Kush Patel

Comp. 3040

HW 1

1.) Sets: Write a formal description of the set containing the string aba.

$\{aba\}$

2.) Sequences and Tuples: What is the power set of $B = \{x, y\}$

$P(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

3.) Give a Boolean expression consisting of only P's, Q's, ¬'s, ∧'s, and ∨'s which is logically equivalent to the Boolean expression below

$\neg(P \leftrightarrow Q)$

P	Q	$P \leftrightarrow Q$	$\neg(P \leftrightarrow Q)$	$P \wedge Q$	$\neg(P \wedge Q)$	$(P \wedge Q) \vee \neg(P \wedge Q)$
0	0	1	0	0	1	1
0	1	0	1	0	0	0
1	0	0	1	0	0	0
1	1	1	0	1	0	1

$\neg((P \wedge Q) \vee \neg(P \wedge Q))$
0
1
1
0

$\neg((P \wedge Q) \vee \neg(P \wedge Q))$

4.) Relations: Give a relation that is Symmetric and Transitive, but not Reflexive

$A = \{1, 2, 3\}$
$R = \{(1,1), (1,2), (2,1), (2,2)\}$

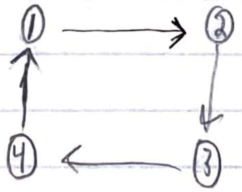
3 is in A. $3 \in A$. But $(1,3)$ $(3,1)$, $(3,2)$, $(2,3)$ or $(3,3)$ is not in R. So the relation is not reflexive. 1 R 3 doesn't exist, 2 R 3 doesn't exist, 1 R 3 doesn't exist

$1, 2 \in A$, aRb,
$(1,2) \in A$ and $(2,1) \in A$
So its symmetric

aRb exists $\rightarrow$ $1R2$
bRa exists $\rightarrow$ $2R1$
This implies aRa, which exists $\rightarrow$ $(1,1)$ or $1R1$. So its transitive

5.) Graphs: Is the statement "For every natural # $n \geq 1$ there exists a directed graph of n vertices for which every vertex has an indegree equal to its outdegree" T or F

This is True because a directed graph is formed. And every vertex in the graph has 1 edge going in and 1 edge going out the other end, or 1 indegree and 1 outdegree.



6.) $\Sigma = \{a, b, c\}$, suppose $x \in \Sigma^*$ and $|x| = 5$. Give a string 'x'' that is a substring of x......

$x' = \{a\ a\ a\ a\ a\}$

$|x'| = 5 \rightarrow |x| = 5$

I think it could also be 5 b's or 5 c's because they are also in Σ.

7.) Prove by induction on n that $C_n = \sum_{i=1}^{n} i^3 = \frac{1}{4} n^2(n+1)^2, \forall n \in \mathbb{N}$

For $i = 0$ $\quad C_n = 0^3 = 0$

$$\frac{k^2(k+1)^2}{4} = \sum_{i=0}^{k+1} i^3 = \sum_{i=0}^{k} + (k+1)^3$$

$(k+1)(k+1)$

$k^2 + k + k + 1$

$k^2 + 2k + 1$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^2$$

$$= \frac{k^2(k+1)^2}{4} + k^2 + 2k + 1$$

$$= \frac{k^2(k^2+2k+1)}{4} + k^2 + 2k + 1$$

$$= \frac{2k^4 + 2k^3 + k^2}{4} + k^2 + 2k + 1$$

$$= \boxed{\frac{k^4 + k^3 + k^2}{2} + k^2 + 2k + 1 = \sum_{i=0}^{k+1} i^3}$$