→ Distributed formation control architecture

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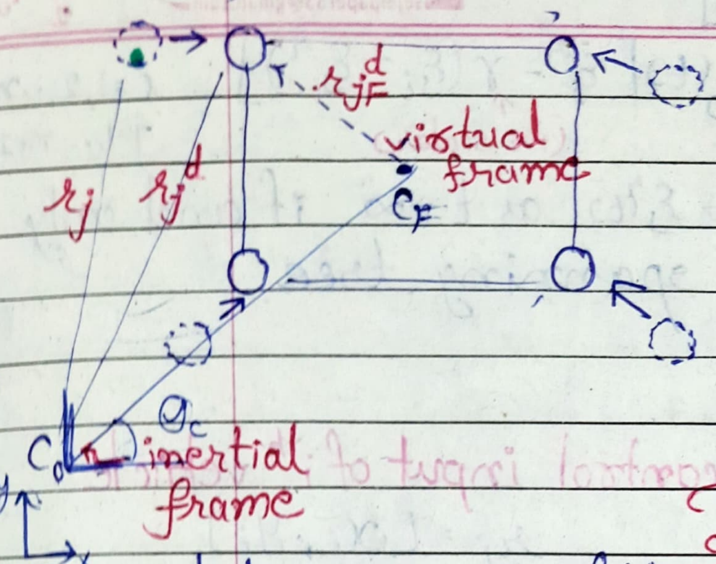


$C_F \cdot (x_c, y_c, \theta_c)$ virtual center (position, orientation)

$r_j = [x_j, y_j]^T$ → jth vehicle → actual pos.

$r_j^d = [x_j^d, y_j^d]^T$ → desired pos.

$r_{jF}^d = [x_{jF}^d \cdot y_{jF}^d]$ → desired deviation rel. to C_F

relation is as follow

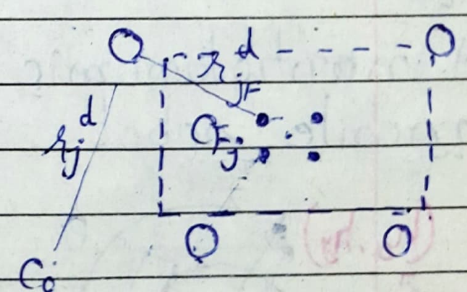
$$\begin{bmatrix} x_j^d(t) \\ y_j^d(t) \end{bmatrix} = \begin{bmatrix} x_c(t) \\ y_c(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \cos[\theta_c(t)] & -\sin[\theta_c(t)] \\ \sin[\theta_c(t)] & \cos[\theta_c(t)] \end{bmatrix}}_{\text{transformation /rotational matrix}} \begin{bmatrix} x_{jF}^d(t) \\ y_{jF}^d(t) \end{bmatrix}$$

Assumption ↳ each robot knows state of virtual co. frame (C_F)

↳ denote $\xi = [x_c, y_c, \theta_c]^T$

(coordination variable of team)

→ Due to dynamically changing situational awareness or limited info. exchange



C_{Fj} → j^{th} vehicle's understanding of virtual coordinate frame

$\xi_j = [x_{cj}, y_{cj}, \theta_{cj}]^T$

See Fig 10.3

$N_i(t)$ → set of vehichles whose instu. of co. variable to veh. i

$J_i(t)$ → position traking errors to vehicle i at t.

$\boxed{\text{Consensus traking module}}$ → drive ξ_i to $\xi^r = [x_c^r, y_c^r, \theta_c^r]^T$

each vehicle applies consensus tracking algo.

$$u_i = \frac{1}{\eta_i(t)} \sum_{j=1}^{n} a_{ij}^c(t)\left[\dot{\xi}_j - \gamma(\xi_i - \xi_j)\right] + \frac{1}{\eta_i(t)} a^c_{i(n+1)}(t)\left[\dot{\xi}^r - \underset{\text{(+ve scalar)}}{\gamma}(\xi_i - \xi^r)\right]$$

$\eta_i(t) = \sum_{1}^{n+1} a^c_{ij}$, $\quad i = 1, 2, \dots n$, $\quad j = 1, \dots n+1$

→ this algo. guarantees $\xi_i(t) \to \xi^r(t)$ as $t \to \infty$ if and only if G^c_{n+1} has a directed spanning tree.

Single integrator dynamics of vehicle → $\underbrace{\dot{r}_i}_{\text{state}} = \underbrace{u_i}_{\text{control input of } i^{th} \text{ vehicle}}$

$r_i = [x_i, y_i]$

$\dot{r}_i = [\dot{x}_i, \dot{y}_i]$

$$u_i = \dot{r}_i^d - \underset{\text{positive scalar}}{\alpha_i}(r_i - r_i^d) - \sum_{j=1}^{n} a^v_{ij}\left[(r_i - r_i^d) - (r_j - r_j^d)\right]$$

a^v_{ij} → (i,j) entry of matrix A^v_n

interaction topology $G^v_n = (V^v_n, \mathcal{E}^v_n)$??

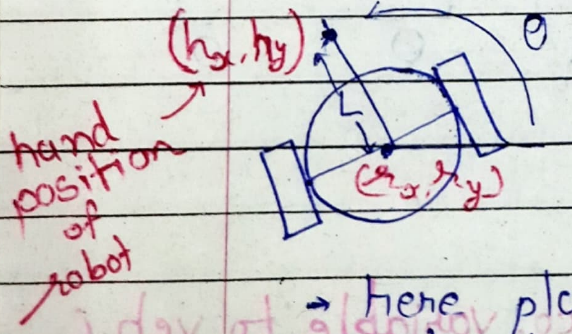
$$\begin{bmatrix} x_i^d \\ y_i^d \end{bmatrix} = \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} + \begin{bmatrix} \cos(\theta_{ci}) & -\sin(\theta_{ci}) \\ \sin(\theta_{ci}) & \cos(\theta_{ci}) \end{bmatrix} \begin{bmatrix} x_{iF}^d \\ y_{iF}^d \end{bmatrix}$$

Control Law → $\boxed{u_i = \dot{r}_i^d - \alpha_i(r_i - r_i^d)}$ → guarantee $r_i(t) \to r_i^d(t)$

* A nonholonomic differentially driven wheeled mobile robot

$h \triangleq [h_x, h_y]^T$

$r \triangleq [r_x, r_y]^T$



Kinematics of hand position are holonomic for $L \neq 0$.

→ here plan is to coordinating hand positions instead of coordinating their centra position.

$(r_{x_i}, r_{y_i}) \rightarrow$ inertial position

$\theta_i \rightarrow$ orientation

$(v_i, \omega_i) \rightarrow$ linear & angular speed

$$\dot{r}_{x_i} = v_i \cos(\theta_i)$$
$$\dot{r}_{y_i} = v_i \sin(\theta_i)$$
$$\dot{\theta}_i = \omega_i$$

Hand position $\begin{bmatrix} h_{x_i} \\ h_{y_i} \end{bmatrix} = \begin{bmatrix} r_{x_i} \\ r_{y_i} \end{bmatrix} + L_i \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}$

$$\begin{bmatrix} \dot{h}_{x_i} \\ \dot{h}_{y_i} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -L_i \sin\theta_i \\ \sin\theta_i & L_i \cos\theta_i \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

Let

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\frac{1}{L_i}\sin\theta_i & \frac{1}{L_i}\cos\theta_i \end{bmatrix} \begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix}$$

gives $\boxed{\begin{bmatrix} \dot{h}_{x_i} \\ \dot{h}_{y_i} \end{bmatrix} = \begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix} = u_i}$ Kinematics Equation

Apply

→ ref state of virtual coordinate fram $\xi^r = [x_c^r, y_c^r, \theta_c^r]^T$ satisfies

$$\dot{x}_c^r = v_c^r \cos(\theta_c^r)$$
$$\dot{y}_c^r = v_c^r \sin(\theta_c^r)$$
$$\dot{\theta}_c^r = \omega_c^r,$$

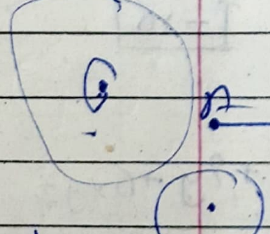
$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix}$$

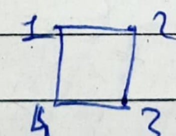
$v_c^r, \omega_c^r, x_c^r, y_c^r, \theta_c^r$

L_i → given

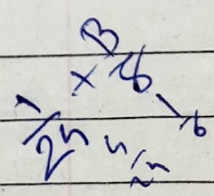
$$x_{jF}^d = l_j \cos\phi_j$$
$$y_{jF}^d = l_j \sin\phi_j$$

r_j^d

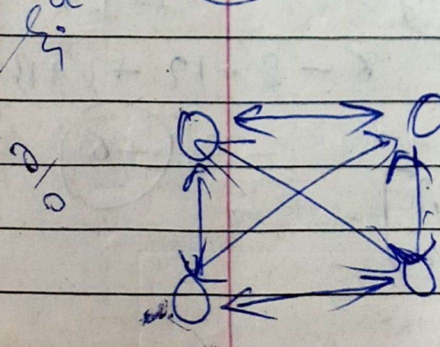


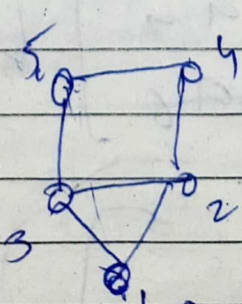


$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$



ξ_i^d





$V = AS$

$A =$

Laplacian matrix

L_{ij}