Washington Experimental Mathematics Lab

Stability Spectrum for PDEs

Faculty Mentor: Dr. Bernard Deconinck
Graduate Mentor: Jeremy Upsal
Team Members: Kush Gupta, Ryan Bushling

Department of Mathematics
University of Washington

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Stability Spectrum for PDEs

Motivation To determine the stability of solutions to certain PDEs, including the MKdV equation

Problem To determine the eigenvalues of a linear operator such that the associated eigenfunctions are bounded

Methods Taking advantage of the periodicity of the coefficients using Floquet theory and Fourier series

Example

Let $\mathcal{L} = -\partial_{xx}$, and consider the eigenvalue problem $\mathcal{L}(y) = \lambda y$, or equivalently, $\partial_{xx} y + \lambda y = 0$.

$$\implies y'' + \lambda y = 0$$

$$\implies y = c_1 \mathrm{e}^{\sqrt{\lambda} x} + c_2 \mathrm{e}^{-\sqrt{\lambda} x}$$

In general, $\sqrt{\lambda} \in \mathbb{C}$, so let $\sqrt{\lambda} = \alpha + \mathrm{i}\beta$ for some $\alpha, \beta \in \mathbb{R}$.

Example (ct'd.)

Letting $\sqrt{\lambda} = \alpha + \mathrm{i}\beta$, rewrite $y = c_1 \mathrm{e}^{\sqrt{\lambda}x} + c_2 \mathrm{e}^{-\sqrt{\lambda}x}$ as

$$\begin{aligned} y &= c_1 \mathrm{e}^{(\alpha+\mathrm{i}\beta)x} + c_2 \mathrm{e}^{-(\alpha+\mathrm{i}\beta)x} \\ &= c_1 \mathrm{e}^{\alpha x} \mathrm{e}^{\mathrm{i}\beta x} + c_2 \mathrm{e}^{-\alpha x} \mathrm{e}^{-\mathrm{i}\beta x} \\ &= c_1 \mathrm{e}^{\alpha x} [\cos(\beta x) + \mathrm{i} \sin(\beta x)] + c_2 e^{-\alpha x} [\cos(\beta x) - \mathrm{i} \sin(\beta x)] \\ &= \end{aligned}$$

Progress

What's worked

What hasn't

Pictures

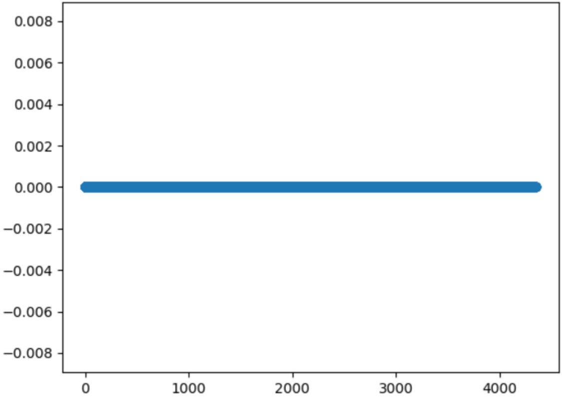

Figure: Spectrum of the operator $\mathcal{L} = -\partial_x^2 + 2q\cos(2x)$ for $q = 0.5$.

Future goals

Next steps Determining the stability of the MKdV equation

Challenges