RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical & Computer Engineering Program: Biomedical Engineering

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Instructor	Dr. Saba Sedghizadeh

Lab/Tutorial Report NO.	1
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Report Title	Transient Response and Stability in 2nd and 3rd Order Systems

Submission Date	March 13, 2020
Due Date	March 13, 2020

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(Note: remove the first 4 digits from your student ID)

^{*}By signing above you attest that you have contributed to this submission and confirm that all work you have contributed to this submission is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a "0" on the work, an "F" in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/policies/pol60.pdf.

Part 1:

A1 & A2:

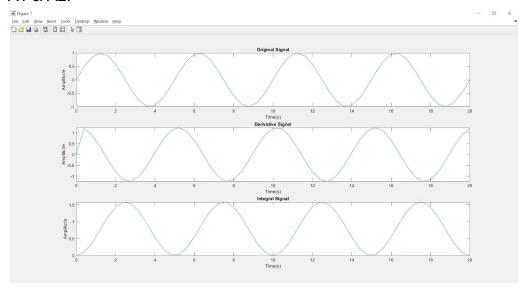


Figure 1. Inputted sine wave with outputs of both a system which is an integrator and a system which is a differentiator.

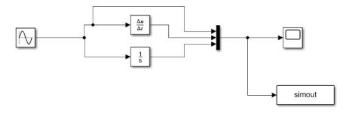


Figure 2. Simulink diagram for A1 & A2

The inputted signal for this part is a normal sine wave with a frequency of 0.2 Hz. Simple calculus tells us that the derivative of a sin(x) is cos(x), and the integral of sin(x) is -cos(x). Therefore, if the system is a differentiator the output wave will be a cosine wave, but if the system is an integrator the output wave will be a cosine wave that is flipped about the y-axis. This corresponds to the waveforms in Figure 1. We can see that the waveform for the integrator is a cosine wave that has been flipped, and similarly the differentiator has a normal cosine wave.

A3:

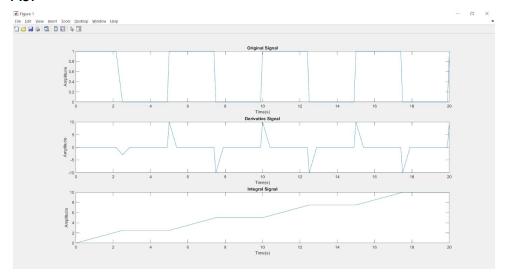


Figure 3. Input signal is a pulse wave with frequency of 0.2Hz and pulse width of 50%. Outputs are an integrator and differentiator waveforms.

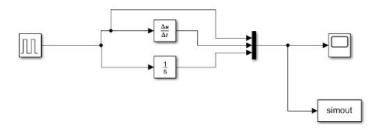


Figure 4. Simulink diagram for A3

A4:

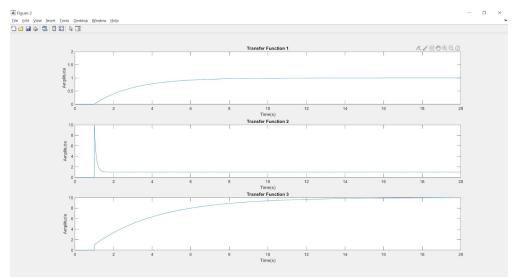


Figure 5. Input is a unit step function shown in Figure 4. Outputs are three different systems with transfer functions: (0.5/s+0.5), (s+1/0.1s+1), (s+3/s+0.3) respectively.

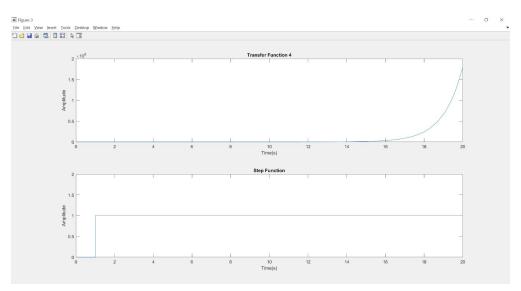


Figure 6. Input is a unit step function. Output is a system with the transfer function (1/s-1). Bottom graph is the input/unit step function.

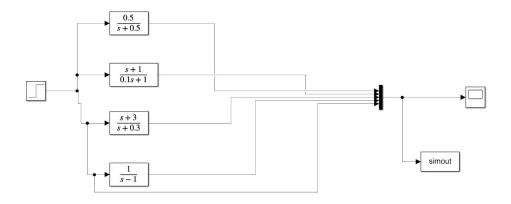


Figure 7. Simulink diagram for A4

A4)
$$\frac{0.5}{S+0.5}$$
 $\frac{S+1}{0.1s+1}$ $\frac{S+3}{s+0.3}$ $\frac{1}{s-1}$

Poles: $s=-0.5$ Poles: $s=-10$ Poles: $s=-0.3$ Poles: $s=1$

Zeros: $s=\infty$ Zeros: $s=-1$ Zeros: $s=-3$ Zeros: $s=\infty$

The first three systems output signals which plateau. This would mean that these systems are stable. But this can also be determined by the location of their poles. The first three systems have poles which are, if put on the s-plane, on the left side of the imaginary axis which indicates that the system is stable. Contrary to the other systems, the fourth system is unstable because it's pole is to the right of the imaginary, and when you look at the output signal it resembles an exponential function.

B1:

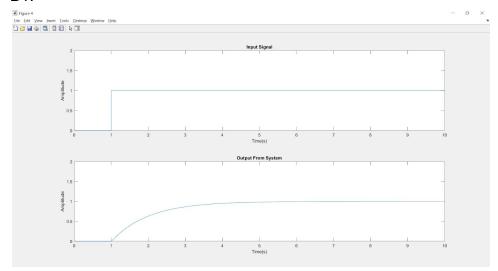


Figure 8. Input is a unit step function. Output is a system with a transfer function of (1/s+1).

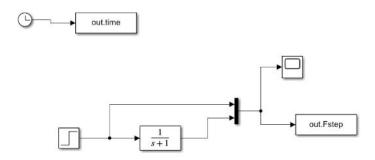


Figure 9. Simulink diagram for B1

81)
$$V(s) = G(s)U(s)$$

$$= \left(\frac{1}{s+1}\right)\left(\frac{1}{s}\right)$$

$$= \frac{1}{s(s+1)} = \frac{4}{s} + \frac{8}{s+1}$$

$$= \frac{1}{s(s+1)} + \frac{8}{s}$$

$$= \frac{1}{s(s+1)} + \frac{8}{s}$$

$$= \frac{4(s+1)}{s} + \frac{8}{s}$$

$$= \frac{4(s+1)$$

The theoretical value of τ calculated is 1 and the DC Gain is 1. When we input this system into Matlab we can see that the output of the system also reaches a y-value of 1, which is consistent to the calculated DC Gain.

B2:

B2)
$$y(s) = G(s) U(s)$$

$$= \frac{k}{(Ts+1)} \left(\frac{1}{5}\right)$$

$$= \frac{k}{s(Ts+1)} = \frac{A}{s} + \frac{B}{Ts+1}$$

$$k = A(Ts+1) + Bs$$

$$(Ss=0=) k = A$$

$$(Ss=0=) k = A$$

$$(Ss=1=) k = A (-T+1) - B$$

$$k = -k(-T+1) - B$$

$$k = -k + kT = -B$$

$$k = k + kT = B$$

$$k = k + kT = -B$$

If we were to move the poles to the right side of the imaginary axis, we would begin to see oscillations. This would move the poles, and if it's on the right side of the imaginary axis the system becomes unstable which could cause oscillations.



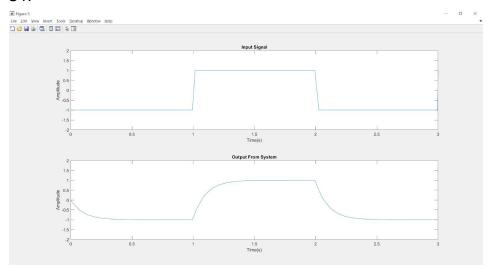


Figure 10. Input is a square wave signal with amplitude of 1 and 0.5Hz. The system has a transfer function of (1/0.1s+1).

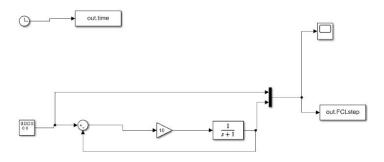


Figure 11. Simulink diagram for AC1

C1)
$$\frac{\sqrt{s}}{R(s)} = \left(\frac{s+1}{o.ls+1}\right) \left(\frac{1}{s+1}\right)$$

$$= \frac{1}{o.ls+1}$$

$$\frac{k}{\tau s+1} \qquad e_{ss} = 1-y_{s}$$

$$= 1-k$$

$$\therefore \tau = 0.1 \qquad = 1-1$$

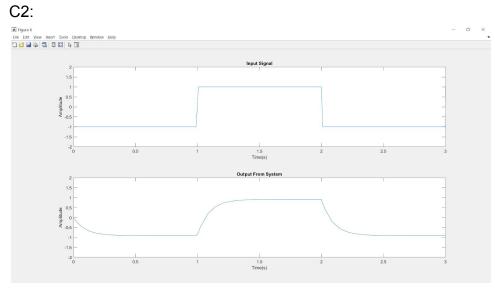


Figure 12. Input is a square wave signal with amplitude of 1 and 0.5Hz. The system has a transfer function of (10/s+11).

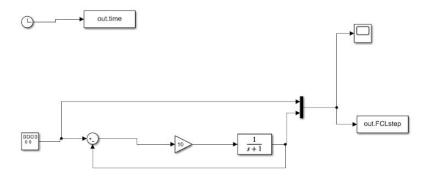
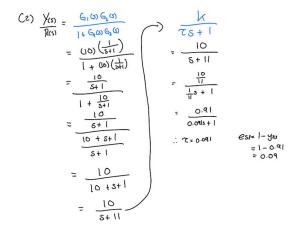


Figure 13. Simulink diagram for C2



C3:

Table 1. Steady state error and time constants for C1 & C2 systems

	Time Constant	Steady State Error
Open-Loop Approach	0.1	0
Closed-Loop Approach	0.091	0.09

The open-loop approach can speed up the could increase the speed exactly 10 times. This approach is also more accurate, just by looking at the error which is 0.



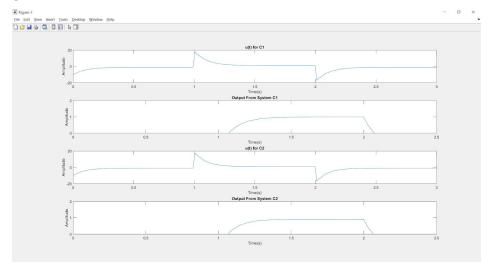


Figure 14. Control signal and output signal comparisons for both system C1 & C2.

Both of the output signal plots are quite similar. But there are two small differences. The first plot has a slightly steeper incline at the beginning compared to the second plot. Also they have different max values. The first plot stops at 1 and the second plot ends around 0.91.

C5:

Table 2. Gain, time constant, and steady state error for C1 & C2 systems with new main system

Main System (1/s+0.5)	Y(s)/R(s)	Time Constant (τ)	Steady-State error (ess)
Open-Loop Approach	(s+1)/(0.1s ² +1.1s+0.05)	Second order system therefore no time constant	0
Closed-Loop Approach	10/s+10.5	0.095	0.048

Table 3. Gain, time constant, and steady state error for C1 & C2 systems with new main system

Main System	Y(s)/R(s)	Time Constant (τ)	Steady-State error
(0.5/s+1)			(ess)

Open-Loop Approach	0.5/0.1s+1	0.1	0.5
Closed-Loop Approach	5/s+6	0.167	0.167

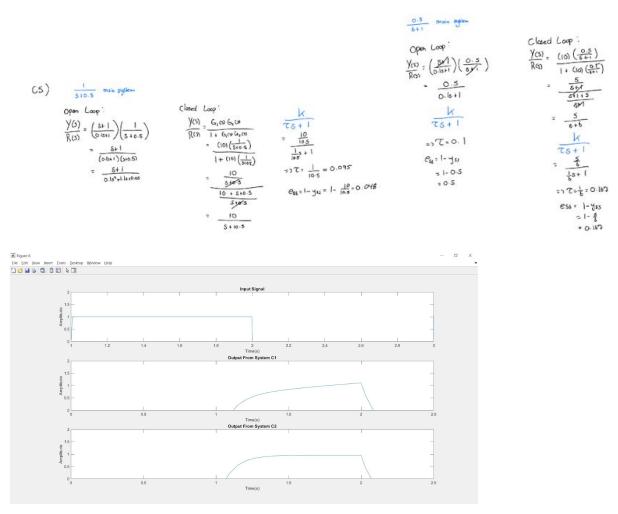


Figure 15. Input is a square wave function with amplitude of 1 and a frequency of 0.5Hz. The two output signals are based off of the C1 & C2 system diagrams but now with a main system transfer function of (1/2+0.5).

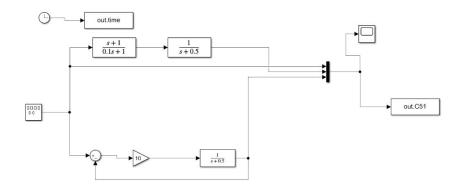


Figure 16. Simulink diagram for C5

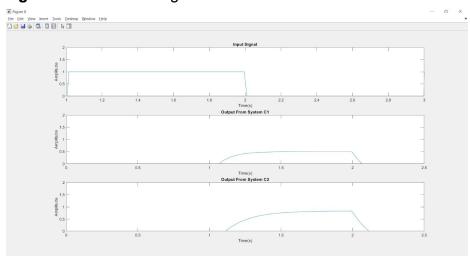


Figure 17. Input is a square wave function with amplitude of 1 and a frequency of 0.5Hz. The two output signals are based off of the C1 & C2 system diagrams but now with a main system transfer function of (0.5/s+1).

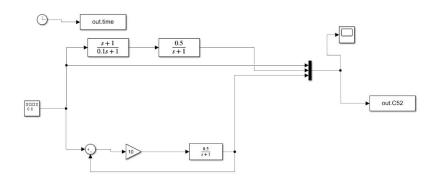


Figure 18. Simulink diagram for C5

When comparing the two methods, the open and closed loop systems, there is one which seems to produce a more consistent output. Which in turns means it is more robust. The closed loop approach is the more robust method. It produces a more consistent output.

PART 2

A1.

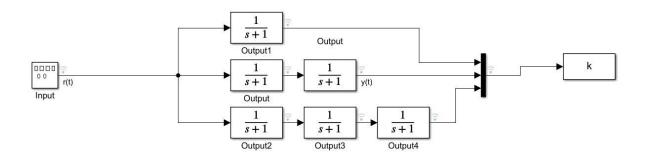


Figure 19: Simulink model of the three systems

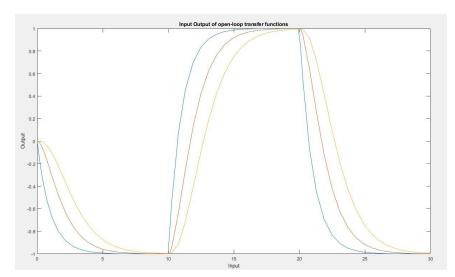


Figure 20: Titled 'Input vs Output of the transfer function" with legends of first order, second order, third order, where yellow line indicates first order, blue line indicates second order, and red line indicates third order.

```
%% Gain of 10
o = k.data(:,1);
d = k.data(:,2);
i = k.data(:,3);
t = k.time;

figure(1);
plot(t,o);
xlabel('Input');
ylabel('Output');
hold on;
plot(t,d);
hold on;
plot(t,i);
title('Input output of open-loop transfer functions');
```

A2. The main difference between the order of systems is the settling time. The third order has the highest settling time compared to the lower order. The first order time constant is 1, meaning the settling time ts is also 5. This means the higher order takes longer to reach within a certain percentage of the final values. Since the higher orders have longer characteristic equations, when the values such as DC gain, undamped frequency decreases the time constant which decreases the settling time, and thus a longer time to reach the percentage of their values.

B1.

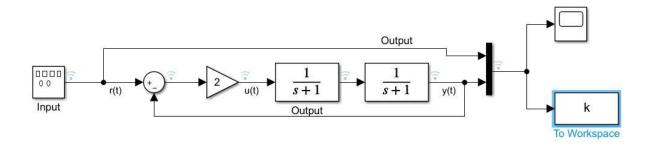


Figure 21:: Gain 2 simulink model with a transfer function of 1/(s+1)^2

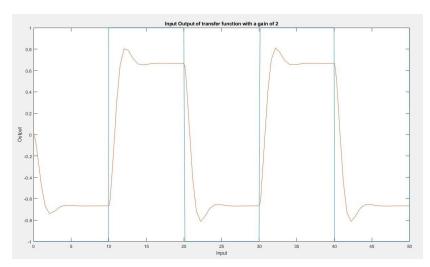


Figure 22: Input output graph of the transfer function with a gain of two.

```
%% Gain of 2

o = k.data(:,1);

d = k.data(:,2);

t = k.time;

figure(1);
plot(t,o);
xlabel('Input');
ylabel('Output');
hold on;
plot(t,d);
title('Input output of transfer function with a gain of 2');
```

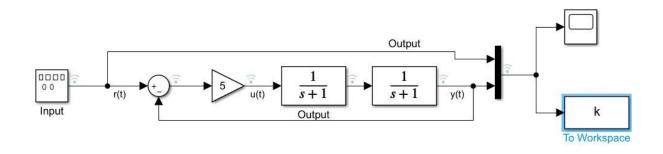


Figure 23: Simulink model of the transfer function with gain 5.

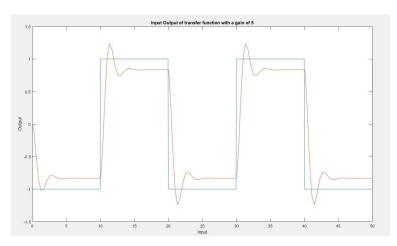


Figure 24: Input vs output of transfer function with gain 5.

```
%% Gain of 5

o = k.data(:,1);
d = k.data(:,2);
t = k.time;

figure(1);
plot(t,o);
xlabel('Input');
ylabel('Output');
hold on;
plot(t,d);
title('Input output of transfer function with a gain of 5');
```

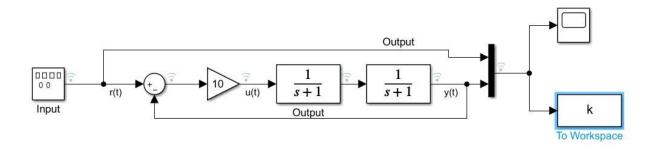


Figure 25: Simulink model of the transfer function with gain 10.

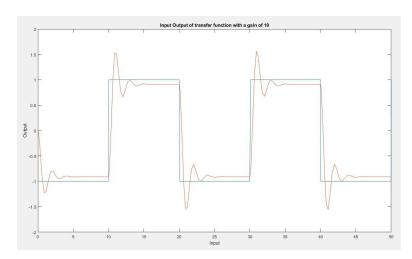


Figure 26: Input output of transfer function with a gain of 10.

```
%% Gain of 10
o = k.data(:,1);
d = k.data(:,2);
t = k.time;

figure(1);
plot(t,o);
xlabel('Input');
ylabel('Output');
hold on;
plot(t,d);
title('Input output of transfer function with a gain of 10');
```

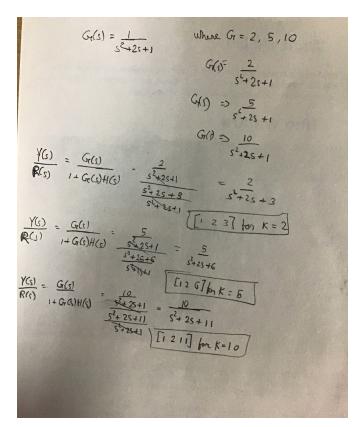


Figure 27: Calculation for transfer function for gain = 2,5,10 with 2nd system order.

Poles:

```
>> x1 = roots([1 2 3])

x1 =

-1.0000 + 1.4142i
-1.0000 - 1.4142i

>> x2 = roots([1 2 6])

x2 =

-1.0000 + 2.2361i
-1.0000 - 2.2361i

>> x3 = roots([1 2 11])

x3 =

-1.0000 + 3.1623i
-1.0000 - 3.1623i
```

The three poles of x1, x2,x3 which correspond to the gain of 2,5,10 respectively. The three roots will be zeros for all gains.

B3.

	Natural frequency	Damping factor	Rise time	% max overshoot	Settling time	Steady-st ate error
K = 2	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	0.834	10.844	3.404	0.67
K = 5	$\sqrt{6}$	$\frac{1}{\sqrt{6}}$	0.744	24.51	3.434	0.834
K = 10	$\sqrt{11}$	$\frac{1}{\sqrt{11}}$	0.469	36.952	3.39	0.31

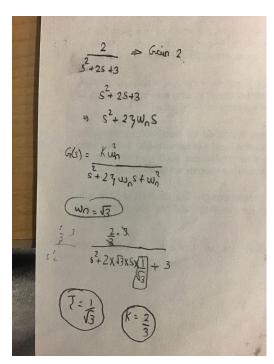


Figure 28: Calculation of natural frequency, DC gain, and damping factor as an example for transfer function with gain 2.

B4. The chart clearly depicts the relation among the different factors. As demonstrated, the DC gain and natural frequency are directly proportional as when the DC gain increased, so did the natural frequency. However, they are both inversely proportional to the damping factor as it decreased. Furthermore, the rise time decreased with an increasing DC gain and so did the percent max overshoot. Lastly, with a higher gain, the steady state error decreased and the Yss increased.

B5. If the gain increases to a certain limit, then the system might become unstable.

C1:

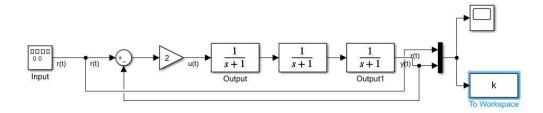


Figure 29: Simulink model of the transfer function with gain of 2.

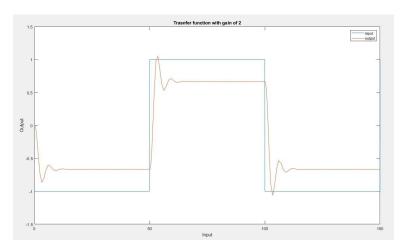


Figure 30: Input output of third order transfer function with a gain of 2.

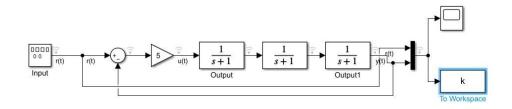


Figure 31: Simulink model of the transfer function with gain of 5.

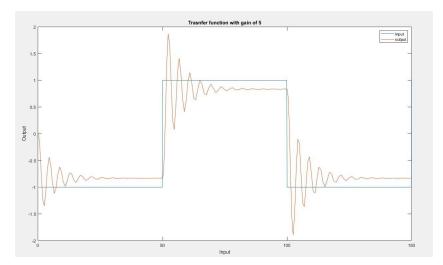


Figure 32: Input output of third order transfer function with a gain of 5.

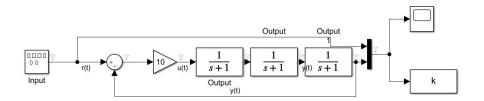


Figure 33: Simulink model of the transfer function with gain of 10.

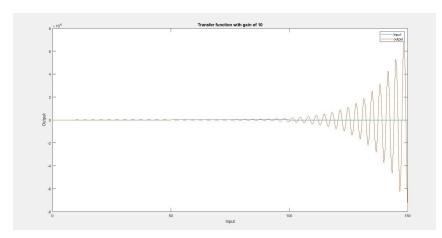


Figure 34: Input output of third order transfer function with a gain of 10.

C.2:

$$\frac{V(s)}{R(s)} = \frac{K}{(s+1)^3} \frac{V(s)}{R(t)} = \frac{G_1(s)}{1 + G_1(s)}$$

$$G_{11}(s) = \frac{2}{(s+1)^3} \stackrel{3}{\Rightarrow} \frac{2}{(s+1)(s^2 + 2s + 1)} \stackrel{2}{\Rightarrow} \frac{2}{s^3 + 2s^2 + 3s + 1}$$

$$G_1(s) = \frac{2}{s^3 + 2s^2 + 3s + 1}$$

$$G_{12}(s) = \frac{5}{s^3 + 2s^2 + 3s + 1}$$

$$G_{13}(s) = \frac{10}{S^3 + 3s^2 + 3s + 1 + 10}$$

$$\frac{V(s)}{R(s)} = \frac{10}{s^3 + 3s^2 + 3s + 1 + 10}$$

$$\frac{V(s)}{R(s)} = \frac{s}{s^3 + 3s^2 + 3s + 1 + 10}$$

$$\frac{V(s)}{R(s)} = \frac{s}{s^3 + 3s^2 + 3s + 1 + 10}$$

$$\frac{V(s)}{R(s)} = \frac{s}{s^3 + 3s^2 + 3s + 1 + 10}$$

$$\frac{V(s)}{R(s)} = \frac{s}{s^3 + 3s^2 + 3s + 1 + 10}$$

Figure 35: Third system order transfer function calculation

```
>> pl = roots([ 1 3 3 3])
p1 =
 -2.2599 + 0.00001
 -0.3700 + 1.0911i
 -0.3700 - 1.0911i
>> p2 = roots([ 1 3 3 6])
p2 =
 -2.7100 + 0.0000i
 -0.1450 + 1.4809i
 -0.1450 - 1.4809i
>> p3 = roots([ 1 3 3 11])
p3 =
 -3.1544 + 0.0000i
  0.0772 + 1.86581
  0.0772 - 1.86581
>>
```

Figure 36: The above figure calculates the poles of the transfer functions with gains k = 2,5,10. The zeroes are none and thus its at infinity.

C.3:

	Rise time	% Max overshoot	Setting time	Steady state error
K = 2	1.35	29.86	10.07	0.164
K = 5	0.866	63.88	26.08	0.34
K = 10	na	na	na	na

MATLAB CODE:

for k = [2 5 10]

```
H = tf([0 0 k], [1 3 3 k+1]);
k
damp(H)
stepinfo(H)
end
```

Figure 37: Matlab code calculating the poles of the third order system.

C.4: There is a limitation in the amount K can be increased before the system becomes unstable. K should be greater than zero for the system to be stable. As seen in the Routh-Hurwitz approach, K must be -1<k<8 to remain stable. Marginal stability is achieved when the values in the first column are equal to zero.

$$(S+1)^{3} = \frac{1}{S^{3} + 3S^{2} + 3S + 1}$$
where
$$S^{3} + 3S^{2} + 3S + 1 = K \text{ is the }$$
characteristic expection.

Ear etablish to occur in the first raw,
$$\frac{8 - K}{3} > C \qquad K + 1 > 0$$

$$K < 8 \qquad K > -1 \qquad S^{3} \qquad 1 \qquad 3$$

$$K < 8 \qquad K > -1 \qquad S^{3} \qquad 1 \qquad 3$$

$$S^{1} \qquad \frac{9 - (1 + 1)}{3}$$

$$S^{0} \qquad K + 1$$

Figure 38: Calculation of the stability boundaries using the Hourtz-Hurwitz approach.

APPENDIX:	plot(t,tr1);
PART 1:	ylim([0 2])
%% A1 & A2 & A3	xlabel('Time(s)');
o = simout.data(:,1);	ylabel('Amplitude');
d = simout.data(:,2);	title('Transfer Function 1');
i = simout.data(:,3);	%Derivative signal
t = simout.time;	subplot(312);
omoditimo,	plot(t,tr2);
figure(1);	ylim([0 10])
%Original signal	xlabel('Time(s)');
subplot(311);	ylabel('Amplitude');
plot(t,o);	title('Transfer Function 2');
xlabel('Time(s)');	%Integral signal
ylabel('Amplitude');	subplot(313);
title('Original Signal');	plot(t,tr3);
subplot(312);	ylim([0 10])
%Derivative signal	xlabel('Time(s)');
plot(t,d);	ylabel('Amplitude');
xlabel('Time(s)');	title('Transfer Function 3');
ylabel('Amplitude');	title (Transfer Farieties 7),
title('Derivative Signal');	figure(3);
%Integral signal	%Integral signal
subplot(313);	subplot(211);
plot(t,i);	plot(t,tr4);
xlabel('Time(s)');	xlabel('Time(s)');
ylabel('Amplitude');	ylabel('Amplitude');
title('Integral Signal');	title('Transfer Function 4');
tite(integral digital),	%Integral signal
%% A4	subplot(212);
tr1 = simout.data(:,1);	plot(t,step);
tr2 = simout.data(:,2);	ylim([0 2])
tr3 = simout.data(:,3);	xlabel('Time(s)');
tr4 = simout.data(:,4);	ylabel('Amplitude');
step = simout.data(:,5);	title('Step Function');
t = simout.time;	and otop i dilonoity,
· · · · · · · · · · · · · · · · · · ·	%% B1
figure(2);	,0,0 01
%Original signal	tB = out.Fstep.time;
subplot(311);	in = out.Fstep.data(:,1);
	Jan. J. J.,

```
out = out.Fstep.data(:,2);
                                                  %% C2
figure(4)
%input signal
                                                  tC2 = out.FCLstep.time;
                                                  in = out.FCLstep.data(:,1);
subplot(211);
plot(tB,in);
                                                  out = out.FCLstep.data(:,2);
ylim([0 2])
xlabel('Time(s)');
                                                  figure(6)
ylabel('Amplitude');
                                                  %input signal
title('Input Signal');
                                                  subplot(211);
%output signal
                                                  plot(tC2,in);
                                                  ylim([-2 2])
subplot(212);
                                                  xlabel('Time(s)');
plot(tB,out);
ylim([0 2])
                                                  ylabel('Amplitude');
xlabel('Time(s)');
                                                  title('Input Signal');
ylabel('Amplitude');
                                                  %output signal
title('Output From System');
                                                  subplot(212);
                                                  plot(tC2,out);
%% C1
                                                  ylim([-2 2])
                                                  xlabel('Time(s)');
tC1 = out.FOLstep.time;
                                                  ylabel('Amplitude');
                                                  title('Output From System');
in = out.FOLstep.data(:,1);
out = out.FOLstep.data(:,2);
                                                  %% C4
figure(5)
%input signal
                                                  t4 = out.C4.time;
                                                  ut1 = out.C4.data(:,1);
subplot(211);
plot(tC1,in);
                                                  out1 = out.C4.data(:,2);
                                                  ut2 = out.C4.data(:,3);
ylim([-2 2])
xlabel('Time(s)');
                                                  out2 = out.C4.data(:,4);
ylabel('Amplitude');
title('Input Signal');
                                                  figure(7)
                                                  %u(t) for C1
%output signal
subplot(212);
                                                  subplot(411);
plot(tC1,out);
                                                  plot(t4,ut1);
ylim([-2 2])
                                                  %ylim([0 2])
xlabel('Time(s)');
                                                  xlabel('Time(s)');
ylabel('Amplitude');
                                                  ylabel('Amplitude');
title('Output From System');
                                                  title('u(t) for C1');
```

%C1	subplot(312);
subplot(412);	plot(t51,out1);
plot(t4,out1);	ylim([0 2])
ylim([0 2])	xlabel('Time(s)');
xlabel('Time(s)');	ylabel('Amplitude');
ylabel('Amplitude');	title('Output From System C1');
title('Output From System C1');	%C2
%u(t) for C2	subplot(313);
subplot(413);	plot(t51,out2);
plot(t4,ut2);	ylim([0 2])
%ylim([0 2])	xlabel('Time(s)');
xlabel('Time(s)');	ylabel('Amplitude');
ylabel('Amplitude');	title('Output From System C2');
title('u(t) for C2');	, ,
%C2	%% C52
subplot(414);	
plot(t4,out2);	t52 = out.C52.time;
ylim([0 2])	in = out.C52.data(:,1);
xlabel('Time(s)');	out1 = out.C52.data(:,2);
ylabel('Amplitude');	out2 = out.C52.data(:,3);
title('Output From System C2');	
	figure(8)
	%input signal
%% C51	subplot(311);
	plot(t52,in);
t51 = out.C51.time;	ylim([0 2])
in = out.C51.data(:,1);	xlabel('Time(s)');
out1 = out.C51.data(:,2);	ylabel('Amplitude');
out2 = out.C51.data(:,3);	title('Input Signal');
	%C1
figure(8)	subplot(312);
%input signal	plot(t52,out1);
subplot(311);	ylim([0 2])
plot(t51,in);	xlabel('Time(s)');
ylim([0 2])	ylabel('Amplitude');
xlabel('Time(s)');	title('Output From System C1');
ylabel('Amplitude');	%C2
title('Input Signal');	subplot(313);
%C1	plot(t52,out2);

```
ylim([0 2])
                                                    %i = k.data(:,3);
xlabel('Time(s)');
                                                    t = k.time;
ylabel('Amplitude');
title('Output From System C2');
                                                    figure(1);
                                                    %Original signal
                                                    plot(t,o);
                                                    hold on;
                                                    xlabel('Input');
                                                    ylabel('Output');
                                                    title('Transfer function with gain of k');
PART 2:
                                                    %where k = 2,5,10
A1:
                                                    %Derivative signal
                                                    plot(t,e);
For Gain = 2, 5, 10
                                                    %plot(t,i);
o = k.data(:,1);
                                                    hold off;
e = k.data(:,2);
                                                    legend('input', 'output')
\%i = k.data(:,3);
                                                    C1:
t = k.time;
figure(1);
                                                    For Gain = 2, 5, 10
%Original signal
                                                    o = k.data(:,1);
plot(t,o);
                                                    e = k.data(:,2);
hold on;
                                                    \%i = k.data(:,3);
xlabel('Input');
                                                    t = k.time;
ylabel('Output');
title('Transfer function with gain of k');
                                                    figure(1);
%where k = 2,5,10
                                                    %Original signal
%Derivative signal
                                                    plot(t,o);
plot(t,e);
                                                    hold on;
                                                    xlabel('Input');
%plot(t,i);
hold off;
                                                    ylabel('Output');
legend('input', 'output')
                                                    title('Transfer function with gain of k');
                                                    %where k = 2,5,10
B1:
                                                    %Derivative signal
                                                    plot(t,e);
For Gain = 2, 5, 10
                                                    %plot(t,i);
                                                    hold off;
o = k.data(:,1);
                                                    legend('input', 'output')
e = k.data(:,2);
```