Independent Study

Simulation Model for Options Gamma Hedging

- Final Paper -

Contents

Abstract

In this paper we develop a simulation model which allows a trader to run a predefined number of simulated options gamma hedging scenarios. We then show how the use of this model can give the trader the ability to trial a variety of hedging methodologies before entering into the trade, in order to inform future hedging decisions. The simulation model will have the functionality to specify, upfront, many of the decisions that traders make on a daily basis when hedging options. For purposes of this paper we restrict our analysis to “plain-vanilla” equity options that can be priced using the Black-Scholes Option Pricing Model.

There are many papers which cover options pricing, options modelling, options greeks, market movements of the underlying asset (trend moves, jump moves, ranges, etc.), relationships between options volatility and stock prices, options skew, and options trading, including options gamma trading. Despite this, there are relatively few simulation models available which allow a trader to run forecasted gamma hedging scenarios. Stated another way, the author has yet to find any open source models which allow a trader to choose upfront how they will gamma hedge an option before entering into the actual trade, run market simulations on these choices, and generate results1. More specifically, two important parameters we add in our simulation model are the ability to roll beyond the first option into multiple consecutive options, and the ability to set stop-loss and target-gain levels on the strategy upfront. For example, it is required that, when rolling into multiple consecutive options upon expiration (or a pre-specified time to expiration) of the current option, that a new option with a prospective at-the-money strike is created in order to properly simulate a medium to long-term gamma hedging strategy, as opposed to a single option trade. Additionally, looking at simulated results of a gamma-hedging strategy without allowing for a stop-loss or target-gain to be applied upfront is not realistic as most options traders are required to have these limits. This, and the results of our simulation model, are what this paper looks to add to the body of research done to date.

[1] As far as public and non-public models go, those that allow for simulations and / or backtesting are primarily for the underlying securities themselves, not for options. There are many backtesters available online, primarily to test stock-based strategies, and far fewer simulation models. The array of available tools for options backtesting and simulation is even further narrowed. The author arrived at this conclusion after trading options for 30 years at large investment banks, but for purposes of this paper, performed further verification by contacting the following companies who provide more complex financial product backtesters: Deltix, SourceForge, Tradestation, Amibroker, Wealthlab, Axioma, Trading Blox, Ninja Trader, QuantyCarlo, alpha-5, Quantiopian, oscreener, Quandl/Optionworks, OptionNet Explorer, OptionMetrics, Hypervolatility, www.getvolatility.com, and LIM’s XIMM. There are larger company’s that, if they have capabilities for pre-simulated gamma hedging, are private. These include Bloomberg, Factset, OneTick, and KDB. Some other backtesters which do not have these capabilities are Zacks Research Wizard, GuruFocus, OldSchool Value, AmiBroker, TradeStation, WealthLab, Equities Lab, FinViz, Profitspi, Portfolio123, Stock Rover, and VectorVest. Henry Crutcher,“A Comparison of Backtesting Tools”, AAII Journal, December 2017

1 Intro duction

The purpose of this research is twofold: to develop a model-based stochastic process for the purpose of both equity price and equity option hedging simulations. The intent is to illustrate how a trader would use the output of these simulations to inform hedging decisions before a trade is done. More specifically, the simulation model will generate simulated stock prices, then based on pre-defined hedging actions, will output the resulting P/L (profit & loss) of these scenarios along with the number of days the strategy is outstanding. We will compare the results of multiple combinations of these pre-defined actions to come to conclusions around which options hedging actions serve which trading purposes best.

It is intended that being able to analyze different hedging scenarios upfront will remove the bulk of "emotion-based" minute-by-minute trading decisions that tend to sabotage long-term returns2, along with allowing the trader to "autopilot" his or her equity options rebalancing so that the bulk of the traders time can be spent searching for new opportunities. The most important underlying assumption of this paper is that, by hedging periodic movements in the stock price, traders who are long options try to generate more in profits than the original cost of purchasing the option3.

Options are financial instruments that allow the holder (buyer) to buy or sell an underlying asset, such as a stock, at a pre-specified "strike" price. They are a derivative product - their value is based on the value of underlying security (e.g. the stock price itself). Unlike futures, the options holder is not required to buy or sell the asset if they choose not to. Call options allow the holder to buy the asset at the strike price within a specific time frame (between today, denoted T0, and the options expiration date). Put options allow the holder to sell the asset at a stated price within a specific time frame. For purposes of this paper, we will value options using the Black-Scholes Model ("the model"), described in more detail in section 1.2 below. We used R [6] in our analysis, and we used the RQuantlib package to calculate the option prices [8].

Hedging is the act of offsetting a financial asset (i.e. equity) position with the opposite position for the purpose of reducing market risk. In terms of derivatives more specifically, the term hedging is generally used to mean offsetting not an entire position but one of the underlying valuation parameters of the derivative itself. For example, as we will see below, an equity option has many parameters that determine its price. One of these parameters is the price of the underlying stock

[2] While not empirically tested, it is widely believed both in the markets and in the field of Behavioral Finance (of which this is one of many topics researched) that swings in emotions generally work to the detriment of a trader, causing him/her to exit long positions after they have fallen, and enter long positions after they have risen.

[3] More technically, a profitable hedger will earn more from trading the changes in option delta (defined in section 1.1) than he/she loses from the "decaying" of the option premium, the result being a net gain from the gamma hedging strategy.

on which the option is based. If this price moves significantly, the delta (defined in section 1.1 below) of the option, which was originally equal to the amount of underlying shares held, will now be different than the amount of underlying shares held. The strategy will no longer be neutral to price changes in the underlying stock - i.e. the trader will now have an open delta position, and will be at risk of losing money if the stock price moves against them. In this case, the trader will have adjust their underlying equity position by buying or selling the stock in an amount that will get the strategy back to a position where there is no exposure to the underlying stock price move. For purposes of this paper the term hedging will mean the act of offsetting the option's risk to movements in only the price of the underlying stock4. Additionally, note that a hedge action can happen at different frequencies as preferred by the trader. In our simulation model, it can be input as any frequency expressed in days (1d, 2d, 7d, 30d, etc.).

1.1 Terms

Following are definitions of options-related terms that are both relevant to our research and widely used in the financial markets trading industry. They can be found in any standard options textbook, for instance see Options Volatility and Pricing, Second Edition, Sheldon Natenberg, 2015 [3].

Delta is the first derivative of the option's price with respect to the underlying stock price. Delta measures the sensitivity of an option's theoretical value to a change in the price of the underlying security [3]. In this paper, it can be thought of as the change of the price of the option given the change in the price of the underlying stock. For example, if the price of the option moves 50 cents for every $1 move in the stock, we would say that the option's delta is 50%, or .5. Note that when hedging options, the phrase delta-neutral will sometimes be used. This means that the delta of the option is fully offset by an equal and opposite position in the underlying stock, resulting in a net position that has zero delta. Said another way, the net position is insensitive to local price movements in the stock.

Gamma is the second derivative of the option's price with respect to the underlying stock price (i.e. the rate of change in the option's delta for every one-point move in the underlying stock price). Gamma hedging refers to the rebalancing of this change in delta in order to get the full strategy back to delta-neutral. More specifically, gamma hedging is a trading strategy that, over the life of

[4] All of the other parameters that go into the pricing of an option will be assumed to remain unhedged over the life of the simulation. There are many reasons a trader may decide to execute a hedge at any point in time. In the authors experience, the most common are: a stock movement large enough to capture a local period's time decay, a pre-specified time frame for each hedge (daily, weekly, monthly, etc.), a desire to keep some specific open delta exposure, a pre-specified profit / loss for the period, or even something as subjective as a "feeling" that the stock is at a high or low for the some local period. This is certainly not an exhaustive list, but covers a subset of drivers behind a traders decision to hedge.

the trade, targets a constant delta in the option and underlying stock combination. This targeted delta is most often (and for the purposes of this paper, always) one that is delta-neutral.

Volatility is a trader's term for standard deviation. Throughout this paper, this term will specifically represent the standard deviation of the percent price changes on the underlying stock. Volatility can be thought of as a measure of the speed and magnitude of market moves. Intuitively, a stock that has large up and down moves (i.e. large positive and/or negative returns) will have a higher volatility than a stock that has small up and down moves. From a practical perspective, if the market for an underlying equity does not move at a sufficient speed and magnitude, options on that contract will have less value because they will have a reduced likelihood of the stock going through the option's strike price [3]. In general, when discussing options, there are two types of volatility. Realized volatility ("RV") is the annualized standard deviation of percent price changes on an underlying contract (i.e. a stock) over some period of time [3]. This can be measured based on the price of the stock at any given interval5. In general, the most common intervals are daily (based on daily closing prices), weekly (based on weekly closing prices), or high/low (based on the highs and lows of the stock price each day in the option period). In this paper our measure of volatility will be based on daily changes in the stock price. Implied Volatility ("IV") represents the market's consensus expectations of the future realized volatility of the stock. The performance a gamma hedging strategy can be approximated as the realized volatility the trader earned over the life of the option, less the implied volatility used in the model to calculate the price the trader paid for the option. If the result of this difference is positive, the trader will have made a profit. If it is negative, they will have incurred a loss.

Vega measures the sensitivity of an option's theoretical value to a change in volatility [3] - generally the implied volatility. It can be thought of as the change in the price of the option for a one-point change (e.g. from 20% to 21%) in the IV used to price the option. In general, the buyer of an option is "long vega" since the theoretical value of options increase as the implied volatility input into the black-scholes model increases6. This is somewhat intuitive - as the implied volatility increases, so does the expected range of potential stock prices, and then so is the probability and size of the potential payouts.

[5] The time interval between stock price measurements must be specified for realized volatility to have meaning. Unfortunately this is too often not the case, and the analyst is left to"assume" a time period based on the context of the strategy. However it is important to note that there can be significant differences between, for example, daily, weekly, and monthly realized volatilities.

[6] We say "in general" because there are certain option types that can exhibit the opposite behavior. Some of these options are commonly known as "digital" options, or "bet" options, that pay a prespecified fixed amount if they are in-the-money, and 0 if they are out-of-the-money. A more detailed discussion of these structured options is beyond the scope of this paper.

Theta, or “time decay”, is the sensitivity of an option’s theoretical value to a change in the amount of time remaining to expiration [3]. Note that theta and gamma are inversely related - as gamma increases (the opportunity to hedge larger changes in delta and hence earn more profit), negative theta increases. Large positive gamma occurs with large negative theta, and small positive gamma occurs with small negative theta (and vice versa).

The forward price of a stock is the price at which a buyer and seller would be willing to exchange shares of the stock at some date in the future. The primary consideration when calculating a forward price is, what are the costs and benefits of buying this stock today versus buying it at some future date. This is calculated as the current price of the stock, plus any earnings generated (e.g. dividends), less any costs incurred (e.g. interest costs on the financing of the stock purchase) between today (represented as T0) and the forward date. At this forward price, a buyer would be indifferent between buying the stock today and holding it (all the while earning dividends versus paying interest on the cash borrowed to purchase the stock), or simply agreeing to buy the stock today at some point in the future. For example, if interest rates are 10% and we buy a share of a 5% dividend paying stock today for $100, the “breakeven” price that we would be willing to sell that stock for in one year would be $105, calculated as the purchase price paid today plus dividends received over the year less one year of interest costs to finance the purchase:

$$\$100 - (\$100 * .05) + (\$100 * .10) = \$105$$

As a result, the buyer of the stock would be indifferent between buying the stock today for $100, or agreeing today to buy the stock in one year for $105. For options pricing, this is an important price. It is considered the arbitrage-free price, and as such, represents the expected price of the stock at the forward date (which for options purposes is the end of the option term)[7].

A stop-loss (“stop”) is used to limit a trader’s loss on a financial position. It is the size of the loss incurred at which point the trader unwinds the trade. It is expressed in either percentage or absolute dollar terms. Similarly, a target-profit (“target”) is the size of the gain a trader realizes on a financial position, at which point he/she unwinds the trade. In this paper, both stop and target are expressed as a percentage of the initial investment.

Moneyness describes the strike price of the option relative to the current price of the underlying stock. An option that is at-the-money (atm) has a strike that is equal to the forward price of the

[7] The term arbitrage-free is used to infer that the trader can theoretically employ hedging to “lock-in” the $105 future price of the stock today. They do this by buying the stock today for $100, executing what is called a dividend swap on the stock at 5% for one year thus fixing the $5 dividend to be received, and agreeing to pay a fixed interest rate of $10% for one-year on the $100 purchase price of the stock. The net cashflow at the end of the one-year period is $100 + $5 - $10, or $105. Dividend swaps are beyond the scope of this paper, but in summary, they are derivative contracts which effectively fix the the buyers dividend at 5%, or some other agree-upon dividend yield.

underlying stock. For example if a stock has a forward price at time option expiration of $50/share, and the strike of the option is $50, then the option is said to be “at-the-money”. Note that there are two other related terms: in-the-money (itm) and out-of-the-money (otm), which are as they sound. An itm call is one where the forward price of the stock is above the strike. In this case, if the option were exercised at T_0 there would be a positive payout to the holder. Likewise, an otm call is one where the forward price of the stock is below the strike and so, if exercised at T_0, would have no payout[8]. For purposes of this paper, we will primarily be discussing call options that are initially set at-the-money.

1.2 Black-Scholes

The Black-Scholes model is a standard model used to calculate the price of an option over time. The derivation of this equation is complex and exceeds the scope of this paper, so we simply provide the equation below.

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}, \tag{1}$$

where

$$d_1 = \frac{\ln\frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}},$$

$$d_2 = d_1 - \sigma\sqrt{t},$$

C=Call option price,
N=CDF of the normal distribution,
S_t=Current stock price at time t,
K=Strike Price,
r=Risk-Free Interest Rate,
t=time-to-maturity,
and σ=implied volatility of the stock.

As a quick intuitive explanation of the above equation, the first part - $N(d_1)S_t$ - can be thought of as the average value of all stock prices above the exercise price, at expiration. The second part - $N(d_2)Ke^{-rt}$ - represents the average amount we will pay at expiration if we own the option. If we subtract the latter from the former - the expected amount we will pay for the stock at expiration, less the expected value of the stock we will receive at expiration - we arrive at the current value of the call, C.

[8] A put option is the opposite - itm puts have forward stock prices below their strikes, and otm puts have forward stock prices above their strikes.

It is also worth discussing specifically what N(d1) and N(d2) are and what they represent. n(x) represents the standard normal distribution, which by definition has a mean of 0 and a standard deviation of 1. One of the most significant aspects of this distribution is that the total area under the curve adds up to 1, meaning that this curve represents 100 percent of all occurrences that form a true normal distribution[3]. For options pricing in particular, we need to know what percent of specified occurrences fall within a specific portion of the standard normal distribution. This is given by N(x), the standard cumulative normal distribution. If x is some number of standard deviations, N(x) returns the probability of getting an occurrence higher than x by calculating the area under the standard normal distribution curve between the values of x and ∞[3]. The Black-Scholes model makes all calculations using the probabilities associated with a normal distribution[3]. In the Black-Scholes framework, however, we assume that the prices on the underlying contract are lognormally distributed, primarily because stock prices cannot go negative and at the same time have an "unlimited" upside. For this reason, we make some adjustments, done within the Black-Scholes equation above, to the value of x so that we can use N(x) to generate probabilities associated with a lognormal distribution[3]. In a standard normal distribution, the mean, median, and mode all fall in the same place, the middle of the distribution. However, in a lognormal distribution, the mean, median, and mode all likely fall at different points. In a lognormal distribution, with its elongated right-tail, the mean will fall to the right of the mode. The higher the standard deviation, the longer the right tail, and consequently, the further to the right we must shift the mean[3]. Although the math behind this adjustment is beyond the scope of this paper, it is equal to $\frac{\sigma^2 t}{2}$, the term seen in the numerator of the equation for d1 above. Given that we now know that N(d1) represents the mean value of all stock above the exercise price, we can go on to explain why N(d2) is equal to N(d1) adjusted by $-\sigma\sqrt{t}$. To calculate the simple probability of the stock price being above the exercise price at expiration, we care about N(d2), the median of the lognormal distribution. In the Black-Scholes equation, we multiply the exercise price by this probability to get the average amount we will pay if we own the option. The median of the lognormal distribution will fall to the left of the mean by an amount equal to $-\sigma\sqrt{t}$, and this is the term that we see above as an adjustment to N(d1) to arrive at N(d2) [9].

We provide visualizations that show the properties of an option's greeks, specifically its delta, gamma, and vega. Note that the option used for these figures, 1a, 1b, and 2 are all representations of the actual option we used for purposes of our testing throughout this paper, a 6-month 100-strike call option on a stock with an underlying price of 100 and a forward price of 100[10].

Figure 1a shows how an option's price reacts to changes in the underlying stock price at 5 different

[9] Options traders will generally refer to N(d2) as the "probability of exercise".

[10] For ease of illustration we assumed a non-dividend paying stock funded at a 0% interest rate so that the spot and the forward price are equal.

levels of IV based on the Black-Scholes model. Note that as the price of the underlying stock increases, the price of the call option increases. Note also that the higher the IV of the option, the more its value at any given stock price. For example, we can see that at an underlying stock price of $90, the option valued at a 25% implied volatility (represented by the orange dotted line) is valued at roughly $3, but if the option were instead valued at a 50% implied volatility (represented by the gray line), it would be worth roughly $9. This "premium" persists regardless of the price of the underlying stock - the call(put) with the higher implied volatility is always worth more than the call(put) with the lower volatility11. The solid black line represents the P/L from holding the underlying stock with the assumption that it was bought at the market price of $100. There are two points worth noting here. First, we can see that options values are convex - they have no upperbound, but have a lower bound equal to zero. Second, returns on the stock are linear. The convexity of the option's value versus the linearity of the underlying stock's value is reflected in the convexity of the option's delta versus the fixed delta of the stock (one).

Figure 1b shows how an option's delta reacts to both IV and the underlying stock price. The option in this example is the same as that in figure 1a. Note that as the price of the underlying stock increases, the delta of the call option increases (the opposite would be true for a put option). Note also that the higher the IV of the option, the lower it's change in delta for given changes in the stock price12. This can be seen most evidently by comparing the green dotted line, which represents an option priced at a 0% IV, versus the blue dotted line, which represents an option priced at 100% IV. In the case of the former, the stock price moving from 99 to 101 results in the delta moving from 0% to 100%. In the case of the latter, the same stock price move has an almost no impact on delta. Intuitively, this can be explained by thinking of the 0 IV case. If the stock price is 99 and the call option strike is 100, at a 0% IV the black-scholes model "interprets" that the option is otm now and has no chance of ever moving from that position - it will never have a payout (since a 0% IV stock will not move from its current level). Since the option cannot change in value, its delta is 0%. The case is similar but opposite in direction for an IV of 0% with a stock price of 101. Here, the option is $1 itm, and the model will assume that there is no chance it will ever be otm. In this case its delta is 100% - it is fully itm now and will remain that way until maturity. If we instead compare this same 99 to 101 stock price change for our 100-strike option, but this time assume the option has a very high IV, as in the case of the blue dotted line representing 100% IV, then both

[11] Although the x-axis is not wide enough to show it, at extremely low and extremely high stock prices (relative to the stike price) the option would have a similar if not equal value at 25% IV or at 50% IV. We say here that the option has "zero vega". This is somewhat intuitive in that, if an option is so far otm regardless of it's IV that its probability of moving back to the strike is negligible, then the model would "infer" that the value of the option is 0 now and will remain so until expiration. Likewise, if an option is so far itm regardless of its IV that its probability of moving back to the strike is negligible, the the model would infer that the option will be worth its intrinsic value - the difference between the stock price and the strike price - now and will remain so until expiration.

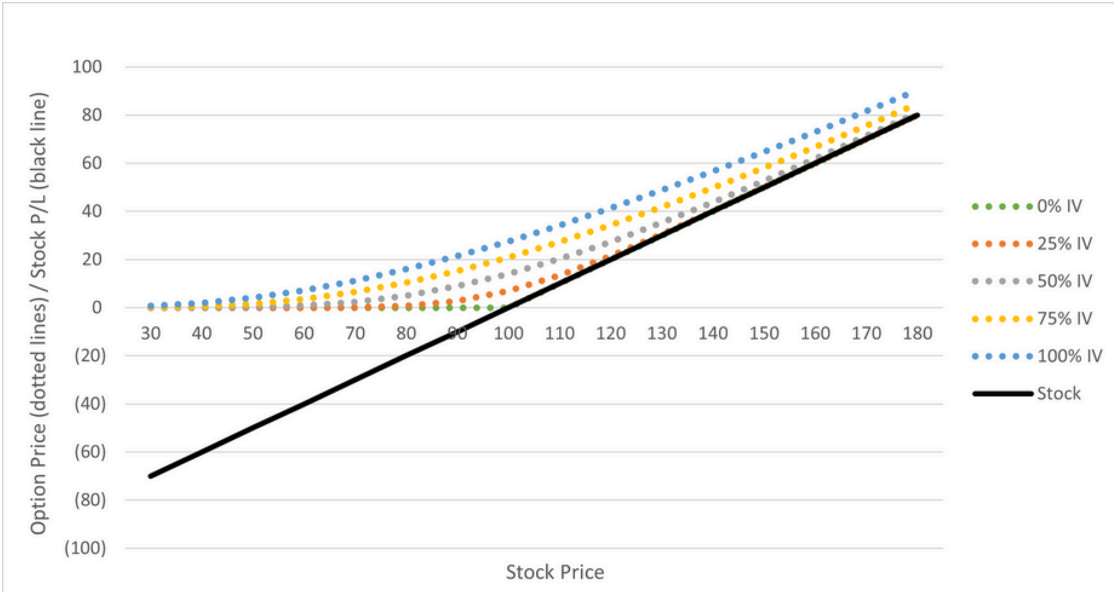
[12] This is the same as saying that at lower IV's an option will have higher gamma.

the in-the-money and the out-of-the-money cases have a much higher probability of the stock price crossing back over the strike price, so the delta will be somewhere around 50% for both the 99 and 101 underlying stock price.13

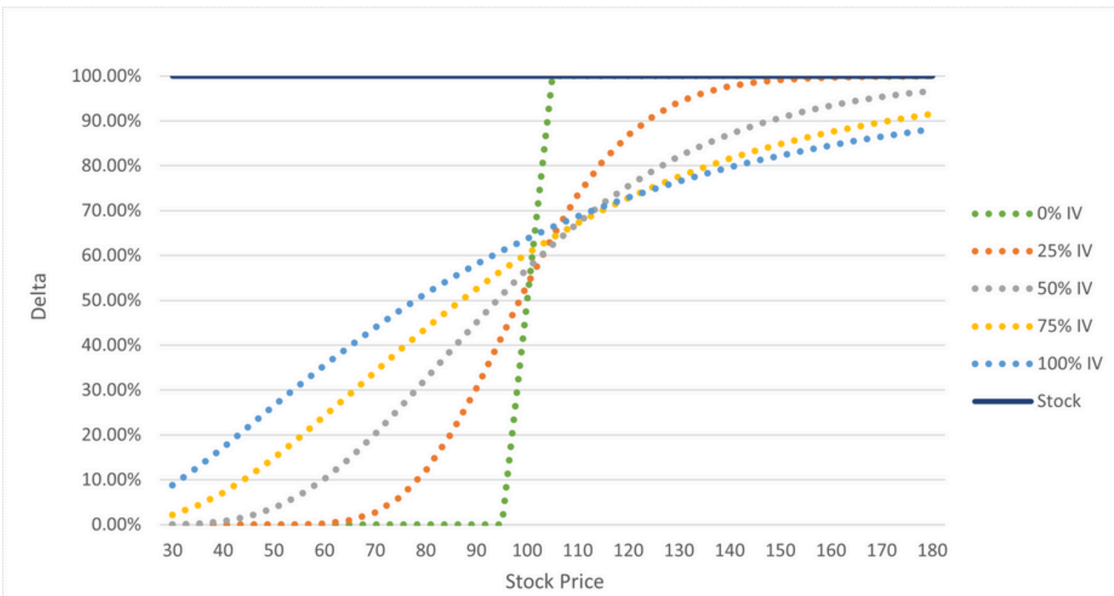
Figure 2 shows how an option's gamma increases over time, as the expiration date of the option gets closer to T0. To calculate the gamma in this graph we use a percentage change in delta for a move in the underlying stock price from 100 to 105. The five individual lines in these graphs, as in the prior figures, each represent five different levels of IV. We can clearly see that in all cases, as we get closer to the expiration date, the rate of change of our delta (i.e. the gamma) increases. Intuitively, if our underlying stock price were to move from 99 to 101 when there were 3-months remaining on its term, the delta would not change much (as we saw earlier), so gamma would be low. If however the price changed from 99 to 101 at one second before expiration, the delta would change from 0 to 1 instantaneously, so the gamma would be very high. The shorter time frame requires larger changes in delta because the probability that there is enough time remaining for an otm option to become itm, or vice versa, lessen with less time remaining on the option. The graph also shows that as volatility increases, the larger changes in gamma occur closer to expiration. Using the same logic as above, at 3-months remaining a stock price move from 99 to 101 will have a significantly lower gamma if that option is valued at 100% IV than if it's valued at 10% IV (as we can see from the gamma of the blue line versus the green line on the far left of the graph). In this example, however, the delta must be 100% at expiration, so the longer the delta remains low (which, for an itm option happens at higher levels of IV), the larger the eventual adjustments required at the expiration of the option.14

[13] In this graph, the 100% IV at-the-money option actually has a delta of 64%. This is due to our assumption of normally distributed stock price returns, which result in right-tailed / lognormal stock price changes, and hence a right-tailed / lognormal distribution of stock prices changes. In other words, the same percentage change in both a high-priced stock and a low-priced stock will result in larger absolute price changes in the former over the latter. These assumptions obviously imply a lower bound of 0 on the underlying stock price, but no upper bound, which is precisely the reason for a delta above 50% when the equity option is technically at-the-money (strike=forward price). Some markets use the opposite terminology, where at-the-money is defined at 50% delta, but for the equity markets, and for purposes of this paper, we consider at-the-money to be defined as an option whose strike price is the same as the forward price of the underlying stock.

[14] Note that, although not shown in the graph and not heavily emphasized in this paper, time decay (theta) increases in an equal and opposite way to gamma. If gamma rises, negative time decay rises. If gamma falls, negative time decay falls. The math behind this is beyond the scope of this paper, but effectively what is happening is the trader is paying more, in terms of theta, for more optionality, in terms of gamma. The larger gamma offers the trader an opportunity for more hedging gains, and the "price" for this opportunity is larger time decay.



(a) Price Scenarios



(b) Delta Scenarios

Figure 1: Equity option price scenarios (top panel), and equity option delta scenarios (bottom panel). Delta is the first derivative of the option's price with respect to the underlying stock, so the bottom panel represents the first derivative of the top panel. In both plots, the x-axis represents the price of the underlying stock, each of the five dotted lines correspond with a different IV for the option - 0%, 25%, 50%, 75%, and 100%, respectively, the solid black line represents a holding of one share of the underlying stock (purchased at $100), and the option itself is a 6-month 100-strike atm call. The y-axis in the top panel represents the price of the option, and the y-axis in the bottom panel represents the delta of the option.

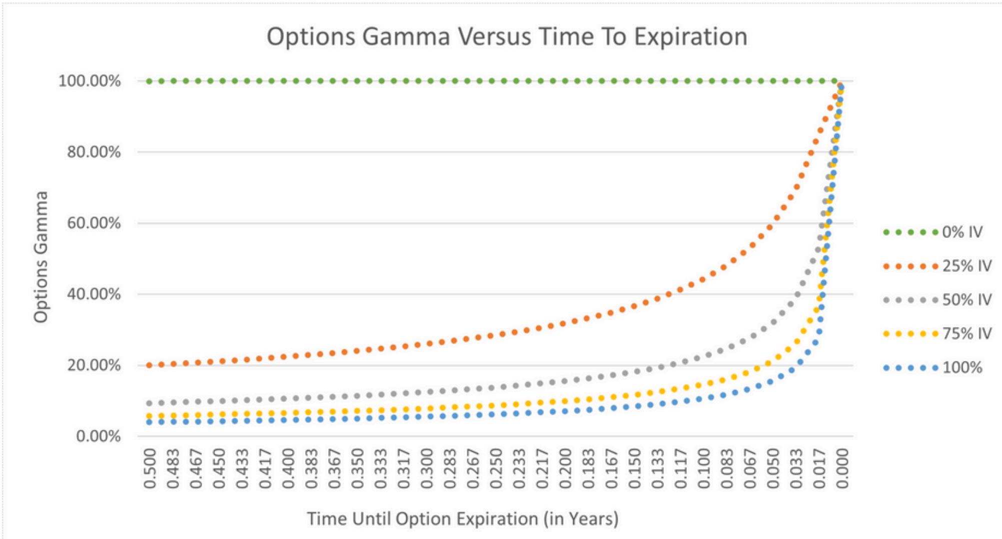


Figure 2: Equity option gamma as the option gets closer to expiry. The y-axis is the percent change in the delta with a forward stock price of 105 versus a forward stock price of 100. Note that for the 0% IV case (green dotted line), the delta is 50% when the forward stock price is 100, and 100% when the forward stock price is 105, so the gamma - represented here as the percentage change in delta, is (1.0/.5) − 1 = 100%.

2 Methodology

We run Monte Carlo simulations of stock prices[15] and evaluate the performance of different gamma hedging scenarios by analyzing the distribution of the P/L. We also look at the distribution of the number of days the strategy was “alive” (or in trader terms, “outstanding”).

One thousand Monte Carlo simulations were run, each comprising four years of stock prices generated daily. Each simulation assumes a normal distribution of stock price returns. Our analysis was conducted in R [6], and simulations were produced using the sde package [7], specifically the Geometric Brownian Motion simulator function “GBM”. In Figure 3 we show the first 100 simulated stock price trajectories. We compare the trajectories assuming a 20% IV (left-panel) and a 50% IV (right panel) on the same scale to show how changes in IV can impact the expectations of future

[15] In the derivatives market, Monte Carlo methods are generally used for “intractable” pricing or valuation issues. It is a numerical method that is useful in many situations when no closed-form solution is available [5]. It can be used to value most types of European (single expiration date) and American (multiple expiration date) options [5]. For example, Monte Carlo simulation are generally used for derivatives where the payoff is dependent on history of the underlying variable, or where there are several underlying variables [4]. When used to value an option, Monte Carlo simulation uses the risk-neutral valuation result. We sample paths to obtain the expected payoff in a risk-neutral world, and then discount this payoff at the risk-free rate [4].

stock prices implied in the Black-Scholes model. We can see that in the case of a 20% IV, the range of ending stock prices is between 0 and 300, whereas with 50% IV, the range is between 0 and 1200. Note that we use a 20% IV for gamma hedge testing in this paper.

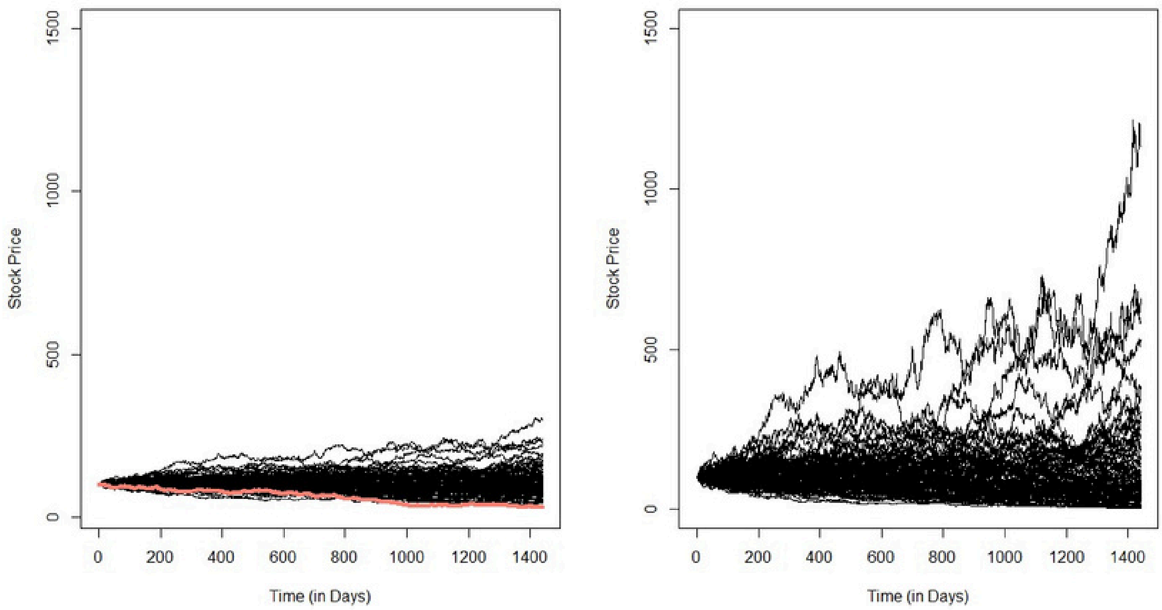


Figure 3: 100 stock price simulations assuming a 20% IV (left panel) and a 50% IV (right panel). The starting price for both is 100. Note the orange simulation in the left panel is simulation 482, which we will discuss in more detail in this section and in section 3 below.

We analyze a total of 15 different gamma hedging scenarios that vary by 1) hedge frequency (daily, weekly, and monthly) and 2) stop/target (25%, 50%, 75%, 100%, and none).

The P/L for each scenario is calculated as follows:

$$ScenarioPL = HedgePL + OptionCosts + OptionsPayouts, \tag{2}$$

where,

$$HedgePL = \sum_{i=1}^{n} \Delta_{i-1} \times (S_i - S_{i-1}) \times nopts_{i-1}, \tag{3}$$

$$OptionCosts = \sum_{j=1}^{p} C_j \times nopts_j, \tag{4}$$

$$OptionsPayouts = \sum_{j=1}^{p} \max(0, S_j - K_j) \times nopts_{j-1}, \tag{5}$$

n = number of days in the scenario,
p = number of option rolls in the scenario,
Δ = option delta,
S = stock price,
C = Black-Scholes call price,
nopts = number of options held.

Simulation 482

In order to show the specific calculation methodology used in our simulation model, we will focus on one specific simulation, number 482 (out of 1000), using the “monthly hedging, 25% stop/target” parameters. This is the only simulation with these parameters that did not hit its stop/target. It can be seen as the single small “bump” at a value of 66 on the x-axis on the upper left panel of figure 6, in between each of the bimodal peaks. We can also see this simulation stand out in the left panel of figure 3 - it is highlighted at the bottom of the plot as the orange line. In table 1, we show a detailed calculation of its P/L. We explain this table in detail in order to show the calculations behind each simulation.

The definitions of each column in the table follow (column number is in parenthesis before the column name):

(1) Month: The number of months from the start date of the scenario.
(2) StockPrice: The underlying stock price for the first day of the month in column (1), as generated by the simulation model.
(3) IV: The implied volatility used to calculate the Black-Scholes option price for the given month.
(4) TimetoExp: The remaining time until expiration, in years, for the current option.
(5) Strike: The strike price for the current option.
(6) d1: Units of standard deviation used to calculate the Black-Scholes delta of the option - N(d1).
(7) d2: Units of standard deviation used to calculate the probability of the option expiring itm - N(d2).
(8) N(d1): The Black-Scholes delta of the option.
(9) N(d2): The implied probability of the option expiring itm.
(10) StN(d1): The average value of all stock prices above the exercise price [3].
(11) Ke−rtN(d2): The average amount the option holder will have to pay on the expiration date of the option [3].
(12) CallPrice: The Black-Scholes value of the call option.
(13) HedgeP/L: The P/L on the underlying stock holdings used to gamma hedge the option.
(14) OptCost: The cost of the new option purchased on the given roll date.
(15) OptPayout: The payout, if any, on the expiration date of the option. The higher of 0 or the (StockPrice - Strike price).

(1) Month	(2) Sto ckPrice	(3) IV	(4) TimetoExp	(5) Strike	(6) d1	(7) d2	(8) N(d1)	(9) N(d2)	(10) St N(d1)	(11) Ke⁻ʳᵗ N(d2)	(12) CallPrice	(13) HedgeP/L	(14) OptCost	(15) OptPayout
0	100	0.	0.5	100	0.0707	-0.0707	0.5282	0.4718	52.8186	47.1814	5.637		-564	
1	95.32	2	0.417	100	-0.3065	-0.4357	0.3796	0.3315	36.183	33.154	3.029	247	0	0
2	90.05	0.	0.333	100	-0.8498	-0.9653	0.1977	0.1672	17.8048	16.7206	1.084	200	0	0
3	92.4	2	0.25	100	-0.7405	-0.8405	0.2295	0.2003	21.206	20.0319	1.174	-46	0	0
4	89.64	0.	0.167	100	-1.2985	-1.3801	0.0971	0.0838	8.7006	8.3773	0.323	63	0	0
5	88.48	2	0.083	100	-2.0914	-2.1492	0.0182	0.0158	1.6142	1.581	0.033	11	0	0
6	91.71	0.	0	100	0	0	0	0	0	0	0	-6	0	0
6	90.94	2	0.5	91	0.0664	-0.075	0.5265	0.4701	47.8789	42.7784	5.1	0	-510	0
7	81.13	0.	0.417	91	-0.8248	-0.9539	0.2047	0.1701	16.6104	15.4759	1.134	517	0	0
8	83.46	2	0.333	91	-0.6916	-0.807	0.2446	0.2098	20.4144	19.0941	1.32	-48	0	0
9	75.92	0.	0.25	91	-1.7618	-1.8618	0.039	0.0313	2.9646	2.8495	0.115	184	0	0
10	79.05	2	0.167	91	-1.6838	-1.7654	0.0461	0.0387	3.6453	3.5261	0.119	-12	0	0
11	81.22	0.	0.083	91	-1.9414	-1.9991	0.0261	0.0228	2.1203	2.0747	0.046	-10	0	0
12	81.86	2	0	91	0	0	0	0	0	0	0	-2	0	0
12	80.01	0.	0.5	80	0.0712	-0.0702	0.5284	0.472	42.2736	37.7609	4.513	0	-451	0
13	78.52	2	0.417	80	-0.0805	-0.2096	0.4679	0.417	36.7394	33.3594	3.38	79	0	0
14	70.87	0.	0.333	80	-0.9918	-1.1072	0.1607	0.1341	11.3859	10.7279	0.658	358	0	0
15	72.27	2	0.25	80	-0.9667	-1.0667	0.1669	0.1431	12.0577	11.4446	0.613	-22	0	0
16	75.91	0.	0.167	80	-0.602	-0.6837	0.2736	0.2471	20.7667	19.7668	1	-61	0	0
17	81.73	2	0.083	80	0.3989	0.3411	0.655	0.6335	53.5322	50.6802	2.852	-159	0	0
18	81.82	0.	0	80	0	0	1	1	81.8185	80	1.818	-6	0	0
18	81.35	2	0.5	81	0.1014	-0.04	0.5404	0.484	43.9617	39.2073	4.754	0	-475	182
19	75.15	0.	0.417	81	-0.5162	-0.6453	0.3029	0.2594	22.7602	21.0093	1.751	335	0	0
20	71.24	2	0.333	81	-1.0545	-1.1699	0.1458	0.121	10.389	9.8022	0.587	118	0	0
21	70.19	0.	0.25	81	-1.3817	-1.4817	0.0835	0.0692	5.8631	5.6056	0.257	15	0	0
22	69.05	2	0.167	81	-1.9135	-1.9951	0.0278	0.023	1.9228	1.8643	0.059	10	0	0
23	63.81	0.	0.083	81	-4.102	-4.1598	0	0	0.0013	0.0013	0	15	0	0
24	65.62	2	0	81	0	0	0	0	0	0	0	0	0	0
24	64.15	0.	0.5	64	0.0875	-0.0539	0.5349	0.4785	34.312	30.6234	3.689	0	-369	0
25	65.12	2	0.417	64	0.1987	0.0696	0.5788	0.5278	37.6878	33.7761	3.912	-52	0	0
26	60.48	0.	0.333	64	-0.4316	-0.547	0.333	0.2922	20.1427	18.6991	1.444	268	0	0
27	55.48	2	0.25	64	-1.3786	-1.4786	0.084	0.0696	4.6604	4.4555	0.205	167	0	0
28	54.94	0.	0.167	64	-1.8297	-1.9113	0.0336	0.028	1.8484	1.7907	0.058	5	0	0
29	51.98	2	0.083	64	-3.5752	-3.6329	0.0002	0.0001	0.0091	0.009	0	10	0	0
30	49.44	0.	0	64	0	0	0	0	0	0	0	0	0	0
30	48.95	2	0.5	49	0.0631	-0.0784	0.5251	0.4688	25.704	22.9697	2.734	0	-273	0
31	46	0.	0.417	49	-0.4256	-0.5547	0.3352	0.2896	15.4184	14.1886	1.23	155	0	0
32	41.5	2	0.333	49	-1.3815	-1.497	0.0836	0.0672	3.4675	3.2927	0.175	151	0	0
33	36.06	0.	0.25	49	-3.0152	-3.1152	0.0013	0.0009	0.0463	0.045	0.001	45	0	0
34	34.16	2	0.167	49	-4.3792	-4.4608	0	0	0.0002	0.0002	0	0	0	0
35	33.06	0.	0.083	49	-6.789	-6.8468	0	0	0	0	0	0	0	0
36	35.24	2	0	49	0	0	0	0	0	0	0	0	0	0
36	35.14	0.	0.5	35	0.0999	-0.0416	0.5398	0.4834	18.9701	16.9198	2.05	0	-205	0
37	35.02	2	0.417	35	0.0682	-0.0609	0.5272	0.4757	18.4595	16.6496	1.81	7	0	0
38	32.73	0.	0.333	35	-0.5225	-0.6379	0.3007	0.2618	9.8418	9.1617	0.68	120	0	0
39	34.84	2	0.25	35	0.0049	-0.0951	0.502	0.4621	17.4895	16.1743	1.315	-63	0	0
40	38.33	0.	0.167	35	1.1539	1.0723	0.8757	0.8582	33.5667	30.0369	3.53	-175	0	0
41	37.24	2	0.083	35	1.1049	1.0471	0.8654	0.8525	32.23	29.8368	2.393	95	0	0
42	37.46	0.	0	35	0	0	1	1	37.4582	35	2.458	-19	0	0
42	37.74	2	0.5	38	0.022	-0.1194	0.5088	0.4525	19.2011	17.1941	2.007	0	-201	246
43	36.63	0.	0.417	38	-0.2189	-0.348	0.4134	0.3639	15.1431	13.8287	1.314	56	0	0
44	35.06	2	0.333	38	-0.6401	-0.7556	0.261	0.2249	9.1519	8.5481	0.604	65	0	0
45	32.71	0.	0.25	38	-1.4478	-1.5478	0.0738	0.0608	2.4156	2.3118	0.104	61	0	0
46	31.37	2	0.167	38	-2.3073	-2.3889	0.0105	0.0084	0.33	0.321	0.009	10	0	0
47	31.16	0.	0.083	38	-3.4058	-3.4635	0.0003	0.0003	0.0103	0.0101	0	0	0	0
48	28.81	2	0.019	38	-9.9145	-9.9424	0	0	0	0	0	0	0	0
Subtotal P/L		0.										2686	-3048	428
Total P/L		2												66
		0.												
		2												

0.
2
0.
2
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2
0.

Table 1: Calculation of gamma hedging simulation number 482 for the “monthly hedging, 25% stop/target” scenario.

To start, we provide some general notes and explanations for table 1. First, there are a total of 49 rows displaying 48 months, ranging from month 0 (the start of the first month) to 49 (the end of the last month) 16. Second, there are a total of 56 rows in the table. The 7 extra rows occur in "roll" months, where the previous option in the simulation ends and the new option is created. Effectively there are two events in these months - the hedge adjustment and the roll. These can be seen at months 6, 12, 18, 24, 30, 36, and 42. Third, and most important, the last 3 columns of the table - HedgeP/L, OptCost, and OptPayout - sum up to the total P/L for the entire simulation, seen in the bottom row of the right-most column of the table (66, meaning this simulation had a net P/L of $66)17. Fourth, Column (1) is meant as a time reference, and is not used anywhere else in the table, and Columns (2) through (5) are simply the details of the option. Last, in our explanations below, we refer to each column either by its name, shown in row 2 of the table, and/or by its column number, shown in row 1 of the table.

The calculated columns in table 1 are columns (6) through (15). The table works from left to right. All of these details feed, in steps explained below, into the two most important columns - the hedge P/L (13) and the options costs (14), the net of which, after adding in any options payouts (15), is the total P/L of the strategy. Specifically, looking at the "Subtotal P/L" row (at the bottom of the table) for columns (13) through (15), we see that: 1) the total hedge p/l for the simulation is $2,686, 2) the total cost of all 7 options purchased (two every year for three years and one in the fourth year) over the life of the simulation is $3,048, and 3) the total payout of the two options that paid were itm at expiration is $428. The total P/L of this simulation is the sum of these three numbers: $2,686 - $3,048 + $428 = $66. The following two paragraphs explain the calculations for column (13) and column (14), respectively.

The hedge P/L shown in column (13) represents the P/L from holding the underlying stock used to hedge of the option's delta. It is calculated as: the previous month's Delta (N(d1) in column 8) times the change in the stock price (column 2) - from the previous month to the current month, times -100, which represents the opposite of the number of options being hedged (since we hold the opposite position in shares to hedge the delta of the call). For example, in month 1, we show a hedge p/l of $247. This is calculated as $.5282 \times (95.32 - 100) \times (-100)$.

The options cost in column 14 is the Black-Scholes model price of the option shown in column (12), multiplied by 100 for the number of options held in the simulation. The important column then is (12), which is the Black-Scholes value of the option. In this table, we present this value as it is presented in equation 1, with column (10) representing the first term on the right-hand side of the

[16] In our actual simulation modeling for this paper, this and all simulations in our model contain 1441 rows, one for each day. Since this table is an example of a monthly hedge strategy, for purposes of readability we only show each month.

[17] We did not add a 16th column for "Total P/L" by month, given space limitations.

equation, and column (11) representing the second term on the right-hand side of the equation. Column (10) shows $SN(d_1)$ - the stock price from column (2) times $N(d_1)$ from column (8)[18].
Column (11) shows $Ke^{-rt}N(d_2)$, - the strike price from column (6) time $N(d_2)$ from column (9).
Column (10) minus column (11) represents the Black-Scholes value of the option. For example, for month 0 (the start of the strategy), the value of the option is 52.8186 - 47.1814 = 5.637 (rounded), as can be seen in the table.

Similarly, the values for d1 and d2 are calculated as in the Black-Scholes equation 1. For example, moving down a row to month 1 (row 2), d1 is calculated as:

$$\frac{\ln\frac{95.32}{100} + \left(0 + \frac{.2^2}{2}\right) * .417}{(.2 * \sqrt{.417})} = -.3065$$

The value for d2 for the same period is calculated as:

$$-.3065 - (.2 * \sqrt{.417}) = -.4357$$

3 Results

As previously stated, every simulation we run starts with a 6-month $100-strike equity call option that is atm at inception (the forward stock price is the same as the strike, $100). Further, we assume that when the current option expires, we roll into a new 6-month atm call option. Specifically, the strike on the new option is equal to whatever the forward stock price is on its start date, so that every new 6-month option starts out atm. We continue this strategy for four years, unless we hit a stop/target before then, in which case we unwind both the option and the underlying stock immediately and the strategy ends early (for that specific simulation).

[18] Note that $N(d_1)$ is also the delta of the option.

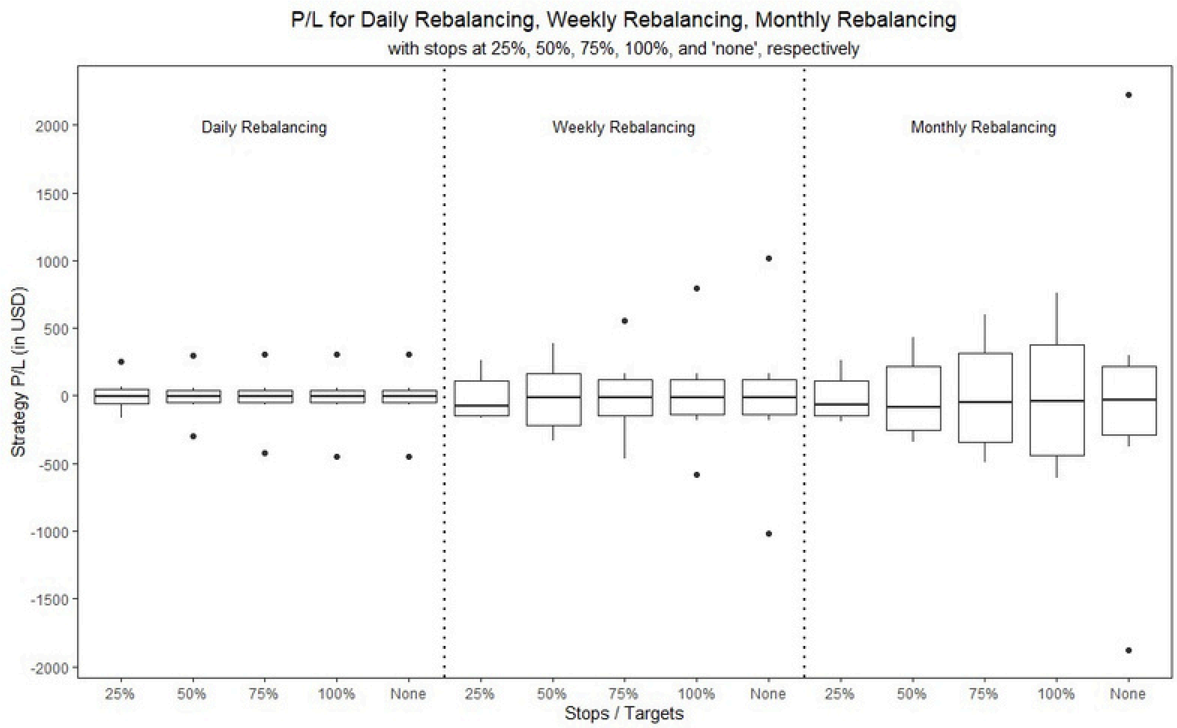

Figure 4: P&L scenarios showing three different hedging frequencies, each with five different stop / target levels. The first third of the graph shows daily hedging, the middle third shows weekly hedging, and the latter third shows monthly hedging.

Figure 4 shows that, overall, the less frequent the hedging (or "rebalancing"), the wider the distributions of potential P&L. We see this by noting wider distributions in the "Weekly Rebalancing" section of the graph than in the "Daily Rebalancing" section, and likewise, the wider distributions in the "Monthly Rebalancing" section of the graph than in the "Weekly Rebalancing" section.

Also note in Figure 4 that, in general, the higher the stop/target, the wider the distribution of potential P&L. We see this by noting that, for each third of the graph, the boxplot distributions get wider as the stop/target increases. This makes sense in that the longer a trade is outstanding (we will see in Figure 5 that the higher the stop/loss, the longer the trade exists), the more time exists for a trade to accumulate larger gains and losses.

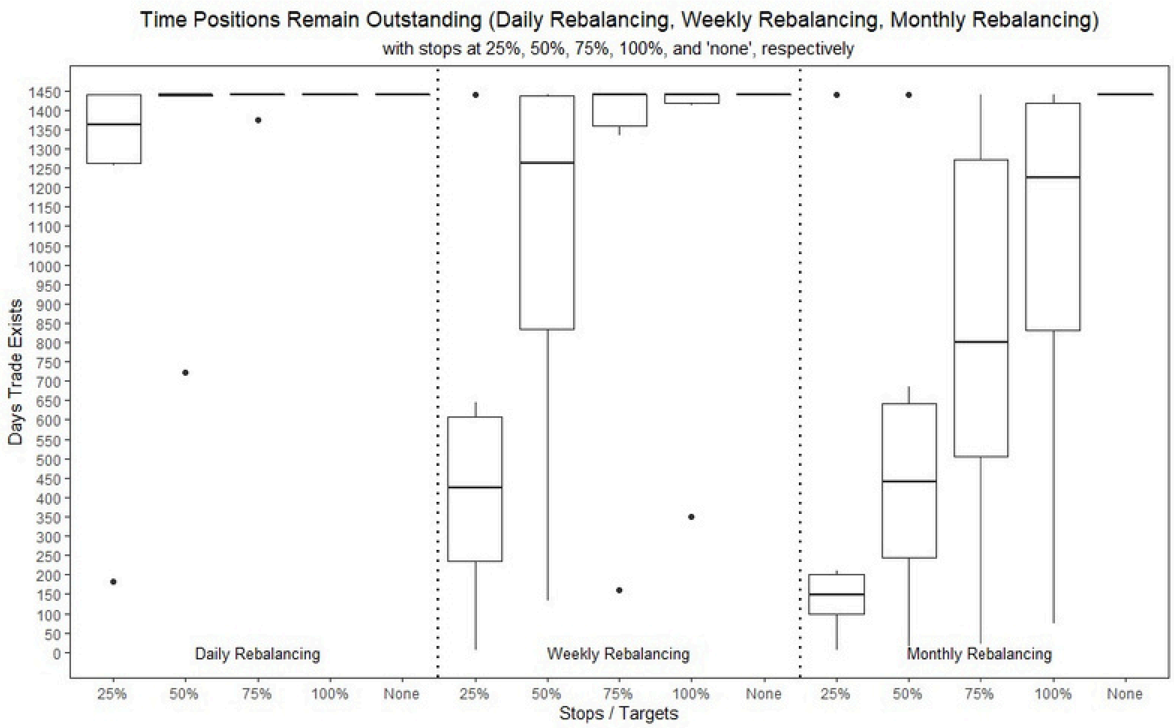

Figure 5: Time Outstanding scenarios showing three different hedging frequencies, each with 5 different stop / target levels. The first third of the graph shows daily hedging, the middle third shows weekly hedging, and the latter third shows monthly hedging.

Figure 5 shows that, when hedged daily, the strategy stays “alive” for almost the full four years regardless of the stop/target level. As can be seen in both the weekly and the monthly hedging scenarios, as hedging becomes more infrequent, the strategies are “alive” for shorter periods of time. The reason is that the less frequently an option is hedged (rebalanced), the larger the open delta can become, resulting in larger p/l swings as the market moves.

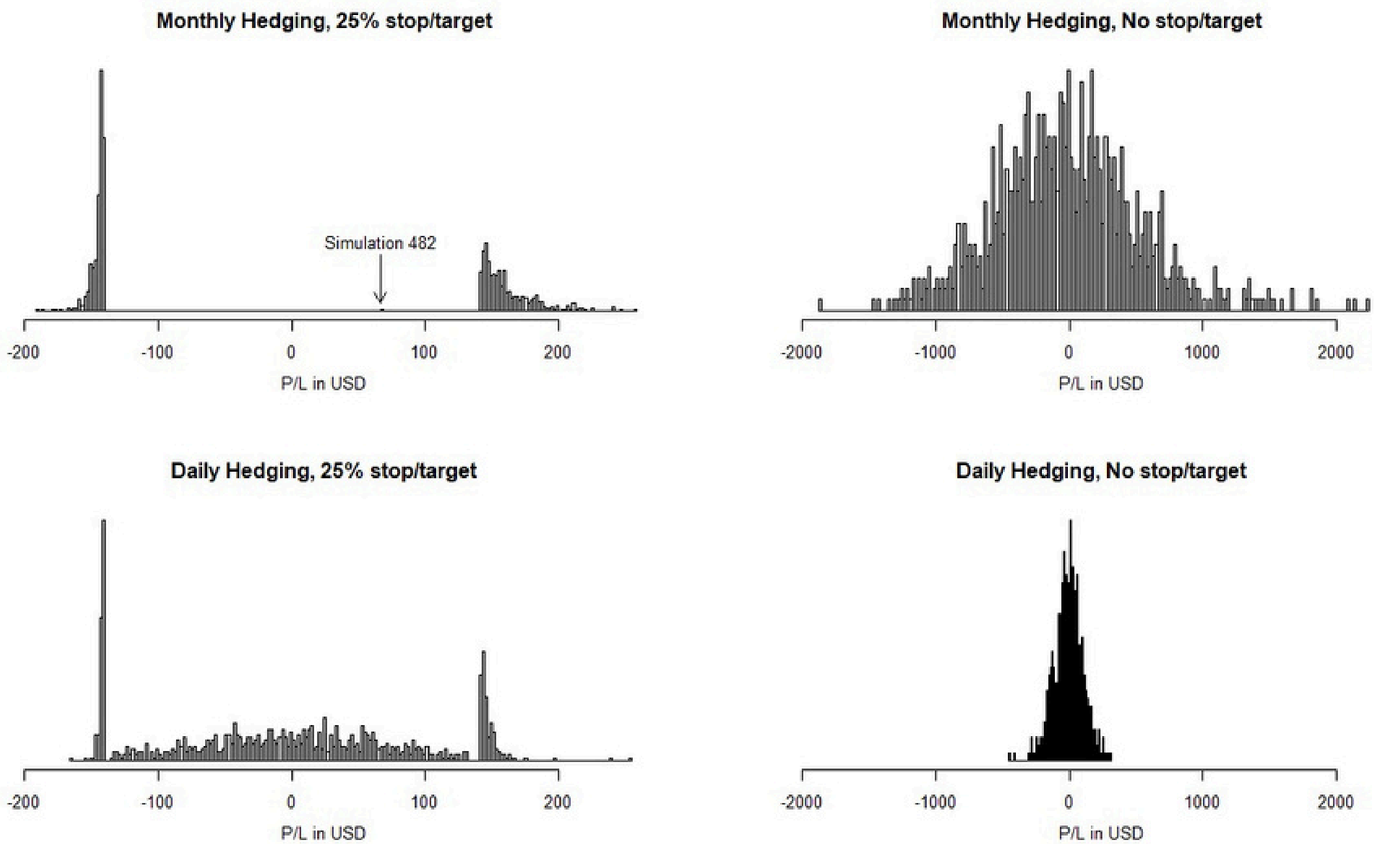

Figure 6: A comparison of extremes. Monthly hedging with a 25% stop/target (upper left panel), monthly hedging with no stop/target (upper right panel), daily hedging with a 25% stop/target (lower left panel), and daily hedging with no stop/target (lower right panel).

Some of the observations seen in figures 4 and 5 above can be explored in detail by comparing four extremes: a 25% stop/target and a no stop target, combined with both monthly hedging and daily hedging, shown in figure 6. First, moving horizontally across the top of figure 6, the left panel shows the monthly hedging scenario with a 25% stop/target and the right panel shows the monthly hedging scenario with no stop/target. In the case of the former, we see a "bimodal" distribution of P/L which results from the combination of large open deltas (we hedge only monthly) and a minimal P/L "allowance" (tight stop/target) causing us to hit the stop or the target in almost every scenario[19]. In the case where we have no stop/target (top right panel) we see a much more "normal" shaped distribution of P/L as the trade has no upper or lower boundary and as such is allowed to exist for the full four years in all simulations. We also note that the possible range of

[19] As mentioned earlier, the small "bump" in the middle of this panel is $66, and represents simulation number 482. We used this simulation to explain the detail of our model's calculations, and we will see it again, analyzed in the Results section of this paper

P/L is much smaller in the 25% stop/target case than the no stop/target case (±200 USD versus ±2000 USD) since the former has P/L limitations but the latter does not. Similarly (although not shown in figure 6), the 25% stop/target simulations result in a much shorter trade, with a median average life outstanding of 129 days, an average of 170 days, a minimum of 6 days, and a maximum of the full 4 years whereas in the no stop/target case, the trade lasts the full four years in all simulations. Second, moving vertically down the left side of figure 6, when we compare the monthly hedging 25% stop/target case (upper left panel) with the daily hedging 25% stop/target case (lower left panel), we see that the latter has a similar bimodal distribution of P/L but with many scenarios that are now inside of the stop/target levels. In other words, with a stop/target of 25% and daily hedging, there are a multitude of scenarios where we do not hit the stop/target at all, which is not the case when hedging monthly. Just as notable, since all of these scenarios "inside" the two peaks of the distribution last for the full 4 years of the trade, we can see that in general, hedging daily will allow a trade to stay "alive" longer than hedging monthly, a direct result of the smaller open delta's of the former. Third, moving vertically down the right side of figure 6, when we compare the monthly hedging, no stop/target case (upper right panel) with the daily hedging, no stop/target case (bottom right panel), we see that they both have similar normal shaped P/L distributions, but with a much different range of P/L outcomes, ±400 USD when hedging daily versus ±2000 USD when hedging monthly. Since there are no P/L constraints for either case, all scenarios remain open for the full four years. However, hedging monthly results in much larger open delta's than hedging daily, and the results are the the wider P/L distribution referenced above. Fourth, moving horizontally across the bottom of figure 6, the left panel versus the right panel shows similar conclusions as when we moved horizontally across the top of figure 6, that the stop/target causes a bimodal distribution of P/L outcomes, although not as severe as when hedging monthly, and that the "no stop/target" case results in a normal distribution of P/L outcomes, although with a much smaller range of outcomes compared to hedging monthly. Last, it is worth noting that when hedging daily, the stop/target is less relevant to both the P/L range and the life of the trade outstanding, as compared to hedging monthly.

An area that we have yet to address is the ex-post RV versus the upfront IV of the gamma hedging strategy. This is generally considered one of the primary indicators of success or failure as measured by P/L. See figure 7 for two different RV scenarios and the resulting P/L's for the monthly hedging, no stop/target case.

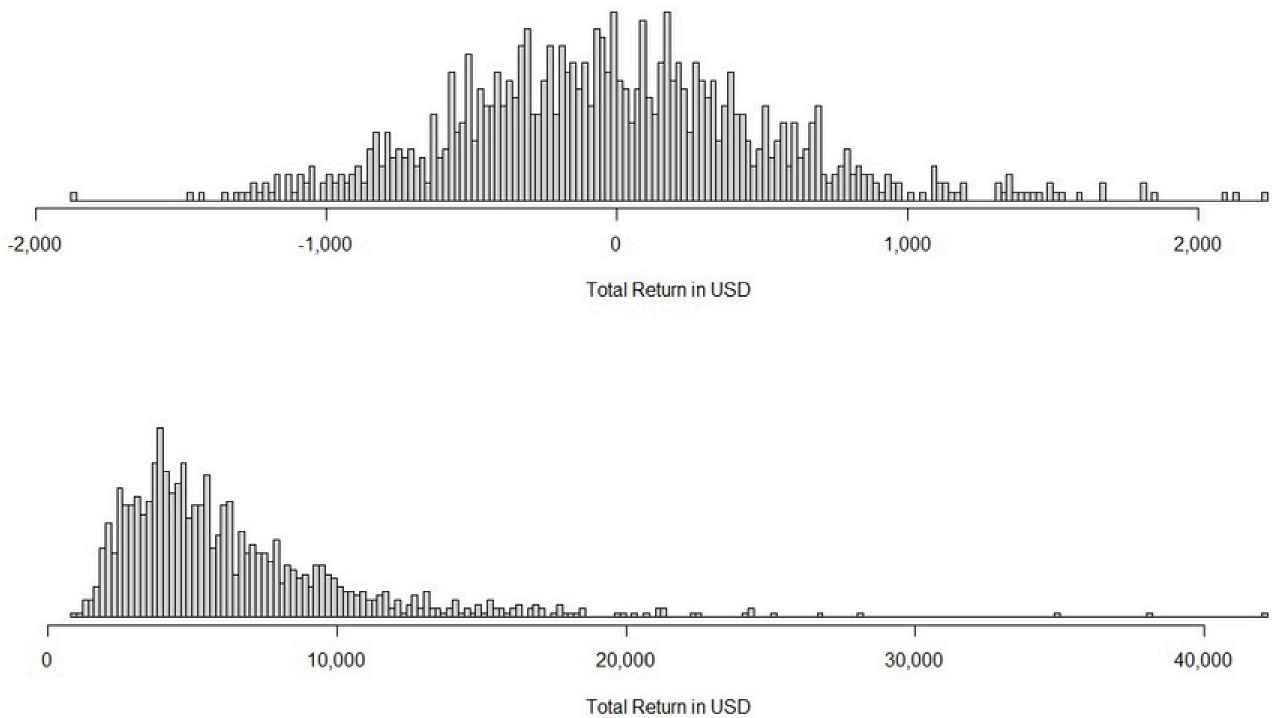

Figure 7: Scenarios showing the distribution of returns with a realized underlying stock volatility of 20% (top panel) versus the distribution of returns with a realized underlying stock volatility of 50% (bottom panel). Both graphs assume no stop or target, so they both run for the full 4-years of simulated prices.

The top half of Figure 7 shows that, for our 1000 simulations, the distribution of returns are “properly” centered around 0, as the option was purchased for an IV of 20%, and the actual RV of the stock price over the 4-year life of the trade is also 20%. Additionally, in the bottom half of Figure 7, we see that at an RV of 50% there are no negative return scenarios. In this latter case, the returns range from a minimum of zero to a maximum of $420k. The range of the lower graph in Figure 7 makes intuitive sense given that the option is purchased at an IV of 20%, but the RV of the stock over the life of the trade was 50%, enabling the gamma hedger to sell the stock at higher highs and buy the stock lower lows (over the 4 year life of the trade) than was implied in the initial purchase price of the option.

Simulation 482 revisited

In section 2 we chose simulation 482 to help detail the calculations used in our model. This specific simulation also provides us with some interesting observations when performing a “micro” analysis of its results. First, even with longer-term (i.e. monthly) hedging periods and a stop/target, an option strategy may "stay alive" for a relatively long period of time if there is a consistent series of relatively small moves in the stock, even if these moves are all in the same direction and the end result is a stock price at the expiration of the strategy which is materially different than the price at T0. Second, IV versus RV may not always be the best indicator of how a gamma hedging strategy has performed over its life. Finally, and the most important of these latter three observations, the path of the stock price may be just as, if not more important, than the actual RV of the stock price in determining the final P/L performance of a gamma hedging strategy. We discuss the detail behind each of these three observations below.

To start, we might ask why is it that simulation 482 is the only one out of 1000 that did not hit its stop/target (again, for the monthly hedging, 25% stop/target scenario). This is the type of question that our model is meant to help answer. In investigating the answer to this question, we note a few things. First, we see in table 1 column 2 that the stock price in this simulation follows a steady, continuing downward trend, from 100 to 28.81 over the 4 years. There are very few "jumps", as can be seen by the relatively consistent decrease in the price, and as such, no sudden high or low strikes "locked-in" for the following six months, which given the stock price "drift" between roll dates, may have resulted in P/L large enough for the trade to hit the stop/target. This information alone would lead us to suspect that 482 could be a "longer-living" scenario. Second, we check the annualized monthly RV of the stock price over the life of the strategy. This comes to $\approx 17.5\%$.

This RV is close enough to the initial IV that, again, we could suspect that this could be a scenario that doesn't hit its stop/target level.

However, the IV and RV numbers raise another important question. If we bought the option for an IV of 20%, an ex-post RV of 17.5% should result in the simulation showing a P/L loss, not a gain, since the RV was lower than the IV[20]. This is another example of a particularly interesting dynamic in scenario 482. The consistent four-year drop in the stock price results in a behavior where each six-month option starts with a high gamma (gamma is generally highest when delta is around 50%, as can be seen when comparing the top and bottom panels of figure 8) that in every case declines to 0. This latter point is best seen in the bottom panel of figure 8, where we show the gamma profile for each of the 8 consecutive options[21]. The differing expectations - expecting

[20] Traders will generally compare ex-post RV with the initial IV to make a "ballpark guess" of the ex-post P/L of a gamma hedging strategy. This simplification is used because, particularly in the inter-dealer community, the trader may have thousands or even tens of thousands of options on the books at any one time, and going through the actual ex-post P/L of even a few of them can be overly time-consuming.

[21] Note that the starting gamma is higher for each consecutive option. This happens because we measure gamma based on the change in delta for a $1 move in the underlying stock price. Since this price starts at $100 and ends at $29, a $1 move initially represents a 1% change in the underlying, but by the end it represents a 3.4% change in the underlying, causing the gamma for a $1 move in the underlying to increase over the life of this simulation.

to lose money based on RV < IV , but actually making money, suggests that comparing the IV to ex-post RV of the stock price moves may not always be the best indicator of whether the strategy made or lost money. There are many possible reasons for this, including some that we do not cover in this paper22, but one key question is: could the “standard” RV versus IV comparison inadvertently increase our expectations of P/L gains when large moves in the stock price occur at the same time that the option has a gamma position close to 0 (and vice versa). For example, if we have a 100-strike call expiring in one second, and the underlying stock price jumps from $1 to $99, the option has zero gamma and zero delta at both prices, but the ex-post RV will be increased when taking this large move into account. We might expect that we made money since the RV was higher than the IV. However, since this large move occurred when the option had 0 gamma, there were no net gains from this large move. We could potentially see an RV that is higher than the IV due primarily to this move, yet have a strategy whose P/L is much lower than the RV would imply since this large move was not hedgeable. Another extreme example would be a stock that has only one move during the option period - a gap up on day 1 of the option period and a gap back down to the original level on day 2. Compare a situation with the same exact move, except this time it occurs much later in the option’s life - 3 days before expiration, reversing back again the next day. As gamma is inversely related to the option’s “time-to-expiration”, the delta change and hedge P/L of the former will be smaller than the delta change and hedge P/L of the latter, but the RV of the stock price over the option period will be the same. Again, the ex-post IV versus RV comparisons of both scenarios will be the same, but the P/L of the former will be lower than the P/L of the latter. This leads us to an interesting theoretical question: how best can we measure the RV of a stock while also taking into account different forms of price changes (i.e. a small number of large moves versus a large number of small moves, random price jumps, etc.) and the path-dependency of these price changes (e.g. early moves versus late moves, etc.). Should we weight the moves by size of gamma of the option23? What impact does path-dependency have on the P/L of a gamma-hedging strategy? These are theoretical considerations that exceed the scope of this paper, but they are examples of more complex questions that our simulation model can help us uncover and analyze.

[22] We refer specifically to the relationship between gamma and time-to-expiration, where gamma increases exponentially as time-to-expiration of the option decreases

[23] A very basic calculation weighting RV of simulation 482 resulted in an RV that was 40% higher than the unweighted RV of 17.5%

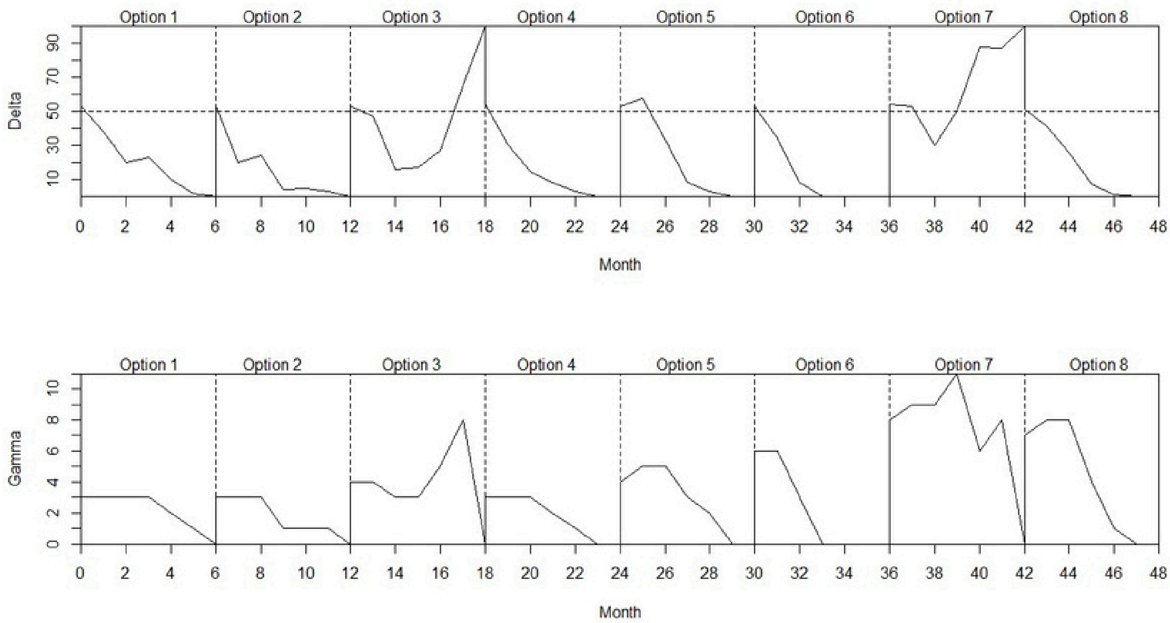

Figure 8: Profile of the delta (top panel) and gamma (bottom panel) for each of the 8 consecutive options in simulation 482.

4 Conclusions and Discussions

In this work, we run a simulation study that can be useful in giving the trader an idea, before entering into the options strategy, of what kind of returns and holding period he or she should expect over the life of the trade. The results show that, first, hedging more frequently allows one to potentially hold the option for a longer period of time, and although it appears to generate a lower distribution of expected P/L, this may be a good way to "rent" an option while waiting for some sort of a market disruption event, or "jump", of some kind, allowing for a large one-time profit. At the same time, the model simulations also show us that one potential downside of frequent (i.e. daily) hedging occurs when there is a large trending move in the underlying stock that persists over the life of the strategy. The frequent hedging will result in less profits than if the trader were to just "let the underlying run", allowing the delta to grow as the stock price trends further in the trader's favor, one of the instances where options traders generally expect to make the most P&L[24]. Second, longer hedging periods (e.g. monthly rather than daily) allow for a wider distribution of P&L, and, in trending markets, where the market moves in one direction for some time while at that same time the delta of the strategy is increasing (due to this trending move), larger profits. Third we see that a critical decision in terms of frequency of hedging is the balance the trader wants between a long-holding period with an ability to benefit from "jumps" (which may or may not happen) versus an ability to take advantage of large trending moves in the underlying stock. The former is more likely to result in a decision to hedge more frequently, the latter to hedge less frequently. It is therefore important for the trader to decide upfront if they have a view about the type of stock movement they expect. Examples of events that inform this kind of decision are future earnings releases, a recent move up or down in the stock price which leads the trader to believe that the stock now contains a material premium or discount, past movement types, future events that the trader foresees outside of normal company operations, future market events that may impact the sector, etc. If the trader expects a trending market, for example, then an infrequent hedging schedule is recommended, and if they expect a volatile but mean-reverting market, then a frequent hedging schedule is preferred. Fourth, the simulation model allows us to see the impact of stops/targets on a strategy. Specifically, it allows us to see the expected size and distribution of the P/L, at what point this distribution goes from normal to bimodal, and how different stops/targets will affect our expected P/L and holding period. Last, by using the simulation model (particularly of extreme scenarios), we may be able to come up with trading insights that are not as easy to see when approaching gamma hedging from a either a macro perspective or a purely theoretical p ersp ective.

There are a few possible future extensions of the simulation study. First, this paper assumed a fixed IV and fixed RV over the term of the option. Instead, one can look to add a parameter for the correlation that often exists between movements in the price of the underlying stock and movements in the IV of that stock. This will obviously encompass reviewing research already done on this topic while also enabling the the model to consider the actual standard and idiosyncratic correlations that exist for each specific stock. Second, one can explore different models for simulating the stock

[24] Something not addressed in this paper but that should be noted is that there may generally be some negative P/L "drag" with an options gamma strategy that is based on daily hedging, due to the propensity for options to trade with a risk premium. This is defined as the premium of implied volatility over realized volatility. In general, when a risk premium exists the IV of the option will be equal to the expected RV until expiration, plus some premium. This premium varies depending on supply and demand, but in the authors experience, and the experience of most research on the topic, it is a long-term persistent attribute of the options market. From a trading perspective, it may result in gamma hedging opportunities to be less than those implied by the initial IV of the option. Note that there are many studies on the reasons for this risk premium, and the topic is beyond the scope of this paper, but the traders job in analyzing the trade upfront is to identify those options where implied volatility is lowest relative to expected realized volatility, and then decide on how to most optimally hedge to capture this difference. This latter part is what our simulation model aims to do.

price, possibly allowing more skewness and taking stock price jumps into account.

Most importantly, it is desirable to add the capability for backtesting[25], taking into account not only all of the preset hedging decisions, but also allowing for both historical stock prices and historical implied volatilities.

[25] Backtesting takes historical data and inputs it into a model in order to see how well a strategy would have performed if it had been executed in the past. Backtesting is one of the many tools traders use to test the efficacy of a given trading strategy.

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