Name: Kushal Sharma
Email: kushals@usc.edu
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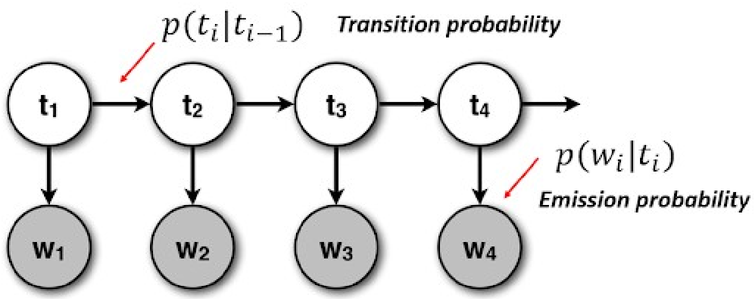
Assignment: HomeWork 7 (Hidden Markov Model)

Hidden Markov Model:

The HMM is based on augmenting the Markov chain. The Markov chain makes a very strong assumption that if we want to predict the future in the sequence, all that matter is the current state. The states before the current state have no impact on future except via the current state.
For an HMM with N hidden states and an observation sequence of T observations, there are N^T possible hidden sequences. Instead of using such an extremely exponential algorithm, we use an efficient $O(N^2 T)$ algorithm called the Viterbi algorithm. The most likely state value path is determined based on initial probability, emission probability and transition probability. Emission probability and Transition probability act as model at every time stamp, means they don't change their value. Initial probability is only used in the beginning which is prior probability of every state in the environment. Emission probability is the conditional probability of the observation w_i given the state value t_i. Transition probability is the conditional probability of the next state t_i given the previous state t_{i-1}.



The Emission probability table is as below with current state in columns and observation in rows -

	1	2	3	4	5	6	7	8	9	10
0	0.33	0	0	0	0	0	0	0	0	0
1	0.33	0.33	0	0	0	0	0	0	0	0
2	0.33	0.33	0.33	0	0	0	0	0	0	0
3	0	0.33	0.33	0.33	0	0	0	0	0	0
4	0	0	0.33	0.33	0.33	0	0	0	0	0
5	0	0	0	0.33	0.33	0.33	0	0	0	0
6	0	0	0	0	0.33	0.33	0.33	0	0	0
7	0	0	0	0	0	0.33	0.33	0.33	0	0
8	0	0	0	0	0	0	0.33	0.33	0.33	0
9	0	0	0	0	0	0	0	0.33	0.33	0.33
10	0	0	0	0	0	0	0	0	0.33	0.33
11	0	0	0	0	0	0	0	0	0	0.33

The Transition probability table is as below with previous state in columns and current state in rows -

	1	2	3	4	5	6	7	8	9	10
1	0	0.5	0	0	0	0	0	0	0	0
2	1	0	0.5	0	0	0	0	0	0	0
3	0	0.5	0	0.5	0	0	0	0	0	0
4	0	0	0.5	0	0.5	0	0	0	0	0
5	0	0	0	0.5	0	0.5	0	0	0	0
6	0	0	0	0	0.5	0	0.5	0	0	0
7	0	0	0	0	0	0.5	0	0.5	0	0
8	0	0	0	0	0	0	0.5	0	0.5	0
9	0	0	0	0	0	0	0	0.5	0	1
10	0	0	0	0	0	0	0	0	0.5	0

The initial prior probability table for the first state is as below -

	1	2	3	4	5	6	7	8	9	10
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

# of valid states = 10
# of valid observations = 12
# of timestamps = 10
The sequence of 10 observations = 8, 6, 4, 6, 5, 4, 5, 5, 7, 9
The most likely state value path = 7, 6, 5, 6, 5, 4, 5, 4, 7, 8

Data Structure:

- Dictionary is used to represent the transition probability and emission probability to explicitly mark the corresponding value of previous state, current state and observations.
- Numpy array is used to print and write the probability tables as well as to find max and argmax value.

Code Level Optimization:

- Use of dictionary helps to track the table operations i.e. multiplications and projection across different indices of state variables and emission variables. Max function helps to find the maximum value as well corresponding key associated with maximum value.

Challenges:

- Careful understanding of flow of Viterbi algorithm and keeping track of associated indices to do table operations.