

# GATE 2023

EE22BTECH11060 - TEJAVATH KUSHAL\*

**Q 20 :** The solution  $x(t)$ ,  $t \geq 0$ , to the differential equation  $\ddot{x} = -k\dot{x}$ ,  $k > 0$  with initial conditions  $x(0) = 1$  and  $\dot{x}(0) = 0$  is: (GATE 2023, IN)

**Ans:**

Differential equation	$\ddot{x} = -k\dot{x}$
Initial conditions	$x(0) = 1$ and $\dot{x}(0) = 0$
$x(t)$	?
Laplace Transform: $\mathcal{L}\{u(t)\} = \frac{1}{s}$	

TABLE 0  
PARAMETER TABLE

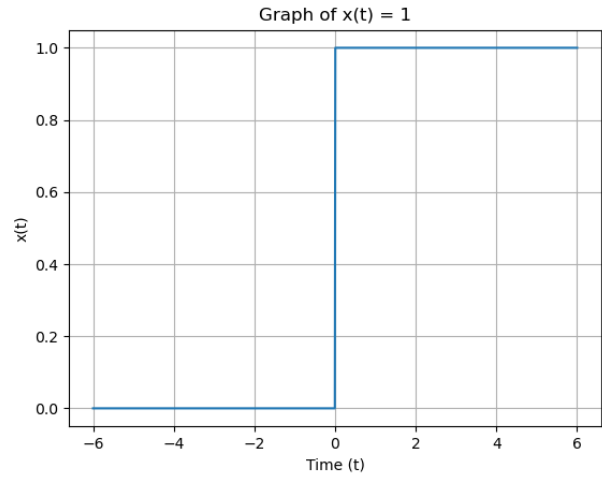


Fig. 0. Stem Plot of  $x(t)$  v/s  $t$

$$\Rightarrow \frac{d^2 x(t)}{dt^2} = -k \frac{dx(t)}{dt} \quad (1)$$

Taking Laplace transform on both sides,

$$\frac{d^2 x(t)}{dt^2} \mathcal{L} \Rightarrow s^2 X(s) - sx(0) - \dot{x}(0) = -k(sX(s) - x(0)) \quad (2)$$

$$\frac{dx(t)}{dt} \mathcal{L} \Rightarrow sX(s) - x(0) = -kX(s) + \frac{1}{s} \quad (3)$$

From Table ,(2) , (3)

$$s^2 X(s) - sx(0) - \dot{x}(0) = -k(sX(s) - x(0)) \quad (4)$$

$$s^2 X(s) - s = -k(sX(s) - 1) \quad (5)$$

$$sX(s)(s + k) = (s + k) \quad (6)$$

$$s \neq -k \quad (7)$$

$$X(s) = \frac{1}{s} \quad (8)$$

$$x(t) = u(t) \quad (9)$$

$$\Rightarrow x(t) = 1 \quad (t \geq 0) \quad (10)$$