LEARNING PATTERNS OF EXPERT BEHAVIOUR IN MULTI-OBJECTIVE DESIGN

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Expert behaviour

- Experts are able to think effectively about problems.
- Experience and practice lead to effective organization of knowledge, that leads to expertise.
- Expert knowledge is 'conditionalised' to the context of applicability.
- With experience, knowledge is chunked into larger units based on functional characteristics.
- Chunking leads to abstraction.
- Expert behaviour in design:
  - Framing the problem.
  - Fixing a principal solution.
  - Identify the aspects of the design problem that need attention.
  - Modify the principal solution according to the design requirements.

Previous work and goals

- [Moss et al., 2004] Uses prior knowledge.
- [Bandaru and Deb, 2010] - Human based extraction of design implicit design principles.
- [Mukerjee and Dabbeeru, 2009] - Symbols emerge from experience.

- Explore discovery of chunks.

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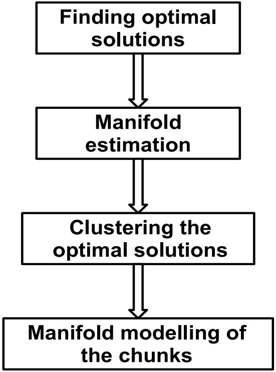
(d) *if the objective function that maps the decision space to the objective space is continuous and well-behaved, this would result in chunks that have a dimensionality $d_c = \mathbf{o}(d)$, i.e. $d_c = kd$, where $k - 1$ is vanishingly small.*

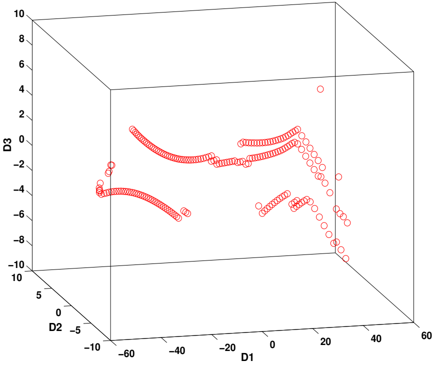
Overview of the chunking process



Estimating the pareto-front manifold

- Dimensionality reduction:
  1. Linear techniques: PCA.
  2. Non-linear techniques: LLE, Isomap*, Laplacian Eigenmaps.

3-d Isomap embedding of the BDCPMM pareto-front



Clusters in BDCPMM problem decision space

Clustering the optimal solution set I

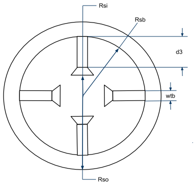
- Clustering in the objective-decision variable space to group solutions which are functionally similar.
- A density based clustering algorithm similar to DBSCAN is used.
- *Core objects* and *Density connectivity*.
- Build a graph based on density connectivity.
- Steps of the algorithm:
  1. Find *MinPoints* nearest neighbors of each point and sort in the increasing order of distance to the point.
  2. A point p is connected to a point n at the top of its nearest neighbor list only if its distance is *similar* to the average neighbor distance in the component the nearest number n belongs to.
  3. Process repeated for *MinPoints* iterations.
- The size and number of components can be controlled through the k parameter.

Experiments

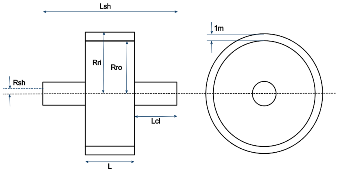
Design problem	D	Cont. variables	$d+1$	Cluster dimensionality	
				Dimensionality	No. of clusters
BDCPM design	5	0	2	1	4
				2	1
Gearbox design (A)	11	10	2	1	11
Gearbox design (B)	29	10	3	2	7
Clutch brake design	5	0	2	1	5
Welded beam design	4	4	2	1	5

Brushless DC permanent magnet motor design Problem I

- Two objectives:
  - (i) Minimize the cost, and
  - (ii) Maximize the peak torque.

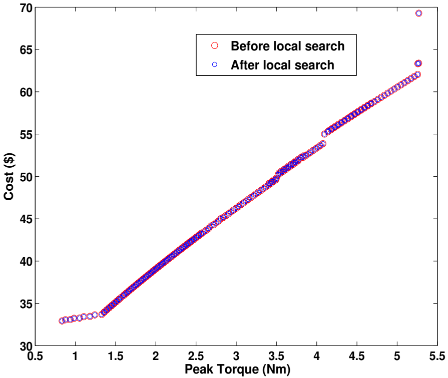


(a) BDCPMM Stator lamination.

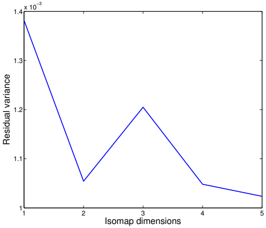


(b) BDCPMM Rotor.

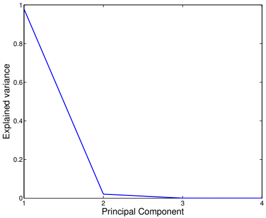
Pareto front for the BDCPMM design problem



Isomap and PCA results for the pareto-front



(c) Isomap residual variance

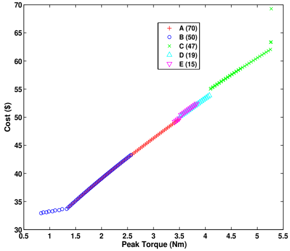


(d) PCA explained variance

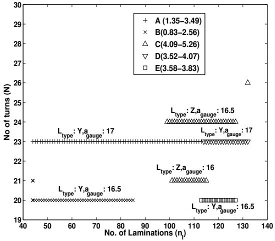
	n_l	N	L_{type}	M_{ph}	a_{gauge}
First PC	0.9993	0.0370	0.0072	0	-0.0024
Second PC	-0.0367	0.9943	0.0165	0	0.0985

Table: First two principal components of the BDCPM data.

Clusters in the BDCPMM pareto-front



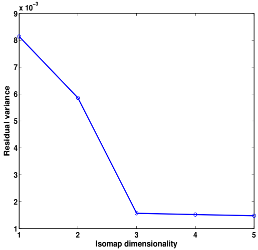
(e) Clusters as seen in the objective space.



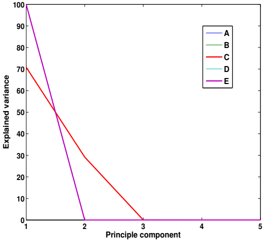
(f) Clusters as seen in the decision space.

Isomap and PCA analysis of the clusters.

- Four of the clusters are one dimensional manifolds.
- Both Isomap and PCA variances show cluster **C** with two dimensions.



(g) Residual variance for the cluster C



(h) Explained variance for the clusters.

Principal components of the clusters

- All the one dimensional clusters are lines parallel to the n_l dimension
- Cluster **C** is embedded in the $n_l - N$ plane.

	n_l	N	L_{type}	M_{ph}	a_{gauge}
A	1	0	0	0	0
B	0.999	0.0077	0	0	-0.019
C	0.789	0.610	0	0	0.066
	-0.613	0.785	0	0	0.074
D	1	0	0	0	0
E	1	0	0	0	0

Design implications

- Y connection should be used in BDCPM designs.
- For low torque motors, laminations with low radial dimensions should be used.
- Thicker wires and small number of turns in the stator coil should be used.
- For high torque motors, laminations with large radial dimension and thick wires for stator winding should be used.

Gearbox design problem

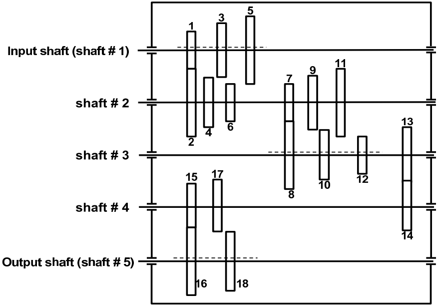


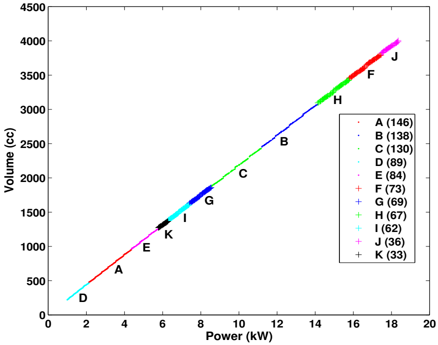
Figure: 18-speed Gearbox schematics.

Optimization problem I

- Two instances of the problem:
  - (A) Fixed gear-teeth layout 11 variable problem in which power and minimization of gearbox weight are the objectives.
  - (B) 29 variable problem with three objectives.

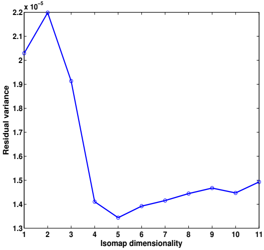
Pareto-front for the fixed gear ratio problem

- 927 points in the pareto-front.
- 11 clusters are obtained in the clustering.

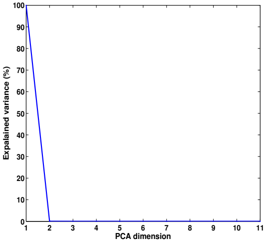


Isomap and PCA results for the pareto-front

- Isomap residual variance shows a manifold dimensionality of four.
- PCA shows a linear dimensionality of one.



(a) Residual variance



(b) Explained variance.

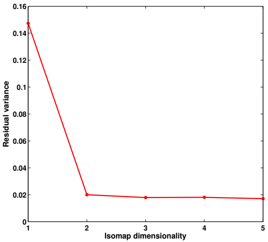
Principal components of the pareto-front

- Thickness of the gear-pairs in the final transmission stage are the most varying.

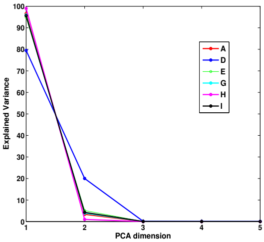
First PC	(t_9) -0.8444	(t_8) -0.4617	(t_7) 0.1837	(t_6) -0.1511	(p) 0.0733
Second PC	(m) 0.9997	(t_5) -0.0124	(t_6) 0.0097	(t_9) 0.0087	(t_8) 0.0087

Isomap and PCA results for the clusters

- Most clusters have negligible residual variances.
- Residual variance for the cluster D shows a manifold dimension of one.
- PCA explained variance shows two significant principal components for cluster D.



(c) Residual variance for the cluster D



(d) Explained variance.

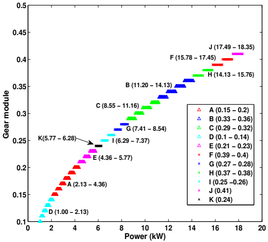
Principal components of the clusters

A	p 0.99	t_8 0.04	t_9 0.02	m 0.02	t_7 0.01	t_4 0.01	t_5 0.01	t_6 0.01	t_1 0.00	t_3 0.00	t_2 0.00
B	p 0.99	t_8 0.01	m 0.01	t_9 0.01	t_7 0.00	t_4 0.00	t_5 0.00	t_6 0.00	t_1 0.00	t_3 0.00	t_2 0.00
C	p 0.99	t_8 0.02	t_9 0.01	m 0.01	t_7 0.01	t_4 0.01	t_5 0.01	t_6 0.01	t_1 0.01	– 0	– 0
D	p 0.99	t_8 0.07	t_9 0.06	t_7 0.03	m 0.03	t_4 0.03	t_5 0.03	t_6 0.02	t_1 0.02	t_3 0.00	t_2 0.00
	t_8 -0.64	t_9 -0.46	t_7 -0.32	t_4 -0.29	t_5 -0.26	t_6 -0.24	t_1 0.17	p 0.12	m 0.02	t_3 0.00	t_2 0.00
E	p 0.99	t_8 0.07	t_9 0.05	t_7 0.03	t_5 0.02	t_4 0.02	t_6 0.02	m 0.01	t_1 0.01	t_3 0.00	t_2 0.00
F	p 0.99	t_8 0.03	t_9 0.02	t_7 0.01	t_4 0.01	t_6 0.01	t_5 0.01	t_1 0.01	m 0.0	t_2 0.0	t_3 0.0
G	p 0.99	t_8 0.07	t_9 0.05	t_7 0.04	t_6 0.03	t_4 0.03	t_5 0.03	t_1 0.02	m 0.01	t_3 0.0	t_2 0.0
H	p 0.99	t_8 0.04	t_9 0.03	t_4 0.02	t_7 0.01	t_5 0.01	t_6 0.01	t_1 0.01	m 0.00	– 0	– 0
I	p 0.98	t_8 0.10	t_9 0.07	t_7 0.05	t_4 0.04	t_5 0.04	t_6 0.04	t_1 0.03	m 0.01	– 0	– 0
J	p 0.97	t_8 0.13	t_9 0.10	t_7 0.06	t_4 0.06	t_6 0.05	t_5 0.05	t_1 0.03	t_3 0.0	t_2 0.0	m 0
K	p 0.85	t_8 0.33	t_9 0.25	t_7 0.17	t_4 0.14	t_5 0.13	t_6 0.13	t_1 0.09	t_3 -0.00	t_2 -0.00	m 0

Gear pair No.	1	8	9	2	4	7	5	6	3
I/O Speed ratio	0.35	0.46	0.54	0.76	0.86	0.97	0.97	1.09	1.71

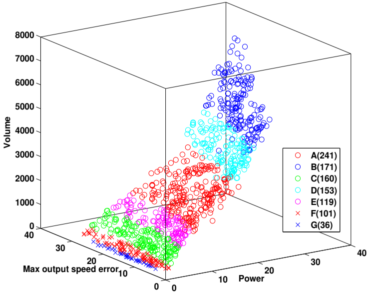
Discussion

- Gear-pairs in the final transmission stage are under the highest stress, hence vary the most with increasing power.
- Gear-pairs with lower input to output speed ratios are at higher stress, hence vary more in the same transmission stage.
- Gearbox with higher power requirements use larger modules.



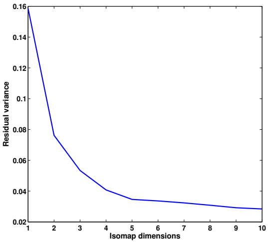
Pareto-front and clusters for the full problem

- 989 optimal solutions
- pareto-frontier is a two dimensional manifold in the objective space
- 7 clusters.

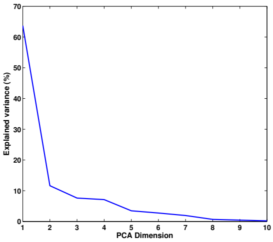


Isomap and PCA analysis

- No clear indication of manifold dimension.
- Four principal components with significant explained variance.



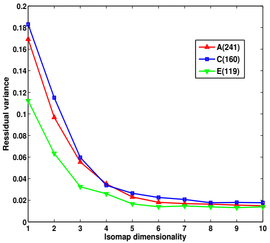
(e) Residual variance



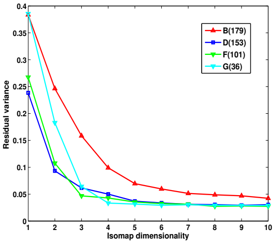
(f) Explained variance.

Isomap residual variances for the clusters

- for most clusters, the largest drop in residual variance is for two or three dimensional embedding.



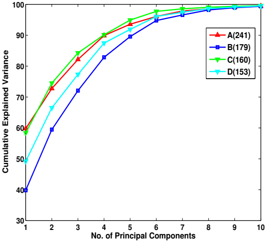
(g) Residual variance for A, C, and E.



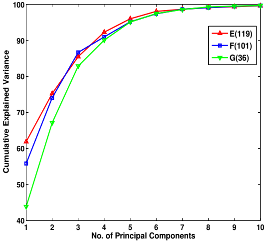
(h) Residual variances for B, D, F and G.

PCA explained variance for the clusters

- Six or seven principal components account for 90% variance.



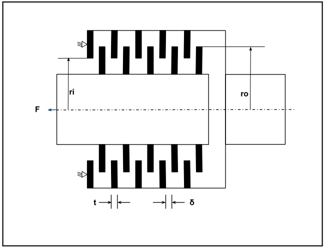
(i) Cumulative explained variances for A, B, C and D.



(j) Cumulative explained variances for E, F and G.

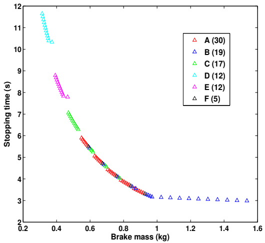
Clutch brake design problem I

- Two objectives:
  (i) minimization of mass, and,
  (ii) minimization of stopping time.

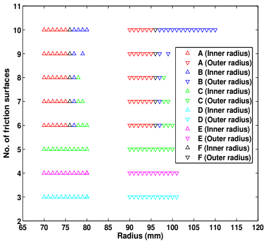


Pareto-front and clusters

- 95 pareto-optimal solutions
- Six clusters.



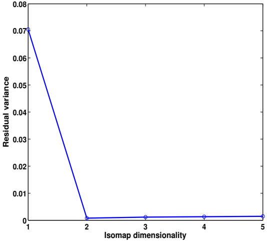
(k) Pareto-front and clusters in the objectives space.



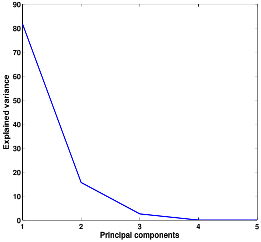
(l) Radius vs. no. of friction surfaces.

Isomap and PCA analysis of the pareto-front

- Pareto-front is a one dimensional manifold.
- Two significant principal components.



(m) Residual variance.



(n) PCA explained variance.

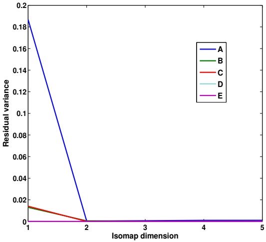
Principal components of the pareto-front

- Disk thickness and force applied are constant.
- Radius variables are ones in which the designs vary.

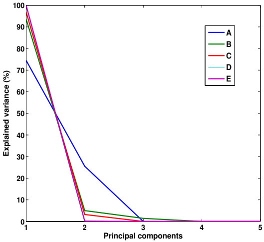
	r_i	r_o	t	F	Z
First PC	0.578	0.806	0	0	0.119
Second PC	-0.207	0.004	0	0	0.978

Isomap and PCA analysis of the clusters

- All the clusters are one-dimensional manifolds.
- Cluster A has two significant principal components.



(o) Residual variance.



(p) PCA explained variance.

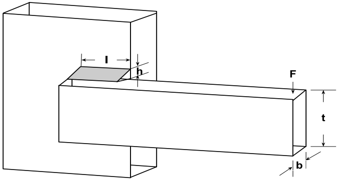
Significant principal components of the clusters

- All the clusters except A are embedded in the $r_i - r_o$ plane.
- Cluster A has two significant principal components, first is in the $r_i - r_o$ plane and the other is parallel to Z.

	r_i	r_o	t	F	Z
A	0.707	0.707	0	0	0
	0	0	0	0	1
B	0.236	0.960	0	0	0.146
C	0.703	0.703	0	0	0.097
D	0.694	0.719	0	0	0
E	0.694	0.719	0	0	0
F	0	0	0	0	1

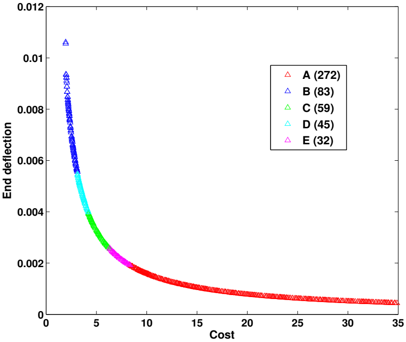
Welded beam design problem I

- Two objectives:
  - (i) Minimization of end deflection and
  - (ii) Minimization of cost.



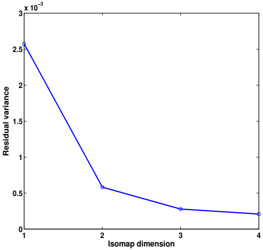
Pareto-front and the clusters for welded beam design problem

- 491 pareto-optimal solutions.
- 5 clusters.

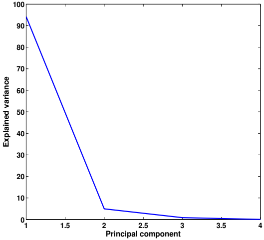


Isomap and PCA results for the pareto-front

- Residual variance shows a one dimensional manifold.
- Linear dimension is one.



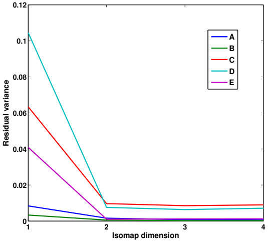
(q) Residual variance.



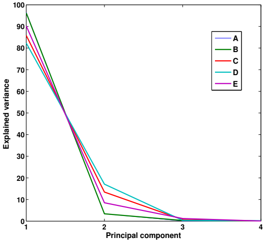
(r) PCA explained variance.

Isomap and PCA results for the clusters

- All the clusters are one dimensional manifolds.
- Three clusters have two significant principal components, others have one.



(s) Residual variance.



(t) PCA explained variance.

Significant principal components of the clusters

- The clusters having one linear dimension vary in thickness (b) and length of the weld (l) the most.
- Clusters with two principal components have significant weights in all variables other than width of the beam (t)

	b	t	l	h
A	0.965	0	-0.06	0.249
B	0.126	0.068	-0.982	0.115
C	0.435	0	-0.755	0.488
	0.897	0.006	0.404	-0.174
D	0.309	0	-0.883	0.351
	0.949	0.011	0.307	-0.06
E	0.381	0.002	-0.616	0.688
	0.924	0.019	0.256	-0.282

Conclusion

- A clustering and dimensionality reduction approach to chunking in design.
- A system can be provided experience by increasing the complexity of the optimization problem.
- Such approach to chunking is applicable in any task where task can be expressed as a multi-objective optimization.
- The example problems suggest the *chunk dimensionality conjecture*.
- More work is needed to substantiate the conjecture.

Questions?

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