Extending Empirical Path Loss Models for Mobile Radio System Simulations in ns-3

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Abstract-Simulations of mobile radio systems require propagation models for determining the channel characteristics for a given arbitrary scenario. Propagation path loss models can be used to predict wireless channel characteristics like received signal strength, multipath effects, and interference. These predictions are an important aspect of simulation of wireless networks and aid in the high-level network planning process. Various models are available for modeling propagation characteristics and they vary in terms of both modeling performance and computational complexity. The work done discusses the implementation of additional propagation loss models for ns-3 network simulation platform, including COST-231 Walfisch Ikegami Model, ECC-33 Model and Stanford University Interim (SUI) models. We present comprehensive performance analysis to validate the applicability of the above models for different propagation environments and its use for 2.5GHz WiMax.

Index Terms-Propagation Loss Models, ns-3, Mobile WiMax

I. INTRODUCTION

Network Simulation is a powerful tool for analysis and evaluation of wireless networks. An important aspect of network simulation of wireless systems is the ability to accurately model the wireless channel effects and to evaluate its impact on system performance. High-level network planning involves tasks like interference calculations, frequency allocation, placement of Base Station Subsystem (BSS) and determination of parameter sets, cell coverage analysis/planning etc. Factors influencing radio propagation behavior include mechanisms like reflection, diffraction, scattering, penetration, shadowing and fading. Therefore the ability to predict propagation path loss is essential for the analysis of coverage areas, determining multipath effects, and performing interference/capacity calculations. Thus, propagation loss models are essential in determining the channel characteristics for any arbitrary installation. Generally, a three stage modeling process as described in [1] is employed to obtain a practical predication model. The first stage involves the digitization of analogue terrain data to obtain digital terrain databases. These databases maintain information regarding terrain height, land usage data, building shapes and surface characteristics. The second stage involves process of defining mathematical approximations for physical propagation mechanisms based on the terrain data information. The third stage will involve the creation of both deterministic and empirical solutions based

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on the approximations for various environments like urban, sub-urban and rural.

Propagation loss models are used extensively for path loss prediction in large and small cell configurations under both Line-of-Sight (LoS) and Non Line-of-Sight (NLoS) conditions. They are useful in conducting feasibility studies during initial deployment and aid in high-level network planning. The ns-3 simulation platform has an implementation of 11 different propagation models for predicting path loss behavior. These models can be classified broadly as Deterministic, Stochastic and Empirical Models. Deterministic models use the governing laws of electromagnetic wave propagation to determine the received signal power at a particular distance from the transmitter. A stochastic model uses a stochastic fading process and models the environment by applying a random fading process on top of a path loss model. The random process accounts for non-deterministic effects caused due to user mobility. These models are generally the least accurate and have low computational complexity for generating predictions. Empirical models, on the other hand, are based on actual measurement and observations.

Empirical models can be time-dispersive (modeling multipath effects, delay spread etc.) like SUI channel models developed by the Institute of Electrical and Electronic Engineers (IEEE) 802.16 working group[2], or non-time dispersive like COST-231 Hata Models, ITU-R etc. These models predict the average path loss as a function of various parameters like distance from the transmitter, transmit/receive antenna heights, rooftop/building information and frequency. In this paper, we present the implementation of three propagation loss models for ns-3 namely: COST-231 Walfisch Ikegami[3], SUI channel models [4] and ECC-33 model[5]. We also present the simulated performance of 2.5GHz WiMax utilizing these channel propagation models.

The remainder of this paper is structured as follows: Section II discusses prior work in the field of propagation loss. We introduce the propagation loss models implemented for ns–3, and a brief description of the newly implemented models in Section III. Section IV discusses our experimental setup and Section V the measured results. Finally, we summarize our conclusions in Section VI.

II. RELATED WORK

Prediction of average received signal strengths and multipath effects in macro-cellular environments have generated significant research interest. Typically, these path-loss predictions are based on the knowledge of topography, land usage

and building height information. The prediction modeling is influenced by various factors including modeling environments (urban, suburban or rural areas), influence of vegetation, large-scale terrain effects, multipath prediction and the combination of the different aspects yielding more general models.

Javornik et.al. [6], provide coverage planning analysis for WiMAX at two carrier frequencies, namely 450 MHz and 3.5 GHz in flat rural, hilly rural and urban environment. The channel model proposed by WiMAX forum has been applied as path loss model at 3.5 GHz for cell coverage prediction, while at 450 MHz the Longley-Rice model for rural areas and Okumura Hata channel models for urban area is used. The cell size prediction strongly suggests to limit the 3.5 GHz frequency band to urban areas, where the higher system capacity is required, while in rural areas the 450 MHz carrier frequency provide good compromise between coverage and system capacity.

The work done by Abhayawardhana et al. [7], compares measurements taken in Cambridge, UK against three empirical propagation models: COST-231 Hata, SUI and ECC-33 Models. The COST-231 Hata model, in general overestimated the path loss, especially at greater antenna heights. The measurement results also show that SUI model exhibited large path loss prediction errors, generally over-predicting the path loss. It also emphasizes the need for optimizing the model parameters to accommodate urban environments. Both models show some scope to be optimized for the 3.5 GHz band. It is to be noted ECC-33 model showed the closest agreement with the measurement results in [7]. However, the model does not provide any correction factors for suburban or rural environments.

Since SUI do not specify prediction models explicitly from urban and suburban environments, it is difficult to compare it with other models for the same corresponding environments. Although most propagation models overestimate the path loss for a given environment, in order to obtain accurate results, measurements in the target environment are always necessary. However, empirical channel models can still be used during initial deployments and feasibility studies as is proved by the assumptions made in [7] and [8] which compare real-world measurements to certain propagation loss models, and unlike empirical models, they exhibit high variations in received signal strength over time or for slight changes in distance.

III. EMPIRICAL MODELS

Path loss (or path attenuation) is the attenuation in power density of an electromagnetic wave as it propagates through space. Path loss is a major component in the analysis and design of the link budget of a telecommunication system. Path loss can be defined as the ratio of the transmitted to received power, usually expressed in decibels. The equation for the path loss at distance d is of the form:

$$PL(d) = PL(d_0) + 10\gamma log_{10}\left(\frac{d}{d_0}\right) \tag{1}$$

where, d_0 is the reference distance and γ is known as the path loss exponent. Note that for Free Space Loss (FSL) the path

loss exponent is equal to two, and thus the environment could be categorized as one having less obstructions and clutter. The choice of γ affects the rate of increase of path loss with respect to distance. We can choose $PL(d_0)$ and γ to minimize $\sigma[4]$. This model will be used as a basis for comparison against widely accepted empirical propagation loss models. An additional three models are implemented for ns-3, namely: COST-231 Walfisch Ikegami, SUI Channel Models and ECC-33 Model. These will be evaluated along with the COST-231 Hata model.

A. COST-231 Walfisch Ikegami Model

The COST 231 model is a path-loss model when small distances exist between the MS and BS, and/or the MS has a small height. The total path-loss for the LOS case is given by:

$$PL = 42.6 + 26\log(d) + 20\log(f_c) \tag{2}$$

for d > 20 m, where again d is in units of kilometers, and f_c is in units of MHz. For the non-LOS case, path-loss consists in the free space path-loss PL_0 , the multiscreen loss L_{msd} along the propagation path, and attenuation from the last roof edge to the MS, L_{rts} (rooftop-to-street diffraction and scatter loss):

$$PL = \begin{cases} PL_0 + L_{rts} + L_{msd} & \text{for } L_{rts} + L_{msd} > 0\\ PL_0 & \text{for } L_{rts} + L_{msd} \le 0 \end{cases}$$
(3)

The free space path-loss is:

$$L_0 = 32.4 + 20\log d + 20\log(f_c) \tag{4}$$

The diffraction loss L_{rts} is given as:

$$L_{rts} = -16.9 - 10 \log w + 10 \log(f_c) + 20 \log \Delta h_m + L_{ori}$$
 (5)

where w is the width of the street in meters, and

$$\Delta h_m = h_{roof} - h_m \tag{6}$$

is the difference between the building height h_{roof} and the height of the MS h_m . Orientation of the street is taken into account by an empirical correction factor L_{ori} :

$$L_{ori} = \begin{cases} -10 + 0.354\varphi & \text{for } 0^{\circ} \le \varphi \le 35^{\circ} \\ 2.5 + 0.075(\varphi - 35) & \text{for } 35^{\circ} \le \varphi \le 55^{\circ} \\ 4.0 - 0.114(\varphi - 55) & \text{for } 55^{\circ} \le \varphi \le 90^{\circ} \end{cases}$$
(7)

where φ is the angle between the street orientation and the direction of incidence in degrees. For the computation of the multiscreen loss L_{msd} ; building edges are modeled as screens. The multiscreen loss is then given as [Walfish and Bertoni 1988]:

$$L_{msd} = L_{bsh} + k_a + k_d \log d + k_f \log f_c - 9 \log b$$
 (8)

where b is the distance between two buildings (in meters). Furthermore:

$$L_{bsh} = \begin{cases} -18\log(1 + \Delta h_b) & \text{for } h_b > h_{roof} \\ 0 & \text{for } h_b \le h_{roof} \end{cases}$$
(9)

$$k_a = \begin{cases} 54 & \text{for } h_b > h_{roof} \\ 54 - 0.8\Delta h_b & \text{for } d \ge 0.5 \text{km and } h_b \le h_{roof} \\ 54 - 0.8\Delta h_b d / 0.5 & \text{for } d < 0.5 \text{km and } h_b \le h_{roof} \end{cases}$$
(10)

where

$$\Delta h_b = h_b - h_{roof} \tag{11}$$

and h_b is the height of the BS. The dependence of path-loss on frequency and distance is given via the parameters k_d and k_f as:

$$k_d = \begin{cases} 18 & \text{for } h_b > h_{roof} \\ 18 - 15\Delta^{h_b/h_{roof}} & \text{for } h_b \le h_{roof} \end{cases}$$
 (12)

$$k_d = -4 + \begin{cases} 0.7 \left(\frac{f_c}{925} - 1 \right) & \text{for medium cities/suburban areas} \\ 1.5 \left(\frac{f_c}{925} - 1 \right) & \text{for metropolitan areas} \end{cases}$$
(13)

Table I gives the validity range for this model.

Table I
VALIDITY RANGE FOR COST-231 WALFISH–IKEGAMI MODEL

carrier frequency	f_c	800-2,000 MHz
BS Antenna Height	h_b	4–50 m
MS Antenna Height	h_m	1-3 m
Distance	d	0.02-5 km

B. Stanford University Interim (SUI) Models

This model was developed under the IEEE 802.16 working group as part of a proposed standard for frequency bands below 11 GHz. The SUI models are divided into three categories of terrains: Category A is associated with maximum path-loss category which is a hilly terrain, Category B represents an intermediate path-loss category and is applicable to mostly flat terrains with moderate to heavy tree densities or hilly terrains with light tree densities, and Category C represents the minimum path-loss category with mostly flat terrain with light tree densities. The basic path loss equation with correction factors is presented in[4], [2]. The median path-loss for the SUI model can be generally written as:

$$PL = A + 10\gamma log_{10} \left(\frac{d}{d_0}\right) + s \tag{14}$$

for $d > d_0$, where $d_0 = 100$ m. The term A (*Intercept*) in the above equation is given by:

$$A = 20\log\left(4\pi d_0/\lambda\right) \tag{15}$$

where λ lambda is the wavelength in meters. The path-loss exponent γ is given by:

$$\gamma = (a - bh_b + c/h_b) + x\sigma_{\gamma}, \ 10\text{m} \ge h_b \ge 80\text{m}$$
 (16)

where, σ_{γ} is the std deviation of gamma.and x is a zero-mean Gaussian variable N[0,1]. The parameters a,b and c depend on the terrain category and are defined in Table II.

Table II Validity range for COST-231 Walfish–Ikegami Model [4]

Parameter	Terrain Category			
i arainetei -	A	В	С	
\overline{a}	4.6	4.0	3.6	
$b \ (\text{in} \ m^{-1})$	0.0075	0.0065	0.0050	
$c \ (\text{in} \ m^{-1})$	12.6	17.1	20.0	
σ_{γ}	0.57	0.75	0.59	
$\mu_{\sigma}^{'}$	10.6	9.6	8.2	
σ_{σ}	2.3	3.0	1.6	

And s is the shadow fading component, given by:

$$s = y\sigma \tag{17}$$

where y is a zero-mean Gaussian variable of unit standard deviation; and σ , the standard deviation of s. Thus

$$\sigma = \mu_{\sigma} + z\sigma_{\sigma} \tag{18}$$

where μ_{σ} is the mean of σ ; σ_{σ} is the standard deviation of σ ; z is a zero-mean Gaussian variable of unit standard deviation N[0,1]. The model is derived from data taken at 1.9 GHz with an omnidirectional terminal antenna at a height of 2 m and is therefore limited to these conditions. Corrective factors are used to extend the model for other frequency and receiver antenna heights and is given by:

$$PL_{sui} = PL + PL_{\Delta f} + PL_{\Delta h} + s \tag{19}$$

where,

$$PL_{\Delta f} = 6\log_{10}(f/2000);$$
 (20)

$$PL_{\Delta h} = \begin{cases} -10.8 \log_{10}(h_r/2000) & \text{for Category A and B} \\ -20 \log_{10}(h_r/2000) & \text{for Category C} \end{cases}$$
 (21)

where, h_r is the receiver antenna height and f is the frequency in MHz.

C. ECC-33 Propagation Model

The ECC-33 model is an extrapolated version of the original measurements made by Okumura [9] with modifications applicable to "medium cities" and is recommended for European cities [10]. Although the Hata- Okumura model is widely used for UHF bands its accuracy is questionable for higher frequencies. The COST-231 model extended its use up to 2 GHz but it was proposed for mobile systems having omnidirectional receiver antennas sited less than 3 m above ground level. The path loss model presented in [5], is referred to here as the ECC-33 model. The path loss is defined as:

$$PL = A_{fs} + A_{bm} - G_b - G_r (22)$$

where, A_{fs} is the free space attenuation in dB, A_{bm} is the basic median path loss in dB, G_b and G_r are the transmitter

and receiver Antenna height gain factors respectively, and are defined as:

$$A_{fs} = 92.4 + 20\log_{10}(d) + 20\log_{10}(f) \tag{23}$$

$$A_{fs} = 20.41 + 9.83 \log_{10}(d) + 7.894 \log_{10}(f) + 9.56 \left[\log_{10}(f)\right]^{2}$$
(24)

$$G_b = \log_{10}(h_b/200) \left\{ 13.958 + 5.8 \left[\log_{10}(d) \right]^2 \right\}$$
 (25)

$$G_r = \begin{cases} [42.57 + 13.7 \log_{10}(f)] [\log_{10}(h_r) - 0.585] & \text{med. city} \\ 0.759h_r - 1.892 & \text{big city} \end{cases}$$
(26)

where, f is the frequency in GHz, d is the distance between BS and MS in km, h_b is the BS antenna height in meters and h_r is the MS antenna height in meters. The medium city model is more appropriate for European cities whereas the large city environment should only be used for cities with highly built-up areas.

IV. EXPERIMENTAL SETUP

We initially designed a simple ns-3 scenario using a wireless ad-hoc network that allows us to perform a comparative analysis of the measured path loss for different propagation models. The ns-3 default simulation configuration parameters are as shown in Table III. The signal frequency for the simulation was set to 2GHz, and Base Station antenna height set to 30m across all propagation models. The measurements were made at the receiver at distances between 50m to 5Km for three different receiver antenna heights of 1.5m, 6m and 10m respectively. The transmission power is set to 20dB. A constant stream of UDP packets are used from a OnOffApplication with a traffic intensity of 15% and a data rate of 700 kb/s. For WiMax[11] simulations, we use an infrastructure network configuration with two subscriber stations (SS) connected to a single base Station (BS). An OFDM channel is used with a simple scheduler. Downlink and uplink service flows classifiers are used to setup flows between the two subscriber stations, with the service flow type set to best effort service. One SS uses a constant speed mobility model, and is used to make the signal strength measurements.

Table III
SUMMARY OF SIMULATION PARAMETERS

Parameter	Value	
Distance	50m to 5000m	
Frequency	2.5 GHz	
BS Antenna Height	30 m	
SS Antenna Height	1.5, 6 & 10 m	
Rooftop Height (for COST-231 WI)	12 m	

V. RESULTS

The path-loss predictions are computed for different environments depending on the chosen propagation model. Results are presented for predictions made with SS antenna heights of 1.5m, 6m and 10m.

Figure 1 also shows the predictions made for all the models in comparison with COST-231 Hata model shipped with ns-3. The SS antenna height, h_r is set to 30m. SUI Category B model is used as an appropriate approximation for urban scenarios. It is to be noted that COST-231 Hata and SUI models generate comparable path-loss predictions, but they overestimate path-loss when compared to ECC-33 and COST-231 WI models.

Figure 1. Comparison for a typical urban environment ($h_r = 1.5$ m)

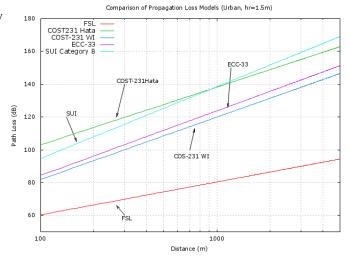
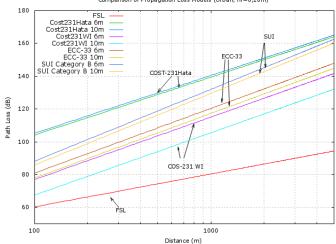


Figure 2 also shows the predictions made by all the empirical models for two different subscriber antenna heights of 6m and 10m respectively for an urban environment. We see that, COST-231 WI model shows the most variation for differing receiver antenna heights.

Figure 2. Comparison for a typical urban environment ($h_r = 6\text{m \& 10m}$)

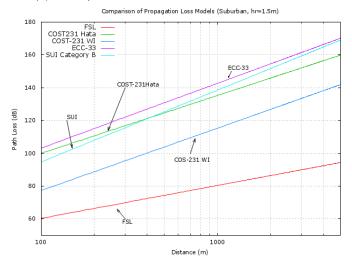
Comparison of Propagation Loss Models (Urban, hr=6.10m)



A comparison of empirical models in the suburban environment is presented in Figure 3. The ECC-33 medium city

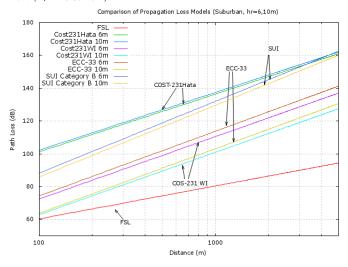
model and the SUI Category B models are used as close approximations of the suburban environments. It is seen that ECC-33 model produces the highest average path-loss when compared to other models. Similar comparisons are made in Figure 4 for SS antenna heights of 6m and 10m respectively.

Figure 3. Comparison of empirical models for a typical suburban environment ($h_T = 1.5$ m)



Here it is seen that changes in antenna heights impacts the prediction performance of the ECC model significantly in comparison to Figure 3.

Figure 4. Comparison of empirical models for a typical suburban environment ($h_T=6~\&~10{\rm m}$)



Comparison of path-loss predictions for rural environments are presented in Figure 5. Note that we compare only two models, i.e. COST-231 Hata and SUI Category C as an approximation for rural environments, as neither COST-231 WI, nor ECC-33 provides an appropriate model for rural environments. The COST Hata models show very little fluctuation for different antenna heights unlike the SUI models.

Figure shows an indicative comparison of all SUI models Categories A, B and C with reference to Free Space Loss model at SS antenna height of 1.5m.

Figure 5. Comparison of SUI Category C Models (rural environment, ($h_T=1.5,\,6,\,10\mathrm{m}$)

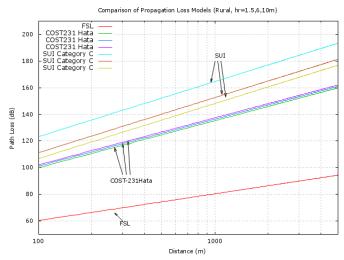
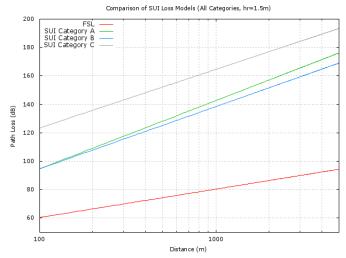


Figure 6. Comparison of SUI Models (All Categories, $h_r = 1.5$ m)



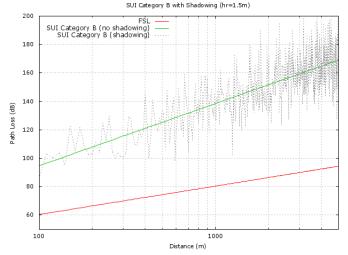
The implemented model for ns3 also provide the ability to enable/disable the shadowing component as described in (Eq. 17) for all SUI model categories. The shadowing information is generated using a zero-mean unit standard deviation Gaussian random variable as described in section 3.2.

Also it is important to note that SUI models do not have classifications for urban scenarios and therefore the closest approximations are used.

VI. CONCLUSIONS AND FUTURE WORK

We present an implementation of three additional propagation loss models for ns-3 simulation platform namely the COST-231 Walfisch Ikegami model, SUI channel models and the ECC-33 model. An optional fading component is implemented with the SUI model and aims to increase the accuracy of the calculated propagation loss by taking into account frequent variations in the communication environment. The implemented models extend ns-3 capabilities and provide additional models for use in initial deployment scenarios to gain preliminary insight into radio channel behavior.

Figure 7. SUI Category B with Shadowing ($h_r = 1.5$ m)



Although, there is no single-fit solution to the proper choice of a propagation loss model, the most appropriate model can always be chosen to perform a preliminary assessment depending on the research goal. Also the propagation models implemented are used with a 2.5GHz WiMax system to model the path-loss of the channel and the simulations validate the implementation performance. Although some of the models overestimate path-loss in certain scenarios, the predictions can be considered as representative of the worst-case channel conditions in different environments. The models implemented above, with the exception of SUI models are generally used for single channel modeling. To accurately represent Multiple Input Multiple Output (MIMO) communication and to improve prediction accuracy, models like 3GPP SCM, WINNER and WiMAX ITU-TDL can be used outdoor scenarios. We are currently working to develop MIMO channel models for the ns-3 platform. The extensions to the ns-3 propagation module¹ has been ported to the latest ns-3 development version and is planned to be merged in next stable version of ns-3.

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¹The code is available at http://github.com/deepaknadig/propagation/