10. Write a computer program for minimizing a multivariate function using the steepest-descent algorithm. Include the following details:

(*a*) Use a back-tracking line search as described in Section 5 of Chapter 10.

(*b*) Accept x as a solution if $\|\nabla f(x)\|/(1+|f(x)|) \leq \epsilon$, or if the number of iterations exceeds ITMAX. Use $\epsilon = 10^{-5}$ and ITMAX $= 1000$.

(*c*) Print out the initial point, and then at each iteration print the search direction, the step length α and the new estimate of the solution x_{k+1}. (If a great many iterations are required, only provide this output for the first 10 iterations and the final 5 iterations.) Indicate if no solution has been found after ITMAX iterations.

(*d*) Test your algorithm on the test problems listed here:

$$f_{(1)}(x) = x_1^2 + x_2^2 + x_3^2, \qquad x_0 = (1, 1, 1)^T$$

$$f_{(2)}(x) = x_1^2 + 2x_2^2 - 2x_1x_2 - 2x_2, \qquad x_0 = (0, 0)^T$$

$$f_{(3)}(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2, \qquad x_0 = (-1.2, 1)^T$$

$$f_{(4)}(x) = (x_1 + x_2)^4 + x_2^2, \qquad x_0 = (2, -2)^T$$

$$f_{(5)}(x) = (x_1 - 1)^2 + (x_2 - 1)^2 + c(x_1^2 + x_2^2 - 0.25)^2, \qquad x_0 = (1, -1)^T$$

For the final function, test three different settings of the parameter c: $c = 1$, $c = 10$, and $c = 100$. The condition number of the Hessian matrix at the solution becomes larger as c increases. Comment on how this affects the performance of the algorithm.

(*e*) Are your computational results consistent with the theory of the steepest-descent method?