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Quiz 1

1.) a) List the set described by the following:

$\{x^2 \mid x \in Z \text{ and } x = 2 \text{ or } x = 9\}$

$\{4, 81\}$ $\quad 2^2 = 4 \quad 9^2 = 81$

b.) Write a formal description of a set of all natural numbers between 5 & 25, inclusive.

$A = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$

2.) a.) Provide an example of 2 sets for which their Cartesian products is not commutative.

$A = \{x, y\}$
$B = \{2, 4\}$

$A \times B = \{(x,2), (x,4), (y,2), (y,4)\}$
$B \times A = \{(2,x), (2,y), (4,x), (4,y)\}$

$A \times B \neq B \times A$

b.) List the power set of $\{2,3,5\}$; that is $P(\{2,3,5\})$

$P(\{2,3,5\}) = \{\emptyset, \{2\}, \{3\}, \{5\}, \{2,3\}, \{2,5\}, \{2,3,5\}, \{3,5\}\}$

3.) True of False

a) The number of strings that can be formed from Σ is finite.

True

b.) A string that can be formed from Σ must be of finite length

True

c.) $(ba)^3 = bbbaaa$

False

d) abc is a substring of $aabcbc$

True

e.) $\{a, bac, cbd\}$ is a subset of the language Σ^*

False

4.) Is the following binary relation R on Z an equivalence relation?

$x R y \leftrightarrow x - y$ is divisible by 5

Is symmetric: $5 R 10 \leftrightarrow 5-10 = -5/5$

$10 R 5 \leftrightarrow 10-5 = 5/5$

Is transitive: $5 R 10 \leftrightarrow 5-10 = -5/5$

$y R z \quad 10 R 15 \leftrightarrow 10-15 = -5/5$

$x R y \quad 5 R 15 \leftrightarrow 5-15 = -10/5$

0 is divisible by 5

This would be reflexive as well; $xRx \rightarrow x=1 \quad 1-1=0 \;/\; 5 = 0$

If x and y is the same value, $x=y$, then all properties are fulfilled making it equivalence.

5.) Prove or disprove the following statement: $\sqrt{3} \in \mathbb{Q}$

$$(\sqrt{3})^2 = \left(\frac{x}{y}\right)^2$$

$$3 = \frac{x^2}{y^2} \rightarrow 3y^2 = x^2$$

↖ x must be divisible by 3.

$$y^2 = \frac{x^2}{3}$$

$$3y^2 = (3x)^2$$

$$3y^2 = 9x^2$$

$$y^2 = 3x^2$$

↖ y must be divisible by 3

$$\frac{y^2}{3} = x^2$$

Since both x & y must be divisible by 3.

This is a contradiction, so $\sqrt{3} \notin \mathbb{Q}$. $\sqrt{3}$ is irrational