Journal Pre-proof

Pricing and Product-bundling Strategies for E-commerce Platforms with Competition

Xiaogang Lin, Yong-Wu Zhou, Wei Xie, Yuanguang Zhong, Bin Cao

PII: S0377-2217(19)30978-6
DOI: https://doi.org/10.1016/j.ejor.2019.11.066
Reference: EOR 16202

To appear in: *European Journal of Operational Research*

Received date: 2 December 2018
Accepted date: 28 November 2019

Please cite this article as: Xiaogang Lin, Yong-Wu Zhou, Wei Xie, Yuanguang Zhong, Bin Cao, Pricing and Product-bundling Strategies for E-commerce Platforms with Competition, *European Journal of Operational Research* (2019), doi: https://doi.org/10.1016/j.ejor.2019.11.066

Highlights

- We consider pricing and product-bundling strategies for competing platforms.
- We investigate the impacts of installed base, mixed bundling, and competition.
- The platforms with installed base and competition will charge the same fees.
- The mixed product-bundling strategy is a dominant equilibrium solution.
- We also provide insights on the value of the mixed-bundling strategy.

Pricing and Product-bundling Strategies for E-commerce Platforms with Competition

Xiaogang Lin[1], Yong-Wu Zhou[2], Wei Xie[2], Yuanguang Zhong[2], Bin Cao[2*]

[1]School of Management, Guangdong University of Technology,
Guangzhou 510520, China.

[2]School of Business Administration, South China University of Technology,
Guangzhou 510640, China

December 3, 2019

Abstract

We consider pricing and product-bundling strategies for two competing platforms with two groups of agents, customers and sellers (independent content developers). In this paper, two such groups are required to pay the platforms fixed (subscription) fees to gain access; each platform also produces its own integrated content. We investigate the impacts of installed base, mixed bundling, and competition on the platforms' pricing and product-bundling strategies. We find that in the presence of both installed base and competition, each platform will charge sellers the same fee, regardless of whether the unbundling or mixed-bundling strategy is chosen. However, if the mixed-bundling strategy is used, the prices charged to customers may be higher relative to the unbundling equilibrium. Importantly, our results reveal that the mixed product-bundling strategy can be used as a strategic competitive tool for the competing platforms to seize more market share and induce the platforms to subsidize customers who buy the bundle by charging customers who only access the platforms (without bundling)

*Corresponding author: bmbinc@scut.edu.cn.

a higher price. Moreover, we find that the proposed mixed product-bundling strategy is a dominant equilibrium solution for two competing platforms. We also show that under certain conditions, the mixed-bundling strategy can always bring larger value for both platforms if the intrinsic value of the integrated content (the fractions of the installed base) becomes smaller (higher) relative to the unbundling strategy. These results offer valuable implications for relevant practices.

Keywords: OR in marketing; competing platforms; installed base; pricing and product-bundling strategies; two-sided market

1 Introduction

Two-sided markets are economic platforms with two distinct user groups that provide each other with benefits.[1] Example markets include video-game consoles (gamers and game developers), search engines (advertisers and users), operating systems (end-users and developers), online marketplaces (buyers and sellers), etc. In these two-sided markets, the benefits to each group exhibit demand economics of scale. For example, gamers prefer video games provided by more developers, while developers prefer video-game consoles owned by more gamers. In reality, two-sided markets are employed by many well-known companies, such as Microsoft (operating systems and game consoles), Nintendo (game consoles), Taobao (a marketplace in China), Baidu (a search engine in China), and others. These companies mainly contain two categories: One category only offers transaction sites, such as Taobao, Baidu, and Google; another category provides products that act as transaction sites, such as Microsoft and Nintendo. A common business strategy in traditional one-sided markets, called *mixed bundling*, is typically used to increase profits for these firms. Under this strategy, customers can either purchase the goods together as a bundle or buy the goods separately.

Nowadays, the mixed-bundling strategy is also commonly adopted in two-sided markets (i.e, platforms). Microsoft, for example, produces Xbox game consoles and integrated video games. Customers need an Xbox console to play console games developed by Microsoft and other content developers. In practice, Microsoft may sell the hardware (Xbox) and

[1] See https://en.wikipedia.org/wiki/two-sidedmarket.

integrated content (video game) both separately and jointly.[2] Sony employs this strategy, but sometimes uses an unbundling strategy.[3] In other words, both Microsoft and Sony can price the hardware (Xbox and WiiU) and the integrated content separately, they can implement the mixed-bundling strategy for the hardware and integrated content. Meanwhile, in practice, two such platforms often compete for two-sided market share (to capture more profit). Motivated by these practices, therefore, the following research questions arise and will be addressed in this paper. Given the competition and installed base, what types of pricing strategies should the two platforms implement? Further, does mixed bundling always affect the proposed pricing strategies? What roles do the mixed-bundling strategy play for the two competing platforms? Which strategies should the two platforms apply at the equilibrium? Finally, how do key parameters, such as the intrinsic values of platforms and corresponding integrated content, installed base, and network externalities, affect the value of the mixed-bundling strategy?

To answer the above questions, in this paper we investigate two competing platforms whose customers are single-homing while sellers are multi-homing. That is, customers can only choose one platform, and sellers wish to join both platforms to achieve maximal network benefits. We assume that there exists an effect of cross-network externality. Then, customer demand for each platform and its integrated content and the number of sellers that will join each platform are characterized. We further analyze the pricing structures under three configurations: (a) both platforms adopt the unbundling strategy, (b) both platforms use the mixed-bundling strategy, and (c) one of the platforms applies the mixed-bundling strategy and the other uses the unbundling strategy.

Our results show that increasing the utility of the integrated content (or network externalities) can induce platforms using the unbundling strategy to reduce their prices and attract more customers who buy access, but charge customers who buy the integrated content a higher price to raise profits. Moreover, whether the access price charged by the unbundling platform decreases or increases in the installed base closely depends on network externalities. As a result, the impact of the customer-side network externality on the platform's profit is also associated with the relative magnitude of network externalities. Interestingly, when the

[2]For example, Microsoft mixed-bundles the Xbox One X (hardware) with Final Fantasy 15 (game). See https://www.microsoftstore.com.cn/xbox/xbox-one/p/mic1712 (accessed Nov. 27, 2018).

[3]These firms price the hardware and the integrated content separately.

seller-side network externality increases, the platform captures more profit. For the effect of the new potential customers' intrinsic value, the opposite result is obtained. We also find that these results apply for the case in which the mixed-bundling strategy is used by both platforms. An exception is that increasing the installed base will bring more profit for the platforms by increasing their bundling prices. In case one platform chooses the unbundled strategy and the other adopts the mixed-bundled strategy, it is interesting to observe that the individual pricing strategy only depends on its own bundling policy, but not the rival's bundling strategy in the symmetric equilibrium. In addition, most of the results obtained in the first two configurations qualitatively remain true in this case.

We also compare the equilibrium prices for the three configurations. The results show that regardless of whether the unbundling or mixed-bundling strategy is selected, each platform will charge the same fee on the seller side. That is, the bundling strategy does not influence the fee charged to sellers. In addition, compared with the unbundling strategy, each platform with mixed bundling will charge customers who only buy access to the platform a higher price. Further, this result is also not affected by the other platform's bundling strategy. This implies that the platforms subsidize customers who buy the bundle and obtain compensation from customers who only want to access to the platforms by charging a higher price. In this regard, our results are in sharp contrast to those found in the literature (e.g., Chao and Derdenger, 2013). Moreover, we extend our model to the case of two platforms competing against each other on the bundling strategy. Our finding implies that if one platform unbundles its integrated content, the other platform can use mixed bundling as a competitive strategy to seize market share and increase its profits. Consequently, the mixed-bundling strategy is preferred by the two competing platforms, which is consistent with practice. Finally, we show that under certain conditions (e.g., the intrinsic value of the integrated content or the fraction of the installed base is relatively small), the mixed-bundling strategy can always bring higher value for both platforms if the intrinsic value of the integrated content (the fractions of the installed base) becomes smaller (higher) relative to the case of the unbundling strategy.

The remainder of the paper is organized as follows. Section 2 reviews prior research in related areas. In Section 3, we introduce the notation, assumptions, and problem formulation. To identify the impacts of mixed bundling and competition on the two-sided market, Section 4 analyzes three models: both platforms are unbundled, both platforms are mixed bundled, and one platform is mixed bundled and the other is unbundled. Section 5 discusses the comparison and sensitivity analyses of equilibrium prices for the three scenarios, an extension that considers the bundling game between the two platforms, and the impacts of the key parameters on the value of the mixed-bundling strategy. Section 6 concludes. All proofs are relegated to the Appendix.

2 Literature review

Our work is closely related to the research strand that examines pricing strategies for bundling in traditional one-sided markets. Early studies (e.g., Stigler, 1963; Adams and Yellen, 1976; Schmalensee, 1984; Pierce and Winter, 1996) focus on price discrimination in bundling, and demonstrate that a pure-bundling or mixed-bundling strategy can increase profits even in the absence of cost advantage to bundling. This price discrimination of bundling has been a fundamental pricing assumption of many subsequent papers associated with different research questions. A large number of studies contribute to this strand, such as Nalebuff (2004); Peitz (2008); Bhargava (2013); and Prasad et al. (2015). Recently, Banciu and Ødegaard (2016) innovatively study pricing of bundling when the underlying valuations of the bundle components are dependent. Importantly, they show that the price of independence can significantly affect the seller's profitability. Moreover, using a simulation model of pure components, pure bundling and mixed bundling, Kopczewski et al. (2018) investigate the impacts of three key elements on the three pricing strategies. Notably, the problems considered in these studies using price discrimination to block entry ignore the impact of network externality. However, network externality is a vital element that influences bundling strategies, which is verified by the literature (e.g., Prasad and Mahajan, 2010; Luo et al., 2017; Xu et al., 2018).

In addition, some papers investigate the bundling problem in a supply chain without the network externality described above. For example, Chakravarty et al. (2013) consider the problem of one retailer and multiple suppliers seeking to decide whether to bundle their products in a decentralized supply chain, and show that bundling can be profitable if the reservation prices are uniformly distributed under certain conditions. Further, Giri et al.

(2017) consider a supply chain with two manufacturers selling two complementary products through a common retailer who decides whether to employ the pure-bundling strategy. They confirm that the pure-bundling strategy is better than unbundling from the perspective of supply chain profitability. Note that these studies are based on a monopoly setting. In contrast, some studies of bundling strategy in duopoly markets, such as Vamosiu (2018a,b), analyze the profitability of bundling for a duopolist with and without imperfect competition. Bhattacharya et al. (2019) compare installed base management with selling with a separate maintenance contract in which the manufacturer bundles repair and maintenance services along with the product. For a comprehensive review of bundling-related literature, readers are referred to Venkatesh and Mahajan (2009).

Different from the aforementioned studies, in this paper we identify the mixed-bundling strategy in two-sided markets in which the installed base effects are considered. In particular, we focus on the relative price effects of mixed bundling with unbundling action as the baseline measure, and optimize the mixed-bundling strategies of two competing platforms.

Another stream of research related to our paper studies pricing strategies for bundling in two-sided markets. Two-sided markets have been studied widely in economics (e.g., Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Parker and Alstyne, 2005; Armstrong, 2006; Halaburda and Yehezkel, 2013). Several papers consider the problems of tying and bundling in two-sided markets. For the former substream, Choi (2010) investigates how tying affects two-sided market competition when customers are multi-homing. Amelio and Jullien (2012) consider a platform on which a negative price on one side of the market is set with preference and opportunistic risk, and find that tying can be used as an implicit subsidy and to avoid undesirable customers. Using a similar market, Choi and Jeon (2016) analyze incentives for a monopolist to tie and identify that tying is profitable under certain conditions. In addition, for the latter case, Chao and Derdenger (2013) consider a monopoly two-sided market where mixed-bundling and installed base effects are analyzed together. They describe a pricing structure that differs from both traditional bundling and standard two-sided markets. (For more details on the main differences between this study and ours, see Section 3.2.) Chen et al. (2016) use a stylized model to investigate the incentives to bundle and its impact on welfare within a two-sided market framework. By studying a monopoly platform's incentive to bundle, Gao (2018) demonstrates that its optimal pricing strategy is determined by simple formulas using familiar price elasticities of demand. Sun (2018) studies the bundling strategies of two platforms in which each platform sells a horizontally differentiated product and a complementary good. Unlike these studies, this paper's focus is the subscription fee and pricing problem of mixed bundling, which allows each platform to sell the right to access and its own content individually or as a bundle. Furthermore, we examine how competition in two-sided markets affects the pricing and product-bundling strategies of e-commerce platforms.

3 Notation, Assumption, and Problem Formulation

Consider two platforms that compete for two groups of agents: customers and sellers (content developers). For convenience, we first summarize the notation adopted in this paper in Table 1. In general, agents can obtain benefits from interactions on the platforms. We assume that cross-network externality exists, i.e., the benefits obtained by one of the groups depend on the size of the other group on the same platform. To focus on our research questions, this paper mainly considers the competitive bottleneck model, in which group c the customers only join one of the platforms (i.e., single-homing), while group s the sellers can join both platforms (i.e., multihoming). In practice, the competitive bottleneck structure is common. One example is the smartphone market, in which all customers are single-homing (they only use one smartphone) and almost all content developers are multi-homing (the apps are available on all types of smartphones). Another is internet trading platforms on which sellers post the same offer on different platforms, while buyers only purchase the product from one platform. Note that we can simply exclude the outside option (the no-purchase option), because this will provide similar analysis and results (the proof is similar, and thus we omit it). Moreover, we assume that customers only join one platform. This is commonly adopted in the literature related to price competition between two platforms (e.g., Armstrong, 2006; Reisinger, 2014). The objectives of the two platforms (denoted by 1 and 2) are to maximize profits at their own individual level.

In this paper, the two platforms also produce integrated content by themselves. To reduce analysis of the complexity, we adopt a simplified yet reasonable assumption that platform i ($i = 1, 2$) only produces one piece of integrated content, and the costs of developing its

Table 1: Notation

Notation	Description
v^i, v^i_g	Intrinsic values of new potential customers for platform i and its integrated content, respectively
α^i	Fraction of the installed base of platform i
a_c, a_s	Marginal benefits each customer and seller who join platform i receive from an additional seller and customer on the platform, respectively
p^i_c, p^i_g	Prices charged to new potential customers for access to platform i and the corresponding content, respectively
p^i_s	Price charged to sellers for developing and selling compatible content for platform i
n^i_o	Number of customers who join platform i but do not purchase its integrated content
n^i_{cb}	Number of customers who both join platform i and purchase its integrated content
n^i_c, n^i_s	Total numbers of customers and sellers who join platform i, respectively
M1, M2, M3	Scenarios in which both platforms use an unbundled strategy; and a mixed-bundle strategy; and platform i adopts a mixed-bundle strategy while platform j uses an unbundled one, respectively
$p^i_{1,c}$, $p^i_{1,g}$, $p^i_{1,s}$ ($p^j_{3,c}$, $p^j_{3,g}$, $p^j_{3,s}$)	Prices charged to new potential customers for access to platform i (j) and the integrated content, and charged to sellers for joining platform i (j) under M1 (M3) scenario, respectively
$n^i_{1,c}$, $n^i_{1,cb}$, $n^i_{1,s}$ ($n^j_{3,c}$, $n^j_{3,cb}$, $n^j_{3,s}$)	Total numbers of customers who join platform i (j) and purchase its integrated content, and sellers who join platform i (j) under M1 (M3) scenario, respectively
$P^i_{2,c}$, $P^i_{2,g}$, $P^i_{2,s}$ ($P^i_{3,c}$, $P^i_{3,g}$, $P^i_{3,s}$)	Prices charged to new potential customers for the access of platform i, and the integrated content, and charged to sellers for joining platform i under M2 (M3) scenario, respectively
$P^i_{2,ge}$, $P^i_{2,B}$ ($P^i_{3,ge}$, $P^i_{3,B}$)	Price charged to new potential customers for the bundle of platform i, and the effective price for the integrated content under M2 (M3) scenario
$N^i_{2,c}$, $N^i_{2,cb}$, $N^i_{2,s}$ ($N^i_{3,c}$, $N^i_{3,cb}$, $N^i_{3,s}$)	Total numbers of customers who join platform i, and purchase the bundle, and sellers who join platform i under M2 (M3) scenario, respectively
Π^i_{M1}, Π^i_{M2}, Π^i_{M3}	Profits of platform i under M1, M2 and M3 scenarios, respectively

platform and content can be ignored. Suppose platform i interacts with the two agents (customers and sellers) by charging price p_c^i to customers for platform access and p_g^i for the corresponding integrated content, while charging a fixed fee p_s^i to sellers to develop and sell compatible contents on the platform. In addition to the unbundled prices p_c^i and p_g^i, platform i can set a price P_B^i for the bundle. Throughout this paper, we use lowercase letters to denote related variables under unbundling and uppercase letters to denote related variables under bundling.

3.1 Customer Side

To derive customer demand for platform i and its integrated content, we consider three groups of customers (the total size is normalized to one).[4] Groups 1 and 2 are the installed bases of platforms 1 and 2, respectively (with fraction α^i, $i = 1, 2$). The remaining customers (new potential customers) with fraction $1-(\alpha^1+\alpha^2)$ are classified into Group 3. The installed bases are the customers who have already purchased access to the platform but not bought the integrated content, whereas the new potential customers are those who will decide which platform to purchase and its corresponding integrated content.[5] Note that customers who purchase the console of platform i can be divided into two groups: customers who purchase the console of platform i but do not buy the integrated content, and customers who purchase both the console and its integrated content.

We first derive demand from Group i for the integrated content of platform i. Denote by $U^i_{installed}$ the gross utility a Group i ($i = 1, 2$) customer obtains from buying the integrated content; then the customers of installed base i will purchase the integrated content from platform i if and only if $U^i_{installed} - p_g^i \geq 0$. Similar to Chao and Derdenger (2013), we assume that customers from Group i can obtain the highest gross utility from buying the integrated content, i.e., $U^i_{installed} = 1$.[6] Because $U^i_{installed} = 1$, one has $1 \geq p_g^i$ in the equilibrium. Thus,

[4] In the following, when we say that "customers join platform i", "customers purchase access to platform i", "customers purchase platform i", or make similar statements, we mean that customers purchase the console (e.g., smartphone or video console) of platform i.

[5] Chao and Derdenger (2013) offer realistic reasons why the installed base cannot be overlooked. A typical reason is that the integrated content may not be available before a considerable number of customers join the platforms.

[6] The reason is twofold. First, the installed base of platform i has already purchased access, and thereby

the demand for platform i's integrated content from Group i is just α^i.

Then, in order to derive demand from Group 3 for platform i and its integrated content, we adopt a widely used approach in discrete choice models (Anderson and de Palma, 1992; Roberts and Lilien, 1993). Specifically, we can treat the product bundles as attributes and characterize customer preferences for these attributes through the associated random parameters. The linear customer utility model, which is a common tool in economics (Caplin et al., 1991; Train, 2003; Hensher and Greene, 2003), is a good candidate. Let n_s^i be the number of sellers who will join platform i. Hence, the utility of a customer in Group 3 who only purchases access to platform i can be expressed as

$$\tilde{U}_o^i = V_o^i + \tilde{\varepsilon}_o^i = v^i + a_c n_s^i - p_c^i + \tilde{\varepsilon}_o^i, \ \ i = 1, 2,$$

and the utility of a customer in Group 3 who joins platform i and also buys the integrated content is given by

$$\tilde{U}_{cb}^i = V_{cb}^i + \tilde{\varepsilon}_{cb}^i = v^i + a_c n_s^i - p_c^i + v_g^i - p_g^i + \tilde{\varepsilon}_{cb}^i, \ \ i = 1, 2,$$

where v^i and v_g^i ($v^i, v_g^i \in [0, 1]$) are the new potential customer's intrinsic values for platform i and the integrated content; p_c^i and p_g^i are the prices charged to the new potential customer for access to platform i and its integrated content, respectively; $a_c \geq 0$ measures the benefit each customer obtains from the interaction with a seller; and, $\tilde{\varepsilon}_o^i$ and $\tilde{\varepsilon}_{cb}^i$ are random components that reflect unobserved effects in the model.

Note that the network externality on the consumer side is actually reflected in the term $a_c n_s^i$, since the above consumer utilities increase in the number of sellers who will join that platform i. The higher number of sellers will lead to higher demand for platform i and its integrated content. It is reasonable to assume that the customer is rational and seeks to purchase the product that can maximize his utility; that is,

$$\max_{i \in \{1,2\}} \{\tilde{U}_o^i, \tilde{U}_{cb}^i\}.$$

they are more likely to be "locked in" than new users because they need its integrated content to better use the platform. Second, the installed base of platform i can be viewed as loyal customers, because they brought access before the integrated content was available, which implies that they are more willing to pay for the integrated content.

For the sake of convenience, we assume that the noise terms $\tilde{\varepsilon}_o^i$ and $\tilde{\varepsilon}_{cb}^i$ follow uniform distribution $U[0,1]$ for any $i \in N$.[7] Note that since $E[\varepsilon_o^i] = E[\varepsilon_{cb}^i] = 1/2$, we can assume that both ε_o^i and ε_{cb}^i follow the uniform distribution $U[-1,1]$ to assure $E[\varepsilon_o^i] = E[\varepsilon_{cb}^i] = 0$. However, this will yield similar analysis and results (the proof is similar, and thus we omit it).

Let n_o^i be the number of customers that will join platform i, but not purchase its integrated content, and n_{cb}^i the number of customers will both join platform i and purchase the integrated content. Thus, following Natarajan et al. (2009) and Mishra et al. (2014), we have

$$\begin{aligned} n_o^i &= \frac{1 - \sum_{j=1}^{2} (V_o^j + V_{cb}^j)}{4} + V_o^i \\ &= \frac{1}{4} + \frac{2(v^i - v^j) - (v_g^i + v_g^j) + 2a_c(n_s^i - n_s^j) - 2(p_c^i - p_c^j) + (p_g^i + p_g^j)}{4}, \end{aligned} \tag{1}$$

and

$$\begin{aligned} n_{cb}^i &= \frac{1 - \sum_{j=1}^{2} (V_o^j + V_{cb}^j)}{4} + V_{cb}^i \\ &= \frac{1}{4} + \frac{2(v^i - v^j) + (3v_g^i - v_g^j) + 2a_c(n_s^i - n_s^j) - 2(p_c^i - p_c^j) - (3p_g^i - p_g^j)}{4}, \end{aligned} \tag{2}$$

where $i \neq j, i, j = 1, 2$. Denote by n_c^i the number of customers who purchase platform i (single-homing). Since n_c^i is the number n_o^i of customers who join platform i but do not purchase its integrated content plus the number n_{cb}^i of customers who both join platform i and purchase the integrated content, i.e., $n_c^i = n_o^i + n_{cb}^i$, we have

$$\begin{aligned} n_c^i &= n_o^i + n_{cb}^i \\ &= \frac{1}{2} + \frac{2(v^i - v^j) + (v_g^i - v_g^j) + 2a_c(n_s^i - n_s^j) - 2(p_c^i - p_c^j) - (p_g^i - p_g^j)}{2}, \end{aligned} \tag{3}$$

where $i \neq j, i, j = 1, 2$. With Equations (1), (2), and (3), we observe that n_o^i, n_{cb}^i, and n_c^i are directly affected by the fees charged by platform j because the customers are single-homing.

[7] In general, these noise terms are assumed to follow a Gumbel distribution. However, the problem is difficult, if not impossible, to solve and analyze.

3.2 Seller Side

As noted earlier, it is practical to assume that independent content developers (sellers)[8] wish to join both platforms (multi-homing) and make their content compatible with each. The utility of a seller who joins platform i is given by

$$U_s^i = a_s[\alpha^i + (1 - \alpha^1 - \alpha^2)n_c^i] - p_s^i - \theta,$$

where θ is the fixed cost of developing compatible content for platform i, which varies across sellers; p_s^i is the fixed fee charged to the seller for developing and selling compatible content on platform i; $a_s \geq 0$ represents the marginal benefit the seller receives from an additional customer on the platform; and $\alpha^i + (1 - \alpha^1 - \alpha^2)n_c^i$ is the total number of customers (including new potential customers and the installed base of platform i) who purchase access to platform i. Note that the network externality of the seller side derives from the term $a_s[\alpha^i + (1 - \alpha^1 - \alpha^2)n_c^i]$, since the above seller's utility increases in the total number of customers who join platform i. Correspondingly, sellers are heterogeneous, and each seller type can be summarized by the fixed cost θ of developing content for both platforms. The total number of potential content developers is normalized to one, and developers decide whether to enter the market and join the platform rather than setting their content prices.

In line with the literature (e.g., Armstrong, 2006; Jeitschko and Tremblay, 2014; Reisinger, 2014), we assume that θ is an independently and identically distributed random variable with a uniform distribution $U[0, 1]$. Moreover, following Armstrong (2006), multi-homing sellers will develop compatible content for platform i if and only if they can obtain non-negative utility, i.e., $U_s^i \geq 0$. In other words, a type θ seller will create and produce content for platform i if and only if $\theta \leq a_s[\alpha^i + (1 - \alpha^1 - \alpha^2)n_c^i] - p_s^i$. Therefore, we obtain the number of sellers who will join platform i as follows:

$$n_s^i = a_s[\alpha^i + (1 - \alpha^1 - \alpha^2)n_c^i] - p_s^i.$$

Because the sellers are multi-homing, n_s^i does not depend on the fixed fees charged by platform j. Then, with the arguments above, two implications can be formed for the independent content. First, potential customers may not need the integrated content, because they can enjoy platforms with independent content. Second, in this setting cross-network effects are

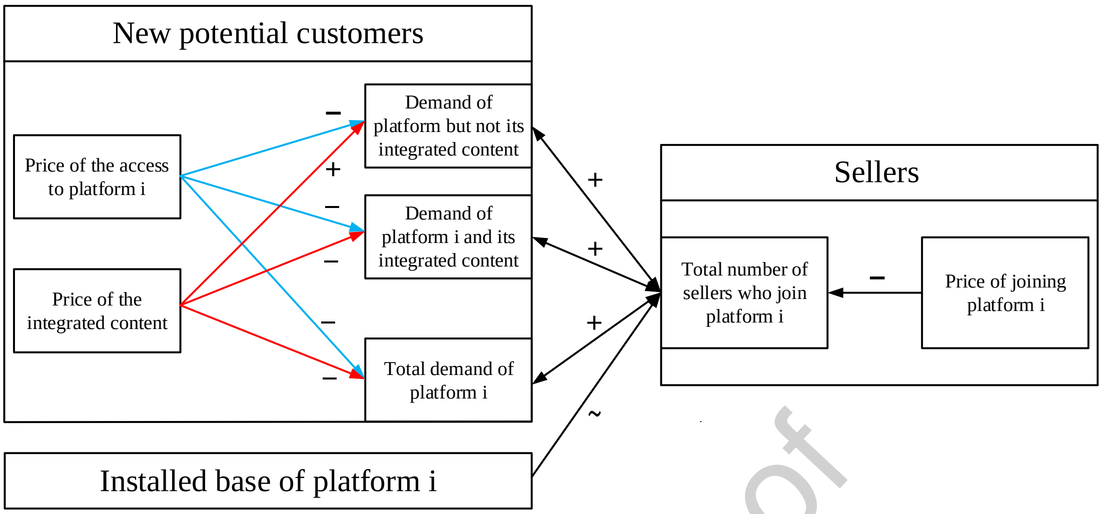
[8] In the subsequent analysis, we will use "content developer" and "seller" interchangeably.

present, since the number of sellers relies on the number of platform users $\alpha^i+(1-\alpha^1-\alpha^2)n_c^i$, and conversely, the amount of users hinges on the number of independent content. Consequently, the market structure is two-sided.

We note that the integrated content offered by platform i may *partially* compete with sellers' content and thereby influence their utility. Actually, when price p_g^i of platform i's integrated content increases, it follows from Equation (1) that more new potential (single-homing) customers who join platform i will only purchase access to platform i but not its integrated content, which can further improve sellers' utility via network externality a_s (the term U_s^i), and thus more sellers join the platform. To some extent, this captures the competition between integrated content and the seller's content, because demand for both access to platform i and its integrated content is reduced, which is shown in Equation (2). However, since the increased price of p_g^i can significantly reduce demand for both access and integrated content, the total number n_c^i of customers who purchase platform i decreases, as shown in Equation (3). As a result, the increased price of the integrated content reduces sellers' utility, and therefore fewer sellers join the platform. Overall, the network externality captures the relationship between integrated content and sellers' content, and its impact on the equilibrium will be discussed in the following.

We use Figure 1 to further illustrate the impacts of prices charged to new potential customers and sellers on the numbers of customers and sellers who join platform i and the multi-directional interactions among participants. Specifically, a higher price charged to new potential customers of platform i leads to lower numbers of customers who purchase access to the platform and its integrated content. As depicted in Figure 1, the demand of platform i positively depends on the number of sellers who join the platform, and vice versa. Furthermore, from the term $n_s^i = a_s[\alpha^i + (1-\alpha^1-\alpha^2)n_c^i] - p_s^i$, it follows that the numbers of sellers and thus customers (due to the network externality) who join the platform are not necessarily monotonic in the installed base of platform i, which may significantly affect the associated decisions of both platforms.

With the descriptions above, we next present the timing of the game as follows. First, two platforms will decide whether to implement mixed bundling or not and set the prices accordingly. There are three possible scenarios: both platforms use an unbundled strategy, both platforms use a mixed-bundle strategy, and platform i adopts a mixed-bundle strategy



Note: Arrows with "+", "−", and "∼" signs indicate that an increase in the factor at the origin leads to an increase, a decrease, or an uncertainty in the factor at the destination, respectively.

Figure 1: Impact of prices on the numbers of customers and sellers who join platform i.

but platform j uses an unbundled one ($i = 1, 2,\ j = 3 - i$). Next, after observing the prices offered by the two platforms, single-homing customers will join one of the platforms and decide whether to purchase the integrated content, and multi-homing sellers will decide whether to enter the market and join the platforms. In this section, for ease of exposition, the first, second, and third scenarios are denoted by Model 1 (M1), Model 2 (M2), and Model 3 (M3), respectively.

It is worth noting that although some settings and assumptions are similarly derived from Chao and Derdenger (2013), our model and research questions stand in sharp contrast to their work in several respects. First, a novel but practical linear customer utility model is used to obtain new potential customers' demand for platform access only and for both the platform access and its integrated content. Second, we consider a commonly observed competitive bottleneck model in practice–that is, customers are single-homing (they only use one platform based on their preferences), and independent content developers are multi-homing (they join both platforms if their utilities are non-negative). Finally, we study the pricing problem of mixed bundling in a duopolistic two-sided market given the intense competition between e-commerce platforms in practice, and mainly analyze the impact of competition on pricing and bundling strategies on the two platforms. Nevertheless, due to the intense competition between e-commerce platforms, which is lack of related analysis in the literature of studying bundling strategies, and the aforementioned differences between monopolistic and duopolistic settings, our competition model is not only a complement to, but also aa advancement of the monopolistic model adopted by Chao and Derdenger (2013).

4 Equilibrium Analysis

In this section, we analyze the equilibrium for each scenario (i.e., M1, M2 and M3). To guarantee the existence of equilibria and for the sake of convenience, we restrict our attention to symmetric platforms and thus assume $v^1 = v^2 = v$, $v_g^1 = v_g^2 = v_g$, and $\alpha^1 = \alpha^2 = \alpha$ in the subsequent analysis.[9] Moreover, we also need the assumption $4-(1-2\alpha)[(a_c+a_s)^2+4a_ca_s] > 0$ to ensure that the profit function of platform i $(= 1, 2)$ is concave, and thereby the equilibria exist. Finally, to avoid trivial cases (i.e., corner solutions), we assume $0 < \alpha \leq 1/4$, i.e., the installed bases are relatively small. Note that the first two assumptions are commonly adopted in traditional literature related to two-sided markets (see, e.g., Rochet and Tirole, 2003; Armstrong, 2006), and the third assumption is not restrictive because in practice few customers purchased access to the platforms before their integrated content become available.

4.1 Two platforms with unbundled strategy

In this scenario, platform i needs to determine price $p_{1,c}^i$ for platform access, price $p_{1,g}^i$ for its integrated content and fixed fee $p_{1,s}^i$ charged to sellers. As indicated in Section 3, demand for its integrated content from the installed base is just α. For new potential customers with fraction $(1-2\alpha)$, the number of customers who only purchase access to platform i is $(1-2\alpha)n_{1,c}^i$, and the number of customers who join platform i and purchase the integrated content is $(1-2\alpha)n_{1,cb}^i$, where

$$n_{1,c}^i = \frac{1}{2} + \frac{2a_c(n_{1,s}^i - n_{1,s}^j) - 2(p_{1,c}^i - p_{1,c}^j) - (p_{1,g}^i - p_{1,g}^j)}{2} \tag{4}$$

[9] This assumption is not essential; most of our main results remain valid if the assumption is relaxed. For brevity, we omit the detailed analysis because one can simply follow the same procedure used in this paper to get a similar analysis and result. It is available, however, from the authors upon request.

and

$$n_{1,cb}^i = \frac{1}{4} + \frac{2v_g + 2a_c(n_{1,s}^i - n_{1,s}^j) - 2(p_{1,c}^i - p_{1,c}^j) - (3p_{1,g}^i - p_{1,g}^j)}{4}. \tag{5}$$

Moreover, the number of sellers who will join platform i is

$$n_{1,s}^i = a_s[\alpha + (1-2\alpha)n_{1,c}^i] - p_{1,s}^i, \tag{6}$$

where $i \neq j, i, j = 1, 2$. Consequently, the profit of platform i in M1 is

$$\Pi_{\mathrm{M1}}^i(p_{1,c}^i, p_{1,g}^i, p_{1,s}^i) = \alpha p_{1,g}^i + (1-2\alpha)(p_{1,c}^i n_{1,c}^i + p_{1,g}^i n_{1,cb}^i) + p_{1,s}^i n_{1,s}^i. \quad \text{(M1-Profit)} \tag{7}$$

The objective of platform i is to determine the optimal prices to achieve its maximum profit. One can check that no asymmetric equilibrium exists. We thus examine the symmetric equilibrium, in which both platforms offer the same price pair $(p_{1,c}^*, p_{1,g}^*, p_{1,s}^*)$.

Proposition 1. *In the presence of the installed base effect, a unique symmetric equilibrium exists, which can be characterized as* $p_{1,c}^* = \frac{1}{2} - \frac{1}{4}v_g - \frac{\alpha}{2(1-2\alpha)} - \frac{1}{4}a_s(a_s - a_c) - a_s a_c(1 - \frac{3}{2}\alpha)$, $p_{1,g}^* = \frac{1}{2}v_g + \frac{\alpha}{1-2\alpha}$ (≤ 1) *and* $p_{1,s}^* = \frac{1}{4}(a_s - a_c) + \frac{1}{2}\alpha a_c$.

Note that the symmetric equilibrium results in $n_{1,c}^1 = n_{1,c}^2 = \frac{1}{2}$, which indicates that the two platforms will share new potential customers equally for access. According to Proposition 1, we then have

$$n_{1,cb}^1 = n_{1,cb}^2 = n_{1,cb} = \frac{1}{4} + \frac{1}{2}v_g - \frac{1}{2}p_{1,g}^* = \frac{1}{4} + \frac{1}{4}v_g - \frac{\alpha}{2(1-2\alpha)} \tag{8}$$

and

$$n_{1,s}^i = n_{1,s}^j = n_s = \frac{1}{2}a_s - p_{1,s}^* = \frac{1}{4}a_s + \frac{1}{4}a_c(1+2\alpha). \tag{9}$$

From Equation (8), one can see that $\frac{\partial n_{1,cb}^i}{\partial \alpha} < 0$, $\frac{\partial n_{1,cb}^i}{\partial v_g} > 0$ and $\frac{\partial n_{1,cb}^i}{\partial a_c} = \frac{\partial n_{1,cb}^i}{\partial a_s} = 0$ hold. This implies that the number of new potential customers, who will purchase both access and integrated content, increases as α decreases or v_g increases. The reason is that when the installed base α increases, the number of new potential customers decreases. Additionally, the number of new potential customers, who will purchase both access and integrated content, will not be affected by the network externalities. Furthermore, from Equation (9), we have $\frac{\partial n_{1,s}^i}{\partial \alpha} > 0$, $\frac{\partial n_{1,s}^i}{\partial v_g} = 0$, $\frac{\partial n_{1,s}^i}{\partial a_c} > 0$, and $\frac{\partial n_{1,s}^i}{\partial a_s} > 0$. This provides some intuitive results that the number of the sellers who join the platforms increases in a_s, a_c, or α, but is independent of v_g. According to Proposition 1, we further get the following corollary.

Corollary 1. (i) $\frac{\partial p_{1,c}^*}{\partial v_g} < 0$, $\frac{\partial p_{1,c}^*}{\partial a_c} < 0$, $\frac{\partial p_{1,c}^*}{\partial a_s} < 0$, $\frac{\partial p_{1,c}^*}{\partial \alpha} < (>)0$ *when* $1 - 3a_c a_s(1-2\alpha)^2 > (<)0$;
(ii) $\frac{\partial p_{1,g}^*}{\partial v_g} > 0$, $\frac{\partial p_{1,g}^*}{\partial \alpha} > 0$;
(iii) $\frac{\partial p_{1,s}^*}{\partial a_c} < 0$, $\frac{\partial p_{1,s}^*}{\partial a_s} > 0$, $\frac{\partial p_{1,s}^*}{\partial \alpha} > 0$.

Corollary 1 has three important implications. First, price $p_{1,c}^*$ for platform access decreases, but price $p_{1,g}^*$ for integrated content increases in utility v_g of the integrated content. This implies that when the utility of the integrated content increases, the platforms can reduce $p_{1,c}^*$ to attract more customers to buy access, while charging customers who buy the integrated content a higher price to raise profits. Second, price $p_{1,c}^*$ decreases in the benefit a_c (or a_s) that each customer (or seller) enjoys from interacting with an additional seller (or customer). Moreover, price $p_{1,s}^*$ charged to the sellers decreases in a_c, but increases in a_s. This is because the larger the a_s, the more benefits the sellers can obtain from interacting. Thus, the platforms can reduce the price for the customer side to enlarge the market but raise the price for the seller side to obtain compensation. Similarly, when a_c increases, customers can obtain more benefits from interacting. Then, the platforms can attract more sellers to join them by reducing the price for the seller side. Meanwhile, platforms can also reduce the price for the customer side so that there are more customers willing to buy the integrated content.

Last, both prices $p_{1,g}^*$ and $p_{1,s}^*$ increase in the installed base α—because when the installed base increases, platforms have larger market shares on both the customer and seller side. However, the price for platform access may decrease or increase in the installed base, which depends on network externalities a_c and a_s. In fact, when network externalities a_c and a_s are relatively small, the platforms should decrease the price of platform access for the installed base. If not, sales will decrease and sellers will not join the platforms due to the small network externalities. On the other hand, when network externalities a_c and a_s are relatively large, the platforms can increase the access fee for the installed base, as larger network externalities can compensate for the loss of sales.

By substituting the symmetric equilibrium solution characterized in Proposition 1 into the platform's profit, we can obtain that

$$\Pi^*_{\mathrm{M1}} = \frac{1}{4}\left[1 + 2\alpha(v_g + \frac{\alpha}{1-2\alpha} - 1)\right] + \frac{1}{4}(1-2\alpha)\left[\frac{1}{2}v_g^2 - 2a_c a_s(1-2\alpha)\right] - \frac{1}{4}\alpha^2 a_c(2a_s - a_c) - \frac{1}{4}(\frac{1}{4} - \alpha)(a_s - a_c)^2. \tag{10}$$

Equation (10) yields the following corollary, which shows the impacts of the network externalities and the integrated content on the platform's profit.

Corollary 2. (i) $\frac{\partial \Pi^*_{M1}}{\partial a_c} < 0$ *if* $a_c < 5a_s$ *and otherwise* $\frac{\partial \Pi^*_{M1}}{\partial a_c} > 0$;
(ii) $\frac{\partial \Pi^*_{M1}}{\partial a_s} < 0$;
(iii) $\frac{\partial \Pi^*_{M1}}{\partial v_g} > 0$.

From Corollary 2, we can see that the platform's profit decreases in network externality a_c if $a_c < 5a_s$ and increases otherwise, which generalizes the result obtained by Armstrong (2006). The latter part of this result is intuitive, while the former is opposite to our intuition and can be explained as follows. Since both prices $p_{1,c}$ and $p_{1,s}$ decrease in a_c (see Corollary 1), when the network externality a_c is not sufficiently large (compared to a_s), the platform's loss incurred by the decrease of $p_{1,c}$ and $p_{1,s}$ cannot be compensated for by its benefits from the increased number of customers who access the platform.

In addition, parts (ii) and (iii) of Corollary 2 intuitively imply that the platform's profit increases in the utility of integrated content v_g. Interestingly, however, as network externality a_s increases, the platform's profit will decrease. The intuition behind this is that when network externality a_s increases, the platform decreases the price for access but increases the price charged to sellers. As a consequence, the profit increment from the seller side cannot compensate for the loss incurred by the increased number of customers who access the platform.

Moreover, it is easy to show that when installed base α increases (i.e., the number of new potential customers decreases), the platform's profit depends on α, a_c, a_s, and v_g. The reason is that the platform unbundles the integrated content, and the price of the integrated content has a reverse impact on the price for access. Consequently, the platform's profit depends on the changes in both prices.

4.2 Two platforms with mixed-bundle strategy

Now we consider the case in which the two platforms simultaneously adopt the mixed-bundle strategy. More formally, under this circumstance, platform i not only charges $P^i_{2,c}$ for the customers for access and $P^i_{2,s}$ to sellers for the right to produce and sell content on the platform, but also sells its own content and access together (the bundle) at a price $P^i_{2,B}$. In addition, customers can also buy access first and then purchase the integrated content at a fixed fee $P^i_{2,g}$ (unbundled). It is reasonable to assume that $P^i_{2,c} + P^i_{2,g} > P^i_{2,B}$, i.e., the bundle is cheaper than the unbundled combination. Hence, if new customers would like to buy the integrated content, they will choose the bundle and never buy the integrated content at $P^i_{2,g}$ (because the content provides zero utility without platform access). Therefore, only the installed base customers, who have not purchased the integrated content, may buy the integrated content at price $P^i_{2,g}$. Since $U^i_{installed} = 1$, the price of the integrated content can be set to $P^i_{2,g} = 1$. Thus, demand for the integrated content from the installed base i is α.

Apparently, new customers either buy the platform access only or buy the bundle. They will consider the two prices $(P^i_{2,c}, P^i_{2,B})$. Thus, under mixed bundling, new customers will determine their purchase decisions based on $P^i_{2,c}$ and the effective price $P^i_{2,ge} \equiv P^i_{2,B} - P^i_{2,c}$ $(i = 1, 2)$ of the integrated content. Replacing $p^i_{1,c}, p^i_{1,g}$ and $p^i_{1,s}$ in Equations (4), (5) and (6) by $P^i_{2,c}, P^i_{2,ge}$ and $P^i_{2,s}$, the demand by new potential customers for platform i and the bundle are respectively given by

$$N^i_{2,c} = \frac{1}{2} + \frac{2a_c(N^i_{2,s} - N^j_{2,s}) - 2(P^i_{2,c} - P^j_{2,c}) - (P^i_{2,ge} - P^j_{2,ge})}{2} \tag{11}$$

and $4N^i_{2,cb} = \frac{1}{4} + \frac{2v_g + 2a_c(N^i_{2,s} - N^j_{2,s}) - 2(P^i_{2,c} - P^j_{2,c}) - (3P^i_{2,ge} - P^j_{2,ge})}{4}$. Furthermore, the number of sellers who join platform i can be given by $N^i_{2,s} = a_s[\alpha + (1-2\alpha)N^i_{2,c}] - P^i_{2,s}$, where $i \neq j, i, j = 1, 2$. Therefore, the corresponding profit of platform i for M2 can be expressed as

$$\Pi^i_{\mathrm{M2}}(P^i_{2,c}, P^i_{2,ge}, P^i_{2,s}) = \alpha \cdot 1 + (1-2\alpha)(P^i_{2,c}N^i_{2,c} + P^i_{2,ge}N^i_{2,cb}) + P^i_{2,s}N^i_{2,s}. \quad \text{(M2–Profit)} \tag{12}$$

Similarly, we consider the symmetric equilibrium, in which each platform offers the same price pair $(P^*_{2,c}, P^*_{2,ge}, P^*_{2,s})$.

Proposition 2. *In the presence of the installed base effect, a unique symmetric equilibrium exists, which can be characterized as* $P^*_{2,c} = \frac{1}{2} - \frac{1}{4}v_g - \frac{1}{4}a_s(a_s - a_c) - a_s a_c(1 - \frac{3}{2}\alpha)$, $P^*_{2,ge} = \frac{1}{2}v_g$, *and* $P^*_{2,s} = \frac{1}{4}(a_s - a_c) + \frac{1}{2}\alpha a_c$.

Under the symmetric equilibrium, it is clear that $N^1_{2,c} = N^2_{2,c} = \frac{1}{2}$; that is, platforms 1 and 2 share the market equally. From Proposition 2 we obtain

$$\begin{cases} N^1_{2,cb} = N^2_{2,cb} = N_{2,cb} = \frac{1}{4} + \frac{1}{2}v_g - \frac{1}{2}P^*_{2,ge} = \frac{1}{4} + \frac{1}{4}v_g, \\ N^i_{2,s} = N^j_{2,s} = N_{2,s} = \frac{1}{2}a_s - P^*_{2,s} = \frac{1}{4}a_s + \frac{1}{4}a_c(1+2\alpha). \end{cases} \tag{13}$$

Further, with Equation (13), it is easy to see that demand from new customers for the bundle increases in v_g, but is independent of the installed base α. This result seems consistent with our intuition. In addition, demand from sellers increases in a_s, a_c, or α. The reason for this result is similar to that in Subsection 4.1. From Proposition 2, the following corollary can be obtained.

Corollary 3. (i) $\frac{\partial P^*_{2,c}}{\partial v_g} < 0$, $\frac{\partial P^*_{2,c}}{\partial a_c} < 0$, $\frac{\partial P^*_{2,c}}{\partial a_s} < 0$, *and* $\frac{\partial P^*_{2,c}}{\partial \alpha} > 0$;
(ii) $\frac{\partial P^*_{2,ge}}{\partial v_g} > 0$, $\frac{\partial P^*_{2,ge}}{\partial \alpha} = 0$;
(iii) $\frac{\partial P^*_{2,s}}{\partial a_c} < 0$, $\frac{\partial P^*_{2,s}}{\partial a_s} > 0$, *and* $\frac{\partial P^*_{2,s}}{\partial \alpha} > 0$.

Corollary 3 shows that if both platforms are mixed bundled, price $P^*_{2,c}$ decreases and price $P^*_{2,ge}$ increases in the utility of the integrated content v_g. Also, price $P^*_{2,c}$ decreases in customer benefit a_c or seller benefit a_s. In addition, price $P^*_{2,s}$ decreases in a_c, but increases in a_s. Finally, both prices $P^*_{2,c}$ and $P^*_{2,s}$ increase in the installed base α, but price $P^*_{2,ge}$ of the integrated content is independent of the installed base. Similar explanations as in Corollary 1 can be applied to Corollary 3.

Finally, substituting the symmetric equilibrium solution in Proposition 2 into the platform's profit function, we yield

$$\begin{aligned} \Pi^*_{\mathrm{M}2} = \frac{1}{4}(1+2\alpha) + \frac{1}{4}(1-2\alpha)\left[\frac{1}{2}v_g^2 - 2a_c a_s(1-2\alpha)\right] - \frac{1}{4}\alpha^2 a_c(2a_s - a_c) \\ - \frac{1}{16}(1-4\alpha)(a_s - a_c)^2. \end{aligned} \tag{14}$$

According to Equation (14), it is not difficult to show the following corollary.

Corollary 4. (i) $\frac{\partial \Pi^*_{M2}}{\partial a_c} < 0$ *if* $a_c < 5a_s$ *and* $\frac{\partial \Pi^*_{M2}}{\partial a_c} > 0$ *otherwise;*
(ii) $\frac{\partial \Pi^*_{M2}}{\partial a_s} < 0$;
(iii) $\frac{\partial \Pi^*_{M2}}{\partial v_g} > 0$ *and* $\frac{\partial \Pi^*_{M2}}{\partial \alpha} > 0$.

As revealed in Corollary 4, when both platforms are mixed bundled, the corresponding profit decreases (increases) in network externality a_c when $a_c < 5a_s$ ($a_c > 5a_s$), decreases in network externality a_s, and increases in the utility of the integrated content v_g. This result can be explained similar to that of Corollary 2. However, when the installed base α increases (i.e., the number of new potential customers decreases), the platforms' profits will become higher. The reason is that the effective price of the integrated content is independent of α as the platform mixed bundles the integrated content. This implies that when the number of new potential customers decreases, the platform can raise the price of the bundle to enhance the profit.

4.3 One platform with mixed-bundled strategy and one unbundled strategy

In this subsection, we investigate the third case; that is, new customers can choose to either join platform i with a mixed-bundling buying option, or join platform j with the unbundled buying option (i.e., the customer can only purchase access and integrated content separately). Similar to the models M1 and M2, platforms i and j interact with customers and sellers by charging prices $P^i_{3,c}$, $p^j_{3,c}$, $P^i_{3,s}$, and $p^j_{3,s}$, respectively. Customers can purchase the integrated content separately for a fixed fee, $P^i_{3,g}$ or $p^j_{3,g}$, as well as the bundle at price $P^i_{3,B}$. Without loss of generality, we assume that $P^i_{3,c} + P^i_{3,g} > P^i_{3,B}$. Hence, if new customers choose to purchase the integrated content, they will buy the bundle. Similar to M2, the target of $P^i_{3,g}$ is the installed base, who already has access to platform i but has not purchased the integrated content. As a result, the price for the integrated content can be set to $P^i_{3,g} = 1$ for $U^i_{installed} = 1$. Consequently, the demand of the independent content from platform i's installed base is α. Following the same procedure, one can derive demand for integrated content from the installed base for platform j, i.e., α.

Under M3, new customers will determine their purchase decisions based on access prices $P^i_{3,c}$ and $p^j_{3,c}$ for the two platforms, and the effective prices $P^i_{3,ge} \equiv P^i_{3,B} - P^i_{3,c}$ and $p^j_{3,g}$ for the integrated content. Similar to the analysis of M1 and M2, the number of customers who will join platform i is $N^i_{3,c} = \frac{1}{2} + \frac{2a_c(N^i_{3,s} - n^j_{3,s}) - 2(P^i_{3,c} - p^j_{3,c}) - (P^i_{3,ge} - p^j_{3,g})}{2}$, the number of customers who will purchase the bundle is $N^i_{3,cb} = \frac{1}{4} + \frac{2v_g + 2a_c(N^i_{3,s} - n^j_{3,s}) - 2(P^i_{3,c} - p^j_{3,c}) - (3P^i_{3,ge} - p^j_{3,g})}{4}$, $i \neq j$, the number of customers who will join platform j is $n^j_{3,c} = \frac{1}{2} + \frac{2a_c(n^j_{3,s} - N^i_{3,s}) - 2(p^j_{3,c} - P^i_{3,c}) - (p^j_{3,g} - P^i_{3,ge})}{2}$, and the number of customers who purchase the integrated content after buying the access of platform j is

$$n^j_{3,cb} = \frac{1}{4} + \frac{2v_g + 2a_c(n^j_{3,s} - N^i_{3,s}) - 2(p^j_{3,c} - P^i_{3,c}) - (3p^j_{3,g} - P^i_{3,ge})}{4}, i \neq j, i, j = 1, 2. \quad (15)$$

Furthermore, the numbers of sellers who will join platforms i and j are $N^i_{3,s} = a_s[\alpha + (1 - 2\alpha)N^i_{3,c}] - P^i_{3,s}$, and $n^j_{3,s} = a_s[\alpha + (1-2\alpha)n^j_{3,c}] - p^j_{3,s}$, respectively. Correspondingly, platform i's profit under M3 is

$$\Pi^i_{M3}(P^i_{3,c}, P^i_{3,ge}, P^i_{3,s}) = \alpha \cdot 1 + (1-2\alpha)(P^i_{3,c}N^i_{3,c} + P^i_{3,ge}N^i_{3,cb}) + P^i_{3,s}N^i_{3,s}, \quad \text{(M3i–Profit)} \quad (16)$$

and platform j's profit under M3 is

$$\Pi^j_{M3}(p^j_{3,c}, p^j_{3,g}, p^j_{3,s}) = \alpha p^j_{3,g} + (1-2\alpha)(p^j_{3,c}n^j_{3,c} + p^j_{3,g}n^j_{3,cb}) + p^j_{3,s}n^j_{3,s}. \quad \text{(M3j–Profit)} \quad (17)$$

Then, it is not difficult to characterize the equilibrium prices as follows.

Proposition 3. *In the presence of the installed base effect, a unique asymmetric equilibrium can be found, respectively, for platforms i and j as* $P^{i*}_{3,c} = \frac{1}{2} - \frac{1}{4}v_g - \frac{1}{4}a_s(a_s - a_c) - a_s a_c(1 - \frac{3}{2}\alpha)$, $P^{i*}_{3,ge} = \frac{1}{2}v_g$ *and* $P^{i*}_{3,s} = \frac{1}{4}(a_s - a_c) + \frac{1}{2}\alpha a_c$; *and* $p^{j*}_{3,c} = \frac{1}{2} - \frac{1}{4}v_g - \frac{\alpha}{2(1-2\alpha)} - \frac{1}{4}a_s(a_s - a_c) - a_s a_c(1 - \frac{3}{2}\alpha)$, $p^{j*}_{3,g} = \frac{1}{2}v_g + \frac{\alpha}{1-2\alpha}$ (≤ 1) *and* $p^{j*}_{3,s} = \frac{1}{4}(a_s - a_c) + \frac{1}{2}\alpha a_c$.

As shown in Proposition 3, platform i under M3 charges customers a higher access fee and a lower integrated content fee due to mixed bundling, whereas platform j charges a lower access fee and a higher integrated content fee due to unbundling. Proposition 3 also indicates that platform i's pricing strategy is identical to its counterpart in M2, and platform j's pricing strategy is the same as its corresponding one in M1. This implies that when the two platforms compete against each other, the individual platform's pricing strategy only depends on its own bundling policy, but not the rival's bundling strategy in symmetric equilibrium. This result is quite interesting and seems counterintuitive. The reason is that neither platform has an incentive to launch a price war and diminish its profits. Specifically, if platform i (with mixed bundling) transforms the pricing strategy when platform j (with unbundling at first) implements the mixed bundling strategy, then platform i will lose its strategic advantage. Thus, platform i can only decrease the prices of access to the platform or integrated content to seize more market share, but this would cause platform j to have an incentive to decrease its corresponding prices to compete with platform i. This cycling process will not stop, and will eventually lead to a price war and diminish profits for both. If platform i changes mixed bundling policy to the unbundling policy, the reason that platform j does not alter pricing strategy is similar. In addition, when two competing platforms adopt different bundling strategies, the platform that is unbundled will suffer a loss. This means that the two platforms should make the same decision—i.e., both should choose the mixed-bundling strategy.

Although we consider the asymmetric equilibrium, $N^i_{3,c} = n^j_{3,c} = \frac{1}{2}$ always holds. According to Proposition 1, one has $N^i_{3,cb} = \frac{1}{4} + \frac{1}{4}v_g = N_{3,cb}$, $n^j_{3,cb} = \frac{1}{4} + \frac{1}{4}v_g - \frac{\alpha}{2(1-2\alpha)} = n_{3,cb}$, and $N^i_{3,s} = n^j_{3,s} = \frac{1}{4}a_s + \frac{1}{4}a_c(1+2\alpha) = N_{3,s}$. This implies that the demand from new potential customers for bundle i increases in v_g, but is independent of the installed base α, which is consistent with the results in M2. For platform j, the demands from new potential customers who will buy both platform access and the integrated content increases as α decreases and v_g increases, which is similar to that of M1. Furthermore, demand from sellers for both platforms increases in a_s, a_c and α, which is similar to that of M1 and M2.

By substituting the equilibrium prices into the profit functions of the two platforms, we obtain that

$$\begin{cases} \Pi^{i*}_{\mathrm{M3}} = \Pi^*_{\mathrm{M2}} = \frac{1}{4}(1+2\alpha) + \frac{1}{4}(1-2\alpha)\left[\frac{1}{2}v_g^2 - 2a_c a_s(1-2\alpha)\right] \\ \qquad\qquad - \frac{1}{4}\alpha^2 a_c(2a_s - a_c) - \frac{1}{16}(1-4\alpha)(a_s-a_c)^2, \\ \Pi^{j*}_{\mathrm{M3}} = \Pi^*_{\mathrm{M1}} = \frac{1}{4}\left[1 + 2\alpha(v_g + \frac{\alpha}{1-2\alpha} - 1)\right] + \frac{1}{4}(1-2\alpha)\left[\frac{1}{2}v_g^2 - 2a_c a_s(1-2\alpha)\right] \\ \qquad\qquad - \frac{1}{4}\alpha^2 a_c(2a_s - a_c) - \frac{1}{16}(1-4\alpha)(a_s-a_c)^2. \end{cases} \tag{18}$$

Then, parallel to Corollaries 2 and 4, similar results for the impacts of the network externalities and integrated content on both platforms' profits can be obtained, and thus we omit them for brevity.

5 Discussion

In this section, we discuss the comparison and sensitivity analyses of equilibrium prices for the three scenarios (that is, models M1, M2 and M3) in Section 5.1, an extension that considers the case in which the two platforms compete for the bundling strategy in Section

5.2, and the impacts of the key parameters (i.e., intrinsic values and integrated content, installed base, and network externalities) on the value of the mixed-bundling strategy in Section 5.3.

5.1 Comparison and sensitivity analysis

According to the results in Propositions 1, 2, and 3, we can compare the equilibrium prices for the three scenarios as follows.

Theorem 1. *The equilibrium prices under the three settings have the following relationships:*

(i) $p^*_{1,s} = P^*_{2,s} = P^{i*}_{3,s} = p^{j*}_{3,s}$;

(ii) $p^*_{1,c} = p^{j*}_{3,c} < P^{i*}_{3,c} = P^*_{2,c}$;

(iii) $1 = P^*_{2,g} = P^{i*}_{3,g} > p^*_{1,g} = p^{j*}_{3,g} > P^*_{2,ge} = P^{i*}_{3,ge}$; *and*

(iv) $p^*_{1,c} + p^*_{1,g} = p^{j*}_{3,c} + p^{j*}_{3,g} > P^*_{2,B} = P^{i*}_{3,B}$.

As shown, we can obtain the following three main results. First, platform i will charge the same fee on the seller side under different bundling strategies (i.e., $p^{i*}_{1,s} = P^{i*}_{2,s} = P^{i*}_{3,s}$ and $p^{j*}_{1,s} = P^{j*}_{2,s} = p^{j*}_{3,s}$). In other words, regardless of whether both platforms employ an unbundling or mixed-bundling strategy, sellers who join platform i are always charged the same fee. In particular, when both platforms are symmetric, the two platforms will charge the same fee on the seller side regardless of their bundling strategy. This implies that the bundling strategy does not influence the fee charged to the sellers. In fact, once one of the platforms raises its price, the number of sellers who access this platform will decrease. Due to the cross-network externality, this will lead to fewer customers joining this platform, and thus a reduction in its profit. Conversely, if one platform reduces the fee charged to sellers, the profit of the other platform will be affected because its fee becomes relatively high. Consequently, neither of the platforms can benefit from the fee reduction.

Second, we find that each platform charges customers who only buy the platform access a higher price under a mixed-bundling than that under an unbundled one, and will not be affected by the other platform's bundling strategy. It is worth mentioning that these two results above are in sharp contrast to the results obtained by Chao and Derdenger (2013). In their paper, both the access price and the royalty rate are lower under a mixed-bundling equilibrium than their respective counterparts in the unbundling equilibrium. The intuition is that the fees on the seller side are the same regardless of the bundling strategy, and moreover, mixed bundling can appeal to more customers through a lower price for integrated content. The platform can thus raise the access price to make more profits under the mixed-bundling equilibrium.

Third, this also indicates that no matter whether the other platform mixes bundles or not, the bundling price P_B is always lower than $p_c + p_g$, which generalizes the result obtained by Chao and Derdenger (2013). Moreover, the combination of the second and third results implies that the platforms subsidize those customers who buy the bundle, and obtain compensation from customers who only want to access the platforms by charging a higher price. In this sense, adopting a mixed-bundling strategy can enhance the customer surplus.

Moreover, with the results of Propositions 1, 2, and 3, we can easily conduct the sensitivity analysis of equilibrium prices under different scenarios, which delivers additional managerial insights as follows. Specifically, (1) the price of the integrated content is independent of the two sides' network externalities. The reason is that the integrated content is often a supplement to platform access, and its demand only depends on its own value and installed base. (2) If marginal benefit a_s (that sellers receive from an additional customer on the platform) increases, the platform will charge sellers a higher price while reducing the price on the customer side, which makes sense. This is because the reduced customer-side price can attract more customers to join the platform, and thereby increase the sellers' benefits; the platform can also raise the seller-side price. (3) If marginal benefit a_c (that customers receive from an additional seller on the platform) increases, the platform will reduce the prices on both sides. The price reduction on the seller side makes sense. As a_c increases customers' benefits from platform interaction, the platform will attract more sellers to join by decreasing the associated price and generate profit from the customer side. However, the fall in platform access fee seems opposite to our intuition. In fact, the platform can attract more customers who are willing to buy the integrated content by reducing the price of access, and thereby compensate for the corresponding loss.

5.2 A bundling game between the two platforms

In this subsection, the two platforms will compete against each other on the bundling strategy. Platform i will be affected by the decision of its competitor platform j. Each platform has two options: mixed bundling and unbundling. Based on the analysis in the previous subsections, one can observe that if both platforms decide to sell the integrated content separately, each can earn a profit Π^*_{M1}; if both platforms decide to mixed bundle the integrated content, each of them can generate a profit Π^*_{M2}; if platform i mixed-bundles and platform j unbundles the corresponding integrated content, the profits achieved are $\Pi^{i*}_{\mathrm{M3}} = \Pi^*_{\mathrm{M2}}$ and $\Pi^{j*}_{\mathrm{M3}} = \Pi^*_{\mathrm{M1}}$, respectively ($i \neq j, i = 1, 2$). Therefore, we have the payoff matrix of the bundling game as follows. In addition, following Equations (10) and (14), since $v_g \in [0, 1]$

Table 2: Payoff matrix of the bundling game.

		Platform 2	
		mixed bundle	unbundle
Platform 1	mixed bundle	$\Pi^*_{\mathrm{M2}}, \Pi^*_{\mathrm{M2}}$	$\Pi^*_{\mathrm{M2}}, \Pi^*_{\mathrm{M1}}$
	unbundle	$\Pi^*_{\mathrm{M1}}, \Pi^*_{\mathrm{M2}}$	$\Pi^*_{\mathrm{M1}}, \Pi^*_{\mathrm{M1}}$

and $\alpha \in (0, 0.25]$, it is not difficult to derive that

$$\Pi^*_{\mathrm{M2}} - \Pi^*_{\mathrm{M1}} = \alpha \left[1 - \frac{1}{2} v_g - \frac{\alpha}{2(1-2\alpha)} \right] > 0, \quad \text{that is,} \quad \Pi^*_{\mathrm{M2}} > \Pi^*_{\mathrm{M1}}.$$

This indicates that if one platform unbundles the integrated content, its rival platform can use mixed bundling as a competitive strategy to seize market share and raise the profit. Therefore, both platforms prefer to adopt the mixed-bundling strategy compared to the unbundled strategy.

Remark: No matter which strategy the competitor uses, the platform will always implement the mixed-bundling strategy–i.e., both platforms have the dominant strategy: mixed bundling.

With the combination of Equations (10), (14), and (18), we can conclude that regardless of the competitor's choice, the equilibrium profits of platform are Π^*_{M2} with mixed bundling and Π^*_{M1} with unbundling. Since $\Pi^*_{\mathrm{M2}} > \Pi^*_{\mathrm{M1}}$, the platform with unbundling has a strong incentive to switch to mixed bundling, i.e., both platforms tend to choose mixed bundling simultaneously, when the two platforms adopt different strategies. In practice, some evidence can be found to support our results. For example, the Xbox One and PS4 always mixed bundle one of their integrated content sets in at least one or all of their distribution channels.[10]

5.3 Value of the mixed-bundling strategy

In the previous section, we demonstrated that the mixed-bundling strategy can bring higher profit compared with the unbundled policy. However, one may wonder under what circumstances the mixed-bundling strategy would bring greater values compared with the unbundled strategy. In this section, we aim to investigate the effects of the key parameters (i.e., intrinsic values v and v_g of platform i and its integrated content, installed base α, and network externalities a_c and a_s) on the value of the mixed-bundling strategy.

Denote by Π_D the profit difference between the mixed-bundling and unbundled strategies, i.e., $\Pi_D = \Pi^*_{\mathrm{M2}} - \Pi^*_{\mathrm{M1}}$. Then, from Section 5.2, we have

$$\Pi_D = \alpha\left[1 - \frac{1}{2}v_g - \frac{\alpha}{2(1-2\alpha)}\right], \tag{19}$$

where $v_g \in [0, 1]$ and $\alpha \in (0, 1/4]$. Accordingly, with Equation (19), it is not difficult to obtain the following result.

Theorem 2. (i) $\frac{\partial \Pi_D}{\partial v_g} < 0$, *and* $\frac{\partial \Pi_D}{\partial v} = \frac{\partial \Pi_D}{\partial a_c} = \frac{\partial \Pi_D}{\partial a_s} = 0$;

(ii) *if* $v_g > \frac{2(1-5\alpha+5\alpha^2)}{(1-2\alpha)^2}$ *and* $\alpha > \frac{\sqrt{5-2v_g}-1}{2\sqrt{5-2v_g}}$, $\frac{\partial \Pi_D}{\partial \alpha} < 0$; *otherwise,* $\frac{\partial \Pi_D}{\partial \alpha} \geq 0$.

Theorem 2 reveals the following implications. First, the profit difference between the mixed-bundling and unbundled strategies is decreasing in the intrinsic value of the integrated content, but is independent of the value of platform i and the network externalities. The former result makes sense, because a higher value of the integrated content would increase new potential customers' demand for the integrated content when platform i employs the unbundled strategy, which can close the profit gap with mixed bundling. However, the latter result seems counterintuitive, and the explanation of this is as follows. Note that the operational advantage of mixed-bundling is that platform access bundled with the integrated

[10]See http://item.jd.com/7722089.html, https://item.jd.hk/27527186887.html, and https://www.microsoftstore.com.cn/xbox/xbox-one/p/mic1712 (accessed Nov. 27, 2018).

content can attract more customers who prefer to purchase both because the bundled price is lower. The values of platform access and the network externalities, however, mainly control the demands for the platform, and thus increases in these values cannot affect the profit difference between the mixed-bundling and unbundled strategies.

Second, when the fraction of the installed base of platform i increases, the profit difference between the mixed-bundling and unbundled strategies is reduced if both the intrinsic value of the integrated content and the fraction of the installed base are sufficiently large. In fact, from part (i) of Theorem 2, it follows that a higher value of the integrated content would close the profit gap between mixed bundling and unbundling. Meanwhile, if the fraction of the installed base is sufficiently large—which implies that more customers have purchased platform access and will buy the corresponding integrated content—it is reasonable that the profit difference would be narrowed when the fraction of the installed base is reduced. However, if the intrinsic value of the integrated content or the fraction of the installed base is relatively small, mixed-bundling can bring higher profit if the fraction of the installed base increases.

In summary, we can obtain from Theorem 2 that under certain conditions, the mixed bundling strategy can always bring higher value for both platforms if the intrinsic values of the integrated contents become smaller, or the fractions of the installed base become higher, relative to the case of using the unbundled strategy.

6 Conclusion

In this paper, we study the content-bundling and pricing strategies for two competing platforms with installed base and cross-network effects. Based on the customer choice model, we present the equilibrium pricing strategies of the two platforms under three scenarios: (1) both platforms are unbundled; (2) both platforms are mixed bundled; and (3) one platform mixed bundles and the other unbundles. We also discuss the impact of platform competition on pricing under each scenario, compare the equilibrium prices for the three scenarios, and finally discuss a content-bundled option game between the two competing platforms. Our results show that in the presence of the installed base and competition, regardless of whether platform i chooses the mixed-bundling strategy or not, it will charge sellers the same fee—i.e., the bundling strategy will not affect the price for sellers. Moreover, under a mixed-bundling strategy, the price for platform access will be higher than that under the unbundled equilibrium. By implementing a mixed-bundling strategy, the platforms will subsidize customers who buy the bundle by charging customers who only access the platforms (without bundling) at a higher price. This implies that mixed bundling increases the surplus of customers who purchase the bundle.

In addition, we find that when the two competing platforms implement an unbundled strategy, their profits are not monotonic in the installed base (that is, the number of loyalty customers). However, the profits are increasing in the installed base if they adopt the mixed-bundling strategy. Interestingly, when one platform is mixed bundling but the other one is unbundled, the individual pricing strategy only depends on its own bundling policy, but not the rival's bundling strategy in symmetric equilibrium. The intuition behind this result is that neither platform has an incentive to launch a price war and diminish their profits. We thus confirm that mixed bundling can be used as a competitive tool for platforms to seize market share and is a dominant and equilibrium strategy for both platforms. Finally, we demonstrate that in comparison with the unbundled strategy, the mixed-bundling strategy can always bring higher value for both platforms if the intrinsic values of the integrated contents become smaller, or the fractions of the installed base become higher, especially when the intrinsic value of the integrated content or the fraction of the installed base is relatively small.

Our study has some limitations that may suggest potential future research directions. First, this paper employs a linear customer utility model but assumes that the noise terms follow the uniform distribution, it would be meaningful to extend our model to a more general distribution (e.g., Gumbel distribution). Second, we assume that the platforms charge a fixed fee on the seller side, which leads to an identical price for mixed bundling and unbundling. Thus, dynamic pricing may be an interesting topic to study. Finally, we only focus on the competitive bottleneck model, in which customers are single-homing and the sellers are multi-homing. It would be interesting to extend our model to a more comprehensive one, i.e., by including situations in which both sides are either single-homing or multi-homing.

Acknowledgments

The authors would like to thank the Editor and two anonymous referees for their valuable comments and suggestions, which significantly improved the quality and presentation of this paper. This work was supported by the National Natural Science Foundation of China (Grant 7190010147, 7151001002, 71571052, 71601079, 71871097), the Ministry of Education of China, Humanities and Social Sciences Project (Grant 19YJC630003) and the Fundamental Research Funds for the Central Universities (Grant 2019MS032).

References

Adams, W. J. and Yellen, J. L. (1976), "Commodity Bundling and the Burden of Monopoly." *Quarterly Journal of Economics*, 90(3), 475–498.

Amelio, A. and Jullien, B. (2012), "Tying and freebies in two-sided markets." *International Journal of Industrial Organization*, 30(5), 436–446.

Anderson, S. P. and de Palma, A. (1992), "The Logit as a Model of Product Differentiation." *Oxford Economic Papers*, 44(1), 51–67.

Armstrong, M. (2006), "Competition in two-sided markets." *Rand Journal of Economics*, 37(3), 668–691.

Banciu, M. and Ødegaard, F. (2016), "Optimal product bundling with dependent valuations: The price of independence." *European Journal of Operational Research*, 255(2), 481–495.

Bhargava, H. K. (2013), "Mixed bundling of two independently valued goods." *Management Science*, 59(9), 2170–2185.

Bhattacharya, S., Robotis, A., and Van Wassenhove, L. N. (2019), "Installed base management versus selling in monopolistic and competitive environments." *European Journal of Operational Research*, 273(2), 596–607.

Caillaud, B. and Jullien, B. (2003), "Chicken and egg: Competition among intermediation service providers." *Rand Journal of Economics*, 34(2), 309–329.

Caplin, A., Nalebuff, B., et al. (1991), "Aggregation and imperfect competition: On the existence of equilibrium." *Econometrica*, 59(1), 25–59.

Chakravarty, A., Mild, A., and Taudes, A. (2013), "Bundling decisions in supply chains." *European Journal of Operational Research*, 231(3), 617–630.

Chao, Y. and Derdenger, T. (2013), "Mixed bundling in two-sided markets in the presence of installed base effects." *Management Science*, 59(8), 1904–1926.

Chen, M., Rennhoff, A. D., and Serfes, K. (2016), "Bundling, à la carte pricing and vertical bargaining in a two-sided model." *Information Economics and Policy*, 35, 30–44.

Choi, J. P. (2010), "Tying in two-sided markets with multi-homing." *Journal of Industrial Economics*, 58(2), 607–626.

Choi, J. P. and Jeon, D.-S. (2016), "A leverage theory of tying in two-sided markets." Working paper.

Gao, M. (2018), "Platform pricing in mixed two-sided markets." *International Economic Review*, 59(3), 1103–1129.

Giri, R. N., Mondal, S. K., and Maiti, M. (2017), "Bundle pricing strategies for two complementary products with different channel powers." *Annals of Operations Research*, 1–25.

Halaburda, H. and Yehezkel, Y. (2013), "Platform competition under asymmetric information." *American Economic Journal: Microeconomics*, 5(3), 22–68.

Hensher, D. A. and Greene, W. H. (2003), "The Mixed Logit model: The state of practice." *Transportation*, 30(2), 133–176.

Jeitschko, T. D. and Tremblay, M. J. (2014), "Homogeneous platform competition with endogenous homing." Working paper.

Kopczewski, T., Sobolewski, M., and Miernik, I. (2018), "Bundling or unbundling? Integrated simulation model of optimal pricing strategies." *International Journal of Production Economics*, 204, 328–345.

Luo, X., Li, M., Feng, H., and Feng, N. (2017), "Intertemporal mixed bundling strategy of information products with network externality." *Computers & Industrial Engineering*, 113, 369–381.

Mishra, V. K., Natarajan, K., Padmanabhan, D., Teo, C. P., and Li, X. (2014), "On theoretical and empirical aspects of marginal distribution choice models." *Management Science*, 60(6), 1511–1531.

Nalebuff, B. (2004), "Bundling as an entry barrier." *Quarterly Journal of Economics*, 119(1), 159–187.

Natarajan, K., Song, M., and Teo, C. P. (2009), "Persistency model and its applications in choice modeling." *Management Science*, 55(3), 453–469.

Parker, G. G. and Alstyne, M. W. V. (2005), "Two-sided network effects: A theory of information product design." *Management Science*, 51(10), 1494–1504.

Peitz, M. (2008), "Bundling may blockade entry." *International Journal of Industrial Organization*, 26(1), 41–58.

Pierce, B. and Winter, H. (1996), "Pure vs. mixed commodity bundling." *Review of Industrial Organization*, 11(6), 811–821.

Prasad, A. and Mahajan, V. (2010), "Optimal bundling of technological products with network externality." *Management Science*, 56(12), 2224–2236.

Prasad, A., Venkatesh, R., and Mahajan, V. (2015), "Product bundling or reserved product pricing? Price discrimination with myopic and strategic consumers." *International Journal of Research in Marketing*, 32(1), 1–8.

Reisinger, M. (2014), "Two-part tariff competition between two-sided platforms." *European Economic Review*, 68(3), 168–180.

Roberts, J. H. and Lilien, G. L. (1993), "Explanatory and predictive models of consumer behavior." *Handbooks in Operations Research & Management Science*, 5, 27–82.

Rochet, J. and Tirole, J. (2003), "Platform competition in two-sided markets." *Journal of the European Economic Association*, 1(4), 990–1029.

Schmalensee, R. (1984), "Gaussian demand and commodity bundling." *Journal of Business*, 57(1), 211–230.

Stigler, G. J. (1963), "United States v. Loew's Inc.: A note on block-booking." *The Supreme Court Review*, 1963, 152–157.

Sun, K. (2018), "Bundling, vertical differentiation, and platform competition." *Review of Network Economics*, 17(1), 1–23.

Train, K. (2003), "Discrete choice methods with simulation," *Cambridge University Press, Cambridge, UK*.

Vamosiu, A. (2018a), "Compatibility and bundling of stand and no stand-alone use complements." *International Journal of Production Economics*, 201, 62–74.

— (2018b), "Optimal bundling under imperfect competition." *International Journal of Production Economics*, 195, 45–53.

Venkatesh, R. and Mahajan, V. (2009), "The design and pricing of bundles: A review of normative guidelines and practical approaches." *Rao V, ed. Handbook of Pricing Research in Marketing (Edward Elgar Publishing, Northampton, MA).*

Xu, B., Yao, Z., and Tang, P. (2018), "Pricing strategies for information products with network effects and complementary services in a duopolistic market." *International Journal of Production Research*, 56(12), 4243–4263.

Appendix

Proof of Proposition 1:

Taking the first derivatives of Π^i_{M1} with respect to $p^i_{1,c}$, $p^i_{1,g}$, and $p^i_{1,s}$, respectively, and setting them to zeros, we have

$$\begin{cases} \frac{\partial \Pi^i_{M1}}{\partial p^i_{1,c}} = (1-2\alpha)(n^i_{1,c} + p^i_{1,c}\frac{\partial n^i_{1,c}}{\partial p^i_{1,c}} + p^i_{1,g}\frac{\partial n^i_{1,cb}}{\partial p^i_{1,c}}) + p^i_{1,s}\frac{\partial n^i_{1,s}}{\partial p^i_{1,c}} = 0, \\ \frac{\partial \Pi^i_{M1}}{\partial p^i_{1,g}} = \alpha + (1-2\alpha)(p^i_{1,c}\frac{\partial n^i_{1,c}}{\partial p^i_{1,g}} + n^i_{1,cb} + p^i_{1,g}\frac{\partial n^i_{1,cb}}{\partial p^i_{1,g}}) + p^i_{1,s}\frac{\partial n^i_{1,s}}{\partial p^i_{1,g}} = 0, \\ \frac{\partial \Pi^i_{M1}}{\partial p^i_{1,s}} = (1-2\alpha)(p^i_{1,c}\frac{\partial n^i_{1,c}}{\partial p^i_{1,s}} + p^i_{1,g}\frac{\partial n^i_{1,cb}}{\partial p^i_{1,s}}) + n^i_{1,s} + p^i_{1,s}\frac{\partial n^i_{1,s}}{\partial p^i_{1,s}} = 0. \end{cases} \tag{20}$$

To obtain the equilibrium solutions, we first respectively determine the derivatives of $n^i_{1,c}$, $n^i_{1,cb}$, and $n^i_{1,s}$ with respect to $p^i_{1,c}, p^i_{1,g}$ and $p^i_{1,s}$. Since $n^j_{1,c} = 1 - n^i_{1,c}$, from Equation (6), one has

$$\begin{aligned} n^i_{1,s} - n^j_{1,s} &= a_s[\alpha + (1-2\alpha)n^i_{1,c}] - p^i_{1,s} - a_s[\alpha + (1-2\alpha)n^j_{1,c}] + p^j_{1,s} \\ &= -a_s(1-2\alpha) + 2a_s(1-2\alpha)n^i_{1,c} - (p^i_{1,s} - p^j_{1,s}). \end{aligned} \tag{21}$$

Suppose that network externality parameters a_s and a_c are relatively small, such that we can focus on the market-sharing equilibria. It turns out that the necessary and sufficient condition for the existence of a market-sharing equilibrium is $4-(1-2\alpha)[(a_c+a_s)^2+4a_ca_s] > 0$, which is assumed to hold in the following analysis.

Let $\sigma = a_s - a_c$, $\rho = a_sa_c(1-2\alpha) \geq 0$ and $\phi = 1 - 2a_sa_c(1-2\alpha)$, where $\phi = 1-2\rho$. Substituting Equation (21) into Equation (4), we have

$$\begin{aligned} 2n^i_{1,c} &= 1 + 2a_c[-a_s(1-2\alpha) + 2a_s(1-2\alpha)n^i_{1,c} - (p^i_{1,s} - p^j_{1,s})] - 2(p^i_{1,c} - p^j_{1,c}) - (p^i_{1,g} - p^j_{1,g}) \\ &= 1 - 2\rho + 4\rho n^i_{1,c} - 2a_c(p^i_{1,s} - p^j_{1,s}) - 2(p^i_{1,c} - p^j_{1,c}) - (p^i_{1,g} - p^j_{1,g}), \end{aligned}$$

and thus

$$n^i_{1,c} = \frac{1}{2} - \frac{2a_c(p^i_{1,s} - p^j_{1,s}) + 2(p^i_{1,c} - p^j_{1,c}) + (p^i_{1,g} - p^j_{1,g})}{2\phi}.$$

With $n^i_{1,c}$, by substituting Equation (21) into Equation (5), one has

$$n^i_{1,cb} = \frac{1}{2}\left\{\frac{1}{2} + v_g - p^i_{1,g} - (\frac{1}{2} + \frac{\rho}{\phi})[2a_c(p^i_{1,s} - p^j_{1,s}) + 2(p^i_{1,c} - p^j_{1,c}) + (p^i_{1,g} - p^j_{1,g})]\right\}$$

and

$$\begin{aligned} n^i_{1,s} &= a_s[\alpha + (1-2\alpha)n^i_{1,c}] - p^i_{1,s} \\ &= \frac{1}{2}a_s - a_s(1-2\alpha)\frac{2a_c(p^i_{1,s} - p^j_{1,s}) + 2(p^i_{1,c} - p^j_{1,c}) + (p^i_{1,g} - p^j_{1,g})}{2\phi} - p^i_{1,s}. \end{aligned}$$

Then, checking the derivatives of $n^i_{1,c}$, $n^i_{1,cb}$, and $n^i_{1,s}$ with respect to $p^i_{1,c}, p^i_{1,g}$ and $p^i_{1,s}$, respectively, yields

$$\begin{cases} \frac{\partial n^i_{1,c}}{\partial p^i_{1,c}} = -\frac{1}{\phi}, \frac{\partial n^i_{1,cb}}{\partial p^i_{1,c}} = -\frac{1}{2} - \frac{\rho}{\phi}, \frac{\partial n^i_{1,s}}{\partial p^i_{1,c}} = -\frac{a_s(1-2\alpha)}{\phi}, \\ \frac{\partial n^i_{1,c}}{\partial p^i_{1,g}} = -\frac{1}{2\phi}, \frac{\partial n^i_{1,cb}}{\partial p^i_{1,g}} = -\frac{3}{4} - \frac{\rho}{2\phi}, \frac{\partial n^i_{1,s}}{\partial p^i_{1,g}} = -\frac{a_s(1-2\alpha)}{2\phi}, \\ \frac{\partial n^i_{1,c}}{\partial p^i_{1,s}} = -\frac{a_c}{\phi}, \frac{\partial n^i_{1,cb}}{\partial p^i_{1,s}} = -\frac{a_c}{2} - \frac{a_c\rho}{\phi}, \frac{\partial n^i_{1,s}}{\partial p^i_{1,s}} = -\frac{\rho}{\phi} - 1. \end{cases} \tag{22}$$

In the symmetric equilibrium, the prices of both platforms are the same, i.e., $p^i_{1,c} = p^j_{1,c} = p_{1,c}$, $p^i_{1,g} = p^j_{1,g} = p_{1,g}$, and $p^i_{1,s} = p^j_{1,s} = p_{1,s}$, which implies that the two platforms' market shares on the customer side are identical, i.e., $n^i_{1,c} = n^j_{1,c} = \frac{1}{2}$, $n^i_{1,cb} = n^j_{1,cb} = \frac{1}{4} + \frac{1}{2}v_g - \frac{1}{2}p_{1,g}$, and that each platform has the same number of sellers, i.e., $n^i_{1,s} = n^j_{1,s} = \frac{1}{2}a_s - p_{1,s}$.

On the other hand, substituting the above equations and Equation (22) into the first-order conditions, we can obtain

$$\begin{cases} 2p_{1,c} + p_{1,g} + 2a_s p_{1,s} = \phi, \\ p_{1,c} + (\frac{5}{2}\phi + \rho)p_{1,g} + a_s p_{1,s} = (\frac{1}{2} + v_g + \frac{2\alpha}{1-2\alpha})\phi, \\ a_c(1-2\alpha)p_{1,c} + \frac{a_c}{2}(1-2\alpha)p_{1,g} + (2\phi + \rho)p_{1,s} = \frac{1}{2}a_s\phi. \end{cases} \tag{23}$$

Hence, solving Equation (23) yields the symmetric equilibrium prices shown by the results in Proposition 1.

We next turn to the second-order conditions. We need to seek the conditions under which the Hessian matrix (3×3 matrix) is negative definite. Because

$$\begin{cases} \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial (p^i_{1,c})^2} = -\frac{2(1-2\alpha)}{\phi}, \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial p^i_{1,c}\partial p^i_{1,g}} = -\frac{(1-2\alpha)}{\phi}, \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial p^i_{1,c}\partial p^i_{1,s}} = -\frac{(a_c+a_s)(1-2\alpha)}{\phi}, \\ \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial p^i_{1,g}\partial p^i_{1,c}} = -\frac{(1-2\alpha)}{\phi}, \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial (p^i_{1,g})^2} = -(1-2\alpha)(\frac{1}{\phi} + 1), \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial p^i_{1,g}\partial p^i_{1,s}} = -\frac{(a_c+a_s)(1-2\alpha)}{\phi}, \\ \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial p^i_{1,s}\partial p^i_{1,c}} = -\frac{(a_c+a_s)(1-2\alpha)}{\phi}, \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial p^i_{1,s}\partial p^i_{1,g}} = -\frac{(a_c+a_s)(1-2\alpha)}{\phi}, \frac{\partial^2 \Pi^i_{\mathrm{M1}}}{\partial (p^i_{1,s})^2} = -(\frac{1}{\phi} + 1), \end{cases} \tag{24}$$

the Hessian matrix is

$$\begin{pmatrix} -\frac{2(1-2\alpha)}{\phi} & -\frac{(1-2\alpha)}{\phi} & -\frac{(a_c+a_s)(1-2\alpha)}{\phi} \\ -\frac{(1-2\alpha)}{\phi} & -(1-2\alpha)(\frac{1}{\phi}+1) & -\frac{(a_c+a_s)(1-2\alpha)}{\phi} \\ -\frac{(a_c+a_s)(1-2\alpha)}{\phi} & -\frac{(a_c+a_s)(1-2\alpha)}{\phi} & -(\frac{1}{\phi}+1) \end{pmatrix}. \tag{25}$$

To ensure that the profit of M1, Equation (7), is concave in these prices, from Equation (25), we need to have

$$\left| -\frac{2(1-2\alpha)}{\phi} \right| < 0, \quad \begin{vmatrix} -\frac{2(1-2\alpha)}{\phi} & -\frac{(1-2\alpha)}{\phi} \\ -\frac{(1-2\alpha)}{\phi} & -(1-2\alpha)(\frac{1}{\phi}+1) \end{vmatrix} > 0$$

and

$$\begin{vmatrix} -\frac{2(1-2\alpha)}{\phi} & -\frac{(1-2\alpha)}{\phi} & -\frac{(a_c+a_s)(1-2\alpha)}{\phi} \\ -\frac{(1-2\alpha)}{\phi} & -(1-2\alpha)(\frac{1}{\phi}+1) & -\frac{(a_c+a_s)(1-2\alpha)}{\phi} \\ -\frac{(a_c+a_s)(1-2\alpha)}{\phi} & -\frac{(a_c+a_s)(1-2\alpha)}{\phi} & -(\frac{1}{\phi}+1) \end{vmatrix} < 0,$$

if and only if $4-(1-2\alpha)[(a_c+a_s)^2+4a_ca_s]>0$ holds. □

Proof of Corollaries 1 and 2: With Proposition 1, one can easily prove the desired results, and thus we omit the proofs here. □

Proof of Proposition 2: This proof is similar to the proof of Proposition 1. Differentiating Π^i_{M2} with respect to $P^i_{2,c}$, $P^i_{2,ge}$, and $P^i_{2,s}$, we obtain the first-order conditions as

$$\begin{cases} \frac{\partial \Pi^i_{\mathrm{M2}}}{\partial P^i_{2,c}} = (1-2\alpha)(N^i_{2,c} + P^i_{2,c}\frac{\partial N^i_{2,c}}{\partial P^i_{2,c}} + P^i_{2,ge}\frac{\partial N^i_{2,cb}}{\partial P^i_{2,c}}) + P^i_{2,s}\frac{\partial N^i_{2,s}}{\partial P^i_{2,c}} = 0, \\ \frac{\partial \Pi^i_{\mathrm{M2}}}{\partial P^i_{2,ge}} = (1-2\alpha)(P^i_{2,c}\frac{\partial N^i_{2,c}}{\partial P^i_{2,ge}} + N^i_{2,cb} + P^i_{2,ge}\frac{\partial N^i_{2,cb}}{\partial P^i_{2,ge}}) + P^i_{2,s}\frac{\partial N^i_{2,s}}{\partial P^i_{2,ge}} = 0, \\ \frac{\partial \Pi^i_{\mathrm{M2}}}{\partial P^i_{2,s}} = (1-2\alpha)(P^i_{2,c}\frac{\partial N^i_{2,c}}{\partial P^i_{2,s}} + P^i_{2,ge}\frac{\partial N^i_{2,cb}}{\partial P^i_{2,s}}) + N^i_{2,s} + P^i_{2,s}\frac{\partial N^i_{2,s}}{\partial P^i_{2,s}} = 0. \end{cases} \tag{26}$$

To obtain the solutions, we first determine the derivatives of $N^i_{2,c}$, $N^i_{2,cb}$, and $N^i_{2,s}$ with respect to $P^i_{2,c}$, $P^i_{2,ge}$, and $P^i_{2,s}$, respectively. Similar to the proof of Proposition 1, it turns out that the necessary and sufficient condition for the existence of a market-sharing equilibrium is $4-(1-2\alpha)[(a_c+a_s)^2+4a_ca_s]>0$. Also,

$$\begin{cases} \frac{\partial N^i_{2,c}}{\partial P^i_{2,c}} = -\frac{1}{\phi}, \frac{\partial N^i_{2,cb}}{\partial P^i_{2,c}} = -\frac{1}{2} - \frac{\rho}{\phi}, \frac{\partial N^i_{2,s}}{\partial P^i_{2,c}} = -\frac{a_s(1-2\alpha)}{\phi}; \\ \frac{\partial N^i_{2,c}}{\partial P^i_{2,ge}} = -\frac{1}{2\phi}, \frac{\partial N^i_{2,cb}}{\partial P^i_{2,ge}} = -\frac{3}{4} - \frac{\rho}{2\phi}, \frac{\partial N^i_{2,s}}{\partial P^i_{2,ge}} = -\frac{a_s(1-2\alpha)}{2\phi}; \\ \frac{\partial N^i_{2,c}}{\partial P^i_{2,s}} = -\frac{a_c}{\phi}, \frac{\partial N^i_{2,cb}}{\partial P^i_{2,s}} = -\frac{a_c}{2} - \frac{a_c\rho}{\phi}, \frac{\partial N^i_{2,s}}{\partial P^i_{2,s}} = -\frac{\rho}{\phi} - 1. \end{cases} \tag{27}$$

In symmetric equilibrium, the prices of both platforms are the same, i.e., $P^i_{2,c} = P^j_{2,c} = P_{2,c}, P^i_{2,ge} = P^j_{2,ge} = P_{2,ge}$ and $P^i_{2,s} = P^j_{2,s} = P_{2,s}$, which implies that two platforms' market shares on the customer side are identical, i.e., $N^i_{2,c} = N^j_{2,c} = \frac{1}{2}, N^i_{2,cb} = N^j_{2,cb} = \frac{1}{4}+\frac{1}{2}v_g-\frac{1}{2}P_{2,ge}$, and that each platform has an equal number of content developers, i.e., $N^i_{2,s} = N^j_{2,s} = \frac{1}{2}a_s - P_{2,s}$.

On the other hand, substituting the above equations and (27) into the first-order conditions and rearranging, we obtain

$$\begin{cases} 2P_{2,c} + P_{2,ge} + 2a_s P_{2,s} = \phi, \\ P_{2,c} + (\frac{5}{2}\phi + \rho)P_{2,ge} + a_s P_{2,s} = (\frac{1}{2} + v_g)\phi, \\ a_c(1-2\alpha)P_{2,c} + \frac{a_c}{2}(1-2\alpha)P_{2,ge} + (2\phi + \rho)P_{2,s} = \frac{1}{2}a_s\phi. \end{cases} \tag{28}$$

By solving Equation (28), we can obtain the symmetric equilibrium prices shown by the results in Proposition 2.

Next, we turn to the second-order conditions. We need to determine under which conditions the Hessian (3×3 matrix) is negative definite. Since the profit of M2's Hessian matrix is identical to M1's, the profit of M2, Expression (12), is concave in these prices if and only if $4 - (1-2\alpha)[(a_c + a_s)^2 + 4a_c a_s] > 0$ holds. □

Proof of Corollaries 3 and 4: With Proposition 2, one can easily prove the desired results, and thus we omit their proofs here. □

Proof of Proposition 3: By differentiating Π^i_{M3} with respect to $P^i_{3,c}$, $P^i_{3,ge}$, and $P^i_{3,s}$, respectively, we obtain

$$\begin{cases} \frac{\partial \Pi^i_{\mathrm{M3}}}{\partial P^i_{3,c}} = (1-2\alpha)(N^i_{3,c} + P^i_{3,c}\frac{\partial N^i_{3,c}}{\partial P^i_{3,c}} + P^i_{3,ge}\frac{\partial N^i_{3,cb}}{\partial P^i_{3,c}}) + P^i_{3,s}\frac{\partial N^i_{3,s}}{\partial P^i_{3,c}} = 0, \\ \frac{\partial \Pi^i_{\mathrm{M3}}}{\partial P^i_{3,ge}} = (1-2\alpha)(P^i_{3,c}\frac{\partial N^i_{3,c}}{\partial P^i_{3,ge}} + N^i_{3,cb} + P^i_{3,ge}\frac{\partial N^i_{3,cb}}{\partial P^i_{3,ge}}) + P^i_{3,s}\frac{\partial N^i_{3,s}}{\partial P^i_{3,ge}} = 0, \\ \frac{\partial \Pi^i_{\mathrm{M3}}}{\partial P^i_{3,s}} = (1-2\alpha)(P^i_{3,c}\frac{\partial N^i_{3,c}}{\partial P^i_{3,s}} + P^i_{3,ge}\frac{\partial N^i_{3,cb}}{\partial P^i_{3,s}}) + N^i_{3,s} + P^i_{3,s}\frac{\partial N^i_{3,s}}{\partial P^i_{3,s}} = 0. \end{cases} \tag{29}$$

Similarly, differentiating Π^j_{M3} with respect to $p^i_{3,c}$, $p^j_{3,g}$, and $p^j_{3,s}$, we have

$$\begin{cases} \frac{\partial \Pi^j_{\mathrm{M3}}}{\partial p^j_{3,c}} = (1-2\alpha)(n^j_{3,c} + p^j_{3,c}\frac{\partial n^j_{3,c}}{\partial p^j_{3,c}} + p^j_{3,g}\frac{\partial n^j_{3,cb}}{\partial p^j_{3,c}}) + p^j_{3,s}\frac{\partial n^j_{3,s}}{\partial p^j_{3,c}} = 0, \\ \frac{\partial \Pi^j_{\mathrm{M3}}}{\partial p^j_{3,g}} = \alpha + (1-2\alpha)(p^j_{3,c}\frac{\partial n^j_{3,c}}{\partial p^j_{3,g}} + n^j_{3,cb} + p^j_{3,g}\frac{\partial n^j_{3,cb}}{\partial p^j_{3,g}}) + p^j_{3,s}\frac{\partial n^j_{3,s}}{\partial p^j_{3,g}} = 0, \\ \frac{\partial \Pi^j_{\mathrm{M3}}}{\partial p^j_{3,s}} = (1-2\alpha)(p^j_{3,c}\frac{\partial n^j_{3,c}}{\partial p^j_{3,s}} + p^j_{3,g}\frac{\partial n^j_{3,cb}}{\partial p^j_{3,s}}) + n^j_{3,s} + p^j_{3,s}\frac{\partial n^j_{3,s}}{\partial p^j_{3,s}} = 0. \end{cases} \tag{30}$$

To obtain the solutions, we first determine the derivatives of $N^i_{3,c}$, $N^i_{3,cb}$, and $N^i_{3,s}$ with respect to $P^i_{3,c}$, $P^i_{3,ge}$ and $P^i_{3,s}$, and the derivatives of $n^j_{3,c}$, $n^j_{3,cb}$ and $n^j_{3,s}$ with respect to

$p^j_{3,c}, p^j_{3,g}$, and $p^j_{3,s}$, respectively. Similar to the proof of Proposition 1, it turns out that the necessary and sufficient condition for the existence of a market-sharing equilibrium is $4-(1-2\alpha)[(a_c+a_s)^2+4a_ca_s]>0$, and

$$\begin{cases} \frac{\partial N^i_{3,c}}{\partial P^i_{3,c}} = \frac{\partial n^j_{3,c}}{\partial p^j_{3,c}} = -\frac{1}{\phi}, \frac{\partial N^i_{3,cb}}{\partial P^i_{3,c}} = \frac{\partial n^j_{3,cb}}{\partial p^j_{3,c}} = -\frac{1}{2} - \frac{\rho}{\phi}, \frac{\partial N^i_{3,s}}{\partial P^i_{3,c}} = \frac{\partial n^j_{3,s}}{\partial p^j_{3,c}} = -\frac{a_s(1-2\alpha)}{\phi}, \\ \frac{\partial N^i_{3,c}}{\partial P^i_{3,ge}} = \frac{\partial n^j_{3,c}}{\partial p^j_{3,g}} = -\frac{1}{2\phi}, \frac{\partial N^i_{3,cb}}{\partial P^i_{3,ge}} = \frac{\partial n^j_{3,cb}}{\partial p^j_{3,g}} = -\frac{3}{4} - \frac{\rho}{2\phi}, \frac{\partial N^i_{3,s}}{\partial P^i_{3,ge}} = \frac{\partial n^j_{3,s}}{\partial p^j_{3,g}} = -\frac{a_s(1-2\alpha)}{2\phi}, \\ \frac{\partial N^i_{3,c}}{\partial P^i_{3,s}} = \frac{\partial n^j_{3,c}}{\partial p^j_{3,s}} = -\frac{a_c}{\phi}, \frac{\partial N^i_{3,cb}}{\partial P^i_{3,s}} = \frac{\partial n^j_{3,cb}}{\partial p^j_{s}} = -\frac{a_c}{2} - \frac{a_c\rho}{\phi}, \frac{\partial N^i_{3,s}}{\partial P^i_{3,s}} = \frac{\partial n^j_{3,s}}{\partial p^j_{3,s}} = -\frac{\rho}{\phi} - 1. \end{cases} \tag{31}$$

It is obvious that we do not have symmetric equilibrium prices. Substituting the above equations into the first-order conditions, we obtain

$$\begin{cases} 4P^i_{3,c} + 2P^i_{3,ge} + 2(a_s+a_c)P^i_{3,s} - 2p^j_{3,c} - p^j_{3,g} - 2a_cp^j_{3,s} = \phi, \\ 2P^i_{3,c} + (1+2\phi)P^i_{3,ge} + (a_s+a_c)P^i_{3,s} - p^j_{3,c} - \frac{1}{2}p^j_{3,g} - a_cp^j_{3,s} = (\frac{1}{2}+v_g)\phi, \\ 2(a_s+a_c)P^i_{3,c} + (a_s+a_c)P^i_{3,ge} + \frac{2(1+\phi)}{1-2\alpha}P^i_{3,s} - 2a_sp^j_{3,c} - a_sp^j_{3,g} - 2a_sa_cp^j_{3,s} = \frac{a_s\phi}{1-2\alpha} \end{cases} \tag{32}$$

and

$$\begin{cases} -2P^i_{3,c} - P^i_{3,ge} - 2a_cP^i_{3,s} + 4p^j_{3,c} + 2p^j_{3,g} + 2(a_s+a_c)p^j_{3,s} = \phi, \\ -P^i_{3,c} - \frac{1}{2}P^i_{3,ge} - a_cP^i_{3,s} + 2p^j_{3,c} + (1+2\phi)p^j_{3,g} + (a_s+a_c)p^j_{3,s} = (\frac{1}{2}+v_g+\frac{2\alpha}{1-2\alpha})\phi, \\ -2a_sP^i_{3,c} - a_sP^i_{3,ge} - 2a_sa_cP^i_{3,s} + 2(a_s+a_c)p^j_{3,c} + (a_s+a_c)p^j_{3,g} + \frac{2(1+\phi)}{1-2\alpha}p^j_{3,s} = \frac{a_s\phi}{1-2\alpha}. \end{cases} \tag{33}$$

Then, solving the first-order conditions for the six prices, the fees, in an asymmetric equilibrium, can be implicitly defined by the results in Proposition 3.

Next, we turn to the second-order conditions. We need to determine under which conditions the Hessian matrices of Π^i_{M3} and Π^j_{M3} (3×3 matrices) are negative definite. Since they are identical to M1's Hessian matrix, the profit of Π^i_{M3} and Π^j_{M3}, Equations (16) and (17), are concave in these prices if and only if $4-(1-2\alpha)[(a_c+a_s)^2+4a_ca_s]>0$ holds. □

Proof of Theorem 1: One can easily prove this result according to Propositions 1, 2 and 3, and thus we omit it here. □

Proof of Theorem 2: From the expression $\Pi_D = \alpha\left[1-\frac{1}{2}v_g-\frac{\alpha}{2(1-2\alpha)}\right]$, where $v_g\in[0,1]$ and $\alpha\in(0,1/4]$, one can easily prove the desired result, and thus we omit it here. □