Akram, Tanweer

Working Paper

Multifactor Keynesian models of the long-term interest rate

Working Paper, No. 991

Provided in Cooperation with:
Levy Economics Institute of Bard College

Working Paper No. 991

Multifactor Keynesian Models of the Long-Term Interest Rate

by

Tanweer Akram*
Wells Fargo

July 2021

*Affiliation is provided for only identification purpose. Disclaimer: Views expressed are solely those of the author. Acknowledgement: The author thanks Ms. Elizabeth Dunn for her copyediting assistance.

Levy Economics Institute
P.O. Box 5000
Annandale-on-Hudson, NY 12504-5000
http://www.levyinstitute.org

ISSN 1547-366X

ABSTRACT

This paper presents multifactor Keynesian models of the long-term interest rate. In recent years there have been a proliferation of empirical studies based on the Keynesian approach to interest rate modeling. However, standard multifactor models of the long-term interest rate in quantitative finance have not been yet incorporated Keynes's insights about interest rate dynamics. Keynes's insights about the influence of the current short-term interest rate are introduced in two different multifactor models of the long-term interest rate to illustrate how the long-term interest rate relates to the short-term interest rate, the central bank's policy rate, inflation expectations, the central bank's inflation target, volatility in financial markets, and Wiener processes.

KEYWORDS: Long-Term Interest Rate; Government Bond Yields; Monetary Policy; Short-Term Interest Rate; Inflation; Inflation Target; John Maynard Keynes

JEL CLASSIFICATION: E12; E43; E50; E58; E60; G10; G12; G41

1. INTRODUCTION

This paper presents multifactor Keynesian models of the long-term interest rate. John Maynard Keynes highlighted the crucial role the current short-term interest rate plays in determining the long-term interest rate. The Keynesian approach to interest rate modeling has been revitalized in several empirical studies in recent years. Nevertheless, benchmark models of the long-term interest rate in quantitative finance have yet to introduce Keynes's useful insight about interest rate dynamics and financial markets. Attempts at integrating Keynes's insights about interest rate dynamics in benchmark models in quantitative finance have focused on one-factor models rather than multifactor models. This paper fills this lacuna by developing two multifactor models of the long-term interest rate based on Keynes's insight that the short-term interest rate is the key driver of the long-term interest rate, while recognizing the role of other macroeconomic factors, such as the central bank's policy rate, inflation expectations, and the central bank's inflation target. The models presented here modify some benchmark models in quantitative finance to convey Keynes's insight regarding interest rate dynamics.

The paper is structured as follows. Section 2 briefly describes Keynes's views on interest rates and the findings of the recent Keynesian empirical literature on interest rate dynamics. Section 3 presents two multifactor models of the long-term interest rate based on the short-term interest rate, the central bank's policy rate, inflation expectations, the central bank's inflation target, and Wiener processes. Section 4 concludes.

2. KEYNES'S VIEWS ON INTEREST RATES

Keynes (1930) firmly maintained that the central bank's policy rate is the key determinant of the long-term interest rate on government bonds. He rejected the loanable funds view of the interest rate as articulated by classical economists. Keynes drew on Riefler's (1930) statistical findings showing the connection between the long-term interest rate and the short-term interest rate in the United States in the 1920s and his own observations during the same period in the United Kingdom.

Keynes's (1930, [1936] 2007) views on interest rates were based on his conceptions of uncertainty, endogenous expectations, financial markets and institutions, money, liquidity preference, and animal spirits. Keynes recognized that: (1) "the outstanding fact is the extreme precariousness of the basis of knowledge on which our estimates of the perspective yield have to made" ([1936] 2007, 149); (2) "a large portion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation" ([1936] 2007, 161); (3) investors' actions are "a result of animals spirits … and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities" ([1936] 2007, 161); and (4) even though "the state of long-term expectation is often steady … human decision affecting the future, whether personal or political or economics, cannot depend on strict mathematical expectation" ([1936] 2007, 162–63).

Recent work lends credence to Keynes's views on government bonds' long-term interest rate dynamics. Akram and Li (2017, 2020a) show the influence of the short-term interest rate on the long-term interest rate on US Treasury securities. Similar patterns hold in other advanced countries. Akram and Li (2020b, 2020c), Akram and Das (2020), and Das and Akram (2020) respectively show that empirical analysis reveals that there are strong positive correlations between the long-term interest rate and the short-term interest rate in Japan, the United Kingdom, Australia, and Canada. Deleidi and Levrero (2021) find the central bank can permanently affect the long-term interest rate through its influence on the short-term interest rate using US data. However, Rahimi (2014) and Rahimi, Chu, and Lavoie (2017) find a significant two-way Granger causality relationship between the short-term interest rate and the long-term interest rate in United States. Using data from six financial centers (US, eurozone, Germany, UK, Japan, and Canada), Gabrisch (2021) reports that Keynes's contention holds. Based on econometric analysis of 17 advanced countries, Kim (2021) argues that the central bank of countries with a sovereign currency can influence government bond yields to a significant extent, irrespective of the government debt ratio and market sentiment.

Econometric analysis of the data in several emerging markets shows that the relationship between the short-term interest rate and the long-term interest rate also holds there. Akram and Das (2019) and Vinod, Chakraborty, and Karun (2014) provide evidence of this for India, while

Akram and Uddin (2020, 2021) do so for two key Latin American countries, namely Brazil and Mexico. The overall evidence from recent empirical studies, which shows that the short-term interest rate and the long-term interest rate are usually strongly correlated, suggests that multifactor Keynesian models can enrich the theoretical understanding of the long-term interest rate's dynamics and their relationships with fundamental macroeconomic and financial variables.

3. KEYNESIAN MODELS OF LONG-TERM INTEREST RATE DYNAMICS

Although there has been a proliferation of empirical studies vindicating the Keynesian approach to understanding the dynamics of the long-term interest rate, benchmark interest rate models in quantitative finance—such as Vasicek (1977), Dothan (1978), Cox, Ingersoll Jr., and Ross (1985), Ho and Lee (1986), Black, Derman, and Toy (1990), Heath, Jarrow, and Morton (1992), Heston (1993), Hull and White (1990), Kalotay, Williams, and Fabozzi (1993), Jamshidian (1995), and Brace, Gaterk, and Museiela (1997)—do not incorporate Keynes's insights on interest rate models. A close reading of Rebonato's (2004) comprehensive discussion of interest rate models shows that Keynes's insights are yet to be comingled with mainstream quantitative finance. The existing multifactor interest rate models, such as Longstaff and Schwartz (1992) and Chen (1996), fail to integrate Keynes's insights. There are a few theoretical models that advance Keynesian views, such as those found in Akram (2021b, 2021c), but these are one-factor interest rate models. However, multifactor interest rate models with a Keynesian perspective can be quite useful for empirical analysis, as well as in policy formulation, implementation, and evaluation. This paper fills a consequential lacuna by developing two multifactor models that embody Keynesian insights on interest rate dynamics. The mathematical concepts deployed in the models presented below are based on Papoulis (1984) and Karatsas and Shreve (1997).

Model 1

In this model, the long-term interest rate is R_{LT}. The short-term interest rate is r_{ST}. The central bank's policy rate is r_{CB}, and the expected inflation is π^E, while $\bar{\pi}$ is the central bank's inflation target. Z_1, Z_2, and Z_3 are Weiner processes. The parameters of the models are $a_1, a_2, a_3, b_1, b_2, c_1, c_2$.

The model is expressed in the following three equations:

$$\boldsymbol{dR_{LT}(t) = (a_1 r_{ST}(t) + a_2 \pi^E(t))dt + a_3 \sqrt{r_{ST}(t)} dZ_1} \quad \textbf{[1]}$$

$$\boldsymbol{dr_{ST}(t) = b_1(r_{CB}(t) - r_{ST}(t))dt + b_2 \sqrt{r_{ST}(t)} dZ_2} \quad \textbf{[2]}$$

$$\boldsymbol{d\pi^E(t) = c_1(\bar{\pi} - \pi^E(t))dt + c_2 \sqrt{\pi^E(t)} dZ_3} \quad \textbf{[3]}$$

Equation [1] relates the dynamics of the long-term interest rate to the short-term interest rate, inflation expectations, and a Weiner process adjusted by the short-term interest rate. Equation [2] ties the dynamics of the short-term interest rate to the difference between the central bank's policy rate and the short-term interest rate and a Weiner process adjusted by the short-term interest rate. Equation [3] connects the dynamics of inflation expectations to the difference between the central bank's target inflation and inflation expectations and a Weiner process adjusted by inflation expectations.

Model 2

In this model, the long-term interest rate is R_{LT}. The short-term interest rate is r_{ST}. The central bank's policy rate is r_{CB}. Expected inflation is π^E, while $\bar{\pi}$ is the central bank's inflation target. χ represents financial market volatility, while $\tau(t)$ is an exogenous shock. W is a Weiner processes. The parameters of the models are $\alpha_1, \alpha_2, a_3, \beta, \gamma, \delta$.

$$dR_{LT}(t) = \left(\alpha_1 r_{ST}(t) + \alpha_2 \pi^E(t)\right)dt + \chi(t)\sqrt{r_{ST}(t)}dW(t) \qquad [4]$$

$$dr_{ST}(r) = \beta(r_{CB}(t) - r_{ST}(t))dt + \chi(t)\sqrt{r_{S(T}(t)}dW(t) \qquad [5]$$

$$d\pi^E(t) = \gamma(\bar{\pi} - \pi^E(t))dt + \chi(t)\sqrt{\pi^E(t)}dW(t) \qquad [6]$$

$$d\chi(t) = \delta(\bar{\chi} - \chi(t))dt + \tau(t)\sqrt{\chi(t)}dW(t) \qquad [7]$$

Equation [4] relates the dynamics of the long-term interest rate to the short-term interest rate and inflation expectations and a Weiner process adjusted by the volatility of financial markets and the short-term interest rate. Equation [5] expresses the dynamics of the short-term interest rate as a function of the difference between the central bank's policy rate and the short-term interest rate and a Weiner process, adjusted by the volatility of the financial markets and the short-term interest rate. Equation [6] connects the dynamics of inflation expectations to the difference between the central bank's inflation target, inflation expectations, and a Weiner process adjusted by the volatility of financial markets and inflation expectations. Equation [7] expresses the dynamics of financial market volatility as a mean reverting process and a Weiner process adjusted by an exogenous shock and financial market volatility.

4. CONCLUSION

This paper has presented two multifactor models of the long-term interest rate embodying Keynes's insight that the central bank's policy rate influences the long-term interest rate primarily through the short-term interest rate. The multifactor models presented here can be useful in translating Keynes's insights into specific functional forms for understanding interest rate dynamics and evaluating which specifications best capture the underlying processes. These models can be extended to examine of the effects of other relevant variables—such as government debt or deficit ratios, market sentiments, the exchange rate, or the growth of industrial production or economic activity—on government bond yields. If the short-term interest rate is a key driver of the long-term interest rate, there are important implications for macroeconomics and finance. The central bank of a country with monetary sovereignty can have a substantial influence over government bond yields through the policy rate and monetary policy actions.

These models show the long-term interest rate as a function of the short-term interest rate, other macroeconomic and financial variables, and Wiener processes. The models are represented as stochastic differential equations to mathematically convey Keynes's insight on interest rate dynamics, which drew on his conceptions of ontological uncertainty, liquidity preference, animal spirits and endogenous expectations, institutional features of capital markets and financial firms, and investors' behavior and psychology. The Keynesian approach to modeling interest rate dynamics has found support in empirical studies in recent years. It is hoped that modifying benchmark models to incorporate Keynes's insights in a multifactor setting will advance the Keynesian approach to understanding the dynamics of the long-term interest rate on government bonds.

REFERENCES

Akram, T. 2021a. "A Note Concerning the Dynamics of Government Bond Yields." *The American Economist*. (Available online first). https://doi.org/10.1177/0569434520988275

———. 2021b. "A Simple Model of the Long-Term Interest Rate." *Journal of Post Keynesian Economics*. (Available online first). https://doi.org/10.1080/01603477.2021.1878906

———. 2021c. "A Keynesian Approach to Modeling the Long-Term Interest Rate." Levy Institute Working Paper No. 988 (June). Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

Akram, T., and A. Das. 2019. "The Long-Run Determinants of Indian Government Bond Yields." *Asian Development Review* 36(1): 168–205.

———. 2020. "Australian Government Bonds' Nominal Yields: A Keynesian Perspective." *Annals of Financial Economics* 15(1): 2050003-1 2050003-20.

Akram, T., and H. Li. 2017. "What Keeps U.S. Long-Term Interest Rates So Low?" *Economic Modelling* 60: 380–90

———. 2020a. "An Inquiry Concerning Long-Term U.S. Interest Rates Using Monthly Data." *Applied Economics* 52(24): 2594–621.

———. 2020b. "JGBs' Chronically Low Nominal Yields: A VEC Approach." *Applied Economics* 52(53): 5873–93.

———. 2020c. "The Empirics of UK Gilts' Yields." Levy Institute Working Paper No. 969 (September). Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

Akram, T., and S. Uddin. 2020. "An Empirical Analysis of Long-Term Brazilian Interest Rates." Levy Institute Working Paper No. 956 (May). Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

———. 2021. "The Empirics of Long-Term Mexican Government Bond Yields." Levy Institute Working Paper No. 984 (February). Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

Black, F., E. Derman, and W. Toy. 1990. "A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options." *Financial Analysts Journal* 46(1): 33–39.

Brace, A., D. Gatarek, and M. Museiela. 1997. "The Market Model of Interest Rate Dynamics." *Mathematical Finance* 7(2): 127–54.

Chen, L. 1996. *Interest Rate Dynamics, Derivatives Pricing, and Risk Management*. New York: Springer.

Cox J., J. Ingersoll Jr, and S. Ross. 1985. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53(2): 385–467.

Das, A., and T. Akram. 2020. "A Keynesian Analysis of Canadian Government Securities' Yields." *PSL Quarterly Review* 73(294): 241–60.

Deleidi, M., and E. S. Levrero. 2021. "Monetary Policy and Long-term Interest Rates: Evidence from the US Economy." *Metroeconomica* 72(1): 121–47.

Dothan, L. U. 1978. "On the Term Structure of Interest Rates." *Journal of Financial Economics* 6(1): 59–65.

Gabrisch, H. 2021. "GARCH Analyses of Risk and Uncertainty in the Theories of the Interest Rate of Keynes and Kalecki." The Vienna Institute for International Economic Studies Working Paper No. 191. Vienna, Austria: The Vienna Institute for International Economic Studies.

Heath, D., R. Jarrow, and A. Morton. 1992. "Bond Pricing and the Term Structure of the Interest Rates: A New Methodology." *Econometrica* 60(1): 77–105.

Heston, S. 1993. "A Close Form-Solution for Options with Stochastic Volatility with Application to Bond and Currency Options." *Review of Financial Studies* 6(2): 327–43.

Ho, T. S. Y., and S. Lee. 1986. "Term Structure Movements and Pricing Interest Contingent Claims." *Journal of Finance* 41(5): 1011–29.

Hull, J. C., and A. White. 1990. "Pricing Interest Rate Derivative Securities." *Review of Financial Studies* 3(4): 573–92.

Jamshidian, F. 1995. "A Simple Class of Square-Root Interest-Rate Models." *Applied Mathematical Finance* 2(1): 61–72.

Kalotay A. J., G. O. Williams, and F. J. Fabozzi. 1993. "A Model for Valuing Bonds and Embedded Options." *Financial Analysts Journal* 49(3): 35–46.

Karatsas, I., and S. Shreve. 1997. *Brownian Motion and Stochastic Calculus*. Second edition. New York: Springer-Verlag.

Keynes, J. M. 1930. *A Treatise on Money*. Volume I and II. London: Macmillan.

———. (1936) 2007. *The General Theory of Employment, Interest, and Money*. New York: Palgrave Macmillan.

Kim, H. 2021. "Sovereign Currency and Long-Term Interest Rates." *International Review of Applied Economics*. (Available online only). https://doi.org/10.1080/02692171.2021.1908237

Longstaff, F. A., and E. S. Schwartz. 1992. "Interest Rate Volatility and the Term Structure: A Two Factor General Equilibrium Model." *Journal of Finance* 47(4): 1259–89.

Papoulis, A. 1984. *Probability, Random Variables, and Stochastic Processes*. Second edition. New York: McGraw-Hill.

Rahimi, A. 2014. "Essays on the Causal Relationship Between Short-Term and Long-Term Interest Rates." PhD dissertation. Ottawa: University of Ottawa.

Rahimi, A., B. M. Chu, and M. Lavoie. 2017. "Linear and Non-Linear Granger Causality Between Short-Term and Long-Term Interest Rates: A Rolling Window Strategy." *Metroeconomica* 68(4): 882–902.

Rebonato, R. 2004. *Volatility and Correlation*: *The Perfect Hedger and the Fox*. Chichester, UK: John Wiley and Sons.

Riefler, W. W. 1930. *Money Rates and Money Markets in the United States*. New York and London: Harper & Brothers.

Vasicek, O. A. 1977. "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics* 5(2): 177–88.

Vinod, H. D., L. Chakraborty, and H. Karun. 2014. "If Deficits are Not the Culprit, What Determines Indian Interest Rates?" Levy Institute Working Paper No. 811 (July). Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.