Artificial Intelligence (CMSC 671) Assignment 4

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PART III. RESOLUTION AND FORMAL LOGIC (30 PTS.)

10. Represent the following knowledge base in first-order logic. (8 pts)

a) Everything that is hardy, watered, and fertilized will be well-rooted.

Ans: ∀x (hardy(x) ∧ watered(x) ∧ fertilized(x)) → rooted(x)

b) Everything that is well-rooted will survive a planting if it is in season.

Ans: ∀x ∀t rooted(x) ∧ in-season(x) → survive (x, t)

c) A plant survives a planting if and only if they don't die.

Ans: survive (x, t) ↔ die (x, t)

d) Every native plant is hardy.

Ans: ∀x native(x) → hardy(x)

e) If a planting isn't in season, every plant will die in the planting.

Ans: ∀x ¬in-season(x) → ∃t die (x, t))

f) My inkberry is a native plant.

Ans: inkberry → native(inkberry) (using inkberry for my inkberry)

g) My holly survived the fall planting.

Ans: holly → survive (holly, fall-planting) (using holly for my_holly and fall-planting for fall planting)

h) I fertilize my inkberry.

Ans: inkberry → fertilized(inkberry)

i) My inkberry gets watered.

Ans: inkberry → watered(inkberry)

11. Convert the KB to conjunctive normal form. (Hint: you will need to define three constants, in addition to the predicates above and variables x and t.) (8 points)

Ans: a) ∀x (hardy(x) ∧ watered(x) ∧ fertilized(x) →rooted(x))

=∀x (**¬**(hardy(x) ∧ watered(x) ∧ fertilized(x)) ∨ rooted(x)) …...using P → Q = ¬P ∨ Q

=∀x (**¬**hardy(x) ∨ **¬**watered(x) ∨ **¬**fertilized(x) ∨ rooted(x))

= **¬**hardy(x) ∨ **¬**watered(x) ∨ **¬**fertilized(x) ∨ rooted(x)……dropping quantifier

b) $\forall x\ \forall t(\text{rooted}(x) \land \text{in-season}(x) \rightarrow \text{survive}(x, t))$

$= \forall x\ \forall t\ (\neg(\text{rooted}(x) \land \text{in-season}(x)) \lor \text{survive}(x, t))$ ……using $P \rightarrow Q = \neg P \lor Q$

$= \forall x\ \forall t\ (\neg \text{rooted}(x) \lor \neg\ \text{in-season}(x) \lor \text{survive}(x, t))$

$= \neg \text{rooted}(x) \lor \neg\ \text{in-season}(x) \lor \text{survive}(x, t)$ ……dropping quantifier

c) $(\text{survive}(x, t) \leftrightarrow \text{die}(x, t))$

$= (\text{survive}(x, t) \longrightarrow \text{die}(x, t) \land \text{die}(x, t) \longrightarrow \text{survive}(x, t))$ ……using $P \leftrightarrow Q = P \rightarrow Q \land Q \rightarrow P$

$= (\neg \text{survive}(x, t) \lor \text{die}(x, t)) \land (\neg \text{die}(x, t) \lor \text{survive}(x, t))$ ……using $P \rightarrow Q = \neg P \lor Q$

d) $\forall x\ \text{native}(x) \rightarrow \text{hardy}(x)$

$= \forall x\ (\neg \text{native}(x) \lor \text{hardy}(x))$ ……using $P \rightarrow Q = \neg P \lor Q$

$= \neg \text{native}(x) \lor \text{hardy}(x)$ ……dropping quantifier

e) $\forall x\ \neg \text{in-season}(x) \longrightarrow \exists t\ \text{die}(x, t)$

$= \neg\ (\forall x\ \neg \text{in-season}(x)) \lor \exists t\ \text{die}(x, t))$ ……using $P \rightarrow Q = \neg P \lor Q$

$= \exists x\ \text{in-season}(x) \lor \exists t\ \text{die}(x, t)$ ……using $\neg\ \forall x = \exists x$

$= \text{in-season}(F(x)) \lor \text{die}(x, F(x))$ ……Skolemization

f) inkberry $\rightarrow$ native(inkberry) (creating variable inkberry for my inkberry)

$= \neg$inkberry $\lor$ native(inkberry) ……using $P \rightarrow Q = \neg P \lor Q$

g) holly $\rightarrow$ survive (holly, fall-planting) (creating variable holly and fall-planting)

$= \neg$ holly $\lor$ survive (inkberry) ……using $P \rightarrow Q = \neg P \lor Q$

h) inkberry $\rightarrow$ fertilized (my inkberry)

$= \neg$inkberry $\lor$ fertilized (inkberry) ……using $P \rightarrow Q = \neg P \lor Q$

i) inkberry $\rightarrow$ watered(inkberry)

$= \neg$inkberry $\lor$ watered (inkberry) ……using $P \rightarrow Q = \neg P \lor Q$

12. Express the negation of this statement: (inkberry) → survive (inkberry, fall-planting) in conjunctive normal form. (2 pts)

Ans: (inkberry) → survive (inkberry, fall-planting)

Converting to CNF:

=¬inkberry ∨ survive (inkberry, fall-planting)

Using Negation:

=¬ (¬inkberry ∨ survive (inkberry, fall-planting))

=inkberry ∧ ¬ survive (inkberry, fall-planting)

13. Prove that (inkberry) → survive (inkberry, fall-planting).

Adding the negated goal to the KB and using resolution refutation to prove that it is true. You may show your proof as a series of sentences to be added to the KB or as a proof tree. In either case, you must clearly show which sentences are resolved to produce each new sentence, and what the unifier is for each resolution step.

Ans:

1. Negation of above statement is as follows: ¬ ((inkberry) → survive (inkberry, fall-planting))

Which states that inkberry will not survive fall planting, need to

2. From KB[f]: inkberry → native(inkberry) and KB[d]: ∀x native(x) → hardy(x)

we can add following statement to our KB as follows: **KB[j] = inkberry →hardy(inkberry)**

3. From

KB[h]: inkberry → fertilized(inkberry)

KB[i]: inkberry →hardy(inkberry)

KB[i]: inkberry →watered(inkberry)

and KB[a]: ∀x (hardy(x) ∧ watered(x) ∧ fertilized(x)) → rooted(x)

we can add following statement to our KB as follows: **KB[k] = inkberry → rooted(inkberry)**

4. From KB[b]: ∀x ∀t rooted(x) ∧ in-season(x) → survive (x, t) ,

And KB[k]: inkberry → rooted(inkberry) and its given that its fall planting therefore we can add following to our KB.

KB[l] = (inkberry) → survive (inkberry, fall-planting)

5.using KB[l] we can prove that ¬ ((inkberry) → survive (inkberry, fall-planting)) is False. Hence proved.