INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

MA 202: MATHEMATICS - IV
Semester–II, Academic Year 2022-23

Tutorial Set -2
Question - 5

By

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➔ In question 5, we are asked to solve the the following set of 2n non-linear algebraic equations using Newton's method, for the unknowns y = [c1, x1, c2, x2, ..., cn, xn]

$$f_k(\mathbf{y}) \equiv \sum_{j=1}^{n} c_j x_j^{k-1} - \int_{-1}^{1} t^{k-1} dt = 0, \qquad k = 1,\ 2,\ \ldots,\ 2n$$

➔ We are supposed to write a code for a general condition instead of for only n=1,2,3 or 4. These equations naturally arise while applying Gauss quadrature to approximately compute integrals. To solve this equation, we have used Newton's method for non algebraic equations. Approximation after the (k+1)th iteration, Newton's method, as we know, is given by:

$$Given: X_0 \text{ an initial guess of the root of } F(x)=0$$

$$Newton's\ Iteration$$

$$X_{k+1} = X_k - \left[F'(X_k)\right]^{-1} F(X_k)$$

$$F(X) = \begin{bmatrix} f_1(x_1, x_2, \ldots) \\ f_2(x_1, x_2, \ldots) \\ \vdots \end{bmatrix}, \quad F'(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \vdots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \\ \vdots & & \end{bmatrix}$$

```
% SUBMITTED BY - KUSH PATEL (20110131)
% Question -5

% Create a fuunction to call the vale of X0(which contains inputs in order x1 , x2 , c1 , c2)
% and return the vale of Xr(which return the all values of variables)
function [Xr] = T25_20110131(n,X0)
X0 = transpose(X0);
% to print the value of x_r up to long decimal digits
format("long")
Xr = X0;
% Initialize the vales of tolerance and error
err = 1000;
tol = 0.00001;
% Run the while loop to run the iterations
while err > tol
    f = Func(n,X0);                    % Define a function to determine the matrix F and G
    G = f.G;                           % Assign a matrix G
    F = f.F;                           % Assign a matrix F
    h = -inv(F)*G;                     % Determine the matrix h
    Xr = X0 + h;                       % Determine the next (i+1) values by applying formula
    err = max(abs((Xr - X0)./(Xr)));   % determine the error by finding the maximum element of ar
    X0 = Xr;                           % Assign the Xr to again as a input
end

% Create the function to find the matrix G and F
function val = Func(n,X)
% For creating a matrix G
```

```
% Create the function to find the matrix G and F
function val = Func(n,X)
% For creating a matrix G
for i=1:2*n
    y=0;
    for j=1:n
        y = y+ (X(j+n)*(X(j)^(i-1)));
    end
    val.G(i,1) = y - ((1-(-1)^i)/i);
end

% For creating a matrix F
% Determine the values of 1 to n arrays
for g=1:2*n
    for k=(n+1):2*n
        val.F(g,k) =  (X(k-n)^(g-1));
    end
end
% Determine the values of (n+1) to 2*n arrays
for o=1:2*n
    for p=1:n
        val.F(o,p) = ((o-1)* X(p+n)*(X(p)^(o-2)));
    end
end
end
end
end
```

- ➔ As we can see in the code above, I have taken the input in order of [x1 , x2 , c1 ,c2]. Then, the main focusing task was to create a matrix G and F in another function. I created a matrix G by observing a pattern and writing it in the code. To create a matric F, I used two loops. One is to determine the half n values of the matrix F and the rest is for another one. As we can see that there is a more interesting pattern present in F.

- ➔ For **n=2**, results are shown below which satisfy the equations completely.

```
>> [Xr] = T25_20110131(2,[1,2,3,4])

Xr =

  -0.577350269189494
   0.577350269189402
   0.999999999999650
   1.000000000000350

>>
```

➔ For **n=3**, results are shown below which satisfy the equation completely but it gives output for only some specific inputs otherwise it is infinite because the matrix becomes singular so its inverse can not be calculated.

```
>> [Xr] = T5_20110131(3,[1.2503,0.2398,-0.6516,0.3502,0.9298,-0.6904])

Xr =

   0.774596669241483
  -0.000000000000000
  -0.774596669241483
   0.555555555555555
   0.888888888888889
   0.555555555555555

>> |
```

➔ For **n=4,** results are shown below which is NaN (Not a Number).

```
Xr =

   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
```