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HW #8

1.) Is $E_{REX} \in \Sigma_0$?

Convert the regular expression to a DFA using a TM. E_{REX} is decidable because it will accept the original inputs/states. Since it accepts, E_{REX} is decidable.

2.) $L_1 = \{\langle M \rangle \mid M$ is a turing machine that halts when started on the empty tape $\}$

Is $L_1 \in \Sigma_0$?

$K' = HALT_{TM}$

L = On Input $\langle M \rangle$, where M is a turing machine:

1.) Use K_1' to construct a tape TM T

T = "On Input $\langle M, w \rangle$, Run TM T

1. Run $\langle M, w \rangle$ until it Halts
2. If it halts than $\langle M, w \rangle$ is an Input on the tape.

But a halt TM is not decidable, So that means that L_1 is also not decidable, Since the halt is undecidable, the input M is not on the tape.

$L_1 \notin \Sigma_0$

I may have mixed up which Σ_0 or Σ_1 means decidable or undecidable.

Examples and exercises from the textbook helped me do this problem.

For problem 3, I think the professor told us to use $Halt_{TM}$.

3.) $L_2 = \{ \langle M \rangle \mid M$ is a turing machine that, at some point writes the character x on the tape when started on the empty tape $\}$
Is $L_2 \in \Sigma_0$?

L = On Input $\langle M \rangle$, where M is a turing machine... :

$K' = HALT_{TM}$

1.) Use K' to construct a tape TM T

T = "On Input $\langle M, w \rangle$, Run TM T

1. Run M on x with empty tape
2. If M accepts, than x will be written on the tape.
3.) If M accepts, than tape isn't empty.

Since M will accept when x is written on the tape, $\langle M, w \rangle \in L_2$, but only if L_2 is decidable. Since the $Halt_{TM}$ is used, the $Halt_{TM}$ is undecidable so that means L_2 is undecidable. $L_2 \notin \Sigma_0$.

Examples and exercises from the textbook helped me do this problem

4.) $A_{101} = \{ \langle M \rangle \mid M$ is a TM and $101 \in L(M) \}$. Use mapping reduction to show $A_{101} \notin \Sigma_0$

F = "On input $\langle M \rangle$ where M is a TM and 101 ---- "

1.) Construct the machine M.

M = "On Input $\langle M, w \rangle$, Run TM T

1. Run K like M
2. If K accepts, than accept. If K rejects, than rejects

2.) Output M

Assuming M accepts, than 101 is accepted. So then,

$\langle M, w \rangle \in A_{101}$ then $M \in A_{101}$.

Because of this,

it means A_{101} is decidable or $A_{101} \in \Sigma_0$

But we have to show that $A_{101} \notin \Sigma_0$ or undecidable.
I am not sure how to use mapping reduction to prove that it is undecidable.