CPSC 323: Compilers and Languages

Chapter 2 – Pumping Lemma

- **Pumping Lemma for RL**
  - **Proof of pumping lemma for nonregular languages**
- **Intro to Syntactic Analysis (*a.k.a.* Parsing)**
- **Context-Free Grammars (CFGs)**

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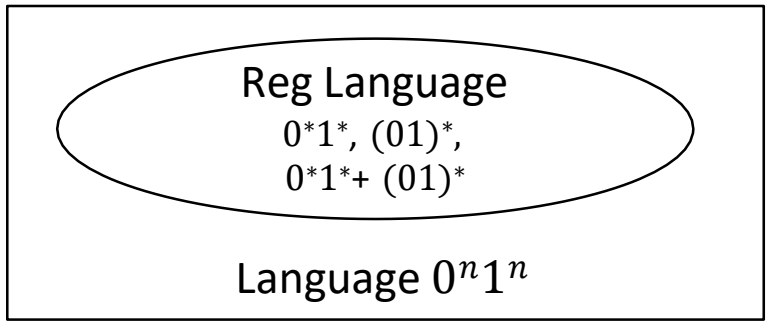
Recap: Regular Languages (RLs)

- A regular language is a formal language that can be defined by a regular expression and is accepted or recognized by FSA (DFA or NFA).
- How to prove a language is regular?
- According to the above definition of RL, it is simply to build FSA (DFA or NFA) that accepts or recognizes the language.
  Regular languages and finite automata can model computational problems that require a very **small amount of memory.**

Nonregular Languages



Not all languages are regular!

- Consider the following example language B

$$B = \{0^n1^n \mid n \geq 0\}$$

Question: Can we build FSA (DFA/NFA) that accepts them? And why?

No. Because that might not be possible using an automaton with finite number of states to recognize language B. This language B of arbitrary amount of information is beyond the capability of FSA. So, language B is not regular.

More example languages: are they regular or nonregular?

Language C = {w is a binary string| w has an equal number of 0's and 1's} -- Nonregular

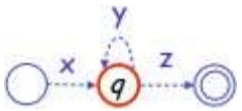
Language D = {w is a binary string| w has an equal number of '01's and '10's} – Regular

The Pumping Lemma

- It is a method or technique for showing or proving that languages are nonregular.

The intuition of generalizing the pumping lemma

- Consider an infinite regular language L
- By definition, it is accepted by some FSA *M*, which again by definition, has a finite number of states.
- But L is infinite, there are strings in L which are very long, and certainly they have more characters than the number of states in *M*.
- Since *M* accepts every strings in L, there must be at least one loop in *M. (exist by the pigeon-hole principle)*
- So, any sufficiently long string *w* ∈ L must traverse some loop, and so can be decomposed into *xyz*, where *y* is **nonempty** and $xy^iz \in$ L for any i ≥ 0.



- We say that the substring *y* of *w* can be pumped.

Pumping Lemma for Regular Languages

- **The Pumping Lemma.** If L is a regular language, there is a positive integer number *p* (called the pumping length) such that any string *w* ∈ L with |w| ≥ *p* can be expressed as *w* = xyz, where substrings x, y, z satisfying the following conditions:

  1. |y| > 0
  2. |xy| ≤ p
  3. xy^iz ∈ L for ∀i ≥ 0

L is regular ⇒ L is pumpable

How to prove that a language *L* is not regular using pumping lemma?

Standard technique for proof : L is not pumpable ⇒ L is not regular

- To prove a language nonregular, we just need to find **one counterexample string**!

- **Towards proof by contradiction:**
  1. Assume that *L* is regular.
  2. Use the pumping lemma to guarantee the existence of a pumping length *p* such that all strings *w* of $|w| \geq p$ in L can be pumped.
  3. Find a string *s* in L that has $|s| \geq p$ but that cannot be pumped. That is, demonstrate every decomposition of *s* into x, y, and z with $|y| > 0$ and $|xy| \leq p$ leads to a string $xy^iz \notin L$ for some $i \geq 0$. The existence of such string *s* contradicts your assumption.

  Hence, L cannot be regular.

- **Things to note:**
  - You cannot choose p - you have to assume this is given by the lemma.
  - You can choose the string s
  - You cannot choose one particular decomposition of s. You have to prove that no possible decomposition of s can satisfy any of the above three conditions.

Example 1: pumping up case

- Show the following example language B = $\{0^n1^n \mid n \geq 0\}$ is not regular.

1, Assume B is regular. Let p be the pumping length given by the lemma.
Pick a string *s* = 0^p1^p
(you may have other string *s*, e.g., $0^{p/2}1^{p/2}$ where p is assumed as even for simplicity).

2, Since *s* ∈ B and $|s| \geq$ p, according to the pumping lemma, *s* can be decomposed into *s* = *xyz* such that $xy^iz \in$ B.

3. Consider all ways as below where s cannot be pumped or cannot be decomposed into substrings x, y, z.

1) Suppose y only contain 0 and at least contain a 0, then xy^2z has more 0's than 1's, so it is not in B.
2) The case where y only contain 1 follows the similar way as above.
3) Suppose y contains '01', then xy^2z has some 1's appear before 0's, so it is not in B.

Hence, there is no decomposition of s leads to $xy^iz \in$ B, which contradicts the assumption.
We can prove B is not regular.

Example 2: pumping down case

- Show the following example language L = $\{0^i 1^j \mid i \geq j\}$ is not regular.

  1, Assume B is regular. Let *p* be the pumping length given by the lemma.
  Pick a string *s* = $0^{p+1}1^p$

  2, Since *s* ∈ B and $|s| \geq$ p, according to the pumping lemma, *s* can be decomposed into *s* = *xyz* such that $xy^iz \in$ L.

  3. According to the condition of |xy| $\leq p$, then y contains only 0. Then xy^0z = *xz* has equal or lesser 0's than 1's. There is no decomposition of s leads to $xy^iz \in$ L, which contradicts the assumption.

  So, we can prove L is not regular.

Intro to Syntactic Analysis (Parsing)

- Example1:

```
distance = velocity * time;
```

- Q1: what is the output of lexical analyzer (lexer) for above example code fragment?

Intro to Syntactic Analysis (Parsing)

- Example 2:

```
height = (width + 56) * factor(foo);
```

- Q1: what is the output of lexical analyzer (lexer) for above example code fragment?

- Q2: what does a parser do in the next phase of a compilation?

Intro to Syntactic Analysis (Parsing)

- Example 2:

```
height = (width + 56) * factor(foo);
```

- Q1: what is the output of lexical analyzer (lexer) for above example code fragment?

  id: height = (id:width + int:56) * id:factor (id:foo) ;

- Q2: what does a parser do in the next phase of a compilation?

  It determines whether the input sequence of tokens forms a valid program.

Lexer and Parser

Phase	Input	Output
Lexer	String of characters	Sequence of tokens
Parser	Sequence of tokens	Syntactic structure of the input program in the form of parse tree

Q3: How does a parser parse?

Context-Free Grammars (CFGs)

- CFGs are used to define context-free languages, which are a class of formal languages with certain characteristics.
- Regular languages have a limited expressive power compared to context-free languages.
- They are much more powerful than regular expressions as they allow for ***recursive structure*** that is a natural structure in the programming languages.

Context-Free Grammars (CFGs)

It typically includes:

•A set of terminal symbols (symbols that appear in the strings generated by the grammar).

•A set of non-terminal symbols (symbols that can be replaced by other symbols using production rules).

•A start symbol (a non-terminal symbol from which the derivation of valid strings begins).

•A set of production rules that specify how non-terminal symbols can be replaced by sequences of terminal and/or non-terminal symbols.

Definition of a CFG

- A context-free grammar (CFG) is a set of **recursive rules** (or *productions*) used to describe *patterns of strings in a language*.

- **A CFG *G* is a 4-tuple of *G* = (*T, N, S, R*):**

where:

- ***T:*** A finite set of terminals over symbols in alphabet Σ^*
- ***N :*** A finite set of non-terminals (*also named variables*)

(*note that **N** and **T** are disjoint sets*)

- ***R :*** A finite set of rules (*also named productions*)
  - Each rule has the form $A \rightarrow \alpha$, where $A \in N$, *and* $\alpha \in (N \cup T)^*$
  - **Note:** rules are written with single arrows
- ***S (S ∈ N) :*** A **unique** start symbol of strings
  - ***S*** is nonterminal symbol of the *first rule* in set ***R***

Definitions of a CFG (cont'd)

- A finite set of terminals ***T***
  - A terminal is a discrete symbol in alphabet Σ^* that can appear in the language, otherwise known as ***a token***. *Examples of terminals: identifiers, keywords, and operators.*
  - We use *lower-case letters* to represent terminals.

- A finite set of non-terminals (also named variables) ***N***
  - A nonterminal (also named a variable) represents a structure that can occur in a language, but is **not** a literal symbol. *Example of nonterminals: declarations, statements, and expressions.*
  - We will use *upper-case letters* to represent non-terminals: *P for program, S for statement, E for expression, etc.*

- The *first rule (or production)* of ***R*** is special: it is the top-level definition of a program, and its nonterminal is known as the *start symbol* ***S*** ($S \in N$).

Examples of CFG

- Consider the following CFG ***G*** that describes **expressions** involving *addition*, *integers*, and *identifiers*:

Grammar G:

1. P → E
2. E → E + E
3. E → ident
4. E → int

where **N** = {P, E}, **Σ** = {ident, int, +}, and **R** = {**P** → E, E → E+E, E → ident, E → int}.

- **Abbreviating grammars.** By convention, a group of productions that have the same left-hand side are shown with the left-hand side written only once. In all but the first production in the group, write | instead of →. For example, the above grammar for expressions is usually written as follows.

  R = {**P** → E, E → E + E | ident | int}

- Grammar G can also be denoted as:

  P → E

  E → E + E

  | ident

  | int

Note: rules (or productions) of G are written with single arrows

Regular languages are context-free

- **Theorem.** For every regular language L, there exists a CFG G such that L=L(G).
- **Corollary.** Every regular language is a CFL.
- **The class of regular languages (RLs) is a proper subclass of CFLs.**