INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

MA 202: MATHEMATICS - IV
Semester–II, Academic Year 2022-23

Tutorial Set -3
Question - 5

By

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➔ In question 5, we are asked to compute how far the jumper travels during the first 8 seconds of free fall using the Composite Simpson's $\frac{1}{3}$ Integration Rule. We have given the equation of the curve as an input as shown below.

$$v(t) = \frac{gm}{c}\left(1 - e^{-(\frac{c}{m})t}\right)$$

➔ The idea behind the Simpson's method is to approximate the integral of a function between two limits, a and b by numerical method. Approximation for Simpson's $\frac{1}{3}$ Integration Rule method, as we know, is given by:

$$\Rightarrow \int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+\ldots..+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$$

where, $\Delta x = \frac{b-a}{n}$

$x_0 = a$ and $x_n = b$

$x_0, x_0, \ldots\ldots\ldots, x_n$ are the ends of the n sub intervals

$$\Rightarrow \text{Error bound} = \frac{M(b-a)^5}{180n^4}$$

where $|f^{(4)}(x)| \leq M$

➔ Here, we are given the equation of the curve so it would be easy to apply the simpson's rule on it. As we can see above, there is an error in this method. So, we need to make sure that our error has to be less than the tolerance given. To find the error, we need the maximum value of the 4th derivative value of a given function at given input points. So, I ran a loop to find a maximum of 4th derivative value. I also made a function that returns both values - function value and its 4th derivative value.

➔ After that I just checked the condition between error and tolerance and then applied Simpson's rule and printed the answer. We can also get a more accurate answer by increasing the value of n.

```
% Simpson's 1/3 intergration rule
function answer = T5_20110131(a,b,n)
format('long')

function [f,ddf] = MyFunc(x)
    g = 9.81;
    m = 80;
    c = 10 ;
    f = ((g*m)/c)*(1 - exp(((-c*x)/m)));
    ddf = diff(f,4);
end

max = 0;
h = (b-a)/n;

if rem(n,2)==1
    disp('Enter a valid n!! (Even Number)')
else
for j =0:n
    k = a + j*h;
    [~,ddf] = MyFunc(k);
    if abs(ddf)>max
        max = abs(ddf);
    else
        continue
    end
end
```

Build a function and compute the error

```
    else
        continue
    end
end

err = (((b-a)^5)/(180*n^4))*max;
tol = 0.001;
if err < tol
    x = a;
    sumo = 0;
    sume = 0;
    for i = 1:1:(n-1)
        x = x+h;
        [f1,~] = MyFunc(x);
        if rem(i,2) == 1
            sumo = sumo + f1;
        else
            sume = sume + f1;
        end
    end
    [fa,~] = MyFunc(a);
    [fb,~] = MyFunc(b);
    answer = (h/3) * (fa + fb + 4*(sumo) + 2*(sume));
else
    disp('Please change the value of n')
end
end
end
```

Code of Simpson's method

```
>> answer = T5_20110131(0,8,100)

answer =

     2.309694283230304e+02
```

Simulated output