INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

MA 202: MATHEMATICS - IV
Semester–II, Academic Year 2021-22
Tutorial Set – 3

Instructions

- In this tutorial set, problems 4(a), 6 and 8 need to be worked on during the tutorial session and **rest of the problems have to be submitted. The deadline for submitting solutions is 11:59 pm on the Sunday, 17th April**. The extra time is provided so that students will have necessary time to prepare a very good report and upload all the files, and submit.
- You should write a computer program to solve the equations. You may use MATLAB or python.
- The report must be in **PDF format**. Please upload the report and program files separately (that is, please do NOT submit all of them together as a single ZIP file). **Solutions to each problem should be submitted as a separate file**. Name each file as: Tutorialproblemnumber_Rollnumber.***. For example, if your roll number is 19110110 and for problem T1, name your report file as T1_20110110.pdf and program file as T1_20110110.m.

T1. The specific volume of a super-heated steam is listed in steam tables for various temperatures. For example, at a pressure of 3000 lb/in^2:

T (°F)	v (ft^3/lb$_m$)
700	0.0977
720	0.12184
740	0.14060
760	0.15509
780	0.16643

Use first- through fourth-order polynomial interpolation to estimate v at $T = 750$°F. Interpret your results.

T2. Prove that the number of steps required to complete Gauss elimination (forward substitution and backward elimination) is $O(n^3)$, for an $n \times n$ system of equations. Argue that the number of steps for Gauss-Seidel iterations is $O(n^2)$ for that same system of equations.

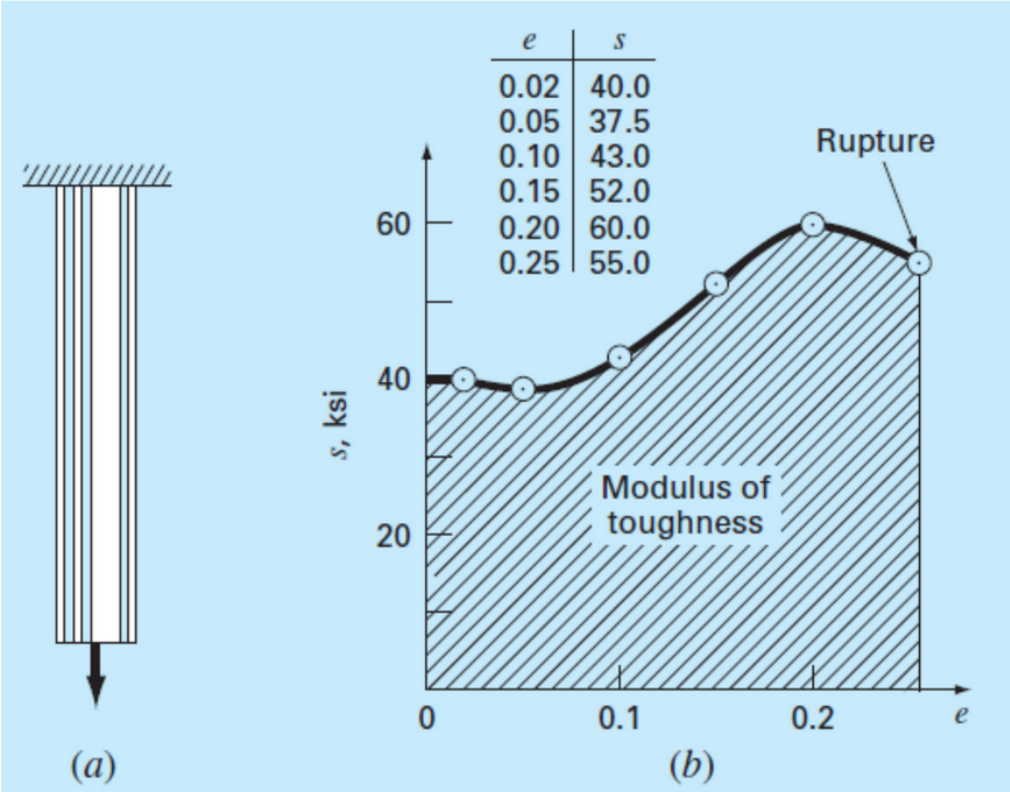

Figure 1:

T3. A rod subject to an axial load will be deformed, as shown in the stress-strain curve (see figure 1). The area under the curve from zero stress out to the point of rupture is called the modulus of toughness of the material. It provides a measure of the energy per unit volume required to cause the material to rupture. As such, it is representative of the material's ability to withstand an impact load. Use Trapezoidal method to compute the modulus of toughness for the shown stress-strain curve.

T4. Perform the following integrals using the Trapezoidal and the Simpson's 1/3 rule, for various values of the arguments (x, y, etc.) of your choice. Ensure that the values that you obtain are accurate upto $\varepsilon_a < 0.001$.

$$\text{(a)} \qquad I(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

$$\text{(b)} \qquad Ei(x) = \int_{x}^{\infty} \frac{e^{-t}}{t} dx$$

$$\text{(c)} \qquad Si(z) = \int_{0}^{z} \frac{\sin(t)}{t} dt$$

T5. Recall that the velocity of the free falling parachutist with linear drag can be computed analytically as

$$v(t) = \frac{gm}{c}(1 - e^{-(c/m)t}),$$

where $v(t)$ is velocity (m/s), t is time (s), $g = 9.81$ m/s^2, m is mass (kg), c is linear drag coefficient (kg/s). Use Composite Simpson's $\frac{1}{3}$-rd integration rule to compute how far the jumper travels during the first 8 seconds of free fall given $m = 80$ kg and $c = 10$ kg/s. Compute the answer to $|\varepsilon_a| < 0.1\%$.

T6. You measure the voltage drop V across a resistor for a number of different values of current i. The results are

i	V
0.25	20.45
0.75	20.6
1.25	0.70
1.5	1.88
2.0	6.0

Use first- through fourth-order polynomial interpolation (Lagrange interpolation) to estimate the voltage drop for $i = 1.15$. Please use $x_0 = 0.25$ as the base point for the linear interpolation. Interpret your results.

T7. Temperature is measured at various points on a heated plate and the data is as follows:

x	T (^{0}C)
0	100
2	85
4	70
6	55
8	40

Estimate the temperatures at (a) $x = 3.2$ and (b) $x = 2.7$ using lagrange interpolating polynomials of various orders. Plot the interpolating polynomial along with the given data points.

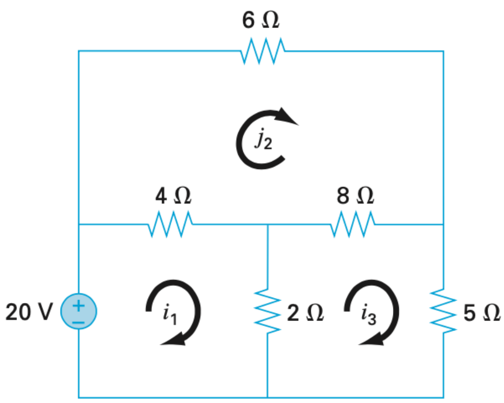

Figure 2:

T8. Use Kirchhoff's voltage law to derive set of equations for calculating currents for the circuit shown in Figure 2. Solve the resulting system of equations numerically, using both Gauss elimination and Gauss Seidel iterations.

T9. As the name implies, indoor air pollution deals with air contamination in enclosed spaces such as homes, offices, work areas, etc. Suppose that you are designing a ventilation system for a restaurant as shown in Figure 3. The restaurant serving area consists of two square rooms and one elongated room. Room 1 and room 3 have sources of carbon monoxide from smokers and a faulty grill, respectively. Steady-state mass balances can be written for each room. For example, for the smoking section (room 1), the balance can be written as

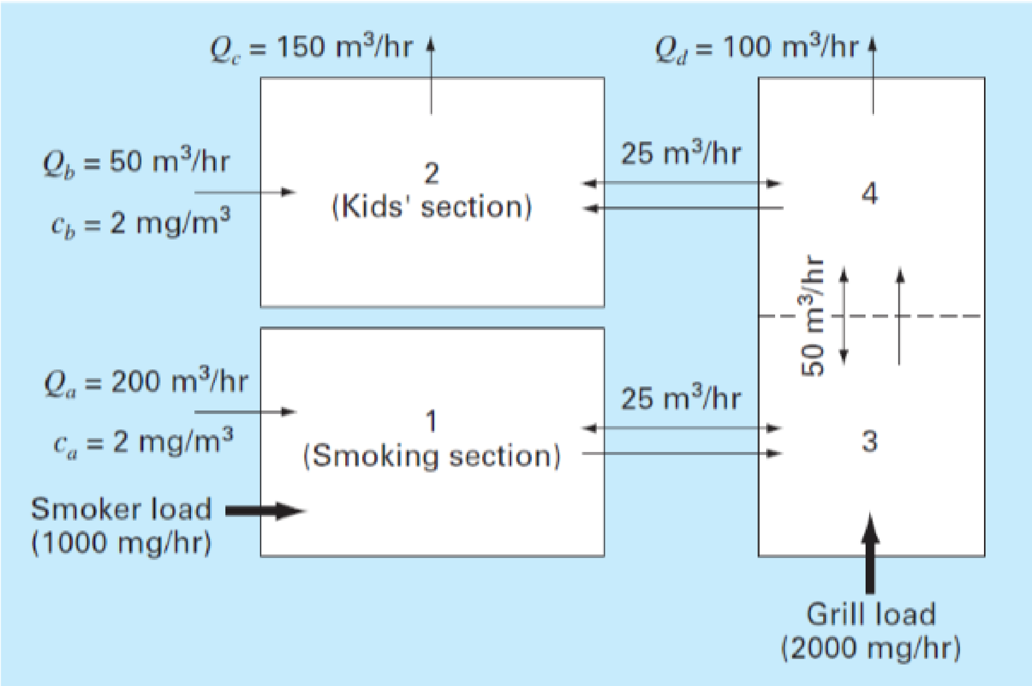

Figure 3:

$$0 = W_{\text{smoker}} + Q_a c_a - Q_a c_1 + E_{13}(c_3 - c_1),$$

or substituting the parameters

$$225c_1 - 25c_3 = 2400.$$

Similar balances can be written for the other rooms.

- Solve for the steady-state concentration of carbon monoxide in each room. Use both Gauss elimination and Gauss-Seidel iterations and compare their results.
- Determine what percent of the carbon monoxide in the kids' section is due to (i) the smokers, (ii) the grill, and (iii) the air in the intake vents.

- If the smoker and grill loads are increased to 2000 and 5000 mg/hr, respectively, use the matrix inverse to determine the increase in the concentration in the kids' section.
- How does the concentration in the kids' area change if a screen is constructed so that the mixing between areas 2 and 4 is decreased to 5 m 3 /hr?

T10. Idealized spring-mass systems have numerous applications throughout engineering. Figure 4 shows an arrangement of four springs in series being depressed with a force of 2000 kg. At equilibrium, force-balance equations can be developed defining the interrelationships between the springs,

$$k_2(x_2 - x_1) = kx_1,$$
$$k_3(x_3 - x_2) = k_2(x_2 - x_1),$$
$$k_4(x_4 - x_3) = k_3(x_3 - x_2),$$
$$F = k_4(x_4 - x_3),$$

where the k's are spring constants. If k_1 through k_4 are 100, 50, 80, and 200 N/m, respectively, compute the x's.

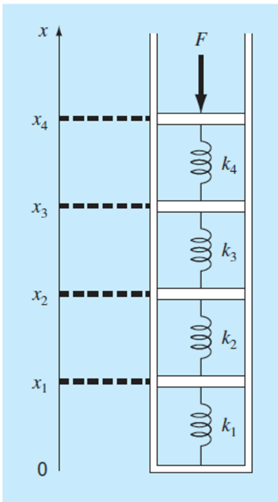

Figure 4:

T11. The outflow concentration from a reactor is measured at discrete times over a 24-hr period and the values are as follows:

t (hr.)	0	1	5.5	10	12	14	16	18	20	24
c, (mg/l)	1	1.5	2.3	2.1	4	5	5.5	5	3	1.2

The outflow rate in m^3/s may be computed using the relation:

$$Q(t) = 20 + 10 \sin\left\{\frac{2\pi}{24}(t - 10)\right\}$$

where t is in seconds. Compute the average concentration leaving the reactor over the 24-hr. period. The avg. concentration is defined as:

$$\bar{c} = \frac{\int_0^t dt' Q(t') c(t')}{\int_0^t dt' Q(t')}$$

T12. It is suspected that the high amounts of tannin in mature oak leaves inhibit the growth of the winter moth (*Operophtera bromata L., Geometridae*) larvae that extensively damage these trees in certain years. The following table (Table 1) lists the average weight of two samples of larvae at times in the first 28 days after birth.

Day	0	6	10	13	17	20	28
Sample 1 average weight (mg)	6.67	17.33	42.67	37.33	30.10	29.31	28.74
Sample 1 average weight (mg)	6.67	16.11	18.89	15.00	10.56	9.44	8.89

Table 1: Average weight of oak leaves (two samples)

The first sample was reared on young oak leaves, whereas the second sample was reared on mature leaves from the same tree.

(a) Use Lagrange interpolation to approximate the average weight on days 12 and 16 for each sample.

(b) Find an approximate maximum average weight for each sample by determining the maximum.