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# Advances in Degradation Modeling

## Applications to Reliability, Survival Analysis, and Finance

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William Q. Meeker in 2009

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## Preface

William Q. Meeker has made pioneering and phenomenal contributions to the general area of reliability and, in particular, to the topics of degradation and accelerated testing. His research publications and the numerous citations he has received over the past three decades provide an ample testimony to this fact.

Statistical methods have become critical in analyzing reliability and survival data. Highly reliable products have necessitated the development of accelerated testing and degradation models and their analyses. This volume has been put together in order to (i) review some of the recent advances on accelerated testing and degradation, (ii) highlight some new results and discuss their applications, and (iii) suggest possible directions for future research in these topics.

With these specific goals in mind, many authors were invited to write a chapter for this volume. These authors are not only experts in lifetime data analysis, but also form a representative group from former students, colleagues, and other close professional associates of William Meeker. All contributions have been peer reviewed and organized into 26 chapters. For the convenience of readers, the volume has been divided into the following six parts:

- Review, Tutorials, and Perspective
- Shock Models
- Degradation Models
- Reliability Estimation and ALT
- Survival Function Estimation
- Competing Risk and Chaotic Systems

It needs to be emphasized here that this volume is not a proceedings, but a carefully and deliberately planned volume comprising chapters consistent with the editorial goals and purposes mentioned above.

Our thanks go to all the authors who have contributed to this volume. Thanks are also due to Mrs. Debbie Iscoe for the excellent typesetting of the entire volume. Special thanks go to Ms. Regina Gorenshteyn and Mr. Tom Grasso (Editor, Birkhäuser, Boston) for their interest and support for this project.

The volume was difficult, it is clear, but Leah (Project Manager at Integra Software Services), Brian, Tom, and Dubby helped us very much to prepare this nice volume!

## XVIII Preface

Like us, all the authors who contributed to this volume have great admiration for the work and accomplishments of William Meeker and therefore provided us with hearty cooperation during the preparation of this volume. It is a great pleasure and honor for all of us to dedicate this volume to William Meeker.

June 2009

**M.S. Nikulin**  
**Nikolaos Linnios**  
**N. Balakrishnan**  
**Waltraud Kahle**  
**Catherine Huber-Carol**



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## William Q. Meeker – Career and Accomplishments

William Meeker received his B.S. degree in industrial management from Clarkson College of Technology in 1972 and M.S. degree in operations research and Ph.D. degree in administrative and engineering systems, both from Union College (Schenectady, New York) in 1973 and 1975, respectively. Soon after getting his Ph.D., he joined as an assistant professor in the Department of Statistics at Iowa State University, Ames. After getting promoted to the ranks of associate professor and professor in 1978 and 1981, respectively, he was appointed there as a distinguished professor of liberal arts and sciences in 1996, and he has been there in this position since. During this period, he also held visiting positions at Global Research Center of General Electric Company (Schenectady, New York), Quality Theory and Systems Department of Bell Laboratories (Holmdel, New Jersey), Louisiana State University (Baton Rouge, Louisiana), and University of Waterloo (Waterloo, Ontario, Canada). He is also a faculty affiliate at Los Alamos National Laboratory (Nevada) since 1999.

At Iowa State University, William Meeker has made invaluable contributions. Since 1989, he has been a principal investigator at the Center for Nondestructive Evaluation. He excelled in teaching a wide array of courses and in fact received the Iowa State University Teaching Excellence Award in 1989 and 1991 for his efforts. He has guided 71 M.S. projects and 11 Ph.D. dissertations and is supervising six graduate students at present.

William Meeker has provided distinguished service to the statistical community at large by his activities in various capacities for professional societies. These include secretary-treasurer of ASA Business and Economics Section (1981–1982), member of Advisory Board of ASA Section on Physical and Engineering Sciences (1984–1986), member of ASA Committee on Publications (1987–1989), member of ASA Ad Hoc Committee for *Journal of Computational and Graphical Statistics* (1987–1990), president of the Iowa Chapter of ASA (1989–1991), chair of the IMS Committee on Statistical Tables (1990–1994), member of ASQC Publications Management Board (1991–1993), chair of *Technometrics* Management Committee (1991–1993), COPSS visiting lecturer (1991–1995), member of ASA Journals Management Committee (1992–1993), representative of Iowa Chapter to ASA Council of Chapters (1995–1997), member of the ISI Committee for Statistics in Business and Industry (1997–2005), member of NSF SCREMS Proposal Review Panel (1998, 2005), program chair of Spring Research Conference (1999), member of ASA Fellows Committee (2001–2003), member of NSF Statistics Research Proposal Review Panel (2001), member of NSF CAREER Proposal

Review Panel (2002), member of NRC Panel on Operational Test Design and Evaluation of the Interim Armored Vehicle (2001–2003), chair of ASA Task Force on Journals Marketing (2003), member of ASA Task Force on Electronic Publications (2002–2004), member of ASA Publications Committee (1998–2002), chair of ASA Publications Committee (2003–2006), and member of NSF Research Experiences for Undergraduates Proposal Review Panel (2006).

In addition, William Meeker has provided tremendous service to many research journals in various capacities. Included in this list are associate editor of *Technometrics* (1979–1986), editor of *Technometrics* (1987–1989), editorial board member of *Selected Tables in Mathematical Statistics* (1981–1989), co-editor of *Selected Tables in Mathematical Statistics* (1990–1994), editorial board member of *International Statistical Review* (1995–1999), editorial board member of *Lifetime Data Analysis* (2001–2009), advisory editor of *Quality Technology & Quality Management* (2003–2009), and guest editor of *Journal of Statistical Planning and Inference* Special Issue on Accelerated Testing (JSPI, 2009).

William Meeker has received numerous distinctions and awards throughout his career. He has been elected a fellow of the American Statistical Association and the American Society for Quality, and an elected member of the International Statistical Institute. Some other notable awards include ASA Outstanding Statistical Application (2001), Frank Wilcoxon Prize for the best practical application paper in *Technometrics* (1987, 1995, 1999), W.J. Youden Prize for the best expository paper in *Technometrics* (1996, 1998, 1999, 2002), William G. Hunter Award from the Statistics Division of the ASQ (2003), and ASQ Shewhart Medal (2006). Moreover, his book *Statistical Methods for Reliability* (co-authored with L. Escobar), published in 1998 by John Wiley & Sons, received the Professional/Scholarly Publishing Division of the Association of American Publishers Award for Excellence and Innovation in Engineering.

William Meeker, through his pioneering and phenomenal research in the area of reliability over the last 35 years, has influenced deeply the trend of research in this area and has provided guidance, inspiration, and encouragement to numerous young researchers. For his exemplary career and immense contributions, he was chosen “Statistician of the Year” in 2006 by the Chicago Chapter of the ASA. It is our sincere hope and wish that he will continue his contributions to the area of reliability and the statistical profession in general with renewed vigor and energy.

This volume includes a number of chapters on the topics of degradation and accelerated testing written by experts who know and appreciate William Meeker and all his contributions!

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