

GATE 2023

EE22BTECH11060 - TEJAVATH KUSHAL*

Q 20 : The solution $x(t)$, $t \geq 0$, to the differential equation $\ddot{x} = -k\dot{x}$, $k > 0$ with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ is: (GATE 2023, IN)

Ans:

Differential equation	$\ddot{x} = -k\dot{x}$
Initial conditions	$x(0) = 1$ and $\dot{x}(0) = 0$
$x(t)$?

TABLE 0
PARAMETER TABLE

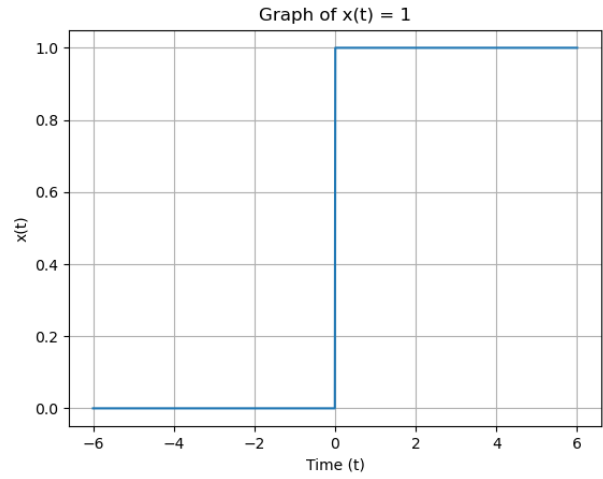


Fig. 0. Plot of $x(t)$ v/s t

$$\Rightarrow \frac{d^2 x(t)}{dt^2} = -k \frac{dx(t)}{dt} \quad (1)$$

Taking Laplace transform on both sides,

$$\frac{d^2 x(t)}{dt^2} \xrightarrow{\mathcal{L}} s^2 X(s) - sx(0) - \dot{x}(0) \quad (2)$$

$$\frac{dx(t)}{dt} \xrightarrow{\mathcal{L}} sX(s) - x(0) \quad (3)$$

From Table (2) , (3)

$$s^2 X(s) - sx(0) - \dot{x}(0) = -k(sX(s) - x(0)) \quad (4)$$

$$s^2 X(s) - s = -k(sX(s) - 1) \quad (5)$$

$$sX(s)(s + k) = (s + k) \quad (6)$$

$$X(s) = \frac{1}{s}, \quad s \neq -k \quad (7)$$

$$x(t) = u(t) \quad (8)$$

$$\Rightarrow x(t) = 1 \quad (t \geq 0) \quad (9)$$