GATE 2021

EE22BTECH11060 - TEJAVATH KUSHAL*

Q.40 : For a unit step input $u[n]$, a discrete-time LTI system produces an output signal $(2\delta[n+1]+\delta[n]+\delta[n-1])$. Let $y[n]$ be the output of the system for an input $\left(\left(\frac{1}{2}^n\right)u[n]\right)$. The value of $y[0]$ is:

(GATE 2021 EC)

Ans.

Input	Output
$u[n]$	$s(n)=2\delta[n+1]+\delta[n]+\delta[n-1]$
$\left(\frac{1}{2}\right)^n u[n]$	$y[n]$

TABLE 0
INPUT-OUTPUT PARAMETER TABLE

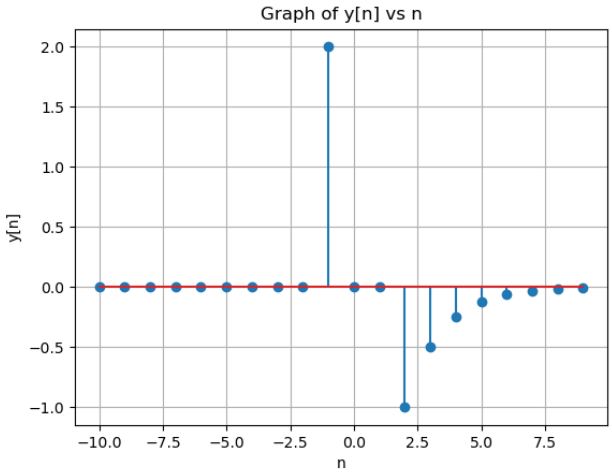

Fig. 0. Plot of $y(n)$ v/s n

For impulse response

$$h(n) = s(n) - s(n-1) \tag{1}$$

$$h(n) = 2\delta(n+1) + \delta(n) + \delta(n-1) - 2\delta(n) - \delta(n-1) - \delta(n-1-1) \tag{2}$$

$$\implies h(n) = 2\delta(n+1) - \delta(n) - \delta(n-2) \tag{3}$$

For input $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$$y(n) = x(n) * h(n) \tag{4}$$

$$= x(n) * [2\delta(n+1) - \delta(n) - \delta(n-2)] \tag{5}$$

$$= 2x(n+1) - x(n) - x(n-2) \tag{6}$$

$$= 2\left(\frac{1}{2}\right)^{n+1} u(n+1) - \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^{n-2} u(n-2) \tag{7}$$

$$y(0) = 2\left(\frac{1}{2}\right)^{0+1} u(0+1) - \left(\frac{1}{2}\right)^0 u(0) - \left(\frac{1}{2}\right)^{0-2} u(0-2) \tag{8}$$

$$\implies y(0) = 0 \tag{9}$$