International Journal of Production Economics 218 (2019) 363–374

Contents lists available at ScienceDirect

International Journal of Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Matching supply and demand on ride-sharing platforms with permanent agents and competition

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ARTICLE INFO

Keywords:
Ride-sharing platforms
Self-scheduling and permanent agents
Subsidy
Competition

ABSTRACT

Ride-sharing platforms, as a new travel service, yield substantial benefit by effectively matching drivers and passengers. We consider ride-sharing platforms that recruit permanent and/or temporary, i.e., self-scheduling, agents to provide on-demand services requested by riders. The permanent agents of a platform are paid a fixed wage by the platform, whereas the temporary agents are self-scheduling and offered a subsidy by the platform for the services they provide. In this paper we assume that the market is supply-dominant, i.e., the number of riders seeking on-demand services is sufficiently large and the price charged to each served rider is determined by the number of agents available on the platforms. In both monopoly and duopoly competition environments, we determine the subsidy level for the temporary agents and/or the employment level of the permanent agents that balance the agents' supply and the riders' demand to maximize the platforms' profits. Furthermore, we examine the impacts of both the employment of permanent agents and platform competition on the platforms' subsidy strategies and profitability, participants' surpluses, and social welfare. We also discuss the managerial implications of the research findings.

1. Introduction

Riders increasingly fulfill their riding needs through third-party services, which can reduce traffic congestion and exhaust emissions. In recent years, there is a phenomenal growth in ride-sharing platforms that provide an alternative transport service. Instead of planning the provider resources (e.g., drivers, cars etc) in advance, ride-sharing platforms operate by effectively matching real-time riders and drivers, i.e., such platforms pool different on-demand services, and subsidize and incentivize agents to provide services on their platforms. Examples of such platforms abound, such as Uber, Lyft, Didi Chuxing, Grab, Gett etc. Ride sharing brings many challenges to platform operations, one of which is the platform's supply and demand balance problem. Take Didi Chuxing, a major ride-sharing platform in China, as an example. Didi provides car-hailing services for riders, attracts drivers to its platform to fulfill riders' on-demand services, and obtains revenue from charging the riders. The drivers using Didi are not employed by or affiliated with Didi, meaning that they are self-scheduling, i.e., temporary agents for Didi.[1] Since the drivers are temporary agents, they can decide whether to join the platform or not, and whether to accept the on-demand service requests from customers or not. To attract more temporary drivers to its platform and accepting the on-demand service requests, Didi offers huge subsidies to the temporary drivers. As we know, if all the service requests are fulfilled by the temporary agents only, Didi can attain good flexibility because any available temporary agent near the location of a service request can offer the service immediately. However, such a scenario may lead to a low service level of the platform because there may not be enough temporary agents available on the platform when service requests occur, especially during the rush hour. According to our survey of Didi, in order to guarantee a certain service level, Didi has recently started the practice of employing some permanent agents that are affiliated with and receive stable wages from Didi. Similar practices have also been adopted by Uber in the USA and Grab in Southeast Asia.[2] Due to the intense competition of ride-sharing platforms (e.g., Uber and Lyft in the USA, and Uber and Gett in Europe), the presence of permanent agents may play a key role in the competing

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[1] In the following we use self-scheduling agents and temporary agents interchangeably.
[2] https://www.wired.com/2016/04/uber-recruiting-tens-thousands-veterans-drivers/and https://www.todayonline.com/singapore/more-young-drivers-joining-grab-uber-full-time.

https://doi.org/10.1016/j.ijpe.2019.07.009
Received 10 December 2018; Received in revised form 5 July 2019; Accepted 11 July 2019
Available online 16 July 2019
0925-5273/

Table 1
A summary of real world examples.

Motivation	Real-world examples
Ride-sharing platforms	Uber, Lyft, Didi Chuxing, Grab, Gett
Competition between ride-sharing platforms	Uber vs Lyft, Uber vs Gett
Ride-sharing platforms with self-scheduling and permanent agents	Uber, Grab, Didi Chuxing

platforms' associated operations.

We summarize the above specific examples in Table 1 to illustrate the motivation for our study. In view of these examples, we pose the following natural and interesting questions:

1. Does employing permanent agents really enhance the service level?
2. Is it beneficial for ride-sharing platforms to employ permanent agents?
3. How do the employment of permanent agents and competition between platforms affect the strategies of the platforms, the corresponding customers'/agents' surpluses, and social welfare?

To answer these questions, we consider ride-sharing platforms such as Didi and Uber that offer on-demand services. We assume that the market is supply-dominant, i.e., the number of riders seeking on-demand services is sufficiently large and the price charged to each served rider is determined by the number of agents available on the platforms. This implies that our focus is on capacity planning under the duopoly scenario, i.e., Cournot competition. We first discuss how to subsidize the self-scheduling agents for a monopoly ride-hailing platform in the absence and presence of permanent agents, and study the impacts of employing permanent agents on the platform's subsidy strategy, customers'/agents' surpluses, and social welfare. Observing that ride-sharing platforms face intense competition for both agents and customers, we analyze how competition affects the platforms' subsidy strategies, customers'/agents' surpluses, and social welfare in the absence and presence of permanent agents, and compare the equilibrium strategies. Finally, we discuss the managerial implications of the research findings.

The contributions of our study are twofold. First, we incorporate permanent and temporary agents into our model for both monopoly and duopoly platforms, in view of the recent practices of such platforms as Didi and Uber that recruit their own permanent agents. Second, we analyze the effects of permanent agents and competition on the platforms' subsidy strategies, participants' surpluses, and social welfare. We find that when the ride-sharing market shifts from a monopoly to a duopoly, employees may be offered lower wages and gain lower surpluses. This finding is contrary to that for the traditional single-sided market.

We organize the rest of the paper as follows: In Section 2 we briefly review the literature related to our research. In Section 3 we analyze a monopoly platform's strategy, while in Section 4 we consider duopoly platforms' equilibrium strategies, under each of two scenarios of employing temporary agents only and employing both temporary and permanent agents. In Section 5 we analyze the effect of competition on the platforms' strategies. We conclude the paper, discuss the managerial implications of the findings, and suggest topics for future research in Section 6. We prsenet the proofs of all the results in the Appendix.

2. Literature review

Our paper is relevant to the research stream concerned with the economic impacts of product sharing. Sun et al. (2016) studied sellers' pricing decisions in the presence of consumers' peer-to-peer sharing of private goods. They showed that when consumer valuation heterogeneity is neither too high nor too low, accommodating sharing brings benefit to the seller, but does not always promote consumer access to the goods. Jiang and Tian (2016) built an analytical framework to study the strategic and economic impacts of product sharing among consumers. One of their main results shows that firms' profits, consumer surplus, and social welfare are not necessarily monotonic in the transaction cost. Benjaafar et al. (2018b) investigated the impacts of peer-to-peer product sharing on ownership, usage, and social welfare, and found that collaborative consumption can result in either lower or higher ownership and usage levels. Fang et al. (2017) focused on the design of prices and subsidies on sharing platforms. They suggested that sharing platforms have a strong incentive to encourage sharing via subsidies. Zha et al. (2016) investigated the competition of ride-sourcing platforms and found that competition does not necessarily lower the price or improve social welfare. Kung and Zhong (2017) studied three pricing policies for a two-sided grocery delivery platform in the presence of network externality. Interestingly, they found that membership pricing, transaction pricing, and cross subsidization are equivalent without time discounting and price-sensitive order frequency. There are also empirical studies on the economic analysis of product-sharing platforms. For example, Zervas et al. (2015), Li et al. (2016), and Zervas et al. (2017) conducted empirical studies on a short-term rental platform called Airbnb. Zervas et al. (2015) revealed that most of Airbnb's properties (nearly 95%) have a high average user-generated rating of either 4.5 or 5 stars; Li et al. (2016) showed that the presence of professional and non-professional agents can considerably affect a profit-maximizing platform's decision; and Zervas et al. (2017) found that the entry of Airbnb may strongly affect hotel revenues. A brief literature review of the origin, development, and economic concerns of the sharing economy can be found in Price and Belk (2016). Different from the above studies, we build an analytical framework to study the impacts of self-scheduling and permanent agents on competing platforms' subsidies, profitability, and welfare. In other words, we focus on the ride-sharing platforms' operational strategies rather than just on the economic analysis of the platforms.

Our work is also related to the growing literature on operations management in the context of the sharing economy. How to coordinate self-scheduling agents and demand is a central issue for most existing studies. Ibrahim and Arifoglu (2015), and Gurvich et al. (2019) adopted a queuing framework and a newsvendor model to study the optimal pricing strategy for a ride-sharing platform with self-scheduling agents, respectively. They both found that allowing self-scheduling is costly to both the platform and customers. Lu et al. (2018) developed a win-win-win coordination mechanism for an on-demand staffing platform with a self-scheduling workforce, and employers with uncertain and possibly time-varying demands. Due to the presence of commonly-seen pricing policies in practice (e.g., surge pricing and static pricing in ride-sharing markets), there are studies on their pros and cons. Cachon et al. (2017), and Guda and Subramanian (2019) investigated and vindicated the merits of the surge pricing policy for on-demand service platforms with one type of service with self-scheduling capacity, while Banerjee et al. (2015), Chen and Hu (2018), and Hu and Zhou (2018) demonstrated that the static pricing policy may perform well versus the surge pricing policy. Lin and Zhou (2019) studied the two pricing policies for ride-sharing platforms (e.g., Uber and Gett) that provide vertically differentiated services with self-scheduling drivers, and found that static pricing may perform better than surge pricing. Zhou et al. (2019) examined the impacts of different commission contracts on a short-term rental platform (e.g., Airbnb) providing horizontally differentiated services with self-scheduling agents. Their main results show that different commission contracts can considerably affect the platform's profit, consumer surplus, and provider surplus. Instead of non-spatial pricing, Bimpikis et al. (2019), Ozkan and Ward (2017), and He et al. (2017) analyzed region-related pricing for ride-sharing platforms. They demonstrated that the prices of rides may depend on the locations they originate from. There is also literature that takes into account waiting time and delay-sensitive customers. Taylor (2018) first explicitly modelled delay-sensitive customers for an on-demand service platform based on the queuing framework, and showed that delay sensitivity has significant impacts on the optimal price and wage. Bai et al. (2018) used a similar model to demonstrate that a dynamic payout ratio/payout ratio can lead to a win-win-win outcome, i.e., the drivers, customers, and platform all become better off. Feng et al. (2017) answered the question of whether the ride-sharing system helps reduce passengers' waiting time and found that it can lead to higher or lower efficiency than the traditional street-hailing mechanism. Benjaafar et al. (2018a) analyzed labour welfare on an on-demand service platform with self-scheduling agents and delay-sensitive customers, and showed that labour welfare is not necessarily monotonic in the size of the labour pool. Cohen and Zhang (2017) may be the first to consider price competition between a ride-sharing platform and a taxi-hailing company, and found that a win-win outcome for them can be achieved under certain conditions.

Our paper differs from the above studies in two aspects. First, we include not only self-scheduling agents but also permanent agents in our model, and study the subsidy strategy for the temporary agents and the employing strategy for the permanent agents in both monopoly and duopoly competition environments. Second, we examine the impacts of employing permanent agents and platform competition on the subsidy strategies, profitability, and social welfare.

3. A monopoly platform

In this section we consider a monopoly platform with customers on one side and agents on the other side. The customers announce their requested services via the platform, which delivers the requested service information to the agents available on the other side. The available agents deliver the services requested by the customers. We study how the platform enhances the service transactions between the agents and customers.

3.1. Platform employing temporary agents only

First, we consider the case where a monopoly platform employs temporary agents only. Such a scenario is common in reality. For example, platforms like Gett and TaskRabbit have temporary agents only. To attract more agents to be active on it, the platform offers a subsidy η for each service undertaken by the temporary agents. Suppose that on the other side of the platform there are homogeneous customers of a total size N that request services during a given period. Each customer requests one service only, which is fulfilled by one agent only. After the service, the customer is charged p by the platform. Fig. 1 is an illustration of this case.

Since all the agents are self-scheduling, they are not affiliated with and dictated by the platform. Therefore, the number of temporary agents available on the platform is not stable. Generally, this number is positively related to the subsidy level provided by the platform. Without loss of generality and to simplify analysis, let $\alpha\eta$ denote the number of temporary agents that join the platform, where $\alpha \geq 1$ represents the sensitivity of the temporary agents to the subsidy η. A greater α suggests that the temporary agents are more sensitive towards the subsidy. Thus, the total supply of agents on the platform A equals the number of temporary agents $\alpha\eta$, i.e.,

$$A = \alpha\eta.$$

We assume that the market concerned is supply-dominant, which means that the total supply of agents can be fully consumed by the customers. For convenience, we summarize and justify our assumptions made in this paper in Table 2, and highlight what may happen if some of the assumptions do not hold in the subsequent analysis.

Therefore, the demand fulfilled equals the total supply of agents on the platform. We adopt a linear demand function to reflect the relationship between the price charged by the platform and the demand fulfilled as follows:

$$p = N - A,$$

which implies that the price is reasonably decreasing in the total supply A of agents because the total supply needs to be fully consumed by the customers.

The platform's revenue comes from charging the services and its cost from paying for the temporary agents. The platform's profit is the difference between its revenue and cost, i.e.,

$$\pi(\eta) = pA - \alpha\eta^2.$$

Lemma 1. *is concave in η .*

The proof of Lemma 1 is straightforward and omitted for brevity. Lemma 1 means that $\pi(\eta)$ has a unique maximum value. The first-order condition that maximizes the platform's profit is $\frac{\partial\pi}{\partial\eta} = 0$, which gives the platform's optimal subsidy η^* for the temporary agents, the optimal total supply of agents A^*, the optimal price charged to customers p^*, and the maximum profit π^*, respectively, as follows:

$$\eta^* = \frac{N}{2(\alpha+1)},\ A^* = \frac{\alpha N}{2(\alpha+1)},\ p^* = \frac{(\alpha+2)N}{2(\alpha+1)},\ \pi^* = \frac{\alpha N^2}{4(\alpha+1)}. \quad (1)$$

From (1), we obtain the following proposition. The proof is straightforward and thus omitted.

Proposition 1. *(i)* $\frac{\partial\eta^*}{\partial N} > 0$, $\frac{\partial p^*}{\partial N} > 0$, $\frac{\partial A^*}{\partial N} > 0$, *and* $\frac{\partial\pi^*}{\partial N} > 0$; *(ii)* $\frac{\partial\eta^*}{\partial\alpha} \leq 0$, $\frac{\partial A^*}{\partial\alpha} \geq 0$, $\frac{\partial A^*}{\partial\alpha} \geq 0$ and $\frac{\partial\pi^*}{\partial\alpha} \geq 0$.

Proposition 1 shows that for a monopoly platform with temporary agents only, *(i)* the subsidy level offered to the temporary agents, the total supply level of agents, the price charged to customers, and the platform profit are intuitively increasing in the potential number of customers N; *(ii)* its subsidy level offered to the temporary agents decreases, but its total supply level of agents and maximum profit increase (although resulting in a lower price charged to customers) as the sensitivity of the temporary agents to the subsidy increases. In other words, when the sensitivity of the temporary agents to the subsidy rises, the platform should adjust its subsidy downwards, which can benefit the platform itself and the customers as well. This result seems at odds with our expectation, but can be explained as follows: Because when the sensitivity of the temporary agents to the subsidy becomes higher, the temporary agents are more easily attracted to the platform by the subsidy. In this case, offering a slightly lower subsidy can still attract more agents to the platform. Since the total supply level of agents on the platform increases, the platform can charge customers a slightly lower price because the total size of the customers is given, which guarantees that the revenue from serving more customers is greater

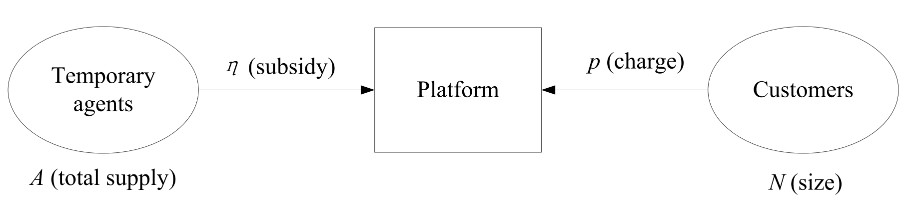

Fig. 1. A monopoly platform with temporary agents only.

Table 2
A summary of the assumptions of this study.

Assumption	Justification
Supply-dominant market	Lack of supply in peak-hour traffic in practice
Without network externality	Partly characterized by the supply-dominant setting because higher supply leads to higher demand and vice versa
Multi-homing customers and agents (in the duopoly scenario)	A commonly seen phenomenon in practice

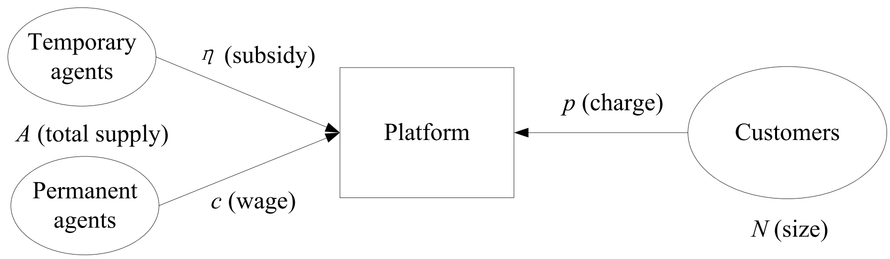

Fig. 2. A monopoly platform with both temporary and permanent agents.

than the cost incurred by subsidizing more agents. As a result, the platform's maximum profit increases.

Next, we analyze the social welfare of the monopoly platform with temporary agents only. The surplus of the customers is

$$CS = \int_{p^*}^{N} A(p)\,dp = \int_{\frac{(\alpha+2)N}{2(\alpha+1)}}^{N} (N-p)\,dp = \frac{\alpha^2 N^2}{8(\alpha+1)^2}. \tag{2}$$

The surplus of the agents is

$$AS = \int_0^{\eta^*} A(\eta)\,d\eta = \int_0^{\frac{N}{2(\alpha+1)}} \alpha\eta\,d\eta = \frac{\alpha N^2}{8(\alpha+1)^2}. \tag{3}$$

Thus, the social welfare, including the surpluses of the customers and agents, and the profit of the platform, is

$$S = CS + AS + \pi^* = \frac{3\alpha N^2}{8(\alpha+1)}. \tag{4}$$

From (2)–(4), we derive Proposition 2.

Proposition 2. *(i)* $\frac{\partial CS}{\partial N} > 0$, $\frac{\partial AS}{\partial N} > 0$, *and* $\frac{\partial S}{\partial N} > 0$; *(ii)* $\frac{\partial CS}{\partial \alpha} \geq 0$, $\frac{\partial AS}{\partial \alpha} \leq 0$, *and* $\frac{\partial S}{\partial \alpha} \geq 0$.

Proposition 2 shows that if a monopoly platform employs temporary agents only, (*i*) the surplus of the customers, the surplus of the agents, and the social welfare are increasing in the potential number of customers *N*; and (*ii*) the surplus of the customers and the social welfare increase, but the surplus of the agents decreases, as the sensitivity of the temporary agents to the subsidy increases. The former result, i.e., (*i*), is consistent with our intuition, whereas the logic behind Proposition 2(*ii*) is as follows: From Proposition 1(*ii*), we know that an increase in the sensitivity of the temporary agents to the subsidy leads to a lower price charged to the customers, which produces a larger surplus of the customers. In addition, an increase in the sensitivity of the temporary agents to the subsidy leads to a lower subsidy offered by the platform. Therefore, the surplus of the agents decreases with decreasing subsidy. With an increase in the sensitivity of the temporary agents to the subsidy, the increases in the surplus of the customers and the profit of the platform may surpass the decrease in the surplus of the agents. Consequently, the social welfare increases with increasing sensitivity of the temporary agents to the subsidy.

3.2. Platform employing both temporary and permanent agents

In this subsection we consider the case where the monopoly platform employs both temporary and permanent agents. Such a scenario also exists in reality. For example, Didi has embarked on employing permanent agents for its platform. In this case, in order to guarantee a basic service level, the platform employs *b* permanent agents and pays each agent *c* in a given period, where *b* is a decision variable that reflects the platform's employment level of the permanent agents and *c* is a constant parameter that denotes the wage level of the permanent agents. On the other hand, to further increase the availability of agents on the platform, the platform also employs temporary agents by offering a subsidy η for each service fulfilled by a temporary agent. Generally, the number of temporary agents willing to join the platform is positively related to the subsidy provided by the platform. Similar to Section 3.1, we assume that the number of temporary agents that join the platform is proportional to the subsidy offered, i.e., $\alpha\eta$, where $\alpha \geq 1$ represents the sensitivity of the temporary agents to the subsidy. Therefore, the total supply level of agents on the platform *A* is

$$A = b + \alpha\eta.$$

Likewise, on the other side of the platform there are homogeneous customers of a total size *N* that request services in a given period. Each customer requests one service only, which is fulfilled by one agent only. After the service, the customer is charged *p* by the platform. Fig. 2 provides an illustration of this case.

Since we assume that the market is supply-dominant, the demand fulfilled is equal to the total supply of agents on the platform. Hence, the price charged to the customers by the platform is given by $p = N - A$, where $A = b + \alpha\eta$. Similarly, the price is reasonably decreasing in the total supply of agents. Thus, in this case the platform's profit is

$$\pi(\eta, b) = pA - \alpha\eta^2 - cb.$$

Lemma 2. *is concave in* η *and* b.

We provide the proof of Lemma 2 in the Appendix. Lemma 2 indicates that $\pi(\eta, b)$ has a unique maximum point (η, b), which is determined by the first-order optimality condition. The first-order condition that maximizes the platform's profit is

$$\begin{cases} \frac{\partial \pi}{\partial \eta} = N - 2(\alpha+1)\eta - 2b = 0, \\ \frac{\partial \pi}{\partial b} = N - 2\alpha\eta - 2b - c = 0. \end{cases}$$

Solving the above equation gives the platform's optimal subsidy η^* for the temporary agents and the optimal employment level b^* of the permanent agents as follows:

$$\eta^* = \frac{c}{2},\ b^* = \frac{N-(\alpha+1)c}{2}. \tag{5}$$

To guarantee the non-negativity of b^*, the condition $c \le \frac{N}{\alpha+1}$ must hold, i.e., the platform employs permanent agents only when the wage level of the permanent agents does not exceed a certain upper bound. This condition is reasonable in reality. From (5), we see that for a monopoly platform with both temporary and permanent agents, the optimal subsidy offered by the platform is independent of the total size N of customers that request services and the sensitivity α of the temporary agents to the subsidy, which is different from the corresponding result for a monopoly platform with temporary agents only. Specifically, the platform sets its optimal subsidy by only referring to the wage of the permanent agents, and its optimal employment level of the permanent agents is less than half of the total size of the customers that request services.

From (5), one can easily derive the platform's optimal total supply of agents A^*, the optimal price charged to the customers p^*, and the maximum profit π^*, respectively, as follows:

$$A^* = \frac{N-c}{2},\ p^* = \frac{N+c}{2},\ \pi^* = \frac{N(N-2c)+(\alpha+1)c^2}{4}. \tag{6}$$

From (5) and (6), we obtain the following proposition.

Proposition 3. *(i)* $\frac{\partial\eta^*}{\partial N} = 0,\ \frac{\partial p^*}{\partial N} > 0,\ \frac{\partial b^*}{\partial N} > 0,\ \frac{\partial A^*}{\partial N} > 0,$ *and* $\frac{\partial\pi^*}{\partial N} > 0$; *(ii)* $\frac{\partial\eta^*}{\partial c} \ge 0,\ \frac{\partial p^*}{\partial c} \ge 0,\ \frac{\partial b^*}{\partial c} \le 0,\ \frac{\partial A^*}{\partial c} \le 0,$ *and* $\frac{\partial\pi^*}{\partial c} \le 0$; *(iii)* $\frac{\partial\eta^*}{\partial\alpha} = 0,\ \frac{\partial p^*}{\partial\alpha} = 0,\ \frac{\partial b^*}{\partial\alpha} \le 0,\ \frac{\partial A^*}{\partial\alpha} = 0,$ *and* $\frac{\partial\pi^*}{\partial\alpha} \ge 0$.

Proposition 3(*i*) reveals that for a monopoly platform with both temporary and permanent agents, the optimal employment level of the permanent agents, the total supply level of agents, and the platform's profit are increasing in the potential number of customers, while interestingly the optimal subsidy charged to customers is independent of the pool of customers. The former statement is intuitive, while the latter one is because the platform can recruit permanent agents to satisfy larger potential demand. Note that this result is in stark contrast to the case where the platform only recruits temporary agents; (*ii*) states that for a monopoly platform with both temporary and permanent agents, the optimal subsidy charged to customers increases, but the optimal employment level of the permanent agents, the total supply level of agents, and the platform's maximum profit decrease in the wage paid to the permanent agents. This result seems to make sense. Because an increase in the wage of the permanent agents means that the labour cost in the labour market increases. Thus, to attract temporary agents to join the platform, it is natural for the platform to increase the subsidy for the temporary agents accordingly. However, increases in both the wage of the permanent agents and the subsidy for the temporary agents lead to a greater expenditure of the platform. To strike a balance, the platform needs to lower the employment level of the permanent agents, thereby increasing the price charged to the customers, which leads to a decline in customers' demand. Because the decline in the agents' supply and the rise in the wage of the permanent agents dominate the increase in the price charged to customers, the platform's profit decreases in the wage of the permanent agents; (*iii*) indicates that for a monopoly platform with both temporary and permanent agents, the platform's profit increases but the employment level of the permanent agents decreases as the sensitivity of the temporary agents to the subsidy increases; while the total supply of agents on the platform and the subsidy for the temporary agents remain unchanged. The former seemingly makes sense. However, the latter seems out of our expectation. This result highlights the following managerial implications for the monopoly platform with both temporary and permanent agents. When the sensitivity of the temporary agents to the subsidy increases, the monopoly platform can still attract more temporary agents to join the platform by keeping the same optimal subsidy as before. Thus, the platform can reduce the number of permanent agents to keep the same total supply of agents as before, which can bring more profit to the platform than before.

In the following we provide an analysis of the social welfare of the monopoly platform. In this case, the surplus of the customers is

$$CS = \int_{p^*}^{N} A(p)\,dp = \int_{\frac{N+c}{2}}^{N} (N-p)\,dp = \frac{(N-c)^2}{8}. \tag{7}$$

The surplus of the agents, including both permanent and temporary agents, is

$$AS = \int_0^{\eta^*} \alpha\eta\,d\eta + \int_0^{c} b^* dc = \int_0^{\frac{c}{2}} \alpha\eta d\eta + \int_0^{c} \frac{N-(\alpha+1)\mu}{2} d\mu = \frac{\alpha c^2}{8} + \frac{c[2N-(\alpha+1)c]}{4(\alpha+1)}. \tag{8}$$

Thus, the social welfare, including the surpluses of the customers and agents, and the profit of the platform, is

$$S = CS + AS + \pi^* = \frac{3N^2}{8} + \frac{c(3\alpha+1)[(\alpha+1)c-N]}{8(\alpha+1)}. \tag{9}$$

From (7)–(9), we derive Proposition 4, whose proof is omitted for brevity.

Proposition 4. *(i)* $\frac{\partial CS}{\partial N} > 0,\ \frac{\partial AS}{\partial N} > 0,$ *and* $\frac{\partial S}{\partial N} > 0$; *(ii)* $\frac{\partial CS}{\partial c} < 0,\ \frac{\partial AS}{\partial c} > 0,$ *and* $\frac{\partial S}{\partial c} < 0$ *if* $0 < c < \frac{N}{2(\alpha+1)}$; *otherwise,* $\frac{\partial S}{\partial c} > 0$; *(iii)* $\frac{\partial CS}{\partial\alpha} = 0,\ \frac{\partial AS}{\partial\alpha} > 0$ *if* $0 < c < \frac{N}{4}$ *and* $2\sqrt{\frac{N}{c}} - 1 < \alpha < \frac{N}{c} - 1$; *otherwise,* $\frac{\partial AS}{\partial\alpha} < 0,\ \frac{\partial S}{\partial\alpha} < 0$ *if* $0 < c < \frac{2N}{3}$ *and* $0 < \alpha < 2\sqrt{\frac{2N}{3c}} - 1$; *otherwise,* $\frac{\partial S}{\partial\alpha} > 0$.

Proposition 4(*i*) indicates that the consumer surplus, the agent surplus, and the social welfare are all increasing in the potential number of customers. The reason is that the platform needs to improve the associated surpluses by employing more temporary and permanent agents and satisfying larger potential demand. As the wage of the permanent agents increases, however, Proposition 4(*ii*) shows that the surplus of the customers decreases but the surplus of the agents increases, whereas the social welfare decreases for a relatively small c (e.g., $0 < c < \frac{N}{2(\alpha+1)}$) but increases for a relatively large c (e.g., $\frac{N}{2(\alpha+1)} \le c \le \frac{N}{(\alpha+1)}$). The former makes sense because raising the wage of the permanent agents means a rise in the labour cost in the labour market, which naturally leads to a rise in the subsidy for the temporary agents. Therefore, the surpluses of both the permanent and temporary agents increase. On the other hand, when the platform raises the wage of the permanent agents, the cost incurred by the platform rises. This leads to a rise in the price charged to the customers by the platform. Consequently, the surplus of the customers or employees declines. However, the latter seems inconsistent with our expectation. One possible reason is that when the wage of the permanent agents rises, i.e., the labour cost in the market rises, the competition in the labour market intensifies. Hence, it is naturally beneficial for the monopoly ride-sharing platform to provide intermediary services between the agents and customers. In addition, the increase in both agents' surpluses may dominate the decrease in the customers' surplus and the platform's profit, so the total social welfare increases in the wage of the permanent agents when it exceeds a certain threshold. Proposition 4(*iii*) indicates that the surplus of the customers is not affected by but the platform's profit increases in the sensitivity of the temporary agents to the subsidy. The reason is that the total supply of agents and the price charged to the customers are not affected by the sensitivity of the temporary agents to the subsidy. Moreover, when the sensitivity of the temporary agents to the subsidy is relatively low, the surpluses of the agents decrease in the sensitivity of the temporary agents to the subsidy; whereas when the sensitivity of the temporary agents to the subsidy is relatively high, it is just the reverse. This observation is mainly due to the fact that the surplus of the temporary agents increases but the surplus of the permanent agents decreases in the sensitivity of the temporary agents to the subsidy. When the sensitivity of the temporary agents to the subsidy exceeds a certain threshold, the increasing rate of the temporary agents' surplus may dominate the decreasing rate of the permanent agents' surplus. Also, when the wage of the permanent agents is relatively low, the social welfare first decreases and then increases in the sensitivity of the temporary agents to the subsidy; whereas when the wage of the permanent agents is relatively high, the social welfare always increases in the sensitivity of the temporary agents to the subsidy. This observation implies that for a monopoly platform with both temporary and permanent agents, the social welfare does not always increase in the sensitivity of the temporary agents to the subsidy, which is different from the counterpart case where the monopoly platform has temporary agents only.

3.3. Comparison between two cases

In the following we compare the optimal strategies of the platform, and the corresponding participants' surpluses and social welfare under the two cases. For convenience, we use the subscript "*M*" to denote the "monopoly platform", and the superscripts "*to*" and "*tp*" to denote "temporary agents only" and "temporary and permanent agents", respectively. We have the following result.

Theorem 1. $\eta_M^{to} < \eta_M^{tp}$, $A_M^{to} < A_M^{tp}$, $p_M^{to} > p_M^{tp}$, *and* $\pi_M^{to} < \pi_M^{tp}$.

Theorem 1 shows that since the monopoly platform can employ permanent agents instead of temporary agents to satisfy demand, the platform can offer a lower subsidy and own a much less total supply of agents, which results in a higher price if it employs temporary agents only than if it employs both temporary and permanent agents. Consequently, the platform obtains much less profit if it employs the temporary agents only than if it employs both temporary and permanent agents. This implies that the monopoly platform has an incentive to employ some permanent agents so as to enhance the service level. This result explains why Didi adopts the policy of employing some permanent agents.

Theorem 2. $AS_M^{to} > AS_M^{tp}$, $CS_M^{to} < CS_M^{tp}$, *and* $S_M^{to} < S_M^{tp}$.

Theorem 2 indicates that the monopoly platform that employs temporary agents only yields more surplus to the temporary agents, but less surplus to the customers, and less social welfare than does the monopoly platform that employs both temporary and permanent agents. Simply, this is because another agent source, i.e., permanent agents, makes the platform more flexible to balance demand and supply. This result implies that employing permanent agents not only benefits the platform itself but also increases the customers' surplus and social welfare.

4. Duopoly platforms

In this section we consider two competing platforms. As intermediaries, they compete against each other for agents and customers. Both customers and agents are multi-homing, i.e., the customers and agents can use the two competing platforms simultaneously. Customers can call for their services on one or the two platforms, which deliver the requested service information to the agents available on platforms. An agent available on one or the two platforms picks up the customer's request and delivers the service. Such competing platforms are common in reality, e.g., UberChina and Didi in China (before UberChina merged with Didi), and Uber and Lyft in America etc.

4.1. Duopoly platforms with temporary agents only

Consider the case where the two platforms employ temporary agents only. Fig. 3 illustrates this case.

Suppose that there are platforms 1 and 2, which attract self-scheduling temporary agents by offering subsidies η_1 and η_2, respectively. Generally speaking, the higher the subsidy offered by a platform, the more are the temporary agents attracted to the platform. To simplify analysis, we assume that the numbers of temporary agents attracted to the two platforms are proportional to their individual subsidies η_1 and η_2, respectively. Let $\alpha\eta_1$ denote the number of temporary agents attracted to platform 1 and $\alpha\eta_2$ the number of temporary agents attracted to platform 2, where $\alpha \geq 1$ represents the attractiveness of the offered subsidies to the temporary agents. A greater α means that the same subsidy can attract more temporary agents.

Since both platforms are competing against each other for the temporary agents, a rise in the subsidy offered by one platform may cause a decline in the supply of temporary agents available on the other platform, and vice versa. Without loss of generality, the reduced number of temporary agents on one platform is positively related to the subsidy offered by the other platform. To simplify analysis, let $\theta\eta_2$ and $\theta\eta_1$ denote the reduced numbers of temporary agents on platforms 1 and 2 due to competition from their individual rival platforms, respectively. $\theta \in [0,1]$ denotes the degree of competition between the two platforms. The assumption that $\alpha \geq \theta$ is to guarantee that the reduced number of temporary agents on one platform caused by its rival platform's competition does not exceed its available temporary agents, i.e., the attractiveness of the subsidy is always greater than or equal to the degree of competition between the two rival platforms.

Therefore, the total supplies of agents on platforms 1 and 2 are A_1 and A_2, respectively, as follows:

$$A_1 = \alpha\eta_1 - \theta\eta_2,\ \ and\ A_2 = \alpha\eta_2 - \theta\eta_1.$$

Assume that during the considered time period, the total size of homogeneous customers that request services on both platforms is *N*. Each customer requests one service only and each agent delivers one such service only. The service demand market considered is supply-dominant, i.e., the total supply of the service can be entirely consumed by the demand. In other words, the demand fulfilled is equivalent to the supply available. Therefore, the price charged to the customers by the platforms are determined by the supplies of agents on the platforms. This implies that we implicitly assume that the model considers the platforms' capacity/quantity decisions. Consequently, according to Cournot's duopoly model, the price charged to the customers by both platforms is

$$p = N - A_1 - A_2 = N - (\alpha - \theta)(\eta_1 + \eta_2),$$

which is reasonably decreasing in the total supply of the service.

The revenues of both platforms come from charging the customers for the services and the costs of both platforms from paying for the temporary agents. The profits of both platforms are the differences between their individual revenues and costs, which are given by

$$\pi_1(\eta_1) = pA_1 - \alpha\eta_1^2,\ \pi_2(\eta_2) = pA_2 - \alpha\eta_2^2. \tag{10}$$

Lemma 3. *is concave in* η_i *for* $i = 1,2$.

Lemma 3 can be easily proved and we omit its proof for brevity. From (10), the first-order condition that maximizes both platforms' profit functions is

$$\begin{cases} \frac{\partial\pi_1}{\partial\eta_1} = \alpha N - 2\alpha(\alpha - \theta + 1)\eta_1 - (\alpha - \theta)\eta_2 = 0, \\ \frac{\partial\pi_2}{\partial\eta_2} = \alpha N - 2\alpha(\alpha - \theta + 1)\eta_2 - (\alpha - \theta)\eta_1 = 0. \end{cases}$$

Solving the above equation yields the equilibrium strategies of two platforms, which are stated in Lemma 4.

Lemma 4. *Platform i's optimal subsidy* η_i^* *for the temporary agents (for* $i = 1,2$*) is given by*

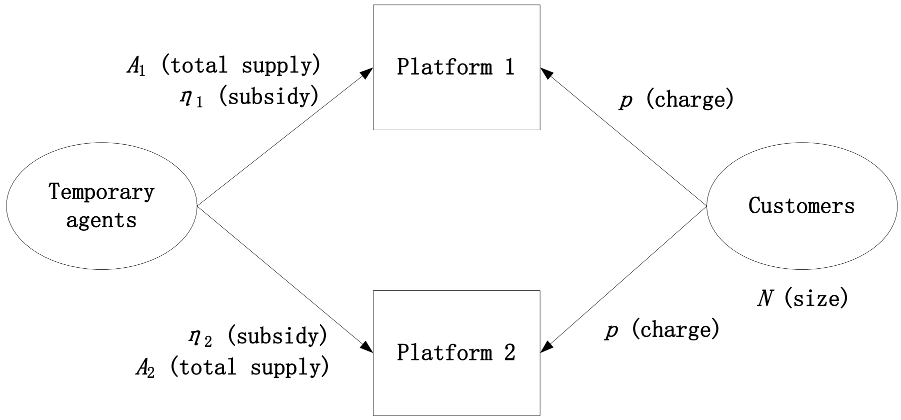

Fig. 3. Duopoly platforms with temporary agents only.

$$\eta_1^* = \eta_2^* = \eta^* = \frac{\alpha N}{(\alpha - \theta)(2\alpha + 1) + 2\alpha}. \tag{11}$$

Therefore, we can obtain each platform's total supply of agents, price charged to the customers, and corresponding profit as follows:

$$\begin{cases} A_1^* = A_2^* = A^* = \frac{\alpha(\alpha-\theta)N}{(\alpha-\theta)(2\alpha+1)+2\alpha}, \\ p^* = \frac{(3\alpha-\theta)N}{(\alpha-\theta)(2\alpha+1)+2\alpha}, \\ \pi^* = \frac{(2\alpha^2+\theta^2-4\alpha\theta)\alpha N^2}{[(\alpha-\theta)(2\alpha+1)+2\alpha]^2}. \end{cases} \tag{12}$$

From Lemma 4 and (12), we obtain Proposition 5.

Proposition 5. *(i)* $\frac{\partial\eta^*}{\partial N} > 0$, $\frac{\partial A^*}{\partial N} > 0$, $\frac{\partial p^*}{\partial N} > 0$ *and* $\frac{\partial\pi^*}{\partial N} > 0$*; (ii)* $\frac{\partial\eta^*}{\partial\alpha} \le 0$, $\frac{\partial A^*}{\partial\alpha} \ge 0$, $\frac{\partial p^*}{\partial\alpha} \le 0$, *and* $\frac{\partial\pi^*}{\partial\alpha} > 0$ *for* $1 \le \alpha \le \alpha_1$ *and* $\frac{\partial\pi^*}{\partial\alpha} < 0$ *for* $\alpha \ge \alpha_1$, *where* α_1 *is the unique root of the equation* $-4\alpha^4 - 4\alpha^3\theta + 26\alpha^2\theta^2 - 6\alpha\theta^3 + 22\alpha^3 - 38\alpha^2\theta + 13\alpha\theta^2 - \theta^3 = 0$*; (iii)* $\frac{\partial\eta^*}{\partial\theta} \ge 0$, $\frac{\partial p^*}{\partial\theta} \ge 0$, $\frac{\partial A^*}{\partial\theta} \le 0$ *and* $\frac{\partial\pi^*}{\partial\theta} < 0$.

Proposition 5(*i*) shows that for duopoly platforms employing temporary agents only, the subsidy paid to the temporary agents, the total supply of temporary agents, and the platform profit are all increasing in the potential number of customers. If the attractiveness of the offered subsidy to the temporary agents increases, Proposition 5(*ii*) demonstrates that the platforms would reduce the subsidy to the temporary agents, but increase the total supply of temporary agents instead, which results in a lower price charged to customers. The will bring higher profits for the platforms with relatively small attractiveness of the subsidy because it would become easier for the platforms to employ temporary agents as the attractiveness increases. Similar explanations as in Proposition 1 can be applied to the above results. Proposition 5(*iii*) indicates that for duopoly platforms with temporary agents only, the subsidies offered to the temporary agents increase in the degree of competition between the two platforms, whereas the total supply of agents and thereby the profits decrease in the degree of competition between the two platforms, which leads to higher prices charged to the customers. Generally speaking, when the competition between the two platforms intensifies, it is natural for both platforms to increase the subsidy for attracting temporary agents. However, when the competition between the two platforms intensifies, much fewer temporary agents would be available on both platforms. This observation seems inconsistent with our expectation. One possible reason is as follows: The competition between the two platforms becomes fiercer, which implies that the size of the supply market of the temporary agents is relatively smaller. In this case, even if both platforms raise their subsidies a little bit, the total supply of temporary agents on both platforms will still decline.

Next, we provide an analysis of the social welfare of the duopoly platforms with temporary agents only. The surplus of the customers is

$$CS = 2\int_{p^*}^{N} A(p)dp = 2\int_{\frac{(3\alpha-\theta)N}{(\alpha-\theta)(2\alpha+1)+2\alpha}}^{N} (N-p)dp = 4.$$
$$\left[\frac{\alpha(\alpha-\theta)N}{(\alpha-\theta)(2\alpha+1)+2\alpha}\right]^2 \tag{13}$$

The surplus of the agents is

$$AS = 2\int_0^{\eta^*} A(\eta)d\eta = 2\int_0^{\frac{\alpha N}{(\alpha-\theta)(2\alpha+1)+2\alpha}} (\alpha-\theta)\eta d.$$
$$\eta = (\alpha-\theta)\left[\frac{\alpha N}{(\alpha-\theta)(2\alpha+1)+2\alpha}\right]^2 \tag{14}$$

Thus, the social welfare is the sum of the surplus of the customers, the surplus of the agents, and the profit of the platform, which is $S = CS + AS + \pi^*$.

From (12)–(14), we can easily derive Proposition 6, whose proof is omitted for brevity.

Proposition 6. *(i)* $\frac{\partial CS}{\partial N} > 0$, $\frac{\partial AS}{\partial N} > 0$, *and* $\frac{\partial S}{\partial N} > 0$*; (ii)* $\frac{\partial CS}{\partial\alpha} > 0$, $\frac{\partial AS}{\partial\alpha} > 0$, *and* $\frac{\partial S}{\partial\alpha} > 0$*; (iii)* $\frac{\partial CS}{\partial\theta} < 0$, $\frac{\partial AS}{\partial\theta} < 0$ *if* $1 \le \alpha \le \frac{3+\sqrt{17}}{4}$ *and* $\frac{\alpha(2\alpha-1)}{2\alpha+1} \le \theta \le 1$*; otherwise,* $\frac{\partial AS}{\partial\theta} > 0$, *and* $\frac{\partial S}{\partial\theta} < 0$.

Proposition 6(*i*) and (*ii*) show that when duopoly platforms compete against each other for the temporary agents, the social welfare and the surpluses of both customers and agents increase in the potential number of customers, and the sensitivity of the temporary agents to the subsidy. Similar explanation of Proposition 2(*i*) can be applied to Proposition 6(*i*). However, the observation of Proposition 6(*ii*) is different from the corresponding one under the monopoly platform with temporary agents only. As shown in Subsection 3.1, for the monopoly platform with temporary agents only, the surpluses of the agents decrease in the sensitivity of the temporary agents to the subsidy. This implies that when duopoly platforms compete for the temporary agents, an increase in the sensitivity of the temporary agents to the subsidy benefits themselves but may harm the duopoly platforms. One possible explanation is that competition between platforms weakens their control over the subsidy. Proposition 6(*iii*) indicates that when duopoly platforms compete for the temporary agents, the surplus of the customers, the two platforms' profits, and the social welfare decrease with the degree of competition between the two platforms, whereas the surpluses of the agents increase with the degree of competition between the two platforms except when θ is relatively large but α is relatively small. This implies that competition between platforms may benefit the agents.

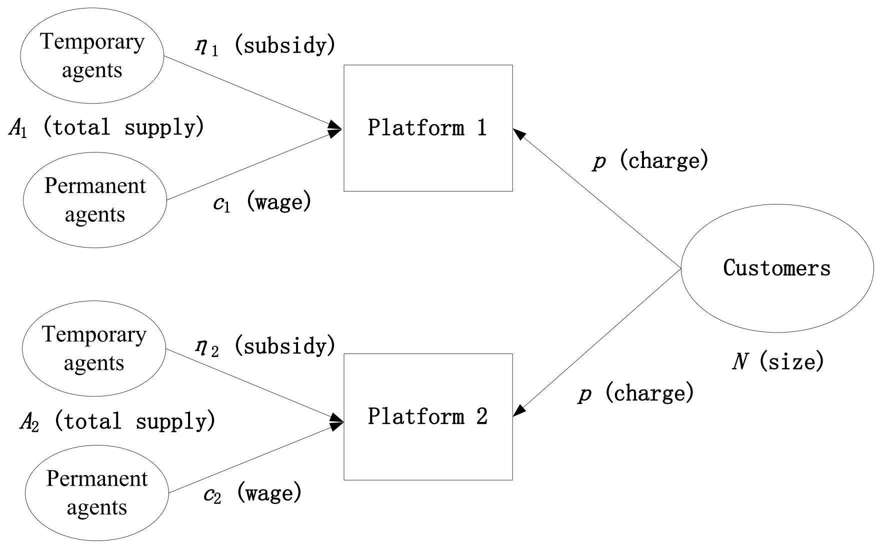

Fig. 4. Duopoly platforms with both temporary and permanent agents.

4.2. Duopoly platforms with both temporary and permanent agents

In this subsection we consider the case where the duopoly platforms employ both temporary and permanent agents. This case is shown in Fig. 4.

To provide a basic service level, platform 1 employs b_1 permanent agents and offers each of them a wage of c_1, whereas platform 2 employs b_2 permanent agents and provides each of them with a wage of c_2. Both b_1 and b_2 are decision variables that represent platform 1's and 2's employment levels of the permanent agents, respectively. c_1 and c_2 are exogenous parameters, which are both assumed not to exceed $N/(\alpha + 1)$ for subsequent comparison. To avoid paying too high wages for the permanent agents, platforms 1 and 2 attract self-scheduling temporary agents by offering subsidies of η_1 and η_2, respectively. To simplify analysis, we assume that the numbers of temporary agents attracted to the two platforms are proportional to their individual subsidies η_1 and η_2, respectively. Let $\alpha\eta_1$ denote the number of temporary agents attracted to platform 1 and $\alpha\eta_2$ the number of temporary agents attracted to platform 2, where $\alpha \geq 1$ represents the attractiveness of the offered subsidy to the temporary agents. A greater α means that the same subsidy can attract more temporary agents.

Since both platforms are competing for the temporary agents against each other, a rise in the subsidy offered by one platform may cause a decline in the supply of temporary agents on the other platform, and vice versa. Similarly, for simplicity, we also assume that the reduced numbers of temporary agents on platforms 1 and 2 due to their individual rival platform's competition are given by $\theta\eta_2$ and $\theta\eta_1$, respectively. Therefore, the total supplies of agents on platforms 1 and 2 are, respectively, as follows:

$A_1 = b_1 + \alpha\eta_1 - \theta\eta_2$ and $A_2 = b_2 + \alpha\eta_2 - \theta\eta_1$.

According to Cournot's duopoly model, the price charged to the customers by both platforms is

$p = N - A_1 - A_2$.

Thus, the profits of the two platforms are

$$\pi_1(\eta_1, b_1) = pA_1 - \alpha\eta_1^2 - c_1 b_1,\ \pi_2(\eta_2, b_2) = pA_2 - \alpha\eta_2^2 - c_2 b_2. \quad (15)$$

Lemma 5. *$\pi_i(\eta_i, b_i)$ is concave in η_i and b_i for $i = 1,2$.*

Lemma 5 indicates that the maximum of platform i's profit function $\pi_i(\eta_i, b_i)$ is determined by the first-order optimality condition of $\pi_i(\eta_i, b_i)$ with regard to η_i and b_i. From (15), the first-order condition for maximizing the two platforms' profit functions are

$$\begin{cases} \frac{\partial \pi_1}{\partial \eta_1} = \alpha N - 2\alpha(\alpha - \theta + 1)\eta_1 - (\alpha - \theta)^2\eta_2 - (2\alpha - \theta)b_1 - \alpha b_2 = 0, \\ \frac{\partial \pi_1}{\partial b_1} = N - (2\alpha - \theta)\eta_1 - (\alpha - 2\theta)\eta_2 - 2b_1 - b_2 - c_1 = 0, \\ \frac{\partial \pi_2}{\partial \eta_2} = \alpha N - 2\alpha(\alpha - \theta + 1)\eta_2 - (\alpha - \theta)^2\eta_1 - (2\alpha - \theta)b_2 - \alpha b_1 = 0, \\ \frac{\partial \pi_1}{\partial b_1} = N - (2\alpha - \theta)\eta_2 - (\alpha - 2\theta)\eta_1 - 2b_2 - b_1 - c_2 = 0. \end{cases} \quad (16)$$

The coefficient matrix of the decision variables $(\eta_1, b_1, \eta_2, b_2)$ in (16) is

$$D = \begin{pmatrix} -2\alpha(\alpha - \theta + 1) & -(2\alpha - \theta) & -(\alpha - \theta)^2 & -\alpha \\ -(2\alpha - \theta) & -2 & -(\alpha - 2\theta) & -1 \\ -(\alpha - \theta)^2 & -\alpha & -2\alpha(\alpha - \theta + 1) & -(2\alpha - \theta) \\ -(\alpha - 2\theta) & -1 & -(2\alpha - \theta) & -2 \end{pmatrix}.$$

It is easy to show that the determinant $det(D) = 12\alpha^2 \neq 0$. Therefore, (16) has a unique solution with regard to its variables $(\eta_1, b_1, \eta_2, b_2)$. Solving (16) yields the equilibrium strategies of the two platforms, which are stated in Lemma 6.

Lemma 6. *Platform i's optimal subsidy η_i^* for the temporary agents and employment level b_i^* of the permanent agents ($i, j = 1,2$ and $i \neq j$) are given by*

$$\begin{cases} \eta_i^* = \frac{(3\alpha - 2\theta)c_i + \theta c_j + \theta N}{6\alpha}, \\ b_i^* = \frac{[\theta^2 - (3\alpha - 2\theta + 4)\alpha]c_i + 2(-\theta^2 + \alpha\theta + \alpha)c_j + (\theta^2 - \theta + 2\alpha)N}{6\alpha}. \end{cases} \quad (17)$$

Therefore, we derive the optimal total supplies of agents on platforms 1 and 2, the price charged to the customers by the two platforms, and the corresponding maximum profits of the two platforms as follows:

$$\begin{cases} A_i^* = \frac{-4\alpha c_i + 2\alpha c_j + [(\alpha - 1)\theta + 2\alpha]N}{6\alpha}, \\ p^* = \frac{\alpha(c_1 + c_2) + [\alpha - (\alpha - 1)\theta]N}{3\alpha}, \\ \pi_i^* = p^* A_i^* - \alpha(\eta_i^*)^2 - c_i b_i^*, \end{cases} \quad (18)$$

where $i, j = 1,2$ and $i \neq j$. From (17) and (18), we derive Proposition 7, whose proof is omitted for brevity.

Proposition 7. *For $i, j = 1,2$ and $i \neq j$, we have (i) $\frac{\partial \eta_i^*}{\partial N} > 0$, $\frac{\partial b_i^*}{\partial N} > 0$, $\frac{\partial A_i^*}{\partial N} > 0$, $\frac{\partial p^*}{\partial N} > 0$ and $\frac{\partial \pi_i^*}{\partial N} > 0$; (ii) $\frac{\partial \eta_i^*}{\partial c_i} > 0$, $\frac{\partial b_i^*}{\partial c_i} < 0$, $\frac{\partial A_i^*}{\partial c_i} < 0$, $\frac{\partial \eta_j^*}{\partial c_i} > 0$,*

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$\frac{\partial \pi_i^*}{\partial c_i} < 0$, $\frac{\partial b_j^*}{\partial c_i} > 0$, $\frac{\partial A_j^*}{\partial c_i} > 0$, $\frac{\partial \pi_i^*}{\partial c_j} > 0$ *and* $\frac{\partial p^*}{\partial c_i} > 0$; *(iii)* $\frac{\partial \eta_i^*}{\partial \alpha} < 0$, $\frac{\partial b_i^*}{\partial \alpha} < 0$, $\frac{\partial A_i^*}{\partial \alpha} > 0$, *and* $\frac{\partial p^*}{\partial \alpha} < 0$, *but* π_i^* *is not necessarily monotonic in* α; *(iv)* $\frac{\partial \eta_i^*}{\partial \theta} > 0$, $\frac{\partial A_i^*}{\partial \theta} > 0$, $\frac{\partial p^*}{\partial \theta} < 0$, *and* $\frac{\partial \pi_i^*}{\partial \theta} < 0$, *but* b_i^* *is not necessarily monotonic in* θ.

From Proposition 7, we deduce the following findings. (*i*) The subsidy and the total supply of the temporary agents, the employment level of the permanent agents, and the profit of platform i are intuitively increasing in the potential number of customers. (*ii*) Once the wage of the permanent agents on one platform rises, this platform would reduce the employment level of the permanent agents, which leads to a higher price charged to customers, and increase the subsidy for its temporary agents, which leads to a decrease in the total supply of its agents and the profit, but its rival platform would increase the employment level of the permanent agents and the total supply level of agents that leads to a higher profit. This finding makes sense. As a rise in the wage of the permanent agents on one platform means a rise in the labour cost, it is certain for the platform to reduce the employment level of the permanent agents and increase the subsidy for the temporary agents accordingly. Since the two platforms compete against each other for agents, the rival platform would increase the employment level of its permanent agents and the subsidy for its temporary agents. These actions ultimately lead to a decrease in the total supply of agents on the two platforms with the wage of the permanent agents on one platform. (*iii*) As the attractiveness of the subsidy for the temporary agents increases, the total supply of agents on both platforms increases, which leads to a lower price, but both the subsidy for the temporary agents and the employment level of the permanent agents decrease. This finding also makes sense. Since when the attractiveness of the subsidy for the temporary agents increases, the temporary agents are more easily attracted to join the platform by the subsidy. In that case, both platforms would necessarily reduce their employment levels of the permanent agents and the subsidy for the temporary agents a little bit. Since the temporary agents are more sensitive to the subsidy, the number of temporary agents joining the platforms still increases even if the subsidy may be reduced a little bit. Consequently, the increased number of temporary agents joining the platforms may exceed the reduced number of permanent agents employed. Therefore, the total supply of the agents increases ultimately, which leads to a decrease in the optimal price charged to the customers. (*iv*) As the degree of competition between the duopoly platforms increases, the subsidy for the temporary agents and the total supply of agents on both platforms increase, which leads a lower price and a lower platform profit. This finding makes sense. However, when the degree of competition between the two platforms increases, changes in the employment levels of the permanent agents on the two platforms are complicated, which depend on all the model parameters. Comparing the above findings with the corresponding ones in Proposition 5, we know that the impacts of both the degree of competition and the attractiveness of the subsidy on the two platforms' subsidy and temporary-agents-employment strategies are not influenced by whether or not the two platforms employ the permanent agents.

In order to facilitate comparison, we analyze the social welfare of the case with a symmetric employment cost, i.e., $c_1 = c_2 = c$. For this special case, the equilibrium strategies of the two platforms are

$$\begin{cases} \eta_1^* = \eta_2^* = \eta^* = \frac{3(\alpha-\theta)c+\theta N}{6\alpha}, \\ b_1^* = b_2^* = b^* = \frac{(4\alpha\theta - 2\alpha - 3\alpha^2 - \theta^2)c + (\theta^2 - \theta + 2\alpha)N}{6\alpha}, \\ A_1^* = A_2^* = A^* = \frac{-2\alpha c + [(\alpha-1)\theta + 2\alpha]N}{6\alpha}, \\ p^* = \frac{2\alpha c + [\alpha - (\alpha-1)\theta]N}{3\alpha}. \end{cases} \tag{19}$$

Under these equilibrium strategies, the surplus of the customers is

$$CS = 2\int_{p^*}^{N} A(p)dp = \int_{\frac{2\alpha c + [\alpha-(\alpha-1)\theta]N}{3\alpha}}^{N} (N-p)dp = \frac{\{[2\alpha + (\alpha-1)\theta]N - 2\alpha c\}^2}{18\alpha^2}. \tag{20}$$

The surplus of the agents, including both the permanent and temporary agents, is

$$AS = 2\int_0^{\eta^*} (\alpha-\theta)\eta d\eta + 2\int_0^{c} b^* dc = \frac{(\alpha-\theta)[3(\alpha-\theta)c+\theta N]^2}{36\alpha^2} + \frac{(4\alpha\theta - 2\alpha - 3\alpha^2 - \theta^2)c^2 + 2(\theta^2 - \theta + 2\alpha)\alpha c N}{6\alpha}. \tag{21}$$

Substituting (19) into $\pi = pA - \alpha\eta^2 - cb$ yields each platform's profit. Thus, totalling the surplus of the customers, the surpluses of the agents, and the profit of each platform yields the social welfare.

Proposition 8. *(i)* $\frac{\partial CS}{\partial N} > 0$, $\frac{\partial AS}{\partial N} > 0$, *and* $\frac{\partial S}{\partial N} > 0$; *(ii)* $\frac{\partial CS}{\partial \alpha} > 0$ *and* $\frac{\partial CS}{\partial \theta} > 0$, *but both AS and S are not necessarily monotonic in* θ *and* α.

Similar to Proposition 6, Proposition 8(*i*) shows that when duopoly platforms compete against each other for the temporary and permanent agents, the social welfare and the surpluses of both customers and agents increase in the potential number of customers. Moreover, Proposition 8(ii) indicates that if the duopoly platforms employ both temporary and permanent agents, the customers' surplus would increase but the monotonicity of both agents' surpluses and social welfare are not clear, as the sensitivity of the temporary agents to the subsidy (or the degree of competition between the platforms) increases. This implies that increases in both the sensitivity of the temporary agents to the subsidy and the degree of competition between the two platforms are beneficial to the customers' surplus, but may be detrimental to the agents' surpluses and social welfare. This finding is different from that under the scenario where the two platforms employ the temporary agents only.

4.3. Comparison between the two cases

For convenience, we use the subscript "D" to denote "duopoly platforms", and the superscripts "*to*" and "*tp*" to denote "temporary agents only" and "temporary and permanent agents", respectively. By comparing the two platforms' optimal strategies, and the corresponding participants' surpluses and social welfare, we have the following results.

Theorem 3. *(i)* $\eta_D^{tp} < \eta_D^{to}$ *if* $N > \frac{3(\alpha-\theta)c\Omega}{6\alpha^2 - \theta\Omega}$; *otherwise,* $\eta_D^{tp} > \eta_D^{to}$; *(ii)* $A_D^{tp} > A_D^{to}$ *if* $1 \le \alpha \le 3$ *and* $N \ge N_1$ *or* $\theta_1 < \theta \le 1$ *and* $N \ge N_1$; *otherwise,* $A_D^{tp} < A_D^{to}$; *(iii)* $p_D^{tp} < p_D^{to}$ *if* $[\alpha - \theta(\alpha-1)]\,\Omega - 3\alpha(3\alpha-\theta) < 0$ *and* $N > N_2$; *otherwise,* $p_D^{tp} > p_D^{to}$;

where $\Omega = (\alpha-\theta)(2\alpha+1) + 2\alpha$, $N_1 = \frac{2\alpha c\Omega}{2\alpha[(\alpha-\theta)(\alpha-1) - 2\alpha] - \theta(\alpha-1)\Omega}$, $N_2 = \frac{2\alpha c\Omega}{3\alpha(3\alpha-\theta)[\alpha - \theta(\alpha-1)]\Omega}$, and $\theta_1 \in (0,1)$ is a unique root of $2\alpha[(\alpha-\theta)(\alpha-1) - 2\alpha] - \theta(\alpha-1)\,\Omega = 0$.

Theorem 3 shows that the duopoly platforms would not always offer a lower subsidy, and own a much less total supply of agents (which results in a higher price) if they employ temporary agents only than if they employ both temporary and permanent agents. Consequently, the duopoly platforms would not always obtain much lower profits if employing temporary agents only than if employing both temporary and permanent agents. This implies that employing some permanent agents is not necessarily beneficial to the duopoly platforms, which is different from the corresponding result for the monopoly platform.

Theorem 4. $CS_D^{tp} > CS_D^{to}$ *if* $\alpha < \frac{3}{3\sqrt{2}-2}$ *and* $N \ge \frac{2\alpha c\Omega}{g(\theta)}$ *or* $\theta > \theta_2$ *and* $N \ge \frac{2\alpha c\Omega}{g(\theta)}$; *otherwise,* $CS_D^{tp} < CS_D^{to}$, *where* $g(\theta) = [2\alpha + (\alpha-1)\theta]\,\Omega - 6\sqrt{2\alpha^2(\alpha-\theta)}$ *and* $\theta_2 \in (0,1)$ *is a unique root of* $g(\theta) = 0$. *Moreover, there are scenarios such that* $AS_D^{tp} < AS_D^{to}$ *and* $S_D^{tp} < S_D^{to}$.

From Theorem 4, for the duopoly platforms, employing both temporary and permanent agents may bring agents and customers much lower surpluses, and may even yield much less social welfare than employing the temporary agents only. In other words, for the duopoly platforms, employing permanent agents may harm the agents and customers, and may reduce the social welfare. Therefore, to some extent, the findings in Theorem 3 and Theorem 4 can seemingly explain the practice of employing temporary agents only for duopoly platforms such as UberChina and Didi, Uber and Lyft in America etc.

5. Effects of competition

In this section we analyze how competition influences the strategies of the platforms. To do so, we need to compare the outcomes presented in Sections 3 and 4. In order to facilitate comparison, we only consider the symmetric equilibrium in Section 4.

5.1. Platforms employing temporary agents only

Theorem 5. *(i)* $\eta_M^{to^*} \geq \eta_D^{to^*}$ *if* $0 \leq \theta \leq \frac{\alpha}{2\alpha+1}$*; otherwise,* $\eta_M^{to^*} < \eta_D^{to^*}$*; (ii)* $p_M^{to^*} \leq p_D^{to^*}$ *if* $1 \leq \alpha \leq \frac{3}{2}$ *and* $\frac{\alpha(2\alpha+1)}{2\alpha+3} \leq \theta \leq 1$*; otherwise,* $p_M^{to^*} > p_D^{to^*}$.

Theorem 5(*i*) shows that if only temporary agents are employed, when the degree of competition between the duopoly platforms is relatively small, the two platforms would offer lower subsidies to the temporary agents than a monopoly platform does; whereas when the degree of competition between the duopoly platforms is relatively large, the two platforms would offer higher subsidies to the temporary agents than a monopoly platform does. The latter seems consistent with our intuition. However, the former seems counter-intuitive, but can be explained as follows: When the degree of competition is small, it is not too different for the two platforms to guarantee sufficient temporary agents by lowering the subsidies, and this can help the platforms increase the price charged to customers and thereby make profits. Theorem 5(*ii*) suggests that when only temporary agents are employed, if the attractiveness of the subsidy to the temporary agents is relatively small but the degree of competition between the two platforms is relatively large, this implies that it is more difficult for the duopoly platforms to recruit sufficient temporary agents to satisfy demands, so the duopoly platforms charge the customers a higher price than a monopoly platform does; otherwise, they charge the customers a lower price than a monopoly platform does.

Theorem 5 implies that for ride-sharing platforms, when they switch from monopoly to duopoly, they do not necessarily have to offer temporary agents a higher subsidy and charge the customers a lower price than a monopoly platform does; quite the contrary, they may even offer the temporary agents a lower subsidy and charge the customers a higher price than a monopoly platform does in our supply-dominant setting. This finding is different from that in the traditional non-sharing market, where competition necessarily leads to a rise in the subsidy and a reduction in the price.

Theorem 6. *(i)* $CS_M^{to} \geq CS_D^{to}$ *if* $\Omega^2 - 32(\alpha+1)^2(\alpha-\theta)^2 \geq 0$ *and* $CS_M^{to} < CS_D^{to}$ *otherwise; (ii)* $\pi_M^{to} \leq \pi_D^{to}$ *if* $4(\alpha+1)(2\alpha^2+\theta^2-4\alpha\theta) - \Omega^2 \geq 0$ *and* $\pi_M^{to} > \pi_D^{to}$ *otherwise; (iii)* $AS_M^{to} \leq AS_D^{to}$ *if* $\Omega^2 - 8(\alpha-\theta)\alpha(\alpha+1)^2 \leq 0$ *and* $AS_M^{to} > AS_D^{to}$ *otherwise; (iv)* $S_M^{to} \leq S_D^{to}$ *if* $8(\alpha+1)[4\alpha(\alpha-\theta)^2 + 3\alpha^2 - 5\alpha\theta + \theta^2] - 3\Omega^2 \geq 0$ *and* $S_M^{to} > S_D^{to}$ *otherwise.*

Theorem 6 shows that if only temporary agents are employed, then when the two-sided market switches from monopoly to duopoly, the surpluses of both customers and agents, the profits of the platforms, and the social welfare may become small if a specified condition holds but may become great otherwise. Due to the difficulty in recruiting temporary agents for the duopoly platforms under certain conditions than the monopoly platform, the intuition is that the duopoly platforms may lower the wage but increase the price, which results in lower profits, surpluses of both customers and agents, so the social welfare. This finding is different from the corresponding one in the traditional single-sided market, where a duopoly always brings customers much more surplus than does a monopoly.

5.2. Platforms employing both temporary and permanent agents

Theorem 7. *if* $\alpha \leq 2$ *and* $\frac{N}{3} \leq c \leq \frac{N}{\alpha+1}$*; otherwise,* $\eta_M^{tp^*} < \eta_D^{tp^*}$*. Moreover* $p_M^{tp^*} > p_D^{tp^*}$ *and* $CS_M^{tp} \leq CS_D^{tp}$.

Theorem 7 indicates that if both temporary and permanent agents are employed, the monopoly platform offers the temporary agents a lower subsidy in most cases, and always charges the customers a higher price than do the duopoly platforms because the monopoly can always guarantee sufficient agents to satisfy demand in the presence of permanent agents. Consequently, the monopoly platform brings customers much less surplus than do the duopoly platforms. These findings make sense. However, Theorem 7 also shows that when α is relatively small but *c* is relatively great, the monopoly platform offers the temporary agents a higher subsidy than do the duopoly platforms, which seems counter-intuitive. This is possibly because when the wage of the permanent agents is relatively high and the attractiveness of the subsidy for the temporary agents of the two platforms is relatively low, the duopoly platforms do not have enough ability to offer higher subsidies to the temporary agents; on the other hand, even when the duopoly platforms have enough ability, the relatively low attractiveness of the subsidy also restrains the duopoly platforms from offering lower subsidies to the temporary agents. In addition, comparing Theorem 5 and Theorem 6 with Theorem 7, we see that the impacts of competition between the platforms on the optimal strategies, agents'/customers' surpluses, and social welfare are affected by the employment level of the permanent agents.

6. Discussion and conclusions

6.1. Managerial implications

Our results provide several implications for decision makers of ride-sharing platforms to improve their operations. First, it is beneficial for managers of a local monopoly platform (e.g., Didi Chuxing in China) to recruit both temporary and permanent agents from the prospective of profitability. Although this practice always harm the agents, consumer surplus and social welfare can be improved. Despite the higher profitability and consumer surplus, the result suggests that the managers should be cautioned never to employ permanent agents if improving agents' surplus plays a key role in the development of the platform. Second, for the platforms engaged in competition for customers and agents (e.g., Uber and Lyft), recruiting both temporary and permanent agents may (may not) lead to a win-win-win outcome, i.e., both platforms, customers, and agents (do not) become better off. More specifically, the managers should or should not recruit both temporary and permanent agents based on the degree of competition, the attractiveness of the subsidies to the temporary agents, the employment cost of the permanent agents, and the pool size of customers. Finally, when the two-sided market switches from monopoly to duopoly, compared with the scenario where only temporary agents are employed, it is beneficial for the platforms to recruit both temporary and permanent agents because a win-win-win outcome can be easily achieved.

6.2. Conclusions

In this paper we consider both a monopoly and duopoly car-hailing platforms that fulfill the on-demand services of the customers. The platforms employ temporary agents that are self-scheduling by offering them subsidies and may also employ permanent agents by paying them fixed wages. From the perspective of maximizing the platforms' profits, we derive the optimal subsidies for the temporary agents, and the employment levels of the permanent agents for both monopoly and duopoly platforms. We also analyze the impacts of the employment of permanent agents and the competition between the platforms on the platforms' optimal strategies, agents'/customers' surpluses, and social welfare.

From our analyses and research findings, we obtain the following insights. First, to make more profit, a monopoly car-hailing platform should offer a higher subsidy that brings a greater supply of temporary agents, so a lower price if it employs both temporary and permanent agents than if it employs temporary agents only. Consequently, the platform can obtain a higher profit and yield greater customers' surplus and social welfare. Second, in the two-sided market, a monopoly platform sometimes may bring more benefit to the customers and agents than duopoly platforms do. Third, the employment of permanent agents can considerably affect the impact of competition on the platforms' optimal strategies, agents'/customers' surpluses, and social welfare.

Due to a few important assumptions we make (see Table 2), our paper has some limitations. First, we assume that the market is supply-dominant. This implicitly implies that we consider Cournot competition, i.e., capacity/quantity competition, in our competition model. In reality, however, there are scenarios where the market may not be supply-dominant. Although relaxing this assumption may change some of our results, the main result that competition can benefit customers and agents might still hold because the intuition is not associated with price competition. However, future research may extend our work to the two-sided market without supply-dominance. Second, for the sake of simplicity, we neglect the network externality of the platforms due to the assumption of the supply-dominant market. It is interesting to incorporate cross-network externality into a new model and check if our main results remain valid. Finally, we assume that both customers and agents are multi-homing, i.e., the customers and agents can use the two competing platforms simultaneously. However, there may exist some customers and/or agents that stick to one of the platforms. The impacts of the presence of these customers and/or agents on the platforms' strategies, participants' surpluses, and social welfare deserve further investigation in future research.

Acknowledgements

We thank the Editor and the two anonymous referees for their constructive and insightful comments, which help us significantly improve the quality of our paper. This work was supported by the National Natural Science Foundation of China [Grants 71871097, 71501077, 71520107001, 71571052], Guangdong Province Universities and Colleges Pearl River Scholar Funded Scheme [Grant 2017], the Fundamental Research Funds for the Central Universities [Grant 2018JQ03], the Guangdong Natural Science Foundation [Grant 2017A030313409]. Cheng was supported in part by The Hong Kong Polytechnic University under the Fung Yiu King - Wing Hang Bank Endowed Professorship in Business Administration.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijpe.2019.07.009.

Appendix

Proof of Lemma 2: The first-order partial derivatives of $\pi(\eta, b)$ with regard to η and b are

$$\begin{cases} \frac{\partial\pi}{\partial\eta} = N - 2(\alpha + 1)\eta - 2b, \\ \frac{\partial\pi}{\partial b} = N - 2\alpha\eta - 2b - c \end{cases}$$

and the second-order partial derivatives of $\pi(\eta, b)$ with regard to η and b are

$$\begin{cases} \frac{\partial^2\pi}{\partial\eta^2} = -2(\alpha + 1)\alpha, \\ \frac{\partial^2\pi}{\partial\eta\partial b} = \frac{\partial^2\pi}{\partial b\partial\eta} = -2\alpha, \\ \frac{\partial^2\pi}{\partial b^2} = -2. \end{cases}$$

Therefore, the Hessian matrix of $\pi(\eta, b)$ with regard to η and b is

$$D = \begin{pmatrix} -2(\alpha + 1)\alpha & -2\alpha \\ -2\alpha & -2 \end{pmatrix}.$$

Since the following determinants

$$D_1 = -2(\alpha + 1)\alpha < 0, D_2 = \begin{vmatrix} -2(\alpha + 1)\alpha & -2\alpha \\ -2\alpha & -2 \end{vmatrix} = 4\alpha > 0,$$

the Hessian matrix D is a negative definite matrix. Therefore, $\pi(\eta, b)$ is concave in η and b.

Proof of Theorem 1: By comparing (1) with (5) and (6), and noting $c \le \frac{N}{\alpha+1}$, one can easily derive the result.

Proof of Theorem 2: Comparing (2)–(4) with (7)–(9), and noting $c \le \frac{N}{\alpha+1}$, we derive the result.

Proof of Lemma 5: The first-order partial derivatives of $\pi_1(\eta_1, b_1)$ with regard to η_1 and b_1 are

$$\begin{cases} \frac{\partial\pi_1}{\partial\eta_1} = \alpha N - 2\alpha(\alpha - \theta + 1)\eta_1 - (\alpha - \theta)^2\eta_2 - (2\alpha - \theta)b_1 - \alpha b_2, \\ \frac{\partial\pi_1}{\partial b_1} = N - (2\alpha - \theta)\eta_1 - (\alpha - 2\theta)\eta_2 - 2b_1 - b_2 - c_1, \\ \frac{\partial\pi_2}{\partial\eta_2} = \alpha N - 2\alpha(\alpha - \theta + 1)\eta_2 - (\alpha - \theta)^2\eta_1 - (2\alpha - \theta)b_2 - \alpha b_1, \\ \frac{\partial\pi_1}{\partial b_1} = N - (2\alpha - \theta)\eta_2 - (\alpha - 2\theta)\eta_1 - 2b_2 - b_1 - c_2 \end{cases}$$

and the second-order partial derivatives of $\pi_1(\eta_1, b_1)$with regard to aη_1nd arb_1e

$$\begin{cases} \frac{\partial^2 \pi_1}{\partial \eta_1^2} = -2(\alpha + 1 - \theta)\alpha, \\ \frac{\partial^2 \pi}{\partial \eta_1 \partial b_1} = \frac{\partial^2 \pi}{\partial b_1 \partial \eta_1} = \theta - 2\alpha, \\ \frac{\partial^2 \pi_1}{\partial b_1^2} = -2. \end{cases}$$

Therefore, the Hessian matrix of $\pi_1(\eta_1, b_1)$ with regard to η_1 and b_1 is

$$E = \begin{pmatrix} -2(\alpha + 1 - \theta)\alpha & \theta - 2\alpha \\ \theta - 2\alpha & -2 \end{pmatrix}.$$

Since the following determinants

$$E_1 = -2(\alpha + 1 - \theta)\alpha < 0,\ E_2 = \begin{vmatrix} -2(\alpha + 1 - \theta)\alpha & \theta - 2\alpha \\ \theta - 2\alpha & -2 \end{vmatrix} = 4\alpha - \theta^2 > 0,$$

the Hessian matrix E is a negative definite matrix. Therefore, $\pi_1(\eta_1, b_1)$ is concave in η_1 and b_1.

Due to the symmetry between $\pi_1(\eta_1, b_1)$and, $\pi_2(\eta_2, b_2)$ we can prove the concavity of wi$\pi_2(\eta_2, b_2)$th regard to andη_2 anal$b_2$ogously.

Proof of Theorem 3: By comparing (11) and (12) with (19), we derive the result of Theorem 3.

Proof of Theorem 4: Comparing (13) with (20), we can derive the result of Theorem 4(i).

Proof of Theorem 5:(i) As known in Sections 3.1 and 4.1, if only the temporary agents are employed, the optimal subsidies offered to the temporary agents by the monopoly and duopoly platforms are $\eta_M^{to^*} = \frac{N}{2(\alpha+1)}$ and $\eta_D^{to^*} = \frac{\alpha N}{(\alpha-\theta)2(\alpha+1)+2\alpha}$, respectively. Then, we have $\eta_M^{to^*} - \eta_D^{to^*} = \frac{[\alpha - \theta(2\alpha+1)]N}{2(\alpha+1)[(\alpha-\theta)(2\alpha+1)+2\alpha]}$. Therefore, when $0 \le \theta \le \frac{\alpha}{2\alpha+1}$, $\eta_M^{to^*} \ge \eta_D^{to^*}$; when $\frac{\alpha}{2\alpha+1} < \theta \le 1$, $\eta_M^{to^*} < \eta_D^{to^*}$.

(ii) From Sections 3.1 and 4.1, we know that if only the temporary agents are employed, the optimal prices charged to the customers by the monopoly and duopoly platforms are $p_M^{to^*} = \frac{(\alpha+2)N}{2(\alpha+1)}$ and $p_D^{tp^*} = \frac{(3\alpha-\theta)N}{(\alpha-\theta)(2\alpha+1)+2\alpha}$, respectively. Then, we have $p_M^{to^*} - p_D^{to^*} = \frac{\alpha[\alpha(2\alpha+1)-\theta(2\alpha+3)]N}{2(\alpha+1)[(\alpha-\theta)(2\alpha+1)+2\alpha]}$. Therefore, if $\alpha > \frac{3}{2}$, then $\alpha(2\alpha+1) - \theta(2\alpha+3) > 0$ for $0 \le \theta \le 1$, i.e., $p_M^{to^*} > p_D^{to^*}$ for $0 \le \theta \le 1$. If $1 \le \alpha \le \frac{3}{2}$, it is easy to see that $\frac{\alpha(2\alpha+1)}{2\alpha+3} < 1$. Thus, we easily derive $p_M^{to^*} > p_D^{to^*}$ for $0 \le \theta < \frac{\alpha(2\alpha+1)}{2\alpha+3}$ and $p_M^{to^*} \le p_D^{to^*}$ for $\frac{\alpha(2\alpha+1)}{2\alpha+3} \le \theta \le 1$.

Proof of Theorem 6: Comparing (2)–(4) with (13)–(14), we can derive the results in Theorem 6.

Proof of Theorem 7:(i) As known in Sections 3.2 and 4.2, if both permanent and temporary agents are employed, the optimal subsidies offered to the temporary agents by the monopoly and duopoly platforms are $\eta_M^{tp^*} = \frac{c}{2}$ and $\eta_D^{tp^*} = \frac{3(\alpha-\theta)c+\theta N}{6\alpha}$, respectively. Then, we have $\eta_M^{tp^*} - \eta_D^{tp^*} = \frac{\theta(3c-N)}{6\alpha}$. Thus, noting that $c \le \frac{N}{\alpha+1}$, we know that $\eta_M^{tp^*} < \eta_D^{tp^*}$ if $\alpha > 2$; otherwise, $\eta_M^{tp^*} < \eta_D^{tp^*}$ for $c < \frac{N}{3}$ and $\eta_M^{tp^*} \ge \eta_D^{tp^*}$ for $\frac{N}{3} \le c \le \frac{N}{\alpha+1}$.

(ii) As known in Sections 3.2 and 4.2, if both permanent and temporary agents are employed, the optimal prices charged to the customers by the monopoly and duopoly platforms are $p_M^{tp^*} = \frac{N+c}{2}$ and $p_D^{tp^*} = \frac{2\alpha c+[\alpha-(\alpha-1)\theta]N}{3\alpha}$, respectively. Then, we have $p_M^{tp^*} - p_D^{tp^*} = \frac{2(\alpha-1)\theta N+\alpha(N-c)}{6\alpha}$. Noting that $c \le \frac{N}{\alpha+1} < N$, we derive $p_M^{tp^*} < p_D^{tp^*}$.

(iii) As known in Sections 3.2 and 4.2, if both permanent and temporary agents are employed, the customers' surpluses for the monopoly and duopoly platforms are $CS_M^{tp} = \frac{(N-c)^2}{8}$ and $CS_D^{tp} = \frac{([2\alpha+(\alpha-1)\theta]N-2\alpha c)^2}{18\alpha^2}$, respectively. Similarly, noting that $c \le \frac{N}{\alpha+1} < N$, we derive $CS_M^{tp} < CS_D^{tp}$.

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