

ASSIGNMENT 4

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Part 1

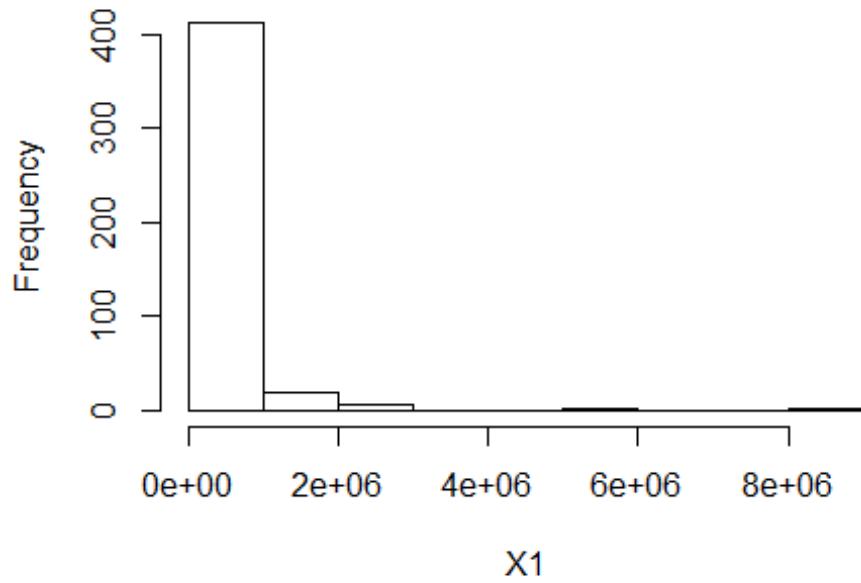
(a)

```
cdi = read.table("D:\\ASU Stuff\\SEM-1\\STP 530\\CDI_data.txt")
Y = cdi$V8
# Model-I
X1 = cdi$V5
X2 = cdi$V4
X3 = cdi$V16

# Model-II
x1 = cdi$V5 / cdi$V4
x2 = cdi$V7
x3 = cdi$V16

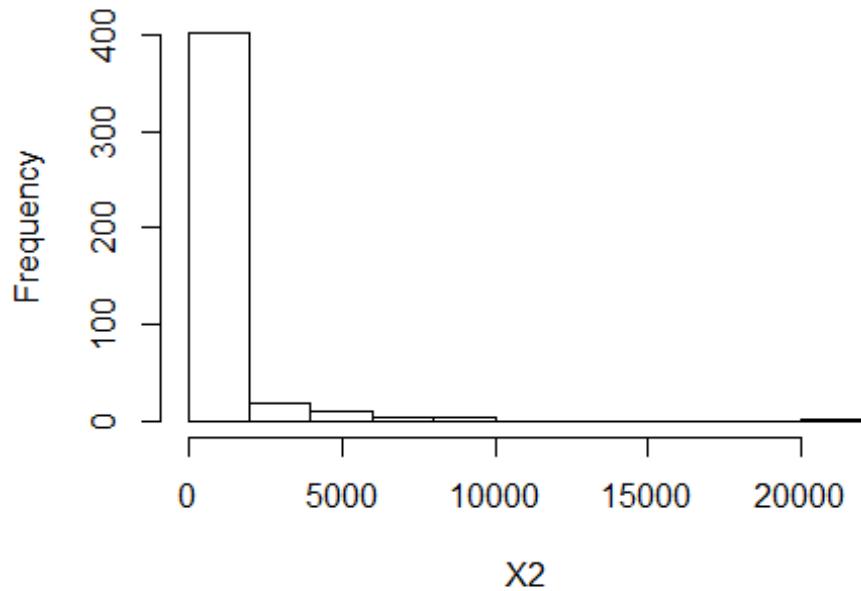
hist(X1)
```

Histogram of X1



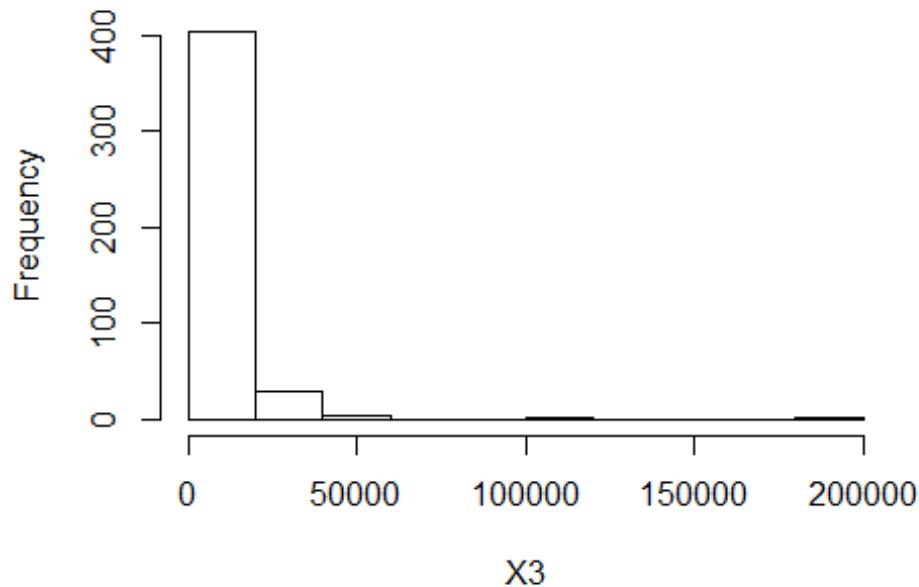
```
hist(X2)
```

Histogram of X2



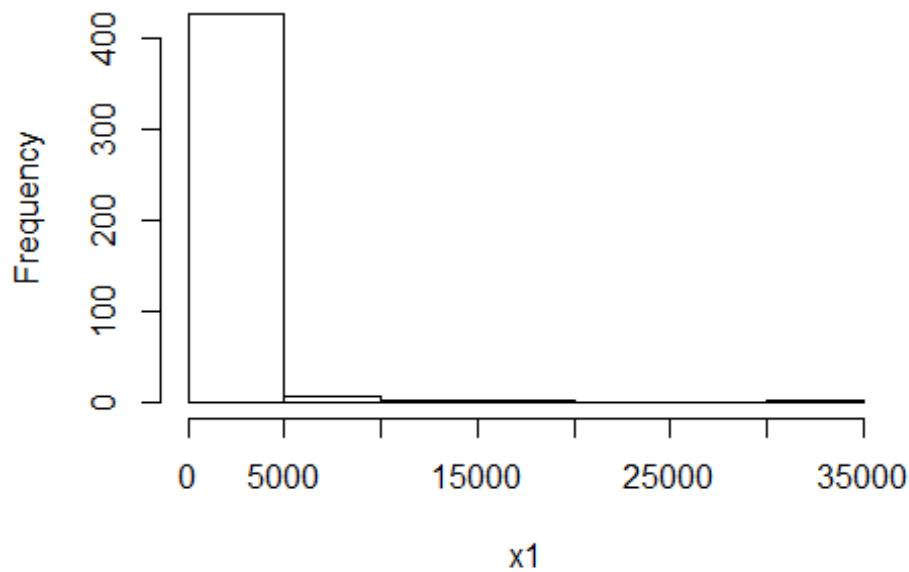
```
hist(X3)
```

Histogram of X3



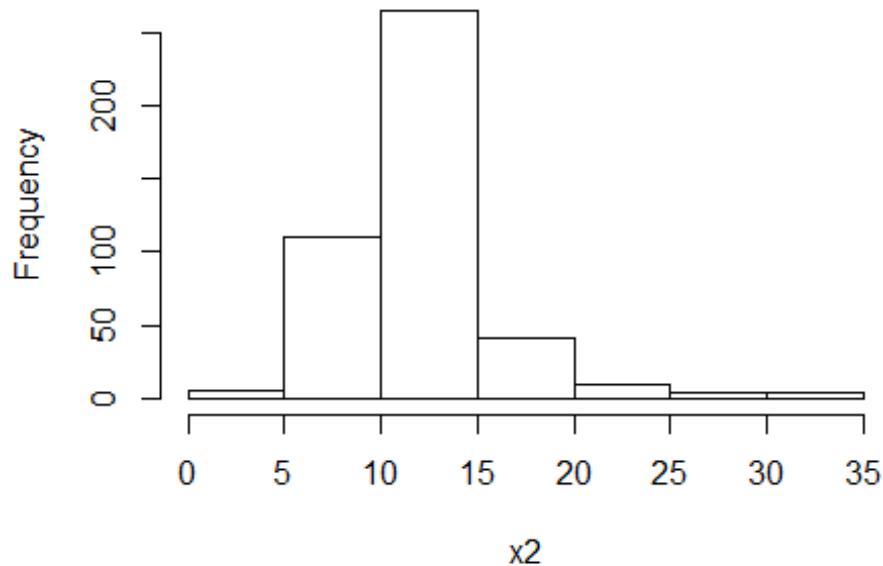
```
hist(x1)
```

Histogram of x1



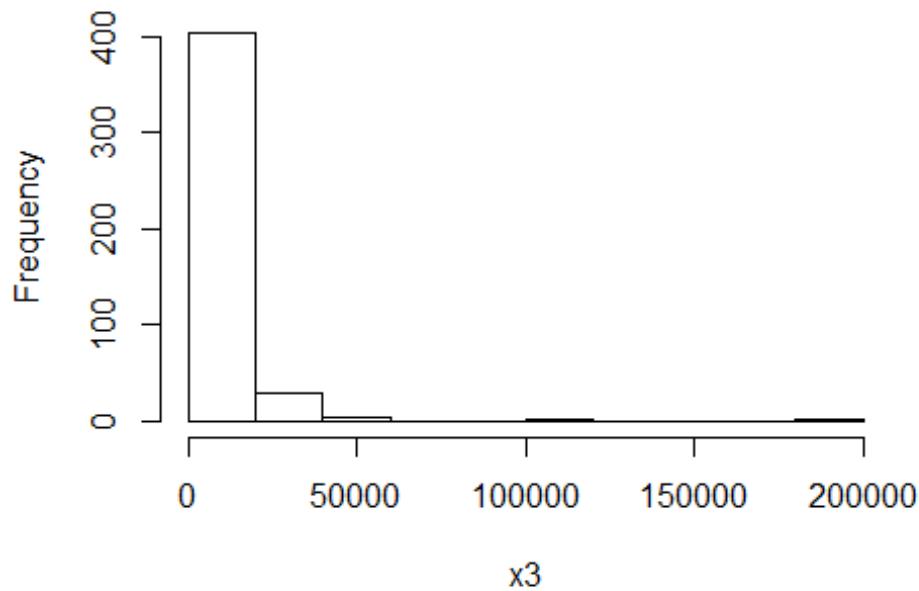
```
hist(x2)
```

Histogram of x2



```
hist(x3)
```

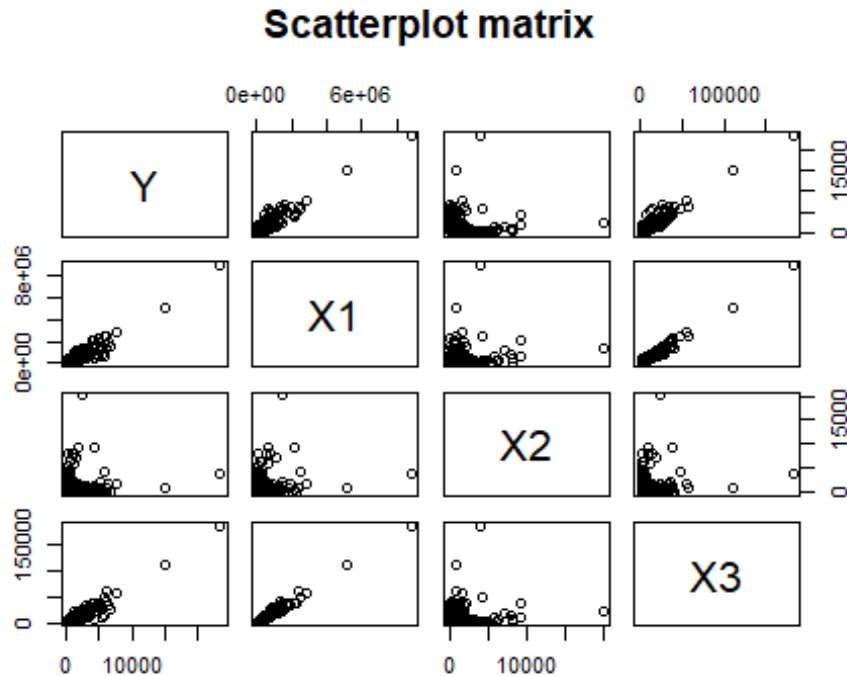
Histogram of x3



From the plots, we can see that the histogram plots for model-I are right-skewed but on the other hand, for model II, the histogram plot of percentage of population greater than 64 years old is approximately normal while the two histogram plots for model II are right-skewed.

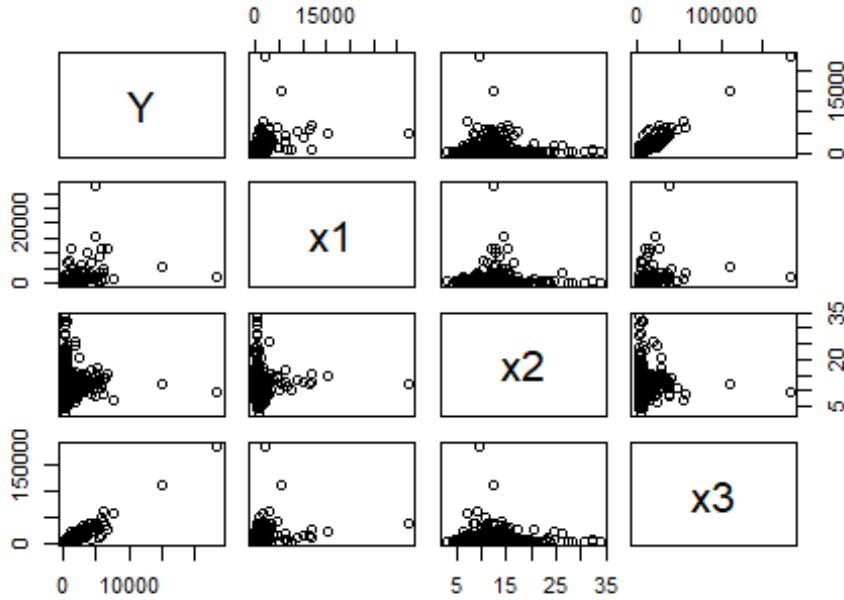
(b)

```
# Scatterplot matrix for model I  
pairs(~Y+X1+X2+X3, data = cdi, main = "Scatterplot matrix")
```



```
# Scatterplot matrix for model II  
pairs(~Y+x1+x2+x3, data = cdi, main = "Scatterplot matrix")
```

Scatterplot matrix



```
# Correlation matrix for model I
C1 = cbind(X1,X2,X3)
cor(C1)

##          X1          X2          X3
## X1 1.0000000 0.1730834 0.9867476
## X2 0.1730834 1.0000000 0.1270743
## X3 0.9867476 0.1270743 1.0000000

# Correlation matrix for model II
C2 = cbind(x1,x2,x3)
cor(C2)

##          x1          x2          x3
## x1 1.0000000 0.02918445 0.31620475
## x2 0.02918445 1.0000000 -0.02273315
## x3 0.31620475 -0.02273315 1.0000000
```

From the scatterplots, we can actually determine the relations between response variables and predictor variables and also between predictor variables and predictor variables. From the scatterplot of modelI, we can see that the number of active physicians vary linearly with total population and total personal income while the same has no specific relation with land area. Clustering is in the starting smaller parts. From the scatterplot of modelII, we can observe that the number of active physicians vary linearly only with the total personal income and there is no specific relation between no of active physicians with population density and no of people greater than 64 years in age. Most of the data are clustered in the starting segments. From the correlation matrix in modelI, we can see that

total population and total personal income are strongly correlated and from the correlation matrix in modelII, we can see that there is a weak correlation between predictor variables.

(c)

```
#Fitting the first order regression model for model I
model1 = lm(Y~X1+X2+X3)
model1

##
## Call:
## lm(formula = Y ~ X1 + X2 + X3)
##
## Coefficients:
## (Intercept)          X1          X2          X3
## -1.332e+01    8.366e-04   -6.552e-02    9.413e-02

#Fitting the first order regression model for model II
model2 = lm(Y~x1+x2+x3)
model2

##
## Call:
## lm(formula = Y ~ x1 + x2 + x3)
##
## Coefficients:
## (Intercept)          x1          x2          x3
## -170.57422     0.09616    6.33984    0.12657
```

Thus, the estimated regression function for modelI is $-13.362 + 0.0008366X_1 - 0.06552X_2 + 0.09413X_3$ and the estimated regression function for modelII is $-170.574 + 0.09616X_1 + 6.33984X_2 + 0.12657X_3$.

(d)

```
# Model I
yhat1 = model1$fitted.values
rsq1 = sum((yhat1 - mean(Y))^2) / sum((Y - mean(Y))^2)
rsq1
## [1] 0.9026432

# Model II
yhat2 = model2$fitted.values
rsq2 = sum((yhat2 - mean(Y))^2) / sum((Y - mean(Y))^2)
rsq2
## [1] 0.9117491
```

Thus, the value of R^2 for model I and mdoel II are 0.9026 and 0.9117 respectively. And, yes, model II explained more variance in the dependent variable than model I and thus is more preferable.

(e)

#Residuals for model I

```
e1 = Y - yhat1
```

```
e1
```

```
##          1          2          3          4          5
## -800.6780229  515.4374974  144.3080224 -502.0168730 -1426.2901812
##          6          7          8          9          10
## -684.7626779 -443.0294891 -1360.9624485 1544.2440120 -423.8873542
##         11         12         13         14         15
## 2841.0124665  797.5951318 -605.4050127  414.6857947 1679.0300557
##         16         17         18         19         20
## 723.2396545  1622.3368033 -1030.7944467 1263.2121638 -506.3677568
##         21         22         23         24         25
## -1143.7386407 434.8731762 -1001.9535363 -1305.4463657 441.8063320
##         26         27         28         29         30
## -186.2090800  588.2759550 -1855.6061839  350.6304732 196.1630218
##         31         32         33         34         35
## 336.3725716  1141.9644241  801.1912369 -925.0844361 205.4556199
##         36         37         38         39         40
## -793.2556524 -299.2196182   71.4885924 -740.8141266 426.9275152
##         41         42         43         44         45
## 60.0217210 -1186.0838861  860.8489737 -781.9474635 871.5151300
##         46         47         48         49         50
## -431.6550305 385.0961473  1903.8340727 -419.6122684 3650.8200223
##         51         52         53         54         55
## -655.2217850  495.9573369  2227.3283527 -1263.4377467 442.7440609
##         56         57         58         59         60
## 91.5568522 -786.5672710  897.8965969 -267.2108316 -79.8414346
##         61         62         63         64         65
## -465.6868525 -769.9678548 -606.3505759   5.3918349 944.7007441
##         66         67         68         69         70
## 455.2265167 3689.0318671  930.9682227 -448.9951934 1479.1473259
##         71         72         73         74         75
## 108.8388711  721.5758503  1835.1289457  478.7505792 400.7389421
##         76         77         78         79         80
## -263.1820444 -345.8902146  1068.2615798 -145.1832675 -117.8814126
##         81         82         83         84         85
## -544.7041792 -371.6126273 -278.3762666 -469.0039165 -38.3316199
##         86         87         88         89         90
## -669.2002360 -627.2121311 -75.3158165 -274.1808170 974.0406700
##         91         92         93         94         95
## -684.6233972 -126.6496926 -81.1059375 -222.4729783 1333.9692908
##         96         97         98         99         100
## -251.2116079 200.8083967 -330.2374586  480.1594247 -375.9442506
##        101        102        103        104        105
## 349.5659208 1523.9228936 -550.1349789  264.4093086 -243.1129170
##        106        107        108        109        110
## -459.6841264 173.6895574 -641.3736247 -217.5227987 -618.4775675
##        111        112        113        114        115
```

##	-612.4199864	-649.7198506	-294.7709547	-617.7076351	-348.5233303
##	116	117	118	119	120
##	-455.6045167	-411.6319604	520.1648979	-210.0878816	-61.3127181
##	121	122	123	124	125
##	-354.2444476	-259.6082578	3198.1664897	-401.1169036	-275.0648033
##	126	127	128	129	130
##	-508.1050801	-174.1711953	-215.0313805	-73.6231269	114.4954054
##	131	132	133	134	135
##	-534.3227798	-275.4193874	-172.1815385	-302.4793104	-3.4355447
##	136	137	138	139	140
##	-246.1245702	693.8356105	121.8785117	-564.6835970	-203.2438194
##	141	142	143	144	145
##	46.3223146	-642.3343151	681.3994000	-224.2648514	-289.5409441
##	146	147	148	149	150
##	-374.8212608	-267.5297402	-337.7525386	200.4194523	-300.8438591
##	151	152	153	154	155
##	-115.5506886	-48.7050949	-91.3644550	-393.1528965	-5.4373046
##	156	157	158	159	160
##	-431.4565062	-284.6756249	-178.0483265	-184.8613557	738.7921737
##	161	162	163	164	165
##	481.4197543	-351.4935231	-5.9468959	-112.7121860	-334.0813513
##	166	167	168	169	170
##	362.0083707	57.6770814	1404.3525151	153.8842120	79.7732328
##	171	172	173	174	175
##	-234.0658439	-159.0458246	-202.9317523	-120.2768303	-525.8789591
##	176	177	178	179	180
##	-216.5844605	-399.5984799	-298.7427051	495.0518032	64.9965096
##	181	182	183	184	185
##	-93.3537165	14.7285480	-424.3608842	-69.8540619	-234.7090152
##	186	187	188	189	190
##	-21.8019881	495.7617156	-113.0950041	-157.7463001	-156.5388501
##	191	192	193	194	195
##	307.6385278	544.9971436	-81.1220552	-69.1058734	-294.5020970
##	196	197	198	199	200
##	379.6151725	-136.4227511	-146.6771792	-307.6181045	107.7607221
##	201	202	203	204	205
##	-135.1412583	-71.2652817	-161.8927356	-264.9403712	262.0592436
##	206	207	208	209	210
##	42.7735533	-360.0701259	-196.9680319	-68.8092991	-258.1315756
##	211	212	213	214	215
##	-85.0939048	-275.2369374	681.5943021	-128.8980226	-266.1611190
##	216	217	218	219	220
##	92.6071242	217.3997942	-244.0388034	-60.6552118	444.4187210
##	221	222	223	224	225
##	120.1343082	-8.3183828	-366.0826834	-318.5310799	-79.7123015
##	226	227	228	229	230
##	-331.9162901	-18.4756015	-113.2684895	-151.8920610	-34.7091015
##	231	232	233	234	235
##	48.0618934	131.8285623	-86.6264248	-185.8321847	373.7662387
##	236	237	238	239	240

##	-92.8438525	-64.3447676	-164.5099069	11.4256757	-224.0971655
##	241	242	243	244	245
##	192.3178877	-190.2330205	623.1681594	-41.3671982	94.5649535
##	246	247	248	249	250
##	-194.8643518	-268.1608735	82.3849295	-189.5749215	5.7356403
##	251	252	253	254	255
##	-182.5105535	-317.4092821	166.3595023	-238.7433607	-222.2480291
##	256	257	258	259	260
##	37.3931921	-215.7465891	1494.9832334	824.1678911	-191.8810737
##	261	262	263	264	265
##	-24.2049886	-59.0279654	191.7696107	-236.7131683	-180.8769311
##	266	267	268	269	270
##	100.4101631	-117.2769223	-206.3493050	39.7034913	-38.3038030
##	271	272	273	274	275
##	-282.4303624	0.4433911	-15.7989288	-46.1028720	-221.9515841
##	276	277	278	279	280
##	12.9228637	-86.0744913	20.5454633	-7.4282722	-164.8539812
##	281	282	283	284	285
##	0.8514891	166.1216326	-131.8136270	-53.6656203	-151.0586070
##	286	287	288	289	290
##	69.3508887	-125.3625819	-43.4861311	-169.4909038	-118.4221944
##	291	292	293	294	295
##	-246.6580824	-144.0020720	-134.9786045	-65.5447599	-181.9770908
##	296	297	298	299	300
##	-121.5938245	47.1494074	59.0544155	-179.0223852	-230.2103795
##	301	302	303	304	305
##	-98.6171383	94.4764316	-101.5244237	-160.7050142	14.3339393
##	306	307	308	309	310
##	-123.5796269	-142.5747750	180.1153054	167.0328529	27.0129825
##	311	312	313	314	315
##	121.6108928	-254.0542522	-157.5630236	-124.7414660	-56.3062154
##	316	317	318	319	320
##	-2.6018789	-57.1755356	-81.5077977	84.4097351	-67.6263557
##	321	322	323	324	325
##	-187.9214299	16.7628750	-58.2167056	-74.7589153	38.6159831
##	326	327	328	329	330
##	-61.5369438	-66.3842856	-137.9579561	-105.0506707	69.4464902
##	331	332	333	334	335
##	-107.7151984	-173.6131393	-86.0711844	175.2404678	-117.9326775
##	336	337	338	339	340
##	-196.6094676	116.2247614	139.6472961	387.1146924	54.9251236
##	341	342	343	344	345
##	-137.8465102	-229.9414697	-123.4057365	-26.0361986	-30.0131404
##	346	347	348	349	350
##	-76.7081910	-110.3710143	-169.3200482	-146.2496081	-145.1958686
##	351	352	353	354	355
##	55.8218922	-41.9320193	-158.2875076	-107.8649196	-61.1882704
##	356	357	358	359	360
##	122.8046301	-20.0028693	-159.2743165	-173.9534706	161.4552682
##	361	362	363	364	365

```

## -2.1255604 -154.0298539 -25.2025759 -117.1186250 -86.9336301
##      366          367          368          369          370
## -66.7120151 -93.3540241 169.0956104 28.9057173 -21.5124763
##      371          372          373          374          375
## -20.8220514 -3.2213422 173.8517775 -39.4457349 36.0484935
##      376          377          378          379          380
## 50.6128948 -85.3375051 -74.4521503 -74.4783961 203.9907062
##      381          382          383          384          385
## -61.2793944 27.2766017 -208.4643187 12.8470308 -11.6499804
##      386          387          388          389          390
## 19.9469381 -72.8105368 168.6762385 31.3884229 -99.8710424
##      391          392          393          394          395
## -10.9194535 528.5063685 93.6815688 -123.5463615 -21.7556583
##      396          397          398          399          400
## 241.5601861 -73.9834687 -21.9982666 -30.9561949 78.6870683
##      401          402          403          404          405
## -95.1488613 -96.3401386 -54.9137295 128.0422224 -26.4159808
##      406          407          408          409          410
## -58.1248140 -90.7417731 -93.2855912 300.6861488 -171.6407136
##      411          412          413          414          415
## -58.9915395 430.7587087 -55.9117764 -59.9500228 287.5854315
##      416          417          418          419          420
## -72.0060052 -102.7889942 1575.4436830 -20.1896239 -31.9045732
##      421          422          423          424          425
## -5.9739923 -36.9017809 -56.2983565 -56.5174791 -79.2994529
##      426          427          428          429          430
## -102.5020062 -113.2637389 -103.0180993 228.3865345 -54.9073365
##      431          432          433          434          435
## -173.5040223 -90.2713861 -31.3179387 -108.4031062 -158.0389082
##      436          437          438          439          440
## -74.4021914 -99.3070734 -72.9820103 22.4794857 -65.2826263

```

#Residuals for model II

```
e2 = Y - yhat2
```

```
e2
```

	1	2	3	4	5
##	258.8136730	686.2731418	560.2895393	-243.6790818	-1563.5533974
##	6	7	8	9	10
##	-3055.7527132	-456.6957506	-1083.1459821	1890.9742682	-290.7005108
##	11	12	13	14	15
##	2230.2902607	775.5032344	-670.2285884	-372.9514039	1626.5445331
##	16	17	18	19	20
##	605.9275512	1667.0466356	-1075.5368926	634.5899287	-480.0564398
##	21	22	23	24	25
##	-1156.7320030	751.7296586	-1089.4840866	-1123.1532029	315.1063742
##	26	27	28	29	30
##	-31.4063560	529.1097567	-1951.9848839	489.1888544	296.5805238
##	31	32	33	34	35
##	239.4881474	771.4414698	789.2961874	-1122.0573886	139.2056186

##	36	37	38	39	40
##	-996.0720039	-263.3441870	247.4948888	-1011.8107856	560.9150965
##	41	42	43	44	45
##	-406.5250371	-1379.8431809	851.4282243	-841.2566943	906.9166399
##	46	47	48	49	50
##	-601.2816866	-108.2368674	1711.6753351	-683.8898611	3045.8079345
##	51	52	53	54	55
##	-570.5658780	701.0808218	744.1169218	-1246.0129018	469.5677249
##	56	57	58	59	60
##	153.4941528	-816.2395548	734.7404159	-134.4755644	52.7611820
##	61	62	63	64	65
##	-591.7698585	-799.7972654	-569.5674282	-100.6098087	635.4931441
##	66	67	68	69	70
##	479.7317243	2740.6200137	1021.7694013	-601.9818952	1483.0162391
##	71	72	73	74	75
##	190.2901097	592.8575609	993.6571643	588.1249203	445.2622154
##	76	77	78	79	80
##	-6.3861268	-216.1226481	1106.3318244	-15.6841336	-52.7865711
##	81	82	83	84	85
##	-640.2366694	-1298.0426795	-479.6926405	-494.2465624	-307.7362145
##	86	87	88	89	90
##	-649.4843285	-689.9216580	-32.0621508	-223.8658703	1031.2086335
##	91	92	93	94	95
##	-623.2277594	-58.4612439	-137.8919302	-130.4046498	1281.4959815
##	96	97	98	99	100
##	-664.0985018	57.6023608	-206.8428384	583.8098795	-275.2119119
##	101	102	103	104	105
##	412.7124973	1344.9676097	-501.9796641	305.8630425	-191.6493221
##	106	107	108	109	110
##	-575.6696651	231.7713225	-616.8810818	-209.8850750	-548.9256570
##	111	112	113	114	115
##	-574.6988812	-678.9905843	-183.6155374	-600.7308039	-273.6421229
##	116	117	118	119	120
##	-391.5745212	-516.9475129	545.7642894	-170.3076078	4.6704746
##	121	122	123	124	125
##	-301.4842673	-194.6711887	2729.7141107	-375.9429238	-220.6349264
##	126	127	128	129	130
##	-455.5277930	-181.7291285	-37.9358261	12.8206940	226.7003407
##	131	132	133	134	135
##	-530.6129405	-245.4171547	-78.3225670	-250.6728966	-69.9375101
##	136	137	138	139	140
##	-173.1895286	751.9459308	170.6754744	-476.6392337	-263.0710054
##	141	142	143	144	145
##	36.9810314	-553.7003959	757.4483110	-175.0911662	-183.9047374
##	146	147	148	149	150
##	-326.6817281	-236.1094062	-305.0880547	271.9874162	-317.7591885
##	151	152	153	154	155
##	-77.0427035	-129.7597410	-18.0822672	-345.2643807	-126.8007793
##	156	157	158	159	160
##	-367.8154847	-223.1505925	-144.4870656	-121.8675151	834.5766378

##	161	162	163	164	165
##	502.9337431	-264.0075312	99.7813147	-99.7587925	-278.2987933
##	166	167	168	169	170
##	453.3822449	113.1364632	1456.8582217	16.3401919	172.9629804
##	171	172	173	174	175
##	-258.6295126	-158.5245472	-325.7373601	-51.5055619	-451.2890833
##	176	177	178	179	180
##	-67.2224964	-290.9631247	-214.1492111	529.5133110	-8.1713431
##	181	182	183	184	185
##	-0.5547246	56.9364880	-356.3173448	21.8988860	-157.6743399
##	186	187	188	189	190
##	67.1108826	197.2396105	25.5506853	-125.9461887	-109.3971964
##	191	192	193	194	195
##	17.0305328	625.1995823	-24.5376582	-33.6772942	-242.7511841
##	196	197	198	199	200
##	443.8351941	-102.5291619	-91.9612861	-256.1508271	134.8039160
##	201	202	203	204	205
##	-199.6381031	-22.4305852	-89.6210582	-182.9648531	308.1471186
##	206	207	208	209	210
##	-42.2367450	-306.3976116	-162.0237401	-25.9860768	-196.7931257
##	211	212	213	214	215
##	-14.0539674	-176.3506137	728.7008950	-65.7083448	-263.2510345
##	216	217	218	219	220
##	161.9953299	304.5210401	-191.8774727	-42.1636072	513.3192722
##	221	222	223	224	225
##	25.5054663	47.9742596	-274.6227443	-300.5386534	-15.0937996
##	226	227	228	229	230
##	-243.0442639	21.7445551	-49.1838083	-195.2666565	28.2348263
##	231	232	233	234	235
##	108.9634949	166.2139309	-36.4217189	-157.7807854	79.5823696
##	236	237	238	239	240
##	-49.2807144	-86.7386093	-99.6545867	103.7598545	-171.6826282
##	241	242	243	244	245
##	283.0050153	-118.7636616	700.1326709	10.6023448	-52.4927243
##	246	247	248	249	250
##	-79.9197714	-196.7311998	102.3369369	-227.7664273	-22.9167994
##	251	252	253	254	255
##	-137.8764963	-265.0916807	208.8154879	-169.7317410	-157.5747155
##	256	257	258	259	260
##	30.2978976	-170.2276094	1543.9222522	902.5663296	-133.7975222
##	261	262	263	264	265
##	30.4381107	-26.9226695	220.7194182	-172.9000757	-130.2245693
##	266	267	268	269	270
##	137.2400683	-4.7317165	-213.3749541	108.7495819	-29.4227561
##	271	272	273	274	275
##	-190.8968515	-572.4810717	-104.6794230	15.8063211	-148.3512125
##	276	277	278	279	280
##	101.1885345	-61.8228427	36.5716327	68.5462624	-126.2694564
##	281	282	283	284	285
##	31.5363991	189.1596473	-75.9359947	4.8243688	-161.9975889

##	286	287	288	289	290
##	111.9468605	-57.0564620	33.0993735	-100.2799493	-70.2426895
##	291	292	293	294	295
##	-181.6890249	-134.2218557	-188.9123133	-191.7899199	-138.3974585
##	296	297	298	299	300
##	-39.0653006	81.6555173	98.4374753	-118.1256279	-143.1024159
##	301	302	303	304	305
##	-80.4642732	135.9846326	34.6266818	-99.3444466	8.5712400
##	306	307	308	309	310
##	-84.4164833	-90.8658435	41.5743538	73.5762290	87.1651878
##	311	312	313	314	315
##	37.1162002	-175.9085998	-105.8298250	-95.6838251	2.8001937
##	316	317	318	319	320
##	66.6262284	-15.0981635	-12.7160277	137.3141729	-47.2073887
##	321	322	323	324	325
##	-105.2187899	95.6009340	-27.5149638	-50.6865279	83.0708318
##	326	327	328	329	330
##	1.5590967	-12.6901690	-65.3118089	-52.5733726	70.0313836
##	331	332	333	334	335
##	-55.0483062	-103.5025038	-34.5845173	87.1953230	-224.4152020
##	336	337	338	339	340
##	-127.7910595	72.2994206	9.8296847	459.2248619	87.5469579
##	341	342	343	344	345
##	-72.3345686	-181.5244937	-70.8858546	55.2367816	7.7855807
##	346	347	348	349	350
##	-45.4468325	-37.8895055	-110.0923912	-87.7429860	-95.6877535
##	351	352	353	354	355
##	29.7018976	18.4407918	-91.3131097	-50.0633140	-71.2364737
##	356	357	358	359	360
##	167.9724253	38.2713666	-115.3020389	-122.0152464	107.1440964
##	361	362	363	364	365
##	48.5290346	-86.4241137	83.1881109	-59.1280022	-49.2637874
##	366	367	368	369	370
##	-38.1618682	-58.5546785	15.0416009	69.6033179	5.6480208
##	371	372	373	374	375
##	40.0883182	43.3071528	53.1276045	29.6697583	72.6420398
##	376	377	378	379	380
##	60.5406572	-0.7289711	-15.0627696	-37.0601739	-25.1927242
##	381	382	383	384	385
##	4.3855978	62.4192133	-142.5720933	16.6777555	69.1789896
##	386	387	388	389	390
##	80.2515486	-30.3027707	101.8774485	84.5961540	-34.9795087
##	391	392	393	394	395
##	-4.7450275	603.2706996	41.7493601	-70.8379912	45.7036242
##	396	397	398	399	400
##	-401.4953482	0.2408944	-105.9335559	3.5653732	79.0323440
##	401	402	403	404	405
##	-52.3241972	-48.0240394	-2.2834495	-15.4706488	64.6437361
##	406	407	408	409	410
##	-4.8650241	-36.0602433	-52.4574781	371.3375822	-142.4890365

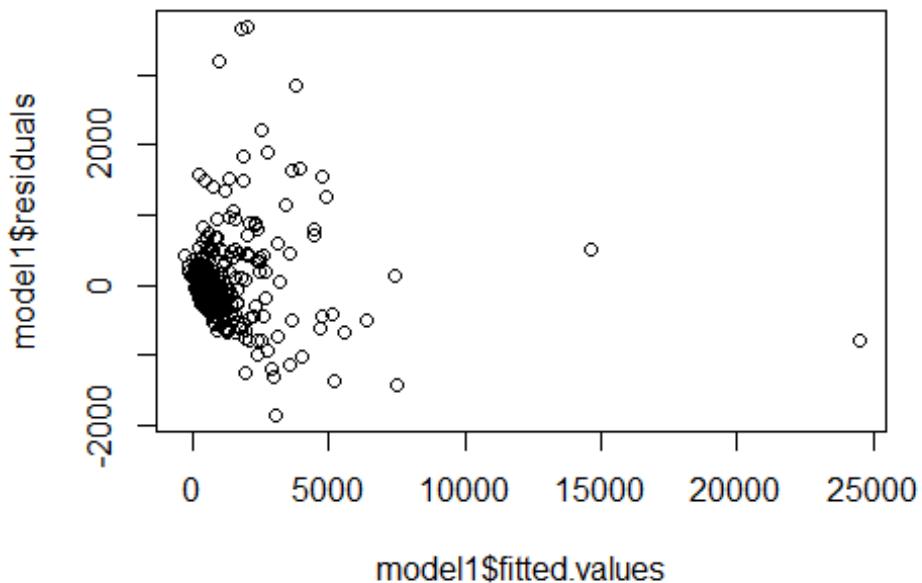
```

##          411          412          413          414          415
## -51.6612442 -54.1479393 -6.3620448 -61.6029302 43.7966233
##          416          417          418          419          420
## -19.8551108 -49.2297990 1629.0761342 26.6511316 48.6915438
##          421          422          423          424          425
## 46.9220277 30.6681366 -7.4602092 23.8945205 -12.1050690
##          426          427          428          429          430
##          431          432          433          434          435
## -71.2551251 -39.4192139 18.1526001 -47.9130275 -72.0451376
##          436          437          438          439          440
## -124.4791644 -174.6343004 22.1129299 47.5723066 -6.5861750

#Plots of residuals against fitted Y values
plot(model1$residuals ~ model1$fitted.values, main = "Residuals vs Fitted
values for model I")

```

Residuals vs Fitted values for model I

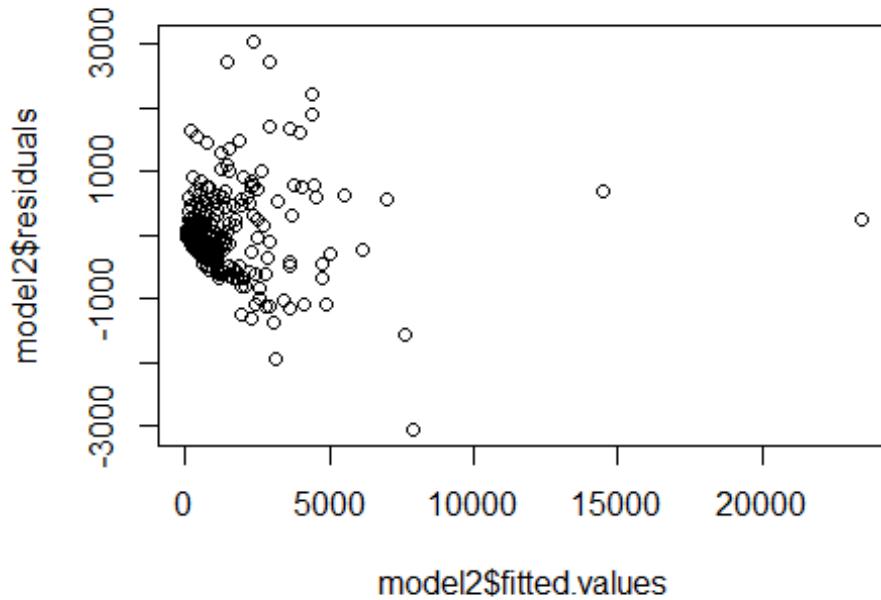


```

plot(model2$residuals ~ model2$fitted.values, main = "Residuals vs Fitted
values for model II")

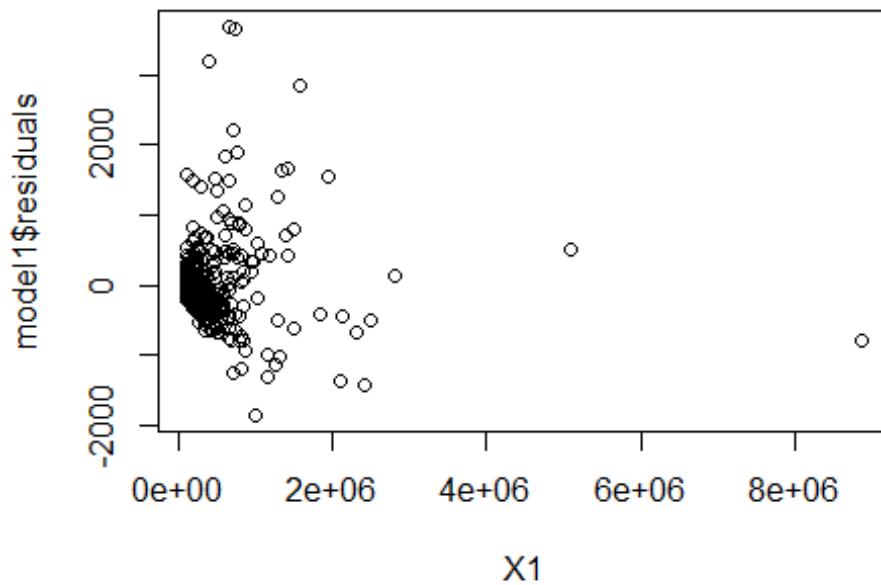
```

Residuals vs Fitted values for model II

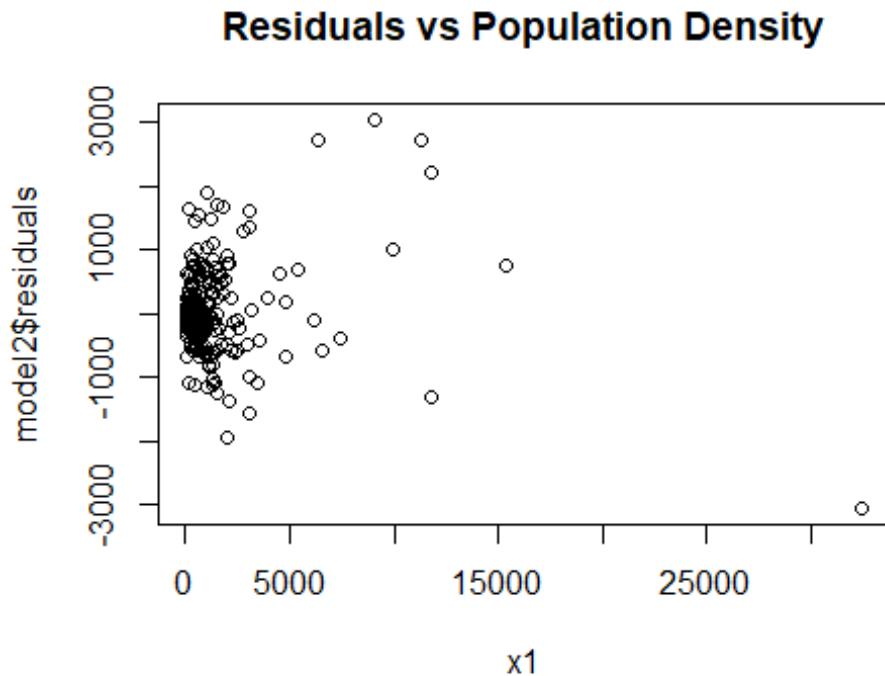


```
#Plots of residuals against predictor variables X1 and x1
plot(model1$residuals~X1, main = "Residuals vs Total population")
```

Residuals vs Total population

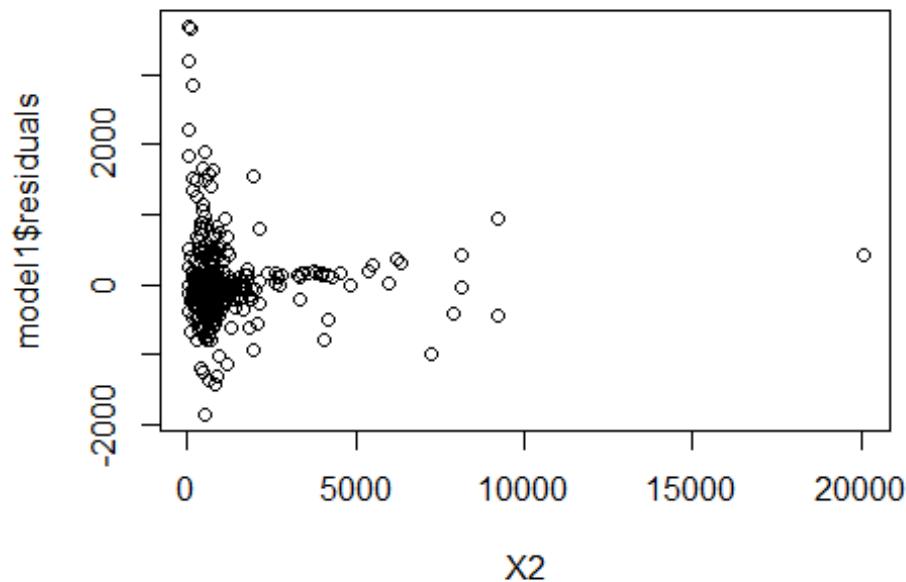


```
plot(model2$residuals~x1, main = "Residuals vs Population Density")
```



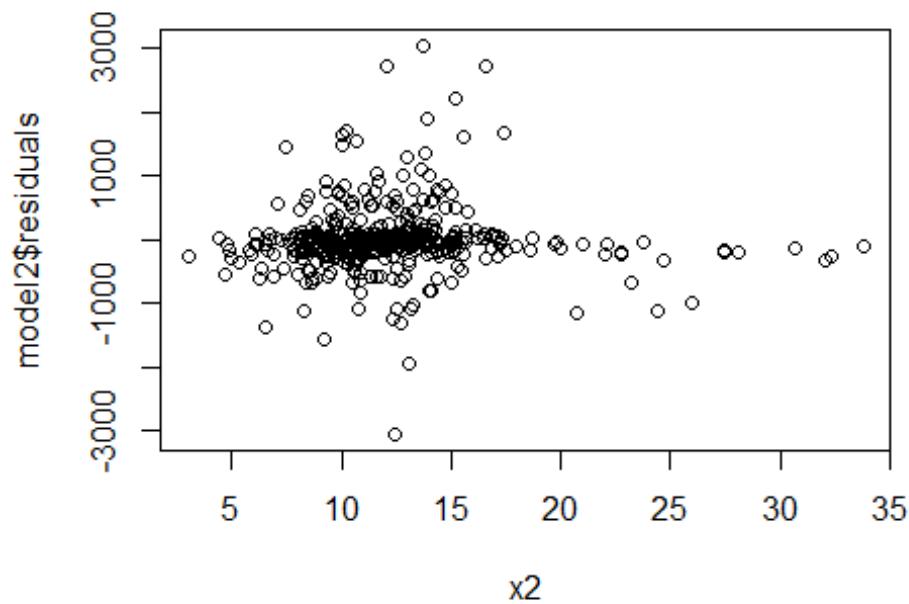
```
#Plots of residuals against predictor variables X2 and x2  
plot(model1$residuals~X2, main = "Residuals vs Land area")
```

Residuals vs Land area



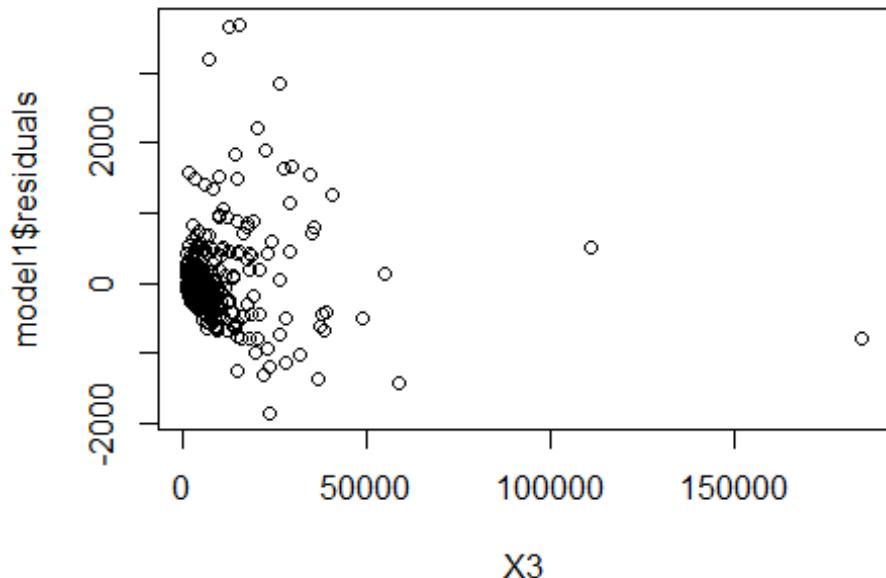
```
plot(model2$residuals~x2, main = "Residuals vs Percent of population greater than 64 years old")
```

Residuals vs Percent of population greater than 64 years old



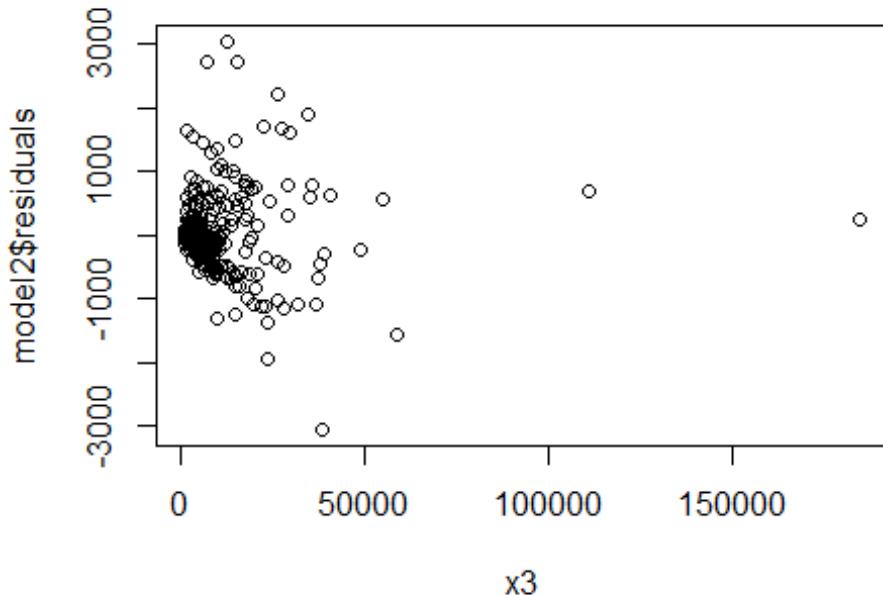
```
#Plots of residuals against predictor variables X3 and x3  
plot(model1$residuals~X3, main = "Residuals vs Total personal income")
```

Residuals vs Total personal income



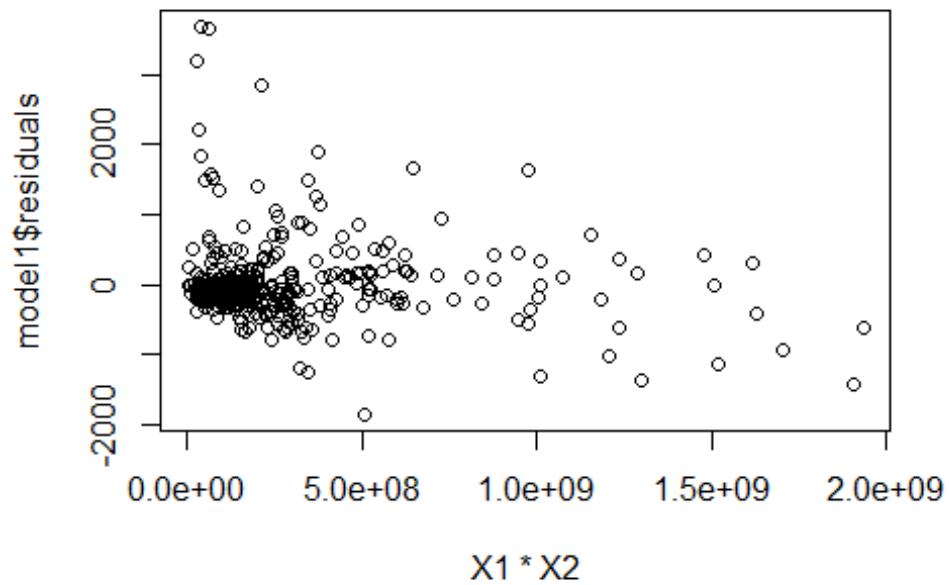
```
plot(model2$residuals~x3, main = "Residuals vs Total personal income")
```

Residuals vs Total personal income

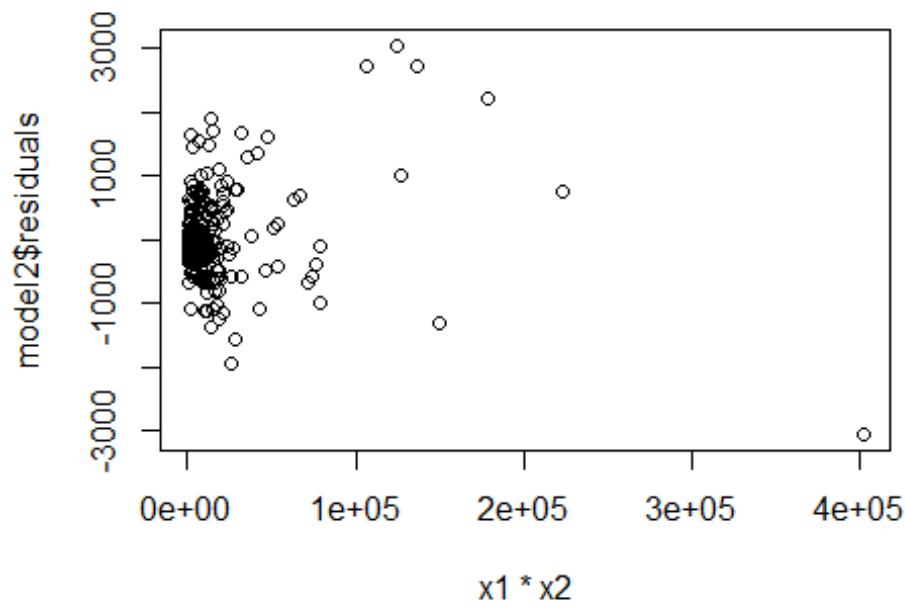


```
#Plots of residuals against predictor variables X1+X2 and x1+x2
plot(X1*X2,model1$residuals)

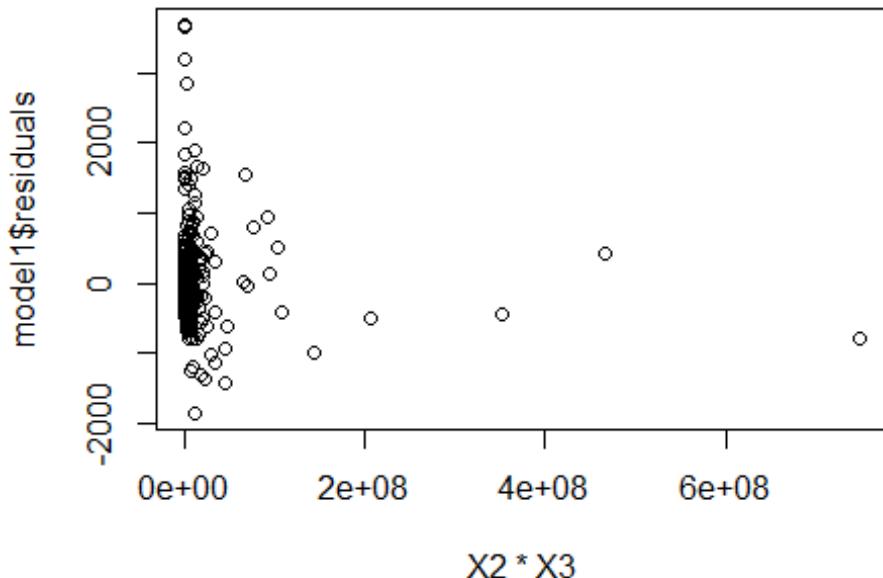
## Warning in X1 * X2: NAs produced by integer overflow
```



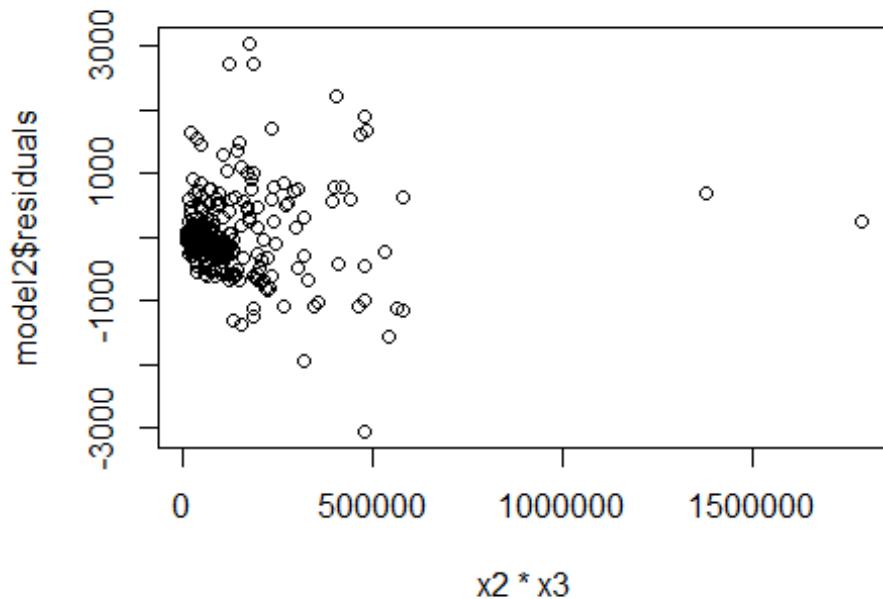
```
plot(x1*x2,model1$residuals)
```



```
#Plots of residuals against predictor variables X2+X3 and x2+x3  
plot(X2*X3,model1$residuals)
```

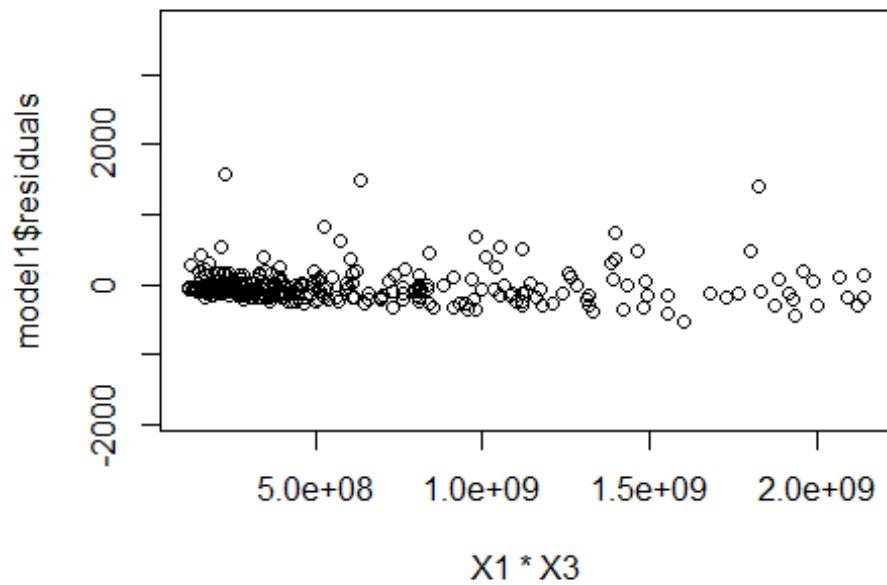


```
plot(x2*x3,model2$residuals)
```

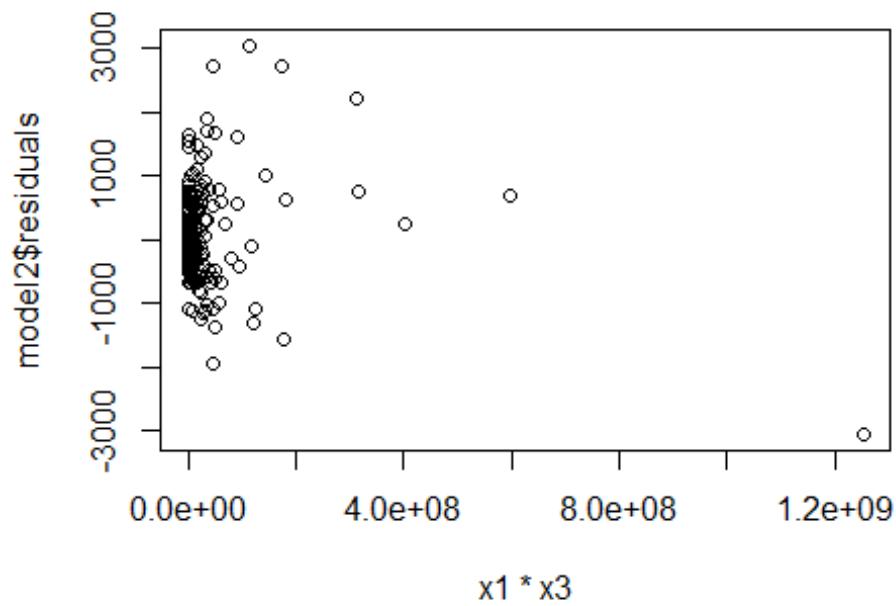


```
#Plots of residuals against predictor variables X1+X3 and x1+x3
plot(X1*X3,model1$residuals)

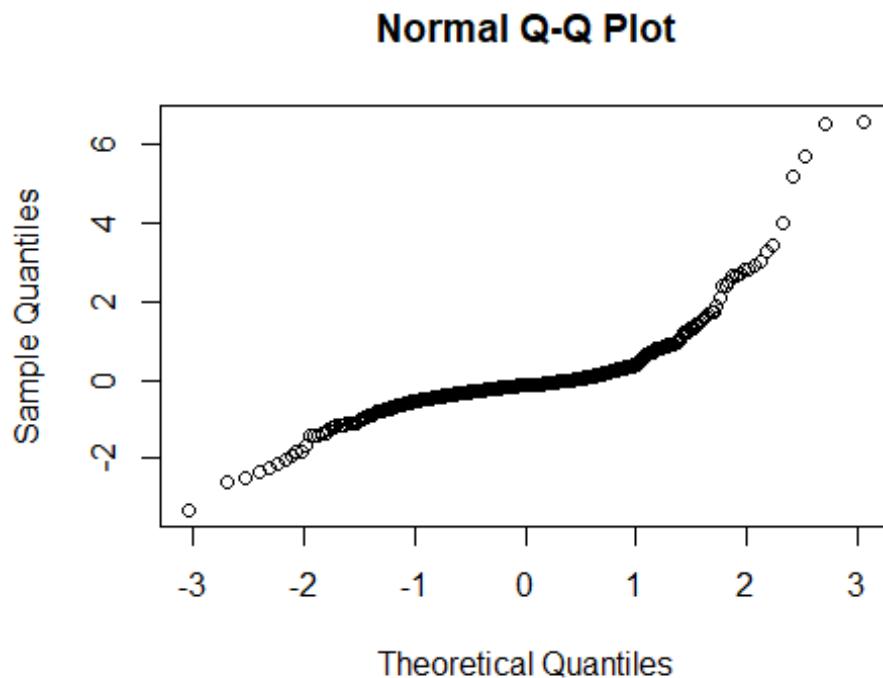
## Warning in X1 * X3: NAs produced by integer overflow
```



```
plot(x1*x3,model1$residuals)
```

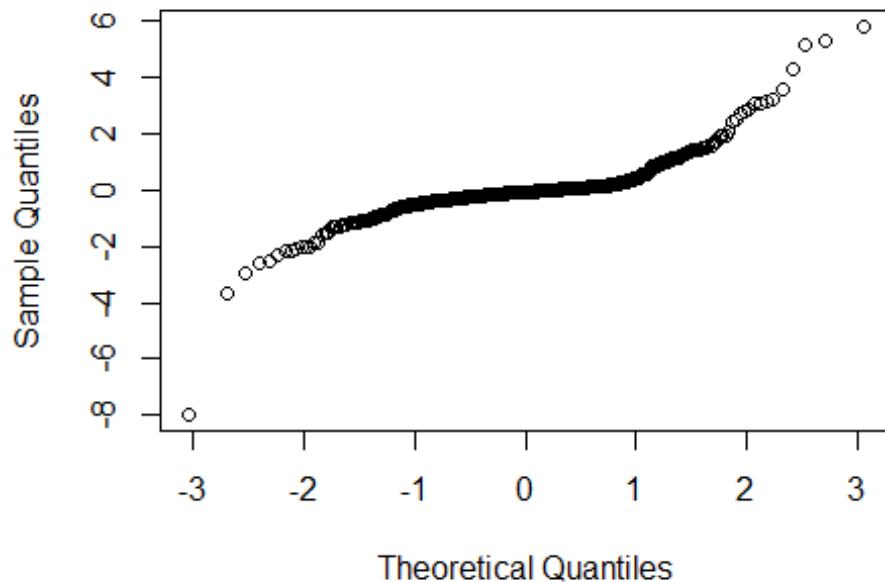


```
#Normal probability plot  
#ModelI  
plot1 = rstandard(model1)  
qqnorm(plot1)
```



```
#ModelII  
plot2 = rstandard(model2)  
qqnorm(plot2)
```

Normal Q-Q Plot



If we observe modelI, and if we specifically see the residuals plots against fitted values, there is no specific trend and there are some outliers as well. Variance and residuals are also not constant. From the plots of residuals vs predictors and two factor interactions, it can be seen that we can actually improve the model by including the variable $X1X2$ and $X1X3$ in our model. From the current predictor variables, the linear model doesn't seem to be appropriate.

If we observe modelII, and if we specifically see the residuals plots against fitted values, the variance for error term and residuals is not constant and there are some outliers as well. From the plots of residuals vs predictors and two factor interaction, the model seems to be appropriate with two factor interaction. Alos, total personal income varies constantly and shows normal distribution and there is no need to involve other variables. from the current predictors, the model is appropriate.

From the normal probability plot, we can see that the plot for both the models is increasing. Overall, the model II is more appropriate as compared to model I.

Part 2

(a)

```
cdi = read.table("D:\\ASU Stuff\\SEM-1\\STP 530\\CDI_data.txt")

model1 = subset(cdi, cdi[,17]==1)
model2 = subset(cdi, cdi[,17]==2)
model3 = subset(cdi, cdi[,17]==3)
model4 = subset(cdi, cdi[,17]==4)

#Model1

Y1 = model1$V10
X1a = model1$V5 / model1$V4
X1b = model1$V15
X1c = model1$V11

mod1 = lm(Y1~X1a+X1b+X1c)
mod1

##
## Call:
## lm(formula = Y1 ~ X1a + X1b + X1c)
##
## Coefficients:
## (Intercept)      X1a          X1b          X1c      
## -64466.231     17.383      -1.406     1182.577

#Model2

Y2 = model2$V10
X2a = model2$V5 / model2$V4
X2b = model2$V15
X2c = model2$V11

mod2 = lm(Y2~X2a+X2b+X2c)
mod2

##
## Call:
## lm(formula = Y2 ~ X2a + X2b + X2c)
##
## Coefficients:
## (Intercept)      X2a          X2b          X2c      
## -4163.2673    33.6193      0.1024     -2.7616
```

#Model3

```
Y3 = model3$V10
X3a = model3$V5 / model3$V4
X3b = model3$V15
X3c = model3$V11

mod3 = lm(Y3~X3a+X3b+X3c)
mod3

##
## Call:
## lm(formula = Y3 ~ X3a + X3b + X3c)
##
## Coefficients:
## (Intercept)      X3a          X3b          X3c
## 38862.667       5.537        1.957      -670.884
```

#Model4

```
Y4 = model4$V10
X4a = model4$V5 / model4$V4
X4b = model4$V15
X4c = model4$V11

mod4 = lm(Y4~X4a+X4b+X4c)
mod4

##
## Call:
## lm(formula = Y4 ~ X4a + X4b + X4c)
##
## Coefficients:
## (Intercept)      X4a          X4b          X4c
## 129323.415      5.717        4.342      -2159.920
```

Thus the estimated regression function for model1 is $Y = -64466.231 + 17.383X_1 - 1.406X_2 + 1182.577X_3$, model2 is $Y = -4163.2673 + 33.6193X_1 + 0.1024X_2 - 2.7616X_3$, model3 is $Y = 38862.667 + 5.537X_1 + 1.957X_2 - 670.884X_3$ and model4 is $Y = 129323.415 + 5717X_1 + 4.342X_2 - 2159.920X_3$.

part (b)

The B_0 values are different for all the four models. It can be seen that B_1 values are similar for 3 and 4. B_2 and B_3 are different for all the four models.

part (c)

```
summary(mod1)
```

```

## 
## Call:
## lm(formula = Y1 ~ X1a + X1b + X1c)
## 
## Residuals:
##    Min     1Q Median     3Q    Max 
## -149353 -3776    339   5091 130462 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -6.447e+04 4.256e+04 -1.515   0.1330    
## X1a          1.738e+01 8.336e-01 20.854  <2e-16 ***  
## X1b          -1.406e+00 7.606e-01 -1.849   0.0674 .    
## X1c          1.183e+03 6.411e+02  1.845   0.0681 .    
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 28060 on 99 degrees of freedom
## Multiple R-squared:  0.8352, Adjusted R-squared:  0.8302 
## F-statistic: 167.2 on 3 and 99 DF,  p-value: < 2.2e-16 

summary(mod2)

## 
## Call:
## lm(formula = Y2 ~ X2a + X2b + X2c)
## 
## Residuals:
##    Min     1Q Median     3Q    Max 
## -148517 -4535   -1011    4321 257651 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -4163.2673 49958.0969 -0.083   0.934    
## X2a          33.6193   3.8553   8.720 4.79e-14 ***  
## X2b          0.1024   1.6708   0.061   0.951    
## X2c         -2.7616  809.2473 -0.003   0.997    
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 32980 on 104 degrees of freedom
## Multiple R-squared:  0.5285, Adjusted R-squared:  0.5149 
## F-statistic: 38.86 on 3 and 104 DF,  p-value: < 2.2e-16 

summary(mod3)

## 
## Call:
## lm(formula = Y3 ~ X3a + X3b + X3c)
## 
## Residuals:

```

```

##      Min     1Q Median     3Q    Max
## -75116 -15583 -9111   3217 217684
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 38862.667  30662.721   1.267  0.2070
## X3a          5.537     2.377   2.329  0.0212 *
## X3b          1.957     1.096   1.787  0.0761 .
## X3c         -670.884   511.472  -1.312  0.1917
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 36970 on 148 degrees of freedom
## Multiple R-squared:  0.09251, Adjusted R-squared:  0.07411
## F-statistic: 5.029 on 3 and 148 DF, p-value: 0.002392

summary(mod4)

##
## Call:
## lm(formula = Y4 ~ X4a + X4b + X4c)
##
## Residuals:
##      Min     1Q Median     3Q    Max
## -101554 -25633 -14428   3976 608081
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 129323.415  95555.937   1.353  0.180
## X4a          5.717     5.774   0.990  0.325
## X4b          4.342     2.731   1.590  0.116
## X4c         -2159.920   1353.579  -1.596  0.115
##
## Residual standard error: 81820 on 73 degrees of freedom
## Multiple R-squared:  0.08665, Adjusted R-squared:  0.04912
## F-statistic: 2.309 on 3 and 73 DF, p-value: 0.08351

```

Model1: MSE: 7.874 $R^2 = SSR/SSTO = 0.8352$

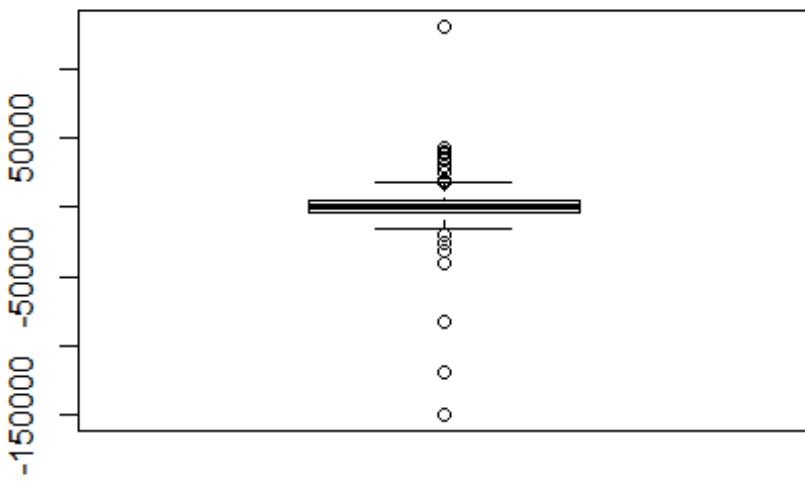
Model2: MSE: $1.08 \times 10^9 R^2 = 0.5280$

Model3: MSE: $1.365 \times 10^9 R^2 = 0.092$

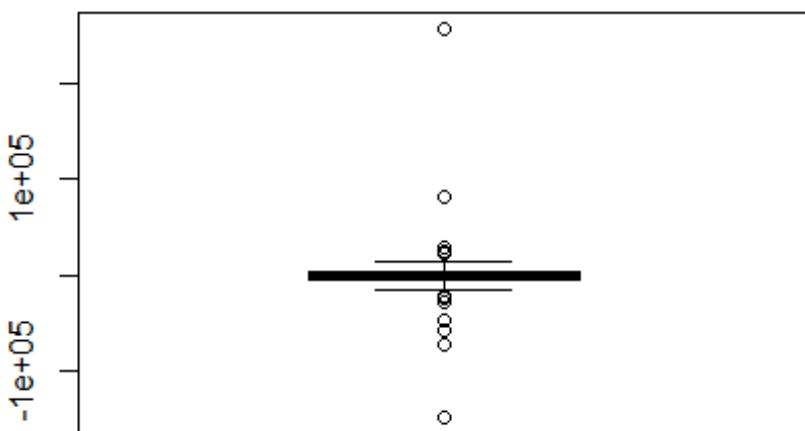
Model4: MSE: $6.69 \times 10^9 R^2 = 0.0867$

part (d)

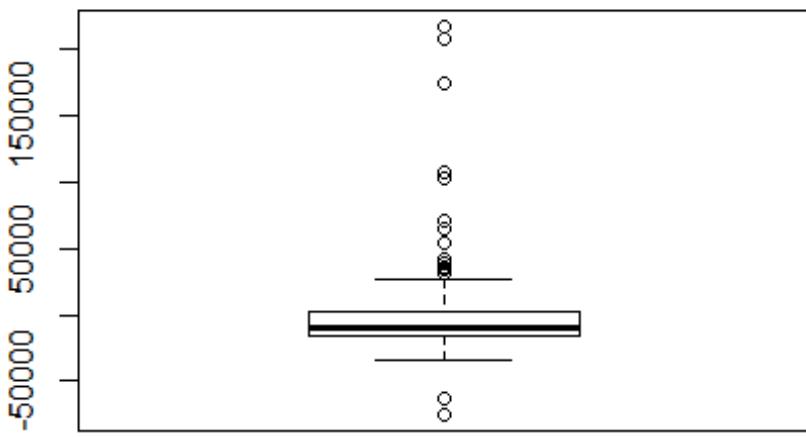
```
boxplot(resid(mod1))
```



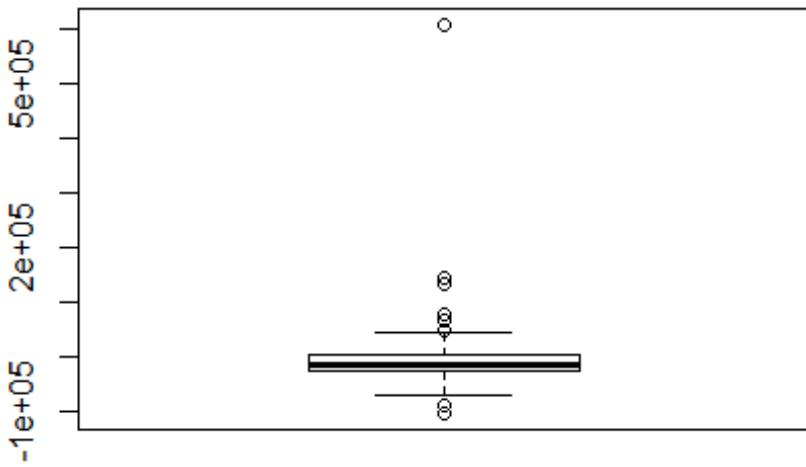
```
boxplot(resid(mod2))
```



```
boxplot(resid(mod3))
```



```
boxplot(resid(mod4))
```



Model1: In this model, we can see that there are a few outliers above as well as in the below region.

Model2: Majority of the outliers lie in the below region.

Model3: Majority of the outliers lie in the above region.

Model4: Majority of the outliers lie in the above region and one is specifically distant from each other.

Part - 3 (10 points) :-

a) $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 \log_{10} x_{i2} + \beta_3 x_{i1}^2 + \epsilon_i$

By comparing this model with the general linear regression model, we can say easily say that it is not a general linear regression model.

But, it can be expressed in the form of (6.7) by making two transformations as follows:-

$$x_{i3} = \log_{10} x_{i2} \quad \text{and} \quad x_{i4} = x_{i1}^2$$

$$\Rightarrow y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i3} + \beta_3 x_{i4} + \epsilon_i$$

b) $y_i = \epsilon_i \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}^2)$

It is not a general linear regression model and also, it cannot be expressed in the form of (6.7) by a suitable transformation.

c) $y_i = \log_{10} (\beta_1 x_{i1}) + \beta_2 x_{i2} + \epsilon_i$

It is not a general linear regression model and also, it cannot be expressed in the form

of (6.7) by a suitable transformation

d) $y_i = \beta_0 \exp(\beta_1 x_{ii}) + \epsilon_i$

It is not a general linear regression model
and also, it cannot be expressed in the
form of (6.7) by a suitable transformation.

e) $y_i = [1 + \exp(\beta_0 + \beta_1 x_{ii} + \epsilon_i)]^{-1}$

It is not a general linear regression model
and it can be expressed in the form of
(6.7) by suitable transformations.

$$y'_i = \log_e \left(\frac{1}{y_i} - 1 \right)$$

$$\Rightarrow y'_i = \beta_0 + \beta_1 x_{ii} + \epsilon_i$$

Part-4 (10 points) :-

Multiple regression model :-

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i1}^2 + \beta_3 X_{i2} + \epsilon_i \quad i=1, \dots, n$$
$$\epsilon_i \sim N(0, \sigma^2)$$

By transformation $X_{i3} = X_{i1}^2$, we can re-write the above equation as:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i3} + \beta_3 X_{i2} + \epsilon_i$$

(a) least squares criterion :-

$$Q = \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i3} - \beta_3 X_{i2})^2$$

The least square estimators are those values of $\beta_0, \beta_1, \beta_2, \beta_3$ that minimizes Q .

least squares Normal Equations:-

Here,

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & X_{11} & X_{13} & X_{12} \\ 1 & X_{21} & X_{23} & X_{22} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & X_{n1} & X_{n3} & X_{n2} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

y_t can be represented in matrix terms as:

$$Y = XB + \varepsilon$$

$n \times 1$ ↓ ↓ ↓
 $n \times 4$ 4×1 $n \times 1$

Now, if we differentiate Q w.r.t $\beta_0, \beta_1, \beta_2$ and β_3 ,

$$X'Xb = X'y$$

$$\therefore b = (X'X)^{-1}(X'y)$$

Also, $X'Xb \approx X'y$ can be expressed in matrix as -

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ x_{11} & x_{21} & \dots & x_{n1} \\ x_{13} & x_{23} & \dots & x_{n3} \\ x_{12} & x_{22} & \dots & x_{n2} \end{bmatrix} \begin{bmatrix} 1 & x_{11} & x_{13} & x_{12} \\ x_{21} & x_{23} & x_{22} \\ \vdots & \vdots & \vdots \\ x_{n1} & x_{n3} & x_{n2} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_{11} & x_{21} & \dots & x_{n1} \\ x_{13} & x_{23} & \dots & x_{n3} \\ x_{12} & x_{22} & \dots & x_{n2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$\therefore \begin{bmatrix} n & \sum x_{ii} & \sum x_{i3} & \sum x_{i2} \\ \sum x_{ii} & \sum x_{ii}^2 & \sum x_{ii}x_{i3} & \sum x_{ii}x_{i2} \\ \sum x_{i3} & \sum x_{i3}x_{ii} & \sum x_{i3}^2 & \sum x_{i3}x_{i2} \\ \sum x_{i2} & \sum x_{i2}x_{ii} & \sum x_{i2}x_{i3} & \sum x_{i2}^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_{ii}y_i \\ \sum x_{i3}y_i \\ \sum x_{i2}y_i \end{bmatrix}$$

Thus, least squares normal equations are:

$$\sum y_i = nb_0 + b_1 \sum x_{ii} + b_2 \sum x_{i3} + b_3 \sum x_{i2}$$

$$\sum x_{ii}y_i = b_0 \sum x_{ii} + b_1 \sum x_{ii}^2 + b_2 \sum x_{ii}x_{i3} + b_3 \sum x_{ii}x_{i2}$$

$$\sum x_{i3}y_i = b_0 \sum x_{i3} + b_1 \sum x_{i3}x_{ii} + b_2 \sum x_{i3}^2 + b_3 \sum x_{i3}x_{i2}$$

$$\sum x_{i2}y_i = b_0 \sum x_{i2} + b_1 \sum x_{i2}x_{ii} + b_2 \sum x_{i2}x_{i3} + b_3 \sum x_{i2}^2$$

(b) Likelihood Function :-

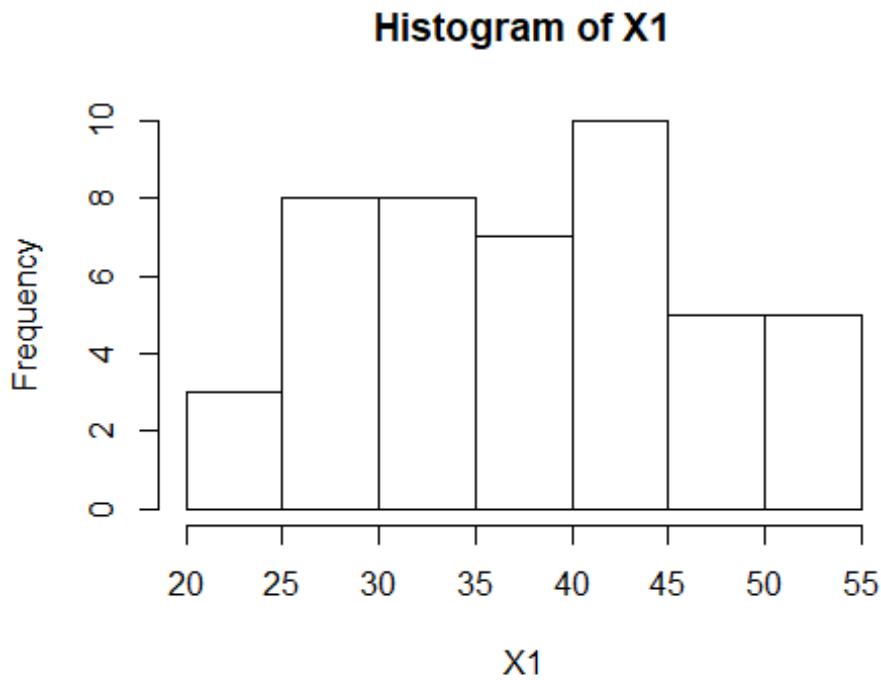
$$L(B_0, B_1, B_2, B_3, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp \left[-\frac{1}{2\sigma^2} \sum_{i=1}^n (Y_i - B_0 - B_1 X_{i1} - B_2 X_{i2} - B_3 X_{i3})^2 \right]$$

Here, maximum likelihood estimators will be the same as the least squares estimators because when the errors are normally distributed like in the normal regression model, the least squares estimators are optimal and fortunately, the maximum likelihood estimator also gives the optimal value when error is distributed normally.

Part 5

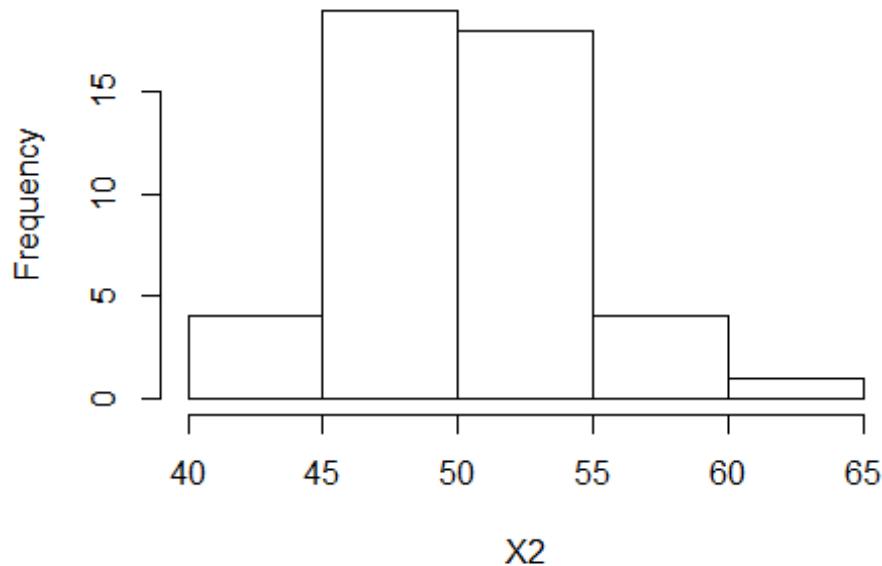
(a)

```
patsat = read.table("D:\\ASU Stuff\\SEM-1\\STP  
530\\Patient_satisfaction.txt")  
Y = patsat$V1  
X1 = patsat$V2  
X2 = patsat$V3  
X3 = patsat$V4  
  
#histogram of Patient's age  
hist(X1)
```



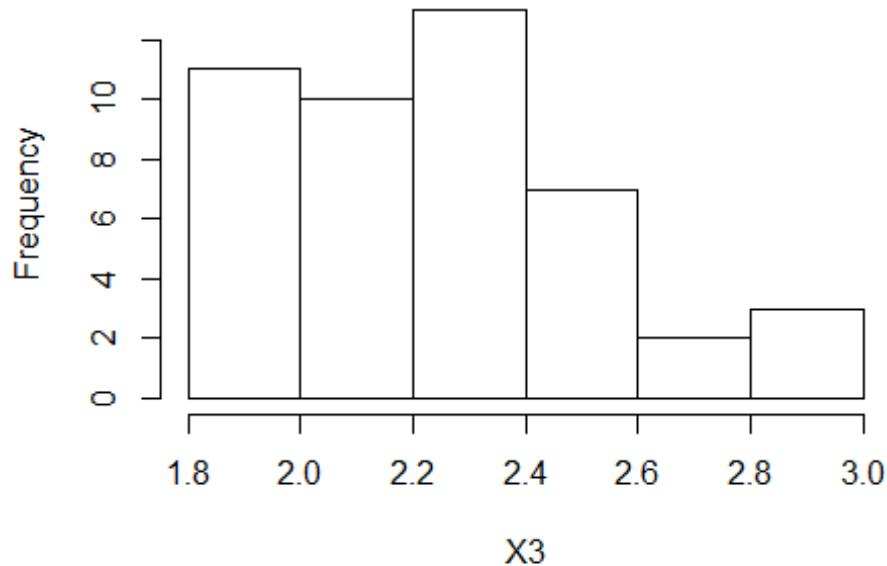
```
#histogram of severity of illness  
hist(X2)
```

Histogram of X2



```
#histogram of anxiety level  
hist(x3)
```

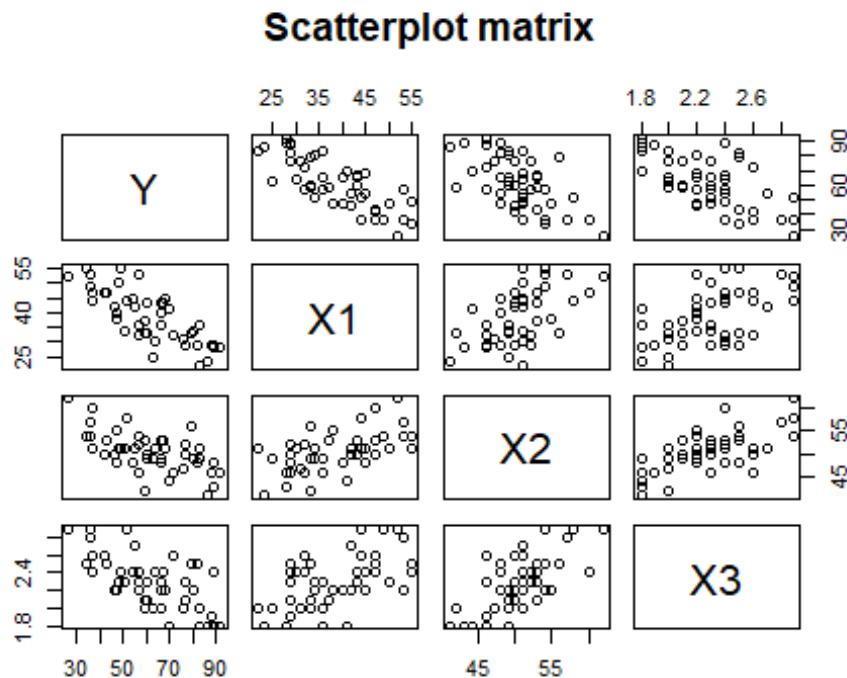
Histogram of X3



From the histogram plots, no noteworthy features are observed for predictor variables X1 and X3, but the plot of variable X2 is approximately normal.

(b)

```
#Scatterplot matrix  
pairs(~Y+X1+X2+X3, data=patsat, main="Scatterplot matrix")
```



```
#Correlation matrix  
C1 = cbind(X1,X2,X3)  
cor(C1)  
  
## X1 X2 X3  
## X1 1.0000000 0.5679505 0.5696775  
## X2 0.5679505 1.0000000 0.6705287  
## X3 0.5696775 0.6705287 1.0000000
```

From the scatterplot matrix, we can see that there is a linear relation between response variable and all the predictor variables i.e. X1, X2 and X3. They are infact linearly decreasing. Also, the data is more clustered together in X1 as compared to X2 and X3. From the correlation matrix, X2 and X3 are strongly correlated than correlation of X1 and X2 and also correlation of X1 and X3.

(c)

```
model = lm(Y~X1+X2+X3)  
model
```

```

## 
## Call:
## lm(formula = Y ~ X1 + X2 + X3)
## 
## Coefficients:
## (Intercept)          X1          X2          X3
##       158.491      -1.142      -0.442     -13.470
b2 = model$coefficients[3]
b2

##          X2
## -0.4420043

```

The estimated regression function is $Y = 158.491 - 1.142X_1 - 0.442X_2 - 13.470X_3$. the Value of b2 seems to be very less in this model.

(d)

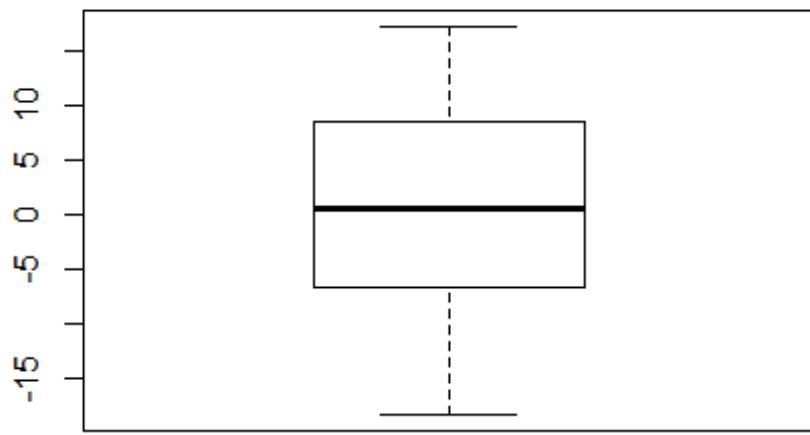
```

e = resid(model)
e

##           1          2          3          4          5          6
##  0.1129334 -9.0796538  4.0237858  2.0093153  5.7263570 -3.6205678
##           7          8          9         10         11         12
## -12.8089820  0.4258777 -6.6596981  2.0030477 17.1600881 13.3526753
##          13         14         15         16         17         18
## -14.1654081 -15.1528562 12.5167654 -2.7946900 16.6095859  8.5409980
##          19         20         21         22         23         24
## -10.8725092  8.1680089  5.5810888  8.4393900  3.6796462 -3.8657107
##          25         26         27         28         29         30
## -4.7338610 -4.1589620 -18.3524203  5.3949478 -9.6470593  3.3681039
##          31         32         33         34         35         36
## -16.3135553 11.5112774  0.6132423 -14.9762142  0.9248761 11.6161190
##          37         38         39         40         41         42
## 11.5071044 -5.3722872 -8.9868475 -5.7128575 11.0056590 -0.8932473
##          43         44         45         46
## -13.6956888 13.0578578 -5.5380448 10.0523698

#Boxplot of residuals
boxplot(resid(model))

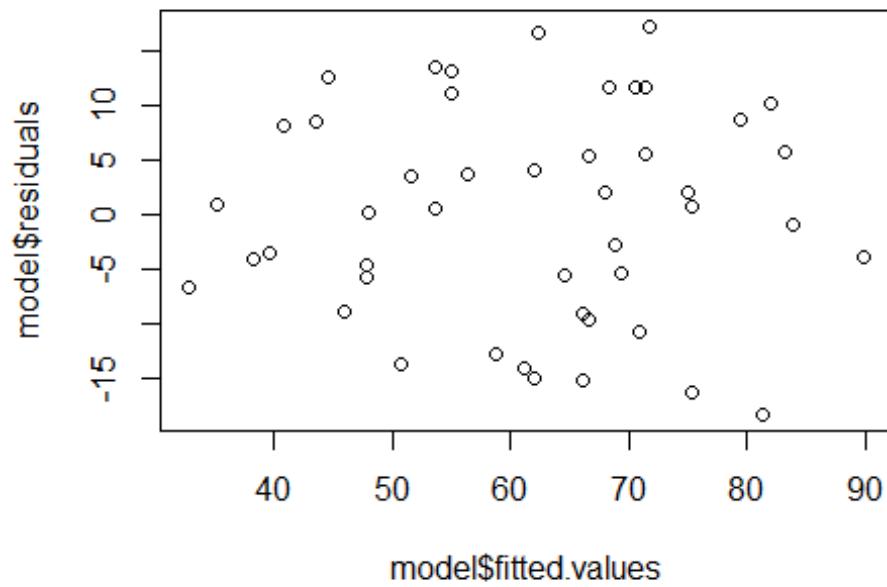
```



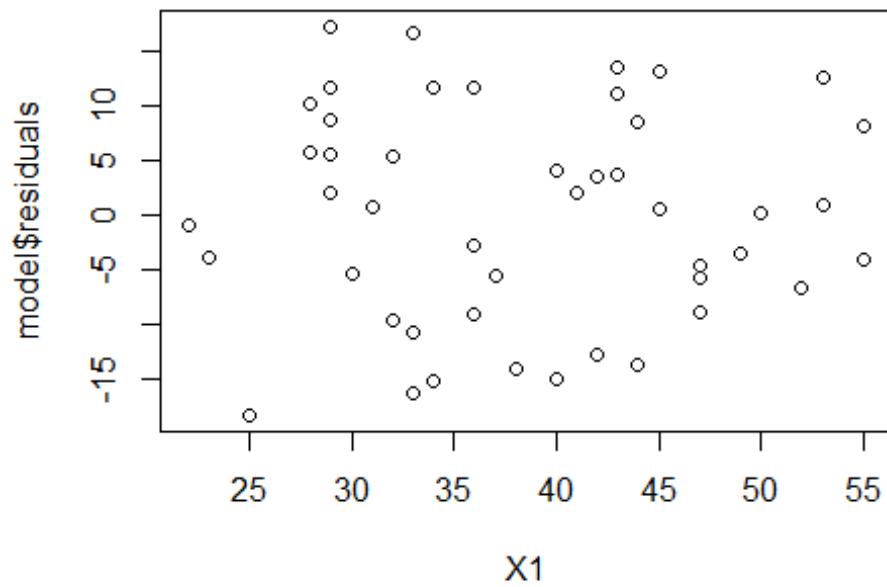
From the box-plot diagram, it is very evident that there are no outliers.

(e)

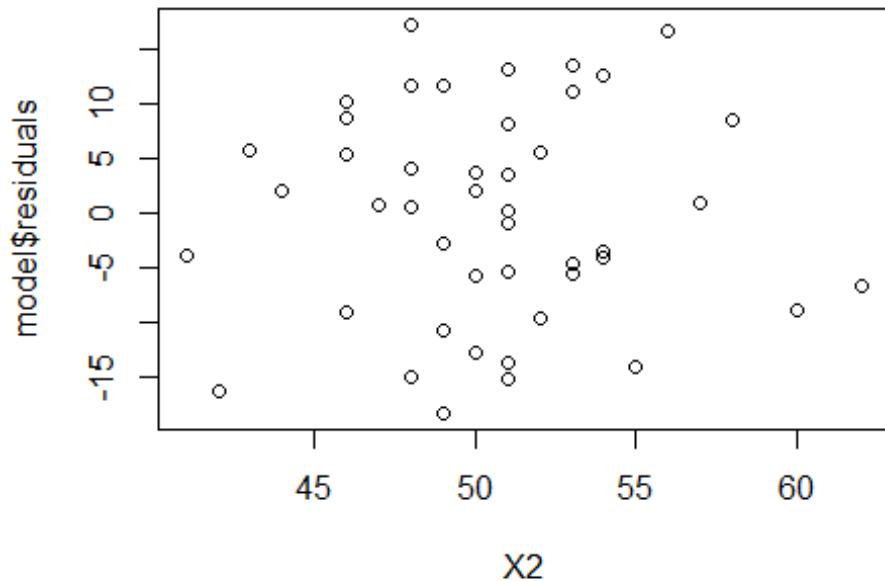
```
#Residuals vs fitted values  
plot(model$residuals~model$fitted.values)
```



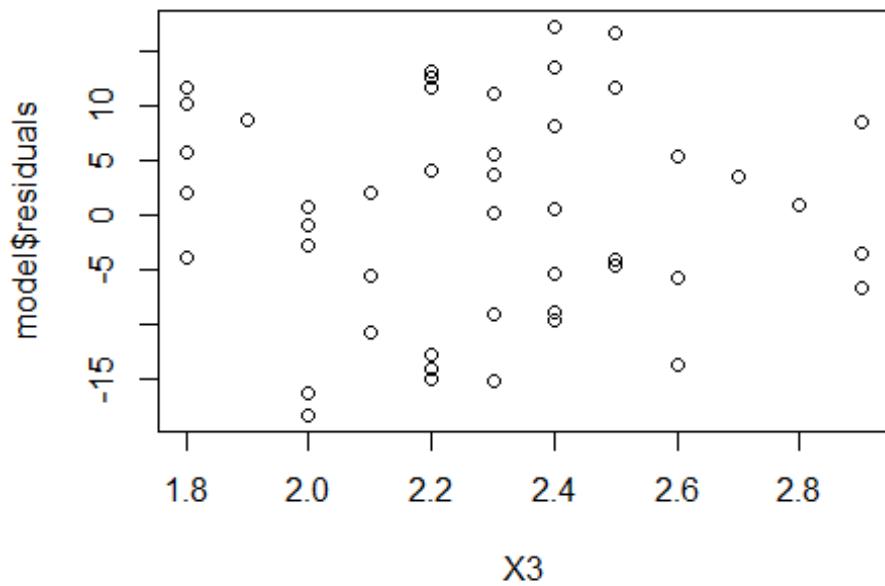
```
#Residuals vs predictor variables  
plot(model$residuals~X1) #with predictor variable X1
```



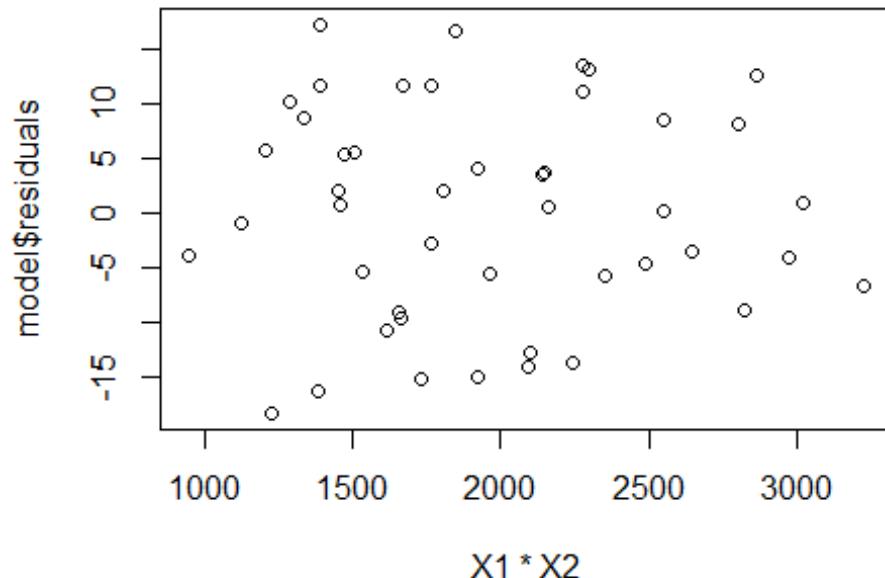
```
plot(model$residuals~X2) #with predictor variable X2
```



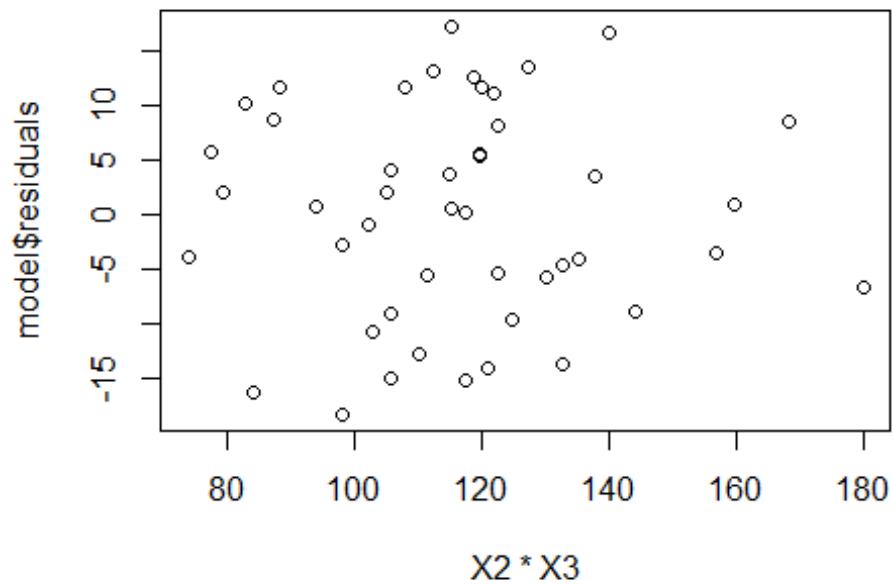
```
plot(model$residuals~X3) #with predictor variable X3
```



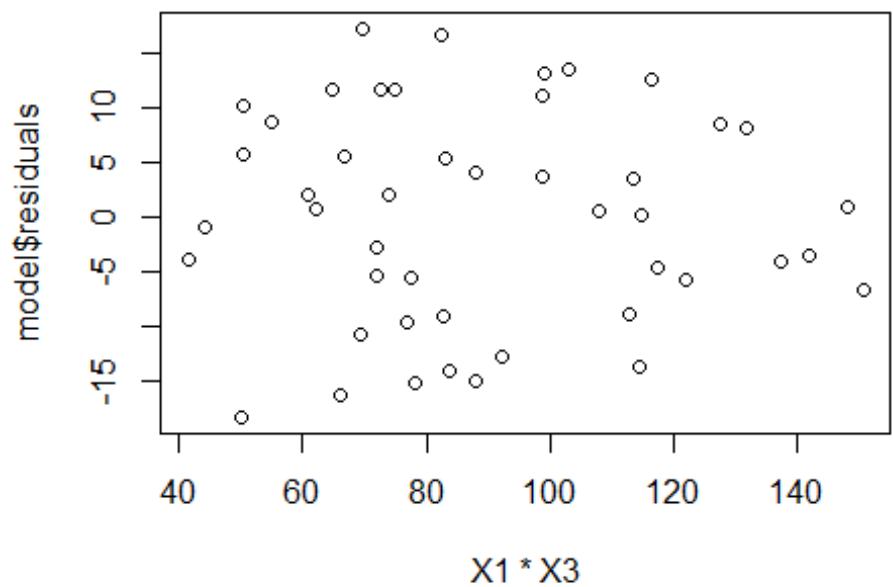
```
#Residuals vs two factor interaction  
plot(X1*X2, model$residuals)
```



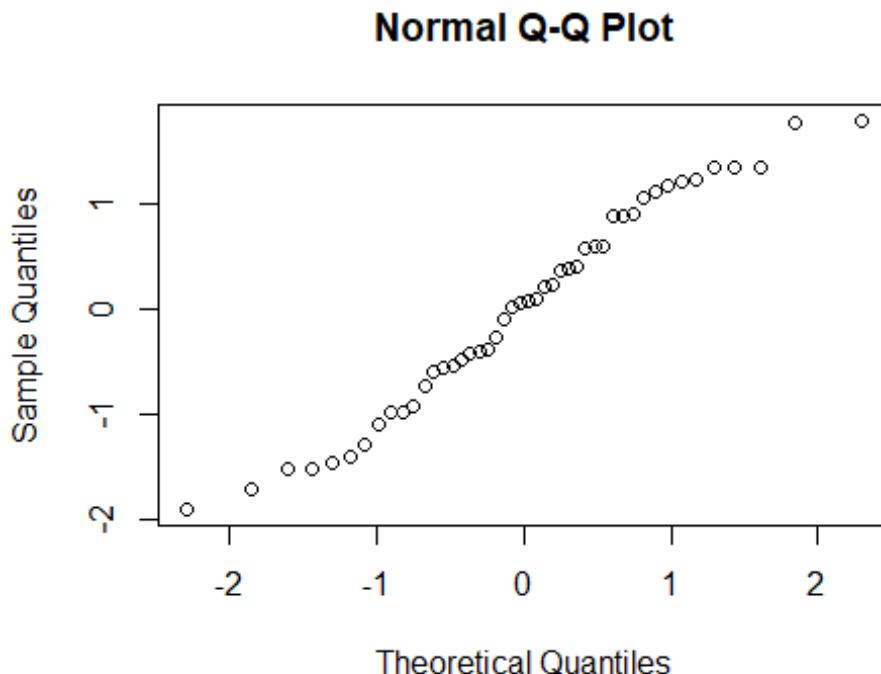
```
plot(X2*X3, model$residuals)
```



```
plot(X2*X3, model$residuals)
```



```
#Normal probability plot
plot1 = rstandard(model)
qqnorm(plot1)
```



From the plots of residuals vs fitted values, it clear that the error term of residuals and variance is constant which in turn gives that the linear model is appropriate. The same is the case with the plots of residuals vs each of the predictor variables and residuals vs two factor interaction.

Also, from the normal probability plot, it can be seen that as the theoretical quantiles increase, so do the sample quantiles.

(f)

When replicate observations are not available, an appropriate lack of fit test can be conducted if there are cases that have similar X_h vectors. In this case, we cannot do the formal test for lack of fit.

(g)

H_0 :the variance is constant; H_1 :The variance is not constant. $SSR = 21,355.5$, $SSE = 4248.8$, $X_B P^2 = (SSR/2)/(4248.8/46)^2 = 1.2516$. If $X_B^2 P \leq 11.34499$, conclude error variance constant, otherwise not constant. Hence, $X_B P^2 \leq X^2(0.99,3)$, therefore, we conclude H_0 .

Part 6

```
patsat = read.table("D:\\ASU Stuff\\SEM-1\\STP  
530\\Patient_satisfaction.txt")  
Y = patsat$V1  
X1 = patsat$V2  
X2 = patsat$V3  
X3 = patsat$V4  
  
mod = lm(Y~X1+X2+X3)  
mod  
  
##  
## Call:  
## lm(formula = Y ~ X1 + X2 + X3)  
##  
## Coefficients:  
## (Intercept) X1 X2 X3  
## 158.491 -1.142 -0.442 -13.470  
  
anova(mod)  
  
## Analysis of Variance Table  
##  
## Response: Y  
## Df Sum Sq Mean Sq F value Pr(>F)  
## X1 1 8275.4 8275.4 81.8026 2.059e-11 ***  
## X2 1 480.9 480.9 4.7539 0.03489 *  
## X3 1 364.2 364.2 3.5997 0.06468 .  
## Residuals 42 4248.8 101.2  
## ---  
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
  
summary(mod)  
  
##  
## Call:  
## lm(formula = Y ~ X1 + X2 + X3)  
##  
## Residuals:  
## Min 1Q Median 3Q Max  
## -18.3524 -6.4230 0.5196 8.3715 17.1601  
##  
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 158.4913 18.1259 8.744 5.26e-11 ***  
## X1 -1.1416 0.2148 -5.315 3.81e-06 ***  
## X2 -0.4420 0.4920 -0.898 0.3741  
## X3 -13.4702 7.0997 -1.897 0.0647 .
```

```

## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.06 on 42 degrees of freedom
## Multiple R-squared:  0.6822, Adjusted R-squared:  0.6595
## F-statistic: 30.05 on 3 and 42 DF,  p-value: 1.542e-10

qf(0.9,3,42)

## [1] 2.219059

B = qt(1-0.1/6,42)

beta0 = mod$coefficients[1]
beta1 = mod$coefficients[2]
beta2 = mod$coefficients[3]
beta3 = mod$coefficients[4]

sbeta1 = 0.2148
sbeta2 = 0.4920
sbeta3 = 70.997

beta1a = beta1-B*sbeta1
beta0aa = beta1+B*sbeta1
beta0a = beta2-B*sbeta2
beta0aa = beta2+B*sbeta2
beta0a = beta3-B*sbeta3
beta0aa = beta3+B*sbeta3

yh = beta0 + beta1*35 + beta2*45 + beta3*2.2

dat = data.frame(X1=35,X2=45,X3=2.2)

predict(mod,dat,interval="confidence",level=0.90)

##          fit      lwr      upr
## 1 69.01029 64.52854 73.49204

predict(mod,dat,interval="prediction",level=0.90)

##          fit      lwr      upr
## 1 69.01029 51.50965 86.51092

```

Part - 6

a) $H_0: \beta_1 = \beta_2 = \beta_3 = \dots = 0$

$H_a:$ Not all β_k are 0.

$$MSR = 3040.2, MSE = 101.2$$

$$F^* = \frac{MSR}{MSE} = \frac{3040.2}{101.2} = 30.05$$

If $F^* \leq F(1-\alpha; p-1, n-p)$, conclude H_0 .
 $F^* > F(1-\alpha; p-1, n-p)$, conclude H_a .

$$F(1-0.1, 3, 42) = 2.0837$$

$$\Rightarrow F^* > F(1-\alpha; p-1, n-p)$$

Thus, we conclude H_a .

$$P\text{-value} = 0.4878$$

b) $B = t \left(1 - \frac{\alpha}{2g}, n-p \right)$

$$= t \left(1 - \frac{0.1}{6}, 42 \right)$$

$$= 2.20038$$

$$S_{\hat{b}_1 y} = 0.2148$$

$$S_{\hat{b}_2 y} = 0.4420$$

$$S_{\hat{b}_3 y} = 1.0447$$

•

$$-1.6140 \leq \beta_1 \leq -0.6790$$

$$-0.6394 \leq \beta_2 \leq 0.6400$$

$$-29.0858 \leq \beta_3 \leq 2.145$$

c) Coefficient of Multiple determination

$$R^2 = \frac{SSR}{SSTO} = \frac{9120.50}{133894} = 0.6759$$

R^2 measures the proportionate reduction of total variation of Y w.r.t. predictor variables. Thus, it means there is a 67.59% reduction of total variation of Y.

d) $\hat{Y}_h \pm t(1-\alpha/2, n-p) s_{\text{pred}} \sqrt{3}$

$$\hat{Y}_h = 158.50 - 1.140 X_{1h} - 0.440 X_{2h} - 18.470 X_{3h}$$

$$X_{1h} = 35, X_{2h} = 15, X_{3h} = 2.2$$

$$\hat{Y}_h = 69.010$$

e) Prediction interval:

$$\hat{Y}_h \pm t(1-\alpha/2, n-p) s_{\text{pred}} \sqrt{3}$$

$$61.509 \leq \hat{Y}_{\text{new}} \leq 86.470$$