

MCA 1st Semester Examination, 2022  
MCA T 11: Design and Analysis of Algorithms.  
Full Marks: 70.  
Time: 3 hours.

Write your Name, Exam Roll Number, Subject with code and registration number.

Answer Q1 and any two from the rest.

- Q1. a. Show that if  $y(n) \in O(z(n))$  and  $x(n) \in O(y(n))$  then  $x(n) \in O(z(n))$ . (5)  
b. Show that if  $a(n) \in O(x(n))$  and  $b(n) \in O(y(n))$  then  $a(n)*b(n) \in O(x(n)*y(n))$ . (5)  
c. If  $f(x) = a_p x^p + a_{p-1} x^{p-1} + \dots + a_2 x^2 + a_1 x^1 + a_0$  where  $a_p > 0$  and  $x \geq 1$   
then show that  $f(x) \in O(x^p)$ . (5)  
d. Show that  $\log_a n \in O(\log_b n)$  and  $\log_b n \in O(\log_a n)$  (5)  
e. If  $f(n) = n^2$  and  $g(n) = n \log_2 n$  then show that  $f(n) \notin O(g(n))$ . (5)  
f. Show that  $(2n^2 2^n + n \log n) \in \Theta(n^2 2^n)$ . (5)  
g. Show that  $(33n^3 + 4n^2) \in \Omega(n^3)$ . (5)  
h. Show that  $n! \in O(n^n)$  Hint: use  $n! \approx \sqrt{(2\pi n)} (n/e)^n$ . (5)

Q2. What is a heap data structure? Write an algorithm to sort the following list of numbers using a heap data structure. Also find out the time and space complexity of the method. (15)

4, 5, 2, 9, 8, 7, 1, 3

Q3. What are disjoint set data structures? Write algorithms for the following operations on the disjoint set data structure:

- a. Find (x): Returns the root of the tree containing the element x.
- b. Union (x, y): Replace two sets containing elements x and y respectively by their union.

How the performance of the above operations are improved by using appropriate heuristics. (15)

Let  $\{1\}$ ,  $\{2\}$ ,  $\{3\}$ ,  $\{4\}$ ,  $\{5\}$ ,  $\{6\}$ ,  $\{7\}$  and  $\{8\}$  be singleton sets, each represented by a tree with exactly one node. Use the union find algorithm with union by rank and path compression to find the tree representation of the set resulting from each of the following unions and finds: union(1,2), union(3,4), union(5,6), union(7,8), union(1,3), union(5,7), find(1), union(1,5), find(1). (15)

Q4. Write short notes on:

- a. Amortized Analysis.
- b. Binary Search.