M_s/E < \phi \leq \alpha_b \beta_s^2 M_s/(\alpha_b - \alpha_s)$, the investment further decreases and more loss is caused. To compensate some of the loss, buyers could be charged. If the cost increases to a very high level, i.e. $\phi > \alpha_b \beta_s^2 M_s/(\alpha_b - \alpha_s)$, little investment is available. In this situation, the cross-market network externality becomes the main determinant for pricing again. Subsidising sellers improves their utility, and when $\alpha_s < \alpha_b$ it appeals a larger number of buyers to join the platform, then higher profit can be achieved by charging buyers.

From Proposition 1, by exploring how the thresholds for the marginal investing benefit (cost), we arrive at the following results.

Corollary 1. For $i = b, s$, the thresholds in Table 1 satisfy that:

(1) The threshold for β_s that $E/(\alpha_s - \alpha_b)M_s((\alpha_b - \alpha_s)M_b/F)$ decreases (increases) in M_i when $\alpha_s > \alpha_b$ ($\alpha_b > \alpha_s$);
(2) The threshold for ϕ that $\beta_s^2 M_s/E$ $\left(\alpha_b \beta_s^2 M_s/(\alpha_b - \alpha_s)\right)$ increases in M_i (M_s).

Corollary 1 reveals how the thresholds in Table 1 change with the market size. The threshold $E/(\alpha_s - \alpha_b)M_s((\alpha_b - \alpha_s)M_b/F)$ of the marginal investing benefit becomes smaller (larger) with a larger size market M_i given that $\alpha_s > \alpha_b$ ($\alpha_s < \alpha_b$). Also, for a larger size market, the threshold $\beta_s^2 M_s/E$ of the marginal investing cost is larger.

According to Table 1, the results above demonstrate that the platform turns to subsidise (charge) buyers (sellers) at a smaller (larger) marginal investing benefit β_s, or turns to charge buyers at a higher marginal investing cost. That is, the region in which the platform subsidises users who own the stronger cross-market network externality becomes larger. The reason is that users' demand for the platform increases with a larger size market, with which the platform is able to gain more profit by realising further demand increment. Thus, the platform can expand the region in which to subsidise users who devote higher utility to the other side users.

If the platform has to subsidise sellers to improve their utility when the marginal investing cost is very high (e.g. the last row in Table 1), then for a larger seller market size M_s, the platform turns to subsidise sellers at a larger marginal investing cost. In other words, the region in which the platform charges sellers becomes larger. This phenomenon is easy to understand, a larger size market indicates a larger platform demand of sellers, for whom the platform could use a charging policy to enlarge its profit. So the platform could turn to charge sellers at a higher investment cost.

Except the characterised conditions on which to subsidise or charge users, some sensitive analyses have also been done to explore the special behaviours that the optimal pricing exhibits related to ϕ, β_s and M_i ($i = b, s$). The features are summarised in Propositions 2 and 3.

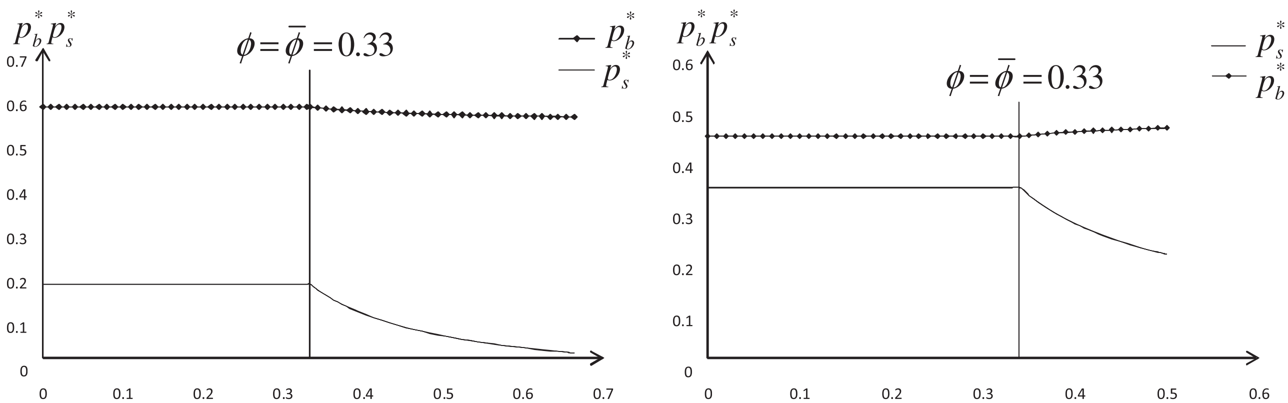
Proposition 2. The following results hold:

In Table 2, Rows 2 and 3 show that when the marginal investing cost is large, i.e. $\phi > \bar{\phi}$, the optimal pricing for sellers decreases in it while the optimal pricing for buyers can either increase or decrease in it. To further illustrate the impact of the marginal investing cost on the optimal pricing, we present two numerical examples. For the ease of presentation, we assume that $M_i = 1$ ($i = b, s$), the Figure 1(a) and (b) give the results.

Intuitively, higher costs for investment lead to higher membership fees. However, the first part of Column 4 in Table 2 reveals that the opposite turns out to be true in our setting, as is shown in Figure 1. In both (a) and (b), the

Table 2. The influence of the marginal investing cost (benefit) on the optimal pricing.

Marginal investing cost	Cross-market network externality	Pricing strategy for buyers	Pricing strategy for sellers
$\phi > \bar{\phi}$	$\alpha_s < \alpha_b$	$\partial p_b^*/\partial\phi < 0$	$\partial p_s^*/\partial\phi < 0$
	$\alpha_s \geq \alpha_b$	$\partial p_b^*/\partial\phi \geq 0$	
$\phi \leq \bar{\phi}$	For all α_b, α_s	$\partial p_b^*/\partial\phi = 0$	$\partial p_s^*/\partial\phi = 0$
For all ϕ	$\alpha_s < \alpha_b$	$\partial p_b^*/\partial\beta_s > 0$	$\partial p_s^*/\partial\beta_s > 0$
	$\alpha_s \geq \alpha_b$	$\partial p_b^*/\partial\beta_s \leq 0$	



(a) $\alpha_s = 0.6, \alpha_b = 0.4, \beta_s = 0.5$ (b) $\alpha_s = 0.4, \alpha_b = 0.6, \beta_s = 0.5$

Figure 1. Optimal pricing in scenario 1 with distinct cross-market network externalities.

optimal pricing for sellers remain constant for any marginal investing cost ϕ below the threshold $\left(\bar{\phi} = 0.33\right)$, but decreases with larger ϕ above the threshold. As Row 4 shows, for all α_b, α_s, $\partial p_i^*/\partial\phi = 0$, $i = b, s$. To interpret, recall that the investment is maximum and the optimal solution is irrelevant to ϕ if $\phi \leq \bar{\phi}$, thus the optimal pricing is constant. With further increase in ϕ, the investment decreases thus the benefit for sellers becomes less, resulting in a demand decrease of sellers. In this situation, charging a lower membership fee enhances sellers' utility and the demand, which improves buyers' utility and demand through cross-market network externality. Consequently, the profit loss caused by higher cost can be mitigated by a larger total user demand.

Comparing Figure 1(a) and (b), we can find the result shown in Row 2, 3, Column 3 in Table 2, that is, the optimal price for buyers increases if $\alpha_s > \alpha_b$ but decreases otherwise. Theorem 1 indicates that the investment decreases with a larger marginal cost if $\phi > \bar{\phi}$. But when $\alpha_s > \alpha_b$, the demand decrease on the buyer side can be lessened, which is, buyers are less effected by a larger cost. So they can be charged to compensate some profit loss associated with the higher cost. Otherwise, buyers get affected more seriously than sellers if $\alpha_s < \alpha_b$, a further demand decrease on the buyer side happens. To mitigate the sharp demand decrease thus to avoid further profit loss, a lower membership fee should be charged for buyers.

In Table 2, the second part of Column 4 shows that the optimal pricing for sellers increases in the marginal investing benefit, while Rows 4 and 5 of Column 3 show that the optimal pricing for buyers can either increase or decrease in it. A larger marginal investing benefit β_s indicates higher utility for sellers from VAS, for which a higher membership fee can be charged. Meanwhile, since the total investment benefit becomes higher, promoting the investment to benefit more sellers can be more profitable. Thus when $\alpha_s > \alpha_b$, buyers can be charged less to appeal more sellers. On the contrary ($\alpha_s < \alpha_b$), a higher benefit from the investment improves buyers' utility indirectly, charging them more raises profit.

Proposition 3. Let $W = \alpha_b\beta_s^2 M_s/(\alpha_b - \alpha_s)$, $Z = 2\beta_s^2/\left(\alpha_b^2 - \alpha_s^2\right)M_b$, the impact of the market size on the optimal pricing can be given in the table below:

In Table 3, Rows 3, 5 and 7 show that when $\alpha_s > \alpha_b$, the optimal pricing for buyers decreases in the market size while the optimal pricing for sellers increases in it. A larger market size indicates a higher user demand base for the platform, then promoting a further demand increase can be more profitable. Since $\alpha_s > \alpha_b$, lowering the price for buyers enlarges sellers' demand indirectly, and higher profit can be achieved by raising the membership fee for sellers.

Table 3. The influence of market size on the optimal pricing.

Marginal investing cost	Cross-market network externality	Pricing strategy for sellers	Pricing strategy for buyers
$\phi \le \bar{\phi}$	$\alpha_s < \alpha_b$	$\partial p_s^*/\partial M_i < 0$	$\partial p_b^*/\partial M_i > 0$
	$\alpha_s \ge \alpha_b$	$\partial p_s^*/\partial M_i \ge 0$	$\partial p_b^*/\partial M_i \le 0$
$\bar{\phi} < \phi \le W\left(\bar{\phi} < \phi \le Z\right)$	$\alpha_s < \alpha_b$	$\partial p_s^*/\partial M_b \ge 0\ \left(\partial p_s^*/\partial M_s \ge 0\right)$	$\partial p_b^*/\partial M_i > 0$
	$\alpha_s \ge \alpha_b$	$\partial p_s^*/\partial M_i > 0$	$\partial p_b^*/\partial M_i \le 0$
$\phi > W\ (\phi > Z)$	$\alpha_s < \alpha_b$	$\partial p_s^*/\partial M_b < 0\left(\partial p_s^*/\partial M_s < 0\right)$	$\partial p_b^*/\partial M_i > 0$
	$\alpha_s \ge \alpha_b$	$\partial p_s^*/\partial M_i > 0$	$\partial p_b^*/\partial M_i \le 0$

Rows 2, 4 and 6 show that the optimal pricing for buyers increases in the market size when $\alpha_s < \alpha_b$. In comparison, given different investment costs, the optimal pricing for sellers can either increase or decrease in the market size. The logic for buyers' pricing is that when $\alpha_s < \alpha_b$, VAS benefits buyers higher indirectly through cross-market network externality, which leads to a higher membership fee. On the seller side, when the marginal investing cost is small, i.e. $\phi \le \bar{\phi}$, the maximal investment benefits sellers significantly. With a larger market size, lowering the membership fee on the seller side attracts a wider range of sellers to benefit from the investment. The profit obtained from the enlarged demand dominates the loss from lowering the price.

When the marginal investing cost increases, i.e. $\bar{\phi} < \phi \le W$ or $\bar{\phi} < \phi \le Z$, relatively less investment will be carried out, but a larger number of sellers from the increased market sizes can still benefit a lot from VAS, then raising the membership fee generates higher profit. However, if the marginal investing cost comes to a very high level, i.e. $\phi > W$ or $\phi > Z$, little investment is available and sellers' demand will experience a sharp decrease. To prevent the decrease, the membership fee can be lowered for more sellers from the larger markets size, which increases the whole users' demand and realises less profit loss.

Besides, worth mentioning is that, when $\phi > \bar{\phi}$, the platform raises the membership fee for sellers if $\alpha_s = \alpha_b$, as is shown in Rows 5 and 7. When $\phi > \bar{\phi}$, the platform turns to decrease the investment and some profit loss happens. To diminish the loss, the membership fee for sellers is raised since VAS creates direct benefit for them, and because $\alpha_s = \alpha_b$, the cross-market utility decrease for buyers is small, thus a large demand decrease on the buyer side wouldn't be led to.

3.2 *The platform invests for buyers*

When the platform invests for buyers, Equation (1) can be reformulated as:

$$u_b = u_k + \alpha_b n_s + \beta_b y - p_b \tag{7}$$

where $y \in [0, 1]$ represents the amount of VAS. Likewise, $y = 1$ corresponds to the maximum amount of VAS. In practice, VAS can be viewed at the maximum for buyers when a mobile bank app contains all the self-help business affairs. Similarly, $\beta_b \in (0, 1)$ measures buyers' utility from an additional unit of VAS.

Likewise, users' demand for the platform can be derived from Equations (2) and (7) when the gross utility is non-negative as:

$$n_b = M_b(1 + \alpha_b n_s + \beta_b y - p_b) \tag{8}$$

$$n_s = M_s(\alpha_s n_b - p_s) \tag{9}$$

Assume that the platform expends cost $\varphi y^2/2$ in providing the amount of VAS y to buyers, the decision problem of the platform can be formulated as:

$$\max \prod(p_b, p_s, y) = p_b n_b + p_s n_s - \varphi y^2/2 \tag{10}$$

Maximising the profit in (10) with (8) and (9), we obtain Theorem 2 as follows (Theorem 2 can be proved in a similar way with Theorem 1, thus we omit the proof in the Appendix 1):

Theorem 2.

Let $G = \varphi I - 2\beta_b^2 M_b$, $\bar{\varphi} = \left(2\beta_b M_b + 2\beta_b^2 M_b\right)/I$, the optimal VAS investment and pricing strategies can be given as:

(1) If $\varphi \leq \bar{\varphi}$, then $y^* = 1$, $p_b^* = [2 - \alpha_s(\alpha_b + \alpha_s)M_bM_s](1 + \beta_b)/I$ and $p_s^* = (\alpha_s - \alpha_b)(1 + \beta_b)M_b/I$;
(2) If $\varphi > \bar{\varphi}$, then $y^* = 2\beta_bM_b/G$, $p_b^* = \varphi[2 - \alpha_s(\alpha_b + \alpha_s)M_bM_s]/G$ and $p_s^* = \varphi(\alpha_s - \alpha_b)M_b/G$.

The theorem demonstrates a one threshold policy as well. Investing at the maximum level when φ is below the threshold $\bar{\varphi}$ is optimum and the optimal pricing is irrelevant to φ. Otherwise, φ becomes a determinant for the optimal pricing and the investment decreases for a larger φ.

Different from Scenario 1, sellers in this scenario can either be charged or subsidised while buyers will always be charged. Recall that without sellers' participation, buyers obtain an intrinsic usage benefit, the total surplus can be well enhanced under VAS investment, thus the platform charges buyers. In comparison, sellers are charged when $\alpha_s > \alpha_b$ as they enjoy considerable cross-market utility from buyers when VAS is developed, but will be subsidised when $\alpha_s < \alpha_b$ because they create more cross-market utility to buyers.

Analogously, in relation to φ and β_b, the optimal pricing also performs some special characteristics, we summarise them in Proposition 4. The impact of the market size M_i ($i = b, s$) on the optimal pricing is similar with the former scenario thus we omit the results here.

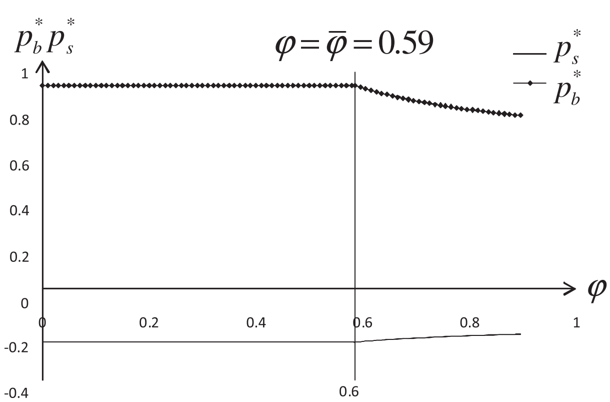
Proposition 4: The following results hold:

The table above reveals similar results with that of Scenario 1. To further illustrate the impact of the marginal investing cost on the optimal pricing, two numerical examples are presented as well, we again assume that $M_i = 1$ ($i = b, s$), the results are as follows:

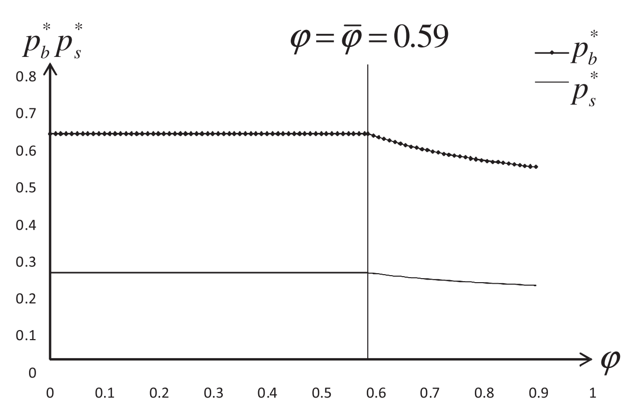
As can be seen in Figure 2, users would be charged a constant price for any marginal investing cost below the threshold ($\bar{\varphi} = 0.59$), as is shown in Rows 2 and 3, Column 4 in Table 4. But buyers would be charged less with further increase in φ, as is shown in the last part of Column 3. The reason is that, the optimal solution is irrelevant to φ when it is below the threshold, thus $\partial p_i^*/\partial\varphi = 0$, $i = b, s$. Otherwise, lowering the price for buyers with a higher φ expands users' utility and the demand, which mitigates the profit loss caused by a higher cost.

cFigure 2(a) and (b) together reveal that, when $\varphi > \bar{\varphi}$, the optimal pricing for sellers decreases if $\alpha_s > \alpha_b$ but increases if $\alpha_s < \alpha_b$, as is shown Rows 4 and 5, Column 4 in Table 4. Sellers' demand would be further diminished when buyers' demand decreases if $\alpha_s > \alpha_b$, as they will be more seriously affected if the cost increases. Charging sellers less enhances their utility thus avoids too much loss. On the contrary, sellers' are less affected that the demand decrement can be lessened when $\alpha_s < \alpha_b$, charging sellers more helps compensate some of the loss associated with a higher cost.

In Table 4, Column 4 shows that the price for sellers could either increase or decrease if the marginal investing benefit β_b is larger. When $\varphi \leq \bar{\varphi}$, given the maximum investment and a large β_b, sellers' utility and demand rises greatly if $\alpha_s > \alpha_b$, for which the platform could charge sellers more to enhance profit. Otherwise, if $\alpha_s < \alpha_b$, charging sellers less improves the their utility and demand, in turn, buyers' demand gets further enlarged and higher profit can be gained from the buyer side. When $\varphi > \bar{\varphi}$, less investment is carried out, but investing with a larger β_b realises higher total users' demand, then charging sellers more diminishes the profit loss associated with a higher cost. In comparison, Column 3 shows that the price for buyers is higher if the marginal investing benefit β_b is larger. This is straightforward since higher utility is devoted to buyers directly when β_b increases.



(a) $\alpha_s = 0.8, \alpha_b = 0.4, \beta_b = 0.5$



(b) $\alpha_s = 0.4, \alpha_b = 0.8, \beta_b = 0.5$

Figure 2. Optimal pricing in scenario 2 with distinct cross-market network externalities.

Table 4. The influence of the marginal investing cost (benefit) on the optimal pricing.

Marginal investing cost	Cross-market network externality	Pricing strategy for buyers	Pricing strategy for sellers
$\varphi \leq \bar{\varphi}$	$\alpha_s \leq \alpha_b$	$\partial p_b^*/\partial\beta_b > 0$, $\partial p_b^*/\partial\varphi = 0$	$\partial p_s^*/\partial\beta_b \leq 0$, $\partial p_s^*/\partial\varphi = 0$
	$\alpha_s > \alpha_b$		$\partial p_s^*/\partial\beta_b > 0$, $\partial p_s^*/\partial\varphi = 0$
$\varphi > \bar{\varphi}$	$\alpha_s \leq \alpha_b$	$\partial p_b^*/\partial\beta_b > 0$, $\partial p_b^*/\partial\varphi < 0$	$\partial p_s^*/\partial\beta_b > 0$, $\partial p_s^*/\partial\varphi \geq 0$
	$\alpha_s > \alpha_b$		$\partial p_s^*/\partial\beta_b > 0$, $\partial p_s^*/\partial\varphi < 0$

4. Comparisons

In this section, we make some comparisons from different aspects. Firstly, for the two scenarios, by assuming that the marginal investing costs are equal, we compare the optimal profits to explore which side to invest can be more profitable, the results are given in Proposition 5. Secondly, we compare the optimal pricing between two cases: with and without VAS investment, and we present the results in Proposition 6. Lastly, Proposition 7 shows on which side the pricing increment is larger when the membership fee increases on both sides after the investment.

Proposition 5. Let $K = 2\beta_b^2\beta_s^2 M_s / \left[4\beta_b^2 - \beta_s^2 M_s^2(\alpha_b + \alpha_s)^2\right]$, if $\phi = \varphi > \max\left[\bar{\phi}, \bar{\varphi}\right]$, then:

(1) If $\beta_s \leq \beta_b$, investing for buyers is more profitable;
(2) When $\beta_b < \beta_s \leq 2\beta_b / M_s(\alpha_b + \alpha_s)$, then if $\max\left[\bar{\phi}, \bar{\varphi}\right] < \phi < K$, investing for sellers is more profitable, if $\phi > K$, investing for buyers is more profitable;
(3) When $\beta_s > 2\beta_b / M_s(\alpha_b + \alpha_s)$, investing for sellers is more profitable.

Given that $\phi = \varphi > \max\left[\bar{\phi}, \bar{\varphi}\right]$, a β_s smaller than β_b suggests higher profit from investing for buyers. In fact, due to the intrinsic usage benefit, the result that investing for buyers yields higher profit when $\beta_s \leq \beta_b$ always holds if $\phi = \varphi$. When β_s is larger than β_b, the investment performs higher attractiveness for sellers. However, for a quite large marginal investing cost ($\phi > K$), little investment is available, investing for buyers achieves more profit as they derive an intrinsic usage benefit and the total utility of buyers can be promoted a lot. If $\max\left[\bar{\phi}, \bar{\varphi}\right] < \phi < K$, relatively more investment can be carried out and investing for sellers is more beneficial. When β_s is quite high, i.e. $\beta_s > 2\beta_b / M_s(\alpha_b + \alpha_s)$, the investment benefits sellers considerably and therefore investing for sellers generates higher profit.

Proposition 6.

Denoted by Δp_b^*, Δp_s^* the optimal pricing changes for buyers and sellers compared with the case without VAS investment, then:

(1) When investing for sellers, then $\Delta p_s^* > 0$; $\Delta p_b^* < 0$ if $\alpha_b < \alpha_s$ and $\Delta p_b^* > 0$ if $\alpha_b > \alpha_s$;
(2) When investing for buyers, then $\Delta p_b^* > 0$; $\Delta p_s^* < 0$ if $\alpha_s < \alpha_b$ and $\Delta p_s^* > 0$ if $\alpha_s > \alpha_b$.

Compared with the case without VAS investment, it is intuitive that the invested user side will be priced higher, because the platform pays to develop VAS to benefit them. Interestingly, the uninvested user side may either be priced higher or lower, depending on the relative magnitude of the mutual cross-market network externalities, i.e. α_b and α_s. In particular, when investing for buyers, the price for sellers rises (decreases) if $\alpha_b < \alpha_s$ ($\alpha_b > \alpha_s$) because they derive (devote) more cross-market utility from (to) buyers. As we discussed before, raising the price for users who derive higher benefit enhances profit, and lowering the price for users who devote higher utility to the other side enlarges the whole users' demand, from which higher profit can be obtained.

Proposition 7. If the pricing increases on both sides after the investment, then:

(1) When investing for sellers, then if $\alpha_b > F/M_s + \alpha_s$, $\Delta p_b^* > \Delta p_s^*$; if $\alpha_s < \alpha_b \leq F/M_s + \alpha_s$, $\Delta p_b^* \leq \Delta p_s^*$;
(2) When investing for buyers, then if $\alpha_s > E/M_b + \alpha_b$, $\Delta p_s^* > \Delta p_b^*$; if $\alpha_b < \alpha_s \leq E/M_b + \alpha_b$, $\Delta p_s^* \leq \Delta p_b^*$.

This proposition reveals that the price increment for uninvested user side can be larger than that for the invested user side. If users on the uninvested side obtain higher cross-market utility from the invested user side but lower than a certain threshold ($\alpha_s < \alpha_b \leq F/M_s + \alpha_s$ or $\alpha_b < \alpha_s \leq E/M_b + \alpha_b$), the optimal pricing increment for users on the invested side is larger because higher benefit is devoted to them by the investment. However, when users on the uninvested side obtain a very large cross-market utility from the invested side ($\alpha_b > F/M_s + \alpha_s$ or $\alpha_s > E/M_b + \alpha_b$), more surplus can be obtained indirectly by the uninvested side users through the cross-market network externality, thus a larger pricing increment for the uninvested side users will be led to.

5. Discussion

The properties of the optimal pricing we explored above indicate that the strength of the mutual cross-market network externalities measures the relative 'importance' of the two sides. To achieve higher profit when providing VAS, platforms should lower the price for the more 'important' side to expand users' demand through cross-market network externality, especially on the less 'important' side. In practice, take credit cards as VAS of banks, card holders can be viewed as the more 'important' users because they are more attractive to merchants. Compared with debit cards, the price for credit cards is lowered since no membership fee is charged for them on condition that card holders make certain transactions per year using the credit cards. Indeed, the condition can easily be satisfied that the credit cards are free actually. Great utility from the credit cards can be obtained by card holders that they are so wildly used all over the world. By charging per-transaction fees, banks' profit can be well enhanced.

Besides, though cost is paid for VAS, subsidising can also be done for platform users to expand users' demand. For further illustration of this implication, we can turn to another real world example. In the taxi market, the mobile application Didi Taxi can be viewed as a typical two-sided platform connecting taxi drivers and passengers. Through incorporating taxi-calling service into some instant messaging applications such as Wechat, the fundamental service contained in the application gets improved, which facilitates passengers to use Didi Taxi more conveniently. Though more benefit is created, passengers still get subsidised for each travel. In this way, users' participation for this application has been widened in a short time.

Synthetically, to provide more benefit for users by investing on VAS, some novel pricing strategies for two-sided platforms can still be employed. For users on the two sides, the relative 'importance' of them plays the key role in determining how to charge both sides when VAS is developed. Pricing the more 'important' side less can be more profitable since it enhances the whole users' demand.

6. Conclusion

We investigate how a monopolistic two-sided platform invests on VAS for each side and how to charge both sides. The results show that the platform tends to invest at the maximum level when the marginal investing cost is below a certain threshold, and to decrease the investment when the marginal investing cost increases above the threshold. Interestingly, we show that for a higher marginal investing cost beyond the threshold, the pricing decreases for invested user side while the pricing for uninvested user side could either decrease or increase; and compared with the case without investment, the price for uninvested user side could either increase or decrease as well, depending on the relative strength of mutual cross-market network externalities. Furthermore, when both sides are priced higher after the investment, the pricing increment for uninvested user side could be larger than that for invested user side.

Our findings imply that subsidising can also be done for platform users through VAS investment. If the marginal investing cost increases above the threshold, lowering the price for invested user side enlarges total users' demand and mitigates the profit loss caused by higher cost. Besides, the relative strength of the mutual cross-market network externalities measures the relative 'importance' of the two sides, the more 'important' side should be priced lower to attract more users on the other side. Furthermore, the relative strength of the mutual cross-market network externalities plays the key role in determining the pricing strategy for uninvested user side. In particular, after developing the investment, platforms should increase (decrease) the price for uninvested user side if they obtain (contribute) more cross-market utility from (to) the invested side users. Furthermore, the price for uninvested side users should be raised to a larger extent than that for invested side users in case that considerable cross-market utility is derived by them from the invested user side.

This study makes an attempt to explore the investment and pricing strategies of a monopolistic two-sided platform when it develops VAS for one side. Still, several limits exist in this paper, which can be improved in future research. For instance, it is also interesting to investigate the optimal investment and pricing strategies when market share or price competition between platforms is considered. Besides, different levels of price information on the two sides can also be allowed for. Other extensions such as incorporating the direct network externality in the same side could also be done to extend the research.

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Appendix 1. Proof of Theorem 1

Equations (4) and (5) can be rewritten as: $p_b = 1 + \alpha_b n_s - n_b/M_b$ and $p_s = \alpha_s n_b + \beta_s x - n_s/M_s$, then the profit function can be rewritten as: $\prod(n_b, n_s, x) = (1 + \alpha_b n_s - n_b/M_b)n_b + (\alpha_s n_b + \beta_s x - n_s/M_s)n_s - \phi x^2/2$. Then we have the first derivatives that $\partial\prod/\partial n_s = \alpha_b n_b + \alpha_s n_b + \beta_s x - 2n_s/M_s$, $\partial\prod/\partial x = \beta_s n_s - \phi x$ $\partial\prod/\partial n_b = 1 + \alpha_b n_s + \alpha_s n_s - 2n_b/M_b$, and Hessian matrix $A = \begin{bmatrix} -2/M_s & \alpha_b + \alpha_s & \beta_s \\ \alpha_s + \alpha_b & -2/M_b & 0 \\ \beta_s & 0 & -\phi \end{bmatrix}$. $|A| = 2\beta_s^2/M_b - \phi\left[4/M_b M_s - (\alpha_b + \alpha_s)^2\right]$, since $I = 4 - (\alpha_b + \alpha_s)^2 M_b M_s > 0$, then:

(1) If $\phi > 2M_s\beta_s^2/I$, then A is negative definite, then the optimality from the first order conditions that:

$$\begin{cases} n_s^* & = \phi(\alpha_b + \alpha_s)M_s M_b/H \\ n_c^* & = \left(2M_b\phi - \beta_s^2 M_b M_s\right)/H \\ x^* & = (\alpha_b + \alpha_s)\beta_s M_s M_b/H \end{cases} \tag{A1}$$

where $H = \phi I - 2\beta_s^2 M_s$ can be assured. Since $x \in [0, 1]$, then we have that $x^* = 1$ if $2M_s\beta_s^2/I < \phi \le \left[(\alpha_b + \alpha_s)\beta_s M_s M_b + 2\beta_s^2 M_s\right]/I$; otherwise, $x^* = (\alpha_b + \alpha_s)\beta_s M_s M_b/H$ if $\phi > \left[(\alpha_b + \alpha_s)\beta_s M_s M_b + 2\beta_s^2 M_s\right]/I$.

Substituting (A1) into Equations (4) and (5), the optimal pricing can be obtained that $p_b^* = \left[2\phi - \phi\alpha_s(\alpha_b + \alpha_s)M_sM_b - \beta_s^2 M_s\right]/H$, $p_s^* = \left[\phi(\alpha_s - \alpha_b)M_b + \alpha_b\beta_s^2 M_bM_s\right]/H$.

(2) If $\phi \le 2M_s\beta_s^2/I$, then A is neither positive definite nor negative definite, since the profit function is continuous and bounded, and the stationary point of the profit function is unique, then the optimal solution is arrived when x is at its boundary, thus we compare the optimal profit of boundary solutions.

Solving (4) (5) and (6) given $x = 1$, by maximising the profit function, the optimal solutions with the maximal investment can be derived as: $n_b^* = [2 + M_s\beta_s(\alpha_b + \alpha_s)]M_b/I$, $n_s^* = [(\alpha_b + \alpha_s)M_b + 2\beta_s]M_s/I$; $p_b^* = [2 - \alpha_s(\alpha_b + \alpha_s)M_bM_s + (\alpha_b - \alpha_s)\beta_sM_s]/I$, and $p_s^* = \{(\alpha_s - \alpha_b)M_b + \beta_s[2 - \alpha_b(\alpha_b + \alpha_s)M_bM_s]\}/I$, where $I = 4 - (\alpha_b + \alpha_s)^2M_bM_s$.

Solving (4)–(6) given $x = 0$, by maximising the profit function, the optimal solutions with no investment can be derived as: $n_s^* = (\alpha_b + \alpha_s)M_bM_s/I$, $n_b^* = 2M_b/I$, $p_b^* = [2 - \alpha_s(\alpha_b + \alpha_s)M_bM_s]/I$, $p_s^* = (\alpha_s - \alpha_b)M_b/I$.

When $\phi = 2M_s\beta_s^2/I$, the optimal profit at $x = 1$ comes to its minimal, based on the optimal solutions when $x = 0$ and $x = 1$ given above, the minimal gap between the optimal profits when $x = 1$ and $x = 0$ can be calculated as: $\prod_{x=1}^* - \prod_{x=0}^* = \frac{(\alpha_b+\alpha_s)M_sM_b\beta_s}{[4-(\alpha_b+\alpha_s)^2M_bM_s]}$, which is positive. Therefore, $x^* = 1$ if $\phi \le 2M_s\beta_s^2/I$.

Synthetically, let $\bar{\phi} = [(\alpha_b + \alpha_s)M_b + 2\beta_s]\beta_sM_s/I$, then $x^* = 1$ if $\phi \le \bar{\phi}$; $x^* = (\alpha_b + \alpha_s)\beta_sM_sM_b/H$ if $\phi > \bar{\phi}$.

Proof of Proposition 1

(1) From the optimal pricing for sellers given in Theorem 1, let $E = 2 - \alpha_s(\alpha_b + \alpha_s)M_bM_s$ and $F = 2 - \alpha_b(\alpha_b + \alpha_s)M_bM_s$, we have that:
   (a) When $\phi \le \bar{\phi}$, $p_s^* = [(\alpha_s - \alpha_b)M_b + \beta_sF]/I$, then:
      (i) If $\alpha_s \ge \alpha_b$, then $p_s^* > 0$ since $F > 0$;
      (ii) If $\alpha_s < \alpha_b$, then if $\beta_s \ge (\alpha_b - \alpha_s)M_b/F$, $p_s^* \ge 0$; otherwise, $p_s^* < 0$.
   (b) When $\phi > \bar{\phi}$, $p_s^* = \left[\phi(\alpha_s - \alpha_b) + \alpha_b\beta_s^2M_s\right]M_b/H$, then:
      (i) If $\alpha_s \ge \alpha_b$, $p_s^* > 0$;
      (ii) If $\alpha_s < \alpha_b$, then if $\phi \le \alpha_b\beta_s^2M_s/(\alpha_b - \alpha_s)$, $p_s^* \ge 0$; otherwise, $p_s^* < 0$.

(2) Form the optimal pricing for buyers given in Theorem 1, it can be derived that:
   (a) When $\phi \le \bar{\phi}$, $p_b^* = [E + (\alpha_b - \alpha_s)M_s\beta_s]/I$, then:
      (i) If $\alpha_b \ge \alpha_s$, then $p_b^* > 0$ since $E > 0$;
      (ii) If $\alpha_b < \alpha_s$, then $p_b^* \ge 0$ if $\beta_s \le E/(\alpha_s - \alpha_b)M_s$; otherwise, $p_b^* < 0$.
   (b) When $\phi > \bar{\phi}$, $p_b^* = \left\{\phi[2 - \alpha_s(\alpha_b + \alpha_s)M_sM_b] - \beta_s^2M_s\right\}/H$, then:
      (i) If $\phi \ge \beta_s^2M_s/E$, $p_b^* \ge 0$;
      (ii) If $\phi < \beta_s^2M_s/E$, $p_b^* < 0$.

For the two thresholds $\alpha_b\beta_s^2M_s/(\alpha_b - \alpha_s)$ and $\beta_s^2M_s/E$, we can derive that:
$\alpha_b/(\alpha_b - \alpha_s) - 1/E = (\alpha_b + \alpha_s)(1 - \alpha_b\alpha_sM_bM_s)/E(\alpha_b - \alpha_s)$.
Since α_i, $M_i \in (0, 1)$, $i = b, s$, thus $\alpha_b\beta_s^2M_s/(\alpha_b - \alpha_s) > \beta_s^2M_s/E$, then we can conclude that when $\phi > \bar{\phi}$:

(1) If $\bar{\phi} < \phi < \beta_s^2M_s/E$, then $p_s^* \ge 0$, $p_b^* < 0$;
(2) If $\beta_s^2M_s/E \le \phi \le \alpha_b\beta_s^2M_s/(\alpha_b - \alpha_s)$, then $p_s^* \ge 0$, $p_b^* \ge 0$;
(3) If $\phi > \alpha_b\beta_s^2M_s/(\alpha_b - \alpha_s)$, then $p_s^* < 0$, $p_b^* \ge 0$.

Proof of Corollary 1

(1) $E/(\alpha_s - \alpha_b)M_s = 2/(\alpha_s - \alpha_b)M_s - \alpha_s(\alpha_b + \alpha_s)M_b/(\alpha_s - \alpha_b)$, thus when $\alpha_s > \alpha_b$ $E/(\alpha_s - \alpha_b)M_s$ is decreasing in M_i.
(2) $(\alpha_b - \alpha_s)M_b/F = (\alpha_b - \alpha_s)M_b/[2 - \alpha_b(\alpha_b + \alpha_s)M_bM_s]$, it's easy to find that $(\alpha_b - \alpha_s)M_b/F$ is increasing in M_s. $\partial[(\alpha_b - \alpha_s)M_b/F]/\partial M_b = 2(\alpha_b - \alpha_s)/F^2$, so when $\alpha_b > \alpha_s$, $(\alpha_b - \alpha_s)M_b/F$ is increasing in M_b.
(3) $\beta_s^2M_s/E = \beta_s^2M_s/[2 - \alpha_s(\alpha_b + \alpha_s)M_bM_s]$, then it's easy to find that $\partial\left(\beta_s^2M_s/E\right)/\partial M_b > 0$. Besides, $\partial\left(\beta_s^2M_s/E\right)/\partial M_s = 2\beta_s^2/E^2 > 0$, thus the threshold $\beta_s^2M_s/E$ is increasing in M_i, $i = b, s$.

Poof of Proposition 2

In Theorem 1, when $\phi > \bar{\phi}$, $p_s^* = \left[\phi(\alpha_s - \alpha_b) + \alpha_b\beta_s^2M_s\right]M_b/H$, $p_b^* = \left(\phi E - \beta_s^2M_s\right)/H$, then we have the first partial derivatives that, $\partial p_s^*/\partial\beta_s = 2\phi\beta_sM_bM_s(\alpha_b + \alpha_s)F/H^2$, $\partial p_b^*/\partial\beta_s = 2\phi(\alpha_b - \alpha_s)(\alpha_b + \alpha_s)\beta_sM_bM_s^2/H^2$, $\partial p_s^*/\partial\phi = -(\alpha_b + \alpha_s)\beta_s^2M_bM_sF/H^2$ and $\partial p_b^*/\partial\phi = (\alpha_s - \alpha_b)(\alpha_b + \alpha_s)\beta_s^2M_bM_s^2/H^2$.

When $\phi \le \bar{\phi}$, $p_s^* = [(\alpha_s - \alpha_b)M_b + \beta_sF]/I$ and $p_b^* = [E + (\alpha_b - \alpha_s)M_s\beta_s]/I$, then we have the first partial derivatives that $\partial p_s^*/\partial\beta_s = F/I$ and $\partial p_b^*/\partial\beta_s = (\alpha_b - \alpha_s)M_s/I$. Therefore:

(1) when $\phi > \bar{\phi}$, $\partial p_s^*/\partial\phi < 0$; $\partial p_b^*/\partial\phi \ge 0$ if $\alpha_s \ge \alpha_b$ and $\partial p_b^*/\partial\phi < 0$ if $\alpha_s < \alpha_b$;
(2) If $\alpha_b > \alpha_s$, then $\partial p_b^*/\partial\beta_s > 0$; if $\alpha_b \le \alpha_s$ then $\partial p_b^*/\partial\beta_s \le 0$.
(3) $\partial p_s^*/\partial\beta_s > 0$.

Proof of Proposition 3

When investing for sellers, from Theorem 1, we have that:

When $\phi \le \bar{\phi}$, $\begin{cases} \partial p_s^*/\partial M_b & = 2(\alpha_s - \alpha_b)[2 + (\alpha_b + \alpha_s)M_s\beta_s]/I^2 \\ \partial p_s^*/\partial M_s & = M_b(\alpha_s^2 - \alpha_b^2)[2\beta_s + (\alpha_b + \alpha_s)M_b]/I^2 \end{cases}$ and $\begin{cases} \partial p_b^*/\partial M_s & = 2(\alpha_b - \alpha_s)[2\beta_s + (\alpha_b + \alpha_s)M_b]/I^2 \\ \partial p_b^*/\partial M_b & = (\alpha_b - \alpha_s)\left[2(\alpha_b + \alpha_s)M_s + M_s\beta_s(\alpha_b + \alpha_s)^2 M_s\right]/I^2 \end{cases}$.

When $\phi > \bar{\phi}$, $\begin{cases} \partial p_s^*/\partial M_b & = 2(2\phi - \beta_s^2 M_s)\left[\phi(\alpha_s - \alpha_b) + \alpha_b\beta_s^2 M_s\right]/H^2 \\ \partial p_s^*/\partial M_s & = \phi(\alpha_b + \alpha_s)M_b\left[2\beta_s^2 + \phi(\alpha_s - \alpha_b)(\alpha_b + \alpha_s)M_b\right]/H^2 \end{cases}$ and $\begin{cases} \partial p_b^*/\partial M_s & = 2\phi^2 M_b(\alpha_b^2 - \alpha_s^2)/H^2 \\ \partial p_b^*/\partial M_b & = \phi(\alpha_b^2 - \alpha_s^2)(\alpha_b + \alpha_s)M_s(2\phi - \beta_s^2 M_s)/H^2 \end{cases}$.

Since when $\phi > \bar{\phi}$, $2\phi > \beta_s^2 M_s$, then:

(1) If $\alpha_s > \alpha_b$, then $\partial p_b^*/\partial M_i < 0$ and $\partial p_s^*/\partial M_i > 0$;
(2) If $\alpha_s = \alpha_b$, then $\partial p_b^*/\partial M_i = 0$ and $\partial p_s^*/\partial M_i > 0$ if $\phi > \bar{\phi}$, $\partial p_s^*/\partial M_i = 0$ if $\phi \le \bar{\phi}$;
(3) If $\alpha_s < \alpha_b$, then $\partial p_b^*/\partial M_i > 0$, when $\phi \le \bar{\phi}$, $\partial p_s^*/\partial M_i < 0$; and when $\phi > \bar{\phi}$:
  (a) If $\phi \le \alpha_b\beta_s^2 M_s/(\alpha_b - \alpha_s)$, then $\partial p_s^*/\partial M_b \ge 0$; otherwise, $\partial p_s^*/\partial M_b < 0$;
  (b) If $\phi \le 2\beta_s^2/\left[(\alpha_b^2 - \alpha_s^2)M_b\right]$, then $\partial p_s^*/\partial M_s \ge 0$; otherwise, $\partial p_s^*/\partial M_s < 0$.

Proof of Proposition 4

In Theorem 2, when $\varphi > \bar{\varphi}$, $p_b^* = \varphi E/G$, $p_s^* = \varphi(\alpha_s - \alpha_b)M_b/G$, where $E = 2 - \alpha_s(\alpha_b + \alpha_s)M_b M_s$, $G = \varphi I - 2\beta_b^2 M_b$, then we have the first partial derivatives that: $\partial p_s^*/\partial\varphi = 2(\alpha_b - \alpha_s)M_b\beta_b^2 M_b/G^2$ and $\partial p_b^*/\partial\varphi = -2\beta_b^2 M_b E/G^2$.

When $\varphi \le \bar{\varphi}$, $p_s^* = (\alpha_s - \alpha_b)(1 + \beta_b)M_b/I$ and $p_b^* = E(1 + \beta_b)/I$, then we have that:

(1) For all φ, $\partial p_b^*/\partial\beta_b > 0$ and $\partial p_i^*/\partial\varphi = 0$, $i = b, s$;
(2) when $\varphi > \bar{\varphi}$, $\partial p_b^*/\partial\varphi < 0$; $\partial p_s^*/\partial\varphi \ge 0$ if $\alpha_b \ge \alpha_s$ and $\partial p_s^*/\partial\varphi < 0$ if $\alpha_b < \alpha_s$;
(3) when $\varphi > \bar{\varphi}$, $\partial p_s^*/\partial\beta_b > 0$; when $\varphi \le \bar{\varphi}$, $\partial p_s^*/\partial\beta_b > 0$ if $\alpha_b < \alpha_s$ and $\partial p_s^*/\partial\beta_b \le 0$ if $\alpha_b \ge \alpha_s$.

Proof of Proposition 5

From Theorem 1 and Theorem 2, the optimal profit in two scenarios are obtained as: $\pi_1^* = (2\phi M_b - \beta_s^2 M_s M_b)/2H$, $\phi > \bar{\phi}$ and $\pi_2^* = \varphi M_b/G$, $\varphi > \bar{\varphi}$. Therefore, when $\phi = \varphi > \max[\bar{\phi}, \bar{\varphi}]$, we have that:

(1) If $\beta_s \le \beta_b$, then $\pi_1^* < \pi_2^*$;
(2) If $\beta_s > \beta_b$, then $\pi_1^* - \pi_2^* = \left\{\varphi\left[\beta_s^2 M_s^2(\alpha_b + \alpha_s)^2 - 4\beta_b^2\right] + 2\beta_b^2\beta_s^2 M_s\right\}M_b^2/2GH$, thus:
  (a) If $\beta_b < \beta_s \le 2\beta_b/M_s(\alpha_b + \alpha_s)$, then when $\max[\bar{\phi}, \bar{\varphi}] < \phi < K$, $\pi_1^* > \pi_2^*$, otherwise, $\pi_1^* < \pi_2^*$, where $K = 2\beta_b^2\beta_s^2 M_s/\left[4\beta_b^2 - \beta_s^2 M_s^2(\alpha_b + \alpha_s)^2\right]$;
  (b) If $\beta_s > 2\beta_b/M_s(\alpha_b + \alpha_s)$, then $\pi_1^* > \pi_2^*$.

Proof of Proposition 6

Denoted by Δp_b^*, Δp_s^* the optimal pricing changes for buyers and sellers compared with no VAS investment that $x = y = 0$, then from the proof of Theorem 1, we have that:

(1) When investing for sellers, then:
  (a) If $\phi \le \bar{\phi}$, $\begin{cases} \Delta_1 p_b^* & = (\alpha_b - \alpha_s)\beta_s M_s/I \\ \Delta_1 p_s^* & = \beta_s F/I \end{cases}$;
  (b) If $\phi > \bar{\phi}$, $\begin{cases} \Delta_2 p_b^* & = (\alpha_b - \alpha_s)(\alpha_b + \alpha_s)\beta_s^2 M_s M_b M_s/HI \\ \Delta_2 p_s^* & = (\alpha_b + \alpha_s)M_b\beta_s^2 M_s F/HI \end{cases}$.

Therefore, $\Delta p_s^* > 0$; $\Delta p_b^* > 0$ if $\alpha_b > \alpha_s$ and $\Delta p_b^* < 0$ if $\alpha_b < \alpha_s$.

(2) When investing for buyers, then:

(a) If $\varphi \leq \bar{\varphi}$, $\begin{cases} \Delta_1 p_b^* & = \beta_b E/I \\ \Delta_1 p_s^* & = \beta_b M_b(\alpha_s - \alpha_b)/I \end{cases}$;

(b) If $\varphi > \bar{\varphi}$, $\begin{cases} \Delta_2 p_b^* & = 2\beta_b^2 M_b E/GI \\ \Delta_2 p_s^* & = 2(\alpha_s - \alpha_b)M_b \beta_b^2 M_b/GI \end{cases}$.

Therefore, $\Delta p_b^* > 0$; $\Delta p_s^* > 0$ if $\alpha_s > \alpha_b$ and $\Delta p_s^* < 0$ if $\alpha_s < \alpha_b$.

Proof of Proposition 7

From the proof of Proposition 6, when Δp_s^* and Δp_b^* are both positive, the gap between the pricing change on the two sides in Scenario 1 and Scenario 2 can be respectively given as:

(1) $\begin{cases} \Delta_1 p_b^* - \Delta_1 p_s^* & = \beta_s[(\alpha_b - \alpha_s)M_s - F]/I \\ \Delta_2 p_b^* - \Delta_2 p_s^* & = [(\alpha_b - \alpha_s)M_s - F](\alpha_b + \alpha_s)\beta_s^2 M_b M_s/HI \end{cases}$,

(2) $\begin{cases} \Delta_1 p_s^* - \Delta_1 p_b^* & = \beta_b[(\alpha_s - \alpha_b)M_b - E]/I \\ \Delta_2 p_s^* - \Delta_2 p_b^* & = 2[(\alpha_s - \alpha_b)M_b - E]M_b \beta_b^2/GI \end{cases}$.

Therefore:

(1) When investing for sellers, if $\alpha_b > F/M_s + \alpha_s$, then $(\alpha_b - \alpha_s)M_s > F$, then $\Delta p_b^* > \Delta p_s^*$; otherwise, $\Delta p_b^* \leq \Delta p_s^*$;
(2) When investing for buyers, if $\alpha_s > E/M_b + \alpha_b$, then $(\alpha_s - \alpha_b)M_b > E$, then $\Delta p_s^* > \Delta p_b^*$; otherwise, $\Delta p_s^* \leq \Delta p_b^*$.