Operations Research

Optimal Production Planning with Emissions Trading

Xiting Gong, Sean X. Zhou

OPERATIONS RESEARCH
Vol. 61, No. 4, July–August 2013, pp. 908–924
ISSN 0030-364X (print) | ISSN 1526-5463 (online)

http://dx.doi.org/10.1287/opre.2013.1189

Optimal Production Planning with Emissions Trading

Xiting Gong
Department of Systems Engineering and Engineering Management, The Chinese University of Hong Kong,
Shatin, N.T., Hong Kong, gongxiting@gmail.com

Sean X. Zhou
Department of Decision Sciences and Managerial Economics, CUHK Business School,
The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, zhoux@se.cuhk.edu.hk

Emissions trading is a market-based mechanism for curbing emissions, and it has been implemented in Europe, North America, and several other parts of the world. To study its impact on production planning, we develop a dynamic production model, where a manufacturer produces a single product to satisfy random market demands. The manufacturer has access to both a green and a regular production technology, of which the former is more costly but yields fewer emissions. To comply with the emissions regulations, the manufacturer can buy or sell the allowances in each period via forward contracts in an outside market with stochastic trading prices while needing to keep a nonnegative allowance account balance at the end of the planning horizon. We first derive several important structural properties of the model, and based upon them, we characterize the optimal emissions trading and production policies that minimize the manufacturer's expected total discounted cost. In particular, the optimal emissions trading policy is a target interval policy with two thresholds that decrease with the starting inventory level. The optimal production policy is established by first determining the optimal technology choice and then showing the optimality of a base-stock type of production policy. We show that the optimal base-stock level is independent of the starting inventory level and the allowance level when the manufacturer trades the allowance or uses both technologies simultaneously. A numerical study using representative data from the cement industry is conducted to illustrate the analytical results and to examine the value of green technology for the manufacturer.

Subject classifications: emissions trading; production planning; technology choice; optimal policy; base-stock; supermodular functions.
Area of review: Operations and Supply Chains.
History: Received January 2011; revisions received January 2012, October 2012, December 2012; accepted March 2013. Published online in *Articles in Advance* July 31, 2013.

1. Introduction

Sustainable development and corporate social responsibility have emerged as important priorities for business corporations. Among various activities to improve the social and environmental consequences of their businesses, the reduction of emissions and pollutants plays an important role, especially for companies in carbon-intensive industries such as cement, power, petrochemicals, and pulp and paper. More stringent environmental regulations have also made emissions reduction an inescapable issue in companies' daily production operations involving technology selection, investment in pollution abatement equipment, production planning, etc.

Emissions trading (also known as cap-and-trade) is a market-based mechanism designed to curb emissions, and it has been implemented in Europe, North America, and several other parts of the world. Two prominent examples are the European Union Emissions Trading Scheme (EU ETS) and the Sulfur Dioxide (SO_2) emissions trading scheme under the Acid Rain Program in the United States. In contrast to traditional regulations, emissions trading schemes provide pollutant emitters with flexibility in how they comply with the regulations. Under such schemes, firms receive some initial emission allowances (sometimes also called emission permits), entitling the firms to emit a specific amount of greenhouse gases (e.g., CO_2) from the regulator; and they can also buy the allowances from or sell them to other companies, brokers, or government agencies in an outside market. If their allowance account balance is negative (because of emissions and/or trade) at the end of a compliance period (e.g., one year), a penalty will be charged. These schemes also provide economic incentives for manufacturing firms to use clean energy and/or adopt green technologies in their production processes, which are often major sources of emissions. These issues create new challenges for firms to adjust their existing production planning strategies to comply with the regulations while maintaining and improving their efficiency and competitiveness. In this study, we develop and analyze a dynamic production model to provide firms with optimal emissions trading, technology selection, and production strategies under such a cap-and-trade emission regulation scheme. We also examine the value regarding emissions and cost reduction that a green production technology can bring to the firms in various scenarios. This could help the firms evaluate their investment on green technologies and emissions abatement.

Specifically, we consider a manufacturer who produces a single product to satisfy random market demands. The manufacturer has two production technologies—a green technology and a regular technology; and it can adopt either or both of them. The green technology produces each unit of the product with fewer emissions (thus having a lower emissions intensity) but with a higher production cost than the regular technology. A typical example is the companies in the pulp and paper industry, which are major polluters and spend more on pollution abatement than most companies in other manufacturing industries (Martin et al. 2000). Pulp and paper mills employ a variety of production technologies that differ in their costs and emissions intensities. For example, coal boilers and natural gas boilers are often used to generate steam. The coal boiler generates 92,900–126,000 kg/TJ of fossil CO_2 emissions, whereas a natural gas boiler generates 56,100–57,000 kg/TJ (NCASI 2005). Natural gas is more expensive than coal if only the cost of energy is considered. One can also consider a scenario where steam is generated by using electricity purchased outside (e.g., a nuclear power plant)[1] or from a combined heat and power generator within the plant. The cement industry is another example, with high energy intensity and an approximately 5% contribution to the total global CO_2 emissions. Cement producers may use coal, natural gas, or other alternative fuels to produce clinkers, which are also heterogeneous in emissions and costs. In addition, different kiln technologies (e.g., wet, dry, precalciner) with different costs and emissions still co-exist in some companies (Choate 2003; Hendriks et al. 1998; Drake et al. 2010a, b). Readers interested in technology choices in cement factories are referred to Drake et al. (2010b). Note that both the pulp and paper and cement industries are currently regulated by the EU ETS.

In each period, the manufacturer first can buy or sell emission allowances in an outside market via forward contracts settled at the end of the planning horizon. As noted by the EU ETS, "*the share of spot trades for immediate delivery has declined, there has been a consequent increase in the share of forward contracts for delivery in December 2009, 2010 and so forth. Companies are hedging against future price developments rather than simply cashing in due to the crisis.*"[2] Trading prices of the forward contracts in different periods are random and modeled as a Markov chain. Then the manufacturer decides on the production quantity from each technology before the random market demands are realized. The unsold product at the end of the period is stocked to the next period and incurs a unit holding cost; unsatisfied demand is backlogged and incurs a unit backlogging cost. The objective of the manufacturer is to minimize the expected total discounted cost over a finite planning horizon.

We formulate the problem as a stochastic dynamic program and derive its structural properties by proving a set of key technical results, which show the preservation of modularity of functions after minimization. Based on those results, we then study the manufacturer's optimal emissions trading and production policies. In particular, we show that the optimal emissions trading policy is a target interval policy with two thresholds decreasing with the starting inventory level. The optimal production policy is established by first determining the optimal technology selection, in which we find that the *additional production cost of green technology per allowance saved* plays a critical role. Then we show the optimality of a base-stock type of production policy. When the firm trades allowances or uses both production technologies, the optimal base-stock level depends only on the trading prices, whereas in other situations it also depends on the starting inventory and allowance levels. Some monotonicity results of the optimal base-stock levels are established. Two special cases where there is a single technology and where the selling and buying prices of the allowances are always identical are also studied.

Based on our analytical results, we conduct a further comprehensive numerical study by using costs and emissions data from the cement industry. We consider different technology combinations and show that the green technology can provide the firm with significant cost and emissions reduction benefits. For example, under certain technology combinations, the cost reduction can be up to 16.64% and the expected emissions reduction can be up to 94.35%. The impacts of the trading price process and other system parameters on the value of green technology are also examined in the numerical study.

The remainder of the paper is organized as follows. In §2, we provide a literature review on the related studies. In §3, we introduce the model and its mathematical formulation in detail. In §4, we study the structural properties of the model; and in §5, we analyze the manufacturer's optimal emissions trading and production policies. A numerical study is conducted in §6 to demonstrate the analytical results and discuss insights. We conclude the paper in §7 with discussions on possible extensions. All the proofs are given in the online electronic companion (available as supplemental material at http://dx.doi.org/10.1287/opre.2013.1189).

2. Literature Review

This work is mainly related to three streams of research, and for clarity we list in Table 1 the related studies in each stream that we will subsequently review. We will also point out how our study differs from these streams of research.

The first stream of research related to our work is on emissions trading. Tietenberg (2006) provides comprehensive discussions on the principles and practices of emissions trading. We mainly review those studies considering both emissions trading and production operations of firms. Laffont and Tirole (1996) analyze the impact of spot and futures markets for tradeable pollution permits on

Table 1. Related literature.

Research streams	Cited papers
Emissions trading w/ or w/o production	Laffont and Tirole (1996), Dobos (2005, 2007), Subramanian et al. (2007), Carmona et al. (2009, 2010), Zhao et al. (2010), Drake et al. (2010a)
Production and capacity management	Angelus and Porteus (2002), Yang (2004), Ye and Duenyas (2007), Chao et al. (2008)
Technology/ supplier selection	Fukuda (1964), Gary and Shadbegian (1998), Stuart et al. (1999), Debo et al. (2005), Federgruen and Yang (2008), Martínez-de-Albéniz and Simchi-Levi (2005)

firms' pollution abatement and production decisions in a two-period deterministic model. Dobos (2005, 2007) study the effect of emissions trading by using the Arrow-Karlin model. Subramanian et al. (2007) develop a three-stage game model in a symmetric oligopoly, where firms decide on the abatement level, bid for emission allowances, and determine production quantity sequentially; and they study how the number of available permits affects the firms' abatement decisions. Carmona et al. (2009) consider an emissions trading and abatement model and they show that the equilibrium carbon price process is a martingale in a multifirm setting. Carmona et al. (2010) further develop a multiperiod oligopoly model in which each firm uses multiple technologies differing in emissions and costs to produce multiple types of goods in each period. They show the existence of a market equilibrium with unique emission allowance prices, and they also discuss the phenomenon of windfall profits. Zhao et al. (2010) propose a nonlinear complementarity model to analyze the efficiency of different initial allowance allocation mechanisms in electronic power markets. Recently, Drake et al. (2010a) consider a two-stage technology choice and capacity investment problem for a firm under emissions cap-and-trade or emissions tax regulations.

Our work differs from the abovementioned studies in that we study a multiperiod production planning problem with emissions trading for a single firm (e.g., a pulp and paper or cement manufacturer) facing stochastic market demands and allowance prices. The unsold product inventory in each period is carried over to the next period. We focus on the structure of the optimal emissions trading, technology selection, and production policies from a single firm's perspective. We also take into account bid-ask price spreads and transaction costs by modeling different selling and buying prices of the allowances.

The second related research stream is on production and capacity management. Angelus and Porteus (2002) consider an integrated capacity and inventory management model where the inventory of a product can or cannot be carried over between periods and derive optimal policies. Yang (2004) develops a multiperiod production model with both a finished goods inventory and a raw material inventory and characterizes the optimal production and raw material trading policies. Ye and Duenyas (2007) incorporate fixed capacity adjustment costs into a capacity management model without considering inventory replenishment decisions. Chao et al. (2008) study a dynamic inventory control problem where the production quantity is constrained by cash on hand, and they show that a capital-dependent base-stock policy is optimal. There are two major differences between our work and these capacity management problems. First, the firm can trade allowances via forward contracts and carry out production irrespective of its allowances on hand during the planning horizon. This feature is unique in our model because it is unrealistic to produce with negative raw materials, production capacity, or working capital. Second, our model considers two technologies with different unit production costs and emissions intensities, which are features often seen in production firms under emissions regulation. We will demonstrate that these differences make our analysis more challenging but also yield new and interesting findings.

The third stream of related research is on technology/supplier selection. Some of the studies have taken into account environmental issues. For example, Gary and Shadbegian (1998) conduct an empirical study on the effects of environmental regulations on technology choices and investment in paper mills. Stuart et al. (1999) develop a mixed-integer programming model to select product and process alternatives while considering tradeoffs such as costs, material consumption, and waste generation. Debo et al. (2005) focus on technology selection and pricing of remanufacturable products, where different technologies incur different costs and lead to different levels of product remanufacturability. For supplier selection, most of the existing studies are focused on the tradeoffs between cost and responsiveness (leadtime) (e.g., Fukuda 1964), reliability (yield) (e.g., Federgruen and Yang 2008), and capacity (e.g., Martínez-de-Albéniz and Simchi-Levi 2005). None of these studies considers integrated emissions trading and dynamic production/inventory management.

3. The Model

Consider a manufacturer that produces a single product to satisfy random market demands over a planning horizon of T periods, indexed by $t = 1, \ldots, T$. The manufacturer is equipped with two production technologies and can choose either or both of them to manufacture the product in each period. The production process of each technology emits greenhouse gases (e.g., CO_2), and the manufacturer is subject to emissions regulations. We assume that the entire planning horizon corresponds to one compliance period (e.g., one year). The manufacturer is endowed with a certain amount of emission allowances at the beginning of the planning horizon, and at the end of the period T it needs to keep a nonnegative balance in its allowances account or otherwise pay a penalty π for each unit of the negative balance. For example, under the EU ETS, a penalty of €40 was levied for each ton of emitted CO_2 not covered by the allowances in the first phase (Jan. 2005–Dec. 2007), and it was raised to €100 in the second phase (Jan. 2008–Dec. 2012).

Each unit of the product produced by technology i incurs cost c_i and emits μ_i units of emissions, $i = 1, 2$. We refer to μ_i as the emissions intensity of technology i. Without loss of generality, we assume that $0 < c_1 < c_2$ and $0 < \mu_2 < \mu_1$ such that neither production technology dominates the other both in production costs and in emissions. Because technology 2 has a lower emissions intensity, it is also referred to as the green technology in contrast to the regular technology of technology 1. The adoption of multiple production technologies with different costs and emissions is exemplified by companies in cement production (e.g., Choate 2003, Hendriks et al. 2004, Drake et al. 2010) and pulp and paper (e.g., NCASI 2005), as mentioned previously. For simplicity, we assume that both technologies have infinite production capacities and that there is no cost for idling each technology or switching cost between the two technologies. The models with finite production capacities and/or the above costs will be discussed in §7.

The manufacturer can buy or sell emission allowances in an outside market in each period. Motivated by the trading practices of the EU ETS mentioned in the previous section, we model trading of emission allowances in each period t via forward contracts settled in period $T+1$ (i.e., the transfer of emission allowances specified by any emissions trading occurs in period $T+1$). Thus, the manufacturer can trade emissions in each period t irrespective of its actual number of allowances on hand. Note that this trading scheme is also adopted by Carmona et al. (2009, 2010). Denote by $\tilde{K}_t$ and $\tilde{k}_t$ the unit buying and selling prices of a forward contract in period t, respectively,[3] which are nonnegative random variables with $P(0 \leqslant \tilde{k}_t \leqslant \tilde{K}_t \leqslant \gamma^{T-t+1}\pi) = 1$, where $0 < \gamma \leqslant 1$ is the one-period discount factor. Although we refer to $\tilde{K}_t$ and $\tilde{k}_t$ as the trading prices, they actually represent the cost and the revenue of buying and selling a unit of allowance, respectively, which include transaction costs. Transaction costs in emissions trading can be significant and have been studied both empirically and theoretically (e.g., Stavins 1995, Woerdman 2001). Bid-ask price spreads, often seen in various trading markets, are another cause of nonidentical selling and buying prices. For instance, the ask and bid prices for ECX EUA (European Union Allowances: carbon credits issued under the EU ETS to CO_2-emitting installations) futures for December 2010 are €15.48 and €14.20 per metric ton (12:00 P.M., Aug. 14, 2010, Hong Kong Time), respectively.[4] We also denote $\tilde{\mathbf{K}}_t = \{\tilde{K}_t, \tilde{k}_t\}$ as the bivariate random variable representing the trading price vector of emission allowances in period t. Because the trading prices in the next period are affected mostly by the prices in the current period, we assume that $\{\tilde{\mathbf{K}}_t, 1 \leqslant t \leqslant T\}$ forms a Markov chain, i.e., $P(\tilde{\mathbf{K}}_t \in B \mid \mathbf{K}_1, \ldots, \mathbf{K}_{t-1}) = P(\tilde{\mathbf{K}}_t \in B \mid \mathbf{K}_{t-1})$ for any given set $B \in \Re^2$. We will use the corresponding notation without the tilde to represent the realization of the random prices.

Because the firm is risk-neutral, if the discounted conditional expected selling (resp., buying) price in a future period is higher (resp., lower) than the buying (resp., selling) price of the current period, then it will buy (resp., sell) an infinite amount of the allowances. To avoid this, we need to impose the following conditions on the trading prices: for $t = 1, \ldots, T$ and $i = 0, \ldots, T-t$,

$$K_t \geqslant \gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t] \quad \text{and} \quad k_t \leqslant \gamma^i \mathsf{E}[\tilde{K}_{t+i} \mid \mathbf{K}_t], \quad \forall \mathbf{K}_t. \tag{1}$$

Specifically, the first (resp., second) inequality requires the unit buying (resp., selling) price of emission allowance in period t be greater (resp., less) than the discounted conditional expected selling (resp. buying) price in period $t+i$. Note that Condition (1) is a necessary condition for our model to be meaningful. In practice, this condition has also be shown valid in most circumstances by several empirical studies (see, e.g., Charles et al. 2010).

For each period t, define z_t as the *the allowance level* of the manufacturer, which equals to the sum of physical allowances and the number of forward contracts (can be either positive or negative) held by the firm *minus* the total emissions from production in the previous $t-1$ periods. Then, z_t can be considered as the firm's allowance account balance, which could be either positive or negative. Hence, z_1 represents the initial allocation of allowances to the firm, and z_{T+1} is the physical allowances the manufacturer holds at the end of the planning horizon. Because compliance and actual settlement of the forward contracts take place in period $T+1$, it suffices to track the allowance level at the beginning of each period.

Market demands $D_1, \ldots, D_T$ for the product in periods $1, \ldots, T$ are independent random variables. In addition, D_t and $\tilde{\mathbf{K}}_t$ are assumed to be independent, which is reasonable because product demand is at the firm level, whereas the allowance prices are formed in the larger emissions trading market. Unsatisfied demand is backlogged and any leftover inventory is carried over to the next period. Denote $G_t(y)$ as the convex expected inventory holding and demand backlogging costs for period t when the inventory level after production is y.

The sequence of events is as follows. At the beginning of each period t, the trading price $\tilde{\mathbf{K}}_t$ is realized and the initial inventory level x_t and allowance level z_t are reviewed. The manufacturer then decides how many emission allowances to buy or sell, and the allowance level is adjusted to $\bar{z}_t$. The manufacturer next decides on how many to produce with each production technology. We denote by y_{1t} and y_{2t} the inventory level after production from technology 1 and the inventory level after production from both technologies, respectively. Hence, $y_{1t} - x_t$ and $y_{2t} - y_{1t}$ are the production quantities using technology 1 and technology 2, respectively. Finally, demand D_t is realized, and all costs are incurred and calculated. The manufacturer's objective is to make optimal emissions trading and production decisions to minimize the expected total discounted cost over the planning horizon.

With the preceding description and notation of the model, after all decisions are made and the demand is realized, the inventory level and the allowance level at the beginning of period $t+1$ are updated as

$$x_{t+1} = y_{2t} - D_t, \quad \text{and}$$
$$z_{t+1} = \bar{z}_t - \mu_1(y_{1t} - x_t) - \mu_2(y_{2t} - y_{1t}),$$

where the transition of the inventory level is standard, while the allowance level z_{t+1} equals to the difference between the post-trading allowance level $\bar{z}_t$ and the total emissions resulting from the production in period t.

Denote $V_t(x_t, z_t, \mathbf{K}_t)$ as the minimum expected total discounted cost from period t to the end of the planning horizon, given the initial inventory level x_t, the allowance level z_t, and the realization of the trading price $\mathbf{K}_t$. Then, we can formulate the firm's optimization problem as the following dynamic program: for $t = 1, \ldots, T$,

$$\begin{aligned} &V_t(x_t, z_t, \mathbf{K}_t) \\ &= \min_{x_t \leqslant y_{1t} \leqslant y_{2t},\, \bar{z}_t} \{C_t(\bar{z}_t - z_t, \mathbf{K}_t) + c_1(y_{1t} - x_t) + c_2(y_{2t} - y_{1t}) \\ &\qquad + G_t(y_{2t}) + \gamma \mathsf{E}_t[V_{t+1}(y_{2t} - D_t, \bar{z}_t \\ &\qquad - \mu_1(y_{1t} - x_t) - \mu_2(y_{2t} - y_{1t}), \tilde{\mathbf{K}}_{t+1})]\}, \end{aligned} \tag{2}$$

where $C_t(z, \mathbf{K}) = Kz^+ - k(-z)^+$ with $x^+ = \max\{x, 0\}$. Because the distribution of $\tilde{\mathbf{K}}_{t+1}$ depends on $\mathbf{K}_t$, for brevity we use $\mathsf{E}_t[\,\cdot\,]$ to denote $\mathsf{E}_{D_t}[\mathsf{E}_{\tilde{\mathbf{K}}_{t+1}}[\cdot \mid \mathbf{K}_t]]$ and to signify its dependence on $\mathbf{K}_t$. The boundary condition is

$$\begin{aligned} &V_{T+1}(x_{T+1}, z_{T+1}, \mathbf{K}_{T+1}) \\ &\quad = \pi(-z_{T+1})^+ + p(-x_{T+1})^+ - p_D x_{T+1}^+, \end{aligned} \tag{3}$$

where the first term represents the penalty when the firm's allowance account balance is negative. When $x \leqslant 0$, a unit shortage cost p is charged; while when $x > 0$, a unit salvage value p_D is yielded. It is intuitive that $p_D \leqslant p$. For simplicity, we assume that there is no salvage value for the emission allowances, but all the subsequent analysis and results remain valid if it is incorporated.

In Equation (2), the decision variables are $\bar{z}_t$, y_{1t}, and y_{2t}. The first term inside the brackets is the cost from emissions trading, the next two terms are the production costs, the fourth term is the expected inventory holding and demand backlogging cost, and the last one is the minimal total expected discounted cost from period $t+1$ to the end of the planning horizon. The constraint requires only a nonnegative production quantity from each production technology. Note that no constraint is imposed on $\bar{z}_t$, since the manufacturer can be at any allowance level and the production is not constrained by the allowance level.

In each period, the manufacturer needs to make several related decisions to balance emissions and production costs and to fulfill random market demands. These include the number of allowances to sell or buy, which production technologies to use, and how many units to produce. By analyzing the structural properties of the problem, we will provide a complete characterization of the optimal policies for the manufacturer.

4. Structural Properties

In this section, we first examine the structural properties of problem (2), which will facilitate the derivation of optimal policies. Define $w_t = \bar{z}_t - \mu_1(y_{1t} - x_t) - \mu_2(y_{2t} - y_{1t})$, representing the ending allowance level of period t. Thus, with given w_t, y_{2t}, and x_t, y_{1t} is uniquely determined as

$$y_{1t} = \frac{\bar{z}_t - w_t + \mu_1 x_t - \mu_2 y_{2t}}{\mu_1 - \mu_2}. \tag{4}$$

For convenience, we will use $\bar{z}_t$, w_t, and y_{2t} as the decision variables instead of $\bar{z}_t$, y_{1t}, and y_{2t}. Note that $\bar{z}_t - w_t$ represents the total emission allowances depleted by production. Thus, the constraint $x_t \leqslant y_{1t} \leqslant y_{2t}$ becomes $w_t \leqslant \bar{z}_t$ and $x_t + \mu_1^{-1}(\bar{z}_t - w_t) \leqslant y_{2t} \leqslant x_t + \mu_2^{-1}(\bar{z}_t - w_t)$. As $(c_2 - c_1)/(\mu_1 - \mu_2)$ will play an important role in our subsequent analysis, for brevity we denote it as C_μ, i.e.,

$$C_\mu = \frac{c_2 - c_1}{\mu_1 - \mu_2}. \tag{5}$$

Note that C_μ is positive and can be interpreted as the *additional production cost of green technology per allowance saved*. For brevity, we hereafter refer to it as the *additional production cost per allowance saved*.

With (4) and (5), the dynamic program (2) can be rewritten as (for notational convenience, we will suppress the subscript t unless confusion would otherwise arise),

$$\begin{aligned} &V_t(x, z, \mathbf{K}) \\ &= \min_{y_2, \bar{z}, w} \bigg\{ C_t(\bar{z} - z, \mathbf{K}) + C_\mu(w - \bar{z}) + G_t(y_2) + \frac{c_2\mu_1 - c_1\mu_2}{\mu_1 - \mu_2} \\ &\qquad \cdot (y_2 - x) + \gamma \mathsf{E}_t[V_{t+1}(y_2 - D_t, w, \tilde{\mathbf{K}}_{t+1})] \bigg\}, \end{aligned}$$

subject to the constraints $w \leqslant \bar{z}$ and $\mu_1^{-1}(\bar{z} - w) \leqslant y_2 - x \leqslant \mu_2^{-1}(\bar{z} - w)$.

For ease of discussion, we denote

$$\begin{aligned} W_t(x, z, \mathbf{K}) = \min_{w, y_2} \bigg\{ &C_\mu(w - z) + G_t(y_2) + \frac{c_2\mu_1 - c_1\mu_2}{\mu_1 - \mu_2} \\ &\cdot (y_2 - x) + \gamma \mathsf{E}_t[V_{t+1}(y_2 - D_t, w, \tilde{\mathbf{K}}_{t+1})] \bigg\}, \end{aligned} \tag{6}$$

subject to $w \leqslant z$ and $\mu_1^{-1}(z - w) \leqslant y_2 - x \leqslant \mu_2^{-1}(z - w)$. Thus,

$$V_t(x, z, \mathbf{K}) = \min_{\bar{z}} \{C_t(\bar{z} - z, \mathbf{K}) + W_t(x, \bar{z}, \mathbf{K})\}. \tag{7}$$

It is intuitive that a higher allowance level will lead to a lower total cost for the firm. Proposition 1 further provides lower and upper bounds of the marginal value of the allowances to the firm.

Proposition 1. *For $t = 1, \ldots, T$ and $z_1 < z_2$,*

$$-\min_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{K}_{t+i} \mid \mathbf{K}_t]\} \leqslant \frac{V_t(x, z_2, \mathbf{K}_t) - V_t(x, z_1, \mathbf{K}_t)}{z_2 - z_1} \leqslant -\max_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t]\}.$$

From Proposition 1, the marginal value of the allowances at the beginning of period t is lower bounded by $\max_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t]\}$ and upper bounded by $\min_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{K}_{t+i} \mid \mathbf{K}_t]\}$, where $\gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t]$ (resp., $\gamma^i \mathsf{E}[\tilde{K}_{t+i} \mid \mathbf{K}_t]$) is the discounted expected revenue (resp., cost) of selling (resp., buying) a unit of allowance in period $t+i$. To understand the lower bound intuitively, note that the firm is risk-neutral and when in period t, it can at least obtain $\gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t]\}$ by selling an allowance in period $t+i$ for any $i = 0, \ldots, T-t$; hence the marginal value of the allowance to the firm in period t should be at least $\max_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t]\}$. The upper bound can be similarly explained. Note that Condition (1) ensures that $\max_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{k}_{t+i} \mid \mathbf{K}_t] \leqslant \min_{0\leqslant i\leqslant T-t}\{\gamma^i \mathsf{E}[\tilde{K}_{t+i} \mid \mathbf{K}_t]\}$.

We next establish the convexity and modularity of the optimal cost functions. To this end, we first provide a set of preservation properties of modularity after minimization in Lemma 1.

Lemma 1. *Suppose $f(x, y): \Re^2 \to \Re$ is convex and for some $0 \leqslant \alpha_1 \leqslant \alpha_2$, define*

$$g(x, y) = \min_{\substack{u\geqslant 0\\ \alpha_1\leqslant\alpha\leqslant\alpha_2}} \{f(x+\alpha u, y-u)\}.$$

Then, the following preservation properties hold:

(a) *If $f(x, y)$ is supermodular in (x, y), then $g(x, y)$ is also supermodular in (x, y);*

(b) *if $f(x, y - x/\alpha_1)$ is submodular in (x, y), then $g(x, y - x/\alpha_1)$ is also submodular in (x, y);*

(c) *if $f(x - \alpha_2 y, y)$ is submodular in (x, y), then $g(x - \alpha_2 y, y)$ is also submodular in (x, y).*

Lemma 1 is the key technical result of the paper. Here we sketch the idea of proving this lemma while giving the detailed proof in the appendix. For (a), we prove that $g(x, y)$ is supermodular by definition, i.e., for any $x_1 < x_2$ and $y_1 < y_2$, we verify that

$$g(x_1, y_1) + g(x_2, y_2) \geqslant g(x_1, y_2) + g(x_2, y_1). \tag{8}$$

Denote $\mathbf{B}(x, y) = \{(x+\alpha u, y-u) \mid u \geqslant 0, \alpha_1 \leqslant \alpha \leqslant \alpha_2\}$. Then, $g(x, y) = \min_{(\tilde{x}, \tilde{y})\in\mathbf{B}(x, y)} f(\tilde{x}, \tilde{y})$. In addition, denote

$$(x_i^*, y_i^*) \in \operatorname*{arg\,min}_{(\tilde{x}, \tilde{y})\in\mathbf{B}(x_i, y_i)} f(\tilde{x}, \tilde{y}), \quad i = 1, 2.$$

Then, $g(x_i, y_i) = f(x_i^*, y_i^*)$, $i = 1, 2$. We show (8) by identifying two points $(\hat{x}, \hat{y}) \in \mathbf{B}(x_1, y_2)$ and $(\bar{x}, \bar{y}) \in \mathbf{B}(x_2, y_1)$ such that $f(x_1^*, y_1^*) + f(x_2^*, y_2^*) \geqslant f(\hat{x}, \hat{y}) + f(\bar{x}, \bar{y}) \geqslant g(x_1, y_2) + g(x_2, y_1)$. To this end, depending on the relative position of (x_1^*, y_1^*) and (x_2^*, y_2^*) on the two-dimensional x-y plane (note that although $f(x, y)$ is supermodular and convex, the relation between (x_1^*, y_1^*) and (x_2^*, y_2^*) can be complicated), we divide the analysis into different cases and construct the desirable points by applying the convexity and supermodularity of $f(x, y)$. For example, when $x_2^* \leqslant x_1^*$ and $y_2^* \leqslant y_1^*$, (8) can be easily verified because $(\hat{x}, \hat{y}) = (x_1^*, y_1^*) \in \mathbf{B}(x_1, y_2)$ and $(\bar{x}, \bar{y}) = (x_2^*, y_2^*) \in \mathbf{B}(x_2, y_1)$. However, if $x_2^* > x_1^*$ and $y_2^* \leqslant y_1^*$, the construction of $(\hat{x}, \hat{y})$ and $(\bar{x}, \bar{y})$ is more involved. We will need to first construct two points $(\hat{x}, \hat{y})$ and $(\bar{x}, \bar{y})$ on the line segment connecting (x_1^*, y_1^*) and (x_2^*, y_2^*) that, respectively, belong to $\mathbf{B}(x_1, y_2)$ and $\mathbf{B}(x_2, y_1)$, and then apply the convexity of $f(x, y)$ to show (8).

For (b), as $g(x, y - x/\alpha_1) = \min_{\alpha_1\leqslant\alpha\leqslant\alpha_2} h(x, y - x/\alpha_1, \alpha)$ with $h(x, y, \alpha) = \min_{u\geqslant 0} f(x+\alpha u, y-u)$, we first show for any given $\alpha \geqslant \alpha_1$ that $h(x, y - x/\alpha_1, \alpha)$ is submodular in (x, y). Based on this result, we again prove by definition that $g(x, y - x/\alpha_1)$ is submodular in (x, y). An analogous idea can prove part (c).

Remark. It is worth noting that by similar approaches, we can also show that if $f(x - \alpha_1 y, y)$ is supermodular in (x, y), so is $g(x - \alpha_1 y, y)$; and if $f(x, y - x/\alpha_2)$ is supermodular in (x, y), so is $g(x, y - x/\alpha_2)$. Moreover, if $f(x, y)$ is defined on $\Re \times [0, +\infty)$ and $g(x, y) = \min_{0\leqslant u\leqslant y,\, \alpha_1\leqslant\alpha\leqslant\alpha_2}\{f(x+\alpha u, y-u)\}$, Lemma 1 and the aforementioned two preservation results also hold.

Lemma 1 and its alternatives stated in the above remark can be applied to show the preservation of various economic relationships between two variables x and y after minimization when regarding y as a resource that can be transformed into a product x in one ($\alpha_1 = \alpha_2$) or two ways ($\alpha_1 < \alpha_2$). The resource level can be any real number such as the allowance level in this paper or restricted to be nonnegative such as production capacity, raw materials (Yang 2004), or available cash (Chao et al. 2008). We expect this lemma and its alternatives to have more applications.

Proposition 2. *For $t = 1, \ldots, T$ and any given $\mathbf{K}$,*

(a) *$V_t(x, z, \mathbf{K})$ and $W_t(x, z, \mathbf{K})$ are convex and supermodular in (x, z);*

(b) *$V_t(x - z/\mu_2, z, \mathbf{K})$ and $W_t(x - z/\mu_2, z, \mathbf{K})$ are both submodular in (x, z);*

(c) *$V_t(x, z - \mu_1 x, \mathbf{K})$ and $W_t(x, z - \mu_1 x, \mathbf{K})$ are both submodular in (x, z).*

Proposition 2 is proved by mainly applying Lemma 1 and will facilitate our analysis on the optimal policies. We sketch the proof of part (a) here. The convexity result can be proved inductively by using the preservation property of convexity after minimization (e.g., Proposition B-4 of

Heyman and Sobel 1984). We focus on the supermodularity. It is easy to check that the result holds for $t = T+1$. Suppose inductively that it holds for $t+1$. Then for t, we first show that $W_t(x, z, \mathbf{K})$ is supermodular. To apply Lemma 1, we need to first conduct variable transformations $\beta = z - w$ and $\alpha = (y_2 - x)/\beta$, and the resulting problem is as follows:

$$W_t(x, z, \mathbf{K}) = \min_{\substack{\beta \geqslant 0 \\ \mu_1^{-1} \leqslant \alpha \leqslant \mu_2^{-1}}} \left\{ \frac{c_2\mu_1 - c_1\mu_2}{\mu_1 - \mu_2}\beta\alpha - C_\mu \beta + G_t(x+\beta\alpha) + \gamma \mathsf{E}_t[V_{t+1}(x+\beta\alpha - D_t, z-\beta, \tilde{\mathbf{K}}_{t+1})] \right\}.$$

Note that the minimand in the above optimization problem is a function of $(x+\beta\alpha, z-\beta)$ and has the same structure as that in Lemma 1 by the inductive assumption. Hence, from part (a) of Lemma 1, $W_t(x, z, \mathbf{K})$ is supermodular in (x, z). Based on this, we can show that the minimand in (7) is submodular in $(-x, z, \bar{z})$. Thus, by applying the preservation of submodularity after minimization (e.g., Theorem 2.7.6 in Topkis 1998), $V_t(-x, z, \mathbf{K})$ is submodular in (x, z) and so $V_t(x, z, \mathbf{K})$ is supermodular in (x, z). The detailed proof of the proposition is given in the electronic companion.

It is worth noting that with Lemma 1, we show the modularity results of the value functions directly before deriving the optimal policies. One will see later that, if the modularity is proved by using the optimal policies, the analysis will be more complicated and tedious because the optimal policies divide the state space into multiple regions.

The convexity and supermodularity of $V_t(x, z, \mathbf{K})$ in (x, z) imply that the marginal values of inventory and allowances both decrease if either the inventory level or the allowance level increases. Furthermore, part (b) implies that μ_2 additional units of emission allowances will not reduce the marginal value of inventory as much as one additional unit of the product; whereas part (c) implies that one additional unit of the product will not reduce the marginal value of allowances as much as μ_1 additional units of allowances. Both results are intuitive, since the amount of allowances used in producing one unit of the product is between μ_2 and μ_1.

5. Optimal Policies

Having established the structural properties of the optimal cost functions, in this section we proceed to study the manufacturer's optimal production and emissions trading policies.

We first characterize the structure of the optimal emissions trading policy, which is summarized in Theorem 1.

Theorem 1. *For $t = 1, 2, \ldots, T$, given state $(x, z, \mathbf{K})$ at the beginning of period t, there exists a pair of thresholds $(L_t(x, \mathbf{K}), U_t(x, \mathbf{K}))$ with $L_t(x, \mathbf{K}) \leqslant U_t(x, \mathbf{K})$ such that the optimal emissions trading policy is a target interval policy, i.e.,*

$$\bar{z}_t^* = \begin{cases} L_t(x, \mathbf{K}), & \text{if } z \leqslant L_t(x, \mathbf{K}); \\ z, & \text{if } L_t(x, \mathbf{K}) < z < U_t(x, \mathbf{K}); \\ U_t(x, \mathbf{K}), & \text{if } z \geqslant U_t(x, \mathbf{K}). \end{cases} \tag{9}$$

Moreover, $-\mu_1 \leqslant (L_t(x_2, \mathbf{K}) - L_t(x_1, \mathbf{K}))/(x_2 - x_1) \leqslant 0$ and $-\mu_1 \leqslant (U_t(x_2, \mathbf{K}) - U_t(x_1, \mathbf{K}))/(x_2 - x_1) \leqslant 0$ for any $x_1 < x_2$.

Theorem 1 shows that the optimal emissions trading policy is a target interval policy characterized by two threshold levels $L_t(x, \mathbf{K})$ and $U_t(x, \mathbf{K})$. That is, if the starting allowance level is less than $L_t(x, \mathbf{K})$, then purchase the allowances to increase the level to $L_t(x, \mathbf{K})$; if it is higher than $U_t(x, \mathbf{K})$, then sell the allowances to reduce the level to $U_t(x, \mathbf{K})$, and do not trade otherwise. In addition, the purchase-up-to level $L_t(x, \mathbf{K})$ is less than the sell-down-to level $U_t(x, \mathbf{K})$. This result is rather intuitive because the buying price K_t is higher than the selling price k_t. Theorem 1 also shows that both target levels are decreasing with the starting inventory level x, and the decreasing rate is bounded by μ_1 since a higher inventory level leads to less need for the allowances and one unit of inventory requires at most μ_1 allowances, which is a direct consequence of Proposition 2. In the subsequent analysis, we will provide a more detailed expression for $L_t(x, \mathbf{K})$ and $U_t(x, \mathbf{K})$ depending on the relationship between C_μ and $\mathbf{K}$.

The optimal production policy that specifies the production quantity from each technology is determined by the optimization problem (6). Although the objective function in (6) is convex, the structure of the optimal policy is not easy to see immediately. With a more thorough analysis, we will be able to characterize the optimal production policy.

In certain circumstances, merely based on the relations between C_μ and the allowance trading price $\mathbf{K}_t$, we can determine the manufacturer's optimal technology choice.

Proposition 3. *For $t = 1, 2, \ldots, T$, given any $\mathbf{K}_t = (K_t, k_t)$,*

(a) *if $C_\mu \geqslant K_t$, it is optimal to use only production technology 1;*

(b) *if $C_\mu \leqslant k_t$, it is optimal to use only production technology 2;*

(c) *if $k_t < C_\mu < K_t$, it is optimal to use only production technology 1 if $z \geqslant U_t(x, \mathbf{K}_t)$; whereas it is optimal to use only production technology 2 if $z \leqslant L_t(x, \mathbf{K}_t)$.*

From Proposition 3, if $C_\mu \leqslant k_t$ or $C_\mu \geqslant K_t$, it is optimal to use only one production technology in period t. The intuition is as follows. When $C_\mu \geqslant K_t$, buying allowances is more economical than using the more expensive green technology (so as to save allowances) and thus the firm will use only the regular technology. If needed, the firm can always buy the amount of allowances used in this period. In contrast, when $C_\mu \leqslant k_t$, the firm can always sell the allowances in this period to obtain more income rather than use the regular technology to deplete the allowances, and therefore it should adopt the green technology. Part (c) shows that when $k_t < C_\mu < K_t$, the technology the manufacturer should deploy depends on the allowance level and its corresponding trading strategy. Thus, the technology choice depends on the future trading price development, which is through the optimal solution $(L_t(x, \mathbf{K}_t), U_t(x, \mathbf{K}_t))$ (see Equations (20) and (21) in the online appendix, available as supplemental material at opre.2013.1189, for the definition). As long as the manufacturer trades emission allowances, it is always optimal to use only one production technology. Specifically, if the manufacturer buys allowances, it should use the green technology because in this case the value of the allowances K_t is higher than the additional production cost per allowance saved; on the other hand, if it sells allowances, it should use the regular production technology because the allowance value k_t is lower than the additional cost from using the green technology.

When $k_t < C_\mu < K_t$ and it is optimal not to trade allowances, additional analysis is required to study the optimal technology selection of the manufacturer. Before that, we provide the structure of the optimal policies in the scenarios presented in Proposition 3(a) and (b), where we have shown the optimal technology choice for the manufacturer.

Theorem 2. *For $t = 1, \ldots, T$, given state $(x, z, \mathbf{K})$ at the beginning of period t, there exist base-stock levels $(S^U_{it}(\mathbf{K}), S^L_{it}(\mathbf{K}))$ and $s_{it}(\mu_i x + z, \mathbf{K})$, $i = 1, 2$, such that the optimal policies $(y^*_{1t}, y^*_{2t}, \bar{z}^*_t)$ are characterized as follows:*

(a) *Suppose $C_\mu \geqslant K$. Then, $y^*_{1t} = y^*_{2t}$. In addition, when $z \geqslant U_t(x, \mathbf{K})$, $y^*_{1t} = \max\{S^U_{1t}(\mathbf{K}), x\}$ and $\bar{z}^*_t = U_t(x, \mathbf{K})$; when $z \leqslant L_t(x, \mathbf{K})$, $y^*_{1t} = \max\{S^L_{1t}(\mathbf{K}), x\}$ and $\bar{z}^*_t = L_t(x, \mathbf{K})$; and when $L_t(x, \mathbf{K}) < z < U_t(x, \mathbf{K})$, $y^*_{1t} = \max\{s_{1t}(\mu_1 x + z, \mathbf{K}), x\}$ and $\bar{z}^*_t = z$. Moreover,*

$$L_t(x, \mathbf{K}) = \mu_1 (S^L_{1t}(\mathbf{K}) - x)^+ + w^L_t(\max\{S^L_{1t}(\mathbf{K}), x\}, \mathbf{K}),$$
$$U_t(x, \mathbf{K}) = \mu_1 (S^U_{1t}(\mathbf{K}) - x)^+ + w^U_t(\max\{S^U_{1t}(\mathbf{K}), x\}, \mathbf{K}).$$

(b) *Suppose $C_\mu \leqslant k$. Then, $y^*_{1t} = x$. In addition, when $z \geqslant U_t(x, \mathbf{K})$, $y^*_{2t} = \max\{S^U_{2t}(\mathbf{K}), x\}$ and $\bar{z}^*_t = U_t(x, \mathbf{K})$; when $z \leqslant L_t(x, \mathbf{K})$, $y^*_{2t} = \max\{S^L_{2t}(\mathbf{K}), x\}$ and $\bar{z}^*_t = L_t(x, \mathbf{K})$; and when $L_t(x, \mathbf{K}) < z < U_t(x, \mathbf{K})$, $y^*_{2t} = \max\{s_{2t}(\mu_2 x + z, \mathbf{K}), x\}$ and $\bar{z}^*_t = z$. Moreover,*

$$L_t(x, \mathbf{K}) = \mu_2 (S^L_{2t}(\mathbf{K}) - x)^+ + w^L_t(\max\{S^L_{2t}(\mathbf{K}), x\}, \mathbf{K}),$$
$$U_t(x, \mathbf{K}) = \mu_2 (S^U_{2t}(\mathbf{K}) - x)^+ + w^U_t(\max\{S^U_{2t}(\mathbf{K}), x\}, \mathbf{K}).$$

$w^L_t(y^*_{2t}, \mathbf{K})$ and $w^U_t(y^*_{2t}, \mathbf{K})$ specify the optimal ending allowance levels when the firm buys and sells the allowances, respectively.

Theorem 2 characterizes the optimal policies when $C_\mu \geqslant K_t$ and $C_\mu \leqslant k_t$ and further provides the specific forms of the optimal thresholds for emissions trading. It shows that the optimal production policy is a base-stock policy with the optimal base-stock level independent of the inventory and allowance levels when the firm trades allowances, whereas the optimal base-stock level depends on the inventory level and the allowance level when it is optimal not to trade. Moreover, the theorem dictates that the thresholds for the emissions trading policy consist of two parts: the allowances needed for the current period and the remaining allowance level for the future periods. In particular, we can see that when the manufacturer produces, $L_t(x, \mathbf{K})$ and $U_t(x, \mathbf{K})$ are linearly decreasing in x with slope $-\mu_i$ when production technology i is chosen.

Again, by applying Proposition 2, we can obtain the properties of $w^j_t(y, \mathbf{K})$, $S^j_{it}(\mathbf{K})$, and $s_{it}(\mu_i x + z, \mathbf{K})$, $i = 1, 2$ and $j = L, U$, in the next result.

Proposition 4. *For $t = 1, \ldots, T$, for any given $\mathbf{K}$,*

(a) $w^L_t(y, \mathbf{K}) \leqslant w^U_t(y, \mathbf{K})$;

(b) *for $y_1 < y_2$, $-\mu_1 \leqslant (w^j_t(y_2, \mathbf{K}) - w^j_t(y_1, \mathbf{K}))/(y_2 - y_1) \leqslant 0$, $j = L, U$;*

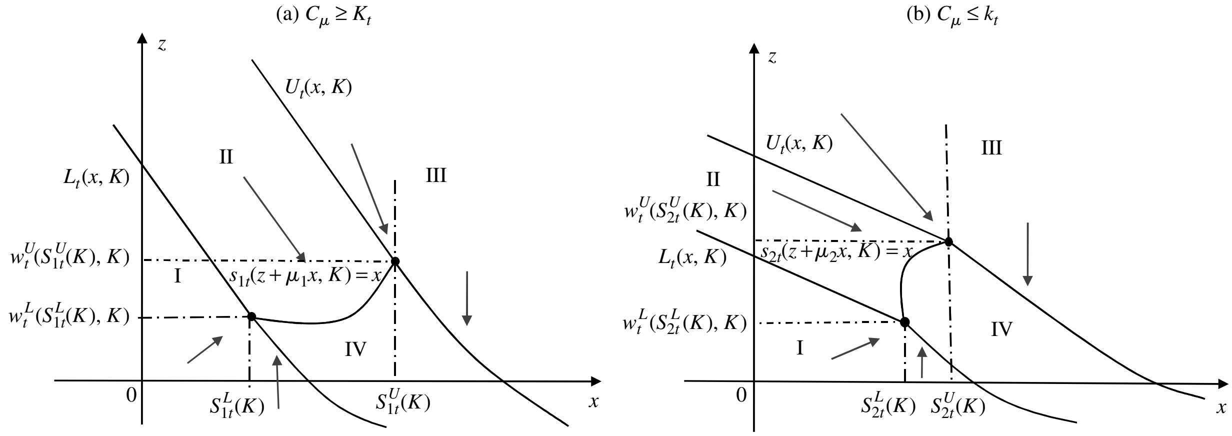
(c) $S^U_{1t}(\mathbf{K}) \geqslant S^L_{1t}(\mathbf{K})$ *and* $w^L_t(S^L_{2t}(\mathbf{K}), \mathbf{K}) \leqslant w^U_t(S^U_{2t}(\mathbf{K}), \mathbf{K})$;

(d) $s_{1t}(\mu_1 x + z, \mathbf{K})$ *is increasing in $\mu_1 x + z$ and $z - \mu_2(s_{2t}(\mu_2 x + z, \mathbf{K}) - x)$ is increasing in $\mu_2 x + z$.*

Because $w^L_t(y, \mathbf{K})$ (resp., $w^U_t(y, \mathbf{K})$) represents the optimal ending allowance level when the post-production inventory level is y and the firm buys (resp., sells) allowances, it is intuitive that $w^L_t(y, \mathbf{K}) \leqslant w^U_t(y, \mathbf{K})$ and $w^j_t(y, \mathbf{K})$ is decreasing in y with rates no more than μ_1. Part (c) dictates that if the manufacturer uses the production technology 1 only, the base-stock level $S^U_{1t}(\mathbf{K})$ when it sells the allowances is higher than the base-stock level $S^L_{1t}(\mathbf{K})$ when it buys the allowances; if the manufacturer uses the production technology 2 only, the optimal ending allowance level $w^U_t(S^U_{2t}(\mathbf{K}), \mathbf{K})$ when it sells the allowances is higher than the ending level $w^L_t(S^L_{2t}(\mathbf{K}), \mathbf{K})$ when it buys the allowances. These relationships are implied by Proposition 2(b) and (c). To explain part (d), note that $s_{it}(\mu_{ix} + z, \mathbf{K})$ is the base-stock level when the firm does not trade the allowances and uses technology i only, and $\mu_{ix} + z$ can be regarded as the effective number of allowances under technology i. Part (d) posits that $s_{1t}(\mu_1 x + z, \mathbf{K})$ and the optimal ending allowance level $z - \mu_2(s_{2t}(\mu_2 x + z, \mathbf{K}) - x)$ increase with their corresponding effective allowance level, which follows directly from Proposition 2(b) and (c).

Figure 1 illustrates the optimal policies in Theorem 2 for a given $\mathbf{K}$. The arrows in the figure point to the optimal produce-up-to level and the optimal ending allowance level after production and trading. For example, in Figure 1(a), if the state (x, z) falls into region I, it is optimal to first purchase additional allowances to increase the allowance level to $L_t(x, \mathbf{K})$ and then produce up to $S^L_{1t}(\mathbf{K})$ (or produce nothing if $x > S^L_{1t}(\mathbf{K})$). Combining these two actions leads to the direction of the arrow pointing to $(S^L_{1t}(\mathbf{K}), w^L_t(S^L_{1t}(\mathbf{K}), \mathbf{K}))$; for (x, z) in region II, it is optimal not to trade allowances while producing up to $s_{1t}(\mu_1 x + z, \mathbf{K})$. For (x, z) in region III, the firm should

Figure 1. Optimal policy: $C_\mu \geqslant K_t$ and $C_\mu \leqslant k_t$.



sell the allowances down to $U_t(x, \mathbf{K})$ and produce up to $S_{1t}^U(\mathbf{K})$; finally, for (x, z) in region IV, it is optimal for the manufacturer to do nothing. Figure 1(b) can be similarly explained.

In the following theorem, we demonstrate that the optimal production policy is also of a base-stock type with base-stock levels independent of the initial inventory and allowance levels when $k_t < C_\mu < K_t$ and the manufacturer trades allowances.

Theorem 3. *For $t = 1, \ldots, T$, given state $(x, z, \mathbf{K})$ at the beginning of period t, if $k < C_\mu < K$, the optimal policies $(y_{1t}^*, y_{2t}^*, \bar{z}_t^*)$ are characterized as follows:*

- *If $z \geqslant U_t(x, \mathbf{K})$, then $y_{1t}^* = y_{2t}^* = \max\{S_{1t}^U(\mathbf{K}), x\}$ and $\bar{z}_t^* = U_t(x, \mathbf{K})$;*
- *if $z \leqslant L_t(x, \mathbf{K})$, then $y_{1t}^* = x$, $y_{2t}^* = \max\{S_{2t}^L(\mathbf{K}), x\}$ and $\bar{z}_t^* = L_t(x, \mathbf{K})$;*

where $L_t(x, \mathbf{K})$ and $U_t(x, \mathbf{K})$ are given by

$$L_t(x, \mathbf{K}) = \mu_2(S_{2t}^L(\mathbf{K}) - x)^+ + w_t^L(\max\{S_{2t}^L(\mathbf{K}), x\}, \mathbf{K}),$$
$$U_t(x, \mathbf{K}) = \mu_1(S_{1t}^U(\mathbf{K}) - x)^+ + w_t^U(\max\{S_{1t}^U(\mathbf{K}), x\}, \mathbf{K}),$$

where $w_t^L(y_{2t}^*, \mathbf{K})$ and $w_t^U(y_{2t}^*, \mathbf{K})$ specify the optimal ending allowance levels when the firm buys and sells the allowances, respectively.

According to Theorem 3, when the manufacturer sells the allowances in period t, it should use the regular technology only to produce up to $S_{1t}^U(\mathbf{K})$; while when it buys the allowances, it should use the green technology only to produce up to $S_{2t}^L(\mathbf{K})$.

Now it remains to study the manufacturer's optimal technology selection and production policy when $k_t < C_\mu < K_t$ and $L_t(x, \mathbf{K}) < z < U_t(x, \mathbf{K})$, which is addressed in the following theorem.

Theorem 4. *For $t = 1, \ldots, T$, given state $(x, z, \mathbf{K})$, if $k < C_\mu < K$ and $L_t(x, \mathbf{K}) < z < U_t(x, \mathbf{K})$, there exist a pair of thresholds $(l_t(x, \mathbf{K}), u_t(x, \mathbf{K}))$ satisfying $L_t(x, \mathbf{K}) \leqslant l_t(x, \mathbf{K}) \leqslant u_t(x, \mathbf{K}) \leqslant U_t(x, \mathbf{K})$, and a base-stock level $S_t^m(\mathbf{K})$. It is optimal not to trade the allowances $\bar{z}_t^* = z$, and the optimal production policies (y_{1t}^*, y_{2t}^*) are characterized as follows:*

(a) If $L_t(x, \mathbf{K}) < z \leqslant l_t(x, \mathbf{K})$, it is optimal to use technology 2 only, and the optimal production policy is a base-stock policy, i.e., $y_{1t}^* = x$ and $y_{2t}^* = \max\{s_{2t}(\mu_2 x + z, \mathbf{K}), x\}$;

(b) if $u_t(x, \mathbf{K}) \leqslant z < U_t(x, \mathbf{K})$, it is optimal to use technology 1 only, and the optimal production policy is a base-stock policy, i.e., $y_{1t}^* = y_{2t}^* = \max\{s_{1t}(\mu_1 x + z, \mathbf{K}), x\}$;

(c) if $l_t(x, \mathbf{K}) < z < u_t(x, \mathbf{K})$, it is optimal to use both production technologies, and the optimal production policy is a base-stock policy with a base-stock level $S_t^m(\mathbf{K})$; the optimal ending allowance level is $w_t^m(S_t^m(\mathbf{K}), \mathbf{K})$. Moreover, $l_t(x, \mathbf{K})$ and $u_t(x, \mathbf{K})$ are given by:

$$l_t(x, \mathbf{K}) = \mu_2(S_t^m(\mathbf{K}) - x)^+ + w_t^m(\max\{S_t^m(\mathbf{K}), x\}, \mathbf{K});$$
$$u_t(x, \mathbf{K}) = \mu_1(S_t^m(\mathbf{K}) - x)^+ + w_t^m(\max\{S_t^m(\mathbf{K}), x\}, \mathbf{K}).$$

Theorems 3 and 4 together characterize the optimal technology selection and production policy when $k_t < C_\mu < K_t$. In particular, if $z \leqslant l_t(x, \mathbf{K})$, then it is optimal to only use the green technology; if $z \geqslant u_t(x, \mathbf{K})$, then it is optimal to only use the regular technology; and otherwise, both technologies should be used. In addition, the optimal production policy is of a base-stock type. Specifically, Theorem 3 states that the optimal base-stock level is independent of the inventory and allowance levels (x, z) when the manufacturer trades the allowances. In contrast, if it is optimal not to trade, Theorem 4 posits that when $L_t(x, \mathbf{K}) < z \leqslant l_t(x, \mathbf{K})$ or $u_t(x, \mathbf{K}) \leqslant z < U_t(x, \mathbf{K})$, the optimal base-stock level depends on $\mu_i x + z$ when technology i is used; whereas if $l_t(x, \mathbf{K}) < z < u_t(x, \mathbf{K})$, the optimal base-stock level is independent of the inventory and allowance levels (x, z).

Note that when $l_t(x, \mathbf{K}) < z < u_t(x, \mathbf{K})$, both technologies are used. This is because, in this case, it is optimal for the manufacturer to keep its post-production inventory level at $S_t^m(\mathbf{K})$ and the ending allowance level at $w_t^m(S_t^m(\mathbf{K}), \mathbf{K})$ simultaneously, i.e., the interior optimal solution of Equation (6) when $\bar{z}^* = z$ is attainable. And $(S_t^m(\mathbf{K}), w_t^m(S_t^m(\mathbf{K}), \mathbf{K}))$ can be attained only by using the regular technology to produce

$$y_{1t}^* - x = \frac{(z - w_t^m(S_t^m(\mathbf{K}), \mathbf{K})) - \mu_2(S_t^m(\mathbf{K}) - x))}{\mu_1 - \mu_2},$$

and the green technology to produce

$$y_{2t}^* - y_{1t}^* = \frac{\mu_1(S_t^m(\mathbf{K}) - x) - (z - w_t^m(S_t^m(\mathbf{K}), \mathbf{K}))}{\mu_1 - \mu_2}.$$

It is clear that both production quantities are positive when $l_t(x, \mathbf{K}) < z < u_t(x, \mathbf{K})$ from the definition of $l_t(x, \mathbf{K})$ and $u_t(x, \mathbf{K})$.

Proposition 5. *For $t = 1, \ldots, T$ and $\mathbf{K} = (k, K)$, if $k < C_\mu < K$, then*

(a) $w_t^L(y, \mathbf{K}) \leqslant w_t^m(y, \mathbf{K}) \leqslant w_t^U(y, \mathbf{K})$;

(b) $l_t(x, \mathbf{K})$, $u_t(x, \mathbf{K})$, *and* $u_t(x, \mathbf{K}) - l_t(x, \mathbf{K})$ *are decreasing in* x;

(c) $S_{1t}^U(\mathbf{K}) \geqslant S_t^m(\mathbf{K})$ *and* $w_t^m(S_t^m(\mathbf{K}), \mathbf{K}) \geqslant w_t^L(S_{2t}^L(\mathbf{K}), \mathbf{K})$.

Proposition 5(a) dictates that, given a post-production inventory level y, the firm would carry a higher (resp., lower) ending allowance level when it sells (resp., buys) the allowances than when it does not trade the allowances. Part (b) shows not only that $l_t(x, \mathbf{K})$ and $u_t(x, \mathbf{K})$ are decreasing in x but also that the region where the manufacturer uses both technologies shrinks when x increases. From Theorem 4, $S_t^m(\mathbf{K})$ and $w_t^m(S_t^m(\mathbf{K}), \mathbf{K})$ are the optimal base-stock level and the optimal ending allowance level, respectively, when the manufacturer does not trade the allowances and uses both technologies. Thus, part (c) posits that in such a case, the manufacturer would keep a lower inventory level than when it sells the allowances and uses technology 1, and keep a higher optimal ending allowance level than when it buys allowances and uses technology 2. Again, most of these results are derived based on Proposition 2.

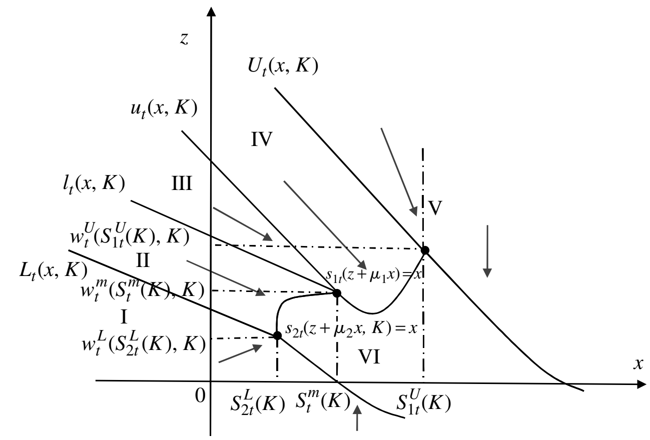
From the preceding analysis, we illustrate the structure of the optimal policy when $k_t < C_\mu < K_t$ in Figure 2. The relative positions and trajectories of the optimal policy parameters are plotted based on Proposition 5.

We next present the result for a special case where there is only a single production technology. That is, each unit of the product depletes $\mu_1 = \mu_2 = \mu$ units of the allowances and incurs the production cost $c_1 = c_2 = c$.

Corollary 1. *Suppose there is a single production technology. For $t = 1, \ldots, T$, given state $(x, z, \mathbf{K})$ at the beginning of period t, the optimal emissions trading policy is a target interval policy characterized by $(L_t(x, \mathbf{K}), U_t(x, \mathbf{K}))$ with $L_t(x, \mathbf{K}) \leqslant U_t(x, \mathbf{K})$; and the optimal production policy is a base-stock policy characterized by $S_t^L(\mathbf{K})$, $S_t^U(\mathbf{K})$, and $s_t(\mu x + z, \mathbf{K})$. Specifically,*

- *if* $z > U_t(x, \mathbf{K})$, *then* $\bar{z}_t^* = U_t(x, \mathbf{K})$ *and* $y_t^* = \max\{S_t^U(\mathbf{K}), x\}$;

Figure 2. Optimal policy: $k_t < C_\mu < K_t$.



- *if* $z < L_t(x, \mathbf{K})$, *then* $\bar{z}_t^* = L_t(x, \mathbf{K})$ *and* $y_t^* = \max\{S_t^L(\mathbf{K}), x\}$;
- *if* $L_t(x, \mathbf{K}) \leqslant z \leqslant U_t(x, \mathbf{K})$, *then* $\bar{z}_t^* = z$ *and* $y_t^* = \max\{s_t(\mu x + z, \mathbf{K}), x\}$;

where $S_t^U(\mathbf{K}) \geqslant S_t^L(\mathbf{K})$ and $s_t(\mu x + z, \mathbf{K})$ is increasing in $\mu x + z$.

5.1. A Special Case: $\tilde{K}_t = \tilde{k}_t$

If the bid-ask price spreads and transaction costs can be ignored, the allowance selling and buying prices become the same. We study this special case here. To avoid introducing additional notation, we denote $\tilde{k}_t$ as the random trading price with $P(0 \leqslant \tilde{k}_t \leqslant \gamma^{T-t+1}\pi) = 1$ and assume that $\{\tilde{k}_t,\ 1 \leqslant t \leqslant T\}$ forms a Markov chain. Because $\tilde{K}_t = \tilde{k}_t$, Condition (1) implies that the trading price process forms a martingale, i.e., $k_t = \gamma \mathsf{E}[\tilde{k}_{t+1} \mid k_t]$ for $t = 1, \ldots, T-1$. The martingale price process has been derived and adopted in emissions trading models (e.g., Carmona et al. 2009, 2010), and several empirical studies have suggested that the martingale model is mostly valid except when some information disclosure systematically changes the market's anticipations (see, e.g., Charles et al. 2010).

With only one trading price, the dynamic program (2) is simplified to

$$\begin{aligned} V_t(x, z, k) &= \min_{\bar{z}}\{k(\bar{z} - z) + W_t(x, \bar{z}, k)\} \\ &= \min_{\bar{z}}\{k\bar{z} + W_t(x, \bar{z}, k)\} - kz, \end{aligned} \tag{10}$$

where

$$\begin{aligned} &W_t(x, z, k) \\ &= \min_{x \leqslant y_1 \leqslant y_2}\{c_1(y_1 - x) + c_2(y_2 - y_1) + G_t(y_2) + \gamma \\ &\quad \cdot \mathsf{E}_t[V_{t+1}(y_2 - D_t, z - \mu_1(y_1 - x_1) - \mu_2(y_2 - y_1), \tilde{k}_{t+1})]\}. \end{aligned}$$

The boundary condition is the same as (3). It is clear that $V_t(x, z, k)$ can be decomposed into two functions, with one being independent of z and the other being $-kz$.

Because the problem above is a special case of problem (2), Proposition 1 continues to hold. If $k_t > \gamma \mathsf{E}[\tilde{k}_{t+1} \mid k_t]$ (resp., $k_t < \gamma \mathsf{E}[\tilde{k}_{t+1} \mid k_t]$), it is easily seen that $k_t \bar{z} + W_t(x, \bar{z}, k_t)$ is strictly increasing (resp., decreasing) in $\bar{z}$ and so the firm will sell (resp., buy) an infinite amount of the allowances. Thus, we need to assume a martingale trading price process to prevent this speculative behavior of the firm.

For this special case, we can show that one of the optimal emissions trading policies is not to trade in the first $T-1$ periods; and at the beginning of period T, as $P(0 \leqslant \tilde{k}_t \leqslant \gamma^{T-t+1}\pi) = 1$, the manufacturer either makes up for the shortfall or sells the additional allowances such that $z_{T+1} = 0$.

Corollary 2 summarizes the optimal emissions trading and production policies.

Corollary 2. *For $t = 1, 2, \ldots, T$, given state (x, z, k) at the beginning of period t,*

(a) *there exist two base-stock levels $S_{1t}(k)$ and $S_{2t}(k)$ such that the optimal production policy is given as follows:*

—*if $C_\mu \geqslant k$, then $y_{1t}^* = y_{2t}^* = \max\{S_{1t}(k), x\}$;*

—*if $C_\mu < k$, then $y_{1t}^* = x$, $y_{2t}^* = \max\{S_{2t}(k), x\}$;*

(b) *one optimal emissions trading policy is given by: when $1 \leqslant t \leqslant T-1$, $\bar{z}_t^* = z_t$; and when $t = T$, $\bar{z}_T^* = \mu_1(S_{1T}(k) - x)^+$ if $C_\mu \geqslant k$; and $\bar{z}_T^* = \mu_2(S_{2T}(k) - x)^+$ otherwise.*

Corollary 2 shows that the firm would use only one technology in each period, and its optimal technology choice depends solely on the relationship between C_μ and the realized trading price k. Once the technology is selected, the optimal production policy is a base-stock policy with the base-stock level independent of the starting inventory and allowance levels. Because the allowance prices form a martingale, it is optimal for the firm to trade only in the last period T to either make up for its shortfall in the allowances or sell the leftover allowances.

6. Numerical Study

In this section, we design numerical experiments with representative data from the cement industry to demonstrate the preceding analytical results and offer additional insights. We study four different technology combinations under settings with and without bid-ask price spread and transaction cost in the emissions trading. We examine the value of green technology by calculating cost and emissions reductions the firm can gain by having the green technology. Furthermore, we investigate the impacts of key parameters in our trading price model. Finally, a sensitivity analysis on the costs and demand distribution parameters is also conducted.

Considering that the cement industry is one of the main motivating examples for this research, we collect and use cost and emission data of four different cement production technologies in this numerical study. We list the unit production costs and emissions intensities of these technologies in Table 2. The data are derived mainly based on Tables 3–8 in Drake et al. (2010b) with the coal and natural gas prices (2.89 €/GJ and 4 €/GJ, respectively) in year 2010 (as we use the ECX EUA prices in 2010). We calculate the cost and emission parameters by using the following logic. The production cost per ton of cement consists of the cost of producing clinker (75%) and other alternatives (25%, e.g., blast furnace slag and fly ash), the cost of fuel, and the cost of electricity depleted (the data from each cost category can be found in Drake et al. 2010b). For technology d, it also includes the operating cost of Oxyfuel CCS technology (an end of pipe CO_2 capture and storage technology). The emissions intensity is calculated by adding up production emissions, emissions from fuel consumption, and electricity consumption. Note that each allowance represents one ton of CO_2 emissions. For ease of computation, we round the emission allowances needed to produce one ton of cement to integer multiples of 0.05.

Table 2. Cost and emission parameters of four existing cement production technologies.

No.	Production technology	€/ton	CO_2 emissions (kg)/ton	Allowances needed/ton
a	Wet kiln (coal)	46.75	896.85	0.90 (0.90)
b	Dry kiln (coal)	41.03	743.25	0.74 (0.75)
c	Dry kiln (natural gas)	44.44	579.25	0.58 (0.60)
d	Precalciner kiln w/ Oxyfuel CCS (coal)	53.00	63.20	0.06 (0.05)

We will test four meaningful combinations of the four technologies (one technology does not dominate the other in both emissions intensity and cost). For example, when comparing dry kiln technology with coal fuel to dry kiln technology with natural gas fuel, we have $(c_1, c_2) = (41.03, 44.44)$ and $(\mu_1, \mu_2) = (0.75, 0.60)$. In particular, technologies 1 and 2 in the subsequent study will be selected from

$$\{(a, d),\ (b, c),\ (c, d),\ (b, d)\},$$

of which the corresponding values of C_μ are equal to 7.35, 22.73, 15.56, and 17.1, respectively.

The other parameters in our model are set as follows:

$$h = 4, \quad b = p = 59, \quad \pi = 40, \quad p_D = 10,$$

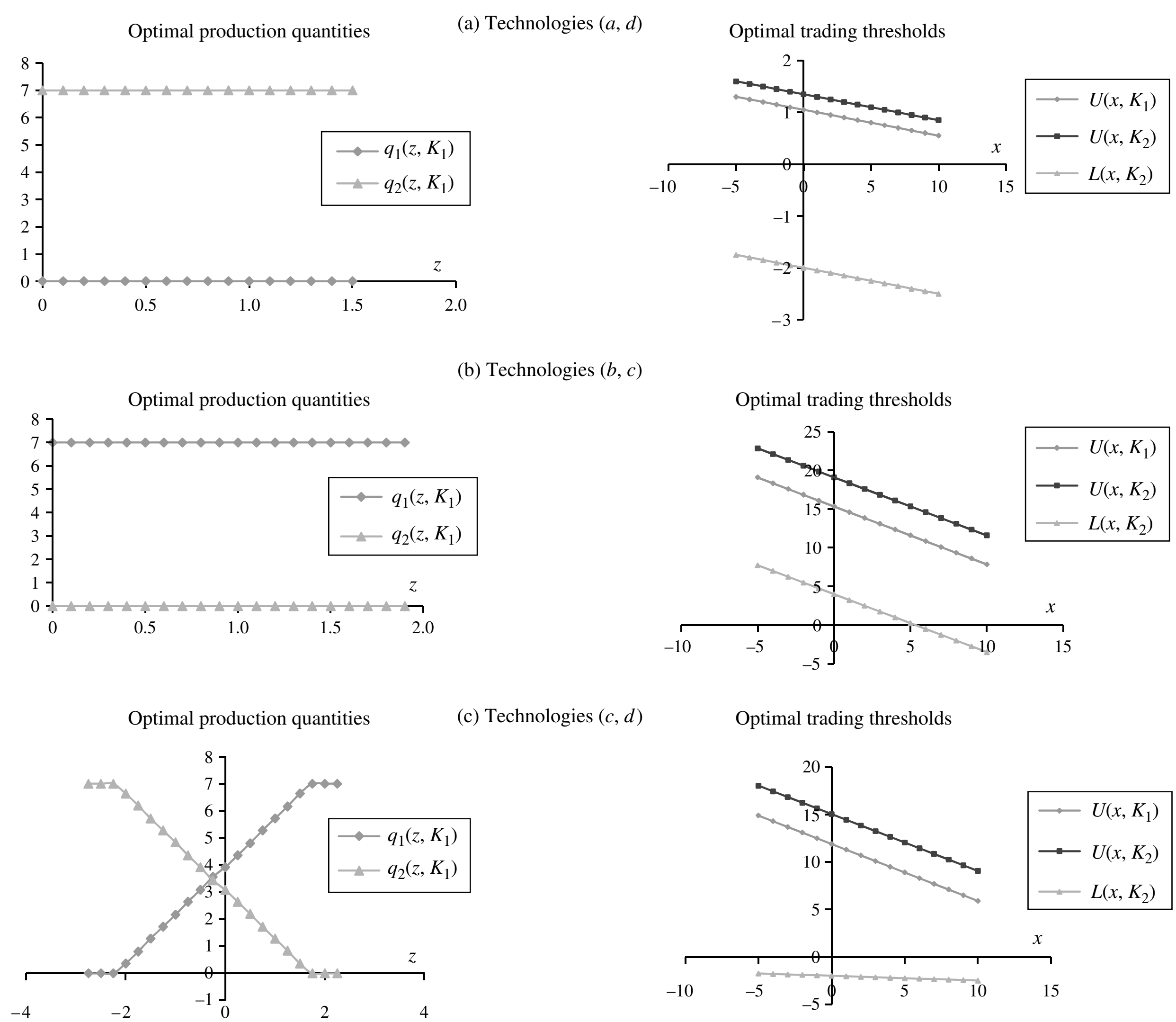
in which $\pi = 40$ follows the penalty in the phase I of the EU ETS program; the holding cost is set at about 10% of the unit production cost and the backlogging cost roughly implies a 94% service level. Product demand in each period is assumed to follow a (truncated) negative binomial distribution with parameters $(r, \bar{p}) = (5, 0.5)$ (mean $= r\bar{p}/(1-\bar{p}) = 5$, variance $= r\bar{p}/(1-\bar{p})^2 = 10$). The discount factor $\gamma = 0.97$.

6.1. Case with $\tilde{K}_t > \tilde{k}_t$

We first consider the general setting where the buying and selling prices of the allowances are different due to the existence of price spread and transaction cost. To model the trading price process, we use a simple Markov chain with two possible states: $\mathcal{K}_1 = (k_1, K_1) = (13.94, 16.64)$ and $\mathcal{K}_2 = (k_2, K_2) = (13.51, 15.87)$, which represent high and low prices, respectively. Each buying (resp., selling) price is calculated based on the average daily ask (resp., bid) price of the ECX EUA forwards in October 2010 and November 2010, respectively, in Nord Pool ASA[5] plus (resp., minus) the estimated transaction cost €1. For this two-state Markov chain, the transition probabilities are specified as $p_{11} = 0.6$, $p_{12} = 0.4$, $p_{21} = 0.7$, and $p_{22} = 0.3$. For example, if the current trading prices are (k_1, K_1), then with probability $p_{11} = 0.6$, the next period's trading prices are still (k_1, K_1). For this setting, the planning horizon has five periods (i.e., $T = 5$). We will test examples with longer planning horizons in the next subsection when we consider the case without price spread.

We illustrate the optimal solutions (production quantities and trading target thresholds) of the technology combinations in Figure 3. Note that in this setting, (b, c) and (b, d) lead to the same solutions because only technology b will be chosen ($P(C_\mu > \tilde{K}_t) = 1$), and so we present only the solutions for (b, c). And as $K_i > \min\{\gamma^t \mathsf{E}[K_t], t = 1, \ldots, 5\}$ for $i = 1, 2$,[6] the firm will not buy any allowance in period 1, so $L_1(x, \mathcal{K}_1) = -\infty$, which we do not plot in the figure. The figure depicts the optimal production quantities and emissions trading thresholds. On the left panel, the starting inventory level $x_1 = 3$. We observe that these figures echo the analytical results in the previous sections. For example, $L_1(x, \mathbf{K}_1)$ and $U_1(x, \mathbf{K}_1)$ are both decreasing in x. For the technology combination (c, d), the firm will simultaneously use both technologies to produce when the allowance level falls into an interval lying between $U_t(x, \mathbf{K}_t)$ and $L_t(x, \mathbf{K}_t)$.

Figure 3. Optimal solutions.



The existence of the green technology increases the flexibility in consuming and trading the emission allowances and thus reduces the cost for the firm. Under the emissions trading scheme, we intend to quantify the cost reduction benefit of green technology (VG) so that the manufacturer can estimate its return on investment in green technologies. To this end, we compare the optimal system cost with and without the green technology and define VG as follows:

$$VG = \text{Average}\left\{ \frac{V_1^r(x,z,\mathbf{K}_1) - V_1(x,z,\mathbf{K}_1)}{|V_1(x,z,\mathbf{K}_1)|} \times 100\% \;\middle|\; x \in [-20,30], z \in [-20,20], \mathbf{K}_1 \in \{\mathcal{K}_1, \mathcal{K}_2\} \right\}, \quad (11)$$

where $V_1^r(x,z,\mathbf{K}_1)$ is the optimal cost when the firm has only the regular technology, so $V_1^r(x,z,\mathbf{K}_1) \geqslant V_1(x,z,\mathbf{K}_1)$. Hence, the higher the VG is, the more valuable the green technology.

The VGs of the four technology combinations under two different values of transaction cost are presented in Table 3, in which we also report the minimum and maximum values of the ratio defined in the brackets of (11) over the ranges of x, z, and $\mathbf{K}_1$.

It can be observed that technology d provides the highest value of green technology when paired with technology a because the combination has the smallest C_μ. Meanwhile, the green technology c in (b,c) and the green technology d in (b,d) do not provide the firm any cost benefit when transaction cost is 1 since their C_μ is so high that the firm tends not to use the green technology. When the transaction cost increases, the VGs of (a,d) and (c,d) decrease because the benefit from trading the allowances decreases, whereas the VG of (b,d) increases because the resulting higher purchasing cost of the allowance makes the green technology d more attractive ($C_\mu = 17.1 < 17.64 = K_1$), i.e., the firm will use technology d to produce in some cases.

Currently, although natural gas is cleaner, it is less attractive than coal as the fuel in production because of its high cost (e.g., in the cement company studied by Drake et al. 2010b, natural gas accounted only for less than 5% of the fossil fuels used). From the results of technology combination (b,c) in Table 3, when producing cement by dry kiln, the firm will use only coal but not natural gas because it brings insufficient economic benefit to the firm, even though it generates fewer emissions. To see how such a situation would change when the price of natural gas drops, we test the instance again but reduce the natural gas price by 10% and 20% (from 4 €/GJ to 3.6 €/GJ and 3.2 €/GJ). The resulting cost per ton of cement using technology c becomes 43.21 and 41.98, respectively. Consequently, the value of green technology under the combination (b,c) increases. In particular, VG becomes 0.28% and 2.08%, respectively. Therefore, if the natural gas price drops by more than 10%, the firm can start to benefit from using both types of fuels in its dry kiln, and the benefit will become even more significant if the price drops by more than 20%. Another interesting question is when the firm will switch to natural gas if the trading price of the allowance increases. This question can actually be answered by our discussion in §5 on optimal technology selection, i.e., as long as $C_\mu < K_t$, the firm may start to use the green technology depending on the allowance level. Under this particular numerical example, if K_1 or K_2 is higher than $C_\mu = 22.73$, the firm may start to use natural gas. By letting the allowance trading price increase by 50% and 70%, the resulting VG is 0.38% and 0.82%, respectively.

We also compare the optimal cost under our model with that when the manufacture selects a single technology at the beginning and uses it throughout the planning horizon. To this end, define the percentage cost reduction VD as follows:

$$VD = \text{Average}\left\{ \frac{\tilde{V}(x,z,\mathbf{K}_1) - V_1(x,z,\mathbf{K}_1)}{|V_1(x,z,\mathbf{K}_1)|} \times 100\% \;\middle|\; x \in [-20,30], z \in [-20,20], \mathbf{K}_1 \in \{\mathcal{K}_1, \mathcal{K}_2\} \right\}, \quad (12)$$

where $\tilde{V}(x,z,\mathbf{K}_1) := \min\{V_1^r(x,z,\mathbf{K}_1), V_1^g(x,z,\mathbf{K}_1)\}$ and $V_1^g(x,z,\mathbf{K}_1)$ is the optimal cost when the firm has only the green technology. That is, VD captures the value of dynamic technology choice. We find that under our current parameter settings, the value VD is close to zero. For example, when the transaction cost equals to 1, under technologies (c,d), $VD = 0.06\%$ with minimum 0.00% and maximum 0.25%. This is intuitive since all the cost parameters and demand are stationary and there are only two possible trading prices. Hence, the dynamic choice of technologies provides only minimum value to the firm and so the optimal static technology choice can serve as an effective heuristic policy. Another interesting finding is that under different initial states (x,z), the optimal static technology choice may be different. For example, when the initial inventory level x is very low while the allowance level z is high, the firm tends to choose the regular technology; whereas it tends to choose the green

Table 3. Cost benefit of green technology ($VG\%$): $\tilde{K}_t > \tilde{k}_t$.

	(a,d)			(b,c)			(c,d)			(b,d)		
Tech.	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
Trans. cost = 1	14.26	1.73	54.62	0.00	0.00	0.00	3.91	0.07	20.65	0.00	0.00	0.00
Trans. cost = 2	11.61	0.92	57.72	0.00	0.00	0.00	0.67	0.09	1.29	0.03	0.01	0.15

technology when x is high but z is low. This can be seen by comparing VD with VG and the average percentage cost improvement when the firm has only the green technology $((V_1^g(x, z, \mathbf{K}_1) - V_1(x, z, \mathbf{K}_1))/|V_1(x, z, \mathbf{K}_1)| \times 100\%)$. For technologies (c, d), these three values are 0.06%, 2.86%, and 0.19%, respectively. In the next subsection, we will investigate more on the value of dynamic technology choice under a more volatile trading price process.

6.2. Case with $\tilde{K}_t = \tilde{k}_t$

In this subsection, we ignore bid-ask price spread and transaction cost in allowance trading. From §5.1, we know that the trading and production decisions can be decoupled in this case, which reduces the computational effort. So we test examples with a longer planning horizon and a more complicated allowance price process.

We model the emission allowance prices as a discrete-time symmetric random walk, which is known to be a martingale. As we solve the problem backward, we specify the possible values of the trading prices in period T as

$$k_T \in \{k_b + (\lfloor (T-1)/2 \rfloor + 1)\Delta, k_b + (\lfloor (T-1)/2 \rfloor) \cdot \Delta, \ldots, k_b, k_b - \Delta, \ldots, k_b - \lfloor (T-1)/2 \rfloor \Delta\}$$

where $\lfloor x \rfloor$ gives the largest integer $\leqslant x$, a real number. That is, $k_{T,i} = k_b + (\lfloor (T-1)/2 \rfloor + 1)\Delta - (i-1)\Delta$, for $i = 1, \ldots, T$, where Δ is the step size of the random walk. For $1 \leqslant t < T$, $k_t \in \{\gamma(0.5k_{t+1,i} + 0.5k_{t+1,i+1}), i = 1, \ldots t\}$. The rationale of using this process is to have a reasonable span of prices on both sides of the price k_b. We choose $k_b = 14.92/\gamma^T$, in which 14.92 is obtained by averaging the trading prices specified in §6.1 and ignoring the transaction cost. Under this price process, it can be shown that the (unconditional) expected trading price in each period increases with k_b or Δ, and the conditional price variance $\mathrm{Var}[\tilde{k}_{t+1} \mid k_t] = \Delta^2$ increases with Δ.

Both the cost and emissions reduction benefit of green technology will be examined. We calculate the total expected emissions under each technology combination. The relative emissions reduction (= (total expected emissions with regular technology only − total expected emissions with both technologies)/total expected emissions with regular technology only × 100%) is used to quantify the emissions reduction and is denoted as EG. The impact of the length of the planning horizon T, the step size Δ of the trading prices, and the price k_b are studied. The parameters are chosen from the following sets:

$$\Delta \in \{0.8, 1.6, 3.2\}, \quad T \in \{3, 6, 12\},$$
$$k_b \in \{0.8, 1.2, 1.4\} \times 14.92/\gamma^T.$$

Default values are $\Delta = 1$, $T = 12$, and $k_b = 14.92/\gamma^T$. For each instance we change one parameter and fix the other two parameters at their default values. Because of larger problem instances, we also expand the range of the initial allowance level from $[-20, 20]$ to $[-30, 30]$. All the other parameters remain the same as those in the previous subsection.

The results are summarized in Table 4. From the table, it is clear that both the cost benefit VG and the emissions reduction EG under different technology combinations are ordered as $(a, d) > (c, d) > (b, d) > (b, c)$, which follows the same ordering of their corresponding C_μ values (from the smallest to the largest). We can further see that VG increases with k_b and the step size Δ, showing that a higher trading price or a more variable trading price process makes the green technology more valuable. Besides, when the planning horizon becomes longer, since the firm is more likely to adopt the green technology under this trading price process, in general both VG and EG increase. In addition to the results reported in Table 4, we also observe in our numerical results that when Δ or k_b increases, the optimal expected total cost becomes higher (resp., lower) when the starting allowance level is low (resp., high), and the optimal base-stock levels and the total expected emissions are lower. These observations can be explained intuitively as follows. From the trading price process we adopt, a larger k_b or Δ implies a higher (unconditional) expected trading price each period. Thus, when k_b or Δ increases and the allowance level is high (resp., low), the optimal cost is lower (resp., higher) due to the allowance trading. Meanwhile, a higher trading price would in effect result in a higher overall production cost (as it includes the cost of consuming allowances that can be sold for revenue or purchased at cost). Therefore, the firm tends to produce less

Table 4. Cost benefit ($VG\%$) and emissions reduction ($EG\%$) of green technology: $\tilde{K}_t = \tilde{k}_t$.

		Planning horizon T			Step size Δ			k_b		
	Tech.	3	6	12	0.8	1.6	3.2	−20%	+20%	+40%
VG	(a, d)	16.08	13.89	16.64	13.51	16.99	19.59	11.17	21.50	27.38
	(b, c)	0	0	0.02	0.01	0.06	0.28	0.00	0.24	0.78
	(c, d)	1.47	2.01	3.20	2.89	3.86	5.62	0.71	6.36	9.51
	(b, d)	0.22	1.04	2.49	2.21	3.21	5.47	0.32	6.36	10.36
EG	(a, d)	93.30	94.22	94.35	94.36	98.36	94.35	94.38	94.33	94.29
	(b, c)	0	0	1.73	1.25	3.81	7.28	0.01	10.13	17.39
	(c, d)	91.67	91.67	91.67	91.67	91.64	90.42	45.35	91.61	91.56
	(b, d)	39.48	59.55	75.52	73.29	75.13	89.05	21.90	93.27	93.25

Table 5. Cost reduction (%): Optimal static vs. dynamic technology choice.

		Planning horizon T			Step size Δ			k_b		
	Tech.	3	6	12	0.8	1.6	3.2	−20%	+20%	+40%
VD	(a, d)	9.70	2.86	0.91	0.91	0.92	0.93	0.85	0.96	1.00
	(b, c)	0	0	0.02	0.01	0.06	0.28	0.00	0.15	0.00
	(c, d)	1.47	1.84	0.91	0.91	0.92	0.94	0.71	0.96	1.00
	(b, d)	0.22	1.04	1.05	1.09	0.94	1.00	0.32	0.96	1.00

and keep a lower base-stock level, which further leads to fewer total emissions.

In Table 5, we present *VD*, the value of the optimal dynamic technology choice over the optimal static one, under the same set of numerical instances. We observe that such value ranges from 0 to 9.70%, with most of the instances being around 1%. So as expected, a more variable trading price process (compared with the two-price setting in the previous section) results in a higher cost benefit from dynamically selecting technologies. Hence, we anticipate that the dynamic technology choice would provide the firm with more cost benefit when there is higher variability between different periods, which can come from either trading price processes or nonstationarity of the cost parameters and demand processes. We also report the percentage cost improvement for the case with both technologies over the one with only green technology in Table 6.

Finally, we conduct a sensitivity analysis for the cost benefit and emissions reduction of the green technology on some other system parameters, viz. backlogging cost b, inventory holding cost h, discount factor γ, and demand $(r, \bar{p})$. In particular, we consider $b \in \{10, 20, 40\}$, $h \in \{2.5, 7.5, 10\}$, $\gamma \in \{0.9, 0.95, 1\}$, and the following demand distribution parameters: $(r, \bar{p}) = (10, 0.5)$, which has mean 10 and variance 20; $(r, \bar{p}) = (15, 0.25)$, which has mean 5 and variance 20/3; and $(r, \bar{p}) = (5/3, 0.75)$, which has mean 5 and variance 20. The results are summarized in Table 7. It can be seen that there is no clear pattern on the impact of the backlogging cost b. Nevertheless, the cost benefit of green technology increases when the holding cost rate h or the discount factor γ decreases. Regarding demand, when mean demand increases while coefficient of variation decreases, i.e., $(r, \bar{p}) = (10, 0.5)$, the cost reduction (e.g., technologies (a, d), 16.64% vs. 16.99%) and emissions reduction are larger (e.g., technologies (a, d), 94.35% vs. 94.40%); when mean demand is fixed while variance increases, both cost and emissions reduction benefits decrease. Moreover, we also find that the optimal base-stock levels, the optimal cost, and the expected emissions increase with the mean or variance of the demand.

7. Conclusion and Discussion

In this paper we study a dynamic production planning problem with emissions trading, where the manufacturer can use a more costly but cleaner green technology or a less costly but more polluting regular technology, or both, to carry out production. The emission allowances are tradeable in each period with stochastic trading prices. We characterize the optimal production and allowance trading policies that minimize the expected total discounted cost over a finite planning horizon. In particular, the optimal allowance trading policy is of a target interval type with two thresholds decreasing in the starting inventory level. For production, we find that in certain cases the optimal technology selection is determined by the relationship between the additional cost per allowance saved and the trading prices, whereas in other cases it also relies on the allowance level. We show that a base-stock type production policy is optimal. Furthermore, we find that the optimal base-stock level is independent of both the inventory and allowance levels only when the firm trades the allowances or uses both production technologies. In other cases, the base-stock level depends on these states and some monotonicity results are provided. A numerical study using representative data from the cement industry is further conducted to illustrate the analytical results and to examine the value of green technology for the manufacturer with respect to both cost and emissions.

Our analysis and results show that the existence of emissions trading and dual production technologies makes the optimal production policies more complicated than a traditional production system (without emissions trading and with a single production technology, the optimal production

Table 6. Cost reduction (%): Green technology only vs. dynamic technology choice.

	Planning horizon T			Step size Δ			k_b		
Tech.	3	6	12	0.8	1.6	3.2	−20%	+20%	+40%
(a, d)	11.94	2.86	0.91	0.91	0.92	0.94	0.85	0.96	1.00
(b, c)	3.52	1.57	1.30	1.64	1.44	0.92	2.51	0.23	0.00
(c, d)	11.94	2.86	0.91	0.91	0.92	0.95	1.53	0.96	1.00
(b, d)	9.54	3.03	1.05	1.09	0.94	1.00	2.90	0.96	1.00

Table 7. Sensitivity analysis.

		Backlogging cost b			Holding cost h			Discount factor γ			Demand $(r, \bar{p})$		
	Tech.	10	20	40	2.5	7.5	10	0.9	0.95	1	(10, 0.5)	(15, 0.25)	(5/3, 0.75)
VG	(a, d)	8.19	16.73	16.10	16.56	14.95	14.38	28.20	18.96	12.28	16.99	16.57	14.79
	(b, c)	0.06	0.44	0.05	0.02	0.02	0.02	1.48	0.24	0	0.02	0.02	0.01
	(c, d)	2.68	3.45	3.28	3.31	3.08	2.96	11.42	5.21	0.88	3.64	3.36	2.85
	(b, d)	2.37	2.62	2.52	2.58	2.43	2.34	12.74	5.01	0.21	2.62	2.65	2.16
EG	(a, d)	87.75	94.32	94.37	94.38	94.34	94.36	94.17	94.34	94.42	94.40	94.37	94.31
	(b, c)	8.53	1.52	1.76	1.69	1.63	1.87	15.18	8.39	0	1.70	1.82	1.50
	(c, d)	91.36	91.67	91.67	91.67	91.62	91.60	91.43	91.56	71.39	91.67	97.58	91.59
	(b, d)	93.32	78.02	76.31	75.51	76.76	77.38	82.46	81.93	20.65	76.43	76.62	73.14

policy under the current cost structure would be a simple state-independent base-stock policy). Because of the high dimensionality of the state space, the computation of optimal solutions would be intensive. Our numerical study suggests that when variability of the trading price process is low and system parameters are stationary, a static technology choice can serve as an effective heuristic. Another possible heuristic is to approximate V_{t+1} by some simpler function (e.g., to use only the expected trading prices as parameters to calculate an approximate V_{t+1} instead of considering all possible sample paths) and to solve the corresponding solutions of period t.

Several possible and important extensions are discussed in the following.

Lost Sales

When unsatisfied demand is lost instead of backlogged, we need to modify the problem (2) by replacing $y_{2t}-D_t$ in V_{t+1} with $(y_{2t}-D_t)^+$. Under the additional assumption $G_t(y) = h_t\mathsf{E}[(y-D_t)^+] + b_t\mathsf{E}[(D_t-y)^+]$ with $h_t+b_t-\gamma c_1 \geqslant 0$, most of the results in the backlog case derived previously will continue to hold. In particular, under this assumption, the convexity and modularities in Proposition 2(a) and (b) continue to hold, whereas Proposition 2(c) may not be true any more. Therefore, the structure of the optimal policies and most of the subsequent results are still valid in the lost sales model, except for those built on Proposition 2(c), i.e., the second parts in Proposition 4(c) and (d) and the second part in Proposition 5(c).

Capacitated Production Technology

Manufacturers often have limited production capacities. For the model in §3, suppose that technology i has a finite production capacity C_i, $i = 1, 2$. Then, the constraints on production in Equation (2) need to be modified to $x_t \leqslant y_{1t} \leqslant \min\{y_{2t}, x_t + C_1\}$ and $y_{2t} \leqslant y_{1t} + C_2$. With this change, the convexity of the value functions still holds, and therefore the optimal emissions trading policy is still of the target-interval type with two thresholds. However, the modularity and other structural results may no longer be true and require further analysis. Hence, it will become more challenging to characterize the optimal technology selection and production policies. However, we expect that with finite production capacity, the manufacturer is more likely to use both technologies simultaneously. One may start the analysis from the case where the capacity constraint is only on the green technology, i.e., $x_t \leqslant y_{1t} \leqslant y_{2t} \leqslant y_{1t} + C_2$.

Switching or Idling Costs of Technologies

In our study, we assume that there is no cost for idling each technology or switching cost between the two technologies. This assumption fits well in situations when such costs do not exist or are relatively minor. One example is that in the pulp and paper industry, when using combined or multi-fuel boilers, the switching cost between different types of fuels such as coal, natural gas, or other alternative fuels should be rather low. In cement production, after installing the carbon capture and storage technology (e.g., the Precalciner Kiln with Oxyfuel CCS), an end-of-pipe method to reduce carbon emissions, we expect that the cost of switching on/off such an end-of-pipe capture and storage system would also be minor.

In other situations, however, the switching or idling cost might be significant because of machine setup or maintenance, and therefore it should not be ignored in production planning. We provide some brief discussions on how to model/incorporate these costs. We can include additional cost terms in Equation (2) to capture the idling cost, i.e., $cI_1\mathbf{1}(y_1 = x) + cI_2\mathbf{1}(y_2 = y_1)$, where cI_i denotes the idling cost of technology $i = 1, 2$ and $\mathbf{1}(A) = 1$ if event A is true and $\mathbf{1}(A) = 0$ otherwise. It is clear that this cost is a fixed cost. To model the switching costs, we need to expand the current state space of the problem to include more variables that indicate whether a technology is used in the previous period. The switching cost is incurred when such state variables change. For example, suppose technology 1 will incur switching cost if it is used in this period but not in the previous period. Then we can use $i = 0$ $(i = 1)$ to denote it is on (off) in the previous period and c_s to denote the switching cost. Then in Equation (2), the state becomes $(x, z, i, \mathbf{K})$ and the switching cost $c_s\mathbf{1}(y_1 > x)i$ needs to be added. Again, the cost is a sort of fixed cost independent of the production quantity. Our analysis and results no longer hold for the models with these new features, which will most likely require very challenging analysis and result in a very complicated optimal policy that is difficult to implement. Therefore, it might be more meaningful to focus on developing efficient heuristic policies. An immediate but commonly used idea is to consider the myopic optimal solutions of the resulting dynamic program and numerically test the performance. We leave this and the study of other possible more sophisticated heuristics to be our future research.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2013.1189.

Endnotes

1. Many greenhouse gas accounting protocols include indirect emissions related to electricity and steam consumption because they are a significant component of a company's total GHG emissions.
2. Page 12, http://ec.europa.eu/environment/news/efe/docs/efe_35_en.pdf.
3. For convenience, we express the trading prices in period t in period-t currency. The conversion of period-$(T+1)$ currency to the period-t currency can be easily done once the discount factor is specified. See Carmona et al. (2010) for a detailed discussion of this.
4. http://www.ecx.eu/market-data.
5. http://www.nasdaqomxcommodities.com/trading/marketprices/.
6. Note that this is different from Condition (1).

Acknowledgments

The authors thank the associate editor and three anonymous referees for their constructive comments and suggestions that have greatly improved the paper. This work was supported in part by the National Science Foundation [Grants CMMI-0927631 and CMMI-1131249] for the first author and by Hong Kong Research Grant Council General Research Fund [Grant CUHK-419411] for the second author.

References

Angelus A, Porteus EL (2002) Simultaneous capacity and production management of short-life-cycle, produce-to-stock goods under stochastic demand. *Management Sci.* 48:399–413.

Carmona R, Fehr M, Hinz J (2009) Optimal stochastic control and carbon price formation. *SIAM J. Control Optim.* 48:2168–2190.

Carmona R, Fehr M, Hinz J, Porchet A (2010) Market design for emissions trading schemes. *SIAM Rev.* 52:403–452.

Chao X, Chen J, Wang S (2008) Dynamic inventory management with cash flow constraints. *Naval Res. Logist.* 55:758–768.

Charles A, Darne O, Fouilloux J (2010) Testing the martingale difference hypothesis in the EU ETS markets for the CO_2 emission allowances: Evidence from phase I and phase II. Working paper, Université de Nantes, Nantes, France.

Choate WT (2003) Energy and emission reduction opportunities for the cement industry. U.S. Department of Energy, Washington, DC.

Debo LG, Toktay LB, Van Wassenhove L (2005) Market segmentation and product technology selection for remanufacturable products. *Management Sci.* 51:1193–1205.

Dobos I (2005) The effects of emissions trading on production and inventories in the Arrow-Karlin model. *Internat. J. Production Econom.* 93–94:301–308.

Dobos I (2007) Tradeable permits and production-inventory strategies of the firm. *Internat. J. Production Econom.* 108:329–333.

Drake D, Kleindorfer P, van Wassenhove L (2010a) Technology choice and capacity investment under emissions regulation. Working paper, INSEAD, Fontainebleau, France.

Drake D, Kleindorfer P, van Wassenhove L (2010b) Heidelberg Cement: Technology choice under carbon regulation. Case 610-014-1, European Case Clearing House, Cranfield University, Cranfield, UK.

Federgruen A, Yang N (2008) Selecting a portfolio of suppliers under demand and supply risks. *Oper. Res.* 56:916–936.

Fukuda Y (1964) Optimal policies for the inventory problem with negotiable leadtime. *Management Sci.* 10(4):690–708.

Gary WB, Shadbergian RJ (1998) Environmental regulation, investment timing, and technology choice. *J. Indust. Econom.* XLVI:235–256.

Hendriks CA, Worrell E, Price L, Martin N, de Jager D, Blok K, Riemer P (1989) Emission reduction of greenhouse gases from the cement industry. *Proc. 4th Internat. Conf. Greenhouse Gas Control Tech. Interlaken, Switzerland.*

Heyman D, Sobel M (1984) *Stochastic Models in Operations Research*, Vol. II (McGraw-Hill, New York).

Laffont J, Tirole J (1996) Pollution permits and compliance strategies. *J. Public Econom.* 32:85–125.

Martin N, Anglani N, Einstein D, Khrushch M, Worrell E, Price LK (2000) Opportunities to improve energy efficiency and reduce greenhouse gas emissions in the U.S. pulp and paper industry. LBNL-46141, Environmental Energy Technologies Division, Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, CA.

Martínez-de-Albéniz V, Simchi-Levi D (2005) A portfolio approach for procurement contracts. *Production Oper. Management* 14:90–114.

National Council for Air and Stream Improvement, Inc. (NCASI) (2005) Calculation tools for estimating greenhouse gas emissions from pulp and paper mills. Version 1.1, July 8. National Council for Air and Stream Improvement, Inc., Research Triangle Park, NC.

Stavins R (1995) Transaction costs and tradeable permits. *J. Environ. Econom. Management* 29:133–148.

Stuart JA, Ammons JC, Turbini LJ (1999) A product and process selection model with multidisciplinary environmental considerations. *Oper. Res.* 47:221–234.

Subramanian R, Gupta S, Talbot B (2007) Compliance strategies under permits for emissions. *Production Oper. Management* 16:763–779.

Tietenberg TH (2006) *Emissions Trading: Principles and Practice* (RFF Press, Washington, DC).

Topkis D (1998) *Supermodularity and Complementarity* (Princeton University Press, Princeton, NJ).

Woerdman E (2001) Emissions trading and transaction costs: Analyzing the flaws in the discussion. *Ecological Econom.* 38:293–304.

Yang J (2004) Production control in the face of random supply, storable raw material, and an outside market. *Oper. Res.* 52:293–311.

Ye Q, Duenyas I (2007) Optimal capacity investment decisions with two-sided fixed-capacity adjustment costs. *Oper. Res.* 55:272–283.

Zhao J, Hobbs B, Pang J-S (2010) Long-run equilibrium modeling of emissions allowance allocation systems in electric power markets. *Oper. Res.* 58:529–548.

Xiting Gong is an assistant professor in the Department of Systems Engineering and Engineering Management at the Chinese University of Hong Kong. His research interests include operations and supply chain management, logistics and inventory control, and game theoretic applications.

Sean X. Zhou is an associate professor in the Department of Decision Sciences and Managerial Economics, CUHK Business School, at the Chinese University of Hong Kong. His primary research interests are inventory control, production planning, pricing, and game theoretic applications in supply chain management.