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Conic CVA and DVA for Option Portfolios

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In this paper we develop the Conic Finance framework for the Credit and Debit Value Adjustments (CVA and DVA) of options and option portfolios. We derive Black's formula for futures options in the Conic Finance setting and suggest a new algorithm for calibrating the distortion (or liquidity) parameter surface from a set of observed option prices. We apply our approach to commodity futures options and option portfolios and show that both CVA and DVA are significantly impacted by the Conic Finance framework, when compared to traditional risk-neutral valuation. In particular, we demonstrate that the DVA decreases significantly when the Conic Finance framework is used, which is in line with the regulatory efforts to rein in a positive DVA resulting from a financial institution's own credit deterioration. Finally, we investigate the robustness of our approach with respect to calibration parameters and show that the calibrated distortion parameter is an excellent explanatory variable for the observed bid-ask spreads.

Keywords: Conic Finance, CVA, DVA, bid and ask pricing, liquidity, futures options, distortion function.

JEL Classification: G10, G21, G32

1. Introduction

The theory of Conic Finance is based on the existence of two prices prevailing in the market: a price at which one may sell to the market (bid price), and a price at which one may buy from the market (ask price). In this way it goes beyond the law of one price and risk neutral valuation, and introduces a two-price economy.

The Conic Finance framework has been introduced by Cherny and Madan (2010), stemming from the seminal results of Cherny and Madan (2009). It is founded on the basis of the concepts of acceptability of stochastic cash flows and distorted expectations. The market is seen as a counterparty which accepts, at zero cost, a *convex cone* of random variables containing the non-negative cash flows, and from this the name *Conic Finance* originates. Conic Finance is inherently related to liquidity effects and risk behavior of financial markets.

Conic Finance is a new and exciting development in quantitative finance, which attracted attention of many researchers in the last few years. It has been applied to several topics in finance; many of them described in the recently published book by Madan and Schoutens (2016a). One such possible application is computing CVA and DVA under Conic Finance assumptions rather than under a traditional risk neutral valuation. In their short note in Wilmott Magazine, Madan and Schoutens (2016b) outline the Conic Finance's impact on a zero-coupon bond, and illustrate how Conic Finance can mitigate the well-known but controversial feature of DVA: seeming profitability of an institution's own credit quality deterioration. Although Madan and Schoutens (2016b) give some intuition about the relation between the Conic Finance and CVA and DVA, their insights cannot be directly applied to a variety of positions or derivatives rather than bonds. So, inspired by their broad insights, we develop the full framework for computing value adjustments under Conic Finance assumptions, and extensively explore the Conic Finance impact on options' CVA and DVA, both theoretically and empirically. We study representative transactions between financial institutions, and investigate the impact of Conic Finance on CVA and DVA for typical option portfolios. Our approach is fundamentally data-driven, e.g., we construct, given observed option prices, a complete surface of the so-called *implied liquidity* measure, also known in Conic Finance as the *distortion parameter*. The philosophy behind constructing this surface is analogous to the well-known implied volatility surface.

Our numerical study, performed on the basis of observed bid and ask prices of commodity futures options, shows the impact of Conic Finance on CVA and DVA on various option positions and portfolios. In particular, we demonstrate that conic CVA and DVA deviate significantly from those obtained by traditional risk neutral valuation. The impact on DVA mitigates DVA's profitability resulting from a decline in one's own credit quality. This indicates that conic valuation of CVA and DVA may be favorable from a regulatory perspective. We also show how conic CVA and DVA depend on the calibrated parameters and, finally, we illustrate how the distortion parameter, calibrated to observed option prices, is an excellent explanatory factor for bid-ask spreads. This emphasizes its relation to market liquidity.[1]

The paper is organized as follows. In the next section, we outline the main concepts of Conic Finance that we need for our analysis. In Section 3 we derive Black's formula for futures options in a conic framework. Section 4 relates to CVA and DVA in a two-price economy. Section 5 takes our approach to the data and demonstrates how the implied distortion parameter surface can be constructed. Section 6 presents an extensive numerical study of the impact Conic Finance has on CVA and DVA of option portfolios, addresses its robustness and relates the calibrated distortion parameter to the observed bid-ask spread. Section 7 concludes.

[1] It is worth mentioning that here we take a *static* approach to Conic Finance, whereas e.g., Bielecki *et al.* (2013) describe a dynamic approach, which uses the theory of dynamic coherent acceptability indices in discrete time.

2. Concepts of Conic Finance

Here we summarize the concepts of the Conic Finance framework that are needed for our analysis.[2]

As in Cherny and Madan (2010), we see a financial market as a passive counterparty that trades any amount of a traded asset with market participants. First we consider the case (to which we refer as the classical framework) of the liquid market. Following Artzner *et al.* (1999), we consider a one-period model $\{0, T\}$, i.e. a set of random cash flows with a known state at the current time 0 and a stochastic realisation X at the future date T. More specifically, we consider stochastic processes of the form (w, X), where X is defined on an atomless probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and w is a real number representing a payment in some fixed unit of account. Given such processes, we define $\mathcal{A}$ as the set of zero-cost cash flows, that is, the set of linear combinations of risky cash flows realizing at time T that can be accessed at zero initial costs. Furthermore, as in Madan and Schoutens (2016a), we assume a constant interest rate r prevailing between 0 and T.

In the classical framework, it is assumed that the set $\mathcal{A}$ is

(i) closed under addition;
(ii) closed under nonnegative scaling;
(iii) closed under negation.

We recognize, as Madan and Schoutens (2011), that a transaction involves an initiating party that needs to trade, whereas the counterparty dictates the terms. The latter is represented by the abstract market. In liquid markets, one may have two parties that want to take opposite positions, so that they reach the prevailing market price. This is the economic intuition that corresponds to assumption (iii). In this classical framework, we accept all zero-cost trades that have non-negative expectation under the *risk-neutral measure* $\mathbb{Q}^0$. Hence, the set of cash flows *acceptable* at zero cost is identified with $\mathcal{A}_0 = \{X | \mathbb{E}^{\mathbb{Q}^0}[X] \geq 0\}$.

However, assumption (iii) does not always hold in practice. This can be observed directly from the fact that bid and ask prices may differ significantly. Retaining the first two, but relaxing the latter assumption, Cherny and Madan (2010) allow the market as a counterparty to distinguish the direction of trade of zero-cost cash flows. This market is, hence, a *two-price market.* Under assumptions (i) and (ii) the set of traded cash flows is a convex cone, from which the name *Conic Finance* originates.

Note that, regardless of investors' risk preferences, a rational agent will always accept a pure arbitrage opportunity, which is represented by a non-negative random variable. Therefore, any cone of acceptable cash flows should contain the set of non-negative random variables, which we denote by $\mathcal{A}_\infty$. In fact, the classical set $\mathcal{A}_0$ is the largest convex cone (namely, a half-space) that includes the set of arbitrage opportunities $\mathcal{A}_\infty$.

In a conic economy the set $\mathcal{A}$ of cash flows, acceptable at zero cost, that contains $\mathcal{A}_\infty$, is potentially much more restricted than the classical set $\mathcal{A}_0$. Following arguments of Artzner *et al.* (1999) and limiting our analysis to bounded random variables on $(\Omega, \mathcal{F}, \mathbb{P})$, it can be shown that there exists a convex set $\mathcal{D}$ of probability measures, absolutely continuous with respect to $\mathbb{P}$, such that

$$\mathcal{A} = \left\{ X \in L^\infty(\Omega, \mathcal{F}, \mathbb{P}) : \inf_{\mathbb{Q} \in \mathcal{D}} E^{\mathbb{Q}}[X] \geq 0 \right\}.$$

Carr *et al.* (2001) refer to the set $\mathcal{D}$ as the set of *test measures.* Our base risk neutral measure $\mathbb{Q}^0$ is assumed to be an element of this set and, thus, $\mathcal{A}_\infty \subseteq \mathcal{A} \subseteq \mathcal{A}_0$.

In two-price markets we recognize that full replication is not possible. This means trading parties hold residual risk. Attitudes of market participants to hold this residual risk may vary over time. Explicitly, we expect that the cone of acceptable cash flows may contract in bearish times whereas it may expand in bullish times. Therefore, it is sensible to let the size of the cone depend on (at

[2] For a more elaborate treatment of this topic, refer to Madan and Schoutens (2016a).

least) one parameter, subject to calibration to observed market values, aiming to capture the stress level of the market.

In the Conic Finance framework, the operator that measures the acceptability of taking on a certain risk is called *index of acceptability*, which is a map $\alpha : L^\infty(\Omega, \mathcal{F}, P) \to \mathbb{R}_+$ that satisfies *quasi-concavity*, *monotonicity*, *scale invariance* and the *Fatou property* (Cherny and Madan 2009). We say that a cash flow X is acceptable at level γ if and only if $\alpha(X) \geq \gamma$. This allows us to link the notion of acceptability index to the set of acceptable cash flows in a conic economy.

Let $\mathcal{P}$ denote the set of probability measures on $(\Omega, \mathcal{F})$, absolutely continuous with respect to $\mathbb{P}$. There exists a family of subsets $(\mathcal{D}_\gamma)_{\gamma\in\mathbb{R}_+}$ of $\mathcal{P}$ with $\mathcal{D}_\gamma \subseteq \mathcal{D}_{\gamma'}$ for $\gamma \leq \gamma'$ such that the following equality holds (cf. Cherny and Madan (2009, Th. 1 and Eq. (12))):

$$\mathcal{A}_\gamma = \{X \in L^\infty(\Omega, \mathcal{F}, \mathbb{P}) : \alpha(X) \geq \gamma\} = \left\{X \in L^\infty(\Omega, \mathcal{F}, \mathbb{P}) : \inf_{\mathbb{Q}\in\mathcal{D}_\gamma} \mathbb{E}^{\mathbb{Q}}[X] \geq 0\right\}.^1 \tag{1}$$

Intuitively, $\mathcal{D}_\gamma$ contains the probability measures that represent investors' risk-preferences given a level of market stress γ.

Now, as a result of Kusuoka's theorem (Kusuoka 2001), it can be proved in a similar way as in Cherny and Madan (2009) that, under the two additional assumptions of *law invariance* and *additive comonotonicity*, α can be represented in terms of a parametric family of concave distortion functions $\Psi^\gamma : [0,1] \to [0,1]$, i.e.

$$\alpha(X) = \sup\left\{\gamma \in \mathbb{R}_+ : \int_{\mathbb{R}} y d\Psi^\gamma(F_X(y)) \geq 0\right\}. \tag{2}$$

Expression (2) is particularly useful, since to determine the acceptability of a risk X given some stress level γ and a distortion function $\Psi^\gamma(\cdot)$ we only need information on the distribution function of X. Therefore, this identity leads to an expression for both its bid price $b_\gamma(X)$ and its ask price $a_\gamma(X)$. In our basic constant interest rate example, following Cherny and Madan (2010), for a given acceptability level γ, the following equalities hold:

$$b_\gamma(X) = e^{-rT} \int_{-\infty}^{\infty} y d\Psi^\gamma(F_X(y)), \tag{3}$$

$$a_\gamma(X) = -e^{-rT} \int_{-\infty}^{\infty} y d\Psi^\gamma(F_{-X}(y)). \tag{4}$$

Equations (3) and (4) enable us to connect acceptability parameters to bid and ask spreads. Observe that the ask price of the cash flow X is the negative of the bid price of $-X$, i.e.

$$a_\gamma(X) = -b_\gamma(-X). \tag{5}$$

3. Conic Black's formula

In this section we derive closed-form formulae for no-default bid and ask prices of call and put options on futures, using the general result of Cherny and Madan (2010). Futures options are very

[1] Expression (1) links the span of acceptable trades in the market to risk attitudes, without explicitly specifying the preferences of market participants. Consequently, the framework of Conic Finance is situated between arbitrage pricing theory on the one hand (which requires little information about agents' behavior, but lacks strong implications for the choice between investment opportunities) and expected utility theory on the other hand (which is better able to distinguish the favorability of possible trades, but imposes strong assumptions on investors' preferences).

Table 1. Cherny and Madan (2010) formulae for no-default bid and ask option prices.

	bid	ask
Call	$b_\gamma(\mathcal{C}_T) = e^{-rT}\int_K^\infty (x-K)d\Psi^\gamma(F_{f_T}(x))$	$a_\gamma(\mathcal{C}_T) = -e^{-rT}\int_K^\infty (x-K)d\Psi^\gamma(1-F_{f_T}(x))$
Put	$b_\gamma(\mathcal{P}_T) = -e^{-rT}\int_0^K (K-x)d\Psi^\gamma(1-F_{f_T}(x))$	$a_\gamma(\mathcal{P}_T) = e^{-rT}\int_0^K (K-x)d\Psi^\gamma(F_{f_T}(x))$

common in commodity markets, and certainly more common than options on physical commodities (Geman 2009).

Given $(\Omega, \mathcal{F}, \mathbb{P})$, we adopt the classical assumption of the absence of arbitrage opportunities on mid prices[1], i.e. there exists a martingale measure $\mathbb{Q}^0$ equivalent to $\mathbb{P}$, named the risk-neutral measure (Harrison and Pliska 1981). Following the Black (1976) model, futures mid price dynamics $(f_t)_{t\in[0,T]}$ are described by the stochastic differential equation

$$df_t = \sigma(K,T)f_t dW_t. \tag{6}$$

Here $\sigma(K,T)$ denotes the volatility of the futures contract's log-returns for maturity T and strike K, while $(W_t)_{t\geq 0}$ is a Brownian motion under the risk-neutral measure $\mathbb{Q}^0$. In practice, equation (6) describes a family of models dependent on both the option's maturity and strike, rather than just a single model. Intuitively, the Black formulae that follow thus act as interpolators across maturities and strikes.

Let us denote the cumulative distribution function of the random variable f_T (under $\mathbb{Q}^0$) by $F_{f_T}(x) = \mathbb{Q}^0(f_T \leq x)$. Cherny and Madan (2010) show how to apply the general formulae (3) and (4) to no-default bid and ask option prices for call and put options. Their results are shown in Table 1. These formulae are used to calculate the bid and ask market values of long positions in call and put options. The respective short counterparts can be computed by using expression (5), i.e. $b_\gamma(-\mathcal{C}_T) = -a_\gamma(\mathcal{C}_T)$, $a_\gamma(-\mathcal{C}_T) = -b_\gamma(\mathcal{C}_T)$, $b_\gamma(-\mathcal{P}_T) = -a_\gamma(\mathcal{P}_T)$ and $a_\gamma(-\mathcal{P}_T) = -b_\gamma(\mathcal{P}_T)$.

Conic Black's formula *The no-default bid price of a European call future option with strike K and maturity T is given by*

$$b_\gamma(\mathcal{C}_T) = e^{-rT}(f_0 e^{-\gamma\sigma\sqrt{T}}\Phi(d_1) - K\Phi(d_2)),$$

where

$$d_1 = \frac{\ln\frac{f_0}{K} + \frac{1}{2}\sigma^2 T - \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

Proof: Consider the distribution of f_T under $\mathbb{Q}^0$:

$$F_{f_T}(x) = \mathbb{Q}^0(f_T \leq x) = \Phi\left(\frac{\ln\frac{x}{f_0} + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right),$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal random variable. Recal that for a fixed and positive real value of γ, the Wang transform (Wang 2000), for a given $u \in [0,1]$, is defined as

$$\Psi^\gamma_{\text{WANG}}(u) = \Phi(\Phi^{-1}(u) + \gamma).$$

[1]Although the Conic Finance framework does not necessarily assume that the mid price is equal to the risk-neutral price, we still use the latter as a proxy for the former.

Table 2. Closed-form formulae for theoretical no-default bid and ask futures option prices.

Option	**Price**	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$
$b_\gamma(\mathcal{C}_T)$	$e^{-rT}(f_0 e^{-\gamma\sigma\sqrt{T}}\Phi(d_1) - K\Phi(d_2))$	$(\ln\frac{f_0}{K} + \frac{1}{2}\sigma^2 T - \gamma\sigma\sqrt{T})/\sigma\sqrt{T}$	$d_1 - \sigma\sqrt{T}$
$a_\gamma(\mathcal{C}_T)$	$e^{-rT}(f_0 e^{\gamma\sigma\sqrt{T}}\Phi(d_1) - K\Phi(d_2))$	$(\ln\frac{f_0}{K} + \frac{1}{2}\sigma^2 T + \gamma\sigma\sqrt{T})/\sigma\sqrt{T}$	$d_1 - \sigma\sqrt{T}$
$b_\gamma(\mathcal{P}_T)$	$e^{-rT}(K\Phi(d_2) - f_0 e^{\gamma\sigma\sqrt{T}}\Phi(d_1))$	$(\ln\frac{K}{f_0} - \frac{1}{2}\sigma^2 T - \gamma\sigma\sqrt{T})/\sigma\sqrt{T}$	$d_1 + \sigma\sqrt{T}$
$a_\gamma(\mathcal{P}_T)$	$e^{-rT}(K\Phi(d_2) - f_0 e^{-\gamma\sigma\sqrt{T}}\Phi(d_1))$	$(\ln\frac{K}{f_0} - \frac{1}{2}\sigma^2 T + \gamma\sigma\sqrt{T})/\sigma\sqrt{T}$	$d_1 + \sigma\sqrt{T}$

Therefore,

$$\Psi^\gamma_{\text{WANG}}(F_{f_T}(x)) = \Phi(\Phi^{-1}(F_{f_T}(x)) + \gamma) = \Phi\left(\frac{\ln\frac{x}{f_0} + \frac{1}{2}\sigma^2 T + \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}}\right).$$

Indicating the call option pay-off as $\mathcal{C}_T$, we derive the theoretical no-default bid price for a call future option by evaluating the integral

$$\begin{aligned} b_\gamma(\mathcal{C}_T) &= e^{-rT}\int_0^\infty x d\Psi^\gamma(F_{\mathcal{C}_T}(x)) = e^{-rT}\int_K^\infty (x-K)d\Psi^\gamma(F_{f_T}(x)) \\ &= e^{-rT}\left(\int_K^\infty x d\Psi^\gamma(F_{f_T}(x)) - \int_K^\infty K d\Psi^\gamma(F_{f_T}(x))\right) \end{aligned} \tag{7}$$

in two parts. Neglecting the discount factor temporarily, the first integral in equation (7) is the expectation on $[K, \infty)$ of a lognormally distributed random variable, i.e,

$$\int_K^\infty x d\Phi\left(\frac{\ln\frac{x}{f_0} + \frac{1}{2}\sigma^2 T + \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}}\right) = f_0 e^{-\gamma\sigma\sqrt{T}}\Phi\left(\frac{\ln\frac{f_0}{K} + \frac{1}{2}\sigma^2 T - \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}}\right). \tag{8}$$

On the other hand,

$$\int_K^\infty K d\Phi\left(\frac{\ln\frac{x}{f_0} + \frac{1}{2}\sigma^2 T + \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}}\right) = K\left(1 - \Phi\left(\frac{\ln\frac{K}{f_0} + \frac{1}{2}\sigma^2 T + \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}}\right)\right). \tag{9}$$

Combining equations (8) and (9), and taking into account the discount factor, we arrive at the no-default bid price of a European call future option with strike K and maturity T:

$$b_\gamma(\mathcal{C}_T) = e^{-rT}(f_0 e^{-\gamma\sigma\sqrt{T}}\Phi(d_1) - K\Phi(d_2)),$$

where

$$d_1 = \frac{\ln\frac{f_0}{K} + \frac{1}{2}\sigma^2 T - \gamma\sigma\sqrt{T}}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}. \qquad \square$$

The result for put options is obtained similarly, and bid and ask values of short option positions can be derived by applying expression (5). Table 2 summarizes these results. Observe that the distortion parameter is the key distinction of the formulae in Table 2 from the classical Black (1976) formulae, which can be retrieved by setting $\gamma = 0$.

4. Conic CVA and DVA

In this section, we first briefly define CVA and DVA.[1] Subsequently, a procedure for calculating CVA and DVA in both the classical and Conic Finance environments using Monte Carlo methods is outlined.

4.1. *The concept of CVA and DVA*

As Zhu and Pykhtin (2007) note, for many years financial institutions have valued their derivatives portfolios neglecting counterparty credit risk. Their values were obtained by taking the expectation of the discounted cash flows under the risk-neutral measure and referred to as risk-free. Since the fall of Lehman Brothers, interest for taking counterparty credit quality, i.e. CVA and DVA, into account quickly rose among financial markets participants, not least by enacted requirements of their regulators to do so.

Brigo *et al.* (2013) define CVA for a (portfolio of) transaction(s) as the difference between its value when traded with default-free counterparties and the value of the same position(s) when traded with real (and thus defaultable) counterparties. DVA, on the other hand, is the adjustment to the portfolio value made by taking into account one's own default probability. In the classical framework it is, thus, the contraposition of CVA: in a bilateral trade, one's DVA is its counterparty's CVA, and vice versa.

DVA has been subject to debate, as it potentially allows a financial institution to record gains when its credit quality deteriorates (however, see Ruiz (2015) for a clear argument in favor of including DVA). The Basel Committee on Banking Supervision has imposed that banks "recognize in the calculation of Common Equity Tier 1, all unrealized gains and losses that have resulted from changes in the fair value of liabilities that are due to changes in the bank's own credit risk". This to avoid that an increase in a bank's credit risk leads to a reduction in the value of its liabilities, and thereby an increase in its common equity.[1] This counterintuitive profitability resulting from credit deterioration is mitigated in a Conic Finance framework (Eberlein *et al.* 2009, Madan and Schoutens 2016b), as we will also show. This makes this approach also interesting from a regulatory perspective.

Assuming independence between exposure and counterparty credit risk[2], Zhu and Pykhtin (2007) define unilateral CVA by

$$CVA = (1 - R) \int_0^T EPE_t dPD_t. \tag{10}$$

Here, EPE_t is the discounted[3] *expected positive exposure* with respect to the pricing measure $\mathbb{Q}^0$ at time t, PD_t is the probability that the counterparty defaults before time t and R is the recovery rate taken constant for simplicity.

The expected positive exposure at time t is defined as the expected value of the amount owed by the defaulting party to the non-defaulting party conditional on a default at time $\tau = t$. This value depends on the fair value (at time 0) of the discounted cash-flows that we are to receive or pay from

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[1]There are numerous references that contain a far more elaborate treatment of the topic; see, e.g., Brigo *et al.* (2013).
[1]Refer to Paragraph 75 of the Basel III regulations and the consultative document on this topic below:

> BCBS, Basel III: A global regulatory framework for more resilient banks and banking systems reference. Technical report, BIS, 2010;
>
> BCBS, Application of own credit risk adjustments to derivatives. Technical report, BIS, 2012.

[2]Note that for a more realistic CVA and DVA calculation, the dependency between exposure and credit quality, i.e. *right* and *wrong way risk*, should be considered as well.
[3]For simplicity, interest rates will be assumed to be deterministic functions of time.

time t until the maximum maturity T. Since we deal with future cash-flows that evolve randomly over time, we need a valuation model to determine theoretical intermediate portfolio values and a pricing measure, which is the risk-neutral measure in the classical, and a distorted measure in the Conic Finance framework. Moreover, we should take into account possible *netting* of different traded products, i.e. that transactions with negative value can be offset against those with positive value in case of default. Obviously, netting agreements bound expected positive exposures from above by their non-netted counterparts and can, therefore, mitigate counterparty credit risk.[4]

Following a common discretization method (cf. *Bucketed approximated CVA* in Brigo *et al.* (2013)) we approximate equation (10) by

$$CVA = (1-R)\sum_{n=1}^{N} EPE_{n\Delta t}(PD_{n\Delta t} - PD_{(n-1)\Delta t}), \tag{11}$$

where N denotes the number of sub-intervals in which $[0,T]$ is divided and $\Delta t = \frac{T}{N}$. Note that the calculation of (unilateral) DVA is in fact—*mutatis mutandis*—equivalent to equation (10), with the only difference being the perspective of the trading party that calculates it. Succinctly, DVA is defined as per equation (10) with the only difference that expected negative exposures are considered instead of their positive counterparts and, further, one's own default probabilities and recovery rate should be used as inputs. A discretization scheme for DVA can then be defined in a similar way as in equation (11).

4.2. *CVA and DVA in the Conic Finance framework*

Conic Finance has been applied to several areas in finance, such as measuring liquidity for exotic options (Guillaume and Schoutens (2015)) or the valuation of the currently popular contingent convertible bonds ((Madan and Schoutens 2011)). The only attempt so far to relate the concept of Conic Finance to valuation adjustments is outlined in Madan and Schoutens (2016b). More precisely, they provide some intuition on how to compute CVA and DVA for a zero-coupon bond under Conic Finance settings. However, a full computational framework for Conic CVA and DVA that can be used to calculate valuation adjustments on derivative portfolios, is currently lacking. Therefore, inspired by Madan and Schoutens (2016b)'s insight, we propose such an approach for computing (exposure-based) CVA and DVA for option portfolios.

Madan and Schoutens (2016b) recognize that prudent accounting standards should demand trading parties to book their assets against bid prices and liabilities against ask prices. This statement relies on the intuition that, given a contract, these are the values to be received or paid to close out the position, respectively. Therefore, the derivatives in the portfolio for which CVA and/or DVA are calculated should be valued at their bid or their ask price, depending on whether their values are positive or negative. Thus, in our approach we compute exposure profiles starting from theoretical bid and ask values for assets and liabilities, respectively.

Compared to Madan and Schoutens (2016b), our approach differs in that we do not distort survival probabilities. They implicitly argue that for the products they consider (i.e. defaultable bonds), the default probability of the counterparty is determinant for the bid-ask spread. However, we use bid and ask prices of exchange-traded products as inputs. Due to central clearing of such products, no counterparty credit risk is assumed to be reflected in these prices. Therefore, we distort the pricing measure of the underlying instead of the default probabilities.

In our analysis we consider (portfolios of) call and put options. Such products have positive value to the holder by definition. Therefore, we value any long (short) position in an option at its

[4] Collateral agreements as credit mitigants are not considered in this article.

theoretical bid (ask) price. In addition, in netted portfolios that consist of long and short positions, our valuation depends on the signs of the separate product values, and not on the sign of the overall portfolio.[1]

Thus, in the Conic Finance framework, we can use the results from Section 3 to simulate future exposures for a specific trade consisting of call and/or put options using a Monte Carlo procedure. Our approach, comparable to the so-called *direct jump to simulation* (DJS), described in Zhu and Pykhtin (2007), consists of the following steps:

1. Recall that $F_{f_t}(\cdot)$ denotes the cumulative distribution function of f_t. We use this cumulative distribution function, given the (risk-neutral) arbitrage-free volatility parameter $\sigma(K,T)$ (described below), to sample the futures mid price distribution for every time point $n\Delta t$ in our discretization (cf. Equation (11));
2. Given the sample $\{f^1_{n\Delta t}, ..., f^M_{n\Delta t}\}$ of M underlying values at time $n\Delta t$, we estimate the expected exposure of an option's simulated bid (ask) prices by averaging over the bid (ask) values that result from these sampled underlying values. For a European option, the value of such a particular exposure realization is the expected option pay-off at time T, conditional on the underlying value $f^m_{n\Delta t}$, discounted back to time $n\Delta t$, under the relevant pricing measure. This pricing measure is the risk-neutral measure in classical Monte Carlo simulation and a distorted measure in the Conic Finance framework;
3. Note that the exposure realized from a long position in an option $E^{bid,m}_{n\Delta t}$ is exactly the theoretical no-default bid price of the option at time t, for which we have derived the closed-form formulae given in Table 2. Similarly, the exposure realized from a short position $E^{ask,m}_{n\Delta t}$, which is negative by definition, is computed using expression (5). Thus, we compute the samples of exposures from bid and ask option values $\{E^{bid,1}_{n\Delta t}, ..., E^{bid,M}_{n\Delta t}\}$ and $\{E^{ask,1}_{n\Delta t}, ..., E^{ask,M}_{n\Delta t}\}$ by applying the relevant formulae to our sample of underlying values $\{f^1_{n\Delta t}, ..., f^M_{n\Delta t}\}$. If applicable, we sum the exposures of products that are in a netting set to find a net exposure sample;
4. Then, we average over these samples to obtain expected bid and ask exposures for every time point $n\Delta t$ in our discretized grid of $[0,T]$. A plot of such expected exposures for a single European call option are given in Figure 1;
5. Finally, to calculate CVA (DVA) figures, we use the expected positive (negative) exposures based on theoretical bid and ask values.

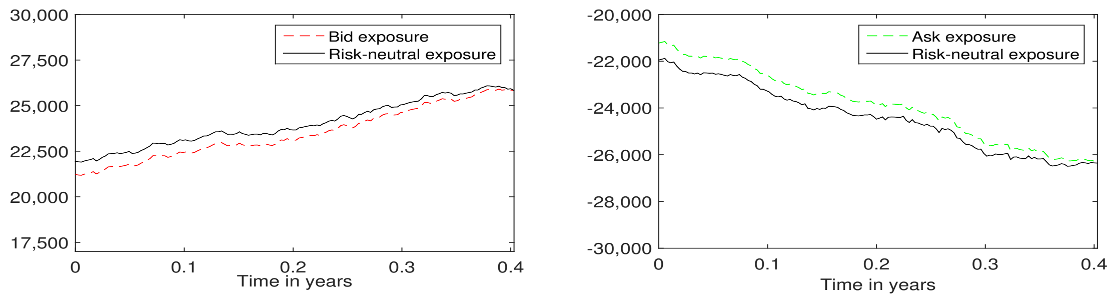
Using the inputs from the procedure described above, we can compare the resulting Conic CVA and DVA figures with the classical ones obtained using the standard risk-neutral valuation approach.

5. Applying the Conic CVA/DVA framework

Now we describe the step-by-step procedure for computing CVA and DVA under conic assumptions. To do so, we assume over-the-counter (OTC) derivative contracts are traded between two representative financial institutions on a particular date, i.e. July 1, 2016. Our reasoning is from the viewpoint of the financial institution to which we will refer as the bank (B), so that we will

[1]The reason for this is that we are interested in calculating CVA and DVA from a risk perspective (compared to, e.g., an accounting perspective). Therefore, this is a conservative approach, since the bid (ask) price of a positive-valued (negative-valued) portfolio is always lower (higher, thus less negative) than the sum of the bid (ask) prices of the separate parts (Zhou 2011). Moreover, our approach is consistent with the BIS reporting standards below, which demand that any option portfolio is reported at gross basis.

BCBS, Guidelines for reporting the BIS international banking statistics - Reporting conventions that apply to both the locational and consolidated banking statistics. Technical report, BIS, 2013.



(a) Long call option

(b) Short call option

Figure 1. Plots of expected risk-neutral, ask and bid exposures that result from trading a single European call option on Natural Gas (Henry Hub) futures.

report valuation adjustments as seen from B's perspective, and refer to the other institution as the counterparty (C). We describe the products that we consider, the data used for our analysis, the procedure for building an arbitrage-free (risk-neutral) volatility surface and the calibration method for the distortion parameter γ.

5.1. *Considered products*

We consider European call and put futures options. Next to long and short positions in single European calls and puts, we analyze the popular *collar* and *butterfly call spread* option strategies, also called *packages* (cf. Hull (2012)).

A collar is an option strategy that consists of a long position in an out-of-the-money put option and a short position in an out-of-the-money call option with the same maturities (cf. Hull (2012), who calls this strategy a *range forward*). A butterfly call spread is an option strategy that consists of long positions in call options with strikes K_1 and K_3, respectively, and of a short position in two call options with strike K_2, where $K_1 < K_2 < K_3$ and $K_2 - K_1 = K_3 - K_2$. These option strategies are of interest here, as they cause two-sided exposures (and, therefore, two-sided credit risk), i.e. both trading parties can suffer from credit loss in the case its counterparty defaults. In such cases, we could face non-zero CVA and DVA at the same time.

The options in the considered packages are assumed to be traded simultaneously with one particular counterparty. Moreover, we assume the options in a package to be in a netting portfolio, so that we can offset the negative exposures from the short positions with the positive exposures from the long positions. However, since we are interested in calculating CVA and DVA from a risk perspective, we make the (implicit) assumption that each package (or portfolio) would be traded component-per-component in case we want to close out the position.[1] That is, we value its constituents separately by considering bid (ask) prices for the long (short) positions that form the portfolio. This is a conservative approach (cf. the footnote on page 9), as opposed to taking a bid or ask price of the portfolio as a whole.[2]

Assume B and C enter an OTC contract to trade the above products on a particular date. For our numerical study, our underlying assets are Natural Gas (Henry Hub) Futures (this choice results from weighing up the availability of European option prices data against the size of bid-ask spreads to obtain meaningful distortion parameter estimates). The options have a maximum maturity of 147 days.[3] Our goal is to compute (unilateral) CVA and DVA for B and C in our

[1]The case where we want to close out the position is the angle from which we perform exposure computations.
[2]For those interested in bid and ask pricing of structured products, we refer to Zhou (2011). Here, within a conic economy, the differences in valuing a structured product as a whole instead of as the sum of its individual components are assessed.
[3]This is the maximum maturity for which Bloomberg quotes bid and ask prices for call or put options on this underlying asset. Bid and ask quotes are necessary for the calibration of the distortion parameter γ.

stylized framework.

5.2. *Data*

We illustrate our approach on European call and put option prices (end-of-day mid quotes), obtained on July 1, 2016, for maturities 07-26-2016, 08-26-2016, 09-27-2016, 10-26-2016 and 11-25-2016. Strikes range from \$1.55 to \$6.45. We use out-of-the-money options and at-the-money calls in our calibration routine (as is done by Guillaume and Schoutens (2015)). So we use call option quotes for moneyness greater than or equal to one (where moneyness is defined as K/F), and put option quotes for moneyness less than one. We calibrate the volatility surface using only call prices as inputs (and outputs), and we compute in-the-money call option prices from the out-of-the-money put option prices using the put-call parity

$$\mathcal{C}(K,T) = \mathcal{P}(K,T) + e^{-rT}(f_T - K). \tag{12}$$

We delete all data points for which the resulting call price is negative. Hence, for a given maturity T_j, the option prices sample is given by

$$\{\mathcal{C}_{mkt}(K_1,T_j),\ldots,\mathcal{C}_{mkt}(K_{N(j)},T_j),\mathcal{C}_{\mathcal{P}}(K_{N(j)+1},T_j),\ldots,\mathcal{C}_{\mathcal{P}}(K_{N(j)+M(j)},T_j)\},$$

where $\{\mathcal{C}_{mkt}(K_i,T_j), 1 \le i \le N(j)\}$ are $N(j)$ not-in-the-money call prices (directly retrieved from the market) and $\{\mathcal{C}_{\mathcal{P}}(K_{N(j)+i},T_j), 1 \le i \le M(j)\}$ are the $M(j)$ call prices implied from out-of-the-money put prices. In total, we obtain 350 data points; for each we have a bid, an ask and a mid price. We will use this sample to calibrate both the volatility $\sigma(K,T)$- and the distortion parameter $\gamma(K,T)$-surfaces.

We refer to the sample call values simply as $\mathcal{C}(i,j)$, where $j \in \{1,\ldots,5\}$ and $i(= i(j)) \in \{1,\ldots,N(j)+M(j)\}$. We will also assume, unless stated otherwise, that the indices i and j will always range within the aforementioned sets. Further, the subscripts mkt and $\mathcal{P}$ will be omitted for the sake of readability.

The assets underlying the options are Natural Gas Futures (although the option itself is financially settled), with deliveries in August, September, October, November and December.[1] Their prices are retrieved from Bloomberg. We obtain the risk-free discount curve using a standard methodology as described, e.g., in Bianchetti (2008). Then, the bootstrapped zero rates are linearly interpolated to obtain their values at any date from July 1, 2016 until November 25, 2016. Furthermore, we obtain mid CDS spreads and related estimated recovery rates for the calculation of default probabilities of the trading parties B and C, by assuming they represent typical large and publicly traded financial institutions.

5.3. *Building the volatility surface*

We employ the smoothing algorithm of Fengler (2009), adjusted for future options, to construct an arbitrage-free implied volatility surface. This algorithm adjusts option call prices in such a way that they adhere to the following classic no-arbitrage conditions, first derived by Merton (1973):

a) The call price function is decreasing in strike. The condition $-e^{-r(T-t)} \le \frac{\partial \mathcal{C}_t(K,T)}{\partial K} \le 0$ is sufficient for this to hold. This restriction avoids the so called *call spread arbitrage*;

[1] To have a complete overview of the contract specifications of both the underlying and the option, refer to `http://www.cmegroup.com/trading/energy/natural-gas/natural-gas_contract_specifications.html` and `http://www.cmegroup.com/trading/energy/natural-gas/natural-gas_contractSpecs_options.html#optionProductId=1352`, respectively.

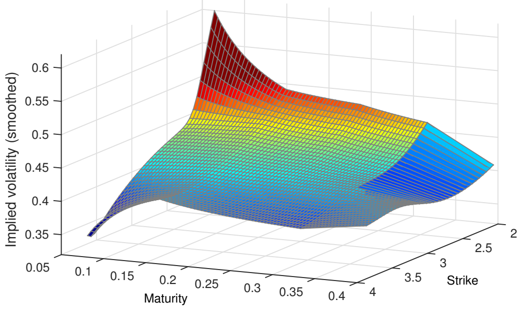


Figure 2. Volatility surface obtained for European Natural Gas futures option prices on July 1, 2016, by the Fengler (2009) algorithm.

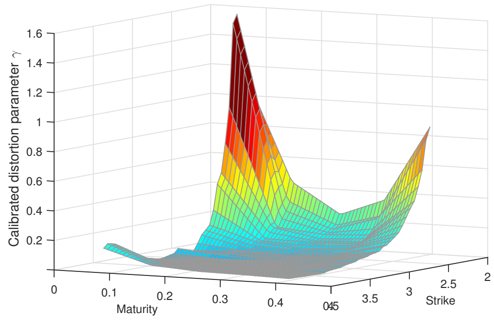


Figure 3. $\gamma(K,T)$-surface calibrated to size-weighted average bid and ask prices of European Natural Gas future options on July 1, 2016.

b) The call price function is a convex function in strike. This restriction is commonly referred to as the avoidance of *butterfly spread arbitrage*. This can be imposed by requiring that its second derivative with respect to strike is non-negative, i.e. $\frac{\partial^2 \mathcal{C}_t(K,T)}{\partial K^2} \geq 0$;
c) The total variance function $v^2(K,T) = \sigma^2(K,T)T$ is a strictly increasing function in T. This condition avoids the so called *calendar spread arbitrage*;
d) The call price at time t is bounded from below by its intrinsic value and from above by its discounted underlying value, i.e. $\max\{e^{-r(T-t)}(f_t - K), 0\} \leq \mathcal{C}_t(K,T) \leq e^{-r(T-t)} f_t$.

Carr *et al.* (2001) show that these four conditions are sufficient to rule out all static arbitrage possibilities.

The approach of Fengler (2009) is to fit, for a given maturity, a natural smoothing cubic spline to the call price function, subject to the linear inequality constraints derived from a) $\div$ d) above. By doing so, one obtains an implied volatility surface that satisfies the above no-arbitrage conditions, but does not necessarily match the original market implied volatilities. However, we use the implied volatilities as inputs for future exposure simulations, and not to exactly reprice existing market quotes. Moreover, we do not use directly observable bid and ask prices but argue that given our purpose of CVA and DVA calculations, we should instead consider bid and ask prices constructed in a way described in Section 5.4 below. Therefore, we consider the Fengler (2009) algorithm an appropriate and computationally cheap method to construct an implied volatility surface that is arbitrage-free for each fixed maturity T_j on mid prices.

Given the market quotes $\mathcal{C}(1,j), \ldots, \mathcal{C}(N(j)+M(j),j)$ as inputs, the Fengler (2009) algorithm now provides us with arbitrage-free (i.e. smoothed) call prices $\widetilde{\mathcal{C}}(1,j), \ldots, \widetilde{\mathcal{C}}(N(j)+M(j),j)$.[1] From these, arbitrage-free volatilities across moneyness levels $\widetilde{\sigma}(K_1,j), \ldots, \widetilde{\sigma}(K_{N(j)+M(j)},j)$ are implied using the Black (1976) model for every j. For each K_i, these volatilities are then interpolated in the maturity direction using the standard linear-in-variance method, i.e. to find an arbitrage-free volatility for a certain T one calculates

$$\widetilde{\sigma}(K_i,T) = \begin{cases} \widetilde{\sigma}(K_i,T_1), & T < T_1, \\ \sqrt{\widetilde{\sigma}^2(K_i,T_j)T_j + \frac{\widetilde{\sigma}^2(K_i,T_{j+1})T_{j+1} - \widetilde{\sigma}^2(K_i,T_j)T_j}{T_{j+1}-T_j}(T-T_j)}, & T_j \leq T < T_{j+1}, j \in \{1,\ldots,4\}, \\ \widetilde{\sigma}(K_i,T_5), & T \geq T_5. \end{cases}$$

The resulting volatility surface is displayed in Figure 2.

[1]We use a smoothing parameter of 0.0004 in the natural cubic splines, where the smoothing spline is defined as in Green and Silverman (1994). This value was obtained by a subjective approach (Green and Silverman 1994), inspecting the fit of the volatility surface for smoothing parameters increasing from 0.

5.4. *Calibrating the distortion parameter*

In the previous section we described how call option mid prices are used to construct an arbitrage-free volatility surface. From these arbitrage-free mid prices, we now artificially construct bid and ask prices by applying average bid-ask spreads to the mid quotes. Specifically, for every option $\mathcal{C}(i,j)$ we consider all the bid $b_q(i,j)$ and ask quotes $a_q(i,j)$, observed on July 1, 2016, and we weight them with bid and ask sizes $s_q^b(i,j)$ and $s_q^a(i,j)$ respectively.[2] So the daily size-weighted bid, ask prices and bid-ask spreads for the option $\mathcal{C}(i,j)$ are given by:

$$wb(i,j) = \frac{\sum_q b_q(i,j) \cdot s_q^b(i,j)}{\sum_q s_q^b(i,j)},$$
$$wa(i,j) = \frac{\sum_q a_q(i,j) \cdot s_q^b(i,j)}{\sum_q s_q^a(i,j)},$$

and

$$wbas(i,j) = \frac{wa(i,j) - wb(i,j)}{(wa(i,j) + wb(i,j))/2}. \tag{13}$$

Here, we select only those data points for which at least one bid and one ask ask price are quoted. The reason for taking weighted average spreads, as done in Ekinci (2008), is to obtain a sensible size of the bid-ask spread throughout the day of interest. We argue that such an average spread is more suitable to be used as an input for the expectation of the future bid-ask spreads in daily simulated exposures than the spread observed for one particular quote, at a particular time, linked to a particular trade size.

Now we compute proxies for end-of-day bid quotes $\tilde{b}(i,j)$ and ask quotes $\tilde{a}(i,j)$ by adjusting the (smoothed) arbitrage-free mid prices $\widetilde{\mathcal{C}}(i,j)$ in such a way that their bid-ask spreads equal to (13), i.e. we define

$$\tilde{b}(i,j) = \widetilde{\mathcal{C}}(i,j)(1 - \frac{1}{2}wbas(i,j)) \tag{14}$$

and

$$\tilde{a}(i,j) = \widetilde{\mathcal{C}}(i,j)(1 + \frac{1}{2}wbas(i,j)). \tag{15}$$

Thus, we construct (smoothed) bid and ask prices that reflect observed size-averaged bid-ask spreads.

To calculate bid and ask option prices, we need the appropriate value of the stress level γ. So we reverse-engineer this question and calibrate γ for every strike and maturity to the (size-averaged) bid and ask quotes in (14) and (15). Specifically, we construct a $\gamma(i,j)$-surface for observed strikes K_i's and maturities T_j's by minimizing the mean relative absolute error, i.e. we define

$$\gamma(i,j) = \arg\min_{\gamma} \left(\left| \frac{\tilde{b}(i,j) - b_\gamma(\widetilde{\mathcal{C}}(i,j))}{\tilde{b}(i,j)} \right| + \left| \frac{\tilde{a}(i,j) - a_\gamma(\widetilde{\mathcal{C}}(i,j))}{\tilde{a}(i,j)} \right| \right),$$

where $b_\gamma(\widetilde{\mathcal{C}}(i,j))$ and $a_\gamma(\widetilde{\mathcal{C}}(i,j))$ are the theoretical (γ-dependent) bid and ask option prices, respectively, computed with strike, maturity and implied volatility associated with quote $\widetilde{\mathcal{C}}(i,j)$. Note

[2]The index q is a function of i and j. However, for readability, this index is denoted as q only, instead of as $q(i,j)$.

that the model-dependence of the distortion parameter γ is similar to that of the implied volatility σ. The $\gamma(K,T)$-parameters can be seen as the *implied liquidity* measures, defined in Corcuera *et al.* (2012) (however, they only report an implied liquidity for a fixed maturity and at-the-money options, while we present here the full $\gamma(K,T)$-surface).

The resulting $\gamma(i,j)$'s are interpolated linearly in both the strike and maturity direction. This is simple to implement, but it also guarantees that the interpolated values always lie between the observed ones (this is not the case, for instance, if natural cubic splines are used). The resulting $\gamma(K,T)$-surface is displayed in Figure 3.

This plot shows that the distortion parameter is the highest for short maturities and low strikes, although for longer maturities (and low strikes) it is also relatively high. For higher strikes, the distortion parameter is low across all considered maturities. This is perhaps a first indication of the lower liquidity in low strike (OTM) call options which is reflected in the distortion parameter.

6. Numerical study

In this section we investigate the quantitative impact Conic Finance has on valuation adjustments, for representative OTC option transactions between counterparties B and C. All results are obtained by Monte Carlo simulations (we use 12,000 runs), which results in relative standard errors below 1.5%.[1] We report all results, unless stated otherwise, as seen from B's perspective. Given short time horizons of the considered options, the default probabilities - and, hence, absolute CVA and DVA values - are small. Therefore, most results are presented in relative terms.

First, we evaluate the impact on B's CVA in case of a long position in a single option. Next, we show the similarities between the impacts on CVA and DVA for a single option position. Then we investigate the Conic Finance' mitigating effect on DVA benefits from credit quality deterioration. We proceed by examining portfolios of long and short option positions, which generate both non-zero positive and negative exposures. Finally, we inspect the sensitivity of our results with respect to calibrated parameters ,and the relationship between the calibrated distortion parameter and the observed bid-ask spreads.

6.1. *CVA of a single long option position*

We start by evaluating the relative impact of Conic Finance valuation on B's CVA that results from buying a call option from C. We compute B's CVA, using various strikes and maturities as inputs.[2] Further, we define *averaged expected positive exposure* (AEPE) as

$$AEPE = \frac{1}{T}\int_0^T EPE_t dt,$$

and *averaged expected negative exposure* (AENE) similarly (cf. Brigo *et al.* (2013)).

[1]Recalling equation (11), if the standard error associated with $EPE_{n\Delta t}$ is denoted with $\epsilon_{n\Delta t}$, we define the standard error associated with CVA calculations as

$$CVA^{err} = (1-R)\sum_{n=1}^{N} \epsilon_{n\Delta t}(PD_{n\Delta t} - PD_{(n-1)\Delta t}).$$

The relative CVA standard error is then defined by $\frac{CVA^{err}}{CVA}$. For DVA calculations, these definitions have been extended in an obvious way.

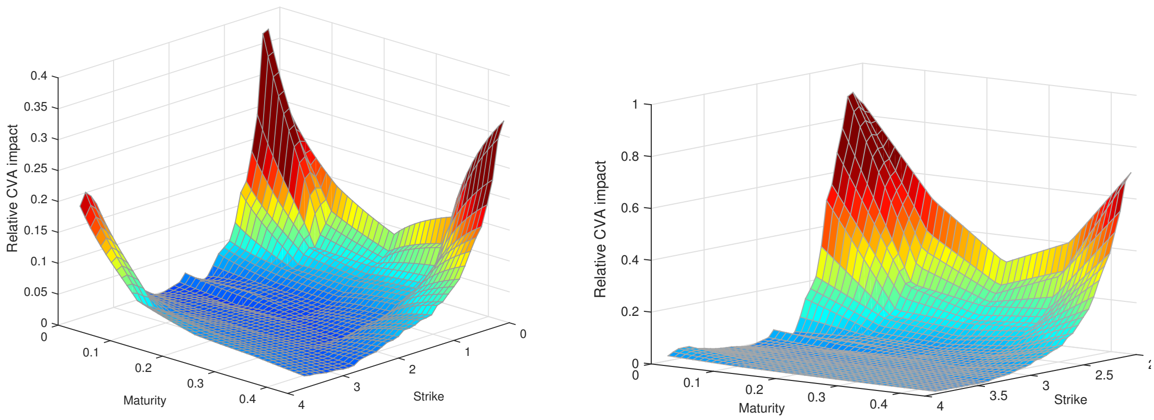
[2]The relative CVA impact is defined as

$$\frac{CVA^{Con} - CVA^{RN}}{CVA^{RN}},$$

where CVA^{RN} (CVA^{Con}) is the CVA under risk-neutral (conic) market assumptions. The relative DVA impact is defined similarly.

Table 3. Relative impact of conic valuation (as compared to risk-neutral valuation) on B's CVA resulting from the purchase of a call option from C. Everywhere it is assumed that B buys, on July 1, 2016, a European call option (with various maturities and moneyness) on a Natural Gas futures. Initial contract values and average expected positive exposures (AEPE) are denominated in USD.

Maturity date	**Moneyness (K/f_0)**	**t_0 contract value (risk neutral)**	**AEPE (risk-neutral)**	**AEPE (conic)**	**Relative impact on CVA**
07-26-2016	0.7	8,711.13	8,789.17	5,472.95	-37.84%
07-26-2016	1.0	1,191.42	1,189.36	1,177.87	-0.97%
07-26-2016	1.3	12.90	12.26	9.58	-21.90%
08-26-2016	0.7	8,437.22	8,739.33	7,508.02	-14.18%
08-26-2016	1.0	1,846.14	1,916.00	1,907.87	-0.43%
08-26-2016	1.3	118.09	124.98	115.14	-7.92%
09-27-2016	0.7	8,301.24	8,782.80	8,091.28	-7.95%
09-27-2016	1.0	2,226.43	2,359.96	2,331.35	-1.22%
09-27-2016	1.3	244.67	261.33	248.33	-5.02%
10-26-2016	0.7	8,428.63	9,050.08	8,270.02	-8.73%
10-26-2016	1.0	2,558.13	2,735.53	2,703.77	-1.18%
10-26-2016	1.3	384.07	409.57	392.09	-4.32%
11-25-2016	0.7	8,699.39	9,549.86	8,250.55	-13.82%
11-25-2016	1.0	2,949.63	3,226.15	3,186.63	-1.25%
11-25-2016	1.3	640.70	695.08	665.94	-4.26%



(a) Long call option

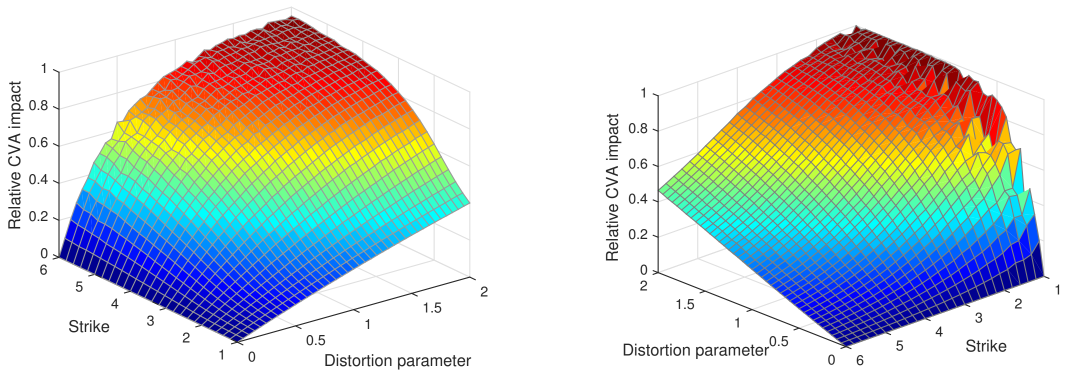
(b) Long put option

Figure 4. Relative impact (in absolute value) of conic valuation, as compared to risk-neutral valuation, on B's CVA. B has a single European option position on Natural Gas futures.

Table 3 reports the relative CVA impact and averaged expected positive exposures, based on risk-neutral (mid) prices and on conic (bid) prices, for different maturities and moneyness. Volatilities and distortion parameters, calibrated as described in Sections 5.3 and 5.4, are used as inputs for valuing the risk-neutral and conic CVAs.

As we can see from Table 3, relative impact on CVA for call options is small for close to at-the-money options (where bid-ask spreads are small), but this changes fast when moving away from the at-the-money point (where spreads increase as well). This becomes even more evident in Figure 4(a), which displays a surface of absolute CVA impacts over different strikes and maturities (we plot there absolute values of CVA impact). Moreover, we observe that the shape of this relative impact surface is rather similar to that of the $\gamma(K,T)$-surface in Figure 3.

Now we consider a long position in a European put option. Figure 4(b) illustrates the relative impact on CVA for such a single put option position (note that here all the impacts are negative; absolute values are shown for plotting convenience). The shape of this surface again follows that of



(a) Long call option (b) Long put option

Figure 5. The sensitivity of the relative CVA impact of conic valuation (compared to risk-neutral valuation) to the distortion parameter γ and strike K. Relative impact is determined for B's CVA resulting from the purchase of a single call option that matures in 147 days. The underlying asset is the Natural Gas December 2016 futures. We assume a fixed volatility of 40%.

the calibrated $\gamma(K,T)$-surface. However, the maximum impact on CVA is much higher in absolute value for put than for call options. Moreover, comparing Figures 4(a) and 4(b) to Figure 3, we see the differences in the CVA impact for different combinations of strikes and $\gamma(K,T)$, i.e. the relative impact is high (in absolute value) for higher strikes for calls but not for puts.

To delve into these dissimilarities further, we assess the relative CVA impact over a range of strikes and values of γ, for a fixed maturity and volatility. In particular, we consider cases where B buys a single European call or put futures option from C with delivery in December 2016. This option matures in 147 days. Instead of using our the calibrated values for both $\sigma(K,T)$ and $\gamma(K,T)$, we use a fixed volatility of 40% over a grid of possible strike and consider the CVA impact for a range of γ values. Figure 5 shows the resulting CVA impacts.

These surfaces show that the relative CVA impact depends on strikes and γ simultaneously. As we can see, for call options the relative impact is largest (in absolute value) for high strikes, i.e. where the options are out-of-the-money and, consequently, their prices are low. For put options, this is the case for low strikes, i.e. again for out-of-the-money options. This relates to our observation on Figures 4(a) and 4(b). The influence of high $\gamma(K,T)$ values at low strikes on put prices is much higher than on call prices. Furthermore, higher $\gamma(K,T)$ values for short maturities and high strikes influence the CVA impact for calls more than for puts.

6.2. *DVA on short option positions*

We now assess the impact of Conic Finance on DVA for short option positions. It turns out that the results are the same as the impact on CVA for the long option positions considered above. This can be explained by the following observations:

- In our conic CVA and DVA computations, only the exposures are impacted by bid-ask spreads;
- If a single option is traded, the expected positive exposure of the buyer of the option is equal to the expected negative exposure of the seller;
- This is also true when bid and ask prices are taken into account. This because of the property (5) and the fact that the exposure resulting from a long position in an option is positive by definition, while negative for a short position;
- Therefore, if we consider a short position in a single call or put option, both the initial value and the risk-neutral averaged expected negative exposure are exactly the negative of the initial value and the risk-neutral averaged expected positive exposure for the option buyer.

Consequently, it is only the default probability of B, compared to that of C, that differentiates the absolute value of DVA from that of CVA, as seen from B's perspective. However, since we consider the relative impact of conic valuation on CVA and DVA, from our discretization method it follows that these are equal up to a reasonable number of digits.

To see this, let the superscripts Con or RN indicate that the relevant quantity is computed under the conic or risk-neutral market assumptions, respectively. Since the shortest tenor for which CDS spreads are quoted exceeds the maximum maturity of the products we consider, we only have one hazard rate for B and one for C that impact valuation adjustments. Let λ denote that hazard rate for B. It follows that (recall that λ, Δt and the option maturity are small):

$$\begin{aligned} PD_{n\Delta t} - PD_{(n-1)\Delta t} &= (1 - e^{-n\Delta t \lambda_{n\Delta t}}) - (1 - e^{-(n-1)\Delta t \lambda_{n\Delta t}}) \\ &= e^{-(n-1)\Delta t \lambda_{n\Delta t}} - e^{-n\Delta t \lambda_{n\Delta t}} \approx \Delta t \lambda_{n\Delta t}. \end{aligned}$$

Therefore,

$$\begin{aligned} \text{Relative Impact}(CVA) &= \frac{(1-R)\sum_{n=1}^{N}(EPE^{Con}_{n\Delta t} - EPE^{RN}_{n\Delta t})(PD_{n\Delta t} - PD_{(n-1)\Delta t})}{(1-R)\sum_{n=1}^{N} EPE^{RN}_{n\Delta t}(PD_{n\Delta t} - PD_{(n-1)\Delta t})} \\ &\approx \frac{\sum_{n=1}^{N}(EPE^{Con}_{n\Delta t} - EPE^{RN}_{n\Delta t})\Delta t \lambda_{n\Delta t}}{\sum_{n=1}^{N} EPE^{RN}_{n\Delta t}\Delta t \lambda_{n\Delta t}} = \sum_{n=1}^{N} \frac{-ENE^{Con}_{n\Delta t} - (-ENE^{RN}_{n\Delta t})}{-ENE^{RN}_{n\Delta t}} \\ &= \text{Relative Impact}(DVA). \end{aligned}$$

Hence, we observe the same impact of conic valuation on DVA for short positions as on CVA for long positions when single-option portfolios are considered. Therefore, we do not explicitly state results for DVA here.

6.3. *Sensitivity of DVA to one's own default probability*

As discussed in Section 4, a potential benefit of the Conic Finance framework, from a regulatory perspective, is that the DVA profitability, resulting from deteriorating own credit quality, is mitigated. Therefore, it is interesting to compare DVA's sensitivity to credit quality, for conic and risk-neutral valuation. For this, we present credit deltas for DVA in Table 4.

We consider an option (either call or put) with strike K and maturity T and the vector $\boldsymbol{S} = (S(t_1), \ldots, S(t_\tau))$ of τ CDS spreads, associated with the tenors $t_1, \ldots, t_\tau$ used for calculating the survival probabilities. This collection of tenors consists of the $\tau - 1$ tenors quoted before the option's maturity T and the infimum of $\{t_i \geq T\}$ (provided that this range of tenors exists). Then, for each $p \in (1, \ldots, \tau - 1)$, the p-th bucketed credit delta is defined by

$$\Delta^p_{DVA}(K,T) = DVA(K,T,\boldsymbol{S} + 1bp \cdot \boldsymbol{e}_p) - DVA(K,T,\boldsymbol{S}), \tag{16}$$

where $\boldsymbol{e}_p$ denotes the basis vector containing a 1 in the p-th coordinate and 0's elsewhere.

The p-th credit delta is, thus, the sensitivity of DVA to a single basis point (bp) increase in the p-th CDS spread of the party of interest. The shortest tenor for which CDS spreads are quoted exceeds the maximum maturity of the considered options. So we only have one hazard rate per trading counterparty that impacts valuation adjustments and, therefore, just one relevant bucketed credit delta. This credit delta is used to evaluate the sensitivity of the DVA to changes in credit quality.[1]

[1]The relative impact on the bucketed credit delta is defined following the conventions introduced in Section 6.1.

Table 4. Comparison of credit delta's of B's DVA resulting from the sale of a call option to C. We assume that B sells a European call option, with maturity and moneyness corresponding to the reported values, on Natural Gas futures.

Maturity date	**Moneyness (K/f_0)**	**Δ_{DVA}^{credit} of B (risk-neutral)**	**Δ_{DVA}^{credit} of B (conic)**	**Relative impact on Δ_{DVA}^{credit}**
07-26-2016	0.7	-0.06493	-0.04036	-37.84%
07-26-2016	1.0	-0.00879	-0.00870	-0.97%
07-26-2016	1.3	-0.00009	-0.00007	-21.90%
08-26-2016	0.7	-0.13922	-0.11948	-14.18%
08-26-2016	1.0	-0.03052	-0.03039	-0.43%
08-26-2016	1.3	-0.00199	-0.00183	-7.92%
09-27-2016	0.7	-0.21478	-0.19771	-7.95%
09-27-2016	1.0	-0.05772	-0.05701	-1.22%
09-27-2016	1.3	-0.00639	-0.00607	-5.02%
10-26-2016	0.7	-0.28888	-0.26367	-8.73%
10-26-2016	1.0	-0.08733	-0.08630	-1.18%
10-26-2016	1.3	-0.01308	-0.01251	-4.32%
11-25-2016	0.7	-0.37619	-0.32421	-13.82%
11-25-2016	1.0	-0.12710	-0.12552	-1.24%
11-25-2016	1.3	-0.02739	-0.02623	-4.26%

In Table 4 we report credit delta values for different maturities and moneyness. We observe a negative sensitivity of DVA to changes in credit quality in both classical and conic valuations. This is exactly the DVA gains caused by a decreasing own credit quality. However, we see that this sensitivity is lower (in absolute terms) for the conic case, confirming that this effect is mitigated in a Conic Finance framework.

6.4. *Option portfolios and the impact of netting*

So far, our analysis has been limited to transactions that involve a single option. In these cases, only one of the parties faces counterparty credit risk. Therefore, if B is long such an option, its DVA will be zero (and CVA is zero for a short option position). In this section, we evaluate the impact of the Conic Finance framework on unilateral CVA and DVA for option strategies that potentially induce two-sided counterparty credit risk.

Our first strategy of interest is the collar (a long position in an out-of-the-money put option and a short position in an out-of-the-money call option with the same maturities). Table 5 reports the impact on B's valuation adjustments if B buys a collar from C. We consider different maturities and moneyness for the call position in the strategy. The moneyness of the put position is adjusted accordingly, to have a contract that is entered with zero initial cost. The constituent products are assumed to be in a netting portfolio. The unilateral CVA and DVA impacts show a similar pattern as we observed for single call option trades, i.e. the impact increases as we move away from the at-the-money level.

Table 7 shows the results of a similar analysis, if B purchases a butterfly call spread from C, with the options that compose this strategy in a netting set. The same pattern as for the collar is observed; however, the relative impact on the valuations adjustments is even higher for this strategy, as more options are involved. Moreover, we observe that, in contrast to the cases analyzed above, DVA is in most cases positively impacted here, which is consistent with the lower averaged expected negative exposures for ask than for risk-neutral prices. This happens due to the exposure netting of option's positions: long positions in this portfolio are away from the at-the-money-point (where the distortion parameter is high), so that they are considerably impacted by the Conic Finance framework. On the other hand, the short positions are initially at-the-money, where the distortion parameter, and therefore the conic valuation impact, are low. The offsetting of exposures, that are each impacted by different values of the distortion parameter, therefore cause the observed DVA

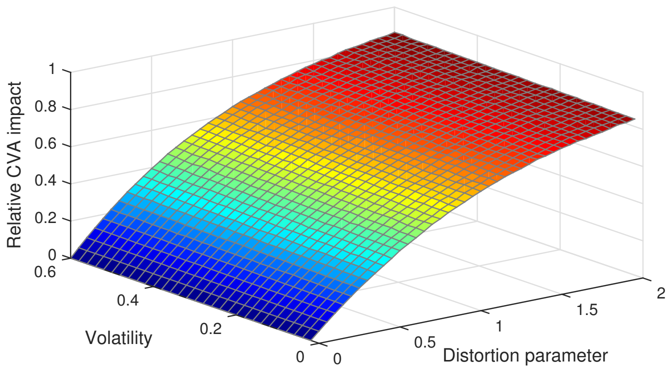


Figure 6. The sensitivity of the relative CVA impact of conic valuation (compared to risk-neutral valuation) to the distortion parameter γ and the volatility σ, for a long call futures option position maturing in 147 days.

impact.

However,the above observations do not contradict our general statement that—*ceteris paribus*—conic DVA is lower in absolute terms than its classic counterpart, as also can be seen from Tables 6 and 8, where the CVA and DVA impact results of the non-netted portfolios are shown. In this case, CVA and DVA are again always negatively impacted by computing them under conic market assumptions. Note that the absence of netting agreements, as expected, leads to higher averaged expected exposures than those found in Tables 5 and 7.

6.5. *Sensitivity of the CVA impact to calibrated parameters*

Our results depend on calibrated values of the volatility and the distortion parameter. So it is interesting to see how sensitive CVA is with respect to changes in these parameters. This can give us an idea of what the impact could be in scenarios where volatility and market stress level are different from those observed on our calibration date.

Consider agian the case of a single call option, i.e. B buys an at-the-money European call futures option from C, which matures in 147 days. Instead of using the calibrated values for $\sigma(K,T)$ and $\gamma(K,T)$, we perform the calculations for a grid of possible values of these parameters, where σ and γ range in $[0, 0.7]$ and $[0, 2]$ respectively. The result of this analysis is depicted in Figure 6.

As can be seen in Figure 6, the relative impact of CVA in a Conic Finance framework is sensitive to the value of the distortion parameter. This suggests that in situations where market stress rises to considerable levels, CVA may be highly impacted in conic framework. The impact of the volatility is much smaller. This implies that the distortion parameter must be re-calibrated as bid-ask spreads change over time.

6.6. *Explanatory power of γ for the bid-ask spread*

We have shown that the Conic Finance framework has a considerable impact on valuation adjustments. The most important parameter there is the distortion parameter $\gamma(K,T)$, which has a natural relationship to liquidity. So we assess the explanatory power of the distortion parameter $\gamma(K,T)$ for the observed daily size-averaged bid-ask spread. We perform an analysis by means of cross-validation and make a comparison to some commonly used proxies for liquidity, such as trading volume and open interest.

Out of the set of available quotes on the grid of strikes and maturities $\{(i,j)\}$, we use half (the training set) to recalibrate γ-values. Now we find γ-values for the other half of the available quotes

Table 5. The relative impact of conic valuation (compared to risk-neutral valuation) on CVA and DVA of B resulting from the purchase of a collar from C. It is assumed that B buys an out-of-the-money put futures option (with moneyness $\frac{K_1}{f_0}$) and sells an out-of-the-money call futures option (with moneyness $\frac{K_2}{f_0}$), with the same maturity. The options are in a netting portfolio. The strike K_1 is set so that the initial contract risk-neutral value is zero. The averaged expected positive and negative exposures (AEPE/AENE) are denominated in USD.

Maturity date	K_1/f_0	K_2/f_0	**AEPE (risk-neutral)**	**AEPE (conic)**	**AENE (risk-neutral)**	**AENE (conic)**	**Relative impact on CVA**	**Relative impact on DVA**
07-26-2016	0.900	1.100	184.95	181.01	-175.26	-171.13	-2.14%	-2.37%
07-26-2016	0.803	1.200	44.33	39.12	-42.00	-37.08	-11.78%	-11.75%
07-26-2016	0.714	1.300	10.56	8.68	-9.69	-7.79	-17.81%	-19.63%
08-26-2016	0.907	1.100	466.17	462.29	-474.75	-469.01	-0.83%	-1.22%
08-26-2016	0.822	1.200	207.66	201.95	-215.83	-209.54	-2.77%	-2.93%
08-26-2016	0.746	1.300	87.12	81.92	-92.77	-86.54	-5.99%	-6.75%
09-27-2016	0.911	1.100	661.85	656.29	-685.47	-676.99	-0.85%	-1.25%
09-27-2016	0.826	1.200	349.23	342.74	-366.83	-358.52	-1.87%	-2.29%
09-27-2016	0.746	1.300	172.98	167.14	-184.64	-176.83	-3.40%	-4.27%
10-26-2016	0.912	1.100	836.92	829.51	-831.34	-820.23	-0.89%	-1.36%
10-26-2016	0.833	1.200	492.66	484.41	-491.81	-480.49	-1.69%	-2.33%
10-26-2016	0.764	1.300	279.34	271.64	-280.24	-270.25	-2.78%	-3.61%
11-25-2016	0.926	1.100	1,081.72	1,074.43	-1,064.81	-1,053.17	-0.68%	-1.11%
11-25-2016	0.861	1.200	714.76	705.55	-700.49	-686.29	-1.30%	-2.06%
11-25-2016	0.805	1.300	465.87	454.89	-453.76	-437.77	-2.38%	-3.58%

Table 6. The relative impact of conic valuation (compared to risk-neutral valuation) on CVA and DVA of B resulting from the purchase of a non-netted long-short portfolio from C. The portfolio is the non-netted counterpart of a collar.

Maturity date	K_1/f_0	K_2/f_0	**AEPE (risk-neutral)**	**AEPE (conic)**	**AENE (risk-neutral)**	**AENE (conic)**	**Relative impact on CVA**	**Relative impact on DVA**
07-26-2016	0.900	1.100	288.67	280.26	-278.98	-270.38	-2.92%	-3.09%
07-26-2016	0.803	1.200	59.46	51.21	-57.13	-49.17	-13.92%	-13.99%
07-26-2016	0.714	1.300	13.13	10.48	-12.26	-9.58	-20.29%	-21.90%
08-26-2016	0.907	1.100	806.20	795.35	-814.78	-802.07	-1.35%	-1.57%
08-26-2016	0.822	1.200	311.37	300.23	-319.54	-307.82	-3.60%	-3.69%
08-26-2016	0.746	1.300	119.33	110.53	-124.98	-115.14	-7.42%	-7.92%
09-27-2016	0.911	1.100	1,188.27	1,171.58	-1,211.88	-1,192.28	-1.42%	-1.63%
09-27-2016	0.826	1.200	551.14	536.85	-568.73	-552.63	-2.62%	-2.86%
09-27-2016	0.746	1.300	249.67	238.64	-261.33	-248.33	-4.46%	-5.02%
10-26-2016	0.912	1.100	1,502.73	1,480.15	-1,497.15	-1,470.86	-1.52%	-1.78%
10-26-2016	0.833	1.200	786.03	766.63	-785.19	-762.70	-2.50%	-2.90%
10-26-2016	0.764	1.300	408.66	393.48	-409.57	-392.09	-3.76%	-4.32%
11-25-2016	0.926	1.100	1,972.55	1,948.96	-1,955.65	-1,927.70	-1.22%	-1.45%
11-25-2016	0.861	1.200	1,174.59	1,150.60	-1,160.33	-1,131.34	-2.08%	-2.54%
11-25-2016	0.805	1.300	707.19	683.06	-695.08	-665.94	-3.47%	-4.26%

Table 7. The relative impact of conic valuation (compared to risk-neutral valuation) on CVA and DVA of B resulting from the purchase of a butterfly call spread from C. It is assumed that B buys one in-the-money and one out-of-the-money call futures options with strikes K_1 and K_3, and sells two at-the-money call futures options. These options are assumed to be in a netting portfolio. Initial contract values and averaged expected positive and negative exposures (AEPE/AENE) are denominated in USD.

Maturity date	K_1/f_0	K_3/f_0	**t_0 contract value (risk-neutral)**	**AEPE (risk-neutral)**	**AEPE (conic)**	**AENE (risk-neutral)**	**AENE (conic)**	**Relative impact on CVA**	**Relative impact on DVA**
07-26-2016	0.900	1.100	1,082.64	1,858.24	1,822.17	-813.24	-816.66	-1.95%	0.42%
07-26-2016	0.800	1.200	3,531.11	3,909.88	2,993.59	-385.29	-495.82	-23.53%	28.85%
07-26-2016	0.700	1.300	6,341.18	6,546.32	3,476.57	-176.94	-402.98	-47.05%	128.60%
08-26-2016	0.900	1.100	668.64	2,300.69	2,255.12	-1,577.84	-1,580.25	-2.00%	0.16%
08-26-2016	0.800	1.200	2,487.15	3,659.34	3,417.05	-1,055.00	-1,099.78	-6.69%	4.30%
08-26-2016	0.700	1.300	4,921.34	5,775.12	4,707.68	-650.56	-782.38	-18.64%	20.51%
09-27-2016	0.900	1.100	523.56	2,675.30	2,653.40	-2,135.90	-2,116.64	-0.83%	-0.91%
09-27-2016	0.800	1.200	2,008.01	3,725.21	3,533.62	-1,618.58	-1,654.91	-5.22%	2.30%
09-27-2016	0.700	1.300	4,168.69	5,523.42	4,968.76	-1,132.13	-1,216.21	-10.19%	7.57%
10-26-2016	0.900	1.100	296.45	2,922.70	2,867.88	-2,649.41	-2,646.75	-1.91%	-0.09%
10-26-2016	0.800	1.200	1,639.53	3,839.66	3,652.06	-2,116.73	-2,160.94	-4.99%	2.16%
10-26-2016	0.700	1.300	3,696.44	5,502.96	4,893.16	-1,562.73	-1,686.56	-11.31%	8.15%
11-25-2016	0.900	1.100	187.89	3,380.86	3,284.34	-3,264.56	-3,276.94	-2.93%	0.41%
11-25-2016	0.800	1.200	1,434.58	4,204.03	3,945.02	-2,714.27	-2,789.16	-6.33%	2.88%
11-25-2016	0.700	1.300	3,440.81	5,773.43	4,789.58	-2,073.51	-2,338.66	-17.48%	13.27%

Table 8. The relative impact of conic valuation (compared to risk-neutral valuation) on CVA and DVA of B resulting from the purchase of a non-netted long-short portfolio from C. The portfolio is the non-netted counterpart of a butterfly call spread.

Maturity date	K_1/f_0	K_3/f_0	**t_0 contract value (risk-neutral)**	**AEPE (risk-neutral)**	**AEPE (conic)**	**AENE (risk-neutral)**	**AENE (conic)**	**Relative impact on CVA**	**Relative impact on DVA**
07-26-2016	0.900	1.100	1,082.64	3,477.04	3,414.45	-2,432.04[a]	-2,408.94	-1.81%	-0.95%[a]
07-26-2016	0.800	1.200	3,531.11	5,956.64	4,906.71	-2,432.04	-2,408.94	-17.68%	-0.95%
07-26-2016	0.700	1.300	6,341.18	8,801.43	5,482.53	-2,432.04	-2,408.94	-37.82%	-0.95%
08-26-2016	0.900	1.100	668.64	4,462.59	4,372.74	-3,739.75	-3,697.87	-2.03%	-1.13%
08-26-2016	0.800	1.200	2,487.15	6,344.09	6,015.14	-3,739.75	-3,697.87	-5.22%	-1.13%
08-26-2016	0.700	1.300	4,921.34	8,864.30	7,623.16	-3,739.75	-3,697.87	-14.09%	-1.13%
09-27-2016	0.900	1.100	523.56	5,192.23	5,123.82	-4,652.84	-4,587.06	-1.33%	-1.43%
09-27-2016	0.800	1.200	2,008.01	6,759.47	6,465.78	-4,652.84	-4,587.06	-4.39%	-1.43%
09-27-2016	0.700	1.300	4,168.69	9,044.13	8,339.61	-4,652.84	-4,587.06	-7.87%	-1.43%
10-26-2016	0.900	1.100	296.45	5,792.70	5,676.64	-5,519.41	-5,455.51	-2.03%	-1.17%
10-26-2016	0.800	1.200	1,639.53	7,242.34	6,946.63	-5,519.41	-5,455.51	-4.14%	-1.17%
10-26-2016	0.700	1.300	3,696.44	9,459.64	8,662.11	-5,519.41	-5,455.51	-8.54%	-1.17%
11-25-2016	0.900	1.100	187.89	6,661.31	6,472.96	-6,545.02	-6,465.57	-2.87%	-1.23%
11-25-2016	0.800	1.200	1,434.58	8,034.77	7,621.42	-6,545.02	-6,465.57	-5.23%	-1.23%
11-25-2016	0.700	1.300	3,440.81	10,244.94	8,916.48	-6,545.02	-6,465.57	-13.18%	-1.23%

[a] Note that the short positions are all at-the-money and hence AENE and relative impact on DVA are constant per maturity in this non-netted case.

Table 9. Output of three different regression analyses of the size-averaged bid-ask spread against moneyness, the square of moneyness and I) standard liquidity factors, II) the distortion parameter γ and III) all of the above. T-statistics are provided in brackets under the variable of interest.

	Inter-cept	**Money-ness**	**Money-ness^2**	**Vol_Opt**	**OpenInt-Option**	**Vol_Fut**	**OpenInt-Future**	γ	R^2	*Adj.* R^2
I	1.69*	-3.17*	1.45*	$-2.56 \cdot 10^{-5}$	$-1.75 \cdot 10^{-6}$	$1.35 \cdot 10^{-6}$*	$5.86 \cdot 10^{-7}$*		0.693	0.682
	(16.2)	(-16.6)	(17.9)	(-1.69)	(-0.71)	(2.28)	(2.34)			
II	0.469*	-0.982*	0.584*					0.368*	0.793	0.789
	(3.70)	(-4.60)	(6.77)					(12.3)		
III	0.574*	-1.34*	0.742*	$-1.74 \cdot 10^{-5}$	$9.48 \cdot 10^{-7}$	$5.94 \cdot 10^{-7}$	$5.76 \cdot 10^{-7}$*	0.335*	0.835	0.828
	(4.73)	(-6.46)	(8.84)	(-1.55)	(0.51)	(1.35)	(3.13)	(11.9)		

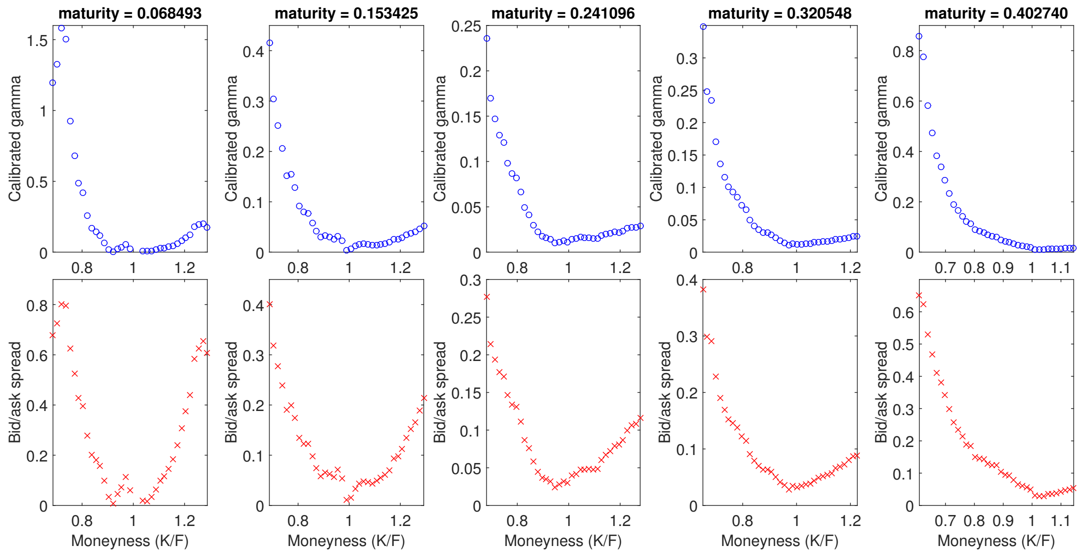


Figure 7. Calibrated gamma values per maturity and size-weighted average bid and ask prices of European futures options.

(the testing set) by linear interpolation, and use this testing set with associated liquidity data to carry out our analysis.

We perform linear least squares regressions of the daily size-averaged bid-ask spread, as defined in Section 5.4, on the liquidity factors for both the option and its underlying assets, as well as on γ, moneyness and—justified by the parabolic shape in Figure 7—the square of moneyness. As we can see in Table 9, in all three regressions the intercept, moneyness, square of moneyness and γ coefficients are highly significant, whereas the volume and open interest of the option are insignificant (at the 1% level). Moreover, we observe a remarkable increase in the (adjusted) R^2 as a result of including γ as a regressor. Also, including γ leads to insignificance of the volume of the underlying future at the 1% level. On the other hand, dropping the other variables does not lead to a convincing decrease in the (adjusted) R^2. This indicates that $\gamma(K,T)$, in combination with moneyness, indeed captures more of the information in the bid-ask spread than the other (liquidity-related) measures.

The results in Table 9 might raise the suspicion of collinearity between moneyness and $\gamma(K,T)$. To check whether this is indeed the case, we inspect the Variance Inflation Factor (VIF) (Kennedy 2008). Given that a regression of $\gamma(K,T)$ on a constant, moneyness and the square of moneyness yields an R^2 of 0.572, we find a VIF of 2.3364, well below levels that could indicate collinearity problems (Kennedy 2008). Thus, our results do not point towards a collinearity between $\gamma(K,T)$

and (the square of) moneyness. We conclude that the two factors together are better able to explain the bid-asks spread than the classic liquidity-related measures.

7. Concluding remarks

In this paper we presented an extensive study of the impact of Conic Finance on CVA and DVA of option positions - single as well as option portfolios. We have shown that the Conic Finance framework has a significant impact on valuation adjustments. Moreover, we demonstrated that, by using Conic Finance valuation, rather than classical risk-neutral valuation, we are able to mitigate the undesirable feature of DVA profitability resulting from one's own credit deterioration. We also developed an implied distortion parameter surface, calibrated to observed bid and ask option prices of various maturities and strikes. Finally, we have shown that the calibrated distortion parameter has a strong connection to liquidity and, together with squared moneyness, it can explain bid-ask spreads better than traditional liquidity measures such as volume and open interest.

To see how our results extrapolate beyond the last maturity for which bid and ask prices are quoted, further research should focus on a term structure of the distortion parameter γ. Also it would be interesting to relate the results of our study to stochastic liquidity models (yet to be developed, as proposed in Corcuera *et al.* (2012)).

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