INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

MA 202: MATHEMATICS - IV
Semester–II, Academic Year 2022-23

Tutorial Set -3
Question - 9

By

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➔ In question 9, we are given the ventilation system of the restaurant as shown below. We are asked to find the steady-state concentration of carbon monoxide in each room using the Gauss Elimination and Gauss-Seidel Method.

➔ The idea of Gaussian elimination is to transform a system of n linear equations in n unknowns to an equivalent system (i.e., a system with the same solution as the original one) with an upper-triangular coefficient matrix, a matrix with all zeros below its main diagonal:

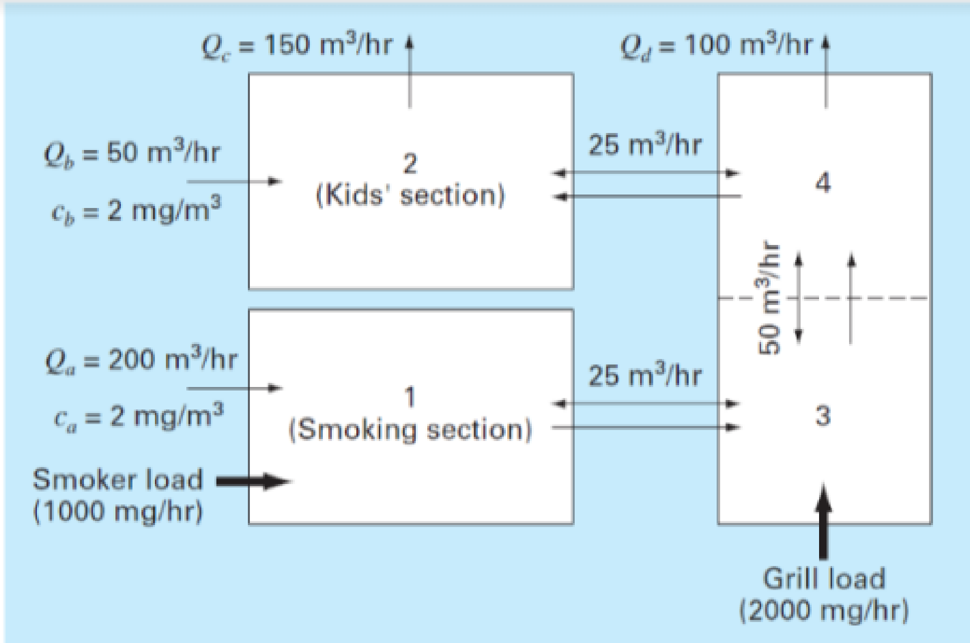

Figure 3:

$$0 = W_{\text{smoker}} + Q_a c_a - Q_a c_1 + E_{13}(c_3 - c_1),$$

or substituting the parameters

$$225c_1 - 25c_3 = 2400.$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned} \quad \Longrightarrow \quad \begin{aligned} a'_{11}x_1 + a'_{12}x_2 + \cdots + a'_{1n}x_n &= b'_1 \\ a'_{22}x_2 + \cdots + a'_{2n}x_n &= b'_2 \\ &\vdots \\ a'_{nn}x_n &= b'_n. \end{aligned}$$

In matrix notations, we can write this as

$$Ax = b \quad \Longrightarrow \quad A'x = b',$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \; b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \; A' = \begin{bmatrix} a'_{11} & a'_{12} & \dots & a'_{1n} \\ 0 & a'_{22} & \dots & a'_{2n} \\ \vdots & & & \\ 0 & 0 & \dots & a'_{nn} \end{bmatrix}, \; b = \begin{bmatrix} b'_1 \\ b'_2 \\ \vdots \\ b'_n \end{bmatrix}.$$

➔ Now, by obtaining the equations for all rooms :

Room 1:

$$0 = w_{\text{smoker}} + Q_a c_A - Q_a c_1 + E_{13}(c_3 - c_1)$$

Room 2:

$$0 = Q_b c_b + \frac{Q_a}{2} c_4 - Q_c c_2 + E_{24}(c_4 - c_2)$$

Room 3:

$$0 = W_{grill} + Q_a c_1 + E_{13}(c_1 - c_3) + E_{34}(c_4 - c_3) - Q_2 c_3$$

Room 4:

$$0 = Q_a C_3 + E_{34}(C_3 - C_4) + E_{24}(C_2 - C_4) - Q_d C_4 - \frac{Q_a}{2} C_4$$

Where..

$$Q_c = Q_b + \frac{Q_a}{2} \text{ and } Q_d = \frac{Q_2}{2}$$

```
% Gauss Elimination Method
% Call the function by giving two matrix as a input
function Xr =T9_20110131(A,B)
% Matrix A(n*n)
% Matrix B(n*1)              % System Ax=B
P = [A B];                   % Constructing the new augmented matrix P
p = size(P);                 % Calculating the size of augmented matrix, P
% Check whether all diagonal elements of Matrix P or Matrix A are non zero
for m = 1:p(1)
    if P(m,m) == 0
       disp('Gauss elimination method can not applicable. Rearrange the equations!!!');
             %Diagonal element zero. Hence pivote can't be calculated.
       return
    end
end
% Run a loop to perform all steps of Gauss Elimination
% Finding zeros of lower triangular matrix.
for m = 1:p(1)-1
    a=P(m,m);
     P(m,:)= P(m,:)/a;               % Devide all elements by its diagonal element
     % run a loop to perform a row opertaion
    for k=m+1:p(1)
      P(k,:)= P(k,:)- P(k,m)* P(m,:);
    end
end
% Perform a operation on last row
 a=P(p(1),p(1));
 P(p(1),:)= P(p(1),:)/a;
```

Gauss Elimination Method

```
% Gauss Seidel Iteration method
% Call the function by giving two matrix as a input
function Xr = GaussSeidel(A,B)
% Matrix A(n*n)
% Matrix B(n*1)               % System Ax=B
n = size(B);                  % Calculating the size of matrix B
p = n(1);                     % Define a number of rows in matrix B
tol = 0.0001;                 % define a value of tolerance
Err = ones(p,1);              % Define a error column matrix
X  = zeros(p,1);              % Define a initial guess of solution
C  = zeros(p,1);              % Define a dummy column matrix for calculations
% Run a loop to find  final result by applying the condition of Gauss
% Seidel ietration method
while max(Err) > tol
    for i = 1:p
        C(i,1) = X(i,1);
        X(i,1) = (1/A(i,i)) * (B(i,1) - sum(A(i,:) * X(:,1)) + A(i,i)*X(i,1));
        Err(i,1) = abs(C(i,1) - X(i,1));
        C(i,1) = X(i,1);
    end
end
disp('Solution by Gauss Seidel Iteration Method is:')
Xr = X;
end
```

Gauss-Seidel Method

On substituting the parameters :

```
%  Matrix A -- [225    0    -25     0
%                 0  175      0  -125
%              -225    0    275   -50
%                 0  -25   -250   275]
% Matrix B -- [1400
%              100
%              2000
%                0 ]
```

This gives

```
>> Xr =T9_20110131([225 0 -25 0;0 175 0 -125;-225 0 275 -50;0 -25 -250 275],[1400;100;2000;0])
Solution by Gauss Elimination Method is:

Xr =

   8.099616858237548
  12.344827586206893
  16.896551724137929
  16.482758620689651

>> Xr = GaussSeidel([225 0 -25 0;0 175 0 -125;-225 0 275 -50;0 -25 -250 275],[1400;100;2000;0])

Xr =

   8.099613753080009
  12.344804459194311
  16.896543296678558
  16.482748856907264
```

b)

We can change the RHS in [1] for grill, smokers and vent as following:

$$\underset{\text{RHS}}{\text{smokers}} = \begin{bmatrix} 1000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Gives $c_2 = 3.45$

$$\underset{\text{RHS}}{\text{grill}} = \begin{bmatrix} 0 \\ 0 \\ 200 \\ 0 \end{bmatrix}$$

Gives $c_2 = 6.90$

$$\underset{\text{RHS}}{\text{vent}} = \begin{bmatrix} 400 \\ 100 \\ 0 \\ 0 \end{bmatrix}$$

Gives $c_2 = 2.0$

$$\text{Smoker } = \frac{3.45}{12.34} \times 100 = 27.6\%$$

$$\text{Grill } = \frac{6.9}{12.34} \times 100 = 55.9\%$$

$$\text{Vent } = \frac{6.9}{12.34} \times 100 = 16.2\%$$

$$Total = 100\%$$

c)

```
A =

     9     0    -1     0
     0     7     0    -5
    -9     0    11    -2
     0     1    10   -11

>> B = [24;0;120;0]

B =

    24
     0
   120
     0
```

Coefficient Matrix A and B

```
>> Xr =T9_20110131([9 0 -1 0;0 7 0 -5; -9 0 11 -2 ;0 1
Solution by Gauss Elimination Method is:

Xr =

   4.652873563218391
  12.413793103448274
  17.875862068965517
  17.379310344827584

>> Xr =GaussSeidel([9 0 -1 0;0 7 0 -5; -9 0 11 -2 ;0 1

Xr =

   4.652870285985149
  12.413768694826992
  17.875853174489265
  17.379300039974513
```

➔ Therefore, the increase in the concentration in the kids section = 12.41379 mg/m3

d)

```
A =

     9     0    -1     0
     0    31     0   -21
    -9     0    11    -2
     0     1    50   -55

>> B = [56;20;80;0]

B =

    56
    20
    80
     0
```

Coefficient Matrix A and B

```
Xr =

   8.074591622189875
  11.048034934497815
  16.671324599708878
  15.356622998544395

>> Xr =GaussSeidel(A,B)

Xr =

   8.074588613391912
  11.048017442974606
  16.671317443270549
  15.356616174663673
```