Two-Sided Platform Competition in a Sharing Economy*

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Abstract

We examine competition between two-sided platforms in a sharing economy. In sharing economies, workers self-schedule their supply based on the wage they receive. The platforms compete for workers as well as consumers. To attract workers, platforms use diverse wage schemes, including *fixed commission rate*, *dynamic commission rate*, and *fixed wage*. We develop a model to examine the impacts of the self-scheduled nature of the supply on competing platforms and the role of the wage scheme in the platform competition. We find that the price competition between platforms is more intense in a sharing economy compared to an economy with a fixed supply of workers if and only if the platforms serve more consumers and workers in the sharing economy than in the traditional economy, regardless of the wage scheme employed by the platforms. Further, any of the three wage schemes can be the best for the platforms and the worst for consumers and workers, depending on the market characteristics. In markets where the competition is more fierce on the demand side than on the supply side, the fixed-wage scheme results in the highest profits for the platforms and lowest surpluses for consumers and workers. In contrast, in markets where the competition on the supply side is more competitive, when the supply is highly (mildly) more competitive, the fixed-commission-rate (dynamic-commission-rate) scheme generates the highest profits for platforms, leading to the lowest surpluses for consumers and workers and the lowest social welfare. The differential impacts of the wage scheme on the price (demand-side) and quantity (supply-side) competition explain our findings.

1 Introduction

A distinct feature of a *sharing economy* is that a platform matches an individual who owns excess capacity of an asset (a *worker*) to one who needs it (a *consumer*) for a fee in a two-sided market (Botsman, 2015). Car sharing, peer-to-peer lending, food delivery, ride sharing, home renting, coworking, and talent sharing are but a few popular examples of such markets. In each of these markets, multiple platforms compete for consumers as well as workers. For instance, in the car-sharing market, GetAround and Turo compete for individuals who are willing to offer vehicles and also consumers that want to rent vehicles; Prosper and Upstart seek to attract both borrowers and lenders in the peer-to-peer lending market; in the food delivery market, DoorDash and Uber Eats compete for both consumers and delivery drivers; Uber and Lyft compete directly with each other for riders and drivers in the ride-sharing market. The nature of platform competition in such two-sided markets in a sharing economy is not well understood because these markets exhibit some unique characteristics.

In sharing-economy markets, a platform receives a payment only when it connects a consumer with a worker. Thus, matching the demand and the supply is a platform's key issue. In addition, the supply is not fixed. Workers in these markets self-schedule the availability of their assets based on various factors such as their own need and the price they can receive for renting an asset. Consequently, each platform's supply depends on the net price workers receive or the commission the platform charges. Analogously, the demand depends on the price charged to consumers. Moreover, the presence of competing platforms adds another layer of complexity. Not only do the platforms compete for the demand, but they also compete for supply. For instance, in the car-sharing economy, both car renters and car owners might check both GetAround and Turo and choose the platform that offers a greater value; in the ride-sharing context, the riders and drivers can flexibly switch between Uber and Lyft; in the food delivery business, the delivery drivers choose between DoorDash and Uber Eats based on economic incentives. Therefore, each platform must balance its demand and supply by considering the competitor's action.

In this research, we examine platform competition in a two-sided market that operates under the sharing-economy model in which platforms set prices on both sides of the market. We model the self-scheduling feature of the sharing economy and focus on the strategic competition related to price and wage. In sharing-economy markets, platforms have experimented with various price and wage schemes. Dynamic pricing on the consumer side has been the standard practice: The platform varies the price charged to consumers based on the demand and supply conditions (Taylor, 2018; Cachon et al., 2017). On the worker side, we have observed platforms offering three different wage schemes (Smith, 2017; Hu and Zhou, 2019): *fixed commission rate* (FCR), *dynamic commission rate* (DCR), and *fixed wage* (FW). Under the FCR scheme, a platform commits to paying its workers a fixed fraction of the prices charged to consumers. Under the FW scheme, a platform commits to paying a fixed wage to its workers for each service sharing, regardless of the price it subsequently charges consumers. Under the DCR scheme, a platform makes no commitment to a fixed commission rate or wage; instead, it determines the wage and price simultaneously when a consumer connects with a worker. The business models used by the two-sided platforms in a sharing economy are diverse and could vary depending on the specific services provided by the platforms. Moreover, two competing platforms that offer similar services could also employ different wage schemes. We present the key characteristics of some well-known two-sided platforms in Table 1 in the appendix to highlight the contexts we examine in this paper.

Our specific research questions relate to the impacts of the self-scheduled nature of the supply and the wage scheme on platform competition in a two-sided market in which the platforms set prices on both sides. Toward that end, we develop a model where two platforms compete for consumers and workers. To examine how the self-scheduled supply affects the competition, we construct a benchmark in which the two platform have a fixed supply of workers and do not compete for them, and we compare the outcomes in this benchmark to those in the sharing context under each wage scheme. To gain insights into the role of the wage scheme, we derive the equilibrium under each wage scheme and then compare the equilibrium outcomes under the three wage schemes.

We find that, as in markets in the traditional economy, platform competition in the sharing market is affected by the degree of platform substitutability. In particular, an increase in platform substitutability on one side (consumer or worker) intensifies the competition on that side (decreasing the consumer price or increasing the worker wage), hurts the platforms, and benefits that side (i.e., increases consumer or worker surplus). We also find that an increase in platform substitutability on one side benefits the other side—an increase in the consumer-side (worker-side) substitutability increases the worker wage and worker surplus (decreases the consumer price and increases the consumer surplus). More importantly, we show that the price competition between platforms is more intense in the sharing economy compared to the benchmark where the platforms have a fixed supply of workers if and only if the platforms serve more consumers and workers in the sharing economy than the benchmark. These results hold regardless of the wage scheme.

Meanwhile, we find that the wage schemes differ in their implications for the players in a sharing economy, and any of the three wage schemes can be the best for the platforms and the worst for consumers and workers, depending on the market characteristics. If the demand side is more competitive than the supply side, then the FW scheme results in the highest platform profits, the lowest consumer and worker surpluses, and the lowest social welfare. If the supply side is more competitive than the demand side, but only mildly more competitive, then the DCR scheme generates the highest platform profits, the lowest consumer and worker surpluses, and the lowest social welfare. On the other hand, if the supply side is much more competitive than the demand side, then the FCR scheme generates the highest platform profits, the lowest consumer and worker surpluses, and the lowest social welfare. Thus, regardless of which wage scheme is adopted by the platforms, a misalignment exists between the platforms' preferences and those of the consumers, workers, and the society.

In addition to offering insights into the platform competition in two-sided markets and a sharing economy, our results extend the results of the literature that examines firm competition using quantity and price. First, competition for supply, as opposed to a fixed supply for each platform, leads to a decrease in price and an increase in consumer surplus and social welfare if the fixed supply constrains the demand significantly. In particular, if supply is a constraint in the traditional economy with a fixed captive supply, the sharing economy model is better for consumers and for society.

Second, in a well-known result, a sequential game in which firms choose quantities/capacities first and then compete on price softens price competition and offers firms higher profits, as compared to a game in which firms compete only on price with no restriction on quantity. However, we show that this result does not necessarily hold in the sharing economy where the platforms compete for quantity (i.e., supply). The result that a capacity-then-price game sequence softens price competition holds in the sharing economy—the FW scheme offers the highest platform profits—only when the demand side is more competitive than the supply side. Intuitively, in a market where the demand side is more competitive than the supply side, choosing the quantity first enables platforms to curb the price competition in the subsequent stage, benefiting the platforms. However, if the supply side is more competitive than the demand side, choosing the quantity (or supply) first and then competing on price results in the lowest platform profits. In these cases, anticipating less-intense price competition in the subsequent stage, platforms have an incentive not to be constrained by supply. Thus, when they choose quantities first, platforms acquire excessive supply, which hurts the platforms.

The rest of the paper is organized as follows. In the next section, we review the related literature. Section 3 presents our baseline model. We then provide an equilibrium analysis of the effects of self-scheduled supply and platform substitutability in Section 4. In Section 5, we compare the three wage schemes and derive the conditions under which each is the best for the platforms and worst for consumers and workers. In Section 6, we consider various model extensions and check the robustness of our main findings. Section 7 concludes the paper with managerial insights.

2 Literature Review

The sharing-economy phenomenon is relatively new, and academic research on this phenomenon is limited. One literature stream has examined how the entry of firms that operate under the sharing-economy model affects firms, consumers, and society under the traditional model. For example, Zervas et al. (2017) examine the economic impact of Airbnb on the traditional lodging industry, and Fraiberger and Sundararajan (2007) show that sharing-economy models enhance consumer welfare in rental markets. Another literature stream has examined the operational aspects of sharing-economy models. For instance, Hu and Zhou (2016) and Allon et al. (2012) investigate demand–supply matching when supply capacity is exogenous, and consumers and workers have their own match preferences. Other papers have studied the impact of a monopoly platform's surge-pricing policy in a dynamic supply and demand environment using queuing models (e.g., Taylor, 2018; Cachon et al., 2017; Riquelme et al., 2015; Gurvich et al., 2016). Garg and Nazerzadeh (2021) propose an incentive-compatible mechanism on the supply side to manage surge pricing. Departing from these studies, our paper focuses on platform competition and its effect on participating players within a sharing economy.

Our work also relates to the literature on platform economics and platform competition (e.g., Basu et al., 2019; Caillaud and Julien, 2003; Rochet and Tirole, 2003; Parker and Van Alstyne, 2005; Armstrong, 2006; Economides and Katsamakas, 2006; Zhu and Iansiti, 2012). For example, Basu et al. (2019) study the interplay between search and authentication services provided by platforms for online matching. Armstrong (2006) investigates pricing strategies in three models of two-sided markets: monopoly, competing platforms with single-homing agents, and competing platforms with multihoming agents. Several studies on platform competition predict a "winner-take-all" outcome—the platform with the largest number of users will dominate the market (e.g., Besen and Farrell, 1994; Caillaud and Julien, 2003). Other studies, however, show that multiple platforms could coexist because of such factors as asymmetric or local network effects (e.g., Lee et al., 2006), costs of adopting multiple platforms (e.g., Eisenmann, 2006), and differentiated consumer preferences (e.g., Armstrong and Wright, 2007). Our context differs from those in the above literature in that a platform in our model generates revenue from one side (i.e., consumers) and pays wages to the other side (i.e., workers).

Our study is closely related to the recent literature examining competition in the ride-sharing economy. In terms of the role of competition, Levin and Skrzypacz (2016) find that competition can lead to inefficiently low prices from the social-welfare perspective; Nikzad (2017) compares different monopoly and duopoly equilibria; Ahmadinejad et al. (2019) investigate the possibility of market failure in a competitive market if platforms deplete shared vehicle resources. A few studies examine the impact of multihoming by riders and drivers on the wait time and social welfare (Bryan and Gans, 2019; Liu et al., 2017). Focusing on interactions of the two sides, Bernstein et al. (2018) model the congestion between demand and supply in ride-sharing markets; Wu et al. (2020) show that the equilibrium outcome depends on the sequence of moves by riders and drivers. Different from these papers, we focus on the different wage schemes and their roles in competition in the sharing-economy context. To the best of our knowledge, no paper has investigated the roles of wage schemes under competition, even though wage scheme is of vital importance to platforms as evidenced by their experimentation with different schemes. Further, few papers model the control of supply through endogenous wages although it is a significant aspect of sharing-economy contexts.

Finally, our work is also related to the economics literature that examines firm competition through quantity and price (e.g., Kreps and Scheinkman, 1983; Davidson and Deneckere, 1986; Osborne and Pitchik, 1986; Van Mieghem and Dada, 1999). One of the major findings of this literature is that a game in which competing firms choose production quantities or capacities first and then engage in Bertrand price competition is equivalent to a game in which they engage in Cournot competition, implying that the quantity-then-price game provides the firms higher profits than the standard Bertrand competition game in which they compete on price with no quantity constraint. The key difference between our work and this literature stream is that we model supply-side competition between firms, whereas prior work assumes that one firm's supply has no impact on the other firm's supply. Furthermore, we consider various wage schemes that have been used in the sharing economy, and deliver new insights regarding the role of the wage scheme.

3 Model

We consider a stylized and parsimonious model of platform competition in a two-sided market. Two platforms, 1 and 2, facilitate a worker's matching with a consumer. For instance, in the car-sharing market, GetAround and Turo are the platforms, car owners are the workers, and car renters are the consumers; in the peer-to-peer lending market, the platforms are Prosper and Upstart, lenders are the workers, and borrowers are the consumers; Uber and Lyft are the platforms in the ride-sharing market, drivers are the workers, and riders are the consumers. We consider a representative time period in which consumers seek workers' services.

Demand/Consumers. The platforms compete for consumers. We use the standard linear model of competition (e.g., Choi, 1991); that is, if platform i, $i \in \{1, 2\}$, charges price p_i, then its demand is given by

$$d_i = a - b(p_i - p_{\bar{i}}) - cp_i, \tag{1}$$

where $\bar{i} = 3-i$, and a, b, and $c \in \mathbb{R}^+$. Parameter a measures the consumer market potential for each platform, and parameter b measures the demand-side substitutability between the two platforms.

Following the procedure used in existing work (e.g., Kwark et al., 2014), the demand function in Equation (1) can be derived using microeconomic utility-based assumptions about consumer behavior. For example, we can assume that each consumer has a unit demand for a service. Some consumers are loyal to one of the platforms—these consumers either use the platform they are loyal to or do not use service sharing at all. Other consumers multihome between the platforms, and choose the platform that offers them a higher surplus. We refer to the former group of consumers as the *loyal segment* and the latter group as the *competitive segment*. Aggregating the consumers' platform choices leads to the demand function in Equation (1). (The details of the derivation can be found in the appendix.) Under this microeconomic framework, parameter b captures the price sensitivity of the competitive-consumer segment, and parameter c captures the loyal segment's price sensitivity.

Supply/Workers. The platforms compete for workers as well. The supply of workers on each platform increases in its own wage and decreases in its competitor's wage. Similar to the demand side, we use a linear model for supply. If platform i offers wage w_i, then its worker supply is given by

$$s_i = -k + l(w_i - w_{\bar{i}}) + mw_i, \tag{2}$$

where k, l, and $m \in \mathbb{R}^+$. The condition $k > 0$ implies that a positive wage is needed for a platform to have a positive worker supply. Parameter l measures the supply-side substitutability between the two platforms. Similar to the demand function, the linear supply function can be derived based on a microeconomic foundation by assuming *loyal* and *competitive* workers. The sensitivity to wage in the competitive and loyal segments are reflected by parameters l and m, respectively.

Platforms. The two platforms compete with each other for consumers and workers via the prices charged to consumers and the wages offered to workers. A transaction happens only when a consumer is matched with a worker. Therefore, the transaction volume on platform i is the minimum of the demand and supply on the platform (i.e., $\min\{d_i, s_i\}$). Noticing the profit margin of a transaction for platform i is $(p_i - w_i)$, we can formulate platform i's profit as

$$\pi_i(p_i, p_{\bar{i}}, w_i, w_{\bar{i}}) = (p_i - w_i)\min\{d_i, s_i\}. \tag{3}$$

Each platform maximizes its profit by choosing the price and wage.

Wage Schemes. We consider three different wage schemes: *fixed wage* (FW), *fixed commission rate* (FCR), and *dynamic commission rate* (DCR). Under the FW scheme, the two platforms announce their worker wage per transaction in the first stage, and decide their consumer price in the second stage. Under the FCR scheme, the platforms announce their commission rates charged to workers (α_i) in the first stage, and decide the consumer prices in the second stage. For each realized transaction, the platform takes its commission from the consumer price according to the preannounced commission rate, and the rest, $(1 - \alpha_i)p_i$, is the worker's wage. Under the DCR scheme, the platforms do not precommit to or announce commission rates or wages before choosing prices charged to consumers. Instead, they choose the prices and wages simultaneously.

We focus on symmetric pure-strategy equilibria. To rule out some trivial cases, we impose the following two technical assumptions.

Assumption 1: $\frac{a}{c} > \frac{k}{m}$.

Assumption 2: $l \leq 2c + \frac{c^2}{b}$.

Assumption 1 is required for matching services to be profitable. A matching service can be profitable only if the price charged to consumers is greater than the wage offered to workers. Moreover, the demand and supply should be positive, which requires the price be less than a/c and the wage be greater than k/m in a symmetric equilibrium, by Equations (1) and (2). Thus, Assumption 1 ensures price is greater than wage. Assumption 2 is somewhat technical: It requires the substitutability on the supply side between the two platforms to be not too high so that a symmetric pure-strategy equilibrium can be sustained. In particular, whether a pure-strategy equilibrium exists under the FW scheme critically depends on this assumption while the pure-strategy equilibria under the other two schemes do not rely on this assumption.

4 Analysis of Platform Competition

We first derive the equilibrium under each of the three wage schemes. We then investigate the impact of self-scheduled variable supply (a function of wage) by comparing the equilibrium in the sharing market with that in a market where supply for each platform is fixed (independent of wage). Finally, we examine how the platform competition and key equilibrium outcomes are affected by consumer-side and worker-side platform substitutability.

4.1 Sharing Economy Equilibria

The equilibrium depends critically on the wage scheme adopted by the platforms. In the main model, we assume a symmetric wage scheme across the two platforms to focus on the nature of competition under each wage scheme. We examine asymmetric wage schemes and the platforms' choice of wage scheme in Section 6.

Dynamic Commission-Rate Scheme: Under the DCR scheme, both platforms decide their prices and commission rates simultaneously to maximize their own profits:[1]

$$\max_{p_i,\alpha_i} \pi_i = \alpha_i p_i \min\{d_i, s_i\}.$$

Based on the platforms' best responses to each other, we derive the equilibrium outcome as follows.

Lemma 1. *Under the DCR scheme, in equilibrium, both platforms charge price p^* and offer commission rate α^*, where*

$$p^* = \frac{k(b+c)(l+m) + a\left[m(l+m) + (b+c)(l+2m)\right]}{c(b+c)(l+2m) + m(b+2c)(l+m)} \tag{4}$$

$$\alpha^* = \frac{(b+c+l+m)(am-ck)}{k(b+c)(l+m) + a\left[m(l+m) + (b+c)(l+2m)\right]}. \tag{5}$$

The DCR equilibrium exhibits two important characteristics. First, we find that demand and supply are matched for each platform in equilibrium. Intuitively, if demand exceeds supply for a platform, the transaction volume is equal to the number of workers on the platform. In this case, if the platform offers the same wage and increases its price a little bit (i.e, by increasing the price and decreasing the commission rate so that the wage remains the same), it could have the same transaction volume and a higher profit margin. Thus, demand cannot exceed supply in equilibrium. Similarly, supply cannot exceed demand in equilibrium, because otherwise the platform could have increased its profit by decreasing the wage by a small amount.

Second, the price and commission rate offered by a platform are strategic complements in the sense that the two are positively associated with each other. On the other hand, we find that the price and wage are strategic substitutes ($w^* = \frac{a+k}{m} - \frac{c}{m}p^*$), implying that the equilibrium commission rate increases more significantly relative to an increase in equilibrium price. The strategic relationship between price and wage (commission rate) is the outcome of the demand matching the supply in equilibrium—an increase in price decreases the demand so the wage must be reduced to decrease the supply as well. Interestingly, such a relationship between price and wage (or commission rate) appears to provide a theoretical explanation for some market phenomena, including in the ride-sharing market the drivers' contention that Uber did not pass on any part of the higher

[1] We could also interpret the DCR scheme as the Dynamic Wage scheme because setting the commission rate is equivalent to setting the wage in this scheme.

price it was charging riders when Uber experimented with the DCR scheme (Stand, 2019).

Fixed Commission-Rate Scheme: Under the FCR scheme, the platforms decide their respective commission rates first. After observing the commission rates, the platforms decide their prices. Specifically, in the first stage, each platform chooses the commission rate to maximize its profit:

$$\max_{\alpha_i} \pi_i = \alpha_i p_i \min\{d_i, s_i\}.$$

In the second stage, given the commission rates, each firm chooses a price to maximize its profit:

$$\max_{p_i} \pi_i = \alpha_i p_i \min\{d_i, s_i\}.$$

We use the subgame-perfect equilibrium as the solution concept. Using backward induction, we first solve the price competition in the second stage, and, considering the subgame equilibrium outcome, we then solve the competition in the first stage. We derive the equilibrium outcome as follows.

Lemma 2. *Under the FCR scheme, in equilibrium, both platforms charge price p^* and offer commission rate α^*, where α^* is the solution to*

$$\frac{(1-\alpha^*)[m(b+2l+m)+c(l+m)]+b(2c+m)+c(c+l+m)}{(1-\alpha^*)m(b+c)(2l+m)+c(2b+c)(l+m)} = \frac{\alpha^*(a+k)}{ma(1-\alpha^*)-ck} \tag{6}$$

and

$$p^* = \frac{a+k}{c+m(1-\alpha^*)}. \tag{7}$$

The FCR equilibrium has some similarities with the DCR equilibrium. For instance, the demand matches the supply in the FCR equilibrium as well. The rationale for this outcome under the FCR scheme is similar to that under the DCR scheme with a subtle difference. Under the FCR scheme, a platform has only one variable (price, as opposed to two variables—price and wage—under the DCR scheme) to match the supply with the demand in the pricing subgame. Moreover, the wage is directly proportional to price as opposed to being fixed. If demand exceeds supply on one platform, by slightly increasing the price, the platform can increase its profit margin and transaction volume, thus increasing its profit. On the other hand, if supply exceeds demand, then a platform will find it profitable to decrease the price and increase demand only if the commission rate is sufficiently high.

Realizing this, platforms set a high-enough commission rate in the first stage so that matching the demand to supply is indeed profitable in the pricing subgame.

The equilibrium price and commission rate are strategically complementary in the FCR scheme as well. Despite the similarities, the equilibrium prices and commission rates are not identical under the two schemes because of the differences in the nature of the game under the two schemes (simultaneous with a single stage under DCR versus two sequential stages under FCR). Furthermore, the drivers' qualm in the ride-sharing economy about DCR that the platforms may not pass any part of higher prices to drivers does not exist under the FCR scheme because the driver wage is directly proportional to the price.

Fixed-Wage Scheme: Under the FW scheme, platforms decide their respective wages first, and, after observing the wages, the platforms decide prices. Specifically, in the first stage, each platform chooses a wage to maximize its profit:

$$\max_{w_i} \pi_i = (p_i - w_i) \min\{d_i, s_i\}.$$

In the second stage, given the wages, each firm chooses a price to maximize its profit:

$$\max_{p_i} \pi_i = (p_i - w_i) \min\{d_i, s_i\}.$$

As with the FCR scheme, we can derive the subgame-perfect equilibrium outcome as follows.

Lemma 3. *Under the FW scheme, in equilibrium, both platforms charge price p^* and offer wage w^*, where*

$$w^* = \frac{b[2a(l+m) + 2ck + k(2l+3m)] + c[(a+2k)(l+m) + ck]}{b[2c(l+2m) + m(2l+3m)] + c[c(l+2m) + 2m(l+m)]} \tag{8}$$

$$p^* = \frac{a + k - mw^*}{c}. \tag{9}$$

As with the DCR and FCR schemes, under the FW scheme, demand matches supply in equilibrium, and wage and price are strategic substitutes. Under the FW scheme, once the wages are set, the platforms' worker supplies are fixed. Thus, the platforms engage in price competition in the second stage given the supply constraint. We find that, in equilibrium, the platforms will set wages such that the supply constraint binds. This result is similar to that in the literature on quantity-then-price competition. However, as stated previously, the platforms also compete for supply in our context. Thus, the supply-side competition does not change the platforms' incentives to constrain quantity in order to curb price competition.

4.2 Impact of Self-Scheduled Supply

We now examine the impact the self-scheduled supply by comparing the sharing-economy equilibria with a corresponding benchmark in which the supply is fixed. In the benchmark, each platform has a fixed supply of workers, s, at its disposal; that is, s is the maximum number of workers that a platform can deploy. We assume s is not too small such that the demand side remains competitive. The demand-side model in the benchmark is the same as that in the main model. Thus, platform i's profit in the benchmark is given by

$$\pi_i = \begin{cases} (p_i - w)\min\{d_i, s\} & \text{if wage is fixed at } w \\ \alpha p_i \min\{d_i, s\} & \text{if commission rate is fixed at } \alpha. \end{cases} \tag{10}$$

Solving the platforms' profit-maximization problems simultaneously, we obtain the equilibrium price in the benchmark when wage is fixed as

$$p_{\mathrm{B}}^* = \begin{cases} \frac{a-s}{c} & \text{if } s \le \frac{(a-cw)(b+c)}{b+2c} \\ \frac{a+(b+c)w}{b+2c} & \text{if } s > \frac{(a-cw)(b+c)}{b+2c}. \end{cases} \tag{11}$$

The equilibrium price in the benchmark when the commission rate is fixed is given by

$$p_{\mathrm{B}}^* = \begin{cases} \frac{a-s}{c}, & \text{if } s \le \frac{a(b+c)}{b+2c} \\ \frac{a}{b+2c} & \text{if } s > \frac{a(b+c)}{b+2c}. \end{cases} \tag{12}$$

where subscript B denotes the equilibrium in the benchmark. Equations (11) and (12) show that when the fixed supply, s, is less than a threshold value, the platforms engage in a quantity-constrained price competition and prices are a function of s. When s is greater than the threshold, the equilibrium price is independent of s because the platforms are no longer constrained by supply when they compete on the demand side.

The benchmark equilibrium involves the exogenous parameter w or α. However, this parameter is endogenous in the sharing equilibria. As a result, the comparison of profits between the benchmark and the sharing context will be driven by the value of the benchmark's exogenous parameter. To make a "fair" comparison, we set w or α to be equal in the benchmark and the sharing equilibrium, depending on the wage scheme used in the sharing context. In particular, for a comparison of the benchmark with the FCR equilibrium, we let α in the benchmark be equal to the equilibrium commission rate under FCR. For a comparison with FW, we let w in the benchmark be equal to the equilibrium wage under FW. For a comparison with DCR, we let w in the benchmark be equal to the equilibrium wage under DCR.[2]

Proposition 1. *The equilibrium price is lower in the sharing economy than in the benchmark if and only if the equilibrium supply in the sharing economy is greater than the fixed supply s in the benchmark.*

Proposition 1 reveals that the price competition is more intense, leading to a lower price, in the sharing economy than in the benchmark scenario with a fixed supply when the fixed supply imposes a severe constraint in the benchmark; if the supply is tight, the platforms set high prices. When the platforms have adequate supply so that they are not constrained by supply, they compete fiercely for demand which leads to a low price in the benchmark. In the sharing economy, on the other hand, the platforms match the demand and supply by controlling both price and wage. While a large supply in the benchmark results in fierce price competition, the platforms control their supply in the sharing economy to curb the price competition. While a small supply in the benchmark induces the platforms to charge high prices and fulfill small demands, the platforms find it profitable in the sharing economy to serve more demand with supply by offering lower prices.

The key takeaway from Proposition 1 is that the sharing economy where the platforms compete for workers alters the nature of competition from that in a traditional market, such as traditional rental service, where firms employ a fixed supply at a fixed cost. Proposition 1 demonstrates that supply-side competition can be a vehicle for softening the demand-side competition (e.g., when firms have adequate fixed supply), which is new to the literature. On the other hand, Proposition 1 is consistent with empirical observations that prices charged to riders in ride-sharing platforms

[2]Alternatively, we can let α in the benchmark be equal to the equilibrium commission rate because of the equivalence between dynamic commission rate and dynamic wage, as explained previously.

are typically lower than those charged by taxi firms and that the volume of transactions in ride-sharing platforms far exceed that of traditional taxi market, suggesting that the supply constraint in the traditional market could contribute to higher prices in the traditional market compared to the ride-sharing economy.

4.3 Impact of Platform Substitutability in the Sharing Economy

In this section, we examine the role of platform substitutability in the sharing economy. The degrees of consumer- and worker-side platform substitutability are captured by parameters b and l, respectively. We thus compute the marginal impacts of these two parameters on key equilibrium outcomes.

Proposition 2. *Under the DCR, FCR, and FW schemes, an increase in the consumer- or worker-side substitutability (i.e., an increase in either b or l)*

(a) decreases the price (i.e., $\frac{\partial p^*}{\partial b} < 0$ and $\frac{\partial p^*}{\partial l} < 0$) and increases the consumer surplus;

(b) increases the wage (i.e., $\frac{\partial w^*}{\partial b} > 0$ and $\frac{\partial w^*}{\partial l} > 0$) and increases the worker surplus;

(c) decreases the platform profit (i.e., $\frac{\partial \pi^*}{\partial b} < 0$ and $\frac{\partial \pi^*}{\partial l} < 0$).

The impacts of b on price, consumer surplus, and platform profit, as shown by Proposition 2(a) and (c), are consistent with well-known results regarding the impacts of demand-side platform (or product) substitutability in a competitive setting. In addition, we find that an increase in the consumer-side substitutability benefits the other side (i.e., workers) by increasing workers' wage and surplus. This result arises because the platforms match the demand with the supply in equilibrium. An increase in consumer-side substitutability increases the demand-side competition and forces the platforms to reduce the consumer price, which increases the overall demand. To match the enhanced demand, the platforms increase supply by increasing the wage offered to workers. A higher wage and a larger number of matched workers increase the overall worker surplus.

An increase in worker-side substitutability (i.e., an increase in l) has qualitatively identical own-side and cross-side impacts as an increase in consumer-side substitutability. When the worker-side platform substitutability increases, more workers switch from one platform to the other when the latter increases the wage by one unit, which increases the competition on the supply side. As a result, an increase in worker-side substitutability increases the equilibrium wages offered by the platforms, leading to a higher worker supply and surplus. The increased supply, in turn, induces platforms to decrease the prices charged to consumers to increase the demand, which benefits consumers. Essentially, each side benefits not only from an increase in platform substitutability on its own side, but also from that on the other side. In contrast, platforms end up hurting when the degree of platform substitutability increases on either side.

In sum, the analysis in this section suggests that the nature and the degree of competition in the sharing economy can be different from those in markets where firms compete only for consumers without competing for supply. Although the impact of consumer-side platform substitutability in the sharing economy could be qualitatively identical to that of demand-side substitutability in traditional markets, the cross-side impacts of platform substitutability are new and they reveal that an increase in substitutability on one side not only benefits that side, but also benefits the other side. Moreover, the effects of platform substitutability are qualitatively identical regardless of the wage scheme adopted in the sharing economy.

5 Comparison of Sharing-Economy Wage Schemes

Given the diverse wage schemes found in sharing economies, a key question is of interest to the platforms, consumers, and workers: Which wage scheme is superior from each perspective? The question is especially interesting in light of the findings of the preceding section that the equilibria in all three wage schemes exhibit similar characteristics—the impact of the self-scheduled worker supply and the impacts of demand-side and supply-side platform substitutability are similar across the three wage schemes. Furthermore, the question is relevant in a competitive scenario because all wage schemes yield the same equilibrium if the platform is a monopoly. We use superscripts D, F, and W to distinguish the equilibrium outcomes under the DCR, FCR, FW schemes, respectively.

5.1 Platforms' Preferences for Wage Schemes

First, we compare the platform profits under the three wage schemes.

Proposition 3. *(a) If $\frac{b}{c} > \frac{l}{m}$, the FW scheme generates the highest platform profit and the DCR scheme generates the lowest profit (i.e., $\pi^{\mathrm{W}*} > \pi^{\mathrm{F}*} > \pi^{\mathrm{D}*}$).*

(b) If $\frac{b}{c} < \frac{l}{m}$ and

$$\frac{b+c+l+m}{(b+c)(l+m)\alpha_i^{\mathrm{D}*}} - \frac{b+\left(1-\alpha_i^{\mathrm{D}*}\right)l}{\left(1-\alpha_i^{\mathrm{D}*}\right)(cl-bm)} > 0, \tag{13}$$

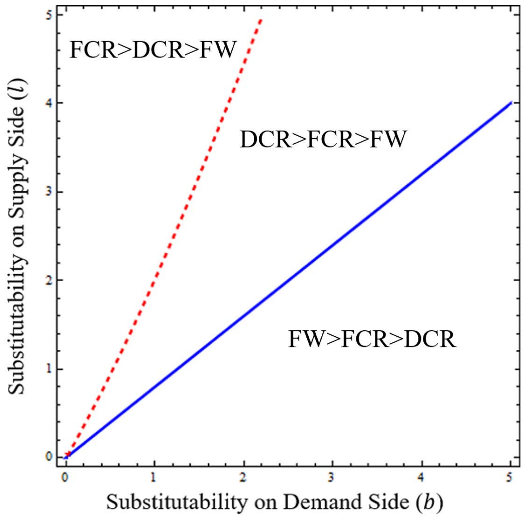
the FCR scheme generates the highest platform profit and the FW scheme generates the lowest profit (i.e., $\pi^{\mathrm{F}*} > \pi^{\mathrm{D}*} > \pi^{\mathrm{W}*}$).

(c) If $\frac{b}{c} < \frac{l}{m}$ and condition (13) does not hold, the DCR scheme generates the highest platform profit and the FW scheme generates the lowest profit (i.e., $\pi^{\mathrm{D}*} > \pi^{\mathrm{F}*} > \pi^{\mathrm{W}*}$).

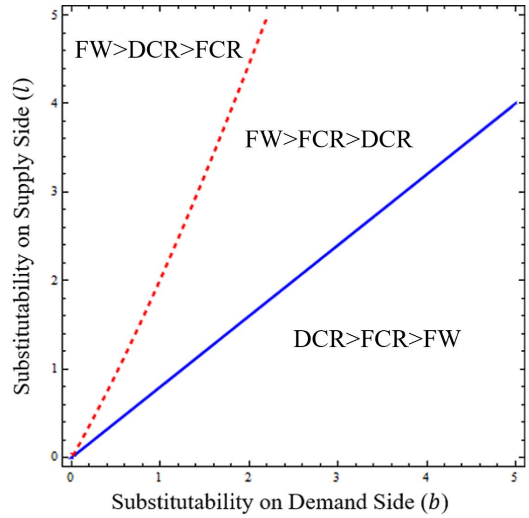
Proposition 3 shows the ordering of platform profit under various conditions. Any of the three wage schemes can generate the highest platform profit, depending on market conditions. The FCR scheme never yields the worst platform profit, and the FW scheme always yields either the highest profit or the lowest profit. Furthermore, we find that the ordering of platform profits under the three wage schemes is identical to the ordering of equilibrium price and the reverse of that of equilibrium wage under the three schemes. Next, we interpret the conditions used in the proposition and then provide the intuition for the results.

Notice that parameter b measures the price sensitivity of the competitive-consumer segment (or the demand-side substitutability), and parameter c captures the price sensitivity of the loyal-consumer segment. We refer to the ratio b/c as the degree of demand-side competition. Analogously, we refer to l/m as the degree of supply-side competition, because l measures the wage sensitivity of the competitive-worker segment and m captures the wage sensitivity of the loyal-worker segment.

When the degree of demand-side competition exceeds that of the supply side (i.e., $b/c > l/m$), if a platform deviates from the equilibrium and decreases its supply and demand by one unit (to keep the demand matched with the supply), the other platform will see a greater increase in its demand than in its supply. Consequently, the other platform will respond more aggressively on price than on wage. On the other hand, when the degree of supply-side competition exceeds that of the demand side (i.e., $b/c < l/m$), a unit decrease in a platform's demand and supply will result in the other platform seeing less increase in its demand than in its supply. In this scenario, the other platform will respond more aggressively on wage than on price. In markets where the degree of supply-side platform competition exceeds that of the demand side, condition (13) further segments the market. An examination of condition (13) reveals that, given other model parameters, the condition is more likely to be satisfied when l is higher rather than lower. Thus, we refer to the



(a) Platforms' Profits



(b) Consumers' and Workers' Surplus

Figure 1: Equilibrium Comparison under the Three Wage Schemes
Note: $a = 15$, $k = 1.5$, $c = 2.5$, and $m = 2$.

scenario where condition (13) is satisfied as one where the degree of supply-side platform competition significantly exceeds that of the demand side and the scenario where condition (13) is not satisfied as one where the degree of supply-side platform competition mildly exceeds that of the demand side. As illustrated in Figure 1, the two conditions provided in the proposition divide the market into three types.

Proposition 3(a) states that when the degree of demand-side competition exceeds that of the supply side (the lower right area in Figure 1(a)), the FW scheme generates the greatest platform profit, followed by the FCR scheme and then the DCR scheme. In these cases, the platforms compete more fiercely for consumers than for workers. Intuitively, under the FW scheme, because the platforms choose and commit to wages first, they fix the supply before engaging in price competition, which lessens the competition for demand in the second stage. Essentially, the FW scheme transforms the price competition into one that is constrained by quantity (supply), which is independent of price. In particular, in the extreme case where the platforms do not compete for supply (i.e., $l = 0$), the FW scheme is in line with the existing literature (e.g., Kreps and Scheinkman, 1983), which shows the supply constraint alters the Bertrand (price) competition into a Cournot (quantity) competition, softening the competition between the platforms. On the other hand, neither the FCR nor the DCR scheme creates a supply constraint in the competition for demand, causing the platforms to engage in a more intense price competition than under the FW scheme. Therefore, the FW scheme yields the highest profit for the platforms.

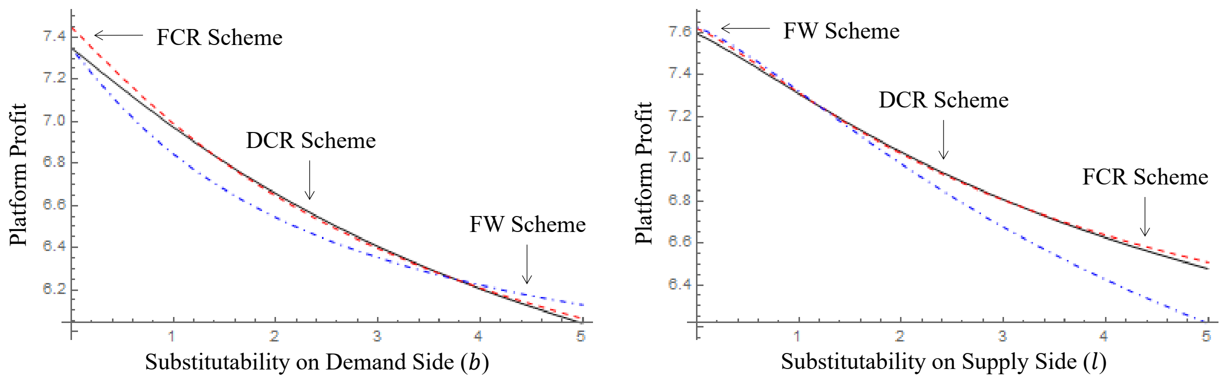
Technically, with the supply commitment in the first stage under the FW scheme, the platforms have incentive to offer a lower wage and set a higher price than under the DCR or FCR scheme. For instance, given Platform 2's wage choice in the first stage under the FW scheme, we consider the marginal effect of Platform 1's unilateral reduction in wage. By unilaterally reducing its wage, the platform secures a lower supply for itself, and can increase its price to account for the decreased supply, which would increase Platform 2's supply and demand. If the degree of demand-side competition exceeds that of the supply side, the increase in Platform's 2's demand exceeds the increase in its supply. As a result, Platform 2 would increase its price, which, in turn, would enhance Platform 1's demand and benefit Platform 1. This additional benefit under the FW scheme, because of precommitment to wage in the first stage, provides the platform incentive to offer a lower wage and charge a higher price, compared to the DCR or FCR scheme.

Further, the FCR scheme bears some similarity to the FW scheme because the platforms choose and commit to commission rates first before engaging in price competition in the second stage. Under the FCR scheme, although the first-stage commission-rate commitment does not fix the supply as under the FW scheme, it restricts the possible change in the platforms' supplies later in the game in a specific manner—an increase (decrease) in price increases (decreases) wage and thus its supply, everything else being equal. Similar to the effect of fixing supply, this restriction can also soften the price competition to some extent. As a result, the price under the FCR scheme is higher than under the DCR scheme, and the FCR scheme yields a higher profit than the DCR scheme does.

In contrast to the cases where the demand-side competition is more intense, when the degree of supply-side competition exceeds that of the demand side, Propositions 3(b) and (c) show that the FW scheme yields the lowest platform profit. In this type of market, the platforms compete more intensely for supply, and the supply-side competition plays a more important role in their profits. Thus, they have relatively less incentive to compete on price. Under the FW scheme, anticipating high prices in the subsequent stage encourages platforms to aggressively acquire supply in the first stage so as to be unconstrained by supply. As a result, the intense supply competition reduces the anticipated benefit from the lucrative business on the demand side, which hurts the platforms. In contrast, under the FCR and DCR schemes, the wage is determined together with price, and the coordination of wage and price enables the platforms to curb supply competition. Therefore, the FW scheme leads to the lowest profits for the platforms among the three wage schemes.

Technically, if the degree of supply-side competition exceeds that of the demand side, the platforms have incentive to offer a higher wage and a lower price under the FW scheme than under the DCR or FCR scheme. Again, we consider the marginal effect of Platform 1's unilateral reduction in wage in the first stage under the FW scheme. By unilaterally reducing its wage, the platform secures a lower supply and can increase its price in the second stage, which would result in a greater increase in supply than in demand for Platform 2. As a result, Platform 2 would decrease its price, which would intensify the price competition and hurt Platform 1, discouraging the platform from reducing the wage. Therefore, the FW scheme leads to the highest wage and lowest platform profit when the degree of supply-side competition exceeds that of the demand side.

Propositions 3(b) and (c) also show that when the degree of supply-side platform competition significantly exceeds that of the demand side, the FCR scheme yields a higher platform profit than the DCR scheme yields. When the former only mildly exceeds the latter, the DCR scheme, rather than the FCR scheme, results in a higher platform profit. To explain these results, we next consider the marginal effect of Platform 1's unilateral increase in the commission rate in the first stage under the FCR scheme. By unilaterally increasing its commission rate, Platform 1 would expect a lower supply for itself and hence would increase its price, which would increase Platform 2's supply and demand. Because the degree of supply-side competition exceeds that of the demand side, Platform 2 would see a greater increase in its supply than demand. As under the FW scheme, Platform 2 then would decrease its price to boost its demand to match the supply. In contrast to the FW scheme, however, a price decrease by Platform 2 might have a positive or a negative effect on Platform 1. On the one hand, Platform 2's price decrease reduces Platform 1's demand and intensifies the price competition, which is negative for Platform 1. On the other hand, Platform 2's price decrease reduces its wage and thus increases Platform 1's supply, lessening the wage competition, which is positive for Platform 1. If the degree of supply-side competition is significantly greater than that of the demand side, the positive effect dominates the negative and the FCR scheme leads to a lower wage and a higher platform profit; If the degree of supply-side competition mildly exceeds that of the demand side, the negative effect dominates and the DCR scheme leads to a higher platform



(a) Effect of Demand-Side Substitutability (b) Effect of Supply-Side Substitutability

Figure 2: Effect of Substitutability on Platform Profits under the Three Wage Schemes
Note: $a = 15$, $k = 1.5$, $c = 2.5$, and $m = 2$; $l = 3$ in (a) and $b = 1.5$ in (b)

profit.

Figure 2 further illustrates the effect of demand-side and supply-side substitutability on platform profits under the three wage schemes. As predicted by Proposition 2, we observe that the platform profit decreases as platform substitutability of either side (i.e., b or l) increases. Consistent with Proposition 3, in a market where the demand-side competition exceeds that of the supply-side (when b is high in Figure 2(a)), the FW scheme generates the highest profits for the platform. As the demand-side substitutability increases, the profit gaps between the FW scheme and the other two schemes increase. Similarly, in a market where the supply-side competition exceeds that of the demand side (when l is high in Figure 2(b)), the FCR scheme generates the highest platform profit. An increase in the supply-side substitutability increases the profit gaps between the FCR scheme and the other two schemes.

5.2 Consumers' and Workers' Wage-Scheme Preferences

We let S_{r} and S_{d} denote the consumer and worker surpluses, respectively, and use superscripts to denote the wage scheme. Proposition 4 summarizes the comparisons of consumer and worker surpluses under the three wage schemes.

Proposition 4. *(a) If $\frac{b}{c} > \frac{l}{m}$, the DCR scheme generates the highest consumer and worker surpluses, and the FW scheme generates the lowest consumer and worker surpluses (i.e., $S_{\mathrm{r}}^{\mathrm{W}*} < S_{\mathrm{r}}^{\mathrm{F}*} < S_{\mathrm{r}}^{\mathrm{D}*}$ and $S_{\mathrm{d}}^{\mathrm{W}*} < S_{\mathrm{d}}^{\mathrm{F}*} < S_{\mathrm{d}}^{\mathrm{D}*}$).*

(b) If $\frac{b}{c} < \frac{l}{m}$ and Condition (13) *holds, the FW scheme generates the highest consumer and*

worker surpluses, and the FCR scheme generates the lowest consumer and worker surpluses (i.e., $S_{\mathrm{r}}^{\mathrm{F}*} < S_{\mathrm{r}}^{\mathrm{D}*} < S_{\mathrm{r}}^{\mathrm{W}*}$ and $S_{\mathrm{d}}^{\mathrm{F}*} < S_{\mathrm{d}}^{\mathrm{D}*} < S_{\mathrm{d}}^{\mathrm{W}*}$).

(c) If $\frac{b}{c} < \frac{l}{m}$ and Condition (13) does not hold, the FW scheme generates the highest consumer and worker surpluses, and the DCR scheme generates the lowest consumer and worker surpluses (i.e., $S_{\mathrm{r}}^{\mathrm{D}*} < S_{\mathrm{r}}^{\mathrm{F}*} < S_{\mathrm{r}}^{\mathrm{W}*}$ and $S_{\mathrm{d}}^{\mathrm{D}*} < S_{\mathrm{d}}^{\mathrm{F}*} < S_{\mathrm{d}}^{\mathrm{W}*}$).

Proposition 4 reveals, as also illustrated by Figure 1(b), that the FCR scheme is never the most preferred scheme for consumers or workers, and the FW scheme is either the least preferred scheme or the most preferred scheme. Two additional observations are worth highlighting.

First, the ordering of the workers' and consumers' surpluses are identical. Two factors explain this observation: (i) Under all schemes, the platforms are induced to balance demand with supply in the equilibrium. Consequently, the number of workers who enjoy a positive surplus is equal to the number of consumers who enjoy a positive surplus. (ii) Any reduction in equilibrium price is accompanied by an increase in wage to match demand and supply. A reduction in price increases the consumer surplus, and an increase in wage increases the worker surplus. As a result, a platform's equilibrium strategy has the same qualitative impact on both workers and consumers.

Second, comparing Propositions 3 and 4, we find that the preferred order for consumers and workers regarding the wage scheme is opposite the platforms' preferred order. Intuitively, on the one hand, a wage scheme offers a higher platform profit than another if and only if the former scheme results in a higher price and lower wage than the latter scheme. On the other hand, the scheme that induces a higher price and lower wage hurts consumers and workers.

5.3 Social-Welfare Comparison of Wage Schemes

We consider social warfare as the sum of consumer surplus, worker surplus, and platform profits. We use SW to represent social welfare. Proposition 5 summarize the social welfare comparison under the three wage schemes.

Proposition 5. *(a) If $\frac{b}{c} > \frac{l}{m}$, the DCR scheme generates the highest social welfare and the FW scheme generates the lowest social welfare (i.e., $SW^{\mathrm{W}*} < SW^{\mathrm{F}*} < SW^{\mathrm{D}*}$).*

(b) If $\frac{b}{c} < \frac{l}{m}$ and Condition (13) holds, the FW scheme generates the highest social welfare and the FCR scheme generates the lowest social welfare (i.e., $SW^{\mathrm{F}*} < SW^{\mathrm{D}*} < SW^{\mathrm{W}*}$).

(c) If $\frac{b}{c} < \frac{l}{m}$ and Condition (13) *does not hold, the FW scheme generates the highest social welfare and the DCR scheme generates the lowest social welfare (i.e., $SW^{\mathrm{D}*} < SW^{\mathrm{F}*} < SW^{W*}$).*

Comparing Propositions 4 and 5, we find that the ordering of the wage schemes with respect to social welfare is identical to that with respect to worker and consumer surpluses. Intuitively, because prices and wages do not affect social welfare—they are simply transfer of money from one party to another within the society—the social welfare depends only on the volume of transactions. The volume of transactions is higher when the equilibrium price is lower. Consequently, the social welfare is higher when the price is lower. Note that the consumer surplus (and the worker surplus) also increases when the price decreases. Therefore, the comparisons of the three wage schemes have the same order with respect to the social welfare and the consumer and worker surpluses.

The results about two-sided competition between the platforms in a sharing economy presented in this section are new to the literature. Moreover, the results also extend the literature that has examined quantity-then-price competition. The extant quantity-then-price competition literature assumes that firms choose quantity first and then engage in price competition, but they do not compete for quantity. The literature finds that price competition is less severe in quantity-then-price competition compared to when quantity is not chosen first. Our model is richer than the traditional quantity-then-price models because competition exists on both sides in our model. Our model reduces to the quantity-then-price model if we set l to zero, which essentially means no competition on the worker side. When $l = 0$, our analysis shows that the platforms prefer the FW scheme, and consumers (and workers) prefer the DCR scheme; that is, platforms prefer a sequential game in which supply is fixed in the first stage and firms compete on price in the second stage, and the consumers prefer the game where there is only a pricing game. These results are consistent with those in the literature. In sharp contrast, when the platforms also compete for the supply, we find that these results do not necessarily hold and may actually be reversed. In particular, in markets where platforms compete more fiercely on the supply side than on the demand side, a game where the quantity and price are chosen simultaneously can result in less intense competition and higher platform profits, and, more importantly, the quantity-then-price game can yield the least platform profits.

6 Model Extensions

In this section, we extend our baseline model in different directions, examine the robustness of our results, and provide additional insights.

6.1 Cross-Side Network Effects

Cross-side network effects are likely to exist in many two-sided markets. For instance, in the ride-sharing context, a rider might have a higher valuation for a platform with more drivers because the rider might have to wait less to find a driver. In the same vein, a driver is likely to have a higher valuation for a platform with more riders. In this section, we extend the baseline model by incorporating such cross-side network effects.

We introduce the cross-side network effects into the baseline model by letting the demand for a platform to be affected not only by prices, but also by the worker supply it has. Analogously, the supply for a platform is affected not only by wages, but also by the consumer demand it has. Specifically, following the existing literature (e.g., Parker and Van Alstyne, 2005), we model the demand and supply for platform i as

$$d_i = a - b(p_i - p_{\bar{i}}) - cp_i + \lambda s_i \tag{14}$$

$$s_i = -k + l(w_i - w_{\bar{i}}) + mw_i + \mu d_i, \tag{15}$$

where λ and μ capture the cross-side network effects, $\lambda, \mu \in [0, 1)$. When $\lambda = 0$ and $\mu = 0$, this extension reduces to the baseline model. We assume that the network effects are not excessive such that λ and μ are bounded away from 1; Otherwise, everything else being equal, adding one worker (consumer) would attract one or more additional consumers (workers) because of the excessive network effects. We can combine the above equations and rewrite the demand and supply as functions of prices and wages as follows.

$$d_i(p_1, p_2, w_1, w_2) = \frac{a - p_i(b + c) + bp_{\bar{i}} - k\lambda + \lambda w_i(l + m) - \lambda l w_{\bar{i}}}{1 - \lambda\mu} \tag{16}$$

$$s_i(p_1, p_2, w_1, w_2) = \frac{-k + w_i(l + m) - lw_{\bar{i}} + a\mu - \mu p_i(b + c) + b\mu p_{\bar{i}}}{1 - \lambda\mu} \tag{17}$$

The assumption that the cross-side network effects are not excessive also ensures the demand and supply functions are well behaved. Specifically, as in the baseline model, a platform's demand decreases in its own price but increases in its competitor price, and its supply increases in its own wage but decreases in its competitor's wage. Further, it is reasonable to assume that the marginal effect of its price on its demand is greater than that of its wage, and the marginal effect of its wage on its supply is greater than that of its price. Similarly, the competitor's price (wage) has greater marginal effect on the platform's demand (supply) than the competitor's wage (price). Altogether, we impose the following conditions on the magnitude of the cross-side network effects, or parameters λ and μ.

$$\begin{cases} \lambda < \min\{\dfrac{b+c}{l+m}, \dfrac{b}{l}, 1\} \\ \mu < \min\{\dfrac{l+m}{b+c}, \dfrac{l}{b}, 1\} \end{cases} \tag{18}$$

Following the same approach as in the baseline model, similar to that in Lemmas 1, 2, and 3, we can derive the equilibrium outcome under the three wage schemes. We provide the analysis and equilibrium in the appendix. Comparing the equilibrium platform profits under the three wage schemes, we can derive the relative profit-generating superiority of these wage schemes. As in Proposition 3, Proposition 6 summarizes this comparison.

Proposition 6. *(a) If $\frac{b}{c} > \frac{l}{m}$, the FW scheme generates the highest platform profit and DCR generates the lowest profit.*

(b) If $\frac{b}{c} < \frac{l}{m}$ and

$$\frac{(1-\mu)(b+c)+(1-\lambda)(l+m)}{(b+c)(l+m)\alpha^{\mathrm{D}*}} - \frac{b(1-\mu)+\left(1-\alpha^{\mathrm{D}*}\right)(1-\lambda)l}{\left(1-\alpha^{\mathrm{D}*}\right)(cl-bm)} > 0, \tag{19}$$

the FCR scheme generates the highest platform profit and the FW scheme generates the lowest profit.

(c) If $\frac{b}{c} < \frac{l}{m}$ and Condition (19) does not hold, the DCR scheme generates the highest platform profit and the FW scheme generates the lowest profit.

As does Proposition 3 in the baseline model, Proposition 6 shows the ordering of wage schemes with respect to the platform profit: When the demand-side competition exceeds that of the supply side, the FW scheme is the best. When the supply side is significantly (mildly) more competitive than the demand side, the FCR (DCR) scheme is the best. Evidently, our main insights are unchanged by the presence of cross-side network effects. Quantitatively, their presence shifts the boundary that separates the regions in the parameter space where "supply side is significantly more competitive than the demand-side" and "supply side is mildly more competitive than the demand-side," as revealed by Condition (19) (vs. Condition (13) in the baseline model). When $\lambda = 0$ and $\mu = 0$, Condition (19) reduces to Condition (13) in the baseline model.

Further, as in the baseline model, we can show the consumers', workers', and a social planner's preference of the wage schemes are in the inverse order of the platforms'; that is, the wage scheme that generates the highest platform profit generates the lowest consumer surplus, lowest worker surplus, and lowest social welfare. Therefore, the cross-side network effects do not change our insights about platform competition and the role of wage scheme in the sharing-economy context.

6.2 Asymmetric Demand or Supply

The baseline model assumes symmetric demand and supply functions for the two platforms. In this extension, we allow the platforms to have different primary demands and supplies. For ease of exposition, we present the key result for the model in which Platform i's market potential on the demand side is a_i, $i \in \{1, 2\}$, instead of being a for both platforms from the baseline model. Other model aspects remain the same as in baseline model. (The model in which the two platforms have different market potentials on the supply side can be similarly analyzed and yields qualitatively similar results as the one presented here.)

As in the baseline model, we can derive the equilibrium outcome under the three wage schemes. We provide the analysis and equilibrium in the appendix. Comparing the equilibrium platform profits under the three wage schemes, we can derive the relative profit-generating superiority of these wage schemes. As in Proposition 3, Proposition 7 summarizes this comparison, in which $\alpha_i^{\mathrm{D}*}$ denotes platform i's equilibrium commission rate under the DCR scheme.

Proposition 7. *(a) If $\frac{b}{c} > \frac{l}{m}$, the FW scheme generates the highest profit and the DCR scheme generates the lowest profit for both platforms.*

(b) If $\frac{b}{c} < \frac{l}{m}$ and

$$\frac{b+c+l+m}{(b+c)(l+m)\alpha_i^{\mathrm{D}*}}\left[cl\left(1-\alpha_{\bar{i}}^{\mathrm{D}*}\right)-bm\left(1-\alpha_i^{\mathrm{D}*}\right)+bl\left(\alpha_i^{\mathrm{D}*}-\alpha_{\bar{i}}^{\mathrm{D}*}\right)\right]-\left[b+\left(\left(1-\alpha_{\bar{i}}^{\mathrm{D}*}\right)\right)l\right]>0 \quad (20)$$

for $i \in \{1,2\}$, the FCR scheme generates the highest profit and the FW scheme generates the lowest profit for both platforms.

(c) If $\frac{b}{c} < \frac{l}{m}$ and

$$\frac{b+c+l+m}{(b+c)(l+m)\alpha_i^{\mathrm{D}*}}\left[cl\left(1-\alpha_{\bar{i}}^{\mathrm{D}*}\right)-bm\left(1-\alpha_i^{\mathrm{D}*}\right)+bl\left(\alpha_i^{\mathrm{D}*}-\alpha_{\bar{i}}^{\mathrm{D}*}\right)\right]-\left[b+\left(1-\alpha_{\bar{i}}^{\mathrm{D}*}\right)l\right]<0 \quad (21)$$

for $i \in \{1,2\}$, the DCR scheme generates the highest profit and the FW scheme generates the lowest profit for both platforms.

(d) Otherwise, either the FCR or DCR scheme generates the highest profit and the FW scheme generates the lowest profit for both platforms.

Under asymmetric demands, the two platforms have different profits under each wage scheme. However, Proposition 7 reveals that the platforms' preference order for the three wage schemes remains qualitatively the same as in the baseline model. For example, as in Proposition 3, when the demand-side competition exceeds that of the supply side, the FW scheme generates the highest profit for both platforms. Moreover, as in the baseline model, the consumers', workers', and a social planner's preferences for the wage schemes are in the inverse order of the platforms': The wage scheme that generates the highest platform profit generates the lowest consumer surplus, lowest worker surplus, and lowest social welfare. Therefore, our insights about platform competition and the role of wage scheme in the sharing-economy context from the baseline model are robust to asymmetric demand or supply. The quantitative difference from the baseline model lies in that, when $b/c < l/m$, but neither (20) nor (21) is satisfied, depending on market characteristics, either the FCR or DCR scheme can generate the highest platform profit (Proposition 7(d)).

6.3 Platforms' Wage-Scheme Choices

In the baseline model, we assume that the two platforms use the same wage scheme. They may use the same scheme because, for example, they may simply want to follow the prevailing practice at the time or have to abide by government regulations. Theoretically, the wage scheme can be a platform's strategic choice and the two platforms could potentially choose different schemes. In this extension, we examine the platforms' choices regarding the wage scheme by adding Stage 0 to the game in the baseline model where the platforms choose their wage schemes simultaneously. We denote (μ_1, μ_2) as the platforms' choice or action profile in Stage 0, where $\mu_i \in \{\text{FW, FCR, DCR}\}$, representing platform i's wage-scheme choice, $i \in \{1, 2\}$.

We note that each platform has three wage-scheme choices, leading to a 3×3 payoff matrix in Stage 0. With the complexity already in the baseline model, this super game is technically intractable. In particular, we are unable to analytically compare the platform profits for choice (FW, FCR) (or (FCR, FW) because of symmetry) with other choices. Hence, we focus on and provide the equilibrium result for the case where each platform chooses between FW and DCR. The appendix provides the closed-form equilibrium results for this case, and also illustrates the equilibrium wage-scheme choices based on numerical computation when each platform considers all three schemes.

When each platform chooses between FW and DCR in Stage 0, the equilibrium analysis for the subgames in which the platforms choose the same wage scheme remains the same as in the baseline model. In the subgames in which Firm i chooses FW and Firm $\bar{i}$ chooses DCR, we assume that Platform i decides its wage in Stage 1, and, in Stage 2, Platform i decides its price and Platform $\bar{i}$ chooses its price and wage. As in the baseline model, we can similarly derive the equilibrium competition outcome and the resulting platform profits.

Proposition 8. *When each platform can choose between FW and DCR both platforms, in equilibrium, choose the FW scheme if*

$$\frac{(2b+c)(b(2c+m)+c(c+l+m))}{c(b+c)(b+c+l+m)(c(2b+c)l+2(c(c+l)+b(2c+l))m+(3b+2c)m^2)^2} > \frac{Y^2}{4X^2}, \tag{22}$$

where X and Y are defined in the appendix. Otherwise, one platform chooses the FW scheme and the other chooses the DCR scheme.

Figure 3(a) illustrates Condition (22) under which both platforms choose the FW scheme in equilibrium: when the demand side is significantly more competitive than the supply side, (FW, FW) arises as an equilibrium. As discussed in the baseline model, when the demand-side competition exceeds that of the supply side (below the grey dashed line in Figure 3(a)), the wage-scheme profile (FW, FW), under which the platforms commit to the quantities first, enables them to curb the excessive price competition in the subsequent stage, benefiting the platforms. Further, this benefit depends on the degree of demand-side competition. Meanwhile, differentiation in wage scheme can

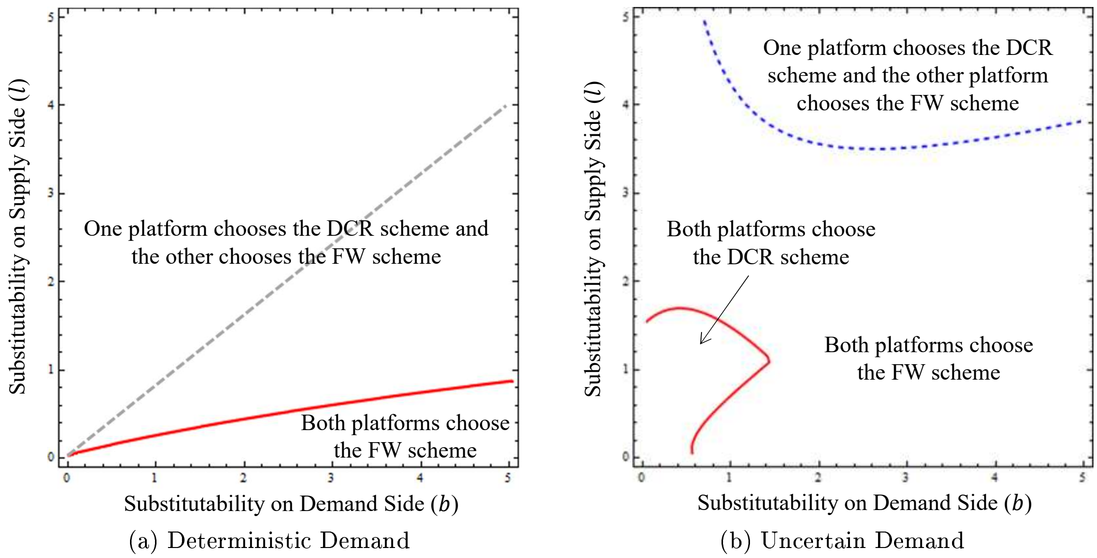


Figure 3: Equilibrium Wage-Scheme Choices
Note: $a = 15$, $k = 1.5$, $c = 2.5$, and $m = 2$; $\epsilon = 0.3$ in (b)

also soften competition and benefit the platforms. When the supply side is more competitive, the asymmetric scheme is more effective than quantity commitment in softening competition, and thus (FW, FW) does not arise as an equilibrium.

Proposition 8 reveals that (DCR, DCR) is not an equilibrium, which suggests that when one platform chooses the DCR scheme, the other platform has an incentive to choose a different wage scheme. Intuitively, asymmetric wage-scheme choices, or differentiation in wage scheme, can soften the subsequent competition between the two platforms on one or both sides of the market compared to the (DCR, DCR) case.

It is worth noting that the fact that (DCR, DCR) cannot arise as an equilibrium is an artifact of specific model assumptions. The DCR scheme gives platforms the flexibility to adjust wages offered to workers without any commitment, and it has its own embedded advantage, especially when the demand is uncertain. For the purpose of illustration, we consider that the platforms face uncertain demand in Stage 1 of the game. Specifically, the demand functions are given by

$$\begin{cases} d_1 = A - b(p_1 - p_2) - cp_1 \\ d_2 = A - b(p_2 - p_1) - cp_2, \end{cases} \tag{23}$$

where the value of consumer market potential A is not observed until Stage 2 of the game. We assume that A is equal to either $a + \epsilon$ or $a - \epsilon$ with equal probability, and the platforms know this distribution in Stage 1 based on historical data. If both platforms adopt the DCR scheme, such demand uncertainty does not affect our analysis because the platforms make decisions only in Stage 2, although the equilibrium outcomes would depend on the realized value for demand. If a platform chooses the FW scheme in Stage 0, the platform decides the wage offered to workers without observing the value of the consumer market potential in Stage 1. In Stage 2, the firm observes the realization of the value and decides the price. We find that (DCR, DCR) can arise as the equilibrium in this case. Figure 3(b) illustrates the equilibrium wage choices. The (DCR, DCR) equilibrium is likely to occur when the degree of platform substitution is low on both consumer and worker sides. Intuitively, postponing the wage decision until the realized demand is observed can be beneficial to both platforms if neither the demand-side nor the supply-side competition is intense.

7 Conclusion

Motivated by the increasing popularity of sharing platforms, we examine two-sided platform competition in a sharing-economy context where workers self-schedule supply and a platform profits only when it matches a worker with a consumer. We consider three wage schemes: *fixed commission rate*, *dynamic commission rate*, and *fixed wage*, and examine the implications of the wage scheme on platforms, consumers, workers, and society. We find that platform competition is more intense in sharing markets than markets with a fixed supply if the sharing market serves more consumers than the fixed supply, regardless of the wage scheme employed by the platforms. Meanwhile, significant differences exist in the implications of the three wage schemes for each involved player, including platforms, consumers, and workers. We find that which wage scheme benefits the platforms (or the consumers and workers) depends critically on whether the supply-side or demand-side competition is more fierce, and that the preferences of the workers, consumers, and society regarding the wage scheme are opposite that of the platforms.

Our findings generate several managerial implications. First, our study highlights the nuanced but important difference between competition in a sharing economy and that in a traditional economy. Given the newness of the sharing economy, it is imperative for platforms and users to understand the market, especially given the complexities associated with the competitive environment and the various wage schemes that have been employed by platforms. For instance, our results suggest that, different from the conventional wisdom, the competition for workers on the supply side in a sharing economy can work to reduce the competition on the demand side as compared to the standard taxi market with employee workers. Consequently, from the perspective of consumers, the sharing economy, while convenient in other respects, may actually hurt them by reducing competition if the transaction volume is limited.

Second, our study underscores the importance of market characteristics in the superiority of different wage schemes for platforms. Although the dynamic-commission-rate scheme gives platforms the flexibility to adjust the wages offered to workers without any commitment, such flexibility does not necessarily benefit platforms—the flexibility benefits platforms only in the markets where the supply is mildly more competitive than the demand. In markets where the supply is highly competitive, platforms are better off with precommitment to commission rates that tie wages offered to workers to prices charged to consumers. In the markets where the demand is more competitive, platforms should even embrace the fixed-wage scheme; it gives them the least flexibility, but it works as a competition reducer.

Third, our study reveals that consumers' and workers' wage-scheme preference order is misaligned with platforms', which calls for the social planner's attention. Some platforms' pricing practices resemble the dynamic commission rate and have generated bad publicity because it seems that the platforms were not passing on any part of surge prices to workers (Smith, 2017). Our finding indicates that the outcry from workers about the dynamic commission rate scheme is not always warranted. In fact, in the markets where the demand is more competitive, workers are better off if platforms adopt a dynamic commission rate. In the markets where the supply is more competitive, workers should seek a fixed wage if they have bargaining power to influence platforms. When workers and consumers have no bargaining power over platforms or when no agreement can be made between them, a government agency such as the Federal Trade Commission (FTC) can provide guidelines or regulations. Our results suggest that the superiority of different wage schemes in generating social welfare, workers' surplus, and consumers' surplus is consistent. To protect platform users' surpluses, the FTC can tailor policies to market characteristics—in the markets where the competition on the demand side is more fierce than on the supply side, a dynamic commission rate could be advocated, whereas in the markets where the supply side is more competitive, the fixed wage scheme could be advocated. Finally, regulations such as a minimum wage have the potential to mitigate the misalignment between the platforms, consumers, and workers.

We have kept the model parsimonious to focus on the impact of self-scheduled supply and wage schemes under platform competition. More importantly, an essential feature of our model is that the platform sets prices on both sides of the market. Such a feature is present in many sharing-economy contexts such as ride sharing, peer-to-peer lending, car sharing, and food delivery. While our parsimonious economic model can be used as a general framework, we note that incorporating idiosyncratic context features would provide additional insights about platform competition in specific sharing-economy contexts.

Our research can be extended in several directions. First, incorporating policy-related factors could lead to additional insights. For instance, the policy of a minimum wage for suppliers is likely to have significant effects on different stakeholders in sharing economies. These merit further investigation. Second, in this paper, we consider profit maximization as the platforms' sole objective. In some special situations (e.g., during a platform launch), the platforms might have different short-term objectives, such as maximizing the number of transactions or the social welfare. Studying these types of markets can complement our current work. Third, other contexts of sharing economies, such as Airbnb where the service providers set prices (instead of platforms), can also be examined to study the impact of self-scheduled supply and competition among providers.

References

Ahmadinejad, AmirMahdi, Hamid Nazerzadeh, Amin Saberi, Nolan Skochdopole, Kane Sweeney. 2019. Competition in ride-hailing markets. Working Paper.

Allon, Gad, Achal Bassamboo, Eren B Çil. 2012. Large-scale service marketplaces: The role of the moderating firm. *Management Science* **58**(10) 1854–1872.

Armstrong, Mark. 2006. Competition in two-sided markets. *The RAND Journal of Economics* **37**(3) 668–691.

Armstrong, Mark, Julian Wright. 2007. Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory* **32**(2) 353–380.

Basu, Amit, Sreekumar Bhaskaran, Rajiv Mukherjee. 2019. An analysis of search and authentication strategies for online matching platforms. *Management Science* **65**(5) 2412–2431.

Bernstein, Fernando, Gregory DeCroix, N Bora Keskin. 2018. Competition between two-sided platforms under demand and supply congestion effects. Working paper.

Besen, Stanley M, Joseph Farrell. 1994. Choosing how to compete: Strategies and tactics in standardization. *The Journal of Economic Perspectives* **8**(2) 117–131.

Botsman, Rachel. 2015. Defining the sharing economy: What is collaborative consumption–and what isn't. *Fastcoexist.com* **27**.

Bryan, Kevin A, Joshua S Gans. 2019. A theory of multihoming in rideshare competition. *Journal of Economics & Management Strategy* **28**(1) 89–96.

Cachon, Gerard P, Kaitlin M Daniels, Ruben Lobel. 2017. The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management* **19**(3) 368–384.

Caillaud, B, B Julien. 2003. Competing in network industries: Divide and conquer. *The RAND Journal of Economics* **34**(6) 309–328.

Choi, S. Chan. 1991. Price competition in a channel structure with a common retailer. *Marketing Science* **10**(4) 271–296.

Davidson, Carl, Raymond Deneckere. 1986. Long-run competition in capacity, short-run competition in price, and the Cournot model. *The Rand Journal of Economics* **17**(3) 404–415.

Economides, Nicholas, Evangelos Katsamakas. 2006. Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry. *Management Science* **52**(7) 1057–1071.

Eisenmann, Thomas R. 2006. Winner-take-all in networked markets. *Harvard Business School Technical Note* 806–131.

Fraiberger, Samuel P, Arun Sundararajan. 2007. Peer-to-peer rental markets in the sharing economy. Working paper.

Garg, Nikhil, Hamid Nazerzadeh. 2021. Driver surge pricing. *Management Science* Published online in Articles in Advance.

Gurvich, Itai, Martin Lariviere, Antonio Moreno. 2016. Operations in the on-demand economy: Staffing services with self-scheduling capacity. Working paper.

Hu, Ming, Yun Zhou. 2016. Dynamic type matching. Working paper.

Hu, Ming, Yun Zhou. 2019. Price, wage and fixed commission in on-demand matching. Working paper.

Kreps, David M, Jose A Scheinkman. 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. *The Bell Journal of Economics* 326–337.

Kwark, Young, Jianqing Chen, Srinivasan Raghunathan. 2014. Online product reviews: Implications for retailers and competing manufacturers. *Information Systems Research* **25**(1) 93–110.

Lee, Eocman, Jeho Lee, Jongseok Lee. 2006. Reconsideration of the winner-take-all hypothesis: Complex networks and local bias. *Management Science* **52**(12) 1838–1848.

Levin, Jonathan, Andrzej Skrzypacz. 2016. Platform pricing for ride-sharing. Working paper, Stanford University.

Liu, Qihong, Oksana Loginova, X Henry Wang. 2017. The impact of multi-homing in a ride-hailing market. Working paper.

Nikzad, Afshin. 2017. Thickness and competition in ride-sharing markets. Working paper.

Osborne, Martin J, Carolyn Pitchik. 1986. Price competition in a capacity-constrained duopoly. *Journal of Economic Theory* **38**(2) 238–260.

Parker, Geoffrey G, Marshall W Van Alstyne. 2005. Two-sided network effects: A theory of information product design. *Management Science* **51**(10) 1494–1504.

Riquelme, C, S Banerjee, R Johari. 2015. Pricing in ride-share platforms: A queueing-theoretic approach. Tech. rep., Working paper, Cornell University.

Rochet, Jean-Charles, Jean Tirole. 2003. Platform competition in two-sided markets. *Journal of the European Economic Association* **1**(4) 990–1029.

Smith, Chris. 2017. Uber has allegedly been cheating both riders and drivers. URL `http://nypost.com/2017/04/07/uber-has-allegedly-been-cheating-both-riders-and-drivers/`. Accessed 10.24.2017.

Stand. 2019. New report: Uber and Lyft take more, pay drivers less. URL `http://www.thestand.org/2019/05/new-report-uber-and-lyft-take-more-pay-drivers-less/`. Accessed 07.09.2019.

Taylor, Terry A. 2018. On-demand service platforms. *Manufacturing & Service Operations Management* **20**(4) 704–720.

Van Mieghem, Jan A, Maqbool Dada. 1999. Price versus production postponement: Capacity and competition. *Management Science* **45**(12) 1639–1649.

Wu, Shining, Shihong Xiao, Saif Benjaafar. 2020. Two-sided competition between on-demand service platforms. Working paper.

Zervas, Georgios, Davide Proserpio, John W Byers. 2017. The rise of the sharing economy: Estimating the impact of Airbnb on the hotel industry. *Journal of Marketing Research* **54**(5) 687–705.

Zhu, Feng, Marco Iansiti. 2012. Entry into platform-based markets. *Strategic Management Journal* **33**(1) 88–106.

A Appendix: Proofs of Main Results

A.1 Key Characteristics of Well-known Two-Sided Platforms

A.2 Demand/Supply Function and Consumer/Worker Surplus

Demand/Consumers. We consider that each consumer has a unit demand and the market consists of loyal and competitive segments of consumers. Each of the two platforms has β_{r} loyal consumers who only use the platform they are loyal to if they use the service. We assume the demand for

Table 1: Features of Some Well-Known Two-Sided Platforms

Name	Business	Prices set by	Wages set by	Wage Scheme
GetAround[a]	Car sharing	The platform[b]	The platform	FCR[c]
Turo	Car sharing	The platform[d]	The platform	FCR[e]
Uber	Ride sharing	The platform	The platform	FCR[f]
Lyft	Ride sharing	The platform	The platform	DCR[g]
Prosper	Peer-to-peer lending	The platform	The platform[h]	DCR[i]
Upstart	Peer-to-peer lending	The platform	The platform	DCR[j]
DoorDash	Food Delivery	The platform	The platform	FW[k]
Uber Eats	Food Delivery	The platform[l]	The platform	FW[m]
Upwork	Multiple categories[n]	Consumer or worker	Consumer or worker	FCR[o]
Freelancer	Multiple categories[p]	Consumer or worker	Consumer or worker	FCR[q]
Amazon Mechanical Turk	Data entry[r]	Consumer	Consumer	FCR[s]
CrowdSpring	Multiple categories[t]	The platform[u]	Consumer	FW

[a] We set in bold the names of the platforms on which the prices and wages are set by the platforms, a key feature of our model.

[b] https://help.getaround.com/hc/en-us/articles/115012600288-Predictive-Pricing-

[c] https://www.getaround.com/terms/fee-commission-schedule

[d] https://support.turo.com/hc/en-us/articles/203991990-Setting-your-vehicle-price-

[e] https://support.turo.com/hc/en-us/articles/203992000-Earnings-and-payment

[f] https://www.uber.com/gh/en/drive/basics/tracking-your-earnings/

[g] https://help.lyft.com/hc/e/articles/115013080008-How-ride-earnings-are-calculated

[h] https://www.prosper.com/loans/rates-and-fees/?refac=CANMB\&refmc=6YRANV\&refd=prosperblog

[i] https://prosper.zendesk.com/hc/en-us/articles/210013543-Are-there-any-fees-to-invest-through-Prosper-

[j] https://files.consumerfinance.gov/f/documents/201709_cfpb_upstart-no-action-letter-request.pdf

[k] https://help.doordash.com/dashers/s/article/Peak-Pay?language=en_US

[l] https://www.nzherald.co.nz/business/surge-charges-introduced-to-uber-eats-why-food-delivery-is-set-to-get-more UC7C4ATCJPY5ET2HWGOG65PO7A/

[m] https://help.uber.com/driving-and-delivering/article/delivery-surge-faq?nodeId=63ead36d-ac64-47d0-ae51-bf1307501265

[n] https://www.upwork.com/services/

[o] https://support.upwork.com/hc/en-us/articles/211062538-Freelancer-Service-Fees

[p] https://www.freelancer.com/job/

[q] https://www.freelancer.com/feesandcharges

[r] https://www.mturk.com/

[s] https://www.mturk.com/pricing

[t] https://www.crowdspring.com/

[u] https://www.crowdspring.com/pricing/

Platform i from its loyal segment is $d_i^l = \beta_\mathrm{r} - cp_i$. We assume the competitive segment of size α_r and use the Hotelling model to capture the competition for this segment. Specifically, we assume that Platform 1 is located at Point 0 and Platform 2 is at Point 1. The consumers are uniformly distributed over the interval $[0,1]$. The distance between a consumer and a platform measures the degree of misfit between them. Each consumer has a valuation v for the service and incurs misfit cost when using a platform. The misfit cost is the distance between the consumer and the platform times unit misfit cost t_r. Therefore, the utilities for the consumer located at x from using each platform can be formulated as

$$\begin{cases} u_1^{\mathrm{r}}(x) = v - t_{\mathrm{r}}x - p_1 \\ u_2^{\mathrm{r}}(x) = v - t_{\mathrm{r}}(1-x) - p_2. \end{cases} \tag{24}$$

By $u_1^{\mathrm{r}}(x^*) = u_2^{\mathrm{r}}(x^*)$, we can derive the indifference point $x^* = \frac{1}{2} - \frac{p_1 - p_2}{2t_{\mathrm{r}}}$. The consumers on the left of the indifference point choose Platform 1 and the others choose Platform 2. Aggregating the demands from the loyal and competitive segments, we can derive the demand for each platform as follows.

$$\begin{cases} d_1 = \left(\frac{\alpha_{\mathrm{r}}}{2} + \beta_{\mathrm{r}}\right) - \frac{\alpha_{\mathrm{r}}}{2t_{\mathrm{r}}}(p_1 - p_2) - cp_1 \\ d_2 = \left(\frac{\alpha_{\mathrm{r}}}{2} + \beta_{\mathrm{r}}\right) - \frac{\alpha_{\mathrm{r}}}{2t_{\mathrm{r}}}(p_2 - p_1) - cp_2 \end{cases} \tag{25}$$

If we denote $a \equiv \frac{\alpha_{\mathrm{r}}}{2} + \beta_{\mathrm{r}}$ and $b \equiv \frac{\alpha_{\mathrm{r}}}{2t_{\mathrm{r}}}$, Equation (25) becomes the demand function in Equation (1).

We next compute consumer surplus. For the loyal segment under price p^*, by the classical theory of monopoly markets (e.g., Tirole, 1988), we can derive consumer surplus as

$$\int_0^{\beta_{\mathrm{r}} - cp^*} \left(\frac{\beta_{\mathrm{r}} - q}{c} - p^*\right) dq = \frac{(\beta_{\mathrm{r}} - cp^*)^2}{2c}. \tag{26}$$

For the competitive segment, if both platforms charge the same price p^* (as in our equilibrium), the consumers located in the interval $[0, \frac{1}{2}]$ choose Platform 1, and these consumers' surplus is

$$\alpha_{\mathrm{r}} \int_0^{\frac{1}{2}} (v - t_{\mathrm{r}}x - p^*)\, dx = \alpha_{\mathrm{r}} \left(\frac{v - p^*}{2} - \frac{t_{\mathrm{r}}}{8}\right). \tag{27}$$

Aggregating the consumer surpluses from both segments on the two platforms, we can derive the total consumer surplus as

$$S_{\mathrm{r}} = \alpha_{\mathrm{r}} \left(v - p^* - \frac{t_{\mathrm{r}}}{4}\right) + \frac{(\beta_{\mathrm{r}} - cp^*)^2}{c}. \tag{28}$$

Supply/Workers. Similarly, we consider that the market consists of loyal and competitive segments of workers. We model Platform i's worker supply from its loyal segment as $s_i^l = -\beta_d + mw_i$, and we use the standard Hotelling model to model the competition in the competitive segment. We denote α_{d} as the size of this segment, t_{d} as the unit misfit cost, and c as each worker's cost of offering the service. As before, Platform 1 is located at Point 0 and Platform 2 is at Point 1, and the workers are uniformly distributed over the interval $[0,1]$. The worker located at the point x derives utility $u_1^{\mathrm{d}}(x) = w_1 - c - t_{\mathrm{d}}x$ if it serves on Platform 1 and $u_2^{\mathrm{d}}(x) = w_2 - c - t_{\mathrm{d}}(1-x)$ if it serves on Platform 2. By $u_1^{\mathrm{d}}(x^*) = u_2^{\mathrm{d}}(x^*)$, we can derive the indifference point $x^* = \frac{1}{2} + \frac{w_1 - w_2}{2t_d}$. The workers to the left of the indifference point choose Platform 1 and the others choose Platform 2. Aggregating the supplies from the loyal and competitive segments, we can derive the supply for each platform as follows.

$$\begin{cases} s_1 = \left(-\beta_{\mathrm{d}} + \frac{\alpha_{\mathrm{d}}}{2}\right) + \frac{\alpha_{\mathrm{d}}}{2t_{\mathrm{d}}}(w_1 - w_2) + mw_1 \\ s_2 = \left(-\beta_{\mathrm{d}} + \frac{\alpha_{\mathrm{d}}}{2}\right) + \frac{\alpha_{\mathrm{d}}}{2t_{\mathrm{d}}}(w_2 - w_1) + mw_2. \end{cases} \tag{29}$$

If we denote $k \equiv \beta_{\mathrm{d}} - \frac{\alpha_{\mathrm{d}}}{2}$ and $l \equiv \frac{\alpha_{\mathrm{d}}}{2t_{\mathrm{d}}}$, Equation (29) becomes the supply function in Equation (2).

We next derive worker surplus. For the loyal segment under wage w^*, similar to consumer surplus, we can derive worker surplus as

$$\int_0^{-\beta_{\mathrm{d}}+mw^*} \left(w^* - \frac{s + \beta_{\mathrm{d}}}{m}\right) ds = \frac{(mw^* - \beta_{\mathrm{d}})^2}{2m}. \tag{30}$$

For the competitive segment, if both platforms offer the same wage w^* (as in our equilibrium), the workers located in the interval $[0, \frac{1}{2}]$ choose Platform 1, and the surplus of these workers is

$$\alpha_{\mathrm{d}} \int_0^{\frac{1}{2}} (w^* - c - t_{\mathrm{d}}x)\, dx = \alpha_{\mathrm{d}} \left(\frac{w^* - c}{2} - \frac{t_{\mathrm{d}}}{8}\right). \tag{31}$$

Aggregating the worker surpluses from both segments on the two platforms, the total worker surplus is

$$S_{\mathrm{d}} = \alpha_{\mathrm{d}} \left(w^* - c - \frac{t_{\mathrm{d}}}{4}\right) + \frac{(mw^* - \beta_{\mathrm{d}})^2}{m}. \tag{32}$$

A.3 Proof of Lemma 1

Proof. We first show $d_i = s_i$ in Platform i's optimal choice because neither $d_i > s_i$ nor $d_i < s_i$ can be optimal. If $d_i > s_i$, Platform i can slightly increase p_i (which slightly reduces the demand) such that $\min\{d_i, s_i\}$ remains unaltered, which increases its profit by Equation (3). If $d_i < s_i$, Platform i can slightly decrease w_i (which slightly reduces the supply) such that $\min\{d_i, s_i\}$ remains unaltered, which increases its profit by Equation (3).

Next, we derive the equilibrium choices. Notice that simultaneously choosing p_i and α_i is equivalent to simultaneously choosing p_i and w_i because $w_i = (1-\alpha_i)p_i$. Because $d_i = s_i$ under the optimal choice, the optimization problem in Equation (3) can be converted to

$$\max_{(p_1,w_1)} (p_i - w_i)\, d_i$$
$$\text{s.t. } d_i = s_i.$$

We use the standard Lagrange multiplier method to solve this constrained optimization problem. The Lagrange function is $L(p_i, w_i, \lambda) = (p_i - w_i)d_i + \lambda(d_i - s_i)$. By the first-order conditions, the optimal choice is given by the following system of equations.

$$\begin{cases} \frac{\partial L}{\partial p_i} = d_i + \frac{\partial d_i}{\partial p_i}(p_i - w_i) + \lambda \frac{\partial d_i}{\partial p_i} = 0 \\ \frac{\partial L}{\partial w_i} = -d_i - \lambda \frac{\partial s_i}{\partial w_i} = 0 \\ \frac{\partial L}{\partial \lambda} = d_i - s_i = 0 \end{cases} \tag{33}$$

Noting that $\frac{\partial d_i}{\partial p_i} = -b - c$ and $\frac{\partial s_i}{\partial w_i} = l + m$, we can solve this equation system and derive Platform i's best response to Platform $\bar{i}$'s choice:

$$p_i^*(p_{\bar{i}}, w_{\bar{i}}) = \frac{a(2b+2c+l+m) + k(b+c) + bp_{\bar{i}}(2b+2c+l+m) + lw_{\bar{i}}(b+c)}{2(b+c)(b+c+l+m)}$$
$$w_i^*(p_{\bar{i}}, w_{\bar{i}}) = \frac{a(l+m) + k(b+c+2(l+m)) + lw_{\bar{i}}(b+c+2(l+m)) + bp_{\bar{i}}(l+m)}{2(l+m)(b+c+l+m)}.$$

Combining the best-response functions of Platforms 1 and 2, we can derive p_i^* and w_i^* as

$$\begin{cases} p_i^* = \frac{k(b+c)(l+m) + a[m(l+m) + (b+c)(l+2m)]}{c(b+c)(l+2m) + m(b+2c)(l+m)} \\ w_i^* = \frac{b[(a+k)(l+m) + ck] + c[(a+2k)(l+m) + ck]}{c(b+c)(l+2m) + m(b+2c)(l+m)}. \end{cases} \tag{34}$$

We can verify that the second-order condition is satisfied. Noticing that $w_i = (1-\alpha_i)p_i$, we can derive p^* and α^* as in the lemma. Assumption 1 ensures $\alpha^* > 0$. □

A.4 Proof of Lemma 2

Proof. We first consider the price competition in the second stage. In this stage, neither platform has incentive to choose p_i such that $s_i < d_i$, because otherwise the platform can slightly increase its price to increase its profit (yielding both higher profit margin and higher transaction volume because the higher price raises the wage paid and therefore the supply). In other words, in equilibrium, $s_i \geq d_i$. We next distinguish different regions of (α_1, α_2) where $s_i > d_i$ for $i \in \{1,2\}$, $s_i = d_i$ for $i \in \{1,2\}$, or $s_i > d_i$ but $s_{\bar{i}} = d_{\bar{i}}$.

(a) In the region where the platforms choose p_i such that $s_i > d_i$, the optimal prices are characterized by the first-order conditions of the profit functions in Equation (3):

$$\begin{cases} \frac{\partial \pi_1}{\partial p_1} = a - b\left(p_1 - p_2\right) - cp_1 - (b+c)p_1 = 0 \\ \frac{\partial \pi_2}{\partial p_2} = a - b\left(p_1 - p_2\right) - cp_2 - (b+c)p_2 = 0. \end{cases} \tag{35}$$

Solving the equations, we can derive the optimal prices as

$$p_1^*\left(\alpha_1, \alpha_2\right) = p_2^*\left(\alpha_2, \alpha_1\right) = p^* = \frac{a}{b+2c}, \tag{36}$$

and thus $d_i = a - cp^*$. Given the commission rate α_i chosen in the first stage, each platform's supply is

$$\begin{cases} s_1 = -k + l\left(\alpha_2 - \alpha_1\right)p^* + m\left(1 - \alpha_1\right)p^* \\ s_2 = -k + l\left(\alpha_1 - \alpha_2\right)p^* + m\left(1 - \alpha_2\right)p^*. \end{cases} \tag{37}$$

Solving $s_i > d_i$, we can derive the following conditions.

$$\begin{cases} \alpha_1 < \frac{a(m-b-c)-k(b+2c)+al\alpha_2}{a(l+m)} \\ \alpha_2 < \frac{a(m-b-c)-k(b+2c)+al\alpha_1}{a(l+m)} \end{cases} \tag{38}$$

The two curves defined in Condition (38) intersect at

$$\alpha_1 = \alpha_2 = 1 - \frac{a(b+c)+k(b+2c)}{am} \equiv \bar{\alpha}. \tag{39}$$

and any α_i in the region defined by Condition (38) satisfies $\alpha_i < \bar{\alpha}$. If and only if (α_1, α_2) is in this region, $s_i > d_i$, $i \in \{1,2\}$, can arise as an equilibrium in the pricing subgame.

(b) In the region where the platforms choose p_i such that $s_i = d_i$, by Equations (1) and (2), we can derive the prices as

$$\begin{cases} p_1^*(\alpha_1,\alpha_2) = \frac{(a+k)[2b+c+(1-\alpha_2)(2l+m)]}{(2-\alpha_1-\alpha_2)[m(b+c)+cl]+(1-\alpha_1)(1-\alpha_2)m(2l+m)+c(2b+c)} \\ p_2^*(\alpha_2,\alpha_1) = \frac{(a+k)[2b+c+(1-\alpha_1)(2l+m)]}{(2-\alpha_1-\alpha_2)[m(b+c)+cl]+(1-\alpha_1)(1-\alpha_2)m(2l+m)+c(2b+c)}. \end{cases} \tag{40}$$

To ensure it is the equilibrium, we must show that neither platform has incentive to increase its price. We next check the nondeviation condition. When increasing its price, Platform i's profit function can be formulated as

$$\pi_i = \alpha_i p_i \min\{d_i, s_i\} = \alpha_i p_i d_i = \alpha_i p_i \left[a - b\left(p_i - p_{\bar{i}}^*\right) - cp_i\right]. \tag{41}$$

By the first-order condition, the solution to the unconstrained optimization problem in Equation (41) is $p_i^{\text{dev}} = \frac{a+bp_{\bar{i}}^*}{2(b+c)}$. Because of the concavity of the objective function, neither platform has incentive to deviate if and only if $p_1^{\text{dev}} \le p_1^*$ and $p_2^{\text{dev}} \le p_2^*$, or, equivalently, if and only if

$$\begin{cases} A_1 = \; a\left[(2-\alpha_1-\alpha_2)\left[m(b+c)+cl\right]+(1-\alpha_1)(1-\alpha_2)m(2l+m)+c(2b+c)\right] \\ \qquad -b(\alpha_1-\alpha_2)(a+k)(2l+m)-(b+2c)(a+k)\left[2b+c+(1-\alpha_2)(2l+m)\right] \le 0 \\ A_2 = \; a\left[(2-\alpha_1-\alpha_2)\left[m(b+c)+cl\right]+(1-\alpha_1)(1-\alpha_2)m(2l+m)+c(2b+c)\right] \\ \qquad -b(\alpha_2-\alpha_1)(a+k)(2l+m)-(b+2c)(a+k)\left[2b+c+(1-\alpha_1)(2l+m)\right] \le 0 \end{cases} \tag{42}$$

In the symmetric case with $\alpha_1 = \alpha_2$, Condition (42) reduces to $\alpha_i \ge \bar{\alpha}$ (where $\bar{\alpha}$ is defined in Equation (39)). Further, we can verify that $\frac{\partial A_i}{\partial \alpha_i} < 0$ and $\frac{\partial A_i}{\partial \alpha_{\bar{i}}} > 0$ for all $\alpha_i \ge \bar{\alpha}$.

(c) In the other regions, supply exceeds demand on one platform and matches on the other platform. When supply exceeds demand on platform i and matches on platform $\bar{i}$, the equilibrium in the subgame requires

$$\begin{cases} a - b\left(p_i - p_{\bar{i}}\right) - cp_i - (b+c)p_i = 0 \\ a - b\left(p_{\bar{i}} - p_i\right) - cp_{\bar{i}} = -k + l\left[\left(1-\alpha_{\bar{i}}\right)p_{\bar{i}} - \left(1-\alpha_i\right)p_i\right] + m\left(1-\alpha_{\bar{i}}\right)p_{\bar{i}}. \end{cases}$$

Solving the system of equations, we derive the equilibrium prices in the subgame as

$$\begin{cases} p_i = \frac{a(2b+c)+bk+a(1-\alpha_{\bar{i}})(l+m)}{b^2+4bc+2c^2+2(1-\alpha_{\bar{i}})(b+c)(l+m)-(1-\alpha_i)bl} \\ p_{\bar{i}} = \frac{a(3b+2c)+2k(b+c)+a(1-\alpha_i)l}{b^2+4bc+2c^2+2(1-\alpha_{\bar{i}})(b+c)(l+m)-(1-\alpha_i)bl}. \end{cases} \tag{43}$$

The required conditions for this equilibrium are that $s_i > d_i$ and platform $\bar{i}$ has no incentive to increase price. Within this region, when $\alpha_i < \alpha_{\bar{i}}$, we can show that these conditions are satisfied. Similarly, when $\alpha_{\bar{i}} < \alpha_i$, we can derive the equilibrium under which supply exceeds demand on platform $\bar{i}$ and matches on platform i.

Next, we consider the wage competition in the first stage and derive the symmetric equilibrium.

If both platforms choose $\alpha < \bar{\alpha}$ (where $\bar{\alpha}$ is defined in Equation (39)), supply exceeds demand in the pricing subgame equilibrium. If one platform slightly increases its commission rate, the price in the subgame does not change and its transaction volume remains the same. Doing so increases the platform's profit margin and profit. Therefore, any commission rate in this region cannot be sustained as an equilibrium.

If both platforms choose $\alpha > \bar{\alpha}$ (where $\bar{\alpha}$ is defined in Equation (39)), supply matches demand in the pricing subgame equilibrium. In this case,

$$\max_{\alpha_i} \pi_i = \alpha_i p_i^* d_i = \left(p_i^* - w_i\right) d_i = \alpha_i p_i^* \left[a - b\left(p_i^* - p_{\bar{i}}^*\right) - cp_i^*\right],$$

where p_i^* is a function of α_i and $\alpha_{\bar{i}}$ defined in Equation (40). By the first-order condition, we have

$$\begin{aligned}
\left.\frac{\partial\pi}{\partial\alpha_i}\right|_{\alpha_1=\alpha_2=\alpha} &= d_i\left(p_i^*+\alpha\frac{\partial p_i^*}{\partial\alpha_i}\right)+\left(p_i^*-w_i\right)\frac{\partial d_i}{\partial\alpha_i} \\
&= d_i\left(p_i^*+\alpha\frac{\partial p_i^*}{\partial\alpha_i}\right)-\left(p_i^*-w_i\right)\left[b\left(\frac{\partial p_i^*}{\partial\alpha_i}-\frac{\partial p_{\bar{i}}^*}{\partial\alpha_i}\right)+c\frac{\partial p_i^*}{\partial\alpha_i}\right] \\
&= \left(-d_i\frac{\partial d_i}{\partial\alpha_i}\right)\left[\frac{p_i^*+\alpha\frac{\partial p_i^*}{\partial\alpha_i}}{-\frac{\partial d_i}{\partial\alpha_i}}-\frac{1}{d_i}\left(p_i^*-w_i\right)\right] \\
&= \left(-d_i\frac{\partial d_i}{\partial\alpha_i}\right)\left[\frac{(1-\alpha)(m(b+2l+m)+c(l+m))+b(2c+m)+c(c+l+m)}{(1-\alpha)m(b+c)(2l+m)+c(2b+c)(l+m)}-\frac{\alpha(a+k)}{a(1-\alpha)m-ck}\right] \\
&= \left(-d_i\frac{\partial d_i}{\partial\alpha_i}\right)\frac{f(\alpha)}{[(1-\alpha)m(b+c)(2l+m)+c(2b+c)(l+m)][a(1-\alpha)m-ck]}=0,
\end{aligned} \tag{44}$$

where the fourth equality is obtained by using $\frac{\partial p_i^*}{\partial w_i}$ and $\frac{\partial p_{\bar{i}}^*}{\partial w_i}$ based on Equation (40) and $a-cp_i^*=-k+m\left(1-\alpha_i\right)p_i^*$, and $f(\alpha)=A\alpha^2+B\alpha+C$ with

$$\begin{aligned}
A &= m(a+k)(b+c)(2l+m)+am(m(b+2l+m)+c(l+m)) \\
B &= -(c+m)\left(a\left(2m(2b+c+2l)+l(2b+c)+2m^2\right)+bk(2l+m)\right) \\
C &= (c+m)(am-ck)(2b+c+2l+m).
\end{aligned}$$

We can verify that $\left(-d_i\frac{\partial d_i}{\partial\alpha_i}\right)>0$, $f(1)<0$, and $f(\bar{\alpha})>0$. Because $f(\alpha)$ is continuous and quadratic, a unique α^* exists in $(\bar{\alpha},1)$ such that $f(\alpha^*)=0$, which leads to Equation (6). Such α^* defines the equilibrium commission rate, under which demand matches supply. Because $d_i=s_i$ in equilibrium, p^* can be derived as a function of α^* as in Equation (7). We can verify that neither platform has incentive to deviate within this region. □

A.5 Proof of Lemma 3

Proof. We first consider the price competition in the second stage. In this stage, neither platform has incentive to choose p_i such that $s_i<d_i$, because otherwise the platform can slightly increase its price to increase its profit (yielding both higher profit margin and higher transaction volume). In other words, in equilibrium, $s_i\geq d_i$. We next distinguish different regions of (w_1,w_2) where $s_i>d_i$ for $i\in\{1,2\}$, $s_i=d_i$ for $i\in\{1,2\}$, or $s_i>d_i$ but $s_{\bar{i}}=d_{\bar{i}}$.

(a) In the region where the platforms choose p_i such that $s_i>d_i$, the optimal prices are characterized by the first-order conditions of the profit functions in Equation (3):

$$\begin{cases} \frac{\partial \pi_1(p_1)}{\partial p_1} = a - b\left(p_1 - p_2\right) - cp_1 - \left(b + c\right)\left(p_1 - w_1\right) = 0 \\ \frac{\partial \pi_2(p_2)}{\partial p_2} = a - b\left(p_2 - p_1\right) - cp_2 - \left(b + c\right)\left(p_2 - w_2\right) = 0. \end{cases} \tag{45}$$

Combining the two equations, we find the equilibrium prices.

$$\begin{cases} p_1^*\left(w_1, w_2\right) = \frac{a(3b+2c)+2w_1(b+c)^2+bw_2(b+c)}{(b+2c)(3b+2c)} \\ p_2^*\left(w_2, w_1\right) = \frac{a(3b+2c)+2w_2(b+c)^2+bw_1(b+c)}{(b+2c)(3b+2c)} \end{cases} \tag{46}$$

We must ensure that supply exceeds demand on both platforms given this equilibrium price, so

$$\begin{cases} -\frac{(b+c)\left(a(3b+2c)-w_1\left(b^2+4bc+2c^2\right)+bw_2(b+c)\right)}{(b+2c)(3b+2c)} - k + w_1(l+m) - lw_2 > 0 \\ -\frac{(b+c)\left(a(3b+2c)-w_2\left(b^2+4bc+2c^2\right)+bw_1(b+c)\right)}{(b+2c)(3b+2c)} - k + w_2(l+m) - lw_1 > 0, \end{cases} \tag{47}$$

which can be simplified to

$$\begin{cases} w_1 > \frac{(3b+2c)(a(b+c)+k(b+2c))+w_2\left(l(b+2c)(3b+2c)+b(b+c)^2\right)}{b^3+b^2(5c+3(l+m))+2bc(3c+4(l+m))+2c^2(c+2(l+m))} \\ w_2 > \frac{(3b+2c)(a(b+c)+k(b+2c))+w_1\left(l(b+2c)(3b+2c)+b(b+c)^2\right)}{b^3+b^2(5c+3(l+m))+2bc(3c+4(l+m))+2c^2(c+2(l+m))}. \end{cases} \tag{48}$$

The two curves defined in Condition (48) intersect at

$$w_1 = w_2 = \frac{a(b+c)+k(b+2c)}{b(c+m)+c(c+2m)} \equiv \bar{w} \tag{49}$$

and any w_i in the region defined by Condition (48) satisfies $w_i > \bar{w}$. If and only if (w_1, w_2) is in this region, $s_i > d_i$, $i \in \{1, 2\}$, can arise as an equilibrium in the pricing subgame.

(b) In the region where the platforms choose p_i such that $s_i = d_i$, by Equations (1) and (2), we can derive the prices as

$$\begin{cases} p_1^*\left(w_1, w_2\right) = \frac{(a+k)(2b+c)-w_1(bm+c(l+m))+w_2(cl-bm)}{c(2b+c)} \\ p_2^*\left(w_2, w_1\right) = \frac{(a+k)(2b+c)-w_2(m(b+c)+cl)+w_1(cl-bm)}{c(2b+c)}. \end{cases} \tag{50}$$

To ensure it is the equilibrium, we must show that neither platform has incentive to increase its price. We next check the nondeviation condition. When increasing its price, Platform i's profit function can be formulated as

$$\pi_i = \alpha_i p_i \min\{d_i, s_i\} = \alpha_i p_i d_i = \alpha_i p_i \left[a - b\left(p_i - p_{\bar{i}}^*\right) - cp_i\right]. \tag{51}$$

By the first-order condition, the solution to the unconstrained optimization problem in Equation (51) is $p_i^{\mathrm{dev}} = \frac{a+bp_{\bar{i}}^*+(b+c)w_i}{2(b+c)}$. Because of the concavity of the objective function, neither platform has incentive to deviate if and only if $p_1^{\mathrm{dev}} \le p_1^*$ and $p_2^{\mathrm{dev}} \le p_2^*$, or, equivalently, if and only if

$$\begin{cases} w_1 < \frac{(2b+c)(a(b+c)+k(b+2c))-w_2(bm(b+c)-cl(3b+2c))}{b^2(2c+m)+bc(3(c+l)+4m)+c^2(c+2(l+m))} \\ w_2 < \frac{(2b+c)(a(b+c)+k(b+2c))-w_1(bm(b+c)-cl(3b+2c))}{b^2(2c+m)+bc(3(c+l)+4m)+c^2(c+2(l+m))}. \end{cases} \tag{52}$$

In the symmetric case with $w_1 = w_2$, the above condition reduces to $w_i \le \bar{w}$ (where $\bar{w}$ is defined in Equation (49)).

(c) In the other regions, supply exceeds demand on one platform and matches on the other platform. When supply exceeds demand on platform i and matches on platform $\bar{i}$, the equilibrium in the subgame requires

$$\begin{cases} a - b\left(p_i - p_{\bar{i}}\right) - cp_i - (b+c)\left(p_i - w_i\right) = 0 \\ a - b\left(p_{\bar{i}} - p_i\right) - cp_{\bar{i}} = -k + l\left(w_{\bar{i}} - w_i\right) + mw_{\bar{i}}. \end{cases}$$

Solving the system of equations, we derive the equilibrium prices in the subgame as

$$\begin{cases} p_i = \frac{2ab+ac+w_i\left((b+c)^2+bl\right)+bk-bw_{\bar{i}}(l+m)}{b^2+4bc+2c^2} \\ p_{\bar{i}} = \frac{a(3b+2c)+2k(b+c)+(b+c)(w_i(b+2l)-2w_{\bar{i}}(l+m))}{b^2+4bc+2c^2}. \end{cases} \tag{53}$$

The required conditions for this equilibrium are that $s_i > d_i$ and platform $\bar{i}$ has no incentive to increase price. Within this region, when $w_{\bar{i}} < w_i$, we can show that these conditions are satisfied. Similarly, when $w_i < w_{\bar{i}}$, we can derive the equilibrium under which supply exceeds demand on platform $\bar{i}$ and matches on platform i.

Next, we consider the wage competition in the first stage and derive the symmetric equilibrium.

If both platforms choose $w_1 = w_2 = w > \bar{w}$ (where $\bar{w}$ is defined in Equation (49)), supply exceeds demand in the pricing subgame equilibrium. If Platform i slightly decreases its wage, supply still exceeds demand on both platforms and its profit becomes $\pi_i = (p_i^*(w_i, w_{\bar{i}}) - w_i)d_i$, where $p_i^*(w_i, w_{\bar{i}})$ is defined in Equation (46). We can derive

$$\frac{\partial \pi_i}{\partial w_i} = -\frac{2(b+c)\left(b^2+4bc+2c^2\right)\left(a(3b+2c) - w_i\left(b^2+4bc+2c^2\right) + bw_{\bar{i}}(b+c)\right)}{(b+2c)^2(3b+2c)^2} \tag{54}$$

and

$$\left.\frac{\partial \pi_i}{\partial w_i}\right|_{w_i = w_{\bar{i}} = \frac{a(b+c)+k(b+2c)}{b(c+m)+c(c+2m)}} = \frac{2(b+c)\left(b^2+4bc+2c^2\right)(ck-am)}{(b+2c)(3b+2c)(b(c+m)+c(c+2m))} < 0, \tag{55}$$

where the inequality is because of Assumption 1. We can verify that $\frac{\partial \pi_i}{\partial w_i}$ decreases in w_i for the symmetric case with $w_1 = w_2 > \bar{w}$. Therefore, Platform i has incentive to decrease its wage and any pair of wages in this region cannot be sustained as an equilibrium.

If both platforms choose $w \leq \bar{w}$ (where $\bar{w}$ is defined in Equation (49)), supply matches demand in the pricing subgame equilibrium. In this case,

$$\max_{w_i} \pi_i = (p_i^* - w_i)\, d_i = (p_i^* - w_i)\left[a - b\left(p_i^* - p_{\bar{i}}^*\right) - cp_i^*\right],$$

where p_i^* is a function of w_i and $w_{\bar{i}}$ is defined in Equation (50). By the first-order condition, we have

$$\begin{aligned}
\left.\frac{\partial \pi}{\partial w_i}\right|_{w_i = w_{\bar{i}} = w} &= d_i\left(\frac{\partial p_i^*}{\partial w_i} - 1\right) + (p_i^* - w_i)\frac{\partial d_i}{\partial w_i} \\
&= d_i\left(\frac{\partial p_i^*}{\partial w_i} - 1\right) - (p_i^* - w_i)\left[b\left(\frac{\partial p_i^*}{\partial w_i} - \frac{\partial p_{\bar{i}}^*}{\partial w_i}\right) + c\frac{\partial p_i^*}{\partial w_i}\right] \\
&= \left(-d_i\frac{\partial d_i}{\partial w_i}\right)\left[\frac{\frac{\partial p_i^*}{\partial w_i} - 1}{-\frac{\partial d_i}{\partial w_i}} - \frac{1}{d_i}(p_i^* - w_i)\right] \\
&= \left(-d_i\frac{\partial d_i}{\partial w_i}\right)\left[\frac{b(2c+m)+c(c+l+m)}{c(2b+c)(l+m)} - \frac{1}{d_i}(p_i^* - w_i)\right] = 0,
\end{aligned} \tag{56}$$

where the last equality is because $\frac{\partial p_i^*}{\partial w_i} = \frac{-(bm+c(l+m))}{c(2b+c)}$ and $\frac{\partial p_{\bar{i}}^*}{\partial w_i} = \frac{(cl-bm)}{c(2b+c)}$ by Equation (50), leading to w^* as in Equation (8). When $w < w^*$, $\frac{\partial \pi}{\partial w_i} > 0$, and when $w > w^*$, $\frac{\partial \pi}{\partial w_i} < 0$.

Notice that w^* derived in Equation (8) is only a necessary condition. First, we must ensure $w^* \leq \bar{w}$, leading to the condition

$$l \leq 3c + \frac{c^2}{b} + \frac{b(2c+m)}{c},$$

which is true under Assumption 2. When this condition is satisfied, neither platform has incentive to deviate within the $s_i = d_i$ region delineated by conditions (52). Second, we can verify that neither platform has incentive to significantly decrease its wage such that its supply matches demand and the competitor's supply exceeds demand. Third, we must check if a platform has incentive to significantly increase its wage such that its supply exceeds demand and the competitor's supply matches demand. If Platform i increases its wage significantly,

$$\pi_i = \frac{(b+c)\left(2ab+ac-w_i\left(2bc-bl+c^2\right)+bk-bw_{\bar{i}}(l+m)\right)^2}{\left(b^2+4bc+2c^2\right)^2} \tag{57}$$

and

$$\frac{\partial \pi_i}{\partial w_i} = \frac{-2(b+c)\left(2bc-bl+c^2\right)\left(2ab+ac-w_i\left(2bc-bl+c^2\right)+bk-bw_{\bar{i}}(l+m)\right)}{\left(b^2+4bc+2c^2\right)^2}. \tag{58}$$

This derivative is nonpositive when $2bc - bl + c^2 \geq 0$ (Assumption 2) and $w_{\bar{i}} = w^*$. Therefore, under Assumption 2, Platform i has no incentive to significantly increase its wage. In the absence of Assumption 2, any pure-strategy symmetric wage offering cannot be sustained as an equilibrium, and the platforms may involve a mixed strategy in equilibrium. □

A.6 Proof of Proposition 1

Proof. We first derive the equilibrium in the benchmark cases and then compare them with the corresponding sharing-economy context. We use superscripts D, F, and W to distinguish the equilibrium outcomes under the DCR, FCR, and FW schemes, respectively.

In the benchmark with the DCR or FW scheme, given the exogenous s and w, each platform's profit function is $\pi_i = (p_i - w)\min\{d_i, s\}$. In the symmetric equilibrium, supply might exceed or match demand. When supply exceeds demand on both platforms, by the first-order conditions, we have

$$\begin{cases} a - (b+c)p_1 + bp_2 - (b+c)(p_1 - w) = 0 \\ a - (b+c)p_2 + bp_1 - (b+c)(p_2 - w) = 0, \end{cases} \tag{59}$$

which leads to $p_1^* = p_2^* = \frac{a+(b+c)w}{b+2c}$. Supply exceeding demand requires

$$s > a - c\frac{a+(b+c)w}{b+2c} = \frac{(b+c)}{b+2c}(a - cw) \equiv s^*(w). \tag{60}$$

When supply matches demand, the equilibrium price is determined by $s = a - cp_i$, which leads to $p_1^* = p_2^* = \frac{a-s}{c}$. If $s \leq s^*(w)$, we can verify that neither platform has incentive to deviate. Altogether, we derive the equilibrium price as in Equation (11).

In the benchmark with the FCR scheme, given the exogenous α and s, each platform's profit function is $\pi_i = \alpha p_i \min\{d_i, s\}$. In the symmetric equilibrium, supply might exceed or match demand. When supply exceeds demand on both platforms, by the first-order conditions, we have

$$\begin{cases} a - (b+c)p_1 + bp_2 - (b+c)p_1 = 0 \\ a - (b+c)p_2 + bp_1 - (b+c)p_2 = 0, \end{cases} \tag{61}$$

which leads to $p_1^* = p_2^* = \frac{a}{b+2c}$. Supply exceeding demand requires

$$s > a - c\tfrac{a}{b+2c} = \tfrac{a(b+c)}{b+2c} \equiv s^*. \tag{62}$$

When supply matches demand, the equilibrium price is determined by $s = a - cp_i$, which leads to $p_1^* = p_2^* = \frac{a-s}{c}$. If $s \leq s^*$, we can verify that neither platform has incentive to deviate. Altogether, we derive the equilibrium price as in Equation (12).

First, we compare the price under the DCR scheme with its benchmark. Based on $s^{\mathrm{D}*} = -k + mw^*$ and $s^*(w^*)$ in Equation (60), we have

$$s^*(w^*) - s^{\mathrm{D}*} = \frac{c(b+c)^2(ak - cm)}{(b+2c)\left(b\left(c(l+2m) + m(l+m)\right) + c\left(c(l+2m) + 2m(l+m)\right)\right)} > 0 \tag{63}$$

because $ak - cm > 0$ by Assumption 1. Under the DCR scheme,

$$p^{\mathrm{D}*} = \tfrac{a - d^{\mathrm{D}*}}{c} = \tfrac{a - s^{\mathrm{D}*}}{c},$$

where the second equality is because demand matches supply in equilibrium. If $s > s^*(w^*)$ in the benchmark, $p_{\mathrm{B}}^* = \frac{a - s^*(w^*)}{c} < p^{\mathrm{D}*}$ because $s^*(w^*) > s^{\mathrm{D}*}$. If $s < s^*(w^*)$ in the benchmark, $p_{\mathrm{B}}^* = \frac{a-s}{c}$, which is less than $p^{\mathrm{D}*}$ if and only if $s \geq s^{\mathrm{D}*}$. We thus prove the result under the DCR scheme.

Second, we compare the price under the FW scheme with its benchmark. Based on $s^{\mathrm{W}*} =$

$-k+mw^*$ and $s^*(w^*)$ in Equation (60), we have

$$s^*(w^*) - s^{\mathrm{W}*} = \frac{(am-ck)\left(b^2(2c+m)+bc(3c-l)+c^3\right)}{(b+2c)\left(2m\left(b(2c+l)+c(c+l)\right)+cl(2b+c)+m^2(3b+2c)\right)} > 0 \tag{64}$$

because $ak-cm>0$ by Assumption 1 and $b^2(2c+m)+bc(3c-l)+c^3>0$ by Assumption 2. Under the FW scheme,

$$p^{\mathrm{W}*} = \tfrac{a-d^{W*}}{c} = \tfrac{a-s^{\mathrm{W}*}}{c},$$

where the second equality is because demand matches supply in equilibrium. If $s>s^*(w^*)$ in the benchmark, $p^*_{\mathrm{B}} = \frac{a-s^*(w^*)}{c} < p^{\mathrm{W}*}$ because $s^*(w^*) > s^{\mathrm{W}*}$. If $s<s^*(w^*)$ in the benchmark, $p^*_{\mathrm{B}} = \frac{a-s}{c}$, which is less than $p^{\mathrm{W}*}$ if and only if $s \geq s^{\mathrm{W}*}$. We thus prove the result under the FW scheme.

Third, we compare the price under the FCR scheme with its benchmark. In the benchmark, when $s<s^*$, demand matches supply in equilibrium. Demand also matches supply under the FCR scheme. Because $s=d=a-cp$ in both cases, $s^*>s^{\mathrm{F}*}$ if $p^*_{\mathrm{B}}(s^*) \leq p^{\mathrm{F}*}$. By Equation (62), $p^*_{\mathrm{B}}(s^*) = \frac{a}{b+2c}$. Under the FCR scheme, $p^{\mathrm{F}*}$ can be sustained as an equilibrium only if neither platform has incentive to increase its price, which requires

$$\left.\frac{d\pi_i}{dp_i}\right|_{p_i=p_{\bar{i}}=p^{F*}} = a-(b+2c)p^{F*} \leq 0 \tag{65}$$

or, equivalently, $p^{F*} \geq \frac{a}{b+2c}$. Therefore, $p^*_B(s^*) \leq p^{F*}$ and thus $s^*>s^{F*}$. We notice that

$$p^{F*} = \tfrac{a-d^{F*}}{c} = \tfrac{a-s^{F*}}{c}.$$

If $s>s^*$ in the benchmark, $p^*_B = \frac{a-s^*(s^*)}{c} < p^{\mathrm{F}*}$ because $s^*>s^{\mathrm{F}*}$. If $s<s^*$ in the benchmark, $p^*_B = \frac{a-s}{c}$, which is less than $p^{\mathrm{F}*}$ if and only if $s \geq s^{\mathrm{F}*}$. We thus prove the result under the FCR scheme. $\square$

A.7 Proof of Proposition 2

Proof. (a) Based on Equation (4), we can derive

$$\frac{\partial p^{\mathrm{D}*}}{\partial b} = \frac{m(l+m)^2(ck-am)}{\left(b\left(c(l+2m)+m(l+m)\right)+c\left(c(l+2m)+2m(l+m)\right)\right)^2} < 0$$
$$\frac{\partial p^{\mathrm{D}*}}{\partial l} = \frac{m(b+c)^2(ck-am)}{\left(b\left(c(l+2m)+m(l+m)\right)+c\left(c(l+2m)+2m(l+m)\right)\right)^2} < 0,$$

where the inequality is due to $ck - am < 0$ by Assumption 1.

Based on Equation (9), we can derive

$$\frac{\partial p^{\mathrm{W}*}}{\partial b} = \frac{m(l+m)(2l+m)(ck-am)}{\left(2m\left(b(2c+l)+c(c+l)\right)+cl(2b+c)+m^2(3b+2c)\right)^2} < 0$$
$$\frac{\partial p^{\mathrm{W}*}}{\partial l} = \frac{m(2b+c)\left(b(2c+m)+c^2\right)(ck-am)}{c\left(2m\left(b(2c+l)+c(c+l)\right)+cl(2b+c)+m^2(3b+2c)\right)^2} < 0,$$

where the inequality is due to $ck - am < 0$ by Assumption 1.

Under the FCR scheme, by differentiating Equation (6) with respect to b, using the implicit-function theorem, we have

$$\left[-\frac{(2l+m)(c+(1-\alpha)m)((1-\alpha)(cl+m(2l+m))+c(l+m))}{((1-\alpha)m(b+c)(2l+m)+c(2b+c)(l+m))^2}\right] + \frac{f'(\alpha)}{[(1-\alpha)m(b+c)(2l+m)+c(2b+c)(l+m)]^2[a(1-\alpha)m-ck]^2}\frac{\partial \alpha^{\mathrm{F}*}}{\partial b} = 0,$$

where $f(\alpha)$ is defined in Equation (44) and $f'(a) < 0$ (implied by the proof of Lemma 2). Notice the term in the first bracket is negative. Therefore, $\frac{\partial \alpha^{\mathrm{F}*}}{\partial b} < 0$. Because demand matches supply (i.e., $a - cp^* = -k + (1-\alpha^*)p^*$), equilibrium price increases in commission rate, which implies $\frac{\partial p^{\mathrm{F}*}}{\partial b} < 0$. Similarly, by differentiating Equation (6) with respect to l, we have

$$\left[-\frac{(2b+c)(c+(1-\alpha)m)\left((1-\alpha)m(b+c)+b(2c+m)+c^2\right)}{((1-\alpha)m(b+c)(2l+m)+c(2b+c)(l+m))^2}\right] + \frac{f'(\alpha)}{[(1-\alpha)m(b+c)(2l+m)+c(2b+c)(l+m)]^2[a(1-\alpha)m-ck]^2}\frac{\partial \alpha^{\mathrm{F}*}}{\partial l} = 0.$$

Therefore, $\frac{\partial \alpha^{\mathrm{F}*}}{\partial l} < 0$ and $\frac{\partial p^{\mathrm{F}*}}{\partial l} < 0$.

By Equation (25), increasing b is equivalent to decreasing t_r, and byEquation (29), increasing l is equivalent to decreasing t_{d}. By Equation (28), S_{r} increases in b and l because of the decrease in prices (and in t_{r}).

(b) In equilibrium $s_i = d_i$ under all wage schemes, or, equivalently, $-k + mw^* = a - cp^*$.

Therefore, w^* increases when p^* decreases. By Equation (25), increasing b is equivalent to decreasing t_{r}, and by Equation (29), increasing l is equivalent to decreasing t_{d}. By Equation (32), S_{d} increases in b and l because of the increase in wages (and the decrease in t_{d}).

(c) Because $s_i = d_i$ in equilibrium under all wage schemes, we have $-k + mw^* = a - cp^*$ and

$$\pi^* = (p^* - w^*)(a - cp^*) = \left(\frac{m+c}{m}p^* - \frac{a+k}{m}\right)(a - cp^*). \tag{66}$$

We notice that

$$\frac{\partial \pi^*}{\partial p^*} = d_i^*\left(1 + \frac{c}{m}\right) - c(p^* - w^*) = d_i^* c\left[\frac{1}{c} + \frac{1}{m} - \frac{1}{d_i^*}(p_i^* - w_i^*)\right]. \tag{67}$$

By the chain rule, $\frac{\partial \pi^*}{\partial b} = \frac{\partial \pi^*}{\partial p^*}\frac{\partial p^*}{\partial b}$. Because $\frac{\partial p^*}{\partial b} < 0$ and $\frac{\partial p^*}{\partial l} < 0$ by part (a), if $\frac{\partial \pi^*}{\partial p^*} > 0$, then $\frac{\partial \pi^*}{\partial b} < 0$ and $\frac{\partial \pi^*}{\partial l} < 0$. We next show $\frac{\partial \pi^*}{\partial p^*} > 0$ under each wage scheme.

Under the DCR scheme, by Equation (33), we have

$$\left.\frac{\partial L}{\partial p_i}\right|_{p_i = p_{\bar{i}} = p^*} = d_i^* + \frac{\partial d_i}{\partial p_i}(p_i^* - w_i^*) + \lambda\frac{\partial d_i}{\partial p_i} = d_i^*(b+c)\left[\frac{1}{b+c} + \frac{1}{l+m} - \frac{1}{d_i^*}(p_i^* - w_i^*)\right] = 0. \tag{68}$$

By comparing Equations (67) and (68), we conclude $\frac{\partial \pi^*}{\partial p^*} > 0$ because $\frac{1}{c} + \frac{1}{m} > \frac{1}{b+c} + \frac{1}{l+m}$.

Under the FCR scheme, by comparing Equations (67) and (44), we can verify that

$$\left.\frac{p_i^* + \alpha\frac{\partial p_i^*}{\partial \alpha_i}}{-\frac{\partial d_i}{\partial \alpha_i}}\right|_{\alpha_i = \alpha_{\bar{i}} = \alpha^*} = \frac{\left(1 - \alpha^{\mathrm{F}*}\right)\left(m(b + 2l + m) + c(l+m)\right) + b(2c+m) + c(c+l+m)}{\left(1 - \alpha^{\mathrm{F}*}\right)m(b+c)(2l+m) + c(2b+c)(l+m)} < \frac{1}{c} + \frac{1}{m},$$

and thus $\frac{\partial \pi^*}{\partial p^*} > 0$.

Under the FW scheme, by comparing Equations (67) and (56), we can verify $\frac{b(2c+m)+c(c+l+m)}{c(2b+c)(l+m)} < \left(\frac{1}{c} + \frac{1}{m}\right)$ and thus $\frac{\partial \pi^*}{\partial p^*} > 0$. □

A.8 Proof of Proposition 3

Proof. First, we show that the ordering of the profits under the three wage schemes is identical to the ordering of the equilibrium prices. By Equation (66), we have

$$\frac{\partial \pi^*}{\partial p^*} = \frac{m+c}{m}(a - cp^*) - c\left(\frac{m+c}{m}p^* - \frac{a+k}{m}\right) = \frac{(m+c)a}{m} + \frac{c(a+k)}{m} - \frac{2c(m+c)p^*}{m}.$$

Because $\frac{\partial \pi^*}{\partial p^*}$ decreases in p^*, $\frac{\partial \pi^*}{\partial p^*} > 0$ if only if $p^* < \arg \frac{\partial \pi^*}{\partial p^*} = 0$. By the proof of part (c) of Proposition 2, $\frac{\partial \pi^*}{\partial p^*} > 0$ at the equilibrium price under each wage scheme. Therefore, a higher equilibrium price leads to a higher equilibrium profit.

We next compare the equilibrium prices under different wage schemes. First, we compare the equilibrium price under DCR and FW. By Equations (4) and (9), we have

$$p^{\mathrm{W}*} - p^{\mathrm{D}*} = \frac{bm(l+m)(am-ck)(bm-cl)}{c(b(c(l+2m)+m(l+m))+c(c(l+2m)+2m(l+m)))(b(2c(l+2m)+m(2l+3m)))+c(c(l+2m)+2m(l+m)))}.$$

Therefore, $p^{\mathrm{W}*} - p^{\mathrm{D}*} \geq 0$ if and only if $bm \geq cl$.

Second, we compare the equilibrium price under FCR and DCR. Notice that $\left.\frac{\partial \pi^{\mathrm{F}}}{\partial \alpha_i}\right|_{\alpha_1=\alpha_2=\alpha}$ in Equation (44) is positive for $\alpha < \alpha^{\mathrm{F}*}$ and is negative for $\alpha > \alpha^{\mathrm{F}*}$. Therefore, $\alpha^{\mathrm{D}*} < \alpha^{\mathrm{F}*}$ if and only if $\left.\frac{\partial \pi^{\mathrm{F}}}{\partial \alpha_i}\right|_{\alpha_1=\alpha_2=\alpha^{D*}} > 0$. By Equation (68), we have $\left.\frac{1}{d_i^*}\left(p_i^* - w_i^*\right)\right|_{\mathrm{DCR}} = \frac{1}{b+c} + \frac{1}{l+m}$. Therefore, we have

$$\begin{aligned}\left.\frac{\partial \pi^{\mathrm{F}}}{\partial \alpha_i}\right|_{\alpha_1=\alpha_2=\alpha^{\mathrm{D}*}} &= \left(-d_i \frac{\partial d_i}{\partial \alpha_i}\right)\left[\frac{\left(1-\alpha^{\mathrm{D}*}\right)(m(b+2l+m)+c(l+m))+b(2c+m)+c(c+l+m)}{(1-\alpha^{\mathrm{D}*})m(b+c)(2l+m)+c(2b+c)(l+m)} - \left(\frac{1}{b+c} + \frac{1}{l+m}\right)\right] \\ &= \left(-d_i \frac{\partial d_i}{\partial \alpha_i}\right) \frac{N}{[(1-\alpha^{\mathrm{D}*})m(b+c)(2l+m)+c(2b+c)(l+m)](b+c)(l+m)}.\end{aligned}$$

By replacing $\alpha^{\mathrm{D}*}$ with that in Equation (5) and simplifying N, we have

$$N = (mb-cl)\frac{b^2m(2a+k)(l+m)+bcm(2a+k)(l+m)+amb(l+m)^2-c^2l(a+2k)(l+m)-clk(b+c)^2-bcl(l+m)(a+2k)}{k(b+c)(l+m)+a(m(l+m)+(b+c)(l+2m))}.$$

$N > 0$ can be rewritten to the condition in Inequality (13). Because demand matches supply (i.e., $a - cp^* = -k + (1-\alpha^*)p^*$), the equilibrium price increases in commission rate. When Inequality (13) is satisfied, $\alpha^{\mathrm{D}*} < \alpha^{\mathrm{F}*}$ and thus $p^{\mathrm{D}*} < p^{\mathrm{F}*}$.

Third, we compare the equilibrium price under FCR and FW. As in the comparison between the FCR and DCR schemes, $\alpha^{\mathrm{W}*} < \alpha^{\mathrm{F}*}$ if and only if $\frac{\partial \pi^{\mathrm{F}}}{\partial \alpha_i}|_{\alpha_1=\alpha_2=\alpha^{\mathrm{W}*}} > 0$. By Equations (8) and (9), we have

$$\alpha^{\mathrm{W}*} = 1 - \frac{c(b(2a(l+m)-2ck-k(2l+3m))+c((a-2k)(l+m)-ck))}{a(b(2c(l+2m)+m^2)+c(c(l+2m)+m(l+m)))-ck(2b+c)(l+m)}.$$

By Equation (56), we have $\frac{1}{d_i^*}\left(p_i^* - w_i^*\right)\Big|_{\mathrm{FW}} = \frac{b(2c+m)+c(c+l+m)}{c(2b+c)(l+m)}$. Therefore, we have

$$\begin{aligned}\frac{\partial \pi^{\mathrm{F}}}{\partial \alpha_i}\Big|_{\alpha_1=\alpha_2=\alpha^{W*}} &= -d_i \frac{\partial d_i}{\partial \alpha_i}\left[\frac{\left(1-\alpha^{\mathrm{W}*}\right)(m(b+2l+m)+c(l+m))+b(2c+m)+c(c+l+m)}{(1-\alpha^{W*})m(b+c)(2l+m)+c(2b+c)(l+m)} - \frac{b(2c+m)+c(c+l+m)}{c(2b+c)(l+m)}\right] \\ &= -d_i \frac{\partial d_i}{\partial \alpha_i} \frac{c(a+k)(am-ck)(cl-bm)(cl(2b+c)+bm(2l+m))[b(2a(l+m)+2k(c+l)+3km)+c((a+2k)(l+m)+ck)]}{D},\end{aligned}$$

where D can be verified to be positive and $am - ck > 0$ by Assumption 1. Therefore, if and only if $cl - bm > 0$, $\alpha^{\mathrm{W}*} < \alpha^{\mathrm{F}*}$ and thus $p^{\mathrm{W}*} < p^{\mathrm{F}*}$.

Altogether, we conclude the comparison results in Proposition 3. □

A.9 Proof of Proposition 4

Proof. As shown in the proof of Proposition 3, platform profit increases in the equilibrium price under the three wage schemes. By Equation (28), S_{r} decreases in p^*. Therefore, the ordering of the consumer surplus under the three wage schemes is the opposite of the ordering of the platform profit.

Because demand matches supply in equilibrium (i.e., $-k + mw^* = a - cp^*$) under each wage scheme, the equilibrium wage decreases in the equilibrium price, which implies platform profit decreases in the equilibrium wage under the three wage schemes. By Equation (32), S_{d} increases in w^*. Therefore, the ordering of the worker surplus under the three wage schemes is the opposite of the ordering of the platform profit. □

A.10 Proof of Proposition 5

Proof. Based on the micro foundation discussed in A.2 ("Demand/Supply Function and Consumer/Worker Surplus"), we can write the equilibrium demand on each platform as $\frac{\alpha_{\mathrm{r}}}{2} + \beta_{\mathrm{r}} - cp^*$ and equilibrium supply as $\frac{\alpha_{\mathrm{d}}}{2} - \beta_{\mathrm{d}} + mw^*$. Because demand matches supply in equilibrium, each platform's profit can be formulated as

$$\pi^* = (p^* - w^*)\left(\frac{\alpha_{\mathrm{r}}}{2} + \beta_{\mathrm{r}} - cp^*\right) = p^*\left(\frac{\alpha_{\mathrm{r}}}{2} + \beta_{\mathrm{r}} - cp^*\right) - w^*\left(\frac{\alpha_{\mathrm{d}}}{2} + mw^* - \beta_{\mathrm{d}}\right). \tag{69}$$

The social welfare is the sum of consumer surplus, worker surplus, and platform profits. Based on Equations (28), (32), and (69), we have

$$SW = S_{\mathrm{r}} + S_{\mathrm{d}} + 2\pi^* = \alpha_{\mathrm{r}}\left(v - \frac{t_{\mathrm{r}}}{4}\right) + \frac{\beta_{\mathrm{r}}^2 - (cp^*)^2}{c} + \alpha_{\mathrm{d}}\left(-c - \frac{t_{\mathrm{d}}}{4}\right) + \frac{\beta_{\mathrm{d}}^2 - (mw^*)^2}{m}. \tag{70}$$

Because $-k + mw^* = a - cp^*$ in equilibrium, we have $\frac{\partial w^*}{\partial p^*} = -\frac{c}{m}$. Therefore,

$$\frac{\partial SW}{\partial p^*} = -2cp^* - 2mw^*\left(-\frac{c}{m}\right) = -2c(p^* - w^*) < 0.$$

Because SW decreases in p^*, the ordering of social welfare under the three wage schemes is the opposite of the ordering of the platform profit. $\square$