Business Models in the Sharing Economy: Manufacturing Durable Goods in the Presence of Peer-to-Peer Rental Markets

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Abstract

Business models that focus on providing access to assets rather than on transferring ownership of goods have become an important industry trend, representing a challenge for incumbent firms. This paper analyzes the interaction of a peer-to-peer (P2P) rental market and an original equipment manufacturer (OEM). The analysis highlights the important role of consumer heterogeneity in usage rates in determining which business model would be preferred by the OEM. The introduction of a P2P rental market creates an equalizing effect, which leads to purchases from low-usage consumers. P2P rentals act as a discrimination device, allowing the OEM to implicitly segment consumers and extract a larger fraction of surplus, which might hurt consumers. Furthermore, the OEM is better off with P2P rentals when the heterogeneity in usage rates is intermediate, while the consumers are better off with P2P rentals when the heterogeneity is sufficiently high. This paper examines different business models such as the sales-only OEM, the OEM offerings rentals in addition to sales (the "dual" firm), introducing a P2P platform to the market (the "P2P-sponsoring" firm), and a mixed structure in which the OEM competes against a P2P by introducing its own direct rentals (the "dual-plus-P2P" firm). Consumer heterogeneity in usage rates continues to play a fundamental role in market outcomes. When usage rates and heterogeneity in usage rates are sufficiently large, the OEM is better off offering sales and facilitating a P2P rental market. In contrast, if heterogeneity in usage rates is too low, the OEM prefers to operate as a sales-only monopoly. If heterogeneity is too high and usage rates are below a threshold, the OEM prefers to operate as a dual firm that offers both sales and rentals directly to consumers. If P2P rentals are unavoidable, the OEM would not necessarily be better off by introducing its own rentals to compete against P2P. Overall, contrary to what could be expected, the OEM has an incentive to facilitate P2P rentals in a large variety of cases.

Keywords: business models, sharing economy, peer-to-peer marketplaces, rentals, manufacturing

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1. Introduction

Business models that focus on providing access to assets rather than on transferring ownership of goods have become one of the most fundamental recent industry trends (Bardhi and Eckhardt 2012). The trend is reflected, for example, in what has been called the *sharing economy* (Sundararajan 2016). Industry reports have indicated that the five key sectors for the sharing economy—travel, product sharing, finance, staffing, and music and video streaming—have the potential to increase their global revenues from $15 billion in 2015 to approximately $335 billion by 2025 (PricewaterhouseCoopers [PWC] 2015). The same survey established that 81% of consumers consider it less expensive to share goods than to own them individually, whereas 43% agree that "owning today feels like a burden," reflecting the relevance of this consumer trend.

The proliferation of sharing-economy companies can represent a challenge for traditional firms. It is sensible to postulate that traditional firms should adapt to compete in this new scenario (Cusumano 2015). However, it is unclear whether the effect of the sharing economy on incumbent firms is necessarily harmful, and also uncertain how the latter should react. Peer-to-peer (P2P) rental markets are part of the sharing economy phenomenon, hereafter referred to in short as "P2P rentals". In such markets, consumers who own assets (such as cars, bikes, and tools) can not only use them whenever needed, but also monetize ownership by renting them when they are not in use to other consumers willing to pay for temporary access to the assets. Examples of such markets include Turo and Getaround (P2P car rentals); SpinLister (P2P bike rentals); Boatsetter (P2P boat rentals); Airbnb (P2P home rentals); and Zilok (P2P rentals for products and tools). The entry of such P2P rental could affect consumer ownership preferences and, hence, incumbent manufacturers. From the perspective of original equipment manufacturers (OEMs), it could be hypothesized that P2P rentals may cannibalize sales. But it is also possible that P2P rentals make ownership more attractive—for consumers with low usage rates for whom ownership may not even be a possibility without the option of renting products from P2P rental markets. This suggests that usage rates (i.e., usage frequency) can play an important role in explaining market outcomes in the sharing economy.

Uncertainty in usage has been consistently highlighted as a main driver of the sharing economy in practice. For example, Brunswick (a boat manufacturer) partnered with the P2P boat rental platform Boatbound (now Boatsetter), based on the common belief that "the P2P boat rental model encourages boating participation and trial, while allowing boat owners to offset some of their ownership costs". Boatbound offers Brunswick "access to an emerging segment of boating consumers" (Brunswick 2014).

The automobile industry has also considered P2P rentals expanding ownership to low-usage consumers. One example is Tesla's master plan, in which it states that P2P rentals can "dramatically lower the true cost of ownership to the point where almost anyone could own a Tesla" (Musk 2016). Tesla CEO Elon Musk recently provided details on the expected Tesla Network and his vision that people sharing cars while they are not using them "is the obvious thing that's going to happen" (Dickey 2018). In the hospitality sector, real estate developers such as Natiivo and Niido are partnering with Airbnb to sell Airbnb-ready condos to owners who otherwise would not consider the ownership option. In the words of a developer, "We're creating the first-ever building designed and built for home sharing... Many condo owners come to the city on a regular basis but are not year-round residents. They cannot justify the fact that [the unit] is empty ... it is costing them money ... We are creating a product for that kind of buyer. We call it flexible ownership" (Lyons 2019). These examples of P2P rentals increasing interest in ownernship among low usage-rate consumers illustrate the importance of usage rate heterogeneity in different industries.

In practice, some OEMs have embraced the idea of P2P rentals. One early example is General Motors partnering with RelayRides (rebranded as Turo in 2015) to facilitate participation of its customers in the P2P rental marketplace in 2013. Similarly, Porsche partnered with Turo to improve the Porsche P2P rental experience. More recently, some OEMs have even considered launching their own P2P rental platforms, arguing that they could make ownership more attractive. Perhaps the best illustration of this is the aforementioned master plan for Tesla, which states that Tesla owners "will also be able to add your car to the Tesla shared fleet just by tapping a button on the Tesla phone app and have it generate income for you while you're at work or on vacation, significantly offsetting and at times potentially exceeding the monthly loan or lease cost" (Musk 2016). Other OEMs have explored a different strategy of offering direct rentals (non-P2P) to consumers. For example, in line with the plan outlined by Toyota's president in 2018 to switch from being exclusively a car-maker to a mobility company that includes services (Holmer 2018), Toyota now allows direct rentals in the U.S. market through its "Rent a Toyota" program, available only through Toyota dealers. This illustrates that although incumbent firms have been actively reacting to the access-based consumption trend, how to better adjust to it is far from obvious for OEMs. It remains largely unclear which business model is the best and under what conditions.

Motivated by this trend, this paper analyzes the interaction of a P2P rentals with a durable goods manufacturer under alternative market structures. The first line of inquiry the paper pursues is to understand the impact of the entry of a P2P rentals market. The paper investigates the conditions under which a monopolistic manufacturer and its consumers are better/worse off with the presence of P2P rentals. In line with industry insights, the analysis reveals the important role played by heterogeneity in usage rate on the observed market outcomes. As an illustrative example, if consumers are homogeneous in their usage rates, both the firm and the consumers are generally worse off with the entry of a P2P rental market. This surprising result arises from the rational expectations equilibrium. Although those who prefer to rent in the presence of P2P rentals are better off, those who continue to prefer to own are worse off, because the OEM is able to extract all the rental revenues via a higher sales price. Furthermore, P2P rentals allow the OEM to segment consumers into buyers and renters while offering only one product in the market.

In contrast, when consumers are heterogeneous in their usage rates, both the firm and consumers can be better off with the entry of a P2P rental market. One key mechanism that explains this result is that P2P rentals create an *equalizing effect*. Indeed, P2P rentals reduce the disparity in the willingness to pay (WTP) for the good among consumers with heterogeneous usage rates. This paper is the first to document this equalizing effect in the context of P2P rentals. Interestingly, due to the reduced disparity in WTP, the equalizing effect enables the OEM to extract a larger fraction of total surplus with a P2P rental market than it could otherwise.

As a second line of inquiry, motivated by the question of whether and how OEMs should adjust their business models, the paper investigates the equilibrium outcomes under different market structures. To that end, in addition to investigating the sales-only OEM ("monopoly" operating in the absence of a P2P rental market) and the "P2P-facing" OEM (the monopolist OEM operating in the presence of a P2P rental market), the paper investigates three alternative scenarios. One, it analyzes the setting where the OEM directly offers sales and (non-P2P, direct) rentals to consumers (the "dual" firm). Two, the paper considers an OEM that itself introduces a P2P platform to the market (the "P2P-sponsoring" firm). Three, it analyzes a mixed structure in which the OEM competes against a P2P by introducing its own direct rentals (the "dual-plus-P2P" firm).

Consumer heterogeneity in usage rates continues to play a fundamental role in market outcomes. When usage rates and heterogeneity in such rates are sufficiently large, the OEM firm is better off offering direct sales and also facilitating a P2P rental market. In contrast, if heterogeneity in usage rates is too low, the OEM prefers to operate as a sales-only monopoly. If heterogeneity is too high and usage rates are below a threshold, the OEM prefers to operate as a dual firm that offers both sales and rentals directly to consumers. If the presence of P2P rentals is unavoidable, the OEM would not always be better off by introducing its own rentals to compete against P2P. The OEM would prefer to not offer direct rentals when consumers are more homogeneous and such rentals would substantially cannibalize sales.

Contrary to what might be expected, this paper shows that OEMs have an incentive to facilitate transactions of P2P rental markets in a large variety of cases. Although heterogeneity in usage frequency is a factor commonly acknowledged in the popular press as one of the most important elements for sharing-economy companies (e.g., Badger 2014), the academic literature on P2P rentals has not emphasized its role in determining market outcomes. This paper fills that gap and shows that usage-rate heterogeneity leads to the equalizing effect of P2P rentals. Overall, this paper contributes to the existing literature by:

(i) Introducing a new mechanism—the equalizing effect—which occurs when there is heterogeneity in consumer usage rates in the market, and which explains how OEMs can benefit from the P2P rentals.

(ii) Showing that the OEM can use P2P rentals as a segmentation device. The presence of P2P rentals allows the firm to discriminate between the higher and lower-value consumers, by selling the cars to the higher-value users, but allowing the lower-value users to rent from P2P rentals, allowing it to extract a higher portion of consumers' surplus. This aspect of P2P rentals is not only interesting but we believe is unique to sharing economy markets and is fundamentally new in the broader literature.

(iii) Being the first paper in the sharing economy literature to analyze the business model problem for the OEM, which is perhaps the most fundamental question for incumbent firms in practice. In particular, we show how our proposed mechanism (the equalizing effect) drives the firm's optimal strategy toward acting as a sales-only OEM, sales+rentals OEM, or P2P-sponsoring OEM.

(iv) Providing a contrast between P2P rentals and traditional (non-P2P) rental markets in the context of their interaction with product sales.

The rest of the paper is organized as follows. Section 2 presents a literature review. Section 3 introduces the model of the P2P rental market, followed by the impact of P2P rentals on a monopoly OEM in Section 4. Section 5 presents and solves the OEM's business model problem. Extensions and robustness checks are presented in Section 6, and Section 7 concludes.

2. Related Literature

Rental markets have been the focus of considerable attention in information systems, operations management, and marketing. Several studies on the topic have focused on aspects such as optimal capacity (Savin et al. 2005), pricing (Gans and Savin 2007), and new product introductions (Bassamboo et al.

2009). Some literature has contrasted leasing versus selling (Desai and Purohit 1999, Bhaskaran and Gilbert 2005, Agrawal et al. 2012) and the interplay between sales and rentals (Purohit 1997, Gilbert et al. 2014).

Although the roots of the literature on rental markets may go back to early developments in the literature on durable goods and secondary markets (e.g., Coase 1972, Bulow 1982), there has been a renewed interest in rental markets for at least two reasons. First, manufacturing-as-a-service business models (servicizing) have become popular in industry, receiving increasing attention in the literature both in terms of analytical models (Agrawal and Bellos 2017, Bellos et al. 2017) and empirical work (Guajardo et al. 2016, Guajardo 2018). Second, rentals are at the core of the sharing economy (Einav et al. 2016). Analytical models formulating new theories include Cachon et al.'s (2017) analysis of the role of surge pricing for matching supply and demand in Uber-type services; Taylor's (2018) analysis of operational trade-offs in on-demand service platforms; and Benjaafar et al. (2019) and their characterization of ownership, usage, and matching frictions in P2P sharing platforms. Our paper is different from the latter in that if focuses on the interaction between P2P and the OEM, whereas Benjaafar et al. (2019) focuses on characterizing different aspects of P2P rental markets but ignoring their interaction with OEM's decisions. Thus, while selling prices and quantities are completely exogenous in their model, in our analysis we endogenize these decisions by fully considering the OEM's problem.

The Information Systems literature has had a longstanding interest in markets where consumers can share goods, primarily from an intellectual property perspective. For example, Leibowitz (1985) shows that sharing can have a positive impact on sellers, since they can appropriate revenues from users who are not original purchasers. Similarly, Bakos et al. (1999) show that sellers of information goods can benefit from sharing in several scenarios. More recently, sharing in the domain of transportation has received particular attention in the Information Systems community. For example, Fraiberger and Sundararajan (2015) developed a dynamic equilibrium model of a sharing market that distinguishes which segments of the market are more prone to switching to rentals; Zheng et al. (2019) analyzed the impact of sales promotions on two-sided platforms in the context of a transportation network company; Filippas et al. (2019) empirically examine the impact of different pricing mechanisms in a transportation sharing economy platform. There has also been interest in how and when sharing markets themselves develop, and the behavior of market-making intermediaries, as analyzed in Razeghian and Weber (2016) and Zimmerman et. al. (2018). These differ from our paper in their focus on the intermediary market itself rather than on the interaction of these P2P rental markets and incumbent OEM manufacturers. Finally, our study has connections with the secondary markets literature. We note, however, that the temporary nature of transactions in P2P rental markets is distinct from the traditional secondary markets literature, which has explored conditions under which secondary markets can hurt or benefit primary markets (e.g., Hendel and Lizzeri 1999, Chen et al. 2013). Indeed, in P2P rental markets assets are shared between consumers even for a short period of time and *without the need of a change in ownership*; this is different from traditional secondary markets, where consumers typically need to become owners of goods in order to use them.

Despite the growing stream of literature attempting to characterize key aspects of the sharing economy, much less is known about how the introduction of P2P rental markets affects incumbent firms. One exception is Weber (2016), who analyzed consumer purchase decisions with and without the presence of secondary sharing markets, with mixed results depending on the product type. In addition, Jiang and Tian (2018) investigated how the entrance of a P2P rental market affects the incumbent's pricing and quality decisions. Their results show that a firm's profits and quantity sold are not monotonically related to the marginal cost of production and the transaction costs in the sharing market, showing that P2P can be good news for an OEM when marginal costs of production are large. The current paper extends this finding by showing that even in the absence of production costs, an OEM (and sometime even consumers) can be better off with P2P rentals when consumers are heterogeneous in their usage rates. Moreover, the equalizing effect that our paper identifies is still valid in the presence of production costs. Finally, Horton and Zeckhauser (2016) also examined the economic properties of P2P rental markets. Note that although Horton and Zeckhauser (2016) allow for heterogeneity in usage patterns, their results do not refer to customer heterogeneity as a driver of market outcomes. While most of the papers have focused on the P2P market, none of them have explicitly considered how it changes the OEM's decisions. Hence, we extend the literature by analyzing the pertinent but hitherto unaddressed business model problem for manufacturers.We believe that the new insights, emerging from consideration of the business model for the OEM and the incorporation of heterogeneity in usage rates as a driver of market outcomes, significantly advance the understanding of P2P rental markets.

3. Model Setup

3.1 Consumer Problem

We begin with the consumer side. There is a unit mass of consumers in the market and infinite, discounted consumption periods. At the beginning of time, each consumer i (she) realizes her per-use valuation of using the durable good, $v_i \sim \mathbb{U}(0, 1)$. Hence, for a given customer, this valuation is time-invariant, but there is heterogeneity across consumers in their per-use valuation for an instance of consumption. We also model a second variation in consumers, consumers having a stochastic usage of the durable good, with consumer i realizing a usage rate of $\lambda_i \in (0,1)$. In each time period, each consumer i realizes a draw of a Bernoulli(λ_i) random variable to determine whether she experiences a need for consumption.

We assume that there are two types of consumers in the market, depending on their usage profiles: high-usage and low-usage consumers $\{H, L\}$, as characterized by their usage rates $\lambda_i \in \{\lambda_H, \lambda_L\} = \{\lambda, \phi\lambda\}$, where $\phi \in (0,1]$. The low-usage rate is a fraction of the high-usage rate. Hence, ϕ is a measure of the market heterogeneity, where $\phi = 1$ denotes a market with homogeneous usage rates. In the base model, we also set the two consumer segments to be of equal size.[1] [2]

In the absence of a rental market, at the beginning of time, a consumer decides whether she wants to purchase the durable good based on her usage rate λ_i, per-use valuation v_i, and the sales price of durable good p_s^M (subscript s for sales, superscript M to denote the "monopoly" firm, in absence of P2P rentals).[3] In the presence of a rental market, in all periods, she has an additional option: to rent the durable good when the stochastic need of usage arises at the per-period rental price p_r^P (subscript r for rentals, superscript P to denote the "P2P-facing" firm). In this section, for simplicity, since there is only one business model with a rental price, the per-period rental price notation is p_r instead. (In Sections 5 and 6, the superscript will be used again for clarity.) When a consumer rents a product, her benefit is μv_i, where the factor $\mu \in (3/4, 1)$ captures the consumer-side inconvenience for the rental transaction.[4] The parameter μ captures any potential frictions of the P2P rentals market. In the base model, we do not consider additional degradation or transaction cost parameters. In Section 6, we consider several extensions where additional degradation and transaction costs are incorporated in the analysis, and where rental frictions are dependent on market size. The results remain qualitatively similar.

Without a P2P rental market, a product sits idle when the owner does not need to use it. In the presence of P2P, in each period, the owner has the additional option of renting out the product to derive a payoff of p_r (p_r is the market clearing rental price that emerges in equilibrium—see details in Section

[1]For analytical simplicity, we adopt equal market size for the two segments. The results do not change qualitatively when we consider two segments of different sizes (see Section 6).

[2]In Section 6, we also consider alternative ways of modeling heterogeneity in usage rates.

[3]Even though we allow the consumers to purchase only in the first period, this model is identical to an infinite horizon setting where consumers arrive in multiple period and can purchase the good when they arrive. Also, if the consumers do not purchase in the first period, there is no change in their utility function that will cause them to purchase later.

[4]We assume $\mu > 3/4$ to ensure analytical tractability. Note that the assumption of an inconvenience costs proportional to valuation is also used in Gilbert et al. (2014) in a similar context, and by Gurvich et al. (2018) in a different context; this assumption permits two-dimensional heterogeneity.

3.4). The per-period utility of a product buyer i is given by

$$u_i^B = \begin{cases} \max\{v_i, p_r\} & \text{if owner demands the product} \\ p_r & \text{otherwise.} \end{cases}$$

Similarly, the per-period utility of a renter i is given by

$$u_i^R = \begin{cases} \mu v_i - p_r & \text{if consumer rents the product} \\ 0 & \text{otherwise.} \end{cases}$$

P2P rentals change the consumer's problem in two distinct ways: (1) They increase the utility owners could derive when they don't need to use a good, from potentially renting it out, and (2) they increase the utility of a non-owner, by having the option to temporarily rent a good. There is full information, and consumers share the same expectations of the rental price. Given a consumer's λ_i and v_i, along with the sales price p_s^P and expected rental price p_r, the overall expected utility that consumer i obtains—in the presence of P2P rentals—from buying ($U_i^{B,P}$), renting ($U_i^{R,P}$), or not participating ($U_i^{0,P}$) are given by

$$\begin{aligned} U_i^{B,P} &= \frac{\mathbb{E}\left[u_i^B\right]}{1-\delta} - p_s^P = \frac{\lambda_i \max\{v_i, p_r\} + (1-\lambda_i)p_r}{1-\delta} - p_s^P, \\ U_i^{R,P} &= \frac{\mathbb{E}\left[u_i^R\right]}{1-\delta} = \frac{\lambda_i[\mu v_i - p_r]^+}{1-\delta}, \end{aligned} \tag{1}$$

where δ denotes the inter-temporal discount rate common to all consumers.[5] The outside option has value 0. In contrast, without P2P rentals, denoted by superscript M, the expected utilities are: $U_i^{B,M} = \frac{\lambda_i v_i}{1-\delta} - p_s^M; U_i^{R,M} = 0$.

Typically, consumers rent out their goods using a platform that charges a fee. In the basic model, for the sake of simplicity, we have ignored platform fees. We introduce such a fee in Section 5, which shows that a platform fee does not qualitatively change the insights. In fact, we show that it can be optimal for an OEM to undercut a P2P rental platform with its own integrated platform and charge no transactions fees as a part of its platform.

[5]The function $[x]^+$ is equal to x if $x > 0$, or 0 otherwise.

3.2 Equalizing Effect of P2P Rentals

Before discussing the firm's problem, we describe the four distinct effects of the availability of P2P rentals. There is a *cannibalization effect* because some consumers may switch from purchasing to renting. On the other hand, the availability of P2P rentals leads to a *participation effect* because P2P allows participation from consumers who were unable to consume when sales were the only option (consider, for example low usage, high per-use valuation customers). P2P rentals also generate a *value effect*, increasing the value of ownership as a result of P2P rentals providing revenue streams for owners. These first three effects are similar to those present in the literature on secondary markets. The value effect has also been documented in the P2P literature (e.g., by Benjaafar et al. 2019).

There is a fourth effect, unique to the sharing economy, which we call the *equalizing effect.* When consumers are heterogeneous in their usage rates, the equalizing effect is the reduction in the disparity of WTP for owning the good among the heterogeneous consumers. The intuition is that lower-usage consumers benefit more from the availability of P2P rentals than higher-usage consumers do, as they have more periods when they could rent out their goods.

To formalize this effect, this section introduces some definitions. Consumer i's WTP to own the good when there is no P2P rental market is $W_i^{B,M} = \frac{\lambda_i v_i}{1-\delta}$ (with a superscript B for "buying", superscript M for buying from the monopoly firm). Consumer i's WTP in the presence of a P2P rental market is, in contrast, $W_i^{B,P} = \frac{\lambda_i \max\{v_i, p_r\} + (1-\lambda_i)p_r}{1-\delta}$. The first term in $W_i^{B,P}$ depends on whether v_i is sufficiently high that the consumer will utilize the good if they demand it.

To see the equalizing effect, consider any two consumers with the same valuation v_i but heterogeneous usage rates λ and $\phi\lambda$. Without a P2P rental market, the disparity in their WTP is: $\Delta W^{B,M}(v_i) = W_\lambda^{B,M} - W_{\phi\lambda}^{B,M} = \frac{\lambda(1-\phi)}{1-\delta} v_i$. In the presence of a P2P rental market, this disparity in their WTP is instead: $\Delta W^{B,P}(v_i) = \frac{\lambda(1-\phi)\max\{v_i - p_r, 0\}}{1-\delta}$, which is smaller than $\Delta W^{B,M}(v_i)$. This reduction in disparity is the equalizing effect. It is formalized in the following lemma.

Lemma 1. *When consumers have heterogeneous usage rates, the availability of P2P rentals reduces the disparity in the WTP among these heterogeneous consumers, termed the equalizing effect. This reduction is given by*

$$\tau(v_i) = \Delta W^{B,M}(v_i) - \Delta W^{B,P}(v_i) = \frac{\lambda(1-\phi)\max\{v_i, p_r\}}{1-\delta}.$$

It is always positive and becomes stronger as the heterogeneity in usage rate increases (i.e., $\partial\tau/\partial\phi < 0$) or the usage rates increase (i.e., $\partial\tau/\partial\lambda > 0$).

The term $\max\{v_i, p_r\}$ captures whether the consumer wants to use the product for her own consumption or rent it out to the market for a higher rental price.

The equalizing effect is an important characteristic of the sharing economy. It occurs only when consumers are heterogeneous in their usage rates. To be clear, the equalizing effect is not reflecting the increase in consumers' WTP —that is the aforementioned, well-known value effect—but rather the reduction in the disparity of WTP among consumers. Incorporation of this effect leads to key insights, which, to our knowledge, has not been established in the existing P2P literature. Note that the equalizing effect is generalizable under any form of heterogeneity in usage rates in the market.

Figure 1 presents an example of the equalizing effect with high and low-usage consumers. The x-axis represents the continuum of consumers arranged by their per-use valuation $v_i \in [0,1]$, and the y-axis is each consumer's corresponding WTP for purchasing. At any x-axis value of v_i, the difference between the solid and dotted lines represents the disparity in WTP between a high-usage and low-usage consumer with the same per-use valuation. It's clear that the WTP of consumers is much closer together in the presence of P2P rental markets than in the absence of P2P rentals. Figure 2 represents buying

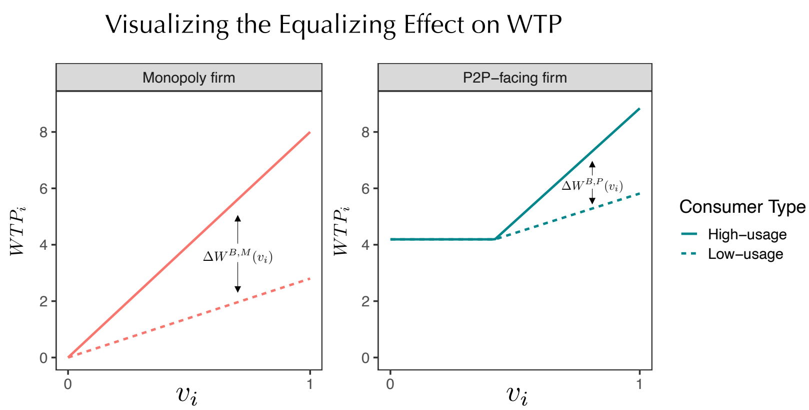


Figure 1: *A comparison of WTP of consumers between the presence or absence of P2P rental markets. The x-axis shows the range of consumer valuations. The left panel is the benchmark $W^{B,M}$, in the absence of P2P, and the right panel is the consumer $W^{B,P}$ after the entry of a P2P rental market. The figure shows that $\Delta W^{B,P}(v_i)$ is smaller than $\Delta W^{B,M}(v_i)$ for all v_i, so thus, the equalizing effect is reflected in their difference, $\tau(v_i) > 0$. $W^{B,P}$ is the same for high-usage and low-usage consumers with valuations $v_i < p_r$. (In this example, $\{\lambda = 0.8, \phi = 0.35\}$.)*

and renting decisions between consumers of different usage rates. There are a higher proportion of high-usage consumer who purchase compared to the lower-usage consumers. Furthermore, the per-use rental utility is the same irrespective of the usage rate.

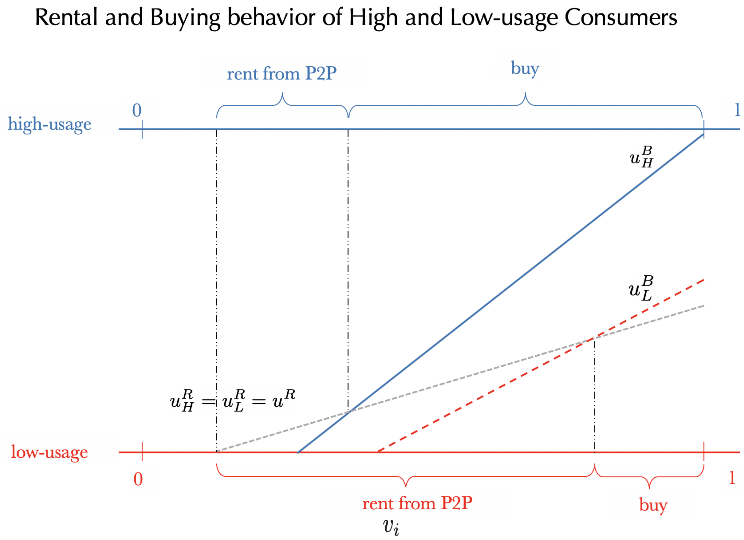


Figure 2: Distribution of buying and renting decisions for high and low-usage consumers.

3.3 OEM's Problem

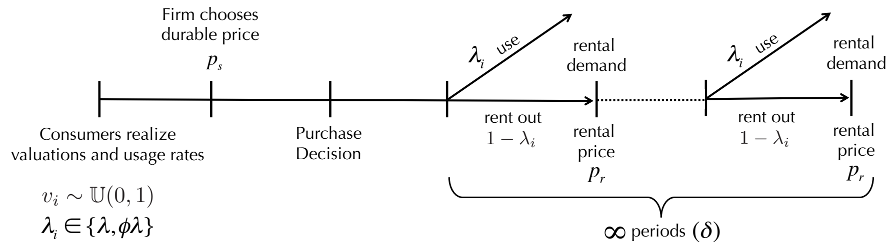
This section turns to the problem facing the manufacturer of the homogeneous durable good, assuming there are two types of consumers in the market as defined earlier. For a P2P-facing firm, the consumer indifferent between buying and the best alternative can be characterized by their per-use valuation: v_θ, where $\theta \in \{H, L\}$ denotes the type of consumer by their usage rate. This indifferent valuation is $v_\theta = \min\left\{1, \frac{(1-\delta)p_s^P - p_r}{\lambda_\theta(1-\mu)}\right\}$. All consumers with $v_{i,\theta} > v_\theta$ will purchase. In our model then, v_H^P and v_L^P denote, in the presence of P2P rentals, the valuations of the high-usage and low-usage consumers respectively, who are indifferent between purchasing and the alternative. It is easy to show that $v_L = \min\left\{1, \frac{v_H}{\phi}\right\}$, and hence $v_L > v_H$ (see the appendix for an exact expression of v_H and v_L in equilibrium). The P2P-facing manufacturer chooses a durable price p_s^P to maximize its profits given by

$$\pi_s = D_b^P p_s^P, \ where \ D_b^P = \frac{1}{2}\left\{(1 - v_H^P) + (1 - v_L^P)\right\} \tag{2}$$

where D_b^P represents the quantity of durable goods bought from the OEM in the presence of a P2P rental market.[6]

The model's timeline in the case with a P2P rental market is reflected in Figure 3. Consumers realize their fixed, per-use valuation v_i and their type $\lambda_i : \{\lambda, \phi\lambda\}$, then the OEM sets the price of the durable good, p_s^P. Based on this information, consumers determine the expected rental price and decide

[6]The main model does not consider production costs, in order to focus the analysis on the main drivers of interest. Section 6 extends the analysis to show the impact of production costs on the results.



whether to purchase or not, and then each period's utility is realized.[7]

3.4 Analysis of the P2P Rental Market

This section analyzes the P2P rental market before deriving the main results. First, the analysis will use a rational expectations equilibrium (REE) as the solution concept for modeling consumer choices. This equilibrium concept has been used commonly in the existing literature (e.g., Cachon and Swinney 2009, Su 2010, Jerath et al. 2010; Muth 1961 provides further contextual background). Assume that consumers predict that the equilibrium rental price is $\hat{p}_r$. Conditional on the predicted rental price $\hat{p}_r$ and the purchase price p_s^P, consumers evaluate whether they want to buy or rent the product based on Equation (1). At the time of purchase, the per-use valuations of the indifferent consumer reflect the predicted rental price: $v_\theta = \min\left\{1, \frac{(1-\delta)p_s^P - \hat{p}_r}{\lambda_\theta(1-\mu)}\right\} \forall \theta \in \{L, H\}$.

There are two possible cases: $v_H \geq \hat{p}_r$ or $v_H < \hat{p}_r$.[8] In the former case, the per-period utility of the marginal high-type purchaser is greater than the expected market rental price. Thus, all owners, of any usage rate, anticipate using the good if they demand it. The latter case, $v_H < \hat{p}_r$, as proven in the appendix, is not a stable equilibrium. Intuitively, under this paper's solution concept, if the expected market rental price is higher than the marginal buyer's per-period usage utility, this would attract additional purchasers, which would lead to a lower expected market rental price, until the marginal buyer's per-period usage utility follows the first case, $v_H \geq \hat{p}_r$. Thus, in equilibrium, product owners will always use the product if they demand it. This simplifies the per-period utility of owners to be v_i if the owner demands the good, rather than $\max\{v_i, p_r\}$.

The demand and supply of rentals ($D_r(p_r, p_s^P)$, $S_r(p_r, p_s^P)$) in the P2P rental market are as follows:

$$D_r(p_r, p_s^P) = \frac{1}{2}\left[\lambda\left(v_H - \frac{p_r}{\mu}\right)^+ + \phi\lambda\left(v_L - \frac{p_r}{\mu}\right)^+\right],$$

$$S_r(p_r, p_s^P) = \frac{1}{2}\left[(1-\lambda)(1-v_H) + (1-\phi\lambda)(1-v_L)\right].$$

[7]We incorporate multiple periods to capture different usage realizations, but we are not examining the evolution of the P2P market from a market penetration perspective. We focus instead on the steady state analysis since we are interested in the long-term strategies of the OEM.

[8]v_L is ignored because it is always true that $v_L \geq v_H$.

In equilibrium, the supply and demand of P2P rentals should coincide, an assumption which we use to derive the equilibrium price[9] . In other words, the equilibrium market rental price is determined

$$D_r(p_r, p_s^P) = S_r(p_r, p_s^P) \tag{3}$$

Depending on whether $v_H > p_r/\mu$ or $v_H \leq p_r/\mu$, the high-usage consumers may rent or may forgo rentals. The market clearing rental prices for the P2P platform are thus given by

$$p_r = \begin{cases} \mu\left(\lambda(1+\phi) - (2 - v_H - v_L)\right)/\,\lambda(1+\phi) & \text{when } v_H > p_r/\mu, \\ \mu\left(\lambda(1+\phi) - \lambda v_H - (2 - v_H - v_L)\right)/\,\lambda\phi & \text{otherwise.} \end{cases}$$

For the REE to hold, the realized rental price in the P2P market should be equal to the predicted rental price, i.e., $p_r = \hat{p}_r$.

The OEM decides the price of the durable good to maximize its profits (see Equation (2)). By deciding p_s^P, the firm effectively "chooses" the segments of consumers to whom it will sell. Table 1 presents the four feasible outcomes for the overall market. Based on the utilities presented in Equation (1) and the incentive compatibility constraints, it is easy to show that if anyone purchases, it will be the high-usage consumers, whereas, if anyone rents, it will be the low-usage consumers. It is easy to show that the firm will always want to sell to at least some high-usage consumers. If both types participate

Table 1: *Different Market Outcomes*

	High-usage Buy and Rent	High-usage Buy Only
Low-usage Buy and Rent	C_1	C_3
Low-usage Rent Only	C_2	C_4

in the rental market (outcomes C_1 and C_2), the equilibrium demand for the durable good after solving the REE is: $v_H = \frac{(1-\delta)p_s^P - p_r}{\lambda(1-\mu)}$, $v_L = \min\left\{1, \frac{v_H}{\phi}\right\}$. If only low-usage consumers participate in the rental market (outcomes C_3 and C_4), then $v_H = \frac{(1-\delta)p_s^P - p_r(1-\lambda)}{\lambda}$, $v_L = \min\left\{1, \frac{v_H}{\phi}\right\}$. The appendix details the exact expressions for the rental prices and marginal buyers among the four possible market outcomes. The equilibrium characterization of these regions is presented in Proposition 2.

[9]This is a common assumption in the P2P literature, e.g., Weber (2016), Jiang and Tian (2018) and Zimmermann et al. (2018). Note that this assumption is consistent with platforms that allow consumers to set their own prices (e.g., Turo). In section 6 we explore alternative assumptions.

4. Impact of P2P Rentals on an OEM

This section presents results for an OEM operating in the presence of P2P rentals, compared to a benchmark scenario where the OEM operates in the absence of P2P rentals. In this section, we refer to the OEM in these cases as the "P2P-facing" firm and the "monopoly" firm, respectively. The section first discusses the case where consumers are homogeneous in usage rates ($\phi = 1$) to develop the intuition for how P2P rentals affect consumer decisions and to highlight the importance of usage-rate heterogeneity. Subsequently, the section develops the case of heterogeneous usage rates. The proofs for all lemmas and propositions are in the appendix.

4.1 Homogeneous Usage Rates

Consider the simplified case of a homogeneous usage rate λ common to all consumers. Proposition 1 describes the equilibrium outcomes.

Proposition 1. *The presence of P2P rentals result in lower firm profits and consumer welfare (relative to the monopoly case). A fraction $q^P = 1 - \hat{v}$ of consumers purchases the product and a fraction $\hat{v} - \tilde{v}$ rents the product in equilibrium, where*

$$\hat{v} = \frac{1}{2} + \frac{(1-\lambda)\mu}{2\left(\lambda^2(1-\mu)+\mu\right)}, \quad \text{and} \quad \tilde{v} = \frac{1}{2} - \frac{\lambda(1-\lambda)(1-\mu)}{2(\lambda^2(1-\mu)+\mu)}.$$

Since $q^P \leq 1/2$, the quantity purchased is lower than the quantity purchased under monopoly ($q^M = 1/2$), but $\tilde{v} < \frac{1}{2}$, so a larger fraction of the consumers is able to use a product when they need to.

The presence of P2P rentals is expected to benefit consumers, as it allows owners to rent out their product when they are not using them (*value effect*) and non-owners to rent when they need to (*participation effect*). However, consumers are actually worse off in the sharing economy. To understand this counterintuitive result, first note that the lifetime utility of ownership can be decomposed into value from ownership use plus value from rental revenue. Each buyer expects to receive $(1-\lambda)p_r$ in rental revenue each period, with a lifetime expected rental revenue $(1-\lambda)p_r/(1-\delta)$. This rental revenue is identical across all owners. When consumers are homogeneous, the firm is able to raise the durable good price to extract all the value effect. Due to this optimal response by the P2P-facing firm, the buyers with valuations $v_i \in (\hat{v}, 1]$ are worse off with the sharing economy. Although the consumers who switch from purchasing to renting face a decrease in their per-period consumption utility by a factor of μ, they face a relatively lower price from rentals. As a consequence, consumers whose valuations $v_i \in [\tilde{v}, \hat{v}]$ are better off with the sharing economy. This points to an interesting feature of the sharing economy, in which some consumer surplus from higher-valuation consumers is redistributed to lower-valuation consumers, as illustrated in Figure 4. Relative to a monopoly firm, there are additional consumers participating in the market (because $\tilde{v} < 1/2$), reflecting the aforementioned *participation effect*. However, the negative impact on buyers is larger than the benefit to those who shift to renting, so consumers are overall worse with the sharing economy.

It is important to note that the firm uses P2P rentals to indirectly segment consumers based on their valuations. The OEM increases prices to sell only to higher-valuation consumers ($\hat{v} > 1/2$), while earning rents from lower-valuation consumers indirectly through inflated durable goods prices. Even though a P2P-facing firm has a better ability to indirectly segment the market, its profits decrease, relative to a monopoly firm. This is because some consumers are now consuming via rentals rather than from purchases (*cannibalization effect*), which as noted earlier, provides less per-use utility ($\mu < 1$), all else equal. Figure 4 provides a visualization of the comparison.[10]

Comparison of Firm and Consumer Welfare with Homogeneous Usage Rates — (in the absence and presence of a P2P rental market)

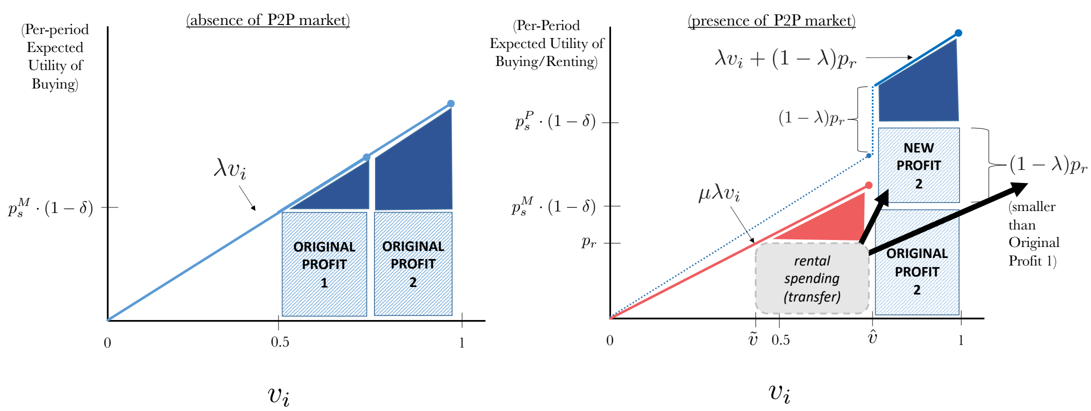


Figure 4: *In the presence of a P2P rental market, a smaller fraction of consumers buys, compared to a monopolistic setting in the absence of P2P, but at a higher price because the firm raises the price to reflect the constant value effect across owners.*

4.2 Heterogeneous Usage Rates

When consumers are heterogeneous in their usage rates, the following lemma and proposition summarize purchasing outcomes for a monopoly firm and P2P-facing firm, respectively. The purchasing outcomes

[10]Note that one could posit a Pareto optimal equilibrium when no owners rent out goods in the P2P market and the market degenerates to the monopoly case. However, this is not an equilibrium, as owners who do not need to use the product will always rent out their products in the P2P market.

for the monopoly firm presented in the following lemma serve as the benchmark comparison in this section.

Lemma 2. *The monopoly firm sells to both high and low-usage consumers when heterogeneity is sufficiently small ($\phi \geq 1/3$) at a price $p_s^M = \frac{\lambda\phi}{(1-\delta)(\phi+1)}$, but under high heterogeneity ($\phi < 1/3$), it sells only to high-usage consumers at a price $p_s^M = \frac{\lambda}{2(1-\delta)}$.*

Next, consider the purchasing and renting behavior of consumers with a P2P-facing firm. Note that when consumers are heterogeneous in their usage rates, as shown in Lemma 1, the equalizing effect of P2P rentals reduces the disparity in WTP among high-usage and low-usage consumers. In light of this, we examine how P2P rentals affect the consumption choices of the low-usage and high-usage consumers in equilibrium, which is presented in the proposition below.

Proposition 2. *In a market with heterogeneous usage rates a fraction of high-usage consumers will always purchase the durable good, and low-usage consumers will always rent from the sharing market. The low-usage consumers purchase the good only when their usage rate is sufficiently high, i.e., $\phi \geq \hat{\phi}$ (regions C_1 and C_3). Some high-usage consumers will always rent for small λ. However, when λ is sufficiently large, high-usage consumers cease to rent for intermediate values of ϕ (regions C_3 and C_4). These regions are illustrated in Figure 5.*

A fraction of high-usage consumers always purchase the durable good, as the OEM will always serve these consumers in equilibrium. Similarly, a fraction of low-usage consumers will always rent from the P2P rental market when they need the good. As the usage rate of the low-usage consumers ($\phi\lambda$) decreases, they gradually move from both buying and renting to only renting. Below a threshold $\hat{\phi}$ (i.e. $\phi < \hat{\phi}$), none of the low-usage consumers buy the product, because their usage rate is too low (conditions C_2 and C_4). This threshold $\hat{\phi}$ is decreasing in λ because a higher value of λ not only reduces the available rental supply (pushing up the rental price), but also leads to increased overall usage, incentivizing purchases from low-usage consumers. It is important to note that there are conditions where some low-usage consumers make purchases in the presence of P2P rentals where they otherwise would have been left out of the market under the monopoly firm (when $\phi < 1/3$ as shown in Lemma 2).

If the usage rate λ is low, there is considerable rental supply from the owners in each period. Since a high rental supply depresses rental prices, at least some high-usage consumers always rent from the P2P rental market when λ is sufficiently low. To those consumers, this low P2P rental price makes renting more attractive than purchasing. However, when λ is sufficiently high, high-usage consumers rent only in the extreme values of ϕ (conditions C_1 and C_4). As ϕ decreases from $1 \to 0$, low-usage consumers move from buying to renting, which puts downward pressure on the rental supply and increases rental demand. As a result, the low-valuation, high-usage consumers who do not buy are priced out of P2P rentals (conditions C_3 and C_4). However, when ϕ becomes sufficiently small and low-usage demand very infrequent, the rental demand in the P2P market is no longer driven by low-usage consumers, and then some high-usage consumers will rent again. Now that we have examined the consumption behavior by

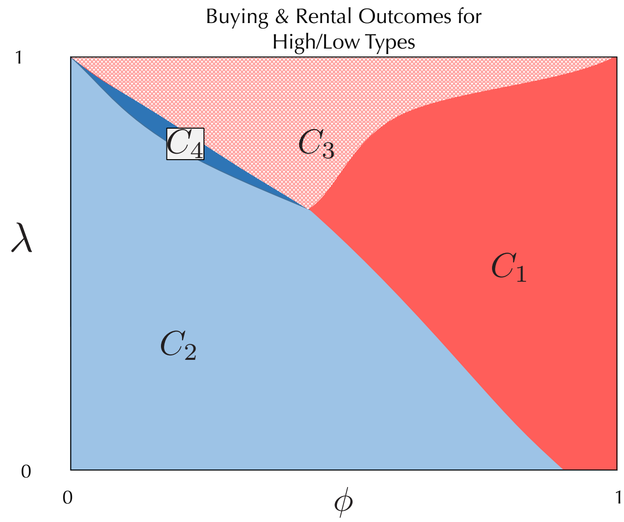


Figure 5: *Illustration of consumer buying and renting behavior (assuming $\mu = 0.8$). C_1 represents the region where both high and low-usage consumers buy and rent. C_2 represents the region where low-usage consumers do not rent as $\phi\lambda$ is relatively low, but high-usage consumers both buy and rent. C_3 represent regions where the rental price is sufficiently high so that the low-value, high-usage consumers are priced out of the market. C_4 represent the region where low-usage consumers do not buy and high-usage consumers do not rent.*

different types of consumer, we first focus on how surplus generated in the presence of P2P rentals is shared between the OEM and the consumers. Introduction of P2P rentals allows consumers to choose between renting and purchasing, and hence creates competition for the durable good. However, the firm can capture some of the rental revenue by charging a higher sales-price p_s^P and can internalize a significant portion of the benefit generated by P2P rentals. This intuition is formalized in the following proposition.

Proposition 3. *The OEM is able to use P2P rentals as a discrimination device and extract a larger fraction of the total surplus compared to the benchmark monopoly firm when it faces heterogeneity and sells to both high and low-usage consumers simultaneously ($\phi \geq 1/3$).*

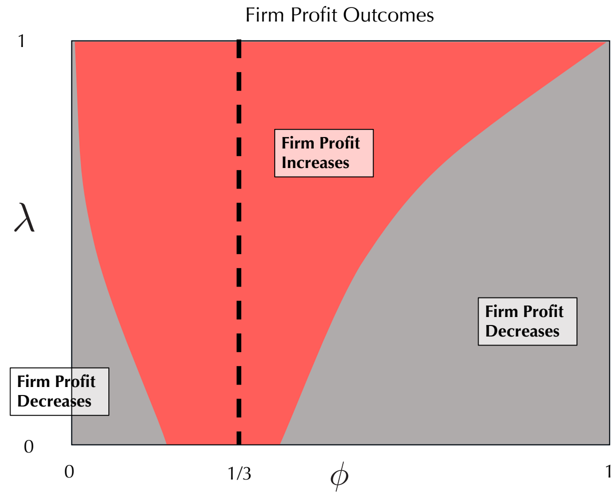


Figure 6: *Firm's profit in the presence of P2P rentals (relative to monopoly)*

The monopoly firm can only charge exactly one price to both the high and low-usage consumers (when $\phi \geq 1/3$) and is unable to extract a significant fraction of the consumer surplus. Interestingly, P2P rentals allow the firm to implicitly segment consumers into buyers (with relatively high v_i and λ_i) and renters (with relatively high v_i and low λ_i, or relatively low v_i) despite having just one product in the market. Further, the equalizing effect reduces the disparity of WTP among consumers, allowing the firm to price facing a relatively more homogeneous market. These factors enable an OEM to extract a significant portion of the surplus from the buyers, which would not have been possible without the P2P market. Hence, contrary to common intuition, P2P rentals can act as a discrimination device and might be more advantageous for the OEM than for consumers. The equilibrium profit outcomes are presented in the Figure 6 and discussed in the following proposition.

Proposition 4. *The OEM's profits are higher in the presence of P2P rentals than in the monopoly case, when ϕ assumes intermediate values, i.e., $0 \leq \phi_1^\pi \leq \phi \leq \phi_2^\pi < 1$. The region $[\phi_1^\pi, \phi_2^\pi]$ is increasing in λ.*

The P2P-facing firm is more profitable than the monopoly firm when the level of heterogeneity in the market is intermediate. Although the entry of a P2P market can cannibalize sales, a P2P market also increases an OEM's ability to segment consumers into buyers (high $\lambda_i v_i$) and renters (intermediate $\lambda_i v_i$) as shown in Proposition 3. In addition, the equalizing effect (Lemma 1) brings WTP of high and low-usage consumers closer together. A P2P-facing firm can then sell to some of *both* high and low-usage owners at a high price as opposed to a relatively lower price under the benchmark monopoly scenario (when they sell to both types). Here, the double-whammy can help the P2P-facing firm generate more

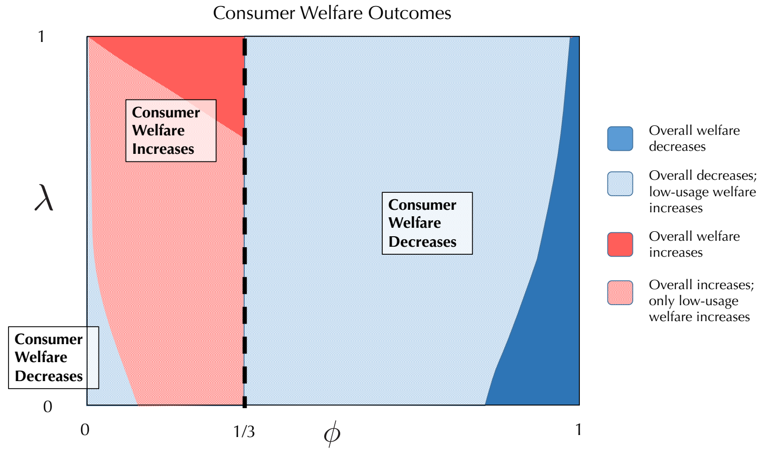


Figure 7: *Consumer welfare in the presence of P2P rentals (relative to under a monopoly firm).*

profits; when the cannibalization effect becomes relatively small due to heterogeneous consumers, the P2P-facing firm is more profitable than a monopoly.

Even when usage rates of low-usage consumers are too low, such that the equalizing effect does not result in low-usage consumer purchases (see conditions C_2 and C_4 in Proposition 2), a P2P-facing firm can benefit from a large participation effect. Low-usage consumers can rent instead of forgoing consumption in the absence of P2P rentals. When $\phi > \phi_1^\pi$, the participation effect is strong enough to overcome the cannibalization of high-usage sales caused by the entry of a P2P market and the P2P-facing firm is better than a monopoly. When $\phi < \phi_1^\pi$, the usage demand from the low-usage segment is so infrequent that neither the equalizing effect nor the participation effect is relevant. Under this condition, the P2P-facing firm is less profitable than the monopoly firm as the market degenerates to an approximately homogeneous market consisting of only high-usage consumers, where market outcomes are again similar to those presented in Section 4.1. Finally, as the usage rate increases, the owners use their good more often. This decreases rental supply as pointed out earlier and incentivizes purchase from low-usage customers, making the P2P-facing OEM more profitable than a monopoly.

In summary, the equilibrium profit outcomes are driven by the tradeoff between cannibalization and the inefficiency of P2P rentals ($\mu < 1$) versus the benefits of segmentation and the equalizing/participation effects. Proposition 4 demonstrates that an OEM can be better off in the presence of P2P rentals, even without the efficiency gains obtained in the presence of positive production costs. Next, we discuss the effect of P2P rentals on consumer welfare, presented in Proposition 5. An illustration is provided in Figure 7.

Proposition 5. *When $\phi_0 < \phi < 1/3$, consumer welfare is larger in the presence of P2P rentals than in* *the absence of P2P rentals under a monopoly OEM. Otherwise, consumer welfare decreases with P2P rentals.* ϕ_0 *is decreasing in* λ*. Moreover, the entry of a P2P rental market reduces the disparity in consumer welfare between low-usage and high-usage consumers, under all conditions. The surplus of the low-usage consumers always increases due to the availability of P2P rentals except when they are sufficiently similar to high-usage consumers (high value of* ϕ*), whereas high-usage welfare can decrease due to the availability of P2P rentals, as shown in Figure 7.*

This proposition highlights an interesting win-lose scenario that arises with the entry of P2P rentals. Although a firm would benefit from increased profits in many cases, the effect on consumers depends on the level of heterogeneity and usage rate. When the benchmark monopoly firm sells only to the high-usage segment ($\phi < 1/3$), consumers benefit overall from the entry of P2P rentals. This rental participation among consumers who were previously priced out of the market results in a net increase in total consumer welfare as long as the participation effect is sufficiently high ($\phi_0 < \phi < 1/3$). If $\phi < \phi_0$ though, the usage demand of low-usage consumers is too rare and the market devolves to an approximately homogeneous market with only high-usage consumers, leading to a reduction in consumer welfare as already shown in Proposition 1. When both usage rate and heterogeneity are both high (low ϕ, high λ), both types of consumers can benefit from P2P rentals. The high-usage consumers are benefiting in this scenario because the large equalizing effect in this parameter region leads to substantial purchasing demand from low-usage consumers, which puts downward pressure on the durable good price. Overall, this shows that both consumers and an OEM can be better off even in the absence of marginal production costs.

When $\phi \geq 1/3$, the benchmark monopoly firm sells to both consumer types, so high-usage consumers benefit greatly from the relatively low purchase price. With the entrance of P2P rentals, the firm extracts more rents as shown in Proposition 3. This consumer segmentation reduces high-usage consumer welfare significantly, dominating the overall consumer welfare outcome.

Interestingly, the entry of P2P rentals always reduces disparities in welfare between low-usage and high-usage consumers. Low-usage consumers benefit from both the participation and equalizing effects, therefore being better off under most conditions as shown in Figure 7 (unless the market is sufficiently homogeneous, see Proposition 1). Since the presence of P2P rentals allows the OEM to extract a larger surplus from high-usage consumers due to its discrimination ability, the surplus of high-usage reduces and becomes closer to the low-usage consumers.

5. The OEM's Business Model Problem

The analysis so far has focused on comparing outcomes for a monopolistic OEM operating in the absence or presence of P2P rentals. This section analyzes three alternative business model strategies for an OEM. First is a "P2P-sponsoring" firm, where an OEM directly operates a P2P platform. This is in the spirit of the aforementioned Tesla network; it is also consistent with the example of Airbnb's direct involvement with real estate developments mentioned in the introduction. A P2P-sponsoring firm derives revenue both from the sales of the durable good and from any transaction fees it charges in the P2P platform. Note that the rental price continues to be the market clearing price and is *not* set by the firm, which instead has the ability to capture a fraction of the rental revenues by setting a transaction fee for each P2P rental. The second strategy is a *"dual"* firm offering both sales and direct rentals (non-P2P), in which case the OEM sets the price for both. As mentioned earlier, the "Rent a Toyota" program by Toyota can serve as an illustration of this model. Discussed further in Section 5.2, if the dual firm only chooses to offer sales, this is the same as the monopoly firm of Section 4; for exposition clarity, offering only sales is referred to going forward as the "sales-only monopoly" firm strategy. Finally, we consider the scenario where an OEM offers direct rentals to compete against P2P rentals (the "dual+P2P" firm).

This section investigates the conditions under which these alternative strategies (P2P-sponsoring, dual, dual+P2P) are most profitable for an OEM. The intention is to identify the best business model for an OEM. Even though an OEM might not be able to completely "choose" the ultimate market structure (e.g, it may not be feasible to prevent P2P rentals from arising), several OEMs and insurance companies have discussed technologies and contractual restrictions that prevent sharing from happening. Independently of that, this analysis is useful to identify the key forces that a firm should keep in mind while deciding the market structure. For example, a firm can choose whether to use a dual strategy or a P2P-sponsoring strategy to segment consumers; as the results illustrate, it should choose the latter when the equalizing effect is stronger.

5.1 The P2P-sponsoring Firm

An OEM can proactively introduce its own P2P rental platform. The P2P-sponsoring firm obtains profits from both sales and P2P rental operations, charging a percentage fee α per P2P rental ($\alpha \in [0, 1)$). Using the superscript N (for "integrated") to denote the P2P-sponsoring strategy, the firm's profit function is thus:

$$\pi^N = p_s^N q_s^N + \frac{\alpha p_r^N q_r^N}{1-\delta},$$

where $\{p_s^N, q_s^N\}$ are the durable good price and quantity sold, and $\{p_r^N, q_r^N\}$ are the equilibrium rental price and quantity of rentals in each period, respectively. We assume that the fee α is incurred by the owners of the good. Thus, the utility of buying is now

$$U_i^{B,N} = \frac{\lambda_i \max\{v_i, p_r^N\} + (1-\lambda_i) p_r^N (1-\alpha)}{1-\delta} - p_s^N.$$

The utility of renting remains the same for consumers. When there is no platform fee ($\alpha = 0$), this model is identical to the P2P-facing scenario described in previous sections. The α represents a mechanism for the P2P-sponsoring firm to influence the market rental price.

Surprisingly, it is optimal for the firm to not charge any platform fee for P2P rentals (i.e., $\alpha = 0$); see the proof in the appendix. To understand why, note that the firm is already able to capture a significant part of the rental revenues indirectly, by raising the durable good price. When the firm charges a fee for P2P rentals, the low-usage owners are hurt disproportionately, as there is a larger reduction in their value effect $\big(\text{size of reduction} = (1-\phi\lambda)p_r^N(1-\alpha)/(1-\delta)\big)$. This also reduces the equalizing effect, making the P2P rental market less efficient. Hence, it is optimal for the firm to provide the platform for free. An increase in the platform fee would also hurt consumer welfare, reducing sales to low-usage consumers and reducing the rental supply.

5.2 The Dual Firm

The dual OEM offers sales and rentals directly to consumers. Under this business model (using superscript D for "dual"), the firm sets the prices p_s^D for the durable good and p_R^D for the per-use rental. The capitalized R subscript (instead of lowercase r) denotes that the dual firm directly sets the rental price. The consumer problem is similar to the one described for a P2P-facing firm. However, in this case, durable good owners derive value only from their own use and do not gain additional utility from rental revenue. The dual firm's direct rentals also have per-use rental frictions, represented as ν, where consumers receive νv_i utility per rental, with no additional transaction costs associated with the supply of rentals. In the comparison of strategies in Section 5.3, we assume that $\nu = \mu$ to keep the comparison parsimonious between the dual firm and the P2P-sponsoring strategy; we relax this assumption in Section 5.4. The consumer utility for the different choices is thus given by

$$\begin{aligned} U_i^{B,D} &= \frac{\lambda_i v_i}{1-\delta} - p_s^D, \\ U_i^{R,D} &= \frac{\lambda_i [\nu v_i - p_R^D]^+}{1-\delta}, \quad U_i^0 = 0. \end{aligned} \tag{4}$$

Let q_R^D denote the equilibrium number of rental units in the market for the dual firm. The dual firm's profit is given by $\pi^D = \frac{1}{2}p_s^D\left[(1-v_L^D)+(1-v_H^D)\right]+\frac{p_R^D q_R^D}{1-\delta}$. Here, $\{v_L^D, v_H^D\}$ are the valuations of the marginal low-usage and high-usage purchasers under the dual firm, such that all low-type (high-type) consumers with valuation $v_i \geq v_L^D\ (v_H^D)$ purchase the good. The firm simultaneously chooses $\{p_s^D, p_R^D\}$. As indicated in Lemma 3 below, the firm offers both sales and rentals only when consumer usage rates are sufficiently heterogeneous.

Lemma 3. *The dual firm chooses to offer both sales and rentals when the difference in usage rates across consumers is sufficiently large: when* $\phi \leq \tilde{\phi}^D = \frac{3-\nu-\sqrt{(9-\nu)(1-\nu)}}{2\nu}$. *Under this condition, it segments high-usage consumers into sales and low-usage consumers into rentals. Otherwise, the dual firm chooses a sales-only operation and sells to both consumer groups.*

The introduction of direct rentals allows a dual firm to segment the market by targeting sales of the durable good to high-usage consumers and per-use rentals to consumers who have low-usage rates but a high valuation from consumption. However, rentals can cannibalize the sales of durable goods. When the difference between the high-usage consumers and low-usage consumers is small ($\phi \geq \tilde{\phi}^D$), the optimal rental price will still cannibalize some sales to high-usage consumers and is less profitable than offering no direct rentals. Hence, the firm decides to offer only sales and act as the sales-only monopoly firm. On the other hand, with significant heterogeneity in usage rates ($\phi < \tilde{\phi}^D$), the firm may be able to realize the benefits of segmentation, offering both sales (to high-usage consumers) and rentals (to low-usage consumers).

Rentals Only The dual firm is also able to have a rentals-only strategy, where it does not offer any sales and instead only provides rentals to consumers. All analyses and extensions consider this; but except for the case of very high production costs, a rentals-only strategy is suboptimal to the dual firm's other options.

5.3 Comparison Between the Different Strategies

Having outlined two alternative strategies that a firm can adopt, we now compare the conditions under which a firm would prefer each different business model. Note that, because it is optimal to charge no platform fees, the analysis of the P2P-sponsoring firm is equivalent to the analysis of the P2P-facing firm in Section 4 (which was simplified by assuming a fee-less P2P platform); hence, the subsequent discussion will refer to only the P2P-sponsoring firm.

This section first highlights the comparative advantages of each strategy. The unique benefit of adopting the dual strategy is segmenting high and low-usage consumers into separate markets, by directly controlling both the sales and rental prices (*direct pricing effect*). The unique benefit of adopting the P2P-sponsoring strategy is the equalizing effect of P2P rentals, resulting in a more homogeneous set of consumer WTP. The sales-only monopoly strategy uniquely benefits from no cannibalization effect. The choice between these three strategies depends on which of these comparative advantages is stronger, which in turn depends on the level and heterogeneity in usage rates in the market (as determined by ϕ and λ). The comparison between the different strategies is formalized in the following proposition.

Proposition 6. *The OEM never prefers a P2P-sponsoring strategy when the usage rate λ is sufficiently small ($\lambda < \lambda_{min}$). Under this condition, the OEM prefers the sales-only monopoly when the heterogeneity in usage rate is sufficiently low ($\phi > \tilde{\phi}^D$) and the dual strategy when ϕ is small ($\phi < \tilde{\phi}^D$), as discussed in Lemma 3. For sufficiently large usage rates ($\lambda \geq \lambda_{min}$), as ϕ decreases from $1 \to 0$, the OEM moves from a sales-only monopoly to a P2P-sponsoring strategy and then might adopt a dual strategy when the direct pricing effect dominates the equalizing effect.*

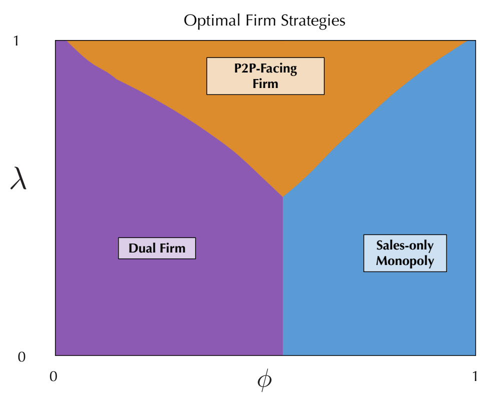


Figure 8: *Optimal firm strategies. In the illustration, $\mu = \nu = 0.8$.*

Figure 8 illustrates the optimal firm strategy. Considering $\lambda < \lambda_{min}$, a P2P-sponsoring strategy is dominated, regardless of heterogeneity, because the low usage rates lead to a large amount of rental supply that competes with durable sales. Instead, following Lemma 3, the OEM prefers either to act as a monopoly when consumers are sufficiently similar (to avoid the cannibalization effect) or the dual strategy when consumers are sufficiently different such that they can be segmented into sales and rentals.

Considering $\lambda \geq \lambda_{min}$, the P2P-sponsoring model is best when the equalizing effect dominates (which occurs when λ is large, and there is a sufficient level of heterogeneity, consistent with Lemma 1). The dual strategy is more profitable when the direct pricing effect dominates (which occurs when heterogeneity between the two consumer groups is sufficiently large), and the monopoly is more profitable when heterogeneity is small. The profitability of the P2P-sponsoring firm here is due to two factors: first, the equalizing effect leading to additional purchases from low-usage consumers, and second, the low-usage purchasers avoiding rental frictions (by owning instead). If the level of usage heterogeneity is sufficiently high but the usage rates are below a threshold, then the equalizing effect will lead to relatively fewer purchases and the direct pricing of sales and rentals separately dominates, so the dual strategy is preferred. Lastly, if there is insufficient usage-rate heterogeneity, any type of rentals (firm or P2P) that are attractive to some portion of low-usage consumers would also attract a sizable portion of high-usage consumers, significantly cannibalizing sales and introducing more rental frictions. Furthermore, the equalizing effect is limited when heterogeneity is low, because the gap in WTP is already small. Hence, for high values of ϕ, the monopoly strategy is most profitable. Table 2 of the appendix provides a numerical illustration of market outcomes under low ($\phi = 0.9$), intermediate ($\phi = 0.5$), and high ($\phi = 0.1$) levels of heterogeneity in usage rates.

With respect to consumer welfare, note first that the results of Section 4 (Proposition 5) show that consumer welfare is generally worse off with a P2P-sponsoring firm versus a benchmark monopoly firm. In contrast, when compared to a benchmark dual firm, total consumer welfare is higher under a P2P-sponsoring firm if at least some low-usage consumers purchase (outcomes C_1 and C_3). On the other hand, if usage rates are low or heterogeneity is high, such that low-usage consumers do not purchase, then total consumer welfare is lower under a P2P-sponsoring firm. The cause of this result is two-fold: (1) If low-usage consumers are able to purchase, they do not face the rental frictions from μ per use, and (2) the demand from low-usage consumers puts downward pressure on the durable price to be attractive to both groups, benefiting high-usage consumers.

5.4 Dual Firm in the Presence of P2P

The analysis so far has considered three business models: sales-only in the absence of P2P (sales-only monopoly), sales plus direct rentals in the absence of P2P (dual), and sales-only with integrated P2P rentals (P2P-sponsoring). However, the presence of P2P rentals may be unavoidable. In such a scenario, an OEM can also consider offering its own rentals *to compete against* P2P rentals. This section considers this additional scenario and refers to it as the "dual+P2P" strategy, because the firm is offering both sales and rentals in the presence of P2P rentals. In this case, we relax the assumption that $\nu = \mu$ and instead assume that the dual firm's rental frictions are lower than the P2P rental frictions (i.e., $\mu < \nu < 1$). This is an important distinction to make because it leads to a scenario in which the dual firm is more competitive against P2P rentals.[11] This distinction was not made in earlier subsection 5.3, in order to parsimoniously analyze the trade-off between the dual and P2P-sponsoring strategies.

Using superscript E (being the letter that follows "D"), a consumer with valuation v_i and usage λ_i thus has the following consumer problem:

$$\begin{aligned} U_i^{B,E} &= \frac{\lambda_i v_i + (1-\lambda_i)p_r^E}{1-\delta} - p_s^E, \\ U_i^{R,E,firm} &= \frac{\lambda_i[\nu v_i - p_R^E]^+}{1-\delta}, \qquad (5) \\ U_i^{R,E,P2P} &= \frac{\lambda_i[\mu v_i - p_r^E]^+}{1-\delta}, \quad U_i^0 = 0. \qquad (6) \end{aligned}$$

The firm chooses the prices $\{p_s^E, p_R^E\}$ to maximize its combined profit from sales and direct rentals, in the same manner as the dual firm in Section 5.2, taking the expected market P2P price p_r^E into consideration. In addition to the marginal valuation of consumers who purchase ($\hat{v}_\theta^E$ for consumer usage type θ) and the marginal valuation of those who rent from P2P ($\tilde{v}^E$, which does not depend on usage rate), there is now an marginal valuation for those indifferent between renting directly from the dual firm and renting from the P2P rental market ($\bar{v}^E$), defined by $U_i^{R,E,firm} = U_i^{R,E,P2P}$. This indifferent valuation is $\bar{v}^E = \frac{p_R^E - p_r^E}{\nu - \mu}$. The result is that the market rental prices are related by $p_R^E = p_r^E + \bar{v}^E(\nu - \mu)$. Additionally, it is possible that $\tilde{v}^E < \hat{v}_H^E < \bar{v}^E$, such that no high-usage consumers rent directly from the firm (they either purchase, rent from P2P, or do not consume).

If the OEM could choose any business model, using numerical analysis[12], we find that the dual+P2P strategy is preferred to the dual and P2P-sponsoring strategies in some conditions. In such cases, if $(\nu - \mu)$ is sufficiently large (i.e., the OEM's own rentals must have sufficiently fewer rental frictions to be viable competition), the dual+P2P strategy represents a "bridge" strategy between the dual and the P2P-sponsoring strategies. This occurs in situations where the amount of purchases from low-usage consumers is low, but not low enough such that the dual strategy is optimal. An example of this is shown in Figure 9(A).

To understand the intuition behind this observation, consider the choice between adopting a P2P-

[11] We believe this assumption is the most reasonable in a market with direct rentals competing with P2P rentals. From a trust perspective, it'd seem more reasonable that, all else equal, a consumer would prefer to rent a car directly from Toyota than from a stranger. In addition, established brands may provide better service standards. Also, dealing directly with an established brand like Toyota could lead to lower coordination costs. Indeed, these very same factors are advertised by Toyota in their webpage: "Rent a Toyota is Toyota dealer based. This means that not only do you have an opportunity to rent the latest model Toyota, but you get the quality customer service of a conveniently-located Toyota dealer". Overall, these reasons lead us to believe that the assumption is most reasonable. We note that this is also the most conservative assumption in terms of when P2P is preferred for the firm; the alternative would lead to even larger regions where the OEM benefits from P2P.

[12] The model is analytically intractable.

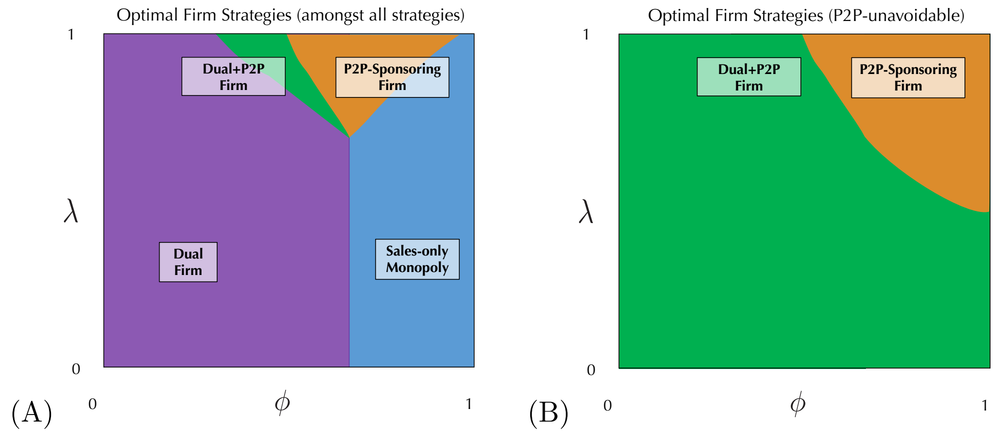


Figure 9: *In the illustration,* $\mu = 0.8$ *and* $\nu = 0.9$*. Each color represents the equilibrium outcomes in the corresponding parameter space.*
(A) when the OEM can control the emergence of the P2P market, or
(B) when P2P market emergence is unavoidable.

sponsoring strategy versus a dual+P2P strategy. At intermediate heterogeneity and sufficient usage λ, the P2P-sponsoring strategy is preferable because there are substantial low-usage consumers who purchase due to the equalizing effect. Introducing a second form of rentals would further cannibalize those sales. However, as the level of heterogeneity increases, the number of purchases from low-usage consumers decreases and a larger fraction of low-usage consumers rent. Once the number of low-usage buyers is low, the OEM prefers to forgo some of its sales to low-usage consumers in exchange for introducing its own direct rentals. These OEM direct rentals are attractive to many low-usage renters because they have lower rental frictions. As a result, the dual+P2P firm would have a higher rental price than a dual firm would ($p_R^E > p_R^D$ in equilibrium), but it also would sell fewer goods than a P2P-sponsoring firm would. The average rental price paid by consumers is lower though.

If an OEM can control the market structure, then it would see a market equilibrium identical to the one presented in Figure 9(A) as a consequence of the P2P rental equilibrium. This is not an unlikely scenario as several OEMs and insurance companies have discussed technologies and contractual restrictions that can prevent P2P rentals from occurring. They can also push for regulations that prevent the sharing of goods for profit, similar to the media industry. However, an OEM may not be able to prevent P2P rentals from emerging, in which case it can evaluate whether or not to introduce direct rentals to compete against P2P. The paper now turns to that scenario, and continues to refer to the OEM as the P2P-sponsoring firm, consistent with prior nomenclature. Figure 9(B) illustrates the outcome of that scenario. In the presence of an unavoidable P2P rentals, an OEM often adopts a dual+P2P strategy to compete against P2P rentals. However, as shown in Figure 9(B), when the usage rate is high but the heterogeneity in usage rates is small, introducing direct rentals cannibalizes sales. In that case, a P2P-sponsoring strategy can dominate the dual+P2P strategy, extending the threshold shown in Figure 9(A) to cover a larger region. This occurs as long as the difference between ν and μ is not too large.

In short, regardless of whether an OEM could fully control the market structure or not, it would not necessarily be better off by introducing its own rentals to compete against P2P and might prefer to forgo that option in a meaningful number of cases.

6. Extensions

In this section we consider a variety of extensions to examine market outcomes under alternative modeling assumptions.

6.1 Transaction Costs

Transaction costs in the sharing economy can arise in many different forms. For owners, they can represent a fee charged by a P2P platform (as in Benjaafar et al. 2019), a moral hazard cost (as in Jiang and Tian 2018), or a depreciation cost caused by increased usage (as in Fraiberger and Sundararajan 2015). With a P2P rental market, goods can be used by both owners and renters. For some goods, this may cause faster degradation. Faster degradation or depreciation can be modeled as a flat cost per use (for example, representing either expected costs of repair or future replacement to restore the value of the good to original levels). Renters can also face transaction costs because of inconvenience (μ) or the presence of commission fees paid to a P2P platform. Overall, such frictions reduce the incentive for owners and consumers to participate in the P2P rental market.

In the prior analysis, we have already considered the inconvenience cost on the renter side (μ). This section includes in addition a transaction cost, tc, on the owner side when renting out the product. The owner's per-period utility in this case is given by

$$u_i^{B,P} = \begin{cases} \max\{v_i, p_r - tc\} & \text{if owner demands the product} \\ p_r - tc & \text{otherwise} \end{cases}$$

The new expected utility from purchasing is given by $U_i^{B,P} = \frac{\lambda_i \max\{v_i, p_r^P - tc\} + (1-\lambda_i)(p_r^P - tc)}{1-\delta} - p_s^P$. We examine this scenario numerically, obtaining results that are qualitatively similar. Higher transaction costs shrink the parameter region where the P2P-sponsoring firm is the most profitable strategy, because a transaction cost will reduce purchase incentives. However, the main insights from the analysis remain the same, where a P2P-sponsoring strategy is best for sufficiently high levels of heterogeneity and usage rates (so that the equalizing effect is meaningful).

6.2 Production Costs

Until now, we have assumed zero production costs in the analysis of a P2P-sponsoring firm. This section turns to the scenario where a firm incurs a positive production cost. In such a case, the firm's profit function is given by $\pi_s^P = (p_s^P - c)\left\{\left[(1-v_H^P)+(1-v_L^P)\right]/2\right\}$, where c represents the marginal cost of production. P2P rentals help redistribute these costly goods when they are not being used by the owner.

First, consider the case where consumers have homogeneous usage rates. Under that condition, when the production costs (c) are sufficiently high, both firm profits and consumer welfare increase due to the P2P economy. That occurs when the production cost $c \geq \frac{\lambda}{(1-\delta)(1+\lambda)} - \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}}$. Hence, Proposition 1 holds only if production costs are not high. However, P2P rentals still allow the firm to segment consumers into renters and buyers, which can lead to a decrease in the consumer surplus of high-valuation (v_i) consumers.

When consumers are heterogeneous in usage rates, this redistribution of goods induced by P2P rentals acts in addition to the equalizing effect and implicit segmentation power of P2P rentals (which both continue to hold even when $c > 0$). All findings continue to hold. Propositions 2 and 3 continue to hold, as the buying behavior of consumers in the presence of P2P rentals is hardly affected by production costs, and a P2P-sponsoring firm continues to extract more welfare than the monopoly firm that sells to both consumer groups. Furthermore, the efficient use of goods with P2P rentals (when $c > 0$) leads to an increase in the parameter region where the entry of P2P rentals leads to high firm profits, and the intuition behind Proposition 4 continues to hold. It is easy to show that the effect of P2P rentals on consumer surplus is similar, and Proposition 5 continues to hold. There is still a substantial region of the parameter space where the P2P-sponsoring firm is able to extract a substantial portion of the surplus generated. Comparing business models when $c > 0$, Proposition 6 continues to hold and the P2P-sponsoring strategy is preferred in a larger parameter region.

6.3 Market Size

In the base model, all consumers potentially participate in the P2P rental market. However, it is possible that in reality only a fraction of consumers might be willing to participate in P2P rentals, because of factors like risk-aversion, lack of physical proximity to sharing markets, psychological factors, etc. To examine this, we extend our base model to allow only a γ fraction of the consumers to participate in P2P rentals, i.e., a fraction (1-γ) of consumers are not affected by the entrance of P2P rentals (consumers in the latter group would not participate in P2P rentals either as supply or demand, regardless of the economics of that market). The case of $\gamma = 1$ analyzed so far is thus a special case when the whole population is part of the sharing economy. The firm's profit is given by

$$\pi_s = p_s \left\{ (1-\gamma)\frac{[(1-v_H^m)+(1-v_L^m)]}{2} + \gamma \frac{\left[(1-v_H^p)+(1-v_L^p)\right]}{2} \right\}, \tag{7}$$

where $\{v_H^p, v_L^p\}$ denote the valuations of the participating consumers who are indifferent between purchasing and renting/not consuming. Similarly, $\{v_H^m, v_L^m\}$ represent the indifferent consumers among those who are not participating in P2P rentals. As a special case, one can consider the homogeneous usage rate case (homologously to Section 4.1). The next proposition shows how in this case the sharing economy can benefit consumers even if there is no usage-rate heterogeneity in the market.

Proposition 7. *When consumer have a homogeneous usage rate ($\phi = 1$) and the fraction of consumers not participating in P2P rentals is sufficiently large, $\gamma < \tilde{\gamma} = \frac{4\lambda^2\mu^2 - 5\lambda^2\mu + \lambda^2 - 4\mu^2 + \mu}{4\lambda^2\mu^2 - 4\lambda^2\mu - 4\mu^2 + \mu}$, then consumer welfare increases under the sharing economy as compared to the sales-only monopoly case.*

Intuitively, this occurs because a larger non-sharing market mediates the firm's increase in the durable good price caused by P2P rentals. It is easy to see that consumers in the sharing segment are better off when $\gamma < 1$ because the firm is not able to increase the durable price as much as than if $\gamma = 1$. If $\gamma < 1$, consumers participating in the sharing market benefit in two ways. Consumers purchasing the product retain more surplus because their rental revenues are no longer extracted completely by the OEM. There is also an increase in the number of consumers who rent, which also increases consumer surplus. Consumers in the non-participating segment are slightly hurt by a price increase, but we show that the consumer surplus gained by the participating segment outweighs the lost surplus in the non-participating segment when $\gamma < \tilde{\gamma}$.

When the usage rates are heterogeneous, we show numerically that if $\gamma < 1$, the parameter region of $\{\lambda, \phi\}$ where consumer welfare is larger under a P2P-sponsoring firm than a dual firm is expanded. The same intuition follows: if a fraction of consumers do not participate in P2P rentals and only choose whether to buy or not, this mediates the P2P-sponsoring firm's durable price increase, also mediating the rent extraction from consumers. Correspondingly, if $\gamma < 1$, the parameter region where the P2P-sponsoring firm is the most profitable business model shrinks.

6.4 Rental Cost as a Function of Suppliers

In the main model, we consider that rental inconveniences, reflected in μ, are not affected by the number of durable goods available in the market. However, a sharing market with a small number of durable goods may lead to larger inconvenience for renters, as the number of owners acting as suppliers is reduced. In this extension, we examine how our results change when the rental inconvenience depends on the number of durable goods that are sold. We add a new term $\eta(1 - D_b^P)$ to the utility of renting, where D_b^P is the number of owners and η is the sensitivity of rental frictions to the market size. If D_b^P is relatively large, then the rental frictions will be relatively low, and vice versa. The utility of renting is thus changed to

$$U_i^{R,P} = \frac{\mathbb{E}\left[u_i^R\right]}{1-\delta} = \frac{\lambda_i[\mu v_i - p_r - \eta(1 - D_b^P)]^+}{1-\delta}.$$

With this change in rental utility, when $\eta > 0$ instead of $\eta = 0$, the parameter region where the P2P-facing firm is more profitable than the sales-only monopoly firm shrinks. This is as expected since η is a form of additional rental frictions that reduces available welfare to the P2P-facing firm. Increasing η, or i.e., increasing the impact of market size, reduces the profitability region for the P2P-facing firm for lower values of λ and for any given value of λ, the range of intermediate ϕ values between where the P2P-facing firm is more profitable than the sales-only monopoly firm. While the parameter region shrinks, the parameter region is still significant and results are qualitatively similar.

6.5 Correlation between λ and ϕ

In the main model, we consider that the usage rate and the per-use valuations are independent. However, consumers who have a higher valuation might also have higher usage rate. In this extension, we examine how our results change when there is a correlation between λ and ϕ. Let $corr(\lambda, \phi) = \rho$. We continue to assume that low-usage consumers have a valuation $v_i \sim U[0, 1]$, whereas high-usage consumers have a valuation $v_i \sim U[b, b + 1]$, where c is a constant. As shown in the appendix, the correlation can be expressed as a function of c, as follows:

$$\rho = \frac{\sqrt{3}b}{\sqrt{1+3b^2}}.$$

As long as there are overlapping segments of high and low-usage consumers, i.e., $b < 1$ (or $\rho < \sqrt{3}/2$), our results remain qualitatively similar to the main results. The region when P2P rentals are profitable for the OEM adjusts when usage rates and per-use valuations are correlated. At moderate correlations, the size of region is of a similar magnitude to the base results, but shifted to the right. For large correlations between λ and ϕ, the parameter region begins to shrink for lower values of λ and for given

λ, the range of the intermediate values of ϕ shrinks. When $\rho \geq \sqrt{3}/2$ ($b \geq 1$), the OEM does not care about the low-usage consumers at all and the situation devolves to a case with only one dimension of heterogeneity (just v_i). Under this condition, the results are similar to the homogeneous case presented in Section 4.1.

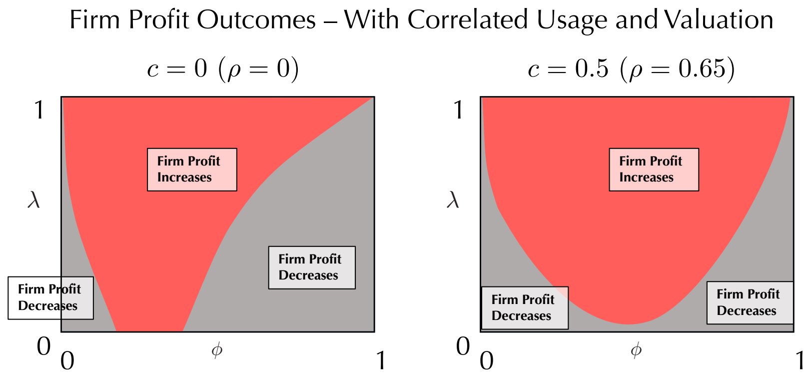


Figure 10: Comparison of monopoly versus P2P-facing firm profits under correlated λ and ϕ.

6.6 Matching Frictions

In the main model, we assume that an owner of a good can always find a renter in periods where they would like to rent out the good. In this extension we consider the case where owners are not guaranteed to find a renter for their good, but instead only have a ψ chance of matching with a P2P renter in each period ($0 < \psi \leq 1$). In the main model, $\psi = 1$, but in this extension, we consider how the results differ if $0 < \psi < 1$. If $\psi < 1$, this affects the utility of ownership and the rental supply available. The utility of ownership is now

$$U_i^{B,P} = \frac{\lambda_i \max\{v_i, p_r\} + (1 - \lambda_i)\psi p_r}{1 - \delta} - p_s^P$$

where the expected rental revenue is affected by ψ. Due to matching frictions, the expected rental supply in each period is now given by

$$S_r(p_r, p_s^P) = \frac{1}{2}\psi\left[(1 - \lambda)(1 - v_H) + (1 - \phi\lambda)(1 - v_L)\right].$$

where the rental supply is equal to the expected number of owners who will not be using the good each period multiplied by ψ.

The results do not change qualitatively as a result of considering matching frictions. There are still three substantial parameter regions where sales-only monopoly, P2P-sponsoring, and dual firm strategies are each preferred, but the region where P2P-sponsoring is preferred reduces due to the inefficiency created by rental frictions. In Figure 11, we compare the business model results of our model for $\psi \in \{0.5, 1\}$. In this comparison, we continue with the main model and consider that $\mu = \nu$, but that only P2P rentals face P2P matching frictions, while the dual firm's rentals do not. When P2P matching frictions are higher, this results in a higher rental price and a larger amount of durable good purchases.

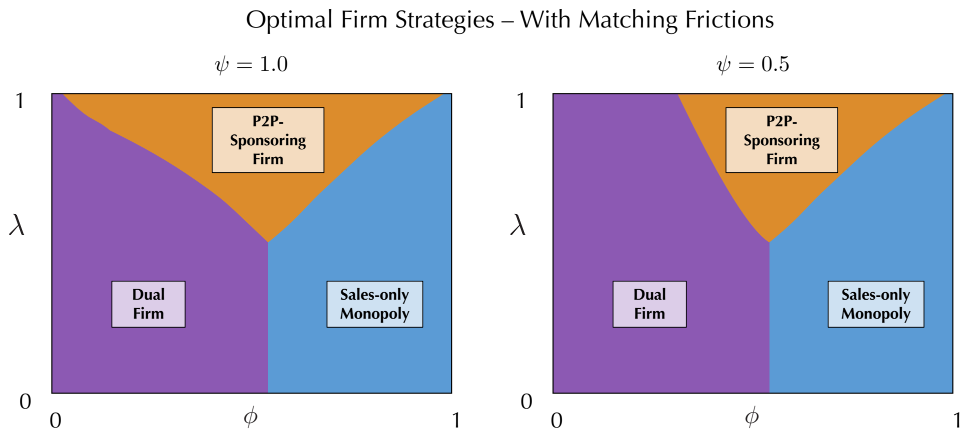


Figure 11: Comparison of monopoly versus P2P-facing firm profits when there are market-size dependent rental frictions (assuming $\mu = \nu = 0.8$).

6.7 Other Extensions

We considered several alternative modifications to the model or extensions to the strategies. For example, we considered a case with different proportions of high and low-usage consumers (i.e., markets with a split different than 50% of each type), and the results hold qualitatively. Also, we considered a model without multiplicative rental frictions by setting $\mu \overset{lim}{\to} 1$ for P2P-sponsoring and dual firms, but including positive flat transaction costs instead. In that simplified case, the qualitative insights of Sections 4 and 5 continue to hold; the parameter region where P2P-sponsoring is optimal is larger but sensitive to transaction costs. In addition, we also considered other models of consumer usage heterogeneity. For example, a model where high-usage consumers have a usage rate $\lambda + \phi$, and low-usage consumers have a usage rate of $\lambda - \phi$; or a model where high-usage consumers have a usage rate of ${}^{\lambda}/_{\phi}$, and low-usage consumers have a usage rate of $\phi\lambda$. The results continue to be robust to these alternative models of consumer heterogeneity and all the main insights apply. Moreover, our main model is preferred because it is more general and captures a superset of the possible market conditions represented by these formulations.

7. Conclusion

The sharing economy represents a substantial change in the structure of a market because consumers also become the providers of goods and services. Our analysis showed that a manufacturer's profit-maximizing responses depend a great deal on the heterogeneity in consumer usage rates. When consumers are homogeneous in usage rates, surprisingly, both the firm and consumers can be worse off with a P2P rental market. However, when there is heterogeneity in usage rates, P2P rentals make high and low-usage consumers' WTP more similar (the equalizing effect). This effect is maximized when usage rates and level of usage rate heterogeneity are above a threshold, and it explains why a firm benefits most from P2P rentals in such cases. The benefit of P2P rentals are realized by the consumers when the heterogeneity in usage rates is significantly high, and they can participate in the market. We also consider the business model problem for a manufacturer and characterized the conditions under which an OEM would prefer to operate offering sales only, sales plus direct rentals, and sales only plus P2P rentals. To the best of our knowledge, this is the first paper to analyze the business model problem for an OEM in the presence of P2P rentals, and to highlight the heterogeneity in usage rates as an important driver of market outcomes in P2P rental markets.

The analysis leads to new insights with important implications for OEMs—for example, deciding when and why to adopt different business models. To illustrate, if an OEM is evaluating the introduction of its own P2P platform and can choose across multiple markets with varying degrees of heterogeneity, it may introduce the P2P option (or equivalently, facilitate the operation of an external P2P platform) in markets where the level of consumer heterogeneity is such that a P2P platform would be beneficial. Alternatively, it can choose to operate a sales-plus-rental operation in markets with very large degree of heterogeneity. Overall, our findings indicate that OEMs can benefit from P2P rentals in a variety of situations, consistent with many industry examples. For instance, some OEMs are promoting car-sharing as a means of increasing sales. In addition to the example of Tesla mentioned earlier, in 2016, BMW Mini announced a feature to make it easier for buyers to share their vehicles when they are not in use (Schwarzenbauer 2016). GM also recently promoted that it would allow peer-to-peer rentals through its Maven platform, "to offer GM owners the opportunity to ... earn income by listing their vehicles" for peer-to-peer rentals (GM corporate newsroom, 2018). Beyond cars, VanMoof (a bicycle manufacturer) partnered with SpinLister (a P2P rental market for bicycles) to sell smart bikes that are easily entered and accessible via SpinLister, and the company marketed this as an added incentive for purchases. In essence, these examples are consistent with the motivation we used to analyze the

P2P-sponsoring strategy in this paper.

More broadly, this paper provides insights for which business models might be most appropriate across different industries. When the usage rates are significantly high and there is little heterogeneity in the market, e.g. in the case of home appliances, the OEM will continue to adopt the sales-only business model. Even though the evidence remains anecdotal, we see that P2P rentals under these conditions have not taken off and startups like Peerby.com have not gained much traction (Morrissey 2015). However, the dual strategy has worked extremely well for retailers like HomeDepot (in the presence of significant heterogeneity), who sell equipment to contractors and heavy duty professionals while operating a very successful rental service for do-it yourself consumers. Industries with an intermediate amount of heterogeneity and sufficiently high usage rate are favorable for the OEM to support the P2P rentals, e.g. in automobiles and heavy equipment. As pointed out earlier, most automobile manufacturers are directly or indirectly supporting P2P rentals. Unicorn startups like EquipmentShare are growing very rapidly supported by OEMs such as Caterpillar (Konrad and Carson, 2019). Hence, empirical evidence suggests that our results are a good representation of the real world outcomes.

The analysis suggested several avenues for future research. For example, the impact of competition between OEMs and the analysis of the interactions between incumbent firms and sharing-economy companies in settings different than P2P rentals (e.g., on-demand services) are two natural extensions to consider. Similarly, although we analyzed and discussed results under multiple extensions, future research can further expand the analysis to consider the impact of alternative costs structures, the evolution of the market dynamics at different stages of the diffusion process (see e.g. Razeghian and Weber 2016 for an existing treatment), and markets where goods are subject to network effects, among others. Finally, empirical work can be useful to test some of the implications from this research. For example, it could be examined empirically whether the entrance of a P2P rental market affects prices, quantities, and consumer segments, in a way consistent with our findings (e.g., whether sales prices indeed increase when a manufacturer introduces a sponsored P2P platform). The entry of sharing-economy companies is challenging traditional incumbent firms in several industries. This paper contributes to a better understanding of the potentially beneficial interactions that traditional firms can experience in this new scenario where consumers are not only always right, but also competing to supply. We hope that this paper can generate interest in understanding how incumbents respond to these new technology-enabled business models.

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Appendix

Numerical Illustration of Proposition 6

In Table 2 of this appendix, we provide a numerical illustration of the various discussed effects of Proposition 6, for markets with low ($\phi = 0.9$), intermediate ($\phi = 0.5$), and high ($\phi = 0.1$) levels of consumer heterogeneity in usage rates. We observe that the firm's profit is highest under the P2P-sponsoring strategy for intermediate ϕ, but dual provides the highest profit under low ϕ, whereas monopoly provides the highest profit under high ϕ.

Table 2: *Market outcomes for Monopoly, Dual, P2P-sponsoring, and Dual+P2P firm strategies* ($\lambda = 0.75, \mu = 0.8, \nu = 0.9$).

Strategy	$1 - \hat{v}_L$ (sales)	$1 - \hat{v}_H$ (sales)	Total Sales	Total Rentals	Durable Price	Rental Price	Firm Profit	Consumer Surplus
$\boldsymbol{\phi = 0.1}$								
Monopoly	0.00	0.50	0.25	–	3.75	–	0.938	0.47
Dual	0.00	0.50	0.25	0.02	3.75	0.450	1.022	0.51
P2P-s	0.00	0.42	0.21	0.05	4.75	0.388	1.008	0.50
D+P2P	0.00	0.40	0.20	0.06	4.75	0.45 / 0.385	1.013	0.33
$\boldsymbol{\phi = 0.5}$								
Monopoly	0.33	0.67	0.50	–	2.50	–	1.25	0.95
Dual	0.00	0.50	0.25	0.09	3.75	0.450	1.36	0.68
P2P-s	0.06	0.51	0.28	0.08	4.69	0.398	1.33	0.67
D+P2P	0.00	0.43	0.21	0.13	4.75	0.450 / 0.389	1.35	0.47
$\boldsymbol{\phi = 0.9}$								
Monopoly	0.47	0.53	0.50	–	3.56	–	1.78	0.90
Dual	0.28	0.54	0.39	0.09	3.55	0.432	1.78	0.90
P2P-s	0.34	0.41	0.37	0.11	4.71	0.382	1.75	0.88
D+P2P	0.34	0.41	0.38	0.11	4.63	0.44 / 0.375	1.75	0.51

Equilibrium Rental Prices and Marginal Purchasers

We detail the exact, analytical equilibrium expressions and marginal purchasers (for low and high-usage consumers), when the firm chooses p_s^P optimally, in terms of the market parameters $\{\delta, \mu, \phi, \lambda\}$. The marginal high-type purchasers are[13]:

$$v_H = \begin{cases} \frac{\phi(\lambda\phi(2\lambda(\mu-1)+\mu)+(\lambda-4)\mu)}{2(\phi+1)(\lambda^2(\mu-1)\phi-\mu)} & if\ C_1 \\ \frac{\lambda(\phi+1)(\lambda(\mu-1)+\mu)-2\mu}{2(\lambda^2(\mu-1)(\phi+1)-\mu)} & if\ C_2 \\ \frac{(\lambda-1)\lambda(\mu-1)\mu\phi(\lambda\phi+\lambda-2)-\mathrm{A1}}{\lambda^3(\mu-1)\phi^2+(\lambda-1)^2\lambda(\mu-1)\mu\phi-\lambda\mu} & if\ C_3 \\ \frac{\lambda^2\phi+(\lambda-1)\mu(\lambda(\phi+2)-2)}{2\lambda^2\phi+2(\lambda-1)^2\mu} & if\ C_4. \end{cases}$$

[13] where $A1 = \frac{\lambda\left(\lambda(\mu-1)\phi^2-\mu\right)\left((\mu-1)\phi^2(\lambda(\mu-2)-\mu)+\mu\phi(\lambda(-\mu)+\lambda+\mu)+\mu\right)}{2(\phi+1)(\mu(\phi-1)-\phi)}$

The marginal low-type purchasers are $v_L = \min\{^{v_H}/_{\phi}, 1\}$, except for outcomes $\{C_2, C_4\}$ where $v_L = 1$ by definition since low-usage consumers are not buying in such circumstances. Similarly, the equilibrium rental price is slightly different depending on the market outcomes, due to variations in supply and demand[14]:

$$p_r = \begin{cases} \frac{\mu(\mu(\phi(2\lambda(\lambda\phi+\lambda-1)-1)-1)-2\lambda\phi(\lambda\phi+\lambda-1))}{2(\phi+1)(\lambda^2(\mu-1)\phi-\mu)} & if\ C_1 \\ \frac{\mu(\lambda(\mu-1)(2\lambda(\phi+1)-1)-\mu)}{2(\lambda^2(\mu-1)(\phi+1)-\mu)} & if\ C_2 \\ \frac{\mu\left((\mu-1)\phi^3(A2)-(\mu-1)\phi^2(A3)+2\lambda^2(1-\mu)^2\phi^4+\mu\phi(1-2(\lambda-2)\lambda(\mu-1))+\mu\right)}{2(\phi+1)(\mu(\phi-1)-\phi)(\lambda^2(\mu-1)\phi^2+(\lambda-1)^2(\mu-1)\mu\phi-\mu)} & if\ C_3 \\ \frac{2\lambda^2\mu\phi+(\lambda-1)\mu((\lambda-1)\mu+\lambda)}{2\lambda^2\phi+2(\lambda-1)^2\mu} & if\ C_4. \end{cases}$$

With these rental prices, it is easy to identify the marginal renter in each outcome; the marginal renter's valuation is $\tilde{v} = \frac{p_r}{\mu}$. Lastly, incorporating these consumer and rental market outcomes, the P2P-facing firm's profits are thus:

$$\pi_s \cdot (1-\delta) = \begin{cases} \frac{\lambda(\mu((2\lambda-1)\phi-1)-2\lambda\phi)^2}{8(\phi+1)(\lambda^2\phi+\mu(1-\lambda^2\phi))} & if\ C_1 \\ \frac{\lambda(\phi+1)(\lambda(-\mu)+\lambda+\mu)^2}{8(\lambda^2(1-\mu)(\phi+1)+\mu)} & if\ C_2 \\ \frac{\lambda\left((\lambda-1)\mu^2(\phi-1)\phi+\mu\left(\lambda\left(\phi-3\phi^2\right)+\phi^2+1\right)+2\lambda\phi^2\right)^2}{8(\phi+1)(\mu(\phi-1)-\phi)(\mu(\lambda^2\phi^2+(\lambda-1)^2(-\phi)-1)-\lambda^2\phi^2+(\lambda-1)^2\mu^2\phi)} & if\ C_3 \\ \frac{\lambda\phi(\lambda+\mu-\lambda\mu)^2}{8(\lambda^2\phi+(\lambda-1)^2\mu)} & if\ C_4. \end{cases}$$

Proof of Lemma 2

The marginal {high-usage, low-usage} buyer the monopoly faces is $\{\frac{p_s(1-\delta)}{\lambda}, \min[\frac{p_s(1-\delta)}{\phi\lambda}, 1]\}$. The demand the monopoly firm faces is $\frac{1}{2}\left(1 - \frac{p_s(1-\delta)}{\lambda} + \max\{1 - \frac{p_s(1-\delta)}{\phi\lambda}, 0\}\right)$. If the firm sets a sufficiently high price, it will sell only to high-usage consumers. Optimal profits when selling to both types is $\frac{\lambda\phi}{2(1-\delta)(1+\phi)}$, and when selling to just high types is $\frac{\lambda}{8(1-\delta)}$. It follows that $\frac{\lambda\phi}{2(1-\delta)(1+\phi)} \geq \frac{\lambda}{8(1-\delta)}$ only when $\phi \geq \frac{1}{3}$. ■

Proof of Section 3.4:

In equilibrium, $\{v_H, v_L\} \geq p_r$ such that all product owners utilize the good in periods when they demand it. We prove this must always hold in equilibrium. v_H is the consumption valuation of the marginal high-type purchaser, and it's clear that $v_L > v_H$, so we only consider v_H. Consider the edge case where $v_H = \hat{p}_r^0$, where $\hat{p}_r^0$ is the consumer expected market rental price. High-usage consumers would not demand rentals, so the rental supply and demand are: $D_r = \phi\lambda\left[v_L - \frac{\hat{p}_r^0}{\mu}\right]^+$, $S_r = (1-\lambda)(1-v_H) + (1-\phi\lambda)(1-v_L)$. The resulting equilibrium market rental price is some p_r^0, which satisfies the rational expectations equilibrium assumption, because $p_r^0 = \hat{p}_r^0$. Now, consider an upward perturbation to the expected rental price $\hat{p}_r^{'} = \hat{p}_r^0 + \epsilon$ while v_H is unchanged. Then $v_H < p_r^{'}$ and ϵ owners rent out the good even in periods when they would like to use it. The rental demand remains unchanged, but the rental supply is now: $S_r^{'} = (1-\lambda)(1-v_H) + (1-\phi\lambda)(1-v_L) + \lambda\epsilon$. Now however, the resulting market rental

[14] where $A2 = \lambda^2(\mu-2)(\mu+1) - 2\lambda\left(\mu^2-1\right) + (\mu-1)\mu$ and $A3 = \lambda^2\left(\mu^2+\mu+2\right) - 2\lambda\left(\mu^2+1\right) + \mu^2 + \mu$.

price does not satisfy the rational expectations equilibrium. The resulting market price is $p_r^{'} = p_r^0 - \frac{2\epsilon\mu}{\phi}$. In fact, $p_r^{'} < v_H < \hat{p}_r^{'}$. Thus, by contradiction, there cannot be an equilibrium market outcome where some owners expect to rent out the good in all periods; that would attract additional consumers to buy into the market and subsequently lower the rental price such that there would no longer be owners who bought purely to rent out. All rational expectations equilibrium will have $v_H \geq p_r$. ■

Proof of Proposition 1

Assume consumers have homogeneous stochastic usage rates, λ. $\hat{v}$ represents the marginal buying consumer, indifferent between buying and renting/outside option; consumers with valuation $v_i > \hat{v}$ purchase. The resulting equilibrium P2P rental price is: $p_r = \frac{\mu(\lambda-(1-\hat{v}))}{\lambda}$. Then, $\hat{v}$ is given by: $\hat{v} = \frac{p_s^{'}-p_r}{\lambda(1-\mu)} = \frac{\lambda p_s^{'}+\mu(1-\lambda)}{\lambda^2(1-\mu)+\mu}$ (where $p_s^{'} = p_s(1-\delta)$). The P2P-facing firm's profit problem is: $\pi^P = (1 + \frac{\mu-\lambda p_s^{'}+\mu(1-\lambda)}{\lambda^2(1-\mu)+\mu})p_s$ and the profit-maximizing durable price is $p_s^P = \frac{1}{2}\left(\lambda(1-\mu)+\mu\right)$. Total sales to the P2P-facing firm are $1-\hat{v} = \frac{1}{2} - \frac{\mu(1-\lambda)}{2(\lambda^2(1-\mu)+\mu)}$, and total sales to the monopoly firm are $\frac{1}{2}$; it's clear that fewer consumers purchase under a P2P-facing firm. The marginal renter is $\tilde{v} = \frac{p_r}{\mu} = \frac{1}{2} - \frac{\lambda(1-\lambda)(1-\mu)}{2(\lambda^2(1-\mu)+\mu)}$, so there is increased participation under a P2P-facing firm than in the absence of a P2P market, i.e., $\tilde{v} < \frac{1}{2}$, so more than half of the consumers now participate. The P2P-facing firm's profit and resulting consumer welfare are also lower. The monopoly firm's profit is: $\pi^M = (1 - \frac{p^M(1-\delta)}{\lambda})p_s^M$, with an optimal price of $p_s^M = \frac{\lambda}{2}(1-\delta)$ and profit of $\frac{\lambda}{4(1-\delta)}$. The optimal profit for a P2P-facing firm is: $\frac{\lambda}{4(1-\delta)}\left(\frac{(\lambda+\mu-\lambda\mu)^2}{(\lambda^2(1-\mu)+\mu)}\right)$. Thus, $\pi^P - \pi^M$ is always negative, because $\frac{(\lambda+\mu(1-\lambda))^2}{(\lambda^2+\mu(1-\lambda^2))} \leq 1$. The monopoly consumer welfare is: $CS^M = \frac{\lambda}{8(1-\delta)}$ and P2P-facing consumer welfare from rentals and purchases is: $CS^P = \frac{\lambda(\lambda+\mu-\lambda\mu)^2}{8(1-\delta)(\lambda^2(1-\mu)+\mu)}$. Following this, $CS^P < CS^M \rightarrow 0 < (1-\lambda)^2\left(\mu+\lambda^2(1-\mu)\right)(1-\mu)\mu$, which always holds true if $0 < \mu < 1$. ■

Proof of Proposition 2

In this proposition, we identify the buying and renting behaviors of consumers under different market conditions $\{\lambda, \phi, \mu\}$, as visualized in the main paper. These are described as four possible market outcomes $\{C_1, C_2, C_3, C_4\}$. These outcomes occur because of the piecewise buying and renting nature of low and high-usage consumers: because $v_L = \min\{1, \frac{v_H}{\phi}\}$ and cases when $v_H < \frac{p_r}{\mu}$. The firm recognizes this and optimally prices to maximize profits. In this proof, we consider ϕ going from $1 \rightarrow 0$ and prove that there exists one threshold in the parameters $\{\lambda, \phi\}$ where low-type consumers stop purchasing and two thresholds bracketing where high-type consumers do not rent. First, note that as ϕ approaches 1 (i.e., $\phi \rightarrow 1$), the market outcome degenerates to C_1, where both types buy and both types rent because low and high-type consumers become homogeneous. Similarly, as $\phi \rightarrow 0$, the market degenerates to condition C_2 because the low-type consumers become nonexistent and thus have no WTP to purchase the good. This ensures the existence of at least one threshold of ϕ where low-usage consumers do not purchase.

We prove there must exist only one such threshold. C_1 and C_3 are market outcomes where low-usage consumers purchase; in those outcomes, low-usage purchases monotonically decrease as ϕ decreases:[15]

$$\frac{\partial \hat{v}_{L,1}}{\partial \phi} = -\frac{2\lambda^4(\mu-1)^2\phi^2 + \lambda^3(\mu-1)\mu(\phi+1)^2 - 2\lambda^2(\mu-1)\mu(4\phi+1) + 4\mu^2}{2(\phi+1)^2\left(\mu - \lambda^2(\mu-1)\phi\right)^2} < 0.$$

$$\frac{\partial \hat{v}_{L,3}}{\partial \phi} = \frac{1}{2}\left(B1 - \frac{\lambda\mu}{\lambda^2(\mu-1)\phi^2 + (\lambda-1)^2(\mu-1)\mu\phi - \mu} - \frac{2(\mu-1)}{(2\mu-1)(\phi+1)^2} + \frac{\mu(\mu-1)}{(2\mu-1)(\mu(-\phi)+\mu+\phi)^2}\right) < 0$$

Thus, for given $\{\lambda, \mu\}$, there exists some $\hat{\phi}$ such that for $\phi < \hat{\phi}$, the firm opts to not sell to low-usage consumers, so the resulting market outcome is C_2 or C_4. This threshold is unique because after the threshold, in market outcomes C_2 or C_4, the rental price is monotonically decreasing as ϕ decreases: $\frac{\partial p_{r,2}}{\partial \phi} = \frac{\lambda^2(1-\mu)\mu(\lambda(1-\mu)+\mu)}{2(\mu-\lambda^2(\mu-1)(\phi+1))^2} > 0$ and $\frac{\partial p_{r,4}}{\partial \phi} = \frac{(\lambda-1)\lambda^2\mu(\lambda(\mu-1)-\mu)}{2(\lambda^2\phi+(\lambda-1)^2\mu)^2} > 0$. Thus for $\phi < \hat{\phi}$, the rental price decreases as ϕ decreases, so the market rental price is always lower than $p_r|_{\phi=\hat{\phi}}$, further reducing incentive for low-usage consumers to purchase. Low-usage consumers thus forego purchases for $\phi \leq \hat{\phi}$.

At sufficiently high values of λ, high-usage consumers do not rent in the market (outcomes C_3, C_4). Again consider ϕ decreasing from $1 \to 0$. This starts with outcome C_1, and in this outcome, there exists a value of $\tilde{\phi}_1$ where, in equilibrium, $\hat{v}_{H,1} = p_{r,1}/\mu$. This value is $\tilde{\phi}_1 = \frac{\lambda(2-3\mu)+3\mu-2\lambda^2(1-\mu)-\sqrt{(\lambda(2\lambda(\mu-1)-3\mu+2)+3\mu)^2-4\lambda\mu^2}}{2\lambda\mu}$. It is easy to show numerically that if λ is sufficiently large, $\tilde{\phi}_1 > \hat{\phi}$, in other words, this rentals threshold is reached first. At that threshold for sufficiently large λ, the market outcome first transitions from C_1 to C_3 as ϕ decreases. Then, as ϕ further decreases for $\phi < \tilde{\phi}_1$, the market first transitions to outcome C_4 (as it is suboptimal to transition to C_1 for $\phi < \tilde{\phi}_1$). Lastly, as ϕ decreases even further and $\phi \to 0$, the market transitions to outcome C_2 as we discussed earlier. This proves the existence of $\tilde{\phi}_2$, such that there exists a region $\tilde{\phi}_2 < \phi < \tilde{\phi}_1$ where high-usage consumers do not rent, if λ is sufficiently large. . If λ is not sufficiently large, then the market simply transitions from C_1 to C_2 as ϕ decreases at $\hat{\phi}$, and due to price monotonicity as shown in the paragraph above, stays in C_2. Lastly, the $\hat{\phi}$ threshold of low-usage consumers purchasing is decreasing in λ, since it is easy to show that $\frac{\partial \hat{v}_{L,1}}{\partial \lambda}$ and $\frac{\partial \hat{v}_{L,3}}{\partial \lambda}$ are both negative for $\{0 < \lambda < 1, 0 < \phi < 1, \frac{3}{4} < \mu < 1\}$. ■

Proof of Proposition 3

The monopoly firm's profits are $\frac{\lambda\phi}{2(1-\delta)(1+\phi)}$ when selling to both high and low-usage consumers; the corresponding consumer welfare is $\frac{\lambda(1+\phi^2-\phi)}{4(1-\delta)(1+\phi)}$. The monopoly firm's profits as a fraction of the total market welfare is defined $pcnt^M = \frac{2\phi}{\phi^2+\phi+1}$. At $\phi = 1$, this is equal to 2/3. For the P2P-facing firm, under condition C_1, when both types buy and rent, its profits are $\frac{\lambda(\mu((2\lambda-1)\phi-1)-2\lambda\phi)^2}{8(1-\delta)(\phi+1)(\lambda^2\phi+\mu(1-\lambda^2\phi))}$ the corresponding consumer welfare is

$$\begin{aligned} CS^{P,1} = {} & \lambda\left(4\lambda^2\phi((\phi-1)\phi+1) + \mu^2\left(\phi\left(4\lambda\left(\lambda\phi^2-(\lambda+1)\phi+\lambda-1\right)-3\phi+10\right)-3\right)\right. \\ & \left. +4\mu(1+\phi(1-\lambda(2\lambda((1-\phi)\phi)+\phi+1)+\phi-2))\right)/ \\ & 16(\delta-1)(\phi+1)\left(\lambda^2(\mu-1)\phi-\mu\right). \end{aligned}$$

Thus, the fraction of total market welfare captured by the firm is: $pcnt^{P,1} = \frac{2(\mu(-2\lambda\phi+\phi+1)+2\lambda\phi)^2}{4\mu(\phi(-2\lambda^2(\phi^2+\phi+1)+3\lambda(\phi+1)+\phi-2)+1)+4\lambda^2\phi(\phi^2+\phi+1)+\mu^2(\phi(4\lambda(\lambda(\phi^2+\phi+1)-3(\phi+1))-\phi+14)-1)}$. At $\phi = 1$,

[15] where $B1 = \frac{\mu(\lambda(\mu-1)\phi((\lambda-2)\lambda(\mu-2)+\mu)+\mu(\lambda(-(\lambda-4)\lambda(\mu-1)-5\mu+3)+2(\mu-1)))}{\left(\lambda^2(\mu-1)\phi^2+(\lambda-1)^2(\mu-1)\mu\phi-\mu\right)^2}$

this is also equal to exactly $2/3$. Then, for all $0 < \phi < 1$, $pcnt^{P,1} > pcnt^{M} \quad \forall\{0 < \phi < 1, 0 < \lambda \leq 1, \frac{1}{2} \leq \mu < 1\}$. In market conditions where low-usage consumers do not purchase (C_2 and C_4), using the profit and consumer surplus equations shown in this appendix and in the proof of Proposition 5, it is clear that the P2P-facing firm is always capturing exactly $2/3$ percent of total welfare. Comparing this to $pcnt^M$, it's clear that $pcnt^M < \frac{2}{3} \quad \forall\{0 < \phi < 1, 0 < \lambda \leq 1, \frac{1}{2} \leq \mu < 1\}$.

Lastly, for condition C_3:

$$\begin{aligned}
pcnt^{P,3} = -2 &\left((1-\mu-)\phi^2(\lambda(\mu-2)-\mu)+\mu\phi(\lambda(-\mu)+\lambda+\mu)+\mu\right)^2 \div \\
&\Big((1-\mu)^2\phi^4\left(-4\lambda^2+(\lambda-1)^2\mu^2-4(\lambda+1)\mu\right)+4\lambda^2(\mu-1)^3\phi^5+ \\
&\ \mu\phi^2\left(\lambda^2(\mu-4)(\mu-1)^2-2\lambda\left(\mu((\mu-6)\mu+3)+2\right)+(\mu-2)(\mu(\mu+4)-2)\right)- \\
&\ 2(\mu-1)\phi^3\left((\lambda(\mu-1)-\mu)\left((\lambda-1)\mu^2+2\lambda+6\mu\right)+2\mu\right)+2\mu\phi\left(\mu(3\lambda(\mu-1)-5\mu+4)-2\right)-3\mu^2\Big)
\end{aligned}$$

Although this is complex, it can be analytically shown that $pcnt^{P,3} > pcnt^{M} \quad \forall\{0 < \phi < 1, 0 < \lambda \leq 1, \frac{1}{2} \leq \mu < 1\}$. Therefore, for all four possible purchasing outcomes, the percentage of total welfare captured by the P2P-facing firm is larger than the percentage captured by the monopoly firm that sells to both high and low-usage consumers. ■

Proof of Proposition 4

To prove Proposition 4, we show that the OEM is more profitable in the presence of a P2P rental market at $\phi = \frac{1}{3}$, but less profitable at $\phi = 1$ and $\phi = 0$. Then, using the monotonicity of the OEM profit functions in the presence/absence of P2P, we use the intermediate value theorem to show the there exists $\{\phi_1^\pi, \phi_2^\pi\}$ such that the OEM is more profitable for $\phi : 0 \leq \phi_1^\pi < \phi < \phi_2^\pi < 1$. First we show the existence and uniqueness of ϕ_2^π in the region of $\phi \in [\frac{1}{3}, 1]$. From Lemma 2, we know that the monopoly firm sells to both high and low-usage consumers when $\phi \in [\frac{1}{3}, 1]$, with a profit of $\frac{\lambda\phi}{2(1-\delta)(1+\phi)}$. This monopoly firm's profit is monotonically increasing in this region of ϕ. The P2P-facing firm has smaller profits than the monopoly firm at $\phi = 1$ (proven in Proposition 1). At $\phi = \frac{1}{3}$, the P2P-facing firm has larger profits than the monopoly firm as shown below for each of the four possible market outcomes:

$$\begin{aligned}
\pi^{P,1} - \pi^{M}|_{\phi=1/3} &= \frac{\lambda\mu((\lambda-4)\lambda(\mu-1)+4\mu-3)}{8(\delta-1)\left(\lambda^2(\mu-1)-3\mu\right)}, \\
\pi^{P,2} - \pi^{M}|_{\phi=1/3} &= \frac{\lambda\mu(4(\lambda-2)\lambda(\mu-1)+4\mu-3)}{8(\delta-1)\left(4\lambda^2(\mu-1)-3\mu\right)}, \\
\pi^{P,3} - \pi^{M}|_{\phi=1/3} &= \frac{\lambda\mu\left(\lambda^2(\mu-1)((\mu+7)\mu+4)-2\lambda(\mu-1)((\mu+12)\mu+8)+((\mu+16)\mu+4)\mu-12\right)}{8(\delta-1)(2\mu+1)\left(\lambda^2(\mu-1)(3\mu+1)-6\lambda(\mu-1)\mu+3(\mu-4)\mu\right)}, \\
\pi^{P,4} - \pi^{M}|_{\phi=1/3} &= \frac{(\lambda-1)\lambda\mu(\lambda(\mu-5)-\mu+3)}{8(\delta-1)\left(\lambda^2+3(\lambda-1)^2\mu\right)},
\end{aligned}$$

each of which is analytically positive for any $\{0 < \lambda < 1, \frac{3}{4} \leq \mu < 1\}$[16]. Thus, the upper envelope of the possible profits at $\phi = \frac{1}{3}$, regardless of market outcome, is greater than the monopoly firm's profit.

[16] For Condition 4, it is easy to prove that it always has lower profits than Condition 2 when $\lambda < \frac{5}{9}$, and Condition 2 is always feasible for the OEM. Thus, for Condition 4, the profit difference only needs to be proved positive for $\frac{5}{9} < \lambda < 1$.

To show that, in $\phi \in [\frac{1}{3}, 1]$, there exists one and only one crossover point of the profits of these two scenarios, ϕ_2^π, we show that the P2P-facing and monopoly firm's profits are monotonically increasing in ϕ. The derivatives of these profit functions with respect to ϕ are

$$
\begin{aligned}
\frac{\partial \pi^{P,1}}{\partial \phi} &= \frac{\lambda\left(2\lambda^3(1-\mu)^2\phi + 2\lambda\mu(1-\mu)(\phi+2) + \mu(\phi+1)(\mu - \lambda^2(1-\mu))\right)(\mu((2\lambda-1)\phi-1) - 2\lambda\phi)}{8(1-\delta)(\phi+1)^2\left(\lambda^2\phi - \mu^2\lambda^2\phi\right)^2}, \\
\frac{\partial \pi^{P,2}}{\partial \phi} &= \frac{\lambda\mu(\lambda+\mu-\lambda\mu)^2}{8(1-\delta)\left(\mu - \lambda^2(\mu-1)(\phi+1)\right)^2}, \\
\frac{\partial \pi^{P,3}}{\partial \phi} &= \frac{\lambda\left((1-\lambda)\mu^2(\phi-1)\phi + \mu\left(\lambda\left(\phi - 3\phi^2\right) + \phi^2 + 1\right) + 2\lambda\phi^2\right)(L_1)}{8(1-\delta)(\phi+1)^2(\mu+\phi-\mu\phi)^2\left(\mu\left(\lambda^2\phi^2 + (\lambda-1)^2(-\phi) - 1\right) - \lambda^2\phi^2 + (\lambda-1)^2\mu^2\phi\right)^2}, \\
&\quad \text{where } L_1 = (\mu-1)^2\phi^4\left(2\lambda^3 + (\lambda-1)^3\mu^3 - (\lambda-1)((\lambda-4)\lambda+1)\mu^2 - (\lambda(\lambda+3)-2)\lambda\mu\right) \\
&\quad + \mu^2\phi\left(\left((5-2\lambda)\lambda^2 - 3\right)\mu^2 + (\lambda-1)^3\mu^3 + \lambda(\lambda(\lambda+2)-16)\mu + \lambda(13-4\lambda) + 9\mu - 4\right) \\
&\quad + (\mu-1)\mu\phi^3\left(\lambda^3(\mu-1)((\mu-5)\mu+7) - \lambda^2(3\mu-4)((\mu-3)\mu+3) + \lambda(\mu-1)(3(\mu-3)\mu+8) - (\mu-4)(\mu-1)\mu\right) \\
&\quad - -3(\mu-1)\mu\phi^2(\lambda(\mu-1) - \mu + 2)\left((\lambda-1)^2\mu^2 - 2(\lambda-1)\lambda\mu + \lambda\right) + \mu^2(\mu((\lambda-4)\lambda(\mu-1) + 3\mu - 1) - 1), \\
\frac{\partial \pi^{P,4}}{\partial \phi} &= \frac{(1-\lambda)^2\lambda\mu(\lambda+\mu-\lambda\mu)^2}{8(1-\delta)\left(\lambda^2\phi + (1-\lambda)^2\mu\right)^2}.
\end{aligned}
$$

The monopoly firm's profit is also increasing in ϕ: $\frac{\partial \pi^M}{\partial \phi}|_{\phi \in [\frac{1}{3},1]} = \frac{\lambda}{2(1-\delta)(\phi+1)^2}$. These five derivatives are positive for $\{0 < \lambda < 1, \frac{3}{4} \leq \mu < 1\}$, which is intuitive because ϕ increases total demand. Thus, the actual firm profit, determined by the upper envelope of these four profit functions when they are valid, $\hat{\pi}^P = \max\{\pi^{P,1}, \pi^{P,2}, \pi^{P,3}, \pi^{P,4}\}$, will also be monotonically increasing in ϕ. These two pieces prove the existence of the threshold ϕ_2^π by the intermediate value theorem.

Second we show the existence and uniqueness of ϕ_1^π in the region of $\phi \in [0, \frac{1}{3})$. From Lemma 2, we know that the monopoly firm sells to only high-usage consumers when $\phi \in [0, \frac{1}{3})$, with a profit of $\frac{\lambda}{8(1-\delta)}$;the monopoly firm's profit is constant in this region of ϕ. It is already proved that the P2P-facing firm's profit is larger than the monopoly firm's profit at $\phi = \frac{1}{3}$. At $\phi = 0$, the P2P-facing firm's market outcome is C_2 (as shown in Proposition 2) where low-usage consumers do not purchase. Thus the profit difference at $\phi = 0$ is $\pi^{P,2} - \pi^M|_{\phi=0} = \frac{-(\lambda-1)^2\lambda(1-\mu)\mu}{8(1-\delta)(\lambda^2(1-\mu)+\mu)}$, and is negative. Again, by the intermediate value theorem and the monotonicity of the profit functions in ϕ, as shown above for $0 < \lambda < 1$ and $\frac{3}{4} \leq \mu < 1$, there must exist one unique crossover point in the profit functions of the P2P-facing firm and the monopoly firm between $\phi = \frac{1}{3}$ and $\phi = 0$. This crossover point is ϕ_1^π. Furthermore, we can show that $\partial\phi_2^\pi/\partial\lambda \geq 0$ and $\partial\phi_1^\pi/\partial\lambda < 0$ using implicit function theorem, indicating that the region where the OEM profits are higher in the presence of P2P rentals increases in λ. ■

Proof of Proposition 5

First, we prove that consumer welfare is higher in the presence of a P2P rental market in a region of $\phi \in [\phi_0, \frac{1}{3}]$, and otherwise lower. In the absence of a P2P rental market, consumer welfare under the monopoly firm is the welfare obtained by purchasers:

$$
CS^M = \begin{cases} \frac{\lambda(1-(1-\phi)\phi)}{2(1-\delta)(1+\phi)} & \phi \in [\frac{1}{3}, 1] \\ \frac{\lambda}{16(1-\delta)} & \phi \in [0, \frac{1}{3}] \end{cases}
$$

Welfare is piecewise because the monopoly firm sells only to high-usage consumers for $\phi \in [0, \frac{1}{3}]$. Consumer welfare in the presence of a P2P rental market is the welfare obtained by both purchasers and renters:

$$
\begin{aligned}
CS^{P,1} &= \lambda\left(4\lambda^2\phi((\phi-1)\phi+1)+\mu^2\left(\phi\left(4\lambda\left(\lambda\phi^2-(\lambda+1)\phi+\lambda-1\right)-3\phi+10\right)-3\right)\right.\\
&\quad \left.+4\mu(1+\phi(1-\lambda(2\lambda((1-\phi)\phi)+\phi+1)+\phi-2))\right)/16(\delta-1)(\phi+1)\left(\lambda^2(\mu-1)\phi-\mu\right),\\
CS^{P,2} &= \frac{\lambda(\phi+1)(\lambda+\mu-\lambda\mu)^2}{16(1-\delta)\left(\lambda^2(1-\mu)(\phi+1)+\mu\right)},\\
CS^{P,3} &= \frac{\lambda\left(2\mu^3(\phi-1)\phi\left(\lambda^2\phi\left(2\phi^2-5\phi+5\right)+\lambda\left(8\phi^2-11\phi-1\right)-5\phi^2+4\phi+3\right)-L_2\right)}{16(\delta-1)(\phi+1)(\mu(\phi-1)-\phi)\left(\mu\left(\lambda^2\phi^2+(\lambda-1)^2(-\phi)-1\right)-\lambda^2\phi^2+(\lambda-1)^2\mu^2\phi\right)},\\
&\quad \text{where } L_2=\mu^2\left(12\lambda^2\phi^5-\left(23\lambda^2-14\lambda+11\right)\phi^4+2\left(13\lambda^2-21\lambda+8\right)\phi^3-\left(11\lambda^2-18\lambda-6\right)\phi^2+2(\lambda-4)\phi+1\right)\\
&\quad +4\mu\phi\left(3\lambda^2\phi^4-\left(4\lambda^2-\lambda+1\right)\phi^3-\left(\lambda^2-\lambda-1\right)\phi+(1-2\lambda)^2\phi^2-1\right)-4\lambda^2\phi^3\left(\phi^2-\phi+1\right)+3(\lambda-1)^2\mu^4(\phi-1)^2\phi^2,\\
CS^{P,4} &= \frac{\lambda\phi(\lambda+\mu-\lambda\mu)^2}{16(1-\delta)\left(\lambda^2\phi+(1-\lambda)^2\mu\right)}.
\end{aligned}
$$

These values are all smaller than CS^M for any $\{0<\lambda<1, \frac{3}{4}\le\mu<1\}$ in $\phi\in[\frac{1}{3},1]$. For the region of $\phi\in[\frac{1}{3},1]$, for any $0<\lambda<1$ and $\frac{3}{4}\le\mu<1$: if market outcomes C_1 or C_3 occur, then consumer welfare is higher under the P2P-facing firm, $\left(CS^{P,1}>CS^M, CS^{P,3}>CS^M\right)$;and if outcomes C_2 or C_4 occur, then consumer welfare is higher if , which occurs when $\frac{(1-\mu)(1-\lambda)^2}{\lambda^2\mu-\lambda^2-2\lambda\mu+2\lambda+\mu}<\phi<\frac{1}{3}$ or $\frac{1-\lambda}{\mu(1-\lambda)+2\lambda}<\phi<\frac{1}{3}$ (respectively). As we showed in Proposition 2, the market outcomes will always be C_2 or C_4, for sufficiently low ϕ when low-usage consumers do not purchase. Thus, for any $\{0<\lambda<1, \frac{3}{4}\le\mu<1\}$, there exists a threshold ϕ_0, where consumer welfare with P2P is larger when $\phi_0<\phi<\frac{1}{3}$, and where consumer welfare with P2P when $\phi<\phi_0$. The value of this threshold is either the thresholds in outcomes C_2 and C_4 identified above, or when the market outcome becomes C_2 or C_4, whichever is smaller. Lastly, this ϕ_0 threshold is decreasing in λ. Proposition 2 shows that the threshold where the market outcome becomes C_2 or C_4 (low-usage consumers no longer purchase in presence of P2P) is decreasing in λ, and it is easy to show to the thresholds in outcomes C_2 and C_4 identified above have negative derivatives with respect to λ.

Second, we prove the second part of Proposition 5, that the difference of welfare between high and low-usage consumers shrinks with a P2P market. Monopoly welfare of high-usage is: $\frac{\lambda}{4(1-\delta)(\phi+1)^2}$. Monopoly welfare of low-usage is: $\frac{\lambda\phi^3}{4(1-\delta)(\phi+1)^2}$. The difference of these is: $\text{welDiff}^M=\frac{\lambda\left(1-\phi^3\right)}{4(1-\delta)(1+\phi)^2}$.

In condition C_1 the difference in welfare is[17]

$$
\text{welDiff}^{P2P,1} = \frac{\lambda\left(-4\lambda^4(\mu-1)^3\phi^5+\mu\phi^3\left(1^A\right)+\lambda^2(\mu-1)\mu\phi^4(4\lambda(\mu-1)+9\mu-8)+\mu\phi\left(1^B\right)+\phi^2\left(1^C\right)+\mu^2(3\mu-4)\right)}{16(\delta-1)(\phi+1)^2\left(\mu-\lambda^2(\mu-1)\phi\right)^2}
$$

$\text{welDiff}^{P2P,3}$ for condition C_3 is too cumbersome to print here, but it can be shown that both $\text{welDiff}^{P2P,1}<\text{welDiff}^M$ and $\text{welDiff}^{P2P,3}<\text{welDiff}^M$ for all $\{0<\lambda<1, 0<\phi<1, 0<\mu<1, 0<\delta<1\}$. For conditions C_2 and C_4 it is analytically cleaner to demonstrate that the ratio of high-usage and low-usage welfare is smaller with a P2P market under some (nonrestrictive) conditions. This is equivalent to showing that the difference is smaller. For conditions C_2 and C_4, the ratio of welfare is always shrinking with a P2P market. The ratios are $\text{welRatio}^{P2P,2}=\frac{\mu-\lambda^2(\mu-1)(\phi+1)^2}{\mu\phi}$ and $\text{welRatio}^{P2P,4}=\frac{\lambda^2\phi}{(\lambda-1)^2\mu}$. Then, $\text{welRatio}^{P2P,2}<\text{welRatio}^M\ \ \forall\{\mu>\frac{\lambda^2\phi^3+\lambda^2\phi^2}{\lambda^2\phi^3+\lambda^2\phi^2-\phi+1}, 0<\phi<1\}$ and $\text{welRatio}^{P2P,4}<\text{welRatio}^M\ \ \forall\{0<\lambda<\frac{1}{\phi^2+1}, \mu>\frac{\lambda^2\phi^4}{\lambda^2-2\lambda+1}\}$. These conditions on μ and λ are not actually restrictive, as these conditions are numerically shown to always be valid whenever C_2 and C_4 are the preferred outcomes $\forall\mu\ge 1/2$.■

[17]where $1^A=-4\lambda^2-3(\lambda(\lambda+4)+1)\mu^2+(\lambda(7\lambda+12)+4)\mu$, $1^B=\lambda^2(-(\mu-1))(9\mu-8)+12\lambda(\mu-1)\mu+\mu(12-13\mu)$, and $1^C=4\lambda^4(\mu-1)^3-4\lambda^3(\mu-1)^2\mu+\lambda^2(\mu-1)\mu(3\mu-4)+\mu^2(13\mu-12)$

Optimal Platform Fees by the P2P-Sponsoring Firm

Under market outcomes C_2 and C_4 when there are no low-usage purchases, optimal firm profits are not affected by α (earnings from α are exactly canceled out by a decrease in the durable good price); hence the profit functions with revenue from α are equal to those shown earlier in this appendix. Otherwise, the firm profits, under C_1, with revenue from α fees, are:

$$\pi^{P,1} = -\frac{\lambda(\mu-1)\left(\mu^2\left(L_3\right)+4\lambda\mu\phi(\phi+1)\left(L_4\right)-4\lambda^2\phi^2(\phi+1)^2\right)}{8(\delta-1)(\phi+1)\left(\mu^2\left(\alpha^2(\phi-1)^2+(\phi+1)^2\left(\lambda^2\phi-1\right)\right)-\mu(\phi+1)^2\left(2\lambda^2\phi-1\right)+\lambda^2\phi(\phi+1)^2\right)},$$
$$\text{where } L_3 = \alpha^2(\phi-1)^2\left(4\lambda^2\phi^3+\left(8\lambda^2-8\lambda-1\right)\phi^2+\left(4\lambda^2-8\lambda+2\right)\phi-1\right)$$
$$-2\alpha(\phi+1)(\phi-1)^2\left(2\lambda^2\phi^2+\left(2\lambda^2-2\lambda-1\right)\phi-1\right)-(\phi+1)^2(-2\lambda\phi+\phi+1)^2,$$
$$\text{and } L_4 = \alpha(\phi-1)^2(\lambda\phi+\lambda-1)+(\phi+1)((2\lambda-1)\phi-1).$$

We find that $\frac{d\pi^{P,1}}{d\alpha} \leq 0$ for any $0 < \lambda < 1$ and $1/2 \leq \mu \leq 1$; hence the optimal value of α is 0. Similarly, the firm profit under C_3 is:

$$\pi^{P,3} = \frac{\lambda\left(\mu^2\left(L_5\right)+L_6\right)}{8(\delta-1)\left(L_7\right)}$$
$$\text{where } L_5 = -\alpha^2(\phi-1)^2\left(4\lambda^2\phi^3+\left(3\lambda^2-6\lambda-1\right)\phi^2-2(\lambda-1)\phi-1\right)$$
$$+2\alpha(\phi-1)\left(4\lambda^2\phi^4-\left(5\lambda^2+2\lambda+1\right)\phi^3+\left(-3\lambda^2+4\lambda+1\right)\phi^2+(2\lambda-1)\phi+1\right)$$
$$+\lambda^2\phi^2\left(13\phi^2-10\phi+1\right)+2\lambda\phi\left(-5\phi^3+3\phi^2-3\phi+1\right)+\left(\phi^2+1\right)^2,$$
$$L_6 = 2(\alpha-1)\mu^3(\phi-1)\phi\left(\alpha(\phi-1)\left(\lambda^2\phi(2\phi+1)-\lambda(2\phi+1)-\phi+1\right)+(\lambda-1)\left(\lambda(3\phi-1)\phi-\phi^2-1\right)\right)$$
$$+(\alpha-1)^2(\lambda-1)^2\mu^4(\phi-1)^2\phi^2+4\lambda\mu\phi^2\left(\alpha\left((\phi-1)^2-\lambda\left(\phi^3-2\phi^2+1\right)\right)+\lambda\left(\phi-3\phi^2\right)+\phi^2+1\right)+4\lambda^2\phi^4,$$
$$L_7 = -\mu^2\left(L_8\right)+\mu\phi\left(\phi^2\left(-2\alpha\lambda^2+2\lambda-1\right)+2\phi\left((\alpha-1)\lambda^2+\lambda-1\right)+2\lambda^2\phi^3-1\right)$$
$$+(\alpha-1)(\lambda-1)^2\mu^3(\phi-1)\phi(\alpha(\phi-1)+\phi+1)-\lambda^2\phi^3(\phi+1),$$
$$\text{and}$$
$$L_8 = \alpha^2(\phi-1)^2\left((\lambda-1)^2\phi+1\right)-2\alpha(\phi-1)\left(\lambda^2\phi^2+(\lambda-1)^2(-\phi)-1\right)+$$
$$(\phi+1)\left(\lambda^2\phi\left(\phi^2-3\phi+1\right)+2\lambda\phi(2\phi-1)-2\phi^2+1\right).$$

Similarly, $\frac{d\pi^{P,3}}{d\alpha} \leq 0$ for any $0 < \lambda < 1$ and $1/2 \leq \mu \leq 1$, hence the optimal value of α is 0. These same proofs also hold when $c > 0$, except that the equations are lengthier. ■

Proof of Lemma 3

The dual firm has different options, depending on how it prices, as well which types of consumers it sells and rents to. We show that among those options, the option to rent to no one and only sell is most profitable for $\phi \geq \tilde{\phi}^D$, and for $\phi < \tilde{\phi}^D$, it is most profitable to sell to high-usage consumers and rent to low-usage consumers.

To clarify the dual firm's possible options: If the firm chooses to sell and rent to both groups, its profits are $\pi^{D,1}$; if the firm chooses to sell to just to high-usage consumers and rents to both groups of consumers, its profits are $\pi^{D,2}$; if the firm chooses to sell to both groups and rent only to low-usage consumers, its profits are $\pi^{D,3}$; and if the firm chooses to sell to just to high-usage consumers and rent only to low-usage consumers, its profits are $\pi^{D,4}$. If consumers are distributed such that the firm is unable to price to achieve one of those outcomes, then the profit is 0.

$$\begin{cases} \pi^{D,1} = \frac{100\lambda^2 p_R{}^2 \phi(\phi+1)+20\lambda\nu p_s\phi(\nu-2p_R-1)+\nu p_s{}^2(\phi+1)}{20\lambda(\nu-1)\nu\phi} & (p_s - 10\lambda\frac{p_R}{\nu}) > 0, \\ & (p_s - 10\lambda\phi(1+p_R-\nu)) < 0 \\ \pi^{D,2} = \frac{10\lambda\left(10\lambda p_R{}^2(-\nu\phi+\phi+1)-2\nu p_R(p_s-5\lambda(\nu-1)\phi)+(\nu-1)\nu p_s\right)+\nu p_s{}^2}{20\lambda(\nu-1)\nu} & (p_s - 10\lambda\frac{p_R}{\nu}) > 0, \\ & 10\lambda\phi(-\nu+p_R+1) < p_s \\ \pi^{D,3} = \frac{100\lambda^2 p_R{}^2\phi^2-20\lambda\nu p_s\phi(-\nu+p_R+1)+\nu p_s{}^2(-\nu\phi+\phi+1)}{20\lambda(\nu-1)\nu\phi} & 10\lambda^2\nu p_R > \lambda\nu^2 p_s, \\ & (p_s - 10\lambda\phi(1+p_R-\nu)) < 0 \\ \pi^{D,4} = \frac{5\lambda p_R\phi(\nu-p_R)}{\nu} - \frac{p_s{}^2}{20\lambda} + \frac{p_s}{2} & 10\lambda^2\nu p_R > \lambda\nu^2 p_s, \\ & 10\lambda\phi(-\nu+p_R+1) < p_s \end{cases}$$

Lastly, if the dual firm only offers sales to both consumers, its profits with the optimal sales price will be the same as the monopoly firm, $\pi^M = \frac{\lambda\phi}{2(1-\delta)(1+\phi)}$. For the above equations, we can identify the optimal profits $\{\pi^{D,1*}, \ldots, \pi^{D,4*}\}$ (ignoring the conditions) by analytically maximizing each profit equation with respect to $\{p_s, p_R\}$, using multivariate optimization (by taking the first-order condition with respect to each price, solving for the zero-point of both, and ensuring the Hessian is negative definite).

First, we consider $\pi^{D,4}$. This segmenting outcome is incentive compatible only if no high-usage consumer who does not buy prefers to rent and no low-usage consumer prefers to buy, i.e. $\frac{p_R}{\mu} < v_H$ & $U^{R,L} > U^{B,L}|_{v_i=1}$. This occurs when consumers are sufficiently heterogeneous: $\phi < \frac{p_s}{10\lambda(1+p_R-\nu)}$. When the dual firm maximizes $\pi^{D,4}$ using prices $p_s = \frac{\lambda}{2(1-\delta)}$ and $p_R = \mu/2$, this value is $\phi < \frac{1}{2-\nu}$. Thus, if consumers are heterogeneous with $\phi < \frac{1}{2-\nu}$, then the dual firm can segment consumers into buyers/renters and achieve the maximal value of $\pi^{D,4}$: which is $\pi^{D,4*} = \frac{\lambda\nu\phi+\lambda}{8-8\delta}$. Then, we solve $\pi^{D,4*} > \pi^M \Rightarrow \frac{\lambda\nu\phi+\lambda}{8-8\delta} > \frac{\lambda\phi}{2(1-\delta)(1+\phi)}$, which holds true when $\nu < \frac{3\phi-1}{\phi^2+\phi}$, or $\phi < \tilde{\phi}^D = \frac{1}{2\nu}\left\{3 - \left(\sqrt{(9-\nu)(1-\nu)} + \nu\right)\right\}$. It is clear that $\tilde{\phi}^D < \frac{1}{2-\nu}$, so $\tilde{\phi}^D$ is the binding threshold of ϕ when comparing $\pi^{D,4*}$ and π^M. In other words: $\phi < \tilde{\phi}^D < \frac{1}{2-\nu}$.

We now know that for $\phi < \tilde{\phi}^D$, profits of $\pi^{D,4*}$ are achievable and $\pi^{D,4*} > \pi^M$. We next consider how $\pi^{D,4*}$ compares the dual firm's optimal values to its other potential pricing options: $\pi^{D,4*} > \pi^{D,1*}|_{\phi\in[0,\tilde{\phi}^D)}$, $\pi^{D,4*} > \pi^{D,2*}|_{\phi\in[0,\tilde{\phi}^D)}$, and $\pi^{D,4*} > \pi^{D,3*}|_{\phi\in[0,\tilde{\phi}^D)}$. These inequalities always hold true in the region of $\phi \in [0, \tilde{\phi}^D)$ (whether the optimal profits of other options are achievable or not). Similarly, for $\phi > \tilde{\phi}^D$, we consider how π^M compares with the dual firm's other potential pricing options: $\pi^M > \pi^{D,1*}|_{\phi\in[\tilde{\phi}^D,1]}$, $\pi^M > \pi^{D,2*}|_{\phi\in[\tilde{\phi}^D,1]}$, and $\pi^M > \pi^{D,3*}|_{\phi\in[\tilde{\phi}^D,1]}$. These inequalities always hold true in the region of $\phi \in [\tilde{\phi}^D, 1]$ (whether the optimal profits of other options are achievable or not). ■

Proof of Proposition 6

We show that there exists a λ_{min}, where with $\lambda < \lambda_{min}$, the P2P-sponsoring strategy is never preferred. Then, for $\lambda > \lambda_{min}$ and when heterogeneity is sufficiently large such that only the P2P-sponsoring and dual strategies are considered ($\phi \le \tilde{\phi}^D$, as proven in Lemma 3), we show that the P2P-sponsoring strategy is first preferred as ϕ decreases and when the equalizing effect dominates, whereas the dual strategy is preferred when the direct segmentation effect dominates. We first prove the latter part, considering the region $\lambda > \lambda_{min}$ and $\phi \le \tilde{\phi}^D$, and then find the value of λ_{min}. (We refer to the proof of

Proposition 4 for $\phi > \tilde{\phi}^D$, when the OEM chooses between the sales-only monopoly and P2P-sponsoring firm.)

If the P2P-sponsoring strategy would result in a market outcome where low types do not buy (C_2 or C_4), then the equalizing effect is dominated by the direct segmentation effect, because the equalizing effect is not resulting to additional purchases. Using the optimal profits derived earlier (e.g., $\pi^{D,4}$, defined in the proof of Lemma 3), the profits for these outcomes are weakly smaller than those of a dual firm in this region:

$$\pi^{P,2} - \pi^{D,4} = -\frac{\lambda(1-\mu)\mu(\lambda\phi+\lambda-1)^2}{8(1-\delta)\left(\lambda^2(1-\mu)(1+\phi)+\mu\right)} \leq 0,$$
$$\pi^{P,4} - \pi^{D,4} = -\frac{\lambda\mu(\lambda\phi+\lambda-1)^2}{8(1-\delta)\left(\lambda^2\phi+(\lambda-1)^2\mu\right)} \leq 0.$$

Conversely, when the equalizing effect dominates the direct segmentation effect, which occurs in outcomes C_1 or C_3 when it results in purchases from low-usage consumers, then the P2P-sponsoring strategy is preferred. The equalizing effect dominates if $\lambda > \tilde{\lambda}_1(\phi,\mu)$ or $\lambda > \tilde{\lambda}_3(\phi,\mu)$, depending on the market outcome. (Note that $\nu = \mu$ in this proof, with μ representing the rental frictions for both dual and P2P-sponsoring firms.) The difference in profits between the P2P-sponsoring strategy and the dual strategy is

$$\pi^{P,3}-\pi^{D,4} = \frac{1}{8(1-\delta)}\left\{\frac{\lambda\left((\lambda-1)\mu^2(\phi-1)\phi+\mu\left(\lambda\left(\phi-3\phi^2\right)+\phi^2+1\right)+2\lambda\phi^2\right)^2}{(\phi+1)(\mu(\phi-1)-\phi)\left(\mu\left(\lambda^2\phi^2-\phi(1-\lambda)^2-1\right)-\lambda^2\phi^2+(1-\lambda)^2\mu^2\phi\right)}+\lambda\mu\phi+\lambda\right\}.$$

There are two roots for this equation: first is when $\lambda = 0$ and the other λ root for this difference is:

$$\begin{aligned}\widetilde{\lambda}_3 = &\left(2\mu^4(\phi-1)\phi+\mu^3\left(2\phi^3-5\phi^2+3\phi-2\right)-\right.\\
&\quad\sqrt{-(\mu-1)^2\mu(\phi+1)^2(\mu(-\phi)+\mu+\phi)^2\left(2\mu^2(\phi-1)\phi+\mu\left(-3\phi^2+3\phi-2\right)-3\phi+1\right)}\\
&\quad\left.+\mu^2\left(-4\phi^3+2\phi^2-4\phi+2\right)+\mu\phi\left(2\phi^2+\phi+3\right)\right)/\\
&\left(2\mu^4(\phi-1)\phi+\mu^3\left(\phi^4+4\phi^3-9\phi^2+3\phi-1\right)+\right.\\
&\quad\left.+\mu^2\left(-2\phi^4-12\phi^3+10\phi^2-3\phi+1\right)+\mu\phi\left(\phi^3+11\phi^2-4\phi+2\right)-3\phi^3+\phi^2\right)\end{aligned}$$

Thus, if $\lambda > \tilde{\lambda}_3$, under market outcome C_3, the P2P-sponsoring strategy is preferred and the equalizing effect dominates the direct segmentation effect. $\tilde{\lambda}_3$, for any $\mu \geq 1/2$, is analytically monotonically decreasing in ϕ. Thus, as ϕ increases, this sufficient value of λ decreases. Similarly, for C_1, the difference in profits between the two strategies is: $\pi^{P,1} - \pi^{D,4} = \frac{1}{8(1-\delta)}\left\{\frac{\lambda(\mu((2\lambda-1)\phi-1)-2\lambda\phi)^2}{(1+\phi)(\mu(1-\lambda^2\phi)+\lambda^2\phi)} - \lambda\mu\phi - \lambda\right\}$. Solving for the root value of λ, where $\pi^{P,1} - \pi^{D,4} = 0$, leads to only one valid value $\in [0,1]$:

$$\tilde{\lambda}_1 = \frac{2\mu^2\phi(\phi+1)+\sqrt{(\mu-1)^2\mu\phi\left(3\phi^2+2\phi-1\right)(\mu\phi+1)}-2\mu\phi(\phi+1)}{(\mu-1)\phi\left(\mu\phi^2+(5\mu-3)\phi+1\right)}.$$

Thus, if $\lambda > \tilde{\lambda}_1$, under market outcome C_1, the P2P-sponsoring strategy is preferred and the equalizing effect dominates the direct segmentation effect.

Finding λ_{min}. Based on the equations shown above, we know that $\pi^{P,1} - \pi^{D,4}$ and $\pi^{P,3} - \pi^{D,4}$ are increasing in ϕ. Thus, in $\phi \leq \tilde{\phi}^D$, the region of interest where the dual versus P2P-sponsoring choice is considered, we know the largest difference in profits must occur at $\phi = \tilde{\phi}^D$. Substituting $\phi = \tilde{\phi}^D$ into

$\pi^{P,1} - \pi^{D,4}$ and solving for the minimum λ at which a P2P-sponsoring strategy is preferred results in:

$$\lambda_{min} = \frac{9 - 5\mu - 3A - \sqrt{6\left(8A - 39\right)\mu - 8\mu^3 - 8\left(A - 11\right)\mu^2 - 54\left(A - 3\right)}}{2\left(\mu^2 + \left(A - 8\right)\mu - 3A + 9\right)},$$

where $A = \sqrt{(1-\mu)(9-\mu)}$. It can be shown that at $\left\{\lambda = \lambda_{min}, \phi = \tilde{\phi}^D\right\}$ and any $\mu \geq \frac{1}{2}$, the P2P-sponsoring firm's market outcome is always C_1, so we do not need to consider outcome C_3. Hence, for any value of $\lambda < \lambda_{min}$, the firm never adopts a P2P-sponsoring strategy, regardless of ϕ. $\blacksquare$

Proof of Proposition 7

With $\gamma < 1$, that reflects the fraction of consumers who participate in the P2P rental market and when consumer usage rate is homogeneous, the valuations of the marginal purchasers are now: $\hat{v}^{P2P} = \frac{\frac{(\lambda-1)\lambda^2\mu(-\gamma+\lambda+1)-\lambda^4}{(\gamma-1)(\lambda^2-1)\mu+\lambda^2}+2(\lambda-1)\mu}{2(\lambda^2(\mu-1)-\mu)}$ and $\hat{v}^{NoP2P} = \frac{(\lambda-1)\mu(\gamma-\lambda-1)+\lambda^2}{2((\gamma-1)(\lambda^2-1)\mu+\lambda^2)}$. The valuation of the marginal renter is

$$\tilde{v}^{P2P} = \frac{\lambda\left(\frac{\lambda}{(\gamma-1)(\lambda^2-1)\mu+\lambda^2} + \frac{1}{\lambda^2(\mu-1)-\mu} + 2\right)}{2(\lambda+1)}.$$

Subsequently, the firm's optimal profit in this case is $\frac{\lambda\left((\lambda-1)\mu(\gamma-\lambda-1)+\lambda^2\right)^2}{4(\delta-1)(\lambda^2(\mu-1)-\mu)((\gamma-1)(\lambda^2-1)\mu+\lambda^2)}$. This profit is smaller than the monopoly profit for all $\gamma : 0 < \gamma \leq 1$, consistent with Proposition 1. For consumer welfare, the welfare of the γ fraction of consumers is $\frac{\lambda^2(2(\lambda-1)\mu-\lambda)\left(-2(\gamma-1)(\lambda-1)^2(\lambda+1)\mu^2+(\lambda-1)\lambda\mu(2\gamma\lambda+\gamma-3\lambda-1)+\lambda^3\right)^2}{8(\delta-1)(\mu-\lambda^2(\mu-1))^2((\gamma-1)(\lambda^2-1)\mu+\lambda^2)^2}$ (from buying) plusz $\frac{(1-\lambda)^2\lambda\mu\left(-2(\gamma-1)(\lambda-1)^2(\lambda+1)\mu^2+(\lambda-1)\lambda\mu(2\gamma\lambda+\gamma-3\lambda-1)+\lambda^3\right)^2}{8(1-\delta)(\mu-\lambda^2(\mu-1))^2((1-\gamma)(\lambda^2-1)\mu+\lambda^2)^2}$ (from renting). The welfare of the $1-\gamma$ fraction of non-exposed consumers is $\frac{\left((1-\lambda)\lambda\mu(2\gamma\lambda+\gamma-\lambda-1)+\lambda^3\right)^2}{8(1-\delta)\lambda((1-\gamma)(1-\lambda^2)\mu+\lambda^2)^2}$. We can show that there is a threshold of γ sufficiently below 1 such that consumer welfare is higher under a P2P-facing firm than a monopoly firm. This sufficiently low value is: $\tilde{\gamma} = \frac{4\lambda^2\mu^2-5\lambda^2\mu+\lambda^2-4\mu^2+\mu}{4\lambda^2\mu^2-4\lambda^2\mu-4\mu^2+\mu}$. $\blacksquare$

Correlation between λ and v

We assume that consumers with a higher usage rate have relatively higher per-use valuation, whereas consumers with lower usage rate have lower per-use valuation. Hence, consumer with usage λ_H have per-use valuation $v \sim U[c, c+1]$, whereas the per-use valuation distribution remains the same for low usage consumers. The correlation $Cor(\lambda, v) = \rho$, where

$$Cor(\lambda, v) \quad = \rho = \quad \frac{\mathbb{E}[(\lambda - \bar{\lambda})(v - \bar{v})]}{\sigma_\lambda \sigma_v} = \frac{\sqrt{3}c}{\sqrt{1+3c^2}}.$$

$\mathbb{E}[(\lambda - \bar{\lambda})(v - \bar{v})] = \lambda_H c(1-\phi)/4$
$\sigma_\lambda^2 = \lambda_H(1-\phi)/2$
$\sigma_v^2 = (1+3c^2)/12$