The leptokurtic crisis and the discontinuous turn in financial modelling

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1. Introduction

Heterodox economics has, since its inception, stressed the extreme importance of financial crises to understand the nature of finance. Both the French school of regulation and the economics of conventions sought to leave the framework of analysis imposed by the dominant paradigm of neoclassical finance, that of the all-powerful exchange in a financial world without crises.

In a completely different way, the fundamental role of financial crises has been emphasized by Benoît Mandelbrot in his work on the mathematical modelling of stock market dynamics with fractals, a new approach which characterised heterodox modelling in financial economics. Initially, heterodox economics and fractals were unaware of each other, while each in its own discipline sought to propose another paradigm for finance. Heterodox economics and heterodox modelling seemed to await a new mathematical tool for rebuilding finance on new foundations. The first works that aimed to bridge the gap between these two attempts at rebuilding finance on alternative bases date back to the early 1990s. These works tried to establish a link between the approach of heterodox economics and the Mandelbrot approach, showing how fractals could both adequately describe the phenomenon of crises and allow the search for the making of an alternative finance than that based on the orthodox economics without crisis. The beginnings of econophysics, for example the winter session of Les Houches School of physics in 1998, established the fertility of the fractal approach in this perspective.

Following Mandelbrot's intuitions, the alternative way of modelling risks (the "fractal modelling"), used other processes than the Brownian motion of the dominant paradigm of neoclassical finance, in particular with non-Brownian stochastic processes. These works challenged the mainstream view of finance by using an internalist approach, whereas other works challenged the mainstream view by using an externalist approach. Heterodox modelling and heterodox economics were in line with their objective: a critical posture of the neoclassical finance arising from orthodox financial theory. The heterodox modelling challenged a very particular and counter-intuitive representation of the dominant paradigm of neoclassical finance, the continuity of stock market fluctuations. With this continuity assumption, whose mathematical financial translation is the Brownian representation of market dynamics, finance refrained from thinking about crises. This finance "empty of crisis" has

[1] This chapter builds on my previous works. To avoid overloading with too many references, I will refer back to these works which contain the complete list of references used, mentioning here only the main ones.

produced the illusion of a taming of the risks of which we understand now how it was associated with the continuity assumption.

In this chapter, I propose to focus on one aspect of these debates that seems to me crucial: the "leptokurtic crisis" and its consequences in challenging the dominant paradigm of neoclassical finance. The observed behaviour of markets exhibits a specific feature: empirical distributions of returns are *leptokurtic*. The word *leptokurtic* comes from the Greek words *leptos*, peaked, and *kurtosis*, curvature. That means that the empirical distributions are more peaked than the Gaussian bell, the theoretical distribution expected in the Brownian representation. Instead of that, the empirical distributions exhibit fat tails and values clustered around the mean, with the result that extreme events like financial crises are more likely than under a Gaussian distribution. "Leptokurtic crisis" is the name I give (Walter, 2002a) to the crisis opened by Benoît Mandelbrot in a series of contributions (Mandelbrot, 1962, 1963, 1967)[2] which introduce the Lévy stable processes to solve the leptokurtic puzzle. The term "crisis" is chosen in reference to Kuhn's 1962 *Structure of scientific revolutions*. In Kuhn's words, "confronted with anomaly of with crisis, scientists take a different attitude towards existing paradigms" (Kuhn, 1962, p. 91). In fact, the Mandelbrot papers exploded the field of financial modelling and launched violent controversies dividing the community of finance academics into two opposite camps: pro and cons the discontinuity. The crisis started the quest for refinements of the Brownian representation that is for new models that could solve the leptokurtic problem with saving the "mild randomness" assumption of Brownian motion.

Two distinct research programmes were currently established in financial modelling to tackle the leptokurtic issue: the first Mandelbrot programme based on stable Lévy processes and the alternative non-stable Lévy processes approach based on the Merton's view. I named (Walter, 2017) these two programmes: the radical programme (RP) and the pragmatic programme (PP). During more than thirty years, these two programmes were incompatible in the sense that Mandelbrot and its opponents speak from "incommensurable" viewpoints in the Kuhn's words. There was no "neutral statistical test" for model-choice. The RP initiated huge controversies in the financial academic field because of the infinite variance of the stable hypothesis. The PP began in the 1970s with explicitly renouncing the stable hypothesis. But the Lévy processes with finite activity and finite variation appeared too restrictive to solve the leptokurtic crisis. In the 1990s a new competitor appeared, called econophysics programme (EP). To untangle these threads and to lighten the issues, I use the Sato classification to describe the competitive programmes. Although the PP and the EP can be traced through separate lines in the academic fields, they shared the use of tempered stable processes and derive from their reliance on Mandelbrot's view. To conclude with a perspective of the history of financial thought, I argue that, for solving the leptokurtic problem, Mandelbrot introduced what I name the "discontinuous turn" in financial modelling.

2. Two competitive representations of financial uncertainty

I have argued elsewhere [note] that some of the key differences between the competitive representations of financial uncertainty can be illuminated by reference to a familiar debate in philosophy over the principle of continuity. Although this philosophical debate may seem to be a scholastic preoccupation within a tight circle of specialists in philosophy of science, far from the financial stakes of modelling and with no impact on concrete financial practices, I have argued on the contrary that the divergent positions about the mindset behind the price changes implicate entirely different views of what it is important to capture and how to model it. Let us emphasize this point.

[2] Mandelbrot's second challenge on independence with long-run correlations is not discussed here.

There are two fundamentally different ways of viewing prices changes in finance. One assumes the principle of continuity, the other doesn't. According to the first view, price movements are modelled by continuous diffusion processes, whose canonical form is the famous Brownian motion. According to the other view, price movements are modelled by discontinuous processes, as for instance Lévy processes. I now elaborate on this, which is of a great importance for contemporary debates in finance and the issue of leptokurtic crisis I aim to address here.

In physics, the principle of continuity states that change is continuous rather than discrete. Leibniz and Newton, the inventors of differential calculus, said "*Natura non facit saltus*" (nature does not make leaps). This same principle underpinned the thoughts of Linné on the classification of species and later Charles Darwin's theory of evolution (1859). In 1890, Alfred Marshall's *Principles of Economics* assumed the principle of continuity, allowing the use of infinitesimal calculus in economics and the subsequent development of neoclassical economic theory. As noticed by Norbert Wiener, "just as primitive peoples adopt the Western modes of denationationalized clothing and of parliamentarism out of a vague feeling that these magic rites and vestments will at once put them abreast of modern culture and technique, so the economist have developed the habit of dressing up their rather imprecise ideas *in the language of the infinitesimal calculus*" (Wiener, 1966, p. 90, our italics).

Modern financial theory grew out of neoclassical economics and naturally assumes the same principle of continuity. One of the great success stories of neoclassical finance was the valuation of derivatives with the replicating portfolio technique and the risk-neutral approach. Examples include the 1973 formulas of Fisher Black, Myron Scholes, and Robert Merton for valuing options and the subsequent fundamental theorem of asset pricing that emerged from the work of Michael Harrison, Daniel Kreps, and Stanley Pliska between 1979 and 1981. As MacKenzie and Spears (2014, p. 401) put it: "it is the strategy of Black-Scholes modelling writ large: find a perfect hedge, a *continuously-adjusted portfolio* of more basic securities that will have the same payoff as the derivative, whatever happens to the price of the underlying asset" (our italics); which means that the continuity principle is at the core of the "market-consistent convention". This specific convention is the "quantification convention" of the dominant paradigm of neoclassical finance which defines the neoclassical financiers' metrology (Chiapello and Walter, 2016). In fact, the twin pillars of neoclassical finance are efficient markets and the theory of asset pricing with no arbitrage. The way probability theory and economics were linked together to create these twin pillars exhibits the strong role of continuity (Walter, 1996). In fact, these twin pillars rest on the principle of continuity.

Following the mathematical breakthrough of Black, Scholes, Merton, Ross, Harrison, Kreps and Pliska, and despite the repeated financial crises following the 1987 stock market crash, the neoclassical finance reaffirmed the principle of continuity. This principle was still predominant in the 1990s despite the emerging evidence of extreme values in the tails of empirical distributions. At the end of the century, many financial techniques such as portfolio insurance or the calculation of capital requirements in the insurance industry still assumed that (financial) nature does not make jumps and therefore promoted continuity. Despite many empirical difficulties arising with attempts to use it practically, and despite academic warnings coming from outside neoclassical finance, the principle of continuity remained vastly more popular than its discontinuous competitors.

One of the cognitive consequences of the continuity principle is a negative spillover about the extreme value issue: the truncation of financial time series into two market regimes. On the one hand, the "normal'" periods, corresponding to the supposed continuous market; on the other hand, the periods of "insanity" where markets are deemed "irrational" and "greedy", corresponding to extreme value behaviours. This cleavage (explained continuity + unexplained jumps) leaves the financiers unable to explain the transition from one period to another. For example, in an editorial in the *Financial Times* (16.3.08), Alan Greenspan commented on the financial crisis of 2007-2008 with these words: “We can never anticipate all discontinuities in financial markets.” For Greenspan, (financial) nature does not make jumps and extreme values are unpredictable outliers. This cognitive bias demonstrates the limits of a continuity-based framework completed with an extreme value approach, and advocates for the need for a global discontinuous framework (Le Courtois and Walter, 2017).

In the 20th century, both physics and genetics abrogated the principle of continuity. Quantum mechanics postulated discrete energy levels while genetics took discontinuities into account. But economics – including modern financial theory – stood back from this intellectual revolution. As early as 1966, Wiener pointed that “here *some recent work of Mandelbrot is much to the point*. He has shown the intimate way in which the commodity market is both theoretically and practically subject to random fluctuations arriving from the very contemplation of its own irregularities is something *much wilder and much deeper than has been supposed*, and that the usual approximations to the dynamics of the market must be applied with much caution than has usually been the case, or not at all” (p. 92, our italics).

3. The leptokurtic phenomenon: the case against neoclassical finance

The concept of “leptokurtic phenomenon” (Walter, 2002a) has been introduced to suggest that it existed in finance a phenomenon to be explained referred to as an “*explanandum phenomenon*” in the Carl Hempel’s sense: “an event occurring at a particular place and time (…) some regularity found” in the markets (Hempel, 1966, p. 50).

3.1 The longstanding leptokurtic problem

The leptokurtic problem is not new. What is new is its institutional acknowledgement: it is common knowledge today that non-Gaussian empirical distributions are a “fact” of the real markets. Yet it took almost forty years for this observable “fact” to become an observed “fact”, as non-normality entered the field of scientific research. The leptokurtic bell of empirical distributions has long been known in the academic community. As early as the 1950s, empirical studies pointed out the problems of the Brownian representation of market dynamics. But, if statisticians highlighted the leptokurtic phenomenon many times, financial academics did not want to consider it. In 1962, the attempt by Mandelbrot to take explicit account of discontinuities on all scales in stock market prices by building a discontinuous global framework for taking account of financial crises led to the leptokurtic crisis.

3.1.1. Statistical evidence: “it’s full of extreme values!”

Let us pick up four examples among others of the pervasiveness of the leptokurtic phenomenon in the long term. In a 1953 landmark paper published in the respected *Journal of the Royal Statistical Society*, Maurice Kendall observed of price data between 1883 and 1934 that “the distributions are accordingly *rather leptokurtic*” (our italics). In 1960, in *Food Research Institute*, Arnold Larson noted that “examination of the pattern of occurrence of all price changes in excess of three standard deviations from zero (…) indicated (…) presence in the data of an *excessive number of extreme values*” (our italics). In a very important article published in 1961 in the *American Economic Review*, Houthakker wrote that “the distribution of day-to-day changes in the logarithms of prices does not conform to the normal curve. It is… *highly leptokurtic*.” The same year in *Industrial Management*

Review, Sydney Alexander emphasized pithily that "a rigorous test… would lead to dismiss the hypothesis of normality; this sort of situation (*leptokurtic*) is frequently encountered in economic statistics" (our italics). Many other examples exist in the early literature of market research.

3.1.2. Financial academics denials: "don't tell me about extreme values!"

Although it has been observed for a long time in the statistical community, the leptokurtic phenomenon was long considered negligible or non-significant in the financial academic community: the 1960s worldview did not include tails in analysis of stock fluctuations. Extreme values were not considered a significant "fact" for proper understanding of price variations. Some academics even indicated that the tails of distribution should be cut off to remove the extreme values. Financial academics were confronted to anomalies (extreme values) but dismissed the "facts"; hence the "facts" quite simply disappeared from the "data". For mainstream financial academics, extreme values were considered as "*outliers*", that is not relevant for modelling. De Bruin and Walter (2017) present and discuss this intriguing episode of history of financial modelling and argue that specific research habits of mainstream had violated epistemic virtues.

3.2 Mandelbrot's challenge: "extreme values are essential!"

The word "outlier" has a precise technical meaning in statistics: it is an observation that is so very different from the other observations that it may be due to variability of the measurement or it may indicate experimental error like accidental foreign contamination. In his memoirs, Mandelbrot gives the example of "astronomical cats": "A classic example concerns astronomical observations that are contaminated by cats residing in the observatory. Yes, cats walking across the observatory floor shook the telescope a bit, causing some orbits to be miscalculated. For two centuries, economists and statisticians have looked for good ways of preserving real data while eliminating *would-be cats*". According to Mandelbrot, in finance, "outliers" are not cats but are crucial to enter the puzzle of financial crises: "the so-called outliers are *essential* in finance. In fact, a common thread of my work is that values far from the norm are the *key* to the underlying phenomenon" (our italics). In Mandelbrot's view, the distribution (fat) tails are not a refuge for "outliers" but on the contrary contained important information for correct understanding of the market's dynamics leading to financial crises.

Mandelbrot's intellectual stance thus constituted a radical change in the way of looking at market dynamics. In the dominant paradigm of neoclassical finance based on continuity, only the fluctuations of the means were considered as interesting: on the contrary, Mandelbrot suggested that attention should be paid to fluctuations in extreme values. It represented a huge challenge to the foundations of the dominant paradigm of neoclassical finance. Unlike the continuous representation of this paradigm, Mandelbrot argued that close attention should be paid to the discontinuities. Hence, the profoundness of the leptokurtic crisis came from the angle of the Mandelbrot's attack: not only he emphasized an *empirical inadequacy* of the continuous representation, but also he argued for an *inadequate grounding* of this representation. It was a frontal attack against the dominant paradigm of neoclassical finance.

4. The leptokurtic crisis: looking for alternative finance

Mandelbrot tackled the leptokurtic phenomenon by driving attention to the cognitive importance of extreme values. He emphasized the necessity of not ignoring the extreme values of the distribution tails by incorporating global discontinuity in the core of the probabilistic model. This challenge to the foundations of the dominant paradigm of neoclassical finance launched the leptokurtic crisis and the heterodox view of finance.

4.1 The hinges of the debates

Now I present the terms of the debates resulting from the leptokurtic crisis. First, Brownian motion increments have the important property of being independent and identically distributed (hereafter IID). The processes with IID increments are called Lévy processes after the French mathematician Paul Lévy. Brownian motion is a specific Lévy process: it assumes continuity. Other Lévy processes do not. Hence, a first epistemic choice is to retain of not the IID framework.

Secondly, Brownian motion entails time scaling of distributions in the sense that a given horizon of return distribution is scaled to another with the *square-root-of-time rule* of scaling. This is the *scaling property* of Brownian motion. This scaling property supports the widely used practice to compute the annual volatility from the weekly one: the annual volatility is equal to the weekly volatility multiplied by square root of 52. This scaling property had been called the “square-root-of-time law” in the Jules Regnault’s 1863 *Calcul des chances et philosophie de la bourse*. This square-root-or-time rule of scaling is a special case of scaling property of stable distributions. Gaussian distribution is a specific stable distribution: it uses a scaling parameter (characteristic exponent) with value 2. Other stable distributions do not. Hence, a second epistemic choice is to retain of not the stable framework.

Thirdly, Brownian motion supports a Gaussian distribution for the marginal distribution of returns. Hence, a third epistemic choice is to retain of not the Gaussian framework. The controversies following the leptokurtic crisis became entangled in the intrication of the marginal distribution view (Gaussian or non-Gaussian), the dynamic processual view (IID or non-IID) and the scaling rule view (stable or non-stable). It is worth noting that this intrication was sometimes a source of confusion in articles of the historical thought literature, some analyses being based on approaches which didn’t distinguished between Lévy distributions and Lévy processes. Here the semantics is misleading.

4.1.1. With or without IID

To solve the leptokurtic problem, the first two possible routes are either to remain in the IID framework or to leave the IID framework. If IID is retained, this epistemic choice leads to change the kind of Lévy process. This epistemic choice thus continues to use the Lévy representation of stock market fluctuations, although without the Brownian characteristic that ensures continuity.

Alternatively, if the Gaussian distribution assumption is retained and the IID assumption is dropped, it introduces a form of temporal dependence on successive stock market fluctuations. This epistemic choice is equivalent to focusing on the conditional distributions of stock market fluctuations, i.e. the market memory. Table 1 below summarises the scaling rules.

<table>
<tr><th>Increments</th><th>Stochastic process</th><th>Years</th><th colspan="2">Scaling rule</th></tr>
<tr><td>non I and non-ID</td><td></td><td></td><td colspan="2">Joint distribution</td></tr>
<tr><td>I and non-ID</td><td>Sato processes</td><td>2000s</td><td colspan="2">Convolution product</td></tr>
<tr><td>IID non-stable</td><td>Lévy processes</td><td>1990s</td><td colspan="2">Convolution product</td></tr>
<tr><td>IID alpha-stable</td><td>Lévy motion</td><td>1962</td><td colspan="2">Fractal invariance</td></tr>
<tr><td>IID 2-stable</td><td>Brownian motion</td><td>1900</td><td>Fractal invariance</td><td>“Square-root-of-time rule” (Regnault, 1863)</td></tr>
</table>

Table 1. Scaling rules as epistemic choices (Walter, 2013)

4.1.2. With or without wild randomness

At this stage, the question arises of whether or not the scaling property assumption should be kept. If the scaling property is maintained, stable Lévy processes have to be chosen. We consider now the second epistemic choice inside the IID representation.

Let us move into the Fourier space. The explicit form of the characteristic exponent of a Lévy process is the Lévy-Khintchine formula which fully defines the process by the following specification:

Characteristic exponent = diffusive component + diffusion coefficient + Lévy measure

In financial words:

Market dynamics = trend of returns + scale of risk + morphology of risk

The scale of risk is the "volatility" of markets. The role of the Lévy measure is decisive. It contains all the information need to characterise the path of the process, apart from its trend and its "volatility. It shapes the morphology of uncertainty, its "roughness". This morphology can be drawn with the notion of "states of randomness", a notion introduced by Mandelbrot to describe the level of roughness of the price charts. He introduced a pivotal distinction between two types of randomness named "mild randomness" and "wild randomness":

> "The traditional Gaussian way of looking at the world begins by focusing on the ordinary and only later deals with exceptions or so-called outliers as ancillaries. But there is also a second way which takes the so-called exceptional as a starting point and deals with the ordinary in a subordinate manner simply because that 'ordinary' is less consequential. These two models correspond to two mutually exclusive types of randomness: mild or Gaussian on the one hand, and wild, fractal or 'scalable power laws' on the other. Measurements that exhibit mild randomness are suitable for treatment by the bell curve or Gaussian models, whereas those that are susceptible to wild randomness can only be expressed accurately using a fractal scale."

In the 1960s two alternatives represented a kind of cardinal choice, in the literal sense of the revolving movement of thought (the word "cardinal" comes from the Latin *cardo*, meaning the hinge of a door) that is required in response to the leptokurtic phenomenon. This was the hinge of financial modelling, summarised in table 2 below.

Hinge: choice between	Route 1	WILD randomness	IID with non-Gaussian distributions	Heterodox finance way
	Route 2	MILD randomness	Non-IID with Gaussian distributions	Neoclassical finance way

Table 2. The hinge of epistemic choices in the 1960s, the 1970s and the 1980s (Walter, 2005)

At the end of the 1960s, in the 1970s and the 1980s, the situation resembled that described in Kuhn's analysis: with this pivotal choice, finance academics have "before [them] a number of competing and incommensurable solutions to these problems, solutions that [they] must ultimately evaluate for [themselves]" (Kuhn, 1962).

4.2 The misfortunes of the stable model

Despite the promising results opened up with this new way of making finance, the adventure of fractal modelling in finance did not displayed a smooth (continuous) history. It was more an eventful (discontinuous) progression of Mandelbrot's assumptions against the evolution of neoclassical finance over forty years, from 1960 until 2000. The Mandelbrot proposal for making finance with wild randomness was qualified as "monster". The most violent debates were related to the "scaling" property of stochastic processes and the Paretian tails of the empirical distributions of returns.

4.2.1. Infinite variance: a probability monster

The first and principal problem was that of the moments of distributions. With a Lévy motion, moments can be infinite. The infinity of the second moment (the variance) horrified the American academics of neoclassical finance. The rejection was massive, violent and total. So strong was the opposition to Mandelbrot's hypothesis that any kind of alternative model was preferred to the idea of infinite variance. At this time, Mandelbrot was working at IBM and "ABM" ("anything but Mandelbrot") became the watchword of the neoclassical finance camp. As Mirowski (1995) observed, "the economics profession dropped the Mandelbrot hypothesis largely for reasons other than empirical adequacy and concise simplicity. […] The only purpose of the negative studies was to refute Mandelbrot". MacKenzie (2006, p. 105) stressed this point by saying that Mandelbrot's model was viewed by the financial academic community as a probability "monster".

According to the old definition of Aristotle, a monster is what deviates from the norm. The norm of the dominant paradigm of neoclassical finance is variance. To imagine that the variance could be infinite was thus to attack frontally one of the pillars of the paradigm of neoclassical finance.

4.2.2. Scalable randomness: scaling anomalies

Underpinned by the desire to reject the infinite variance, the debates shifted to tests of the scaling property of the stable model. The objective was to reject the hypothesis of the infinite variance and since this hypothesis was confused with that of scaling, it was necessary to highlight anomalies in the scaling laws.

A story of this quest was produced with a review over forty years of the research for scaling laws in distributional properties of price variations (Walter, 2002b,XXXX). This review exhibits a turbulent story with fierce controversies which stirred up the academic community with regard to the scaling / non-scaling debate. It ended with the empirical rejection of stable model because the scale invariance principle was found too strong for adequately modelling observed price variations. The stable Lévy processes were abandoned. But the question remains of a partial scale invariance over a given frequency range, the breakdown of scaling. This leads us to consider the tails of distribution.

4.2.3. Power laws: a tale of fat tails

The scaling character of price variations was first established through the study of distribution tails, which brought out the connection between the scaling laws and the appropriate treatment of financial crises. The tails appear to be Paretian, which means the distribution of extreme values followed a power law the origin of which dates back to Pareto. When one rewrites the 1962 stable model with the Lévy-Khintchine representation, the Lévy measure exhibits Paretian tails (details in Walter, 2015, p. 467-68). This Paretian characteristic is precisely the origin of the non-existence of the variance when the Paretian exponent is less than 2.

To remedy the inconvenience of not having any moments for the stable models of the Mandelbrot programme for rebuilding finance, other models were developed with a truncation principle. In fact, a way of avoiding the problem of infinite variance is to weight the Lévy measure by an exponential quantity in order to reduce large fluctuations and therefore recover the moments. This idea corresponds to a class of Lévy processes whose marginal distributions are truncated stable distributions, so-called "tempered stable" models. The stable distributions are truncated by exponential functions. The tails of these distributions, tempered by the truncation, are semi-light.

5. After the leptokurtic crisis: the discontinuous turn

The leptokurtic crisis exploded the field of financial modelling. We conclude this chapter by giving an overview of the posterity of the crisis in the renewal it produced of financial modelling.

5.1 The research programmes resulting from the leptokurtic crisis

On the one hand, after the leptokurtic crisis, two competitive programmes for solving the leptokurtic puzzle were launched in financial modelling: the Mandelbrot programme (heterodox) and the financial academics programme (mainstream). Both programmes investigated the ability of Lévy processes to capture the extreme values of price changes. On the other hand, physicists entered the race in the 1990s by tackling the question of scaling laws. Hence, in the 1990s, research in financial modelling split into two separate communities: that of financial academics and that of physicists.

5.1.1. The pragmatic programme of financial academics: what to do with jumps?

In the 1970s, the mainstream view of price changes made specific assumptions to defend the mathematical tractability of the financial modelling based on continuous diffusion models, by using a compound ad hoc approach, which gained highly recognition in the 1980s: I term this mainstream strand of research the "pragmatic programme", to contrast the Mandelbrot program that I call the "radical program" (Walter, 2017). The pragmatic programme opened a period of model tinkering in neoclassical finance: a situation in which researchers, confronted with descriptive inadequacy, decide to "repair" existing models with new data-driven approach.

This story is characterized by two major stages. First, with the rediscovery of Poisson's law in the late 1960s, a jump component was added to the diffusion process (Brownian motion): this superposition of jump and diffusion processes opened the period of hybrid models known as jump diffusion-processes (1970-1990), which state that prices undergo large jumps followed by small continuous movements. These models were initiated by Press in 1967. It is a simple case of Lévy process with finite activity and finite variation in the jump component. This is the first stage of the pragmatic programme. Then, in the second period, the diffusive component was removed leaving only the jump component, moving to Lévy processes keeping finite variation in the jump component but with infinite activity. This is the second stage of the pragmatic programme. The third stage of the pragmatic programme corresponds to the infiniteness of the variation.

The success of the pragmatic programme in the 1990s results from a reorientation of the mathematical financial research due to European academics who put forward the fruitfulness of infinite activity of the Lévy processes in case of pure jumps models. At the time mathematical financial academics moved to the development of Lévy processes, physicists launched their new strand of research, addressing the scaling properties of tails.

5.1.2. The econophysics programme of physicists: what to do with scales?

In the 1990s, physicists began to propose models combining truncated stable distributions with exponential tails (Mantegna and Stanley, 1994; Koponen, 1995; Bouchaud and Potters, 1997) and physicist research activity enters the financial modelling field. As Mantegna and Stanley (2000) noticed, "since 1990, a research community has begun to emerge". This new community baptized itself with the name "econophysics". Physicists continued along the way paved by Mandelbrot's model, working in particular with the scaling concept: as Mantegna and Stanley (2000) pointed out, financial academics were "trying to determine a characteristic scale for a problem that has no characteristic scale". This is the "econophysics programme".

5.1.3. Sato's classification

Now one turns to the disentanglement of the multiple research programmes resulting from the leptokurtic crisis. A convenient way to navigate the paths of financial modelling after the crisis is to return to Sato's classification. For shaping a Lévy process, either the activity is finite or infinite, or the variation is finite or infinite. The Sato (1999) classification defines a process by its pair (activity, variation) according to the double criterion finite or infinite. The models of the late 1990s and early 2000s used processes with infinite activity and infinite variation but with finite variance. In contrast to these pragmatic programmes, the radical programme proposed by Mandelbrot in 1962 had both infinite activity, infinite variation in the jump component and infinite variance. It was – for this reason – a complete heterodox view.

Table 3 below exhibits the Lévy processes in financial modelling following this double criterion, the variation being that of the jump part of the stochastic process. One has added in the table the variance criterion because the infinite activity case is itself divided into two subgroups, that of the pragmatic approach in which the variance is finite, and the radical approach (Mandelbrot's view) in which the variance is infinite. It appears that there are four types of stochastic processes depending on whether their activity and their variation are finite or infinite, and whether variance is finite or infinite.

Sato's pair				
Activity	**Variation**	**Variance**	**Example of models in financial modelling**	**Research programmes**
Infinite	Infinite	Infinite	Mandelbrot (1962)	Radical
Finite	Finite	Finite	Press (1967), Merton (1976), Cox and Ross (1976)	Pragmatic stage 1
Infinite	Finite	Finite	Madan and Seneta (1990), Madan and Milne (1991)	Pragmatic stage 2
Infinite	Infinite	Finite	Eberlein and Keller (1995), Barndorff-Nielsen (1997), Eberlein, Keller and Prause (1998), Madan, Carr and Chang (1998), Prause (1999), Carr, Geman, Madan and Yor* (2002, 2003) [*depends on exponent]	Pragmatic stage 3

Table 3. Sato's classification and the research programmes after the leptokurtic crisis (Walter, 2017)

5.2 Mandelbrot's legacy: the discontinuous turn in financial modelling

The Mandelbrot programme contributed to a better understanding of the radical discontinuous nature of price change, but the first Mandelbrot's models initially based on stable motions were not accepted by the financial academics community. The pragmatic program succeeded in filling the gaps in the continuous representation of the dominant paradigm without losing the variance that was important for this paradigm. In the 1990s, pure jumps processes became the new way of modelling the discontinuities in mathematical finance.

But the Variance Gamma model of Madan, Carr and Chang and the CGMY model of Carr, Geman, Madan and Yor are special cases of the Koponen model. Here there is an overlap with the physicist's approach: the academic territory of financial modelling is overlapping. The pragmatic programme and the econophysics programme develop similar readings of the Mandelbrot view they shared with the tempered stable family. Hence the financial field of modelling extreme values is not simply divided into two camps: mainstream finance (moving to pragmatic programme) and econophysics.

Despite their separate lines in the academic fields, the econophysics programme and the pragmatic programme derive from their reliance on Mandelbrot's view: two offshoots of what I suggest calling the `discontinuous turn' in financial modelling, introduced by Mandelbrot in 1962.

	Type of Lévy process	Financial models of market dynamics
1900	Brownian motion	Bachelier
1962	Stable motion	Mandelbrot
1967	Brownian motion + compound Poisson process	Press
1995	Hyperbolic motion	Eberlein and Keller
1997	Generalized Hyperbolic motion	Barndorff-Nielsen
1998	Variance Gamma process	Madan *et al*.
2002	Generalized Variance Gamma process	Carr *et al*.
2001	Laplace process	Kotz *et al*.
2010	Generalized Laplace process	Le Courtois and Walter

Table 4. Several generations of Lévy processes in financial modelling

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