INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

MA 202: MATHEMATICS - IV
Semester–II, Academic Year 2021-22
Tutorial Set -2

Instructions

- In this tutorial set, problems 1 and 2 need to be worked on during the tutorial session and **problems 3 - 5 have to be submitted. The deadline for submitting solutions is 11:59 pm on the Sunday, 10th April**. The extra time is provided so that students will have necessary time to prepare a very good report and upload all the files, and submit.
- You should write a computer program to solve the equations. You may use MATLAB or python.
- The report must be in **PDF format**. Please upload the report and program files separately (that is, please do NOT submit all of them together as a single ZIP file). **Solutions to each problem should be submitted as a separate file**. Name each file as: Tutorialproblemnumber_Rollnumber.***. For example, if your roll number is 19110110 and for problem T1, name your report file as T1_20110110.pdf and program file as T1_20110110.m.

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1. Determine the roots of the simultaneous non-linear equations:

$$\begin{aligned}(x-4)^2+(y-4)^2 &= 5\\ x^2+y^2 &= 16\end{aligned}$$

   using the Newton's method. Can you take an initial guess such that $x_0 = y_0$? Explain.

2. Solve the equation $f(x) \equiv -2x^6 - 1.5x^4 + 10x + 2 = 0$ using the Secant Method. Try the initial guess $x_{-1} = 0$ and $x_0 = 1$.

3. Determine the lowest positive root of $f(x) \equiv 7e^{-x}\sin(x) - 1 = 0$. Start with initial guesses, $x_{-1} = 0.5$ and $x_0 = 0.4$.

Definition 1. *Suppose $\{p_n\}_{n=0}^{\infty}$ is a sequence that converges to p, with $p_n \neq p$ for all n. If positive constants λ and α exist with*

$$\lim_{n\to\infty}\frac{|p_{n+1}-p|}{|p_n-p|^{\alpha}} = \lambda,$$

then $\{p_n\}_{n=0}^{\infty}$ ***converges to p of order α, with asymptotic error constant λ.***

In general, a sequence with a high order of convergence converges more rapidly than a sequence with a lower order. Two cases of order are given special attention:

- If $\alpha = 1$ (and $\lambda < 1$), the sequence is **linearly convergent.**
- If $\alpha = 2$, the sequence is **quadratically convergent.**

4. Let $f(x) = e^x - x - 1$. Show that f has a zero of multiplicity 2 at $x = 0$ and that the Newton's method with $p_0 = 1$, converges to this zero but not quadratically. Moreover, show that the modified newton's method improves the rate of converges.

5. Develop a computer program to solve the following set of $2n$ non-linear algebraic equations using Newton's method, for the unknowns $\mathbf{y} = [c_1,\ x_1,\ c_2,\ x_2,\ ...,\ c_n,\ x_n]^T$.

$$f_k(\mathbf{y}) \equiv \sum_{j=1}^{n} c_j x_j^{k-1} - \int_{-1}^{1} t^{k-1} dt = 0, \qquad k = 1,\ 2,\ ...,\ 2n$$

Report your solutions for $n = 2,\ 3$ and 4. Note that you should not write separate codes for three separate values of n; rather just one set of programs should give the output and user can give input for n. These equations naturally arise while applying Gauss quadrature to approximately compute integrals.