Kush Patel

Final Exam

1.) $L_1 = \{0 1^a 0^a 1 \mid a \geq 0\}$ where a is an integer and $\Sigma = \{0,1\}$. Is $L_1 \in REG$?

Using Pumping Lemma

$s = 0101$

$x = 0 \quad y = 1001 \quad z = 1$

$xyz \in A$

$y = 4 > 0$

$xy = 5 \leq p$

$x = 0 \quad y = 10010 \quad z = 1$

$s' = 010011$.

$s' \notin L$

This proves this is not Regular. NO.

s' is not a subset of L. NO

2.) $L_2 = \{1^a 01^a 0^a 10^a \mid a \geq 0\}$ where a is an integer and $\Sigma = \{0,1\}$. Is $L_2 \in REG$?

$s = 101010$

$x = 11 \quad y = 01001 \quad z = 101$

Using Pumping Lemma

$xyz \in A$

$y = 5 > 0$

$xy = 7 \leq p$

$x = 11 \quad y = 01001 \quad z = 101$

$s' = 11010011101$

$s' \notin L$

This proves this is not regular. NO

s' is not a subset of L. NO

3.) $L_1 = \{01^a0^a \mid a \geq 0\}$ where a is an integer and $\Sigma = \{0, 1\}$. Is $L_1 \in CFL$?

$V = \{S, A, B, C\}$
$\Sigma = \{0, 1\}$
$R =$
$S = A$

$S \rightarrow AS \mid BS \mid C$
$A \rightarrow 01S$
$B \rightarrow 0S$
$C \rightarrow 1$

Yes, since I used/created a CFG to prove this problem, it means L_1 is a CFL. Also, the left side, "01^a0^a" has at least one non-terminal symbol, a.

4.) $L_2 = \{1^a01^a0^a10^a \mid a \geq 0\}$ where a is an integer and $\Sigma = \{0, 1\}$. Is $L_2 \in CFL$?

Let $S = 1101000101$

$S = xyz$
$x = 11$
$y = 01000$
$z = 101$

$xy^2z = 11(01000)^3101$
$= 11010000100001000101$
$a = 20$

L_1 is $\notin$ CFL because 20 is not equal to 10, original string length.
NO

$S \rightarrow ABCD$
$A \rightarrow aA$
$B \rightarrow bB$
$C \rightarrow cC$
$D \rightarrow dD$

5.) Using L_2 from the previous problem, is $L_2 \in \Sigma_0$?

L_2 = On input $1^a 0 1^a 0 1^a 0 1^a$, where $a \geq 0$

1.) Construct a TM T

T = "on input $\langle M, w \rangle$, Run TM T

1.' Run $\langle M, w \rangle$

2. If it accepts, than accept. Reject?, th- reject.

Yes, it is decidable because, a language can still be decidable even if its not a CFL. Also, a HALT is not used, and a halt is not decidable. It is decidable because it will be accepted.

6.) Using L_2 from previous problem, is $L_2 \in \Sigma_1$?

Yes, L_2 is recognizable or $\in \Sigma_1$ because any decidable (from 5) language is also recognizable. Tape T recognizes L_2 (from 5) making it recognizable.

7.) $L_5 = \{ \langle M \rangle \mid M$ is a Turing Machine that halts when started on an empty tape $\}$. Is $L_5 \in \Sigma_0$?

$L =$ On input $\langle M \rangle$, where M is a Turing Machine ... :

1.) Use K' to construct a tape TM T

$T =$ "On input $\langle M, w \rangle$

1. Run TM T
2. Run $\langle M, w \rangle$ until it Halts
3. If it halts than $\langle M, w \rangle$ is an input on the tape.

Using $K' = Halt_{TM}$

NO, Since a HALT is used, a HALT is not decidable. Which would mean that L_5 is also not decidable. Than M is not on the tape and $L_5 \notin \Sigma_0$.

8.) FIN, ALL, REG, CFL, $\emptyset$, Σ_0, Σ_1

$\emptyset \subseteq FIN \subseteq REG \subseteq CFL \subseteq \Sigma_0 \subseteq \Sigma_1 \subseteq ALL$