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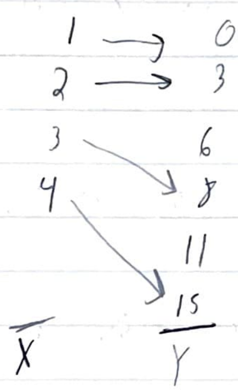
HW #7

1.)

n	f(n)
1	0
2	3
3	8
4	15

a) Is f one-to-one?

Yes, because each value of n goes to only value of f(n). And f(n) has its own value of n.



b.) Onto?

No, because every Y does not have an X.
6 and 15 don't have an n value

c.) Correspondence

No, because it must be both one-to-one and onto.
The function is only one-to-one.

d.) f(5)?

$f(5) = 25$ $\qquad$ $n = 5 \rightarrow f(n) = 25$

$f(n) = n^2 - 1$

2.) a) Is $\langle B, bacc \rangle \in A_{REX}$?

No, because the first input has to be an "a". This has a "b" as the first input.

b.) Is $\langle B, abb \rangle \in A_{REX}$?

Yes, because the input order is acceptable with the expression.

c.) Is $\langle B, B \rangle \in EQ_{DFA}$?

Yes, because a language can accept itself.

3.)

$L = \{ \langle A, B \rangle \mid A \text{ is a DFA and } B \text{ is a reg. exp.} \}$

Used the book as a guide to do this problem

T = "On input $\langle A, B \rangle$, where A is a DFA and B is a reg. exp.}

1.) Convert B into a DFA
2.) Run TM N on $\langle A, B \rangle$
3.) If N accepts, accept; if N rejects, reject.

Wasn't sure which TM to use

4.) Lets say that B is countable, which would mean its a correspondence, both one-to-one and onto.

$x = \{1, 2, \ldots\}$, $f(x) = (a_{x_1}, a_{x_2}, \ldots)$

Since sequences will be infinite, these will be infinite.

x	f(x)		
1	a_{11}	a_{12}	a_{13}
2	a_{21}	a_{22}	a_{23}

← This chart would show a correspondence, which would make it countable. Similar to figure 4.16.

Example: Lets say the i-th digit is 1 in sequence X. Lets also say, $f(2) \to a_2$. They are different.

We can make the i^{th} digit in sequence X be different than the digit in the function $f(x) \to a_x$

Since these are different, X is not a correspondence with a_x, so that means B is not Countable. It also isn't countable because sequence is infinite, opposite of what the book says.

5.) Let $INFINITE_{PDA} = \{\langle M \rangle \mid M$ is PDA and $L(M)$ is an infinite language $\}$
Show that $INFINITE_{PDA}$ is decidable.

I = "On input $\langle M \rangle$, where M is a PDA."

1.) Let j be the number of states in M.
2.) Make a PDA that accepts all strings of j or more.
3.) Test the language by using a decider N on the new PDA
4.) If N accepts, accept; if N rejects, reject.

Not sure which decider to use

I used problem 3 and other examples/exercises from the book to do this problem.

"or more" because its INFINITE

For problem 4, I solved this problem by using Theorem 4.17 in the textbook. I wasn't sure if the book said if the proof by contradiction is part of the diagonalization method, though the theorem was proving that R is untouchable, which is similar to this problem, prove B is untouchable. I hope this is fine.