Journal of the Operational Research Society

ISSN: 0160-5682 (Print) 1476-9360 (Online) Journal homepage: https://www.tandfonline.com/loi/tjor20

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JOURNAL OF THE OPERATIONAL RESEARCH SOCIETY
https://doi.org/10.1080/01605682.2019.1678407

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ABSTRACT
The transport sector is one of the largest contributors to rising greenhouse gas (GHG) emissions in the world. With no tailpipe emissions, electric vehicles (EVs) can be one of the ways to reduce GHG emissions. In recent years, many countries have taken steps to promote the penetration of EVs in the market. Using different policy instruments, in particular, taxes and subsidies are one of the more common approaches. In this paper, using a game-theoretic approach, we study the effect of using a combination of subsidy on EV and green-tax on conventional gasoline vehicles (GV) on overall social welfare, environmental impact, and vehicle stock in monopoly and duopoly forms of market structures. Findings reveal that a combination of subsidy and green-tax can generate higher social welfare as compared to the use of only one of them under both monopoly and duopoly markets. The results provide multifaceted insights for policymakers and governments to design the subsidy and green-tax based on the environmental impact of GVs and EVs along with maximising the social welfare.

ARTICLE HISTORY
Received 17 November 2018
Accepted 25 September 2019

KEYWORDS
Electric vehicle; subsidy; green-tax; utility function; social welfare

1. Introduction

In recent years, managing greenhouse gas (GHG) emissions has emerged as one of the key objectives for governments, businesses, and civil societies globally. The transportation sector contributes about 24% of the global energy-related GHG emissions and is considered a major roadblock in achieving sustainable operations (Denchak, 2016; IEA, 2017). With zero tailpipe emissions, EVs have emerged as a popular choice in recent years, and the market for EVs has expanded globally with most automobile manufacturers launching different EV models. Despite this surge in market expansion, the overall global penetration of EVs continues to remain low. Various reasons ranging from the absence of proper charging infrastructure, the higher initial cost of EVs compared to gasoline vehicles (GVs), and the absence of external stimuli like financial incentives or stringent emission-related regulations have been cited for low penetration of EVs (Madina, Zamora, & Zabala, 2016; Sierzchula, Bakker, Maat, & Van Wee, 2014). To address these reasons and to promote EVs, many governments worldwide have used various mechanisms. Tax incentives, subsidies, and price discounts are some of the most popular policy instruments used worldwide (Axsen & Wolinetz, 2018; Shao, Yang, & Zhang, 2017). Leading EV markets (USA, UK, China, France, Germany, Norway, etc.) typically provide substantial public subsidies or tax reductions for EVs or impose a CO_2-tax (green-tax) on GVs (ACEA, 2017; Yang, Slowik, Lutsey, & Searle, 2016). For example, in the USA, federal tax subsidy varies from \$2500 to \$7500 have been given to promote EVs penetration (Oliver Wyman, 2018). In Japan, subsidies up to \$7800 have been provided to EVs based on the price difference with GVs. In China, subsidies have been provided for EVs that depend on the driving range of EVs. In Spain, sales rebates have been given based on vehicular carbon dioxide emissions. The financial incentives (subsidies or tax incentives) look attractive from both manufacturer's and consumer's point of view. For consumers, the effective price for an EV purchase becomes lower, thereby increasing the demand for EVs while for the manufacturer, it helps to achieve breakeven at lower volumes.

In this article, we consider a vehicle market comprising of both GVs and EVs. The government decides optimal subsidy and green-tax to maximise the social welfare and manufacturer decides the optimal quantity of EVs and GVs to maximise its profits. We analyse optimal price, quantity, and profit of manufactures under two types of market structures (monopoly and duopoly) and different green-tax/subsidy scenarios. We study social welfare as comprising of multiple facets, i.e., manufacturers' profit, green-tax collection, subsidy, environmental impact, and consumer utility. Further, we explore

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which entity should be provided subsidy or levied a green-tax (manufacturers or consumers) to maximise the social welfare. We answer the following research questions:

1. How do subsidy and green-tax influence the demand for GVs and EVs?
2. How does a combination of policy instruments affect social welfare, environmental impact, and vehicle stock in comparison to the case when either of them is used?
3. How does the market structure (monopoly or duopoly) affect the optimal policy decisions in choosing subsidy and green-tax?

Our analysis reveals that the demand for EVs increases with an increase in both subsidy and green-tax, whereas the demand for GVs decreases with the increase in either subsidy or green-tax. We also show that social welfare increases with the use of a combination of policy instruments as compared to using either of them. Further, environmental impact and vehicle stock decrease when two policy instruments are used simultaneously as long as the associated unit environmental impact of the vehicles is low. We also find that the optimal subsidy and green-tax vary with the change in market structures. In case of a duopoly, the optimal subsidy and green-tax will be higher whenever the unit cost of manufacturing an EV is substantially higher than the unit cost of manufacturing a GV.

The paper is organised as follows: Section 2 covers the relevant literature review. Section 3 introduces the problem setting and model description. The analysis of models in the presence of subsidy and green-tax under different market structures are carried out in Sections 4 and 5. Section 6 illustrates the numerical results followed by the discussion and interpretations for the governments, manufacturers, and policymakers. Section 7 provides the conclusion and future research. The "Appendix" presents all proofs.

2. Literature review

The literature review focuses on two broad streams: The first stream covers sustainability in operations management and supply chain. The second stream analyses the role of the government's financial policy on firms' decisions and EVs adoption. Sustainability in operations management includes the impact of sustainability on production decisions (Gunasekaran, Irani, & Papadopoulos, 2014; He, Fan, Li, & Li 2017; Paucar-Caceres & Espinosa, 2011), sustainability aspect of supply chains in different market structures (Antheaume, Thiel, De Corbière, Rowe, & Takeda, 2018; Benjaafar, Li, & Daskin, 2013). Fu, Chen, and Hu (2018) study the role of subsidising strategies in a sustainable supply chain to assess its effects on the manufacturers, suppliers, and EV consumers. New eco-friendly product development has received increased attention in recent years in sustainable supply chain management (Kara, Ibbotson, & Kayis 2014; Zhu & He, 2017). For instance, Zhu and He (2017) study green product design in the supply chain under competition and find that distortion from a non-coordinated network has a counter-intuitive effect on the product greenness.

Policy instruments like tax and subsidy are continuously used to promote the adoption of green technologies despite market inefficiencies that they create (Alizamir, de Véricourt, & Sun, 2016; Chang, Hsu, & Cheng, 2013; Krass, Nedorezov, & Ovchinnikov, 2013). Two of the most important policy instruments used are subsidy and tax, creating a mixed impact on the EVs adoption process. According to some, the subsidy has a positive effect in promoting EVs (Sierzchula et al., 2014; Zhang, 2014) while according to others, CO_2 tax on GVs has a positive effect (Gass, Schmidt, & Schmid, 2014; Gerlagh, Van Den Bijgaart, Nijland, & Michielsen, 2018). The financial incentives can be in many forms like exemptions from registration and annual taxes (Lévay, Drossinos, & Thiel, 2017), sale tax exemptions (Gallagher & Muehlegger, 2011), car purchase tax and feebate policies (Brand, Anable, & Tran, 2013). Similarly, the taxes can be in the form of increased registration taxes, higher fuel taxes, or higher annual road taxes (Gerlagh et al., 2018). The rationale for such policy instruments is to make EVs more attractive in comparison to GVs (Kuppusamy, Magazine, & Rao, 2017; Langbroek, Franklin, & Susilo, 2016). Some recent research has studied the impact of carbon taxes on supply chain decisions (Hammami, Nouira, & Frein, 2018; Ma, Ho, Ji, & Talluri, 2018). Impact of regulations related to carbon emissions on production decisions (Absi, Dauzère-Pérès, Kedad-Sidhoum, Penz, & Rapine, 2013; Benjaafar et al., 2013; He et al., 2017), tradable permits to regulate emissions (Liao, Önal, & Chen, 2009; Montgomery, 1972), and decisions related to replacement of potentially hazardous substances (Kraft, Zheng, & Erhun, 2013) have also been studied extensively.

Another field of literature focuses on studying the impact of using taxes and subsidies in the context of environmental externalities. For example, Bansal and Gangopadhyay (2003) analyse the implications of various tax-subsidies policies in an imperfectly competitive market in the presence of environmentally aware consumers. Some recent

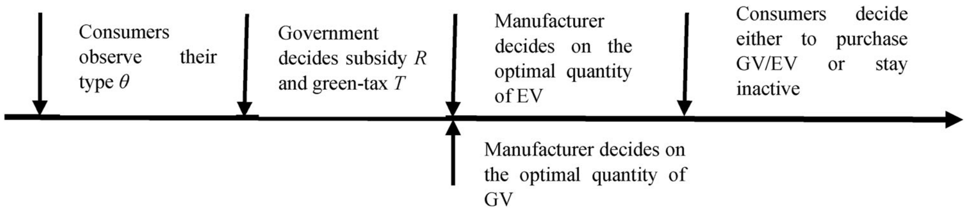

Figure 1. Timeline of the events.

works capture the impact of tax, subsidies, or a combination of both on production decisions (Raz & Ovchinnikov, 2015; Taylor & Xiao, 2019). For example, Raz and Ovchinnikov (2015) have carried out a comparative analysis of consumer/manufacturer subsidy under a single-period newsvendor problem. Findings reveal that that joint strategy might completely coordinate the network while using only consumer subsidy can lead to a small welfare loss. In another work, Taylor and Xiao (2019) have done a comparative study of subsidising commercial and non-commercial channels and found that the optimal subsidy has a non-trivial relationship with consumer awareness of the product. Cohen, Lobel, and Perakis (2016) analyse the impact of demand uncertainty on consumer subsidy while studying green product adoption. Few recent studies use mathematical models to analyse EV adoption process using either subsidy-based, carbon tax-based, or price discount-based schemes (Chemama, Cohen, Lobel, & Perakis, 2019; Shao et al., 2017; Yuyin & Jinxi, 2018). In this article, we consider a combination of subsidy and green-tax to study its impact on EV adoption and overall social welfare under different market structures.

3. Model description

In this section, we introduce the basic setup of the proposed model under both monopoly and duopoly market structures. We start with formulating our problem with the help of consumer utility functions in the presence of the two policy instruments: consumer subsidy for EVs and a green-tax on the GV consumers. The sequence of events is presented in Figure 1.

Each consumer enters the market knowing his (throughout the manuscript, we use *his* for consumer and *her* for the manufacturer) valuation for the services a vehicle provides (θ). We have modelled our problem through a non-cooperative game where the government plays the role of a leader while the manufacturer(s) plays the role of a follower(s). Here the government decides the subsidy and green-tax to maximise social welfare. Based on the subsidy and green-tax, the manufacturer(s) decides the optimal quantity to maximise her profits. Finally, based on the available information on green-tax and subsidy, the consumer evaluates his utility and decides whether to purchase GV or EV or remain inactive. We analyse the market performance of GVs and EVs under monopoly and duopoly market structures. For duopoly markets, we consider the simultaneous movement of GV and EV manufacturers. The summary of the model set-up is presented in Table 1. The notations used for model development and subsequent discussions are presented in Table 2.

3.1. Model assumptions

1. Consumers are heterogeneous about their valuation of the vehicles.
2. The manufacturer decides on the optimal quantity of vehicles to produce of each type.
3. There is no information asymmetry existing across different entities.
4. There is no fixed cost incurred in the production of either type of vehicles and unit cost is constant irrespective of the number of units produced.
5. In the absence of low carbon awareness, a consumer is indifferent between GV and EV regarding technological specifications.
6. θ follows uniform U [0, 1] distribution (Shao et al., 2017).
7. Unit environmental impact for EV is less than that of GV, $i_e < i_g$ (MacKay, 2008).
8. The utility received from an EV is more as compared to GV and utility from a GV is greater than zero, i.e., $U_e \geq U_g \geq 0$.

4. Mathematical model under monopoly

In this case, we consider a single manufacturer, who is selling both GVs and EVs simultaneously. A consumer decides about his choice based on his utility in the presence of available information on consumer subsidy, green-tax and prices of the two types of vehicles. The manufacturer decides the optimal quantity of GVs and EVs to maximise her profits.

Table 1. Model description.

Market structure	Model description
Single manufacturer	A single manufacturer producing both GVs and EVs (*Model M*)
Two manufacturers	One selling only GVs and the other selling only EVs (Manufacturers move simultaneously (*Model D*)

Table 2. Table of notations.

Variables	
R	Subsidy given to EV buyers/manufacturers by the government
T	The green tax imposed by the government on GV buyers/manufacturers
q	The quantity of the vehicles sold
α $(0 \le \alpha \le 1)$	The share of subsidy given to EV manufacturer transferred to EV consumer
β $(0 \le \beta \le 1)$	The share of green tax levied on GV manufacturer transferred to GV consumer
Parameters	
δ	Consumer's low-carbon awareness
θ	Consumer's valuation for the services provided by any vehicle
i	Unit environmental impact of any vehicle
p	The unit price of the vehicle
π	Profit earned by the manufacturer
C	Consumer Surplus
SW	Social Welfare
U	Utility
VS	Vehicle stock
c $(c \le 1)$	The unit cost of a GV
k $(k > 1)$	Cost coefficient of an EV relative to a GV
Superscripts	
Sm	The monopoly setting
Sd	The duopoly setting
Subscripts	
e	For EV
g	For GV
r	Consumer remaining inactive

Referring to the works of Mussa and Rosen (1978), Shao et al. (2017), with the given price of a GV (p_g^{sm}) and an EV (p_e^{sm}) of the monopoly setting, in the presence of subsidy R and green-tax T, we can express the consumers' utility functions in the following way:

$$U_e = (1+\delta)\theta - p_e^{sm} + R \tag{1}$$

$$U_g = \theta - p_g^{sm} - T \tag{2}$$

$$U_r = 0 \tag{3}$$

Here, U_e, U_g, and U_r represent the utility received from using an EV or a GV or remaining inactive respectively. The indifference point between purchasing a GV or an EV can be found by equating U_g and U_e and is given by $\theta_1 = \frac{p_e^{sm} - p_g^{sm} - T - R}{\delta}$. Similarly, by equating U_g and U_r, we can find the second point of indifference, $\theta_2 = p_g^{sm} + T$. So, a consumer with $\theta \in (\theta_1, 1]$ will purchase an EV, a consumer with $\theta \in (\theta_1, \theta_2)$ will purchase a GV, while a consumer with $\theta \in [0, \theta_2)$ will remain inactive. He will, however, remain indifferent between the purchase of a GV or an EV at $\theta = \theta_1$ and indifferent between the purchase of a GV and remaining inactive at $\theta = \theta_2$ (Figure 2).

Thus, we can express the demand function for EVs as $\int_{\theta=\theta_1}^{1} \frac{1}{1-0} d\theta = 1 - \theta_1$. Similarly, the demand function of GVs is $\int_{\theta=\theta_2}^{\theta_1} \frac{1}{1-0} d\theta = \theta_1 - \theta_2$. Substituting the values of θ_1 and θ_2, we get the demand functions as: $q_e^{sm} = \frac{\delta + p_g^{sm} - p_e^{sm} + T + R}{\delta}$ and $q_g^{sm} = \frac{p_e^{sm} - p_g^{sm}(1+\delta) - R - T(1+\delta)}{\delta}$.

We get the inverse demand functions as:

$$p_e^{sm} = 1 + R + \delta\left(1 - q_e^{sm}\right) - q_g^{sm} - q_e^{sm} \tag{4}$$

$$p_g^{sm} = 1 - q_g^{sm} - q_e^{sm} - T \tag{5}$$

The manufacturer's total profit is given as $\pi^{sm} = \left(p_e^{sm} - kc\right) q_e^{sm} + \left(p_g^{sm} - c\right) q_g^{sm}$

Here, the selling price of a vehicle is higher than the cost incurred, i.e., $p_e^{sm} > kc$ and $p_g^{sm} > c$, else it will not be profitable for the firm to sell either type of vehicles.

$$\begin{aligned} \pi^{sm} = {} & (1 + R + (1 - q_e^{sm})\delta - q_g^{sm} - q_e^{sm} - kc) q_e^{sm} \\ & + (1 - T - q_g^{sm} - q_e^{sm} - c)\, q_g^{sm} \end{aligned} \tag{6}$$

The profit function given in (6) is concave at both q_e^{sm} and q_g^{sm}. From the second-order Hessian matrix corresponding to the above profit function, we get $\frac{\partial^2 \pi^{sm}}{\partial (q_e^{sm})^2} \frac{\partial^2 \pi^{sm}}{\partial \left(q_g^{sm}\right)^2} - \left(\frac{\partial^2 \pi^{sm}}{\partial q_e^{sm} \partial q_g^{sm}}\right)^2 = 4\delta > 0$ and $\frac{\partial^2 \pi^{sm}}{\partial (q_e^{sm})^2} = -2$, thus proving the concavity.

We define the consumer surplus as the aggregate utility of all consumers participating in the market and is given as:

$$C^{sm} = \int_{\theta_1}^{1} U_e(\theta) d(\theta) + \int_{\theta_2}^{\theta_1} U_g(\theta) d(\theta) + \int_{0}^{\theta_2} U_r(\theta) d(\theta)$$

Theorem 1. *In the monopoly model, the following holds:*

(i) *The optimal number of vehicles sold:* $q_e^{sm} = \frac{\delta - ck + c + T + R}{2\delta}$, $q_g^{sm} = \frac{ck - \delta(c+T) - c - T - R}{2\delta}$

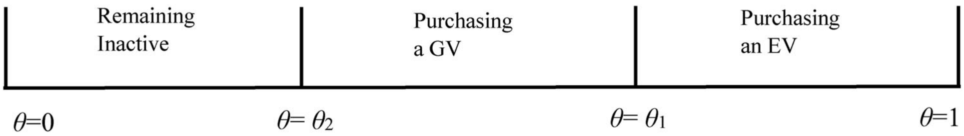

Figure 2. Different behaviours of the heterogeneous customers.

(ii) *The optimal price for the vehicles:* $p_e^{sm} = \frac{1+\delta+ck+R}{2}$, $p_g^{sm} = \frac{1+c-T}{2}$

(iii) *The optimal profit for the manufacturer:*

$$\pi^{sm} = \frac{\delta^2 + (1-2ck + c^2 + 2Tc + T^2 + 2R)\delta + c^2(k-1)^2 + (2c-2ck + R + T)(R+T)}{4\delta}$$

(iv) *The consumer surplus:*

$$C^{sm} = \frac{\delta^2 + (1-2ck + c^2 + 2Tc + T^2 + 2R)\delta + c^2(k-1)^2 - 2ck(R+T) + 2c(R+T) + (R+T)^2}{8\delta}$$

From the above, we can infer as to how the demand for EVs and GVs changes based on subsidy, and green-tax through the following corollary.

Corollary 1.

- *The demand for EVs increases, and GVs decreases as the subsidy or green-tax increases.*
- *The demand for EVs decreases, and GVs increases as the relative cost coefficient of EVs (k) increases.*

The findings are coherent with previous studies, which have confirmed that incentive policies have a positive impact on the EV sales and market share (Melton, Axsen, & Goldberg, 2017; Sierzchula et al., 2014).

We have considered the social welfare function, which as per Varian (1992), is defined as "the social welfare function aggregates the individual utility functions to come up with a social utility." The most reasonable interpretation of such a function is that it represents social decision-makers preferences about how to trade off the utilities of different individuals. The government then decides the subsidy and the green-tax to maximise social welfare by acting as the leader while taking into account the profit-maximising actions of the manufacturer. Social welfare used in this paper is a multidimensional attribute consisting of manufacturer's total profits (π^{sm}), consumer surplus (C^{sm}), total government subsidy (Rq_e^{sm}), total green-tax (Tq_g^{sm}), and environmental impact ($EI^{sm} = i_e q_e^{sm} + i_g q_g^{sm}$). We quantify the per-unit environmental impact of GV and EV as i_g and i_e respectively (Agrawal, Ferguson, Toktay, & Thomas, 2012). The higher per unit environmental impact of GVs and EVs respectively means that the tailpipe emissions are higher. In other words, higher values of per unit environmental impact are worse for the environment. The social welfare can be expressed as:

$$SW^{sm} = \pi^{sm} + C^{sm} - (R + i_e)q_e^{sm} + \left(T - i_g\right)q_g^{sm} \quad (7)$$

Substituting the values of q_e^{sm}, q_g^{sm}, π^{sm} and C^{sm} in the above expression, we get

$$SW^{sm} = \frac{3(\delta^2 - 2ck\delta + c^2\delta + 2Tc\delta + T^2\delta + 2R\delta + \delta(T + R - ck + c)^2)}{8\delta} - \frac{i_g(-c\delta - T\delta + ck - c - T - R)}{2\delta} + \frac{T(-c\delta - T\delta + ck - c - T - R)}{2\delta} - \frac{i_e(\delta - ck + c + T + R)}{2\delta} - \frac{R(\delta - ck + c + T + R)}{2\delta} \quad (8)$$

Here, the first term represents total profit (producer surplus) and consumer surplus, the second term indicates the environmental impact of GV, the third term is green-tax collected, the fourth term is the environmental impact of EV, and the last term is the government subsidy. The social welfare function given by (8) is concave at both R and T. Using second-order Hessian matrix of social welfare function, we get, $\frac{\partial^2 SW^{sm}}{\partial R^2}\frac{\partial^2 SW^{sm}}{\partial T^2} - \left(\frac{\partial^2 SW^{sm}}{\partial R \partial T}\right)^2 = \frac{1}{16\delta} > 0$ and $\frac{\partial^2 SW^{sm}}{\partial R^2} = -\frac{1}{4\delta}$, thus proving concavity. Writing down the first-order conditions for maximising social welfare and then solving the same, we get $\hat{R} = \delta - ck - 2i_e + 1$ and $\hat{T} = 2i_g + c - 1$. Substituting the optimal values of R and T in the social welfare expression, we get

$$SW^{sm} = \frac{(i_e - c)^2 - 2i_g(i_e - c) + c^2(k^2 - 2k) - 2c(i_g - i_e) + {i_g}^2 + \delta^2 + \delta({i_g}^2 + c^2 + 1 + 2ci_g - 2i_e - 2ck)}{2\delta} \tag{9}$$

It is important for the government to understand the impact of δ and k on optimal R and T so that they can devise their policies accordingly. We can understand the relationship with the help of the following corollary.

Corollary 2.

- *As the environmental impact of EV (i_e) increases, subsidy decreases.*
- *As the environmental impact of GV (i_g) increases, green-tax increases.*

Corollary 2 signifies the role of environmental impacts of EVs and GVs for governments during designing of policy instruments like subsidy and green-tax. The optimal values of R and T depend on the environmental impact of EVs and GVs. These environmental impacts act as a tool for the government to decide on subsidy and green-tax since the government cannot decide them arbitrarily and requires some measures related to vehicle emissions. These values of i_g and i_e help the government to decide green-tax and subsidy on GVs and EVs respectively while maximising the social welfare. Hence our further discussions are based on i_g and i_e and their different range of values. We state the following propositions are highlighting the different range of values of i_g and i_e and how they impact R and T.

Proposition 1(a). *In the monopoly setting, for different range of values of i_e (for fixed T), the following holds:*

1) *For, $i_e < i_1^{sm}$, the social welfare increases with an increase in R and is maximised at $R = R_{min}^{sm} = ck - \delta - c - T$, where $i_1^{sm} = \frac{(c+T+1)\delta - 2ck + c + T + 1}{2}$.*
2) *When, $i_1^{sm} \leq i_e \leq i_2^{sm}$, the government will choose $R = \hat{R} = \delta - ck - 2i_e + 1$ to maximise the social welfare, where $i_2^{sm} = \frac{2\delta - 2ck + c + T + 1}{2}$.*
3) *For $i_e > i_2^{sm}$, the social welfare decreases with an increase in R and is maximised at $R = R_{max}^{sm} = ck - c - T\delta - c\delta - T$.*

It is obvious that $q_e^{sm} = \frac{\delta - ck + c + T + R}{2\delta} \geq 0$, and $q_g^{sm} = \frac{ck - \delta(c+T) - c - T - R}{2\delta} \geq 0$. These inequalities can be used to derive the bounds $i_1^{sm} = \frac{(c+T+1)\delta - 2ck + c + T + 1}{2}$ and $i_2^{sm} = \frac{2\delta - 2ck + c + T + 1}{2}$. Here, i_1^{sm} and i_2^{sm} are the two thresholds on the social welfare curve that divide the region into three segments. The social welfare changes in these regions as we pass through these thresholds as defined above. Proposition 1(a) indicates that an increase in the subsidy does not always increase the social welfare, and it can vary for a different range of values of the unit environmental impact of an EV. Here it is important to notice that the government has to choose the subsidy based on the environmental impact and consumer awareness of EV to maximise the social welfare. The combination of R and i_e determines the optimal strategy for the governments to maximise social welfare.

Proposition 1(b). *In the monopoly setting, for a different range of values of i_g (for fixed R), the following holds:*

(1) *For $i_g < i_3^{sm}$, the social welfare increases with an increase in T and the optimal value of SW occurring at $T_{min}^{sm} = ck - \delta - c - R$, where $i_3^{sm} = \frac{1 - \delta + ck - 2c - R}{2}$.*
(2) *When $i_3^{sm} \leq i_g \leq i_4^{sm}$, the government will choose $T = \hat{T} = 2i_g + c - 1$ to maximise social welfare, where $i_4^{sm} = \frac{1 - (2c-1)\delta + ck - 2c - R}{2\delta + 2}$.*
(3) *For $i_g > i_4^{sm}$, social welfare decreases with an increase in T and the optimal value of SW occurring at $T_{max}^{sm} = \frac{ck - c - c\delta - R}{1 + \delta}$.*

Here, i_3^{sm} and i_4^{sm} are the two bounds on the SW curve and changes its trend with an increase in T, as shown in Proposition 1(b). Similar to Proposition 1(a), 1(b) also indicates that an increase in green-tax does not necessarily increase the social welfare and thus vary for a different range of values of i_g. These range of values are important for government and policy-makers to decide green-taxes based on i_g to maximise social welfare. A real-world example of sales rebates based on the environmental impact of GVs can be found in France, where taxes have been imposed based on tailpipe emissions of vehicles (Oliver Wyman, 2018). Thus an optimal green-tax is required for a particular i_g to maximise the social welfare.

Proposition 2(a). *In the monopoly setting, under different range of values of i_e (for fixed T), the following holds:*

$$p_g^{*sm} = \begin{cases} \frac{c - T + 1}{2} & i_e < i_1^{sm} \\ \frac{c - T + 1}{2} & i_1^{sm} \leq i_e \leq i_2^{sm} \\ \frac{c - T + 1}{2} & i_e > i_2^{sm} \end{cases}$$

$$p_e^{*sm} = \begin{cases} \frac{2ck-c-T+1}{2} & i_e < i_1^{sm} \\ \frac{\delta+ck+R+1}{2} & i_1^{sm} \le i_e \le i_2^{sm} \\ \frac{1+\delta+2ck-c\delta-T\delta-c-T}{2} & i_e > i_2^{sm} \end{cases}$$

$$q_g^{*sm} = \begin{cases} \frac{1-c-T}{2} & i_e < i_1^{sm} \\ \frac{ck-\delta(c+T)-c-T-R}{2\delta} & i_1^{sm} \le i_e \le i_2^{sm} \\ 0 & i_e > i_2^{sm} \end{cases}$$

$$q_e^{*sm} = \begin{cases} 0 & i_e < i_1^{sm} \\ \frac{\delta-ck+c+T+R}{2\delta} & i_1^{sm} \le i_e \le i_2^{sm} \\ \frac{1-c-T}{2} & i_e > i_2^{sm} \end{cases}$$

$$\pi^{*sm} = \begin{cases} \frac{(c+T-1)^2}{4} & i_e < i_1^{sm} \\ \frac{\{\delta^2+(-2ck+c^2+2Tc+T^2+2R+1)\delta+c^2k^2-2c^2k-2Tck-2Rck+c^2+2Tc+2Rc+T^2+2RT+R^2\}}{4\delta} & i_1^{sm} \le i_e \le i_2^{sm} \\ \frac{(c+T-1)^2(\delta+1)}{4} & i_e > i_2^{sm} \end{cases}$$

Proposition 2(a) shows that the optimal price for GV is constant for all values of i_e and depends only on the unit price c and green-tax T. Higher value of green-tax results in a price reduction for GV. On the other hand, the optimal price of EV varies with different values of i_e. When $i_1^{sm} \le i_e \le i_2^{sm}$, an increase in the subsidy R increases the price of EV and the optimal quantity for EV but decreases optimal quantity for GV. It can be noted that when, $i_e > i_2^{sm}$, the optimal quantity GV becomes zero as in this case the maximum allowable subsidy is paid for the purchase of EV and hence the effective price of the same reduces substantially, thereby making the purchase of GV less economically viable in comparison to an EV. Similarly, when $i_e < i_1^{sm}$, the least allowable subsidy is paid to the EV consumer, thereby making the purchase of EV less economically viable in comparison to a GV. When either $i_e < i_1^{sm}$ or $i_e > i_2^{sm}$, manufacturer's profit remains independent of R but increases with the green-tax T.

Proposition 2(b). *In the monopoly setting, under different range of values of i_g(for fixed R), the following holds:*

$$p_g^{*sm} = \begin{cases} \frac{\delta-ck+2c+R+1}{2} & i_g < i_3^{sm} \\ \frac{c-T+1}{2} & i_3^{sm} \le i_g \le i_4^{sm} \\ \frac{2c\delta+\delta-ck+2c+R+1}{2(\delta+1)} & i_g > i_4^{sm} \end{cases}$$

$$p_e^{*sm} = \begin{cases} \frac{\delta+ck+R+1}{2} & i_g < i_3^{sm} \\ \frac{\delta+ck+R+1}{2} & i_3^{sm} \le i_g \le i_4^{sm} \\ \frac{\delta+ck+R+1}{2} & i_g > i_4^{sm} \end{cases}$$

$$q_g^{*sm} = \begin{cases} \frac{\delta-ck+R+1}{2} & i_g < i_3^{sm} \\ \frac{ck-\delta(c+T)-c-T-R}{2\delta} & i_3^{sm} \le i_g \le i_4^{sm} \\ 0 & i_g > i_4^{sm} \end{cases}$$

$$q_e^{*sm} = \begin{cases} 0 & i_g < i_3^{sm} \\ \frac{\delta-ck+c+T+R}{2\delta} & i_3^{sm} \le i_g \le i_4^{sm} \\ \frac{\delta-ck+R+1}{2(\delta+1)} & i_g > i_4^{sm} \end{cases}$$

$$\pi^{*sm} = \begin{cases} \frac{(\delta-ck+R+1)^2}{4} & i_g < i_3^{sm} \\ \frac{\delta^2+(c^2+2Tc+T^2+2R+1-2ck)\delta+c^2k^2-2c^2k-2Tck-2Rck+c^2+2Tc+2Rc+T^2+2RT+R^2}{4\delta} & i_3^{sm} \le i_g \le i_4^{sm} \\ \frac{(\delta-ck+R+1)^2}{4(\delta+1)} & i_g < i_4^{sm} \end{cases}$$

Above proposition shows that the optimal price for an EV is constant for all values of i_g and depends only on the unit price c, k, consumer's low carbon awareness and subsidy R. On the other hand, when $i_3^{sm} \le i_g \le i_4^{sm}$ the price of a GV decreases with an increase in T as higher values of green-tax result in an effective price increase for the GV consumers and is counterbalanced by a reduction in the optimal price. It can be noted that when, $i_g > i_4^{sm}$ the optimal quantity GV becomes zero as in this case the maximum allowable tax is paid for the purchase of GV and hence the effective price of the same increases substantially, thereby making the purchase of GV less economically viable compared to EV. Similarly, when, $i_g < i_3^{sm}$ the least allowable tax is paid by the consumer of a GV, thereby making the purchase of EV less economically viable in comparison to a GV. When either $i_g < i_3^{sm}$ or $i_g > i_4^{sm}$, manufacturer's profit remains independent of T but increases with an increase in the subsidy R.

One important aspect of such policy instruments is to comprehend the environmental impact of GVs in current scenarios. This is explained by evaluating the environmental impact of GVs as $q_g^{sm} i_g$. Hence, we have $\frac{\partial\left(q_g^{sm} i_g\right)}{\partial \delta} = -\frac{i_g(ck - i_g + i_e - c)}{\delta^2}$. This gives us to condition on k as defined in the corollary 3 related to consumer awareness.

Corollary 3.

- *Whenever the cost coefficient of producing an EV is greater than $\frac{i_g - i_e + c}{c}$, the increase in consumer awareness decreases the overall environmental impact of GVs.*

From government perspective, it is desirable that the proposed policy instrument should help to reduce the environmental impact of GVs.

We now compare the impact of employing both R and T versus only R or only T. We study the effects on social welfare, environmental impact and vehicle stock (defined here as the total number of GVs and EVs sold).

Proposition 3(a). *The social welfare improves while using both subsidy and green-tax simultaneously as compared to the use of either subsidy or green-tax only.*

Proposition 3(b). *The environmental impact and vehicle stock improve in the presence of both policy instruments as compared to the use of subsidy only when $i_g \ge \frac{1-c}{2}$, and to the use of green-tax only when $i_e \ge \frac{1+\delta-kc}{2}$.*

Propositions 3(a) and 3(b) provide important insights for government and policymakers because the use of both policy instruments (subsidy and green-tax) is beneficial for the government while managing the interest of other stockholders also. The government can achieve more social welfare with the use of both policy instruments. Additionally, environmental impact and vehicle stock also improve under certain conditions of i_g and i_e as defined above.

5. Mathematical models under duopoly (model D)

In this setting, both manufacturers move simultaneously by deciding their optimal production quantities by considering the optimal reaction function of their respective competitors (Cournot form of competition). We have assumed the case of exclusivity implying that GV manufacturer will only produce GVs and the EV manufacturer will only produce EVs.

Following the steps as described in Section 4, the inverse demand functions for GV and EV are expressed as follows:

$p_g^{sd} = 1 - q_g^{sd} - q_e^{sd} - T$ and $p_e^{sd} = 1 + \left(1 - q_e^{sd}\right)\delta - q_g^{sd} - q_e^{sd} + R.$

Profit functions of GV and EV manufacturers in the presence of both R and T are as follows:

$\pi_g^{sd} = \left(p_g^{sd} - c\right) q_g^{sd}$ and $\pi_e^{sd} = \left(p_e^{sd} - kc\right) q_e^{sd}$

Substituting the values of p_g^{sd} and p_e^{sd} in the above expressions, we get

$$\pi_g^{sd} = (1 - q_g^{sd} - q_e^{sd} - c - T) q_g^{sd} \tag{9}$$

$$\pi_e^{sd} = q_e^{sd}\left((1 - q_e^{sd})\delta - q_g^{sd} - q_e^{sd} - ck + R + 1\right) \tag{10}$$

We can easily see that the profit functions of π_e^{sd} and π_g^{sd} are concave with respect to q_e^{sd} and q_g^{sd} respectively as $\frac{\partial^2 \pi_e^{sd}}{\partial \left(q_e^{sd}\right)^2} = -2\delta - 2 < 0$, and $\frac{\partial^2 \pi_g^{sd}}{\partial \left(q_g^{sd}\right)^2} = -2 < 0$, thus proving the concavity.

Theorem 2. *In the duopoly model, when firms move simultaneously, the following holds:*

(i) *The optimal number of vehicles sold:* $q_e^{sd} = \frac{2\delta - 2ck + c + T + 2R + 1}{4\delta + 3}$ *and* $q_g^{sd} = \frac{1 + ck - (2c + 2T - 1)\delta - 2c - 2T - R}{4\delta + 3}$

(ii) *The optimal price charged:* $p_e^{sd} = \frac{2\delta^2 + (2ck + c + T + 2R + 3)\delta + ck + c + T + 2R + 1}{4\delta + 3}$ *and* $p_g^{sd} = \frac{(2c - 2T + 1)\delta + ck + c - 2T - R + 1}{4\delta + 3}$

(iii) *The optimal profit for the manufacturers:*

$\pi_e^{sd} = \frac{(\delta + 1)(2\delta - 2ck + c + T + 2R + 1)^2}{(4\delta + 3)^2}$ *and* $\pi_g^{sd} = \frac{(2c\delta + 2T\delta - \delta - ck + 2c + 2T + R - 1)^2}{(4\delta + 3)^2}$

(iv) *The optimal consumer surplus:*

$$C^{sd} = \frac{(2\delta-2ck+c+T+2R+1)(2\delta^2-2ck\delta-3c\delta-3T\delta+2R\delta+5\delta-3c-3T+3)}{2(4\delta+3)^2}$$

For understanding the impact of subsidy and green-tax on social welfare, we proceed in the same way as shown in the case of monopoly and get the following social welfare function in terms of R and T:

$$\begin{aligned} SW^{sd} = {} & \frac{(2c\delta+2T\delta-\delta-ck+2c+2T+R-1)^2}{(4\delta+3)^2} - \frac{i_e(2\delta-2ck+c+T+2R+1)}{4\delta+3} - \frac{R(2\delta-2ck+c+T+2R+1)}{4\delta+3} \\ & + \frac{(\delta+1)(2\delta-2ck+c+T+2R+1)^2}{(4\delta+3)^2} + \frac{4\delta^3+(13-8ck+4c^2+(8T-8)c+4T^2-8T+8R)\delta^2}{32\delta^2+48\delta+18} \\ & + \frac{\left(4c^2k^2+(-8R-10)ck+5c^2+(10T-12)c+5T^2-12T+4R^2+10R+13\right)\delta+c^2k^2+(2c^2+(2T-2R-4)c)k+c^2+(2T-2R-4)c}{32\delta^2+48\delta+18} \\ & + \frac{T^2+(-2R-4)T+R^2+4R+4}{32\delta^2+48\delta+18} - \frac{i_g(1-(2c+2T-1)\delta+ck-2c-2T-R)}{4\delta+3} + \frac{T(1-(2c+2T-1)\delta+ck-2c-2T-R)}{4\delta+3} \end{aligned} \quad (11)$$

The social welfare function given in Equation (11) is jointly concave in both R and T. This can be proved through the second-order Hessian matrix, which gives: $\frac{\partial^2 SW^{sd}}{\partial R^2}\frac{\partial^2 SW^{sd}}{\partial T^2} - \left(\frac{\partial^2 SW^{sd}}{\partial R \partial T}\right)^2 = \frac{\delta}{(4\delta+3)^2} > 0$ and $\frac{\partial^2 SW^{sd}}{\partial R^2} = -\frac{4\delta+1}{(4\delta+3)^2} < 0$, thus proving the concavity at R and T.

The government's objective is to maximise the social welfare (11) by optimally choosing the policy instruments R and T. Writing down the first-order conditions for maximising social welfare and then solving the same we get $\hat{R} = \frac{\delta^2+(1-ck+i_g-2i_e+c)\delta-ck+i_g-i_e+c}{\delta}$ and $\hat{T} = \frac{(2i_g+c)\delta-ck+i_g-i_e+c}{\delta}$

The optimal social welfare can be written as:

$$SW^{sd} = \frac{(i_e-c)^2-2i_g(i_e-c)+c^2(k^2-2k)-2c(i_g-i_e)+{i_g}^2+\delta^2+\delta({i_g}^2+c^2+1+2ci_g-2i_e-2ck)}{2\delta} \quad (12)$$

As mentioned above, the government can adjust these R and T to optimise social welfare. Further, these policy instruments are linked with i_g and i_e. Considering policy instruments in terms of i_g and i_e helps governments and policymakers to align policies in line with tailpipe emissions of the vehicles. We thus, state a few propositions linking the different range of values of i_g and i_e and how they affect the optimal policy decisions.

Proposition 4(a). *In the duopoly simultaneous move game, for a different range of values of i_e (for fixed T), the following holds:*

1. *For $i_e < i_1^{sd}$, social welfare increases with an increase in R and is maximised at* $R_{min}^{sd} = \frac{1-2\delta+2ck-c-T}{2}$ *where* $i_1^{sd} = \frac{(2c+2T)\delta^2+\left(-2ck+i_g+3c+2T\right)\delta-ck+i_g+c}{2\delta+1}$.
2. *For $i_1^{sd} \le i_e \le i_2^{sd}$, SW is maximised at* $R = \hat{R} = \frac{\delta^2+(-ck+i_g-2i_e+c+1)\delta-ck+i_g-i_e+c}{\delta}$ *where* $i_2^{sd} = \frac{4\delta^2+\left(-4ck+2i_g+3c+T+3\right)\delta-2ck+2i_g+2c}{4\delta+2}$.
3. *For $i_e > i_2^{sd}$, social welfare decreases with an increase in R and is maximised at* $R_{max}^{sm} = 1+(1-2c-2T)\delta+ck-2c-2T$.

Here, i_1^{sd} and i_2^{sd} are the two bounds on the social welfare curve, which divide the region into three segments, as discussed in the case of monopoly. The social welfare changes in these regions as defined above.

Proposition 4(b). *In the duopoly simultaneous move game, for a different range of values of i_g (for fixed R), the following holds:*

1. *For, $i_g < i_3^{sd}$, Social welfare increases with an increase in T and the optimal value of SW*

occurs at $T^{sd}_{min} = 2ck - 2\delta - c - 2R - 1$

where $i^{sd}_3 = \frac{ck+i_e-2\delta^2-(1-2ck+2c+2R)\delta-c}{2\delta+1}$

2. *When* $i^{sd}_3 \le i_g \le i^{sd}_4$, *the government will choose* $T = \hat{T} = \frac{(2i_g+c)\delta-ck+i_g-i_e+c}{\delta}$ *to maximise the social welfare*

   where $i^{sd}_4 = \frac{2ck+2i_e-(4c-1)\delta^2+(1+3ck+2i_e-6c-R)\delta-2c}{4\delta^2+6\delta+2}$
3. *For,* $i_g > i^{sd}_4$, *Social welfare decreases with an increase in* T *and the optimal value of SW occurs at* $T^{sm}_{max} = \frac{1-(2c-1)\delta+ck-2c-R}{2\delta+2}$

Similarly, i^{sd}_3 and i^{sd}_4 are the two bounds on the social welfare curve and vary based on T as illustrated above.

Proposition 5(a). *In the duopoly simultaneous move game, retail price, demand and profit function of GV and EV for a different range of values of* i_e *are expressed as:*

$$p^{*sd}_g = \begin{cases} \frac{c-T+1}{2} & i_e < i^{sd}_1 \\ \frac{(2c-2T+1)\delta+ck+c-2T-R+1}{4\delta+3} & i^{sd}_1 \le i_e \le i^{sd}_2 \\ c & i_e > i^{sd}_2 \end{cases}$$

$$p^{*sd}_e = \begin{cases} ck & i_e < i^{sd}_1 \\ \frac{2\delta^2+(2ck+c+T+2R+3)\delta+ck+c+T+2R+1}{4\delta+3} & i^{sd}_1 \le i_e \le i^{sd}_2 \\ 1-c\delta-T\delta+\delta+ck-c-T & i_e > i^{sd}_2 \end{cases}$$

$$q^{*sd}_g = \begin{cases} \frac{1-c-T}{2} & i_e < i^{sd}_1 \\ \frac{1-(2c+2T-1)\delta+ck-2c-2T-R}{(4\delta+3)} & i^{sd}_1 \le i_e \le i^{sd}_2 \\ 0 & i_e > i^{sd}_2 \end{cases}$$

$$q^{*sd}_e = \begin{cases} 0 & i_e < i^{sd}_1 \\ \frac{2\delta-2ck+c+T+2R+1}{(4\delta+3)} & i^{sd}_1 \le i_e \le i^{sd}_2 \\ 1-c-T & i_e > i^{sd}_2 \end{cases}$$

$$\pi^{*sd}_e = \begin{cases} 0 & i_e < i^{sd}_1 \\ \frac{(\delta+1)(1+2\delta-2ck+c+T+2R)^2}{(4\delta+3)^2} & i^{sd}_1 \le i_e \le i^{sd}_2 \\ (c+T-1)^2(\delta+1) & i_e > i^{sd}_2 \end{cases}$$

$$\pi^{*sd}_g = \begin{cases} \frac{(1-c-T)^2}{4} & i_e < i^{sd}_1 \\ \frac{(1-2c\delta-2T\delta+\delta+ck-2c-2T-R)^2}{(4\delta+3)^2} & i^{sd}_1 \le i_e \le i^{sd}_2 \\ 0 & i_e > i^{sd}_2 \end{cases}$$

Proposition 5(a) shows when $i^{sd}_1 \le i_e \le i^{sd}_2$, an increase in subsidy leads to a decrease in price and quantity of GVs but increase in price and quantity of EVs. This is because a higher subsidy for EV makes it more economically viable compared to GV for environmentally sensitive consumers. Similarly, for the same range of i_e, the profit of the EV manufacturer increases but the profit for the GV manufacturer decreases with an increase in R. It can be noted than when $i_e > i^{sd}_2$ the optimal quantity of GVs becomes zero as in this case the maximum allowable subsidy is paid for the purchase of EVs and hence the effective price of the same reduces substantially, thereby making the purchase of GVs less economically viable compared to EVs. Similarly when $i_e < i^{sd}_1$, the least allowable subsidy is paid to the EV consumer, thereby making the purchase of EV less economically viable in comparison to GV. When $i_e < i^{sd}_1$, GV manufacturer's profit remains independent of R but decreases with an increase in T.

Proposition 5(b). *In the duopoly simultaneous move game, retail price, demand and profit function of GV and EV for different range of values of* i_g *are expressed as:*

$$p^{*sd}_g = \begin{cases} \delta-ck+c+R+1 & i_g < i^{sd}_3 \\ \frac{1+(2c-2T+1)\delta+ck+c-2T-R}{4\delta+3} & i^{sd}_3 \le i_g \le i^{sd}_4 \\ c & i_g > i^{sd}_4 \end{cases}$$

$$p_e^{*sd} = \begin{cases} ck & i_g < i_3^{sd} \\ \dfrac{2\delta^2 + (2ck + c + T + 2R + 3)\delta + ck + c + T + 2R + 1}{4\delta + 3} & i_3^{sd} \leq i_g \leq i_4^{sd} \\ \dfrac{\delta + ck + R + 1}{2} & i_g > i_4^{sd} \end{cases}$$

$$q_g^{*sd} = \begin{cases} \delta - ck + 1 + R & i_g < i_3^{sd} \\ \dfrac{(1-(2c+2T-1)\delta + ck - 2c - 2T - R)}{(4\delta+3)} & i_3^{sd} \leq i_g \leq i_4^{sd} \\ 0 & i_g > i_4^{sd} \end{cases}$$

$$q_e^{*sd} = \begin{cases} 0 & i_g < i_3^{sd} \\ \dfrac{2\delta - 2ck + c + T + 2R + 1}{(4\delta+3)} & i_3^{sd} \leq i_g \leq i_4^{sd} \\ \dfrac{\delta - ck + R + 1}{2(\delta+1)} & i_g > i_4^{sd} \end{cases}$$

$$\pi_e^{*sd} = \begin{cases} 0 & i_g < i_3^{sd} \\ \dfrac{(\delta+1)(2\delta - 2ck + c + T + 2R + 1)^2}{(4\delta+3)^2} & i_3^{sd} \leq i_g \leq i_4^{sd} \\ \dfrac{(\delta - ck + R + 1)^2}{4(\delta+1)} & i_g > i_4^{sd} \end{cases}$$

$$\pi_g^{*sd} = \begin{cases} (1 + \delta - ck + R)^2 & i_g < i_3^{sd} \\ \dfrac{(1 - 2c\delta - 2T\delta + \delta + ck - 2c - 2T - R)^2}{(4\delta+3)^2} & i_3^{sd} \leq i_g \leq i_4^{sd} \\ 0 & i_g > i_4^{sd} \end{cases}$$

Proposition 5(b) illustrates that when $i_3^{sd} \leq i_g \leq i_4^{sd}$, an increase in green-tax leads to a decrease in price and quantity of GVs, but increase in price and quantity of EVs for reasons similar to as explained above. Similarly, for the same range of i_g the profit of the EVs manufacturer increases but the profit for the GVs manufacturer decreases with an increase in R. Again, it can be noted than when $i_g > i_4^{sd}$ the optimal quantity of GVs becomes zero as in this case the maximum allowable tax is paid for the purchase of GVs and hence the effective price of the same increases substantially, thereby making the purchase of GVs less economically viable as compared to EVs. Similarly, when $i_g < i_3^{sd}$, the least allowable tax is paid by the consumer of a GV thereby making the purchase of EV less economically viable in comparison to a GV. When $i_g < i_3^{sd}$ GV manufacturer's profit remains independent of T but increases with an increase in R.

In line with the monopoly case, we compare the benefits while using both policy instruments vs. a single policy instrument in the duopoly setting as follows:

Proposition 6(a). *In the duopoly simultaneous move game, the social welfare improves while using both subsidy and green-tax as compared to using either subsidy or green-tax.*

Proposition 6(b). *In the duopoly simultaneous move game, the environmental impact and vehicle stock improve in the presence of both policy instruments as compared to the use of subsidy only when* $i_g \geq \frac{i_e + ck - c\delta - c}{2\delta + 1}$ *and green-tax only when* $i_e \leq \frac{\delta^2 + (1 - ck + i_g + c)\delta - ck + i_g + c}{2\delta + 1}$ *or* $i_e \geq \frac{i_g\delta + i_g}{2\delta + 1}$ *whenever* $1 \leq k \leq 1 + \frac{\delta}{c}$ *else when* $i_e \leq \frac{i_g\delta + i_g}{2\delta + 1}$ or $i_e \geq \frac{\delta^2 + (-ck + i_g + c + 1)\delta - ck + i_g + c}{2\delta + 1}$ *and vehicle stock when* $i_e \geq \frac{\delta^2 + (1 - ck + i_g + c)\delta - ck + i_g + c}{2\delta + 1}$.

Next, we discuss cases related to whom the green-tax or subsidy is targeted at. In other words, what happens when instead subsidy is provided to EV manufacturer in place of EV consumer, and green-tax is imposed on GV manufacturer in place of GV consumer. In such cases, we can express the consumer's utility function for monopoly market as follows:

$$U_e = (1 + \delta)\theta - p_e^{sm} + \alpha R \tag{12}$$

$$U_g = \theta - p_g^{sm} - \beta T \tag{13}$$

$$U_r = 0 \tag{14}$$

Using methods described above, we obtain $\hat{R} = -\delta - ck - 2i_e - 1 < 0$. Hence, when the subsidy is given to the EV manufacturer instead of EV consumer, we get an infeasible solution. However, if the same subsidy-green-tax model is applied in the duopoly case for manufacturers, we end up getting identical results as obtained in the case when both subsidy and green-tax are given to the consumers.

Proposition 7. *The optimal social welfare obtained in the monopoly case is equal to that of the duopoly case.*

This signifies that the optimal social welfare is the same in both models, although their respective optimal subsidy and green-tax values are different. It means that the government has to utilise policy instruments (R and T) differently in both the cases in order to get the same optimal social welfare.

6. Numerical results and discussion

We conduct numerical experiments to complement our findings. These will help us to understand the effect of per unit environmental impact of GV and EV on various decisions of governments and firms under different market structures. Using the model set-up and values of the model parameters similar to Crane and Mao (2015), Cen, Lo, and Li (2016), and Shao et al. (2017), we get Table 3 for model parameters:

Further, we choose the market potential and subsequently adjust all other real values of unit costs for GV and EV in the interval $[0, 1]$. The variation in social welfare for different values of i_g and i_e for both models are presented in Figure 3.

In Figure 3, we observe that social welfare peaks for some intermediate range of i_g and i_e (deep blue zone) which is consistent with our findings in Propositions 1 and 4. We also notice that when i_g and i_e increases, the social welfare also increases, and after attaining a maximum value, starts decreasing. These combinations of i_g and i_e are important for governments to maximise societal benefits by adjusting both R and T. With the help of Equations (8) and (11), we illustrate the variation of social welfare with respect to R and T by considering an illustrative example. For instance, if we consider the environmental impact of EV and GV to be 0.02 and 0.15 respectively, the variation of social welfare with respect to R and T for both models is as shown below (Figure 4).

Figure 4 indicates that for a fixed value of the environmental impact of EV and GV, social welfare changes based on the different range of values of policy instruments R and T for different market structures. This finding is important for policy-makers to use appropriate combinations of R and T based on the environmental impact of EVs and GVs to maximise social welfare. Proceeding similarly, we get the environmental impact (EI) for both models as presented in Figure 5.

We can see that the environmental impact is minimum (green region) when i_e is at minimum and i_g is at the maximum range (Figure 5). Here green-tax plays a crucial role in minimising the environmental impact of GV, because higher i_g means more green-tax to be levied, thereby discouraging the GV consumers and reducing the overall GV sales. Since the environmental impact is a function of the quantity of the vehicles sold, the overall

Table 3. Table for model parameters.

Model parameters	c	k	δ	i_g	i_e
Values	0.5	1.16	0.055	0.02–0.2	0.02–0.15

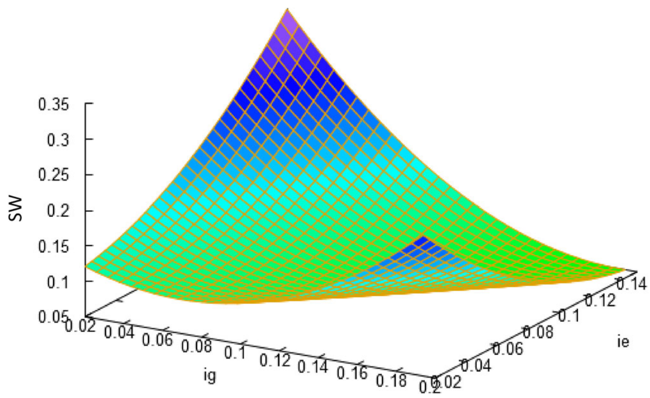

Figure 3. Social welfare curve for different values for i_g and i_e for models M and D.

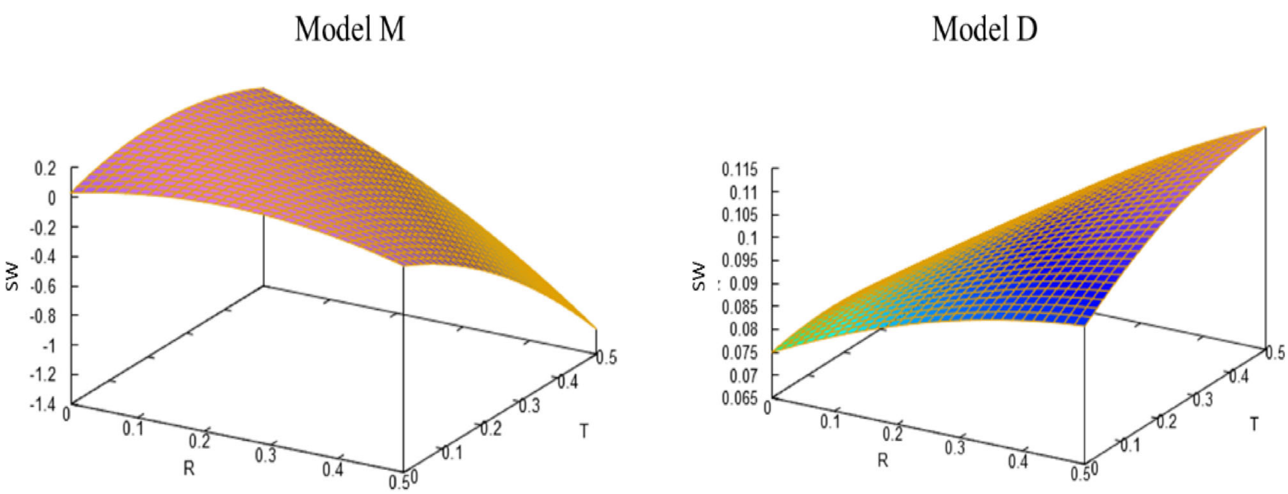

Figure 4. Social welfare curve for different values for R and T for models M and D.

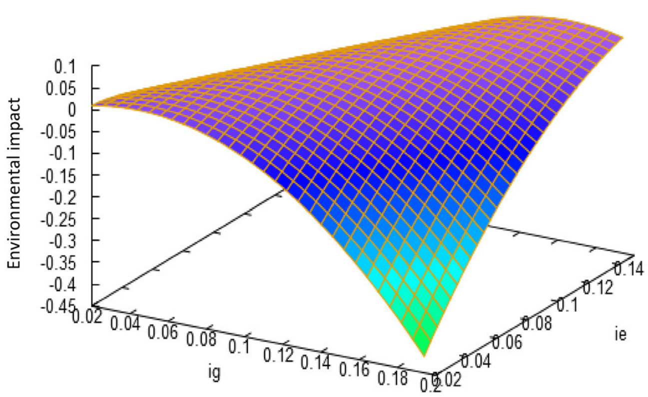

Figure 5. Environmental impact curve for different values for i_g and i_e for models *M* and D.

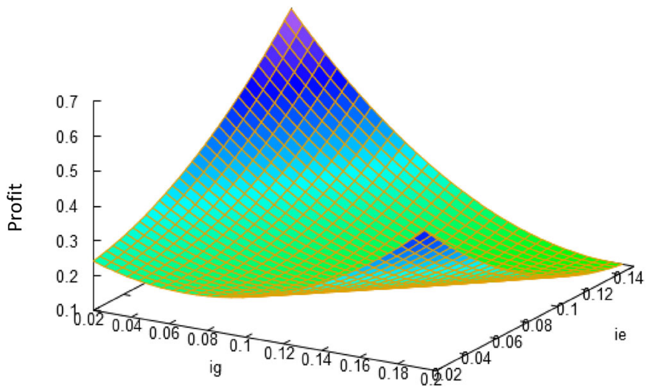

Figure 6. Manufactures profit curve for different values for i_g and i_e (Model M).

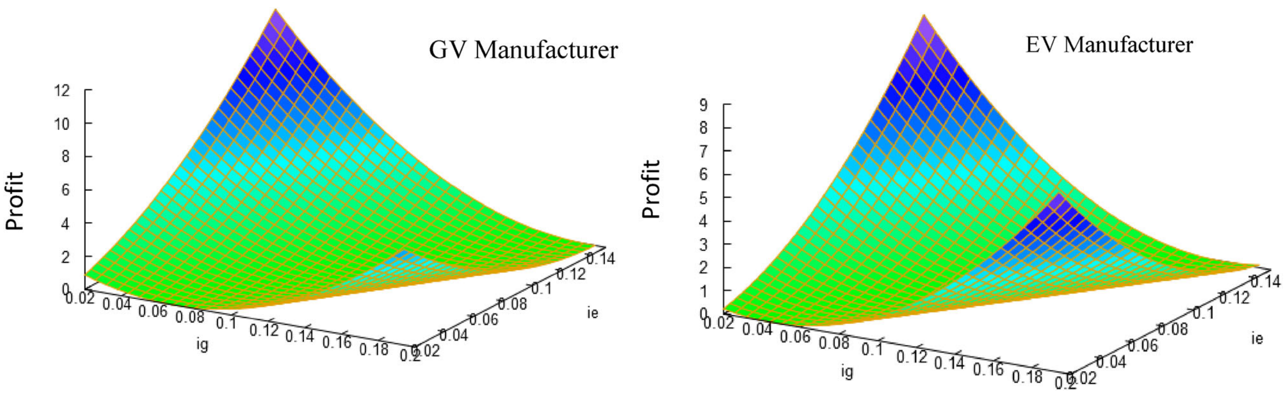

Figure 7. Manufacturers profit curve for different values for i_g and i_e (Model D).

environmental impact reduces. The profit curves for models M and D are shown in Figures 6 and 7, respectively.

From Figure 6, it is clear that the profit of the firm peaks (deep blue zone) where i_e and i_g both are in some intermediate range. As i_e and i_g increase, the manufacturer's profit first increases, then attains a maximum value and finally starts decreasing, which is consistent with our findings in Propositions 2(a) and 2(b). This graph shows how the manufacturer's profit gets maximised under different values of i_e and i_g in a dual policy scheme.

Figure 7 indicates that the GV manufacturer has a higher profit than EV manufacturer for lower values of i_g, whereas the EV manufacturer has higher profits for higher values of i_g and lower values of i_e. The deep blue zone indicates the zone of maximum profit. Further, the profits increase as i_g and i_e increase, attain maximum value, and then decrease with further increase in i_g and i_e. In the case of vehicle stock, it depends only on i_g for both models and follows a linear relation with i_g. When i_g increases, vehicle stock decreases because higher i_g is liable for a higher green-tax, and this helps to reduce the vehicle stock. This finding also provides a rationale for the imposition of green-tax on GV consumers to minimise the vehicle stock along with maximising the social welfare. From the government's perspective, understanding of subsidy amount is significant to devise EV adoption

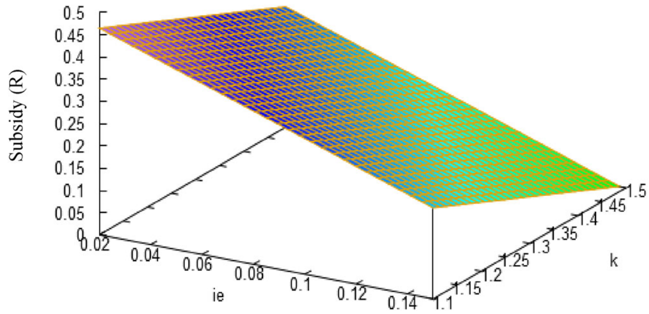

Figure 8. Subsidy curve for different values for i_e and k (Model M).

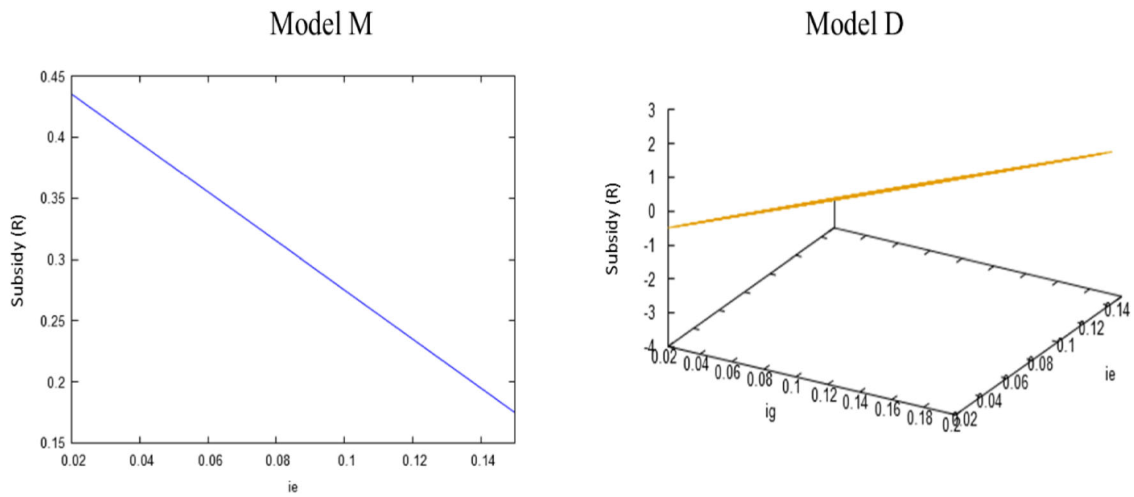

Figure 9. Subsidy variation for different values for i_g and i_e for models M and D.

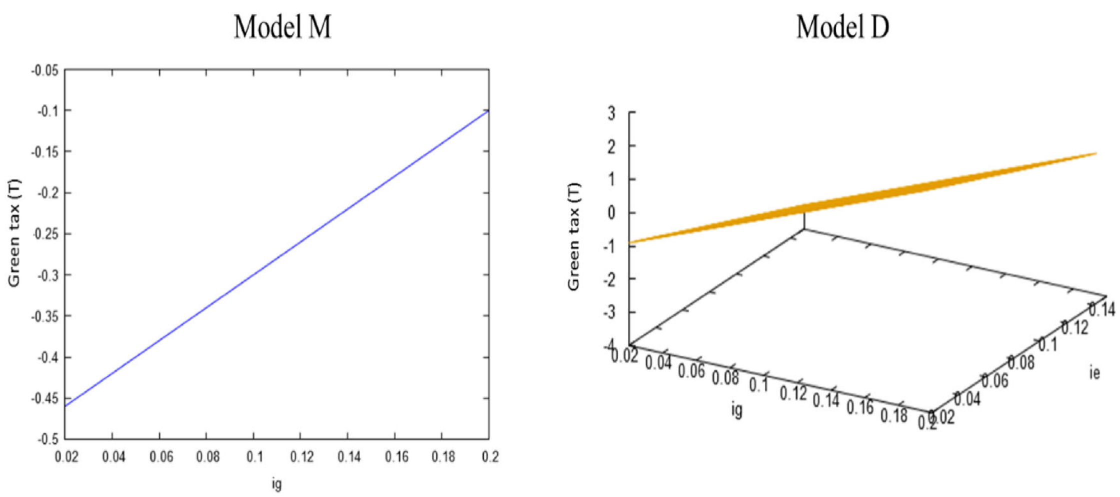

Figure 10. Green tax variation for different values for i_g and i_e for models M and D.

strategy. The optimal subsidy varies for different combinations of i_e and k as depicted in Figure 8. The subsidy is maximum when i_e and k are in the minimum range. Subsidy then decreases when i_e and k both increase and becomes minimum when both i_e and k are maximum in the given range as marked by green zone.

Similarly, the subsidy and the green-tax variation can be studied. We illustrate how optimal subsidy (R) and green-tax (T) vary based on the environmental impact of EV and GV, as shown below (Figures 9 and 10).

The model M illustrates (Figures 9 and 10) that as i_e increases, subsidy decreases, and when i_g increases, green-tax increases in the case of monopoly. On similar lines, for duopoly model D, green-tax and subsidy increase for combinations of i_e and i_g as shown above. Future government policies for reducing vehicular emissions can depend on values of i_g and i_e to differentiate among different vehicle technologies (e.g., GV, EV, hydrogen vehicles, etc.) and different vehicle categories. These findings provide key insights related to subsidy and green-tax for better EV adoption and increased social welfare.

7. Conclusions and future research

Policy instruments are effective tools in influencing many operations management decisions. In the context of EV adoption, subsidy and green-tax combination are shown to have positive results in improving social welfare under different market structures. We have also shown that environmental impact and vehicle stock both get improved under certain conditions for both subsidy and green-tax. An important insight that emerges from our study is that the extent of the environmental impact of EV and GV affecting not just the governments' decisions about subsidy and green-tax but also to manufacturers' decisions related to the optimal quantity of GVs and EVs under both market structures.

Further, under a certain range of unit environmental impact of these vehicles, there could be an exit of either from the vehicle's market due to the demand becoming zero. An implication of the same is that manufacturers of both GVs and EVs can focus more on reducing the environmental impact of both types of vehicles by making changes at the product design stage itself. We found that the overall social welfare is same in the case of duopoly models as compared against the monopoly model. Further, the optimal profits, vehicle stock, and environmental impact also changed with the market structure. Another important insight from our study is that whether one provides a subsidy to consumer or manufacturer or imposes a green-tax on consumer or manufacturer, the overall impact on social welfare, vehicle stock, and environmental impact remain the same.

Our study adds to the current policy and management debate around the role of subsidies and taxes in incentivising higher adoption of EVs. Many governments world over are taking steps to reduce vehicular tailpipe emissions and promote zero-emission vehicles. These tailpipe emissions can be considered as a measure of i_g and i_e to differentiate among vehicle categories.

For a better understanding of the choices for governments and the use of green-tax and subsidies, one can consider recent developments in India. FAME (Faster Adoption and Manufacturing of Hybrid and Electric Vehicles), a national policy introduced by the Government of India in 2015, aimed at providing direct upfront subsidy to EV consumers as part of demand incentives to reduce the EV purchase price and boost EV demand. FAME was revised in 2019 and the revised FAME, called FAME-II, would be effective in India from April 2019 till March 2022. FAME-II continues upfront subsidy to EV as a demand-side policy incentive to boost EV sales. Consumers will have to pay an upfront reduced price, and the Indian government would reimburse the EV manufacturer directly. In another proposal floated in 2018, Society of Manufacturers of Electric Vehicles (SMEV), a leading body of EV manufacturers in India, has proposed taxing diesel vehicles and passing the benefits in the form of subsidies to the consumers. This proposal is similar to our case of imposing both instruments simultaneously (*Financial Express*, 2018). In another development, Germany has expressed reservations about continuing subsidy for EV consumers for a longer time (*Economic Times*, 2018). By studying the role of taxes and subsidies, imposed on consumers and manufacturers separately and jointly, under different market structures, we have studied the impact on social welfare, environmental impact, and vehicle stock.

One of the limitations of our work is the consideration of exclusivity of GV and EV manufacturers. A more realistic scenario would be to incorporate the case when multiple manufacturers are present in the market, selling both GVs and EVs. Another aspect that we can consider as a possible extension of our work is to find the optimal level of investment at the design stage of vehicles that will bring down the unit environmental impact. Further, we have considered the imposition of green-tax on the purchase of GVs, which is a one-time tax. Instead, we can consider another form of tax in the form of annual miles travelled of GVs. The increasing cost differential between the service costs of a GV and an EV can be explored. Collecting relevant industry evidence and empirical analysis can be a part of future research.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix

A1. Proof of Theorem 1

$$\pi^{sm} = ((1-q_e^{sm})\delta - q_g^{sm} - q_e^{sm} - kc + R + 1)q_e^{sm} + (-q_g^{sm} - q_e^{sm} - c - T + 1)\ q_g^{sm}$$

Writing down the first-order conditions w.r.t. q_e^{sm} and q_g^{sm} and then solving the system of equations, we get $q_e^{sm} = \frac{\delta-ck+c+T+R}{2\delta}$ and $q_g^{sm} = \frac{ck-\delta(c+T)-c-T-R}{2\delta}$.

Substituting these values for the expressions of p_e^{sm}, p_g^{sm} and π^{sm}, we get the required results.

For finding consumer surplus, we know $\theta_1 = \frac{p_e^{sm} - p_g^{sm} - T - R}{\delta}$ and $\theta_2 = p_g^{sm} + T$. Substituting the values of p_e^{sm} and p_g^{sm} into above expressions, we get $\theta_1 = \frac{\delta+ck-c-T-R}{2\delta}$ and $\theta_2 = \frac{c+T+1}{2}$

Substituting values for $U_e(\theta)$, $U_g(\theta)$, θ_1 and θ_2 in the above expression and then solving the same, we get the desired result.

A2. Proofs of Propositions 1(a) and 1(b)

It is known that $\frac{\partial^2 SW}{\partial R^2} = -\frac{1}{4\delta} < 0$.

Now, from $q_e^{sm} = \frac{\delta-ck+c+T+R}{2\delta} \geq 0$ and $q_g^{sm} = -\frac{\delta(c+T)-ck+c+T+R}{2\delta} \geq 0$ we have $\delta - ck + c + T + R \geq 0$ and $\delta(c+T) - ck + c + T + R \leq 0$ thereby implying

$R_{max}^{sm} = ck - c - T\delta - c\delta - T$ and $R_{min}^{sm} = ck - \delta - c - T$ for any fixed T.

Now, $\hat{R} - R_{min}^{sm} = \delta - ck - 2i_e + 1 - (ck-\delta-c-T\ ) \geq 0$ gives $i_e \leq \frac{2\delta-2ck+c+T+1}{2} = i_2^{sm}$

Also $\hat{R} - R_{max}^{sm} = \delta - ck - 2i_e + 1 - (ck-c-T\delta-c\delta-T) \leq 0$ gives $i_e \geq \frac{(c+T+1)\delta-2ck+c+T+1}{2} = i_1^{sm}$.

Again it is known that $\frac{\partial^2 SW}{\partial T^2} = -\frac{\delta+1}{4\delta} < 0$.

Now, from above, we have $\delta - ck + c + T + R \geq 0$ and $\delta(c+T) - ck + c + T + R \leq 0$ thereby implying

$T_{max}^{sm} = \frac{ck-c-c\delta-R}{1+\delta}$ and $T_{min}^{sm} = ck - \delta - c - R$ for any fixed R.

Now, $\hat{T} - T_{min}^{sm} = 2i_g + c - 1 - (ck-\delta-c-R\ ) \geq 0$ implies $i_g \geq \frac{1-\delta+ck-2c-R}{2} = i_3^{sm}$.

Also $\hat{T} - T_{max}^{sm} = 2i_g + c - 1 - \left(\frac{ck-c-c\delta-R}{1+\delta}\right) \leq 0$ implies $i_g \leq \frac{1-(2c-1)\delta+ck-2c-R}{2\delta+2} = i_4^{sm}$.

A3. Proofs of Propositions 2(a) and 2(b)

By substituting the required values of R and T, respectively, into the expressions for p_e^{sm}, p_g^{sm}, q_e^{sm}, q_g^{sm} and π_{sm}, we get the desired results.

A4. Proof of Propositions 3(a) and 3(b)

$$SW_{with\ \ (T,R)}^{sm} - SW_{with\ \ (R)}^{sm} = \frac{(2i_g+c-1)^2}{8} > 0$$

Here subscript "with (T, R)" represents the use of both policy instruments, while the subscript "with (R)" indicates the use of only subsidy. Similar notations are used while comparing other benefits. Similarly, $SW_{with\ \ (T,R)}^{sm} - SW_{with\ \ (T)}^{sm} = \frac{(\delta-ck-2i_e+1)^2}{8(\delta+1)} > 0$

$EI_{with\ \ (T,R)}^{sm} - EI_{with\ \ (R)}^{sm} = \frac{i_g(1-c-2i_g)}{2} > 0$ and $VS_{with\ \ (T,R)}^{sm} - VS_{with\ \ (R)}^{sm} = \frac{1-2i_g-c}{2} > 0$ if $i_g < \frac{1-c}{2}$

$EI^{sm}_{with\ (T,R)} - EI^{sm}_{with\ (T)} = \frac{i_e(\delta - ck - 2i_e + 1)}{2(\delta+1)} > 0$ and $VS^{sm}_{with\ (T,R)} - VS^{sm}_{with\ (T)} = \frac{\delta - ck - 2i_e + 1}{2(\delta+1)} > 0$

$$if\ \ i_e < \frac{1 - ck + \delta}{2}$$

B1. Proof of Theorem 2

$$\pi_e^{sd} = q_e^{sd}((1 - q_e^{sd})\delta - q_g^{sd} - q_e^{sd} - ck + R + 1)$$

Writing down the first-order condition w.r.t. q_e^{sd}, we get $(1 - q_e^{sd})\delta + q_e^{sd}(-\delta - 1) - q_g^{sd} - q_e^{sd} - ck + R + 1 = 0$ and $\pi_g^{sd} = (1 - q_g^{sd} - q_e^{sd} - c - T)q_g^{sd}$.

Writing down the first-order condition w.r.t. q_g^{sd}, we get $1 - 2q_g^{sd} - q_e^{sd} - c - T = 0$.

Solving the above system of equations, we get the required values for q_e^{sd} and q_g^{sd}. On substituting the optimal values of q_e^{sd} and q_g^{sd}, we get other results.

$\theta_1 = \frac{2\delta + 2ck - c - T - 2R + 2}{4\delta + 3}$ and $\theta_2 = \frac{(2c + 2T + 1)\delta + ck + c + T - R + 1}{4\delta + 3}$

$$U_e = \frac{(4\delta^2 + 7\delta + 3)\theta - 2\delta^2 + (-2ck - c - T + 2R - 3)\delta - ck - c - T + R - 1}{4\delta + 3},$$

$$U_g = \frac{(4\delta + 3)\theta + (-2c - 2T - 1)\delta - ck - c - T + R - 1}{4\delta + 3}$$

After getting threshold and utility functions, we can easily calculate all other desired results.

B2. Proof of Propositions 4(a) and 4(b)

It is known that $\frac{\partial^2 SW}{\partial R^2} = -\frac{4\delta + 1}{(4\delta + 3)^2} < 0$. As discussed earlier, demand functions are setting the R boundaries as follows. $R^{sd}_{max} = (-2c - 2T + 1)\delta + ck - 2c - 2T + 1$ and $R^{sd}_{min} = \frac{2ck - 2\delta - c - T - 1}{2}$ and $\hat{R} = \frac{\delta^2 + (1 - ck + i_g - 2i_e + c)\delta - ck + i_g - i_e + c}{\delta}$.

Now,
$R^{sd}_{min} - \hat{R} = -\frac{4\delta^2 - 4ck\delta + 2i_g\delta - 4i_e\delta + 3c\delta + T\delta + 3\delta - 2ck + 2i_g - 2i_e + 2c}{2\delta} \leq 0$, implies

$$i_e \leq \frac{4\delta^2 + \left(-4ck + 2i_g + 3c + T + 3\right)\delta - 2ck + 2i_g + 2c}{4\delta + 2} = i_2^{sd}.$$

Similarly,
$R^{sd}_{max} - \hat{R} = -\frac{2c\delta^2 + 2T\delta^2 - 2ck\delta + i_g\delta - 2i_e\delta + 3c\delta + 2T\delta - ck + i_g - i_e + c}{\delta}$, implies

$$i_e \geq \frac{(2c + 2T)\delta^2 + (-2ck + i_g + 3c + 2T)\delta - ck + i_g + c}{2\delta + 1} = i_1^{sd}.$$

Also it is known that $\frac{\partial^2 SW}{\partial T^2} = -\frac{(\delta + 1)(4\delta + 1)}{(4\delta + 3)^2} < 0$, $T^{sd}_{max} = \frac{1 - (2c - 1)\delta + ck - 2c - R}{2\delta + 2}$, and $T^{sd}_{min} = 2ck - 2\delta - c - 2R - 1$ and, $\hat{T} = \frac{(2i_g + c)\delta - ck + i_g - i_e + c}{\delta}$.

$T^{sd}_{min} - \hat{T} = -\frac{2\delta^2 - 2ck\delta + 2i_g\delta + 2c\delta + 2R\delta + \delta - ck + i_g - i_e + c}{\delta} \leq 0$ gives

$$i_g \leq \frac{2ck + 2i_e - (4c - 1)\delta^2 + (1 + 3ck + 2i_e - 6c - R)\delta - 2c}{4\delta^2 + 6\delta + 2} = i_4^{sd}.$$

$T^{sd}_{max} - \hat{T} = -\frac{4i_g\delta^2 + 4c\delta^2 - \delta^2 - 3ck\delta + 6i_g\delta - 2i_e\delta + 6c\delta + R\delta - \delta - 2ck + 2i_g - 2i_e + 2c}{2\delta(\delta + 1)}$ gives

$$i_g \geq \frac{ck + i_e - 2\delta^2 - (1 - 2ck + 2c + 2R)\delta - c}{2\delta + 1} = i_3^{sd}.$$

B3. Proofs of Propositions 5(a) and 5(b)

By substituting the required values of R and T respectively into the expressions for $p_e^{sd}, p_g^{sd}, q_e^{sd}$, q_g^{sd}, π_g^{sd} and π_e^{sd}, we get the desired results.

B4. Proofs of Propositions 6(a) and 6(b)

$$SW^{sd}_{(T,R)} - SW^{sd}_{(R)} = \frac{(2i_g\delta + c\delta - \mathrm{ck} + i_g - i_e + c)^2}{2\delta(4\delta + 1)} > 0$$

$$SW^{sd}_{(T,R)} - SW^{sd}_{(T)} = \frac{(\delta^2 - ck\delta + i_g\delta - 2i_e\delta + c\delta + \delta - ck + i_g - i_e + c)^2}{2\delta(4\delta + 1)(\delta + 1)} > 0$$

$EI^{sd}_{T,R} - EI^{sd}_{T} = -\frac{\left(i_g\delta - 2i_e\delta + i_g - i_e\right)\left(\delta^2 - ck\delta + i_g\delta - 2i_e\delta + c\delta + \delta - ck + i_g - i_e + c\right)}{\delta(\delta + 1)(4\delta + 1)} > 0$ if $i_e \leq \frac{\delta^2 + \left(1 - ck + i_g + c\right)\delta - ck + i_g + c}{2\delta + 1}$ and $i_e \geq \frac{i_g\delta + i_g}{2\delta + 1}$ whenever $1 \leq k \leq 1 + \frac{\delta}{c}$ else when $i_e \leq \frac{i_g\delta + i_g}{2\delta + 1}$ or $i_e \geq \frac{\delta^2 + (-ck + i_g + c + 1)\delta - ck + i_g + c}{2\delta + 1}$

$$EI^{sd}_{T,R} - EI^{sd}_{R} = -\frac{\left(2i_g\delta + i_g - i_e\right)\left(2i_g\delta + c\delta - ck + i_g - i_e + c\right)}{\delta(4\delta + 1)} > 0$$

if $i_g \geq \frac{i_e + ck - c\delta - c}{2\delta + 1}$

$$VS^{sd}_{T,R} - VS^{sd}_{R} = -\frac{2\left(2i_g\delta + c\delta - ck + i_g - i_e + c\right)}{4\delta + 1} > 0 \text{ if } i_g \geq \frac{i_e + ck - c\delta - c}{2\delta + 1}$$

$$VS^{sd}_{T,R} - VS^{sd}_{T} = \frac{\delta^2 - ck\delta + i_g\delta - 2i_e\delta + c\delta + \delta - ck + i_g - i_e + c}{(\delta + 1)(4\delta + 1)} > 0$$

if $i_e \geq \delta^2 + (1 - ck + i_g + c)\delta \ \frac{-ck + i_g + c}{2\delta + 1}$